

## Chapter 2 Component Interconnection and Signal Conditioning

### Solution 2.1

(a):

$$\text{Electrical Impedance} = \frac{\text{Voltage Output}}{\text{Current Input}}$$

$$\text{Mechanical Impedance} = \frac{\text{Force Output}}{\text{Velocity Input}}$$

(b): Both these impedances are frequency response functions (defined in the frequency domain). Both define the resistance provided by the load against the driving force. High-impedance devices need high levels of effort (voltage or force) to drive them (i.e., to pass current through electrical impedances, or to cause movement of mechanical loads). Note that voltage is an across-variable whereas force is a through-variable. Hence, there is an inconsistency in the definitions of “impedance,” with respect to the force-current analogy.

(c): To avoid this inconsistency, we may use the force-voltage analogy, in which voltage and force are termed “effort” variables and velocity and current are “flow” variables, as in the “bond graph” nomenclature.

Note, however, that in order to use general relations for interconnecting basic elements (in forming multicomponent devices or circuits), it is the across-variable and through-variable nomenclature that is applicable. Specifically, when two elements are connected in series, the through-variable is common and the across-variables add; when two elements are connected in parallel, the across-variable is common and the through-variables add. Hence, it is the through- and across-variable nomenclature that is natural with regard to component interconnection. In this context we may define a generalized series element or generalized impedance (to include electrical impedance or mechanical mobility) and a generalized parallel element (to include electrical admittance or mechanical impedance).

(d): The input impedance has to be comparatively high for a measuring device that is connected in parallel, to measure an across variable, whereas the input impedance has to be quite low for a device that connected in series, to measures a through variable. This is essential to reduce loading errors. The output impedance of a measuring device has to be low in order to maintain a high *sensitivity*, and get acceptable signal levels for processing, actuating or recording.

When cascading two devices, in order to reduce the “loading” of one device by the other, and to maintain good frequency characteristics, the output impedance of the first device (which provides the signal) has to be smaller in comparison to the input impedance of the second device (which receives the signal). Otherwise, the signal will be distorted by the second device (the load). If power transfer characteristics are important, however, one impedance should be the complex conjugate of the other. Different matching criteria are used depending on the applications.

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**Solution 2.2**

1. Maximum power transfer
  2. Power transfer at maximum efficiency
  3. Reflection prevention in signal transmission
  4. Loading reduction
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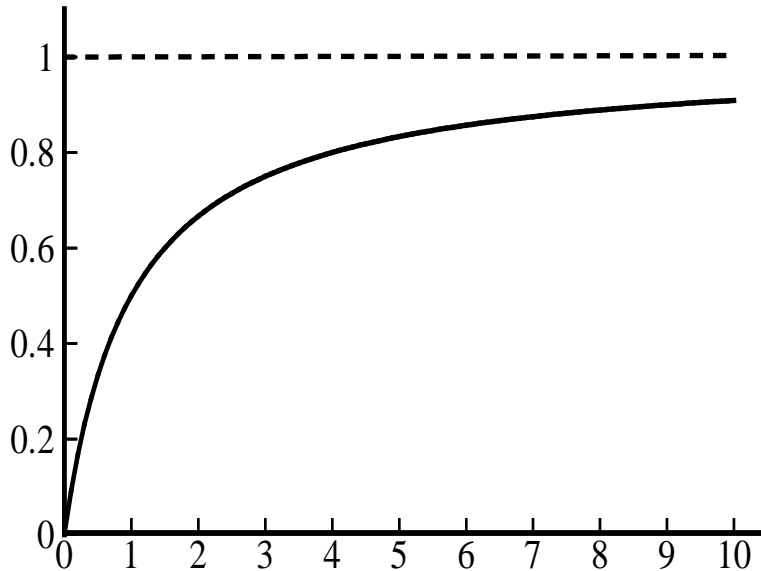
**Solution 2.3**

When a measuring device is connected to a system, the conditions in the system itself will change, as the measured signal flows through the measuring device. For example, in electrical measurements, a current may pass through the measuring device, thereby altering the voltages and currents in the original system. This is called “electrical loading,” and will introduce an error, as the measurand itself is distorted. Similarly, in mechanical measurements, due to the mass of the measuring device, the mechanical condition (forces, motions) of the original system will change, thereby affecting the measurand and causing an error. This is called mechanical loading.

Now consider the system shown in Figure P2.3. We have:  $v_o = K \left[ \frac{Z_i}{Z_s + Z_i} \right] v_i$

For a voltage follower,  $K = 1$  and  $Z_o \ll Z_i$ . Hence,  $v_o = \left[ \frac{Z_i}{Z_s + Z_i} \right] v_i$ , or  $\frac{v_o}{v_i} = \frac{(Z_i / Z_s)}{1 + (Z_i / Z_s)}$ .

This relationship is sketched in Figure S2.3.



**Figure S2.3: Non-dimensional curve of loading performance.**

Some representative values of the curve are tabulated below.

$Z_i / Z_s$	$v_o / v_i$
0.1	0.091
0.5	0.55
1	0.5
2	0.667
5	0.855
7	0.875
10	0.909

**Note:** Performance improves with the impedance ratio  $Z_i / Z_s$ .

#### **Solution 2.4**

Open-circuit voltage at the output port is (in the frequency domain)

$$v_{oc} = \frac{\left[ R_2 + \frac{1}{j\omega C} \right]}{\left[ R_1 + R_2 + j\omega L + \frac{1}{j\omega C} \right]} = v_{eq} \quad (i)$$

*Note:* Equivalent source  $v_{eq}$  is expressed here as a function of frequency. Its corresponding time function  $v_{eq}(t)$  is obtained by using inverse Fourier transform. Alternatively, first

replace  $j\omega$  by the Laplace variable  $s$ :  $v_{eq}(s) = \frac{\left[ R_2 + \frac{1}{sC} \right]}{\left[ R_1 + R_2 + sL + \frac{1}{sC} \right]} v(s)$ . Then obtain the

inverse Laplace transform, for a given  $v(s)$ , using Laplace transform tables.

Now, in order to determine  $Z_{eq}$ , note from Figure P2.4(b) that if the output port is shorted, the resulting short circuit current  $i_{sc}$  is given by:  $i_{sc} = \frac{v_{eq}}{Z_{eq}}$ . Hence,

$$Z_{eq} = \frac{v_{eq}}{i_{sc}} = \frac{v_{oc}}{i_{sc}} \quad (ii)$$

Since we know  $v_{oc}$  (or  $v_{eq}$ ) from equation (i) we only have to determine  $i_{sc}$ . Using the actual circuit with shorted output, we see that there is no current through the parallel impedance  $R_2 + \frac{1}{j\omega C}$  because the potential difference across it is zero. Thus,

$$i_{sc} = \frac{v}{(R_1 + j\omega L)} \quad (\text{iii})$$

Now substituting Equations (i) and (iii) in (ii) we have:

$$Z_{eq} = \left[ \frac{\left[ R_2 + \frac{1}{j\omega C} \right] [R_1 + j\omega L]}{\left[ R_1 + R_2 + j\omega L + \frac{1}{j\omega C} \right]} \right]$$

### **Solution 2.5**

(a) Load power efficiency  $\eta = \frac{R_l}{(R_l + R_s)} = \frac{R_l / R_s}{(R_l / R_s + 1)}$

(b) Load power  $p_l = \frac{v_s^2 R_l}{[R_l + R_s]^2}$ ; Maximum load power (occurs at  $R_l = R_s$ )  $p_{\max} = \frac{v_s^2}{4R_s}$

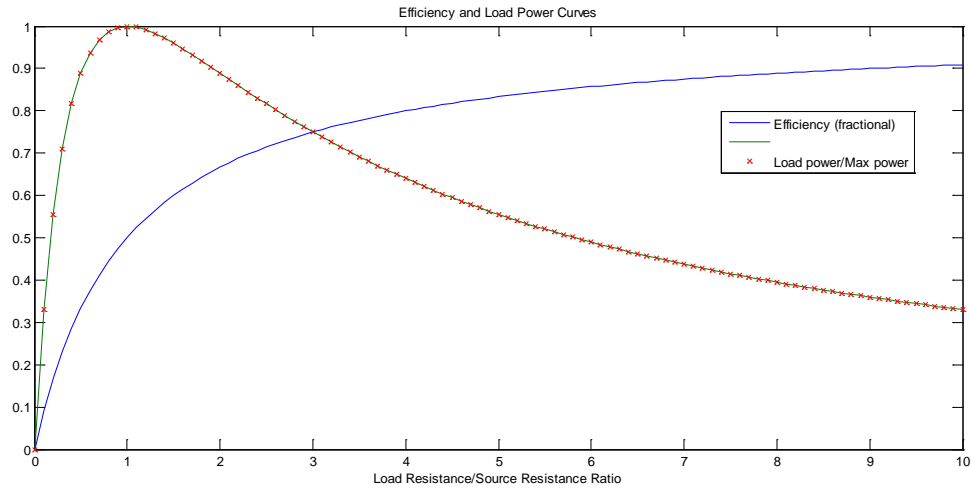
$$\rightarrow p_l / p_{\max} = \frac{4R_l / R_s}{[R_l / R_s + 1]^2}$$

We use the following MATLAB script (.m file) to generate the two curves:

```
% Efficiency and load power curves
lr=[]; eff=[]; pw=[]; % declare vectors
lr=0; eff=0; pw=0; %initialize variables
for i=1:100
a=0.1*i; %load resistance ratio
lr(end+1)=a; % store load resistance
eff(end+1)=a/(a+1); % efficiency
pw(end+1)=4*a/(a+1)^2; % load power
end
plot(lr,eff,'-',lr,pw,'-',lr,pw,'x')
```

The two curves are plotted in Figure S2.5.

It is seen that maximum efficiency does not correspond to maximum power. In particular, the efficiency increases monotonically with the load resistance while the maximum power occurs when  $R_l = R_s$ . Hence, a reasonable trade-off in matching the resistances would be needed when both considerations are important.



**Figure S2.5: Variation of efficiency and maximum power with load resistance.**

### **Solution 2.6**

Voltage is an across variable. In order to reduce loading effects, the resistance of a voltmeter should be much larger than the output impedance of system or load impedance. Then, the voltmeter will not draw a significant part of the signal current (and will not distort the signal). Current is a through variable. The resistance of an ammeter should be much smaller than the output impedance of system or load impedance. Then, the ammeter will not provide a significant voltage drop (and will not distort the signal).

Voltmeter should be able to operate with a low current (due to its high resistance) and associated low torque, in conventional electromagnetic deflection type meters. Low torque means, a torsional spring having low stiffness has to be used to get an adequate meter reading. This makes the meter slow, less robust, and more nonlinear, even though high sensitivity is realized.

Ammeter should be able to carry a large current because of its low resistance. Hence meter torque would be high in conventional designs. This can create thermal problems, magnetic hysteresis, and other nonlinearities. The device can be made fast, robust, and mechanically linear, however, while obtaining sufficient sensitivity.

*Note:* The torque is not a factor in modern digital multi-meters.

### **Solution 2.7**

(a) The input impedance of the amplifier = 500 M $\Omega$ .

$$\text{Estimated error} = \frac{10}{(500 + 10)} \times 100\% = 2\%$$

(b) Impedance of the speaker = 4  $\Omega$ .

$$\text{Estimated error} = \frac{0.1}{(4 + 0.1)} \times 100\% = 2.4\%$$


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**Solution 2.8**

$$v_o = F_1(f_o, f_i) \Rightarrow \delta v_o = \frac{\partial F_1}{\partial f_o} \delta f_o + \frac{\partial F_1}{\partial f_i} \delta f_i \quad (\text{i})$$

$$v_i = F_2(f_o, f_i) \Rightarrow \delta v_i = \frac{\partial F_2}{\partial f_o} \delta f_o + \frac{\partial F_2}{\partial f_i} \delta f_i \quad (\text{ii})$$

In terms of incremental variables about an operating point, we can define the input impedance  $Z_i$  and the output impedance  $Z_o$  as

$$Z_i = \frac{\delta v_i}{\delta f_i} \quad \text{with} \quad \delta f_o = 0 \quad (\text{iii})$$

$$Z_o = \frac{\delta v_o \text{ with } \delta f_o = 0}{\delta f_o \text{ with } \delta v_o = 0} \quad (\text{iv})$$

*Note:*  $\delta f_o = 0$  corresponds to incremental open-circuit condition and  $\delta v_o = 0$  corresponds to incremental short-circuit condition.

From (ii) with  $\delta f_o = 0$  (i.e., open circuit at output) we get  $Z_i = \frac{\partial F_2}{\partial f_i}$ .

Now using the open-circuit by subscript “oc” and the short-circuit by subscript “sc” we have:  
From (i):

$$\delta v_o \big|_{oc} = \frac{\partial F_1}{\partial f_i} \delta f_i \big|_{oc} \quad (\text{v})$$

From (ii):

$$\delta v_i \big|_{oc} = \frac{\partial F_2}{\partial f_i} \delta f_i \big|_{oc} \quad (\text{vi})$$

*Note:*  $\delta v_i$  is an independent increment, which does not depend on whether oc or sc condition exists at the output. But  $\delta f_i$  will change depending on the output condition.

From (v) and (vi):

$$\delta v_o \big|_{oc} = \frac{\partial F_1}{\partial f_i} \bigg/ \frac{\partial F_2}{\partial f_i} \delta v_i \quad (\text{vii})$$

From (i):

$$0 = \frac{\partial F_1}{\partial f_o} \delta f_o \big|_{sc} + \frac{\partial F_1}{\partial f_i} \delta f_i \big|_{sc} \quad (\text{viii})$$

From (ii):

$$\delta v_i = \frac{\partial F_2}{\partial f_o} \delta f_o \big|_{sc} + \frac{\partial F_2}{\partial f_i} \delta f_i \big|_{sc} \quad (\text{ix})$$

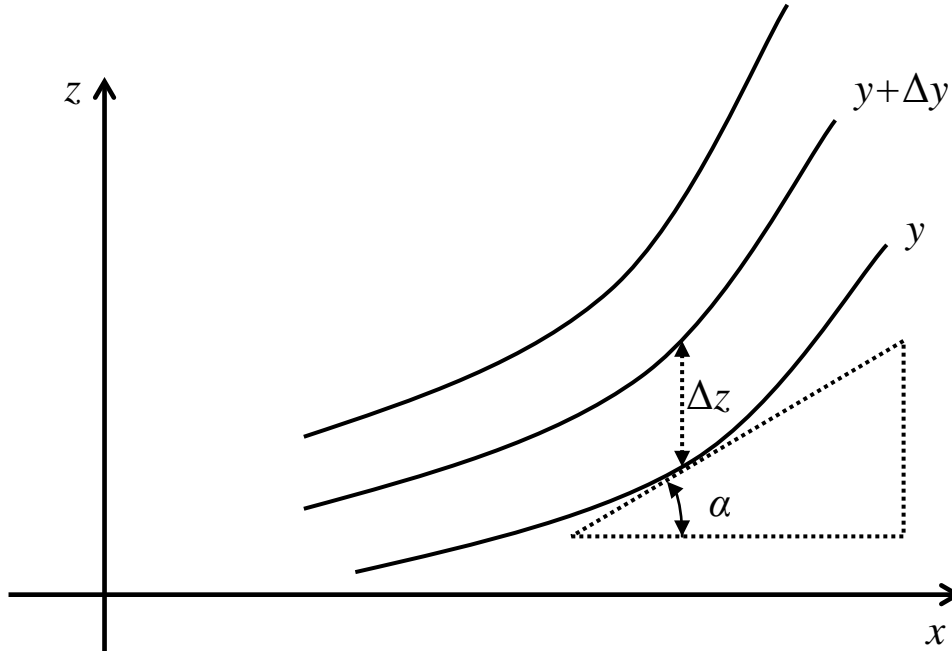
Eliminating  $\delta f_i \big|_{sc}$  from Equations(viii) and (ix) we get,

$$\delta f_o \big|_{sc} = \frac{1}{\left[ \frac{\partial F_2}{\partial f_o} - \frac{\partial F_1}{\partial f_o} \cdot \frac{\partial F_2}{\partial f_i} / \frac{\partial F_1}{\partial f_i} \right]} \delta v_i \quad (\text{x})$$

Substitute (vii) and (x) in (iv):

$$Z_o = \left[ \frac{\partial F_1}{\partial f_i} / \frac{\partial F_2}{\partial f_i} \right] \left[ \frac{\partial F_2}{\partial f_o} - \frac{\partial F_1}{\partial f_o} \cdot \frac{\partial F_2}{\partial f_i} / \frac{\partial F_1}{\partial f_i} \right] = \frac{\partial F_1}{\partial f_i} \cdot \frac{\partial F_2}{\partial f_o} / \frac{\partial F_1}{\partial f_i} - \frac{\partial F_1}{\partial f_o}$$

One way to experimentally determine  $Z_i$  and  $Z_o$  (under static conditions) is to first experimentally determine the two sets of operating curves given by  $v_o = F_1(f_o, f_i)$  and  $v_i = F_2(f_o, f_i)$  under steady-state conditions. For example  $f_o$  is kept constant and  $f_i$  is changed in increments to measure  $v_o$  and  $v_i$  once the steady state is reached. This will give two curves  $f_i$  versus  $v_o$  and  $v_i$  versus  $v_i$  for a particular value of  $f_o$ . Next  $f_o$  is incremented and another pair of curves is obtained. Once these two sets of curves are obtained for the required range for  $f_i$  and  $f_o$ , the particular derivatives are determined from using the general method shown in Figure S2.8, for the case  $z = F(x, y)$  with:  $\frac{\partial z}{\partial x} \cong \alpha$  and  $\frac{\partial z}{\partial y} \cong \frac{\Delta z}{\Delta y}$ .



**Figure S2.8: Computation of local slopes.**

**Solution 2.9****(a)**

We have:  $v_i = Z_c i_i$ ,  $v_t = Z_l i_t$ ,  $v_r = -Z_c i_r$ ,  $v_t = v_i + v_r$ , and  $i_t = i_i + i_r$ , where “ $i$ ” denotes current and the subscript “ $r$ ” denotes “reflected.”

$$\text{Substitute: } v_t = Z_l i_t = Z_l (i_i + i_r) = Z_l \left( \frac{v_i}{Z_c} - \frac{v_r}{Z_c} \right) = \frac{Z_l}{Z_c} (v_i - v_r) = \frac{Z_l}{Z_c} (v_i - (v_t - v_i)) = \frac{Z_l}{Z_c} (2v_i - v_t)$$

$$\Rightarrow \left(1 + \frac{Z_l}{Z_c}\right)v_t = 2 \frac{Z_l}{Z_c} v_i \Rightarrow v_t = \frac{2Z_l}{(Z_l + Z_c)} v_i$$

$$\text{(b) We need } v_t = v_i \Rightarrow \frac{2Z_l}{(Z_l + Z_c)} = 1 \Rightarrow Z_l = Z_c$$

(c) Use a transformer with the required impedance ratio = (turns ratio)<sup>2</sup>

**Solution 2.10**

For the given system,  $\omega_n = \sqrt{\frac{1 \times 10^6}{100}}$  rad/s = 100 rad/s and  $\omega \geq 200$  rad/s. Hence, we have the frequency ratio  $r \geq 2.0$ .

For  $r = 2.0$  and  $|T_f| = 0.5$  we have  $0.5 = \sqrt{\frac{1 + 16\zeta^2}{9 + 16\zeta^2}}$  or,  $\zeta = \sqrt{\frac{5}{48}}$ . Hence,

$$b = 2\zeta\omega_n m = 2\sqrt{\frac{5}{48}} \times 100 \times 100 \text{ N.s/m} \rightarrow b = 6.455 \times 10^3 \text{ N.s/m}.$$

With this damping constant, for  $r \geq 2$ , we will have  $|T_f| \leq 0.5$ . Decreasing  $b$  will decrease  $|T_f|$  in this frequency range.

To plot the Bode diagram using MATLAB, first note that:

$$2\zeta\omega_n = b / m = 6.455 \times 10^3 / 100 = 64.55 \text{ rad/s and } \omega_n^2 = 10^4 \text{ (rad/s)}^2$$

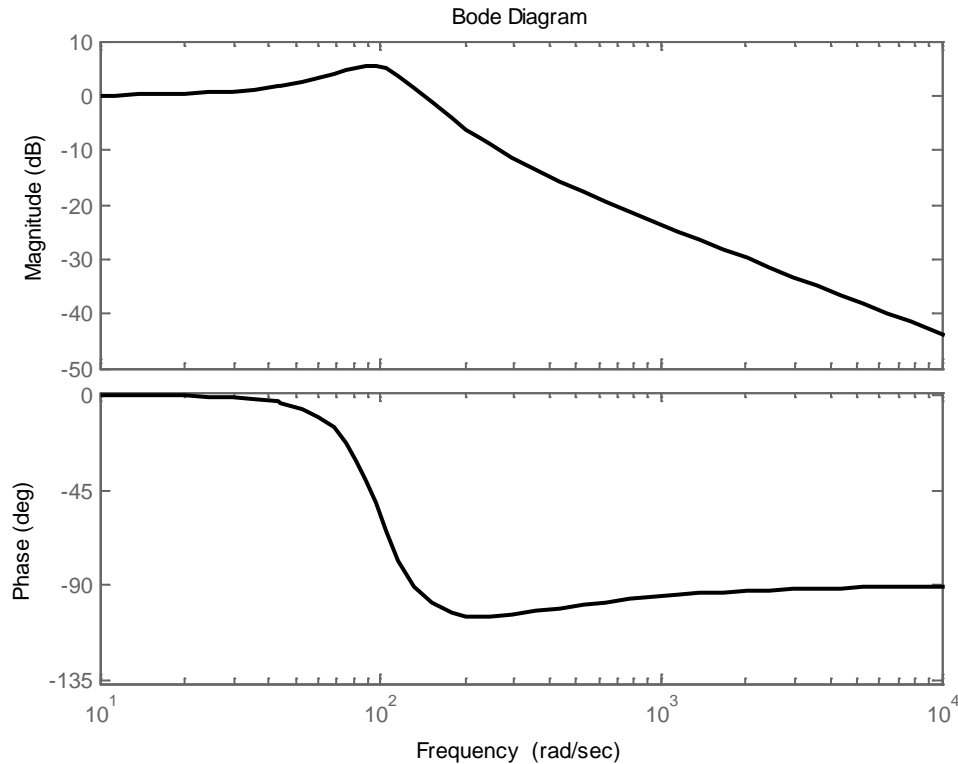
The corresponding transmissibility function is  $T_f = \frac{64.55s + 10^4}{s^2 + 64.55s + 10^4}$  with  $s = j\omega$

The following MATLAB script will plot the required Bode diagram:

```
% Plotting of transmissibility function
clear;
m=100.0;
k=1.0e6;
b=6.455e3;
sys=tf([b/m k/m],[1 b/m k/m]);
bode(sys);
```

The resulting Bode diagram is shown in Figure S2.10. A transmissibility magnitude of 0.5 corresponds to  $20\log_{10} 0.5 \text{ dB} = -6.02 \text{ dB}$ .

Note from the Bode magnitude curve in Figure S2.10.4 that at the frequency 200 rad/s the transmissibility magnitude is less than -6 dB and it decreases continuously for higher frequencies. This confirms that the designed system meets the design specification.



**Figure S2.10: Transmissibility magnitude and phase curves of the designed system.**

### **Solution 2.11**

**(a) Mechanical Loading**

A motion variable that is being measured is modified due to forces (inertia, friction, etc.) of the measuring device.

**(b) Electrical Loading**

The output voltage signal of the sensor is modified from the open circuit value due to the current flowing through external circuitry (load).

Mechanical loading can be reduced by using noncontact sensors, reducing inertia and friction, etc.

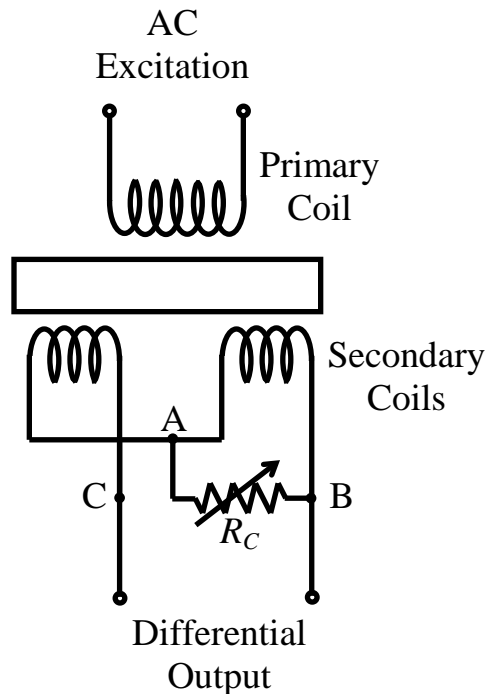
Electrical loading can be reduced by using a low-output-impedance sensor, high-impedance load, impedance transformer, etc. Some typical values of the listed parameters are given in the following table:

Parameter	Ideal Value	Typical Value
Input Impedance	Infinity	1 M $\Omega$
Output Impedance	Zero	50 $\Omega$
Gain	Infinity	$10^6$
Bandwidth	Infinity	10 kHz

### **Solution 2.12**

The differential signal from the secondary windings is amplified by the ac amplifier and is supplied to the demodulator. A carrier signal is used by the demodulator to demodulate the differential ac signal. The modulating signal that is extracted in this manner is proportional to the machine displacement. This signal is filtered to remove high-frequency noise (and perhaps the carrier component left by the demodulator), and then amplified and digitized (using an ADC) to be fed into the machine control computer.

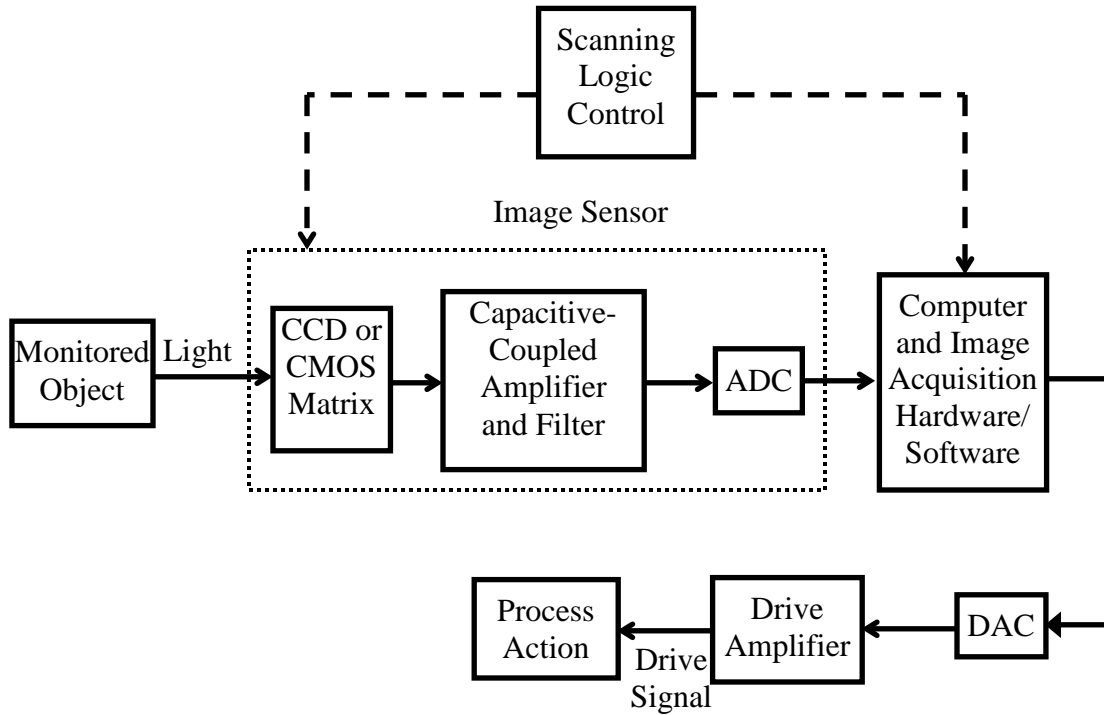
The compensating resistor  $R_c$  may be connected between the points A and B or A and C, as shown in Figure S2.12.



**Figure S2.12: Null compensation for an LVDT.**

**Solution 2.13**

(a)

**Figure S2.13: Monitoring of an industrial process using image processing.**(b) Data rate =  $488 \times 380 \times 8 \times 30$  bits/s = 44.5 Megabits/s

(c) Since hardware processors are faster, we prefer them for this level of high data rates for real-time action. Also, they are cheaper when mass produced. Disadvantages include limitations on algorithm complexity in image processing and memory size.

**Solution 2.14**

Since the open-loop gain  $K$  of an op-amp is very high ( $10^5$  to  $10^9$ ) and the output voltage cannot exceed the saturation voltage (which is of the order of 10 V) the input voltage  $v_i = v_{ip} - v_{in}$  is of the order of a few  $\mu\text{V}$ , which can be assumed zero (when compared with the operating voltages) for most practical purposes. Hence,  $v_{ip} = v_{in}$ . Next since the input impedance  $Z_i$  is very high ( $\text{M}\Omega$ ), the current through the input leads has to be very small for this very small  $v_i$  under unsaturated conditions.

(a) The saturated output of the op-amp must be 14 V in this example. The ac noise (line noise, ground loops, etc.) in the circuit can easily exceed the saturation input (on the order of 10  $\mu\text{V}$ ) of the op-amp, under open-loop conditions. Hence,  $v_i = v_{ip} - v_{in}$  can

oscillate between + and - values of the saturation input. This provides an output, which switches between the +ve saturated output  $+v_{sat}$  and the -ve saturated output  $-v_{sat}$  of the op-amp.

(b)

**Case 1:**  $v_{ip} = -1 \mu\text{V}$ ,  $v_{in} = +0.5 \mu\text{V}$   $\Rightarrow v_i = v_{ip} - v_{in} = -1 - 0.5 \mu\text{V} = -1.5 \mu\text{V}$

$$\Rightarrow v_o = -1.5 \times 5 \times 10^6 \mu\text{V} = -1.5 \times 5 \text{ V} = -7.5 \text{ V}$$

This is valid since the output is not saturated.

**Case 2:**  $v_{ip} = 0$ ,  $v_{in} = 5 \mu\text{V}$   $\Rightarrow v_i = 0 - 5 \mu\text{V} = -5 \mu\text{V}$

$$\Rightarrow v_o = -5 \times 5 \times 10^6 \mu\text{V} = -25 \text{ V}$$

$\Rightarrow$  Op-amp is saturated  $\Rightarrow$  The actual output would be  $v_o = -14 \text{ V}$

### **Solution 2.15**

(a) Offset Current (Typically in nA)

Bias currents are needed to operate the transistor elements in an op-amp IC. These currents  $i_+$  and  $i_-$  flow through the input leads of an op-amp. The offset current is the difference  $i_+ - i_-$ . Ideally, the offset current is zero.

(b) Offset Voltage (Typically in mV or less)

Due to internal circuitry (IC) in an op-amp, the output voltage might not be zero even when the two inputs are maintained at the same potential (say, ground). This is known as the offset voltage at output. Furthermore, due to unbalances in the internal circuitry, the potentials at the two input leads of an op-amp will not be equal even when the output is zero. This potential difference at the input leads is known as the input offset voltage. This is usually modeled as a small voltage source connected to one of the input leads

(c) Unequal Gains (Can range over  $10^5$  to  $10^9$ )

The open-loop gain of an op-amp with respect to the “+” input lead may be different from that with respect to the “-” input lead. This is known as unequal gains.

(d) Slew Rate (Typically about  $0.5 \text{ V}/\mu\text{s}$ )

When the input voltage is instantaneously changed, the op-amp output will not change instantaneously. The maximum rate at which the output voltage can change (usually expressed in  $\text{V}/\mu\text{s}$ ) is known as the slew rate of an op-amp.

Even though  $K$  and  $Z_i$  are not precisely known and can vary with time and frequency, their magnitudes are large. Hence, we can make the basic assumptions: equal potential at the two input leads and zero current through the input leads, under unsaturated conditions. Then, these parameters do not enter the output equations of an op-amp circuit.

**Solution 2.16****(a)**

A voltage follower is an amplifier having a unity voltage gain, a very high input impedance, and a very low output impedance. A simple model for a voltage follower is obtained by connecting the “-” lead of an op-amp to the output (feedback path) and using the “+” lead as the input lead. Under unsaturated conditions we have  $v_o = v_i$ . It is known that the input impedance of a voltage follower is much larger than that of the original op-amp (which itself is quite large—megohm range) and the output impedance of a voltage follower is much smaller than that of the original op-amp (which is also small). Hence, a voltage follower functions primarily as an impedance transformer that provides the ability to acquire a voltage from a high-impedance device, where the current is rather low (e.g., a high-impedance sensor) and transmitting that voltage signal into a low-impedance device, without distorting the acquired voltage.

**(b)**

Consider circuit in Figure P2.16. Since  $v_B = 0$ , we have  $v_A = 0$ .

Hence, current summation at node A gives:  $\frac{v_i}{R} + \frac{v_o}{R_f} = 0$

*Note:* The current through an input lead of an op-amp has to be zero.

Hence,  $\frac{v_o}{v_i} = -\frac{R_f}{R}$  and  $K_v = -\frac{R_f}{R} \rightarrow$  This is an inverting amplifier.

**Solution 2.17**

Slew rate:  $s = 2\pi f_b a$  (i)

where,  $a$  = output amplitude,  $f_b$  = bandwidth (Hz).

The rise time  $T_r$  is inversely proportional to  $f_b$ . Hence,  $f_b = \frac{k}{T_r}$  where,  $k$  = constant.

Substitution gives:  $s = \frac{2\pi k a}{T_r}$  (ii)

From (i): For constant  $s$ , bandwidth decreases as  $a$  is increased.

For a sine signal, substitute the given values in (i):  $f_b = \frac{0.5}{2\pi \times 2.5} \text{ MHz} = 31.8 \text{ kHz}$

Next, for a step input, use  $s = \frac{\Delta y}{\Delta t}$  where,  $\Delta y$  = step size,  $\Delta t$  rise time

Substitute numerical values:  $\Delta t = \frac{\Delta y}{s} = \frac{2.5}{0.5} \mu\text{s} = 5 \mu\text{s}$ .

**Solution 2.18**

(a) Common-mode voltage  $v_{cm}$  = voltage common to the two input leads of a differential amplifier = average of the two inputs.

Common-mode output voltage  $v_{ocm}$  = output voltage of the amplifier due to  $v_{cm}$  (i.e., in the absence of any voltage differential at the input.)

(b) Common-mode gain =  $\frac{v_{ocm}}{v_{cm}}$

(c)  $CMRR = K \frac{v_{cm}}{v_{ocm}} = \frac{K}{\text{common-mode gain}}$

where,  $K$  = amplifier gain (i.e., differential gain or gain at the output for the inferential input).

Specifically:  $v_o = K(v_{ip} - v_{in}) + K_{cm} \times \frac{1}{2}(v_{ip} + v_{in})$

Typically  $CMRR \approx 20,000$ .

When A is closed and B is open, the flying capacitor  $C$  gets charged to the differential voltage  $v_{i1} - v_{i2}$  and hence the common-mode voltage does not enter. When A is open and B is closed, the capacitor voltage, which does not contain the common-mode signal, is applied to the differential amplifier.

**Solution 2.19**

The textbook definition of stability relates to the dynamic model (linear or nonlinear) of a system and hence to its natural response. In particular, in a linear system, if at least one pole (eigenvalue) has a positive real part, the natural response of the system will diverge, and the system is unstable.

Instrumentation stability is linked to the drift associated with change in parameters of the instrument or change in the environmental conditions.

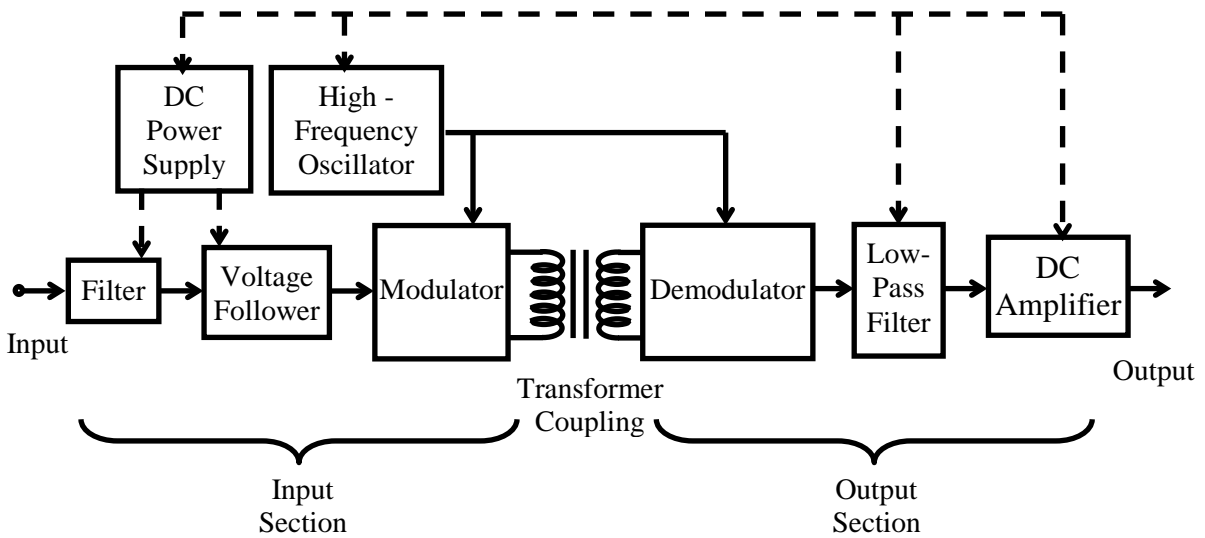
$$\text{Temperature drift} = \frac{\text{Change in output}}{\text{Change in temperature}}$$

assuming that the other conditions and the input are maintained constant.

$$\text{Long term drift} = \frac{\text{Change in output}}{\text{Duration}}$$

assuming that the other conditions and input are the same.

**Ways to Reduce Drift:** Regulate the power supply; Use feedback; Keep the environment uniform; Use compensating elements and circuitry; Recalibrate the device before each use.

**Solution 2.20****Figure S2.20: An isolation amplifier.****Solution 2.21**

Possible causes:

1. Faulty cellphone charger and it not having a ground lead and pin
2. Faulty laptop charger and it not having a ground lead and pin

**Faulty Cellphone Charger:** Due to a short-circuit, the high voltage (110-240 VAC) will leak into its cable and reach the cellphone. If the cellphone is not properly grounded/isolated, the voltage will form a path through the user's body. According to the burns, this path has to include the chest and the ears (possibly through the headphone cable).

**Faulty laptop Charger:** Due to a short, the high voltage (110-240 VAC) will leak from the charger into the DC cable segment that is connected to the laptop. If the laptop is not properly grounded/isolated, the voltage will form a path through the user's body. According to the burns, this path has to include the chest and the ears (possibly through the headphone cable).

On the one hand, the newspaper report indicated that there were inexpensive and non-compliant cellphone chargers in the market. However, since the power consumption of the cellphone charger is relatively low and since the electricity path through the body included the ears (*Note:* The headphones were connected to the laptop, not to the cellphone) the other possibilities of fault need to be investigated as well. Typically, however, the laptop chargers (particularly those provided by reputed laptop

manufacturers) are subjected to rigorous standards, inspection, and quality control (so are cellphone chargers from reputed manufacturers).

---

### **Solution 2.22**

Passive filters are circuits made of passive elements, which do not require an external power supply to operate. These circuits allow through those signal components in a certain frequency range and block off the remaining frequency components.

Advantages and disadvantages of passive filters: See disadvantages and advantages of active filters.

The voltage follower is an impedance transformer. It reduces loading problems by providing a very high input impedance and very low output impedance. Furthermore, it does not change the voltage gain.

---

### **Solution 2.23**

Applications:

- (a) Anti-aliasing filters in digital signal processing
- (b) To remove dc components in ac signals
- (c) As tracking filters
- (d) To remove line noise in signals.

Each single-pole stage will have a transfer function of the form:  $G_i(s) = \frac{k_i(\tau_{zi}s + 1)}{(\tau_{pi}s + 1)}$

Hence, the cascaded filter will have the transfer function:

$$G(s) = \Pi G_i(s) = \Pi k_i \frac{(\tau_{zi}s + 1)}{(\tau_{pi}s + 1)}, \text{ where “}\Pi\text{” denotes the product operation.}$$

Note that the poles are at  $-\frac{1}{\tau_{pi}}$  and these are all real; there are no complex poles. Hence, there cannot be resonant peaks.

---

### **Solution 2.24**

It provides the flattest magnitude over the pass band among all filters of the same order (same pole count).

Also, we prefer a very sharp cutoff (i.e., steep roll-up and roll-down).

---

**Solution 2.25****(a)**

Op-amp properties: 1. Voltages at input leads are equal; 2. Currents through input leads = 0

Op-amp property:

$$v_B = v_P = v_o \quad (i)$$

$$\text{Current Balance at Node A: } \frac{(v_i - v_A)}{Z_c} = \frac{(v_A - v_B)}{Z_c} + \frac{(v_A - v_P)}{R} \quad (ii)$$

$$\text{Current Balance at Node B: } \frac{(v_A - v_B)}{Z_c} = \frac{v_B}{R} \quad (iii)$$

Note:  $Z_c = \frac{1}{Cs}$  = impedance of capacitor

$$\text{Substitute (i) and (iii) in (ii): } \frac{(v_i - v_A)}{Z_c} = \frac{v_o}{R} + \frac{(v_A - v_o)}{R} = \frac{v_A}{R} \rightarrow v_i = \left(1 + \frac{1}{\tau s}\right)v_A \quad (iv)$$

$$\text{Substitute (i) in (iii): } \frac{(v_A - v_o)}{Z_c} = \frac{v_o}{R} \rightarrow v_A = \left(1 + \frac{1}{\tau s}\right)v_o \quad (v)$$

Note:  $\tau = RC$  = time constant

$$\text{Substitute (iv) in (v): } G(s) = \frac{v_o}{v_i} = \frac{(\tau s)^2}{(\tau s + 1)^2}$$

This is a 2nd order transfer function  $\rightarrow$  2-pole filter

**(b)**

$$\text{With } s = j\omega \text{ in } G(s), \text{ we have } G(j\omega) = \frac{-\tau^2 \omega^2}{(1 + \tau j\omega)}$$

$$\text{Filter magnitude } |G(j\omega)| = \frac{\tau^2 \omega^2}{(1 + \tau^2 \omega^2)}$$

The magnitude of the filter transfer function is sketched in Figure S2.25. This represents a high-pass filter.

**(c)**

$$\text{When, } \omega \ll \frac{1}{\tau}: |G(j\omega)| \cong \tau^2 \omega^2$$

$$\text{When, } \omega \gg \frac{1}{\tau}: |G(j\omega)| \cong \frac{\tau^2 \omega^2}{\tau^2 \omega^2} = 1$$

Hence, we may use  $\omega_c = \frac{1}{\tau}$  as the cutoff frequency.

Note:  $|G(j\omega)| \rightarrow 1$  as  $\omega \rightarrow \infty$

For small  $\omega$ : Roll-up slope of  $|G(j\omega)|$  curve is 40 dB/decade.

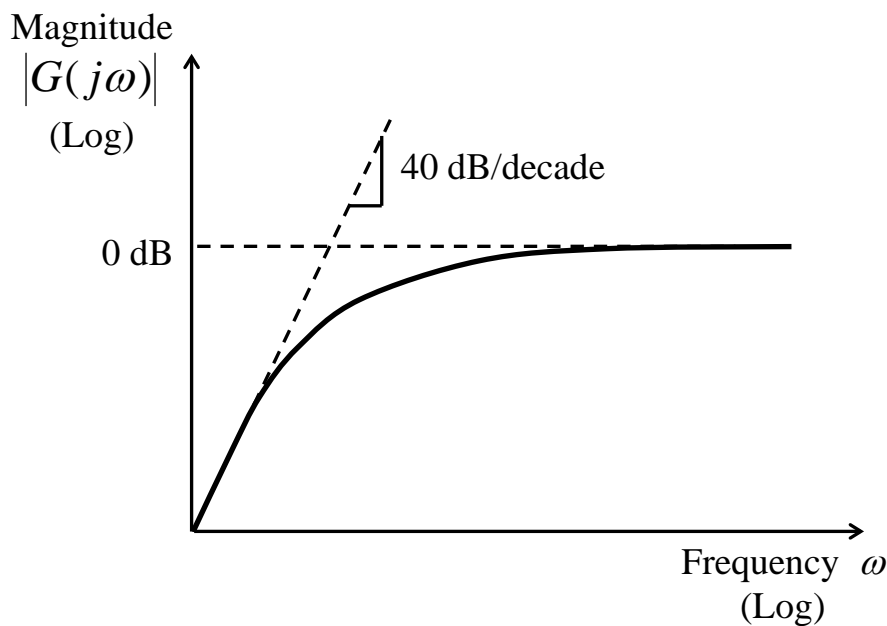


Figure S2.25: Filter transfer function magnitude.

### **Solution 2.26**

**Strain Gauge for force Sensing:** Low-frequency noise due to ambient temperature fluctuations. These may be compensated for (using a bridge circuit) and also through high-pass filtering

**Wearable Ambulatory Monitoring (WAM):** In human mobility monitoring (e.g., in telehealth applications) a popular WAM sensor is a combined accelerometer and gyroscope. Both sensors will be affected by bias, removal of which would need high-pass filtering). High-frequency artifacts may be generated in the sensed signal due to muscle tremor and low-frequency artifacts may be formed due to respiration. These may be removed using band-pass filtering.

**Microphone (Robotic Voice Commands):** A band-pass filter for the human vocal range (80Hz to 1100Hz).

**AC-powered Tachometer for Speed Sensing:** Line noise (60 Hz) may be removed using a notch filter.

### **Solution 2.27**

(a)

The main signal component appears sinusoidal with frequency  $\sim 1$  rad/s (period  $\sim 6.3$  s). From the figure it is not clear whether there is a superimposed sinusoidal signal of high frequency and/or high-frequency noise, even though some oscillations are observed in the noise.

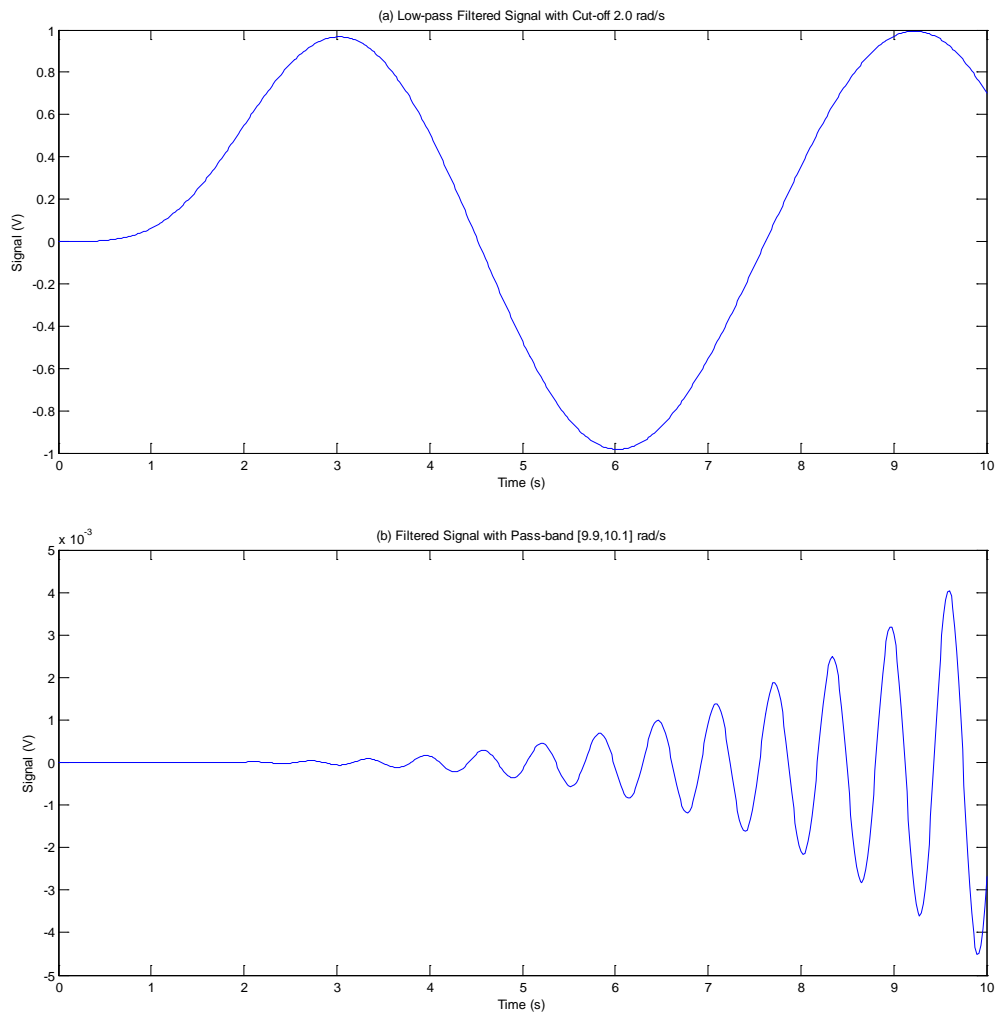
**(b)**

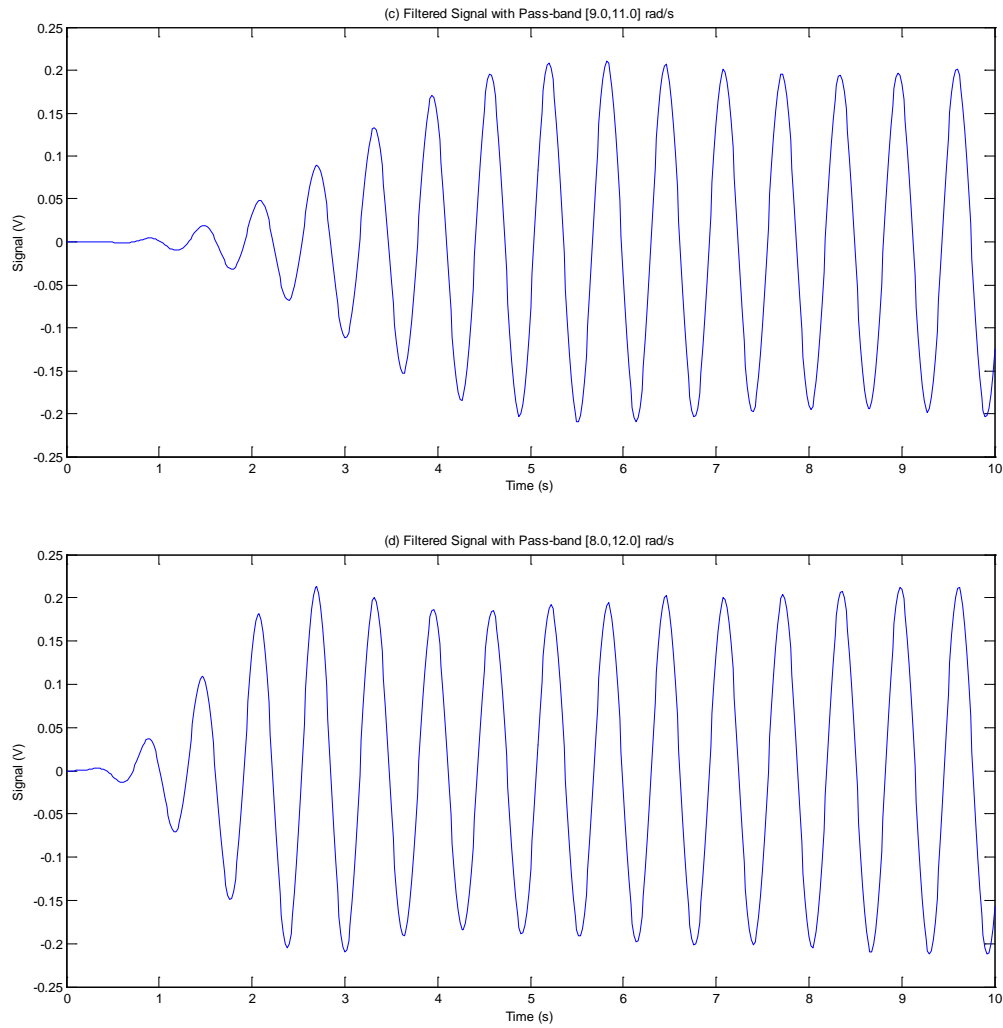
We use the following MATLAB command to obtain the four-pole Butterworth low-pass filter with cut-off frequency at 2.0 rad/s:

```
>> [b,a]=butter(4,2.0,'s')
b =
      0      0      0      0 16.0000
a =
 1.0000  5.2263 13.6569 20.9050 16.0000
```

Then, we use the following MATLAB commands to filter the data signal using this filter, and plot the result shown in Figure S2.27(a):

```
>> y1=lsim(b,a,u,t);
>> plot(t,y1,'-')
```





**Figure S2.27: Filtered signals. (a) Low-pass at 2.0 rad/s; (b) Band-pass over [9.9, 10.1]; (c) Band-pass over [9.0, 11.0]; (d) Band-pass over [8.0, 12.0].**

It is seen that the filtered signal has a frequency of 1.0 rad/s with the correct amplitude (1.0) and negligible phase shift. Initially some signal distortion is seen due to the transient nature of the output. However, the steady-state value is reached in half a period of the signal.

(c)

Band-pass filtering for the three cases are obtained using the following MATLAB commands:

```
>> Wn=[9.9,10.1];
>> [b2,a2] = butter(4,Wn,'bandpass','s');
>> y2=lsim(b2,a2,u,t);
>> plot(t,y2,'-')
```

```

>> Wn=[9.0,11.0];
>> [b2,a2] = butter(4,Wn,'bandpass','s');
>> y2=lsim(b2,a2,u,t);
>> plot(t,y2,'-')

>> Wn=[8.0,12.0];
>> [b2,a2] = butter(4,Wn,'bandpass','s');
>> y2=lsim(b2,a2,u,t);
>> plot(t,y2,'-')

```

The results are shown in Figures S2.27 (b)-(d). The very narrow pass-band produced a filtered result that took a rather long time to reach the steady state of amplitude 0.2 (i.e., the filter had a larger time constant). When the pass-band was increased, the steady state was reached quicker (i.e., smaller filter time constant). However, the amplitude distortion of the filtered signal was noticeable as a result.

---

### **Solution 2.28**

If a characteristic of a signal “*B*” is changed with respect to time, depending on some characteristic parameter of another signal “*A*,” this process is termed *modulation*. The *modulating signal* (or data signal) is the signal *A*. The *carrier signal* is the signal *B*. The output signal of the modulation process is the *modulated signal*. The process of recovering the data signal (*A*) from the modulated signal is known as *demodulation*.

#### **(a) Amplitude Modulation (AM)**

The carrier is a periodic signal (typically a sine wave). The amplitude of the carrier signal is varied in proportion to the magnitude of the data signal. Specifically, the carrier signal is multiplied by the data signal. In one form of AM, the carried signal is added again to the resulting product signal. The AM technique is used in radio transmission and in sensing (e.g., differential transformer). The sign of the data signal is represented by a 180° phase change in the carrier signal.

#### **(b) Frequency Modulation (FM)**

The carrier is typically a sine wave signal. The frequency of the carrier signal is varied in proportion to the magnitude of the data signal. This process is commonly used in radio transmission and data storage. Sign of the data signal is represented by changing the carrier phase angle by 180°.

#### **(c) Phase Modulation (PM)**

The carrier signal is typically a sine wave. The phase angle of the carrier signal is varied in proportion to the magnitude of the data signal. Used in signal transmission. Sign of the data signal is represented by positive or negative phase change in the carrier.

#### **(d) Pulse-width Modulation (PWM)**

The carrier is a pulse signal. The pulse width of the carrier is changed in proportion to the magnitude of the data signal. Both the spacing between the pulses (pulse period) and the pulse amplitude are kept constant. Used in dc motor speed control, other control applications, and digital-to-analog conversion (DAC). Sign of data is accounted for by using both +ve and -ve pulses.

**(e) Pulse-Frequency Modulation (PFM)**

The carrier is a pulse signal. The frequency of the pulses is changed in proportion to the magnitude of the data signal. Pulse width and pulse amplitude are maintained constant (and the pulse period is varied). Used in dc motor speed control. Sign of data is accounted for by using both +ve and -ve pulses.

**(f) Pulse-Code Modulation (PCM)**

Carrier signal is a pulse sequence. The value of the data signal at a given time instant is represented in the binary form and this value is represented in the carrier (of by equally spaced pulses) using the fact that the presence of a pulse can be used to represent binary 0. Then for a given word size, say  $n$  bits, a maximum of  $n$  pulses have to be transmitted. The sign of the data word may be represented by an additional bit, known as the sign bit (using, say 1 to represent “+” and 0 to represent “-”). Separation between one data word and the next may be detected through “framing” a data word using “start bits” and “end bits.” Used in digital communication.

## **Solution 2.29**

### **Intentional AM**

- Radio broadcast  
AM will improve signal communication with reduced distortion by noise and transmission loss. It will also facilitate making several broadcasts simultaneously in the same geographic area (due to the frequency-shifting property of AM)
- Signal conditioning  
AM enables us to exploit advantages of ac signal conditioning hardware (improved stability, reduced drift, etc.). Also, the AM process will improve the signal level and noise immunity as a result of the use of the original signal (to be conditioned) to modulate a high-frequency, high-power carrier signal.

### **Natural AM**

- Any device that uses the transformer action (primary winding and secondary winding with the primary coil being excited by an AC; e.g., linear variable differential transducer or LVDT, ac tachometer).
- A rotating machine with a fault; e.g., a gearbox with a fault on a tooth, a turbine rotor with eccentricity or damaged blade.

Yes. In the first device AM provides the advantages of ac signal conditioning, and improves noise immunity and signal level.

In the second device, the AM principle is useful for fault detection and diagnosis (through identification of the associated frequencies).

---

### **Solution 2.30**

(a) Ball passing frequency = carrier frequency  $f_c = \frac{840 + 960}{2} = 900 \text{ Hz}$

Hence, Estimated shaft speed =  $f_b = 900 - 840 \text{ rev/s} = 60 \text{ rev/s} = 3600 \text{ r.p.m.}$

Number of balls =  $\frac{\text{ball passing frequency}}{\text{shaft speed}} = \frac{900}{60} = 15$

Peak frequencies =  $900 \pm 60 \text{ Hz} = [840 \text{ and } 960] \text{ Hz}$ , (see Figure P2.30).

(b) Yes.

---

### **Solution 2.31**

(a) Phase-Sensitive Demodulation

If the sign of the data signal (modulating signal) is preserved at all times during modulation and is properly recovered in the demodulation process, we have a phase-sensitive demodulation. *Note:* A sign change corresponds to a  $180^\circ$  phase change of the modulated signal; hence, the name.

(b) Half-Wave Demodulation

In half-wave demodulation, an output is generated during every other half-period of the carrier signal.

(c) Full-Wave Demodulation

Here, an output (demodulated value) is generated continuously for every point of the carrier signal.

The rotating frequency (rev/s)  $f_o$  modulates the responses produced by the forcing functions such as gear meshing, ball or roller hammer, and blade passing in rotating machinery. Hence, if the forcing frequency is  $f_c$ , according to the modulation theorem, peaks occur at  $f_c \pm f_o$  instead of at  $f_c$ .

---

### **Solution 2.32**

Resolution corresponds to 1 LSB  $\rightarrow 10/2^8 = 10/256 = 0.04 \text{ V}$

Quantization error (rounding off) corresponds to half this value  $\rightarrow 0.02 \text{ V}$

---

**Solution 2.33****(a)**

The decimal value of the input binary word is given by

$$D = 2^{n-1}b_{n-1} + 2^{n-2}b_{n-2} + \dots + 2^0b_0. \quad (\text{i})$$

The least significant bit (LSB) is  $b_0$  and the most significant bit (MSB) is  $b_{n-1}$ . The analog output voltage  $v$  of the DAC has to be proportional to  $D$ .

Each bit  $b_i$  in the digital word  $w$  will activate a solid-state microswitch in the switching circuit, typically by sending a switching voltage pulse. If  $b_i = 1$ , the circuit lead will be connected to the  $v_{ref}$  supply, providing an input voltage  $v_i = -v_{ref}$  to the corresponding weighting resistor  $2^{n-i-1}R$ . If, on the other hand,  $b_i = 0$ , then the circuit lead will be connected to ground, thereby providing an input voltage  $v_i = 0$  to the same resistor. Note that the MSB is connected to the smallest resistor ( $R$ ) and the LSB is connected to the largest resistor ( $2^{n-1}R$ ). By writing the summation of currents at node  $A$  of the output op-amp, we

get  $\frac{v_{n-1}}{R} + \frac{v_{n-2}}{2R} + \dots + \frac{v_0}{2^{n-1}R} + \frac{v}{R/2} = 0$ .

In writing this equation, we have used the two principal facts for an op-amp: the voltage is the same at both input leads and the current through each lead is zero. Note that the positive lead is grounded and hence node  $A$  should have zero voltage. Now, since  $v_i = -b_i v_{ref}$ , where  $b_i = 0$  or  $1$  depending on the bit value (state of the corresponding switch), we have

$$v = \left[ b_{n-1} + \frac{b_{n-2}}{2} + \dots + \frac{b_0}{2^{n-1}} \right] \frac{v_{ref}}{2} \quad (\text{ii})$$

Clearly, as required, the output voltage  $v$  is proportional to the value  $D$  of the digital word  $w$ .

**(b)**

The full-scale value (FSV) of the analog output occurs when all  $b_i$  are equal to 1. Hence,

$$\text{FSV} = \left[ 1 + \frac{1}{2} + \dots + \frac{1}{2^{n-1}} \right] \frac{v_{ref}}{2}$$

Using the commonly known formula for the sum of a geometric series

$$1 + r + r^2 + \dots + r^{n-1} = \frac{(1-r^n)}{(1-r)} \quad (\text{iii})$$

we get

$$\text{FSV} = \left( 1 - \frac{1}{2^n} \right) v_{ref} \quad (\text{iv})$$

*Note:* This value is slightly smaller than the reference voltage  $v_{ref}$ .

**(c)**

A major drawback of the weighted-resistor DAC is that the range of the resistance value in the weighting circuit is very wide. This presents a practical difficulty, particularly when the size (number of bits  $n$ ) of the DAC is large. Use of resistors with widely different magnitudes in the same circuit can create accuracy problems. For example, since the MSB corresponds to the smallest weighting resistor, it follows that the resistors must have a very high precision.

**Solution 2.34**

For an ADC,

Resolution: This is the change in analog input that corresponds to one LSB change in the digital output.

Dynamic Range: This is the ratio of the span (maximum value - minimum value) to the resolution, expressed in dB.

For an  $n$ -bit ADC,  $DR = 2^n - 1$  as a ratio  $= 20 \log_{10}(2^n - 1)$  dB

Full-scale value: The value of the analog input corresponding to the maximum digital output (i.e., when all the bits are at 1).

Quantization Error: Error introduced due to digitization of a sampled data value. This is equal to: (Signal value corresponding to the digital output) - (Actual sampled value just before conversion).

This error is less than one LSB. With rounding-off, this error is less than  $1/2$  LSB.

**Solution 2.35****(a) Dual-Slope ADC**

A dual-slope ADC is based on timing (i.e., counting the number of clock pulses during) a capacitor-charging process. It uses an RC integrating circuit. Hence, it is also known as an *integrating ADC*. It is simple and inexpensive. In particular, an internal DAC is not utilized and hence, DAC errors will not enter the ADC output. Furthermore, the parameters  $R$  and  $C$  in the integrating circuit do not enter the ADC output. As a result, the device is self-compensating in terms of circuit-parameter variations due to temperature, aging, so on. A shortcoming of this ADC is its slow conversion rate because, for accurate results, the signal integration has to proceed for a longer time in comparison to the conversion time for a successive approximation ADC.

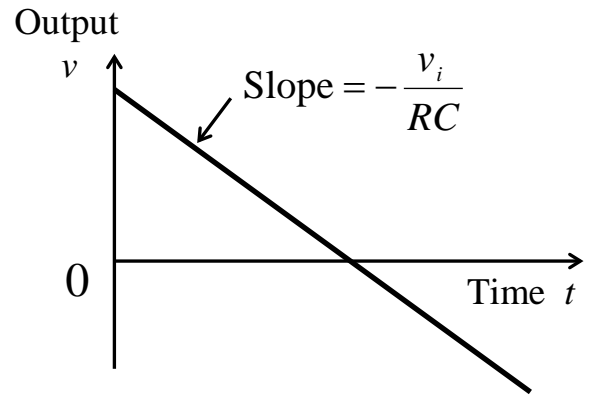
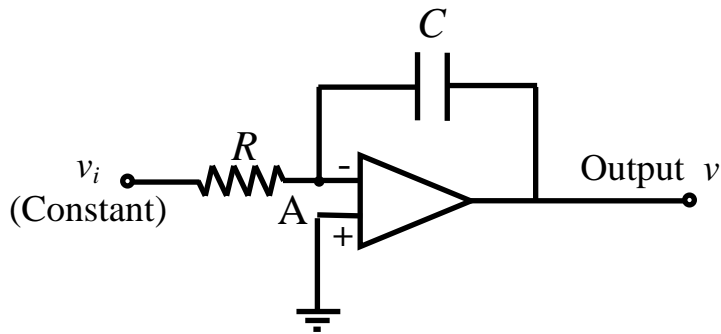
The principle of operation can be explained with reference to the integrating circuit shown in Figure S2.35(a). Here,  $v_i$  is a constant input voltage to the circuit and  $v$  is the output voltage. Since the positive lead of the op-amp is grounded, the negative lead (and node A) also will have zero voltage. In addition, the currents through the op-amp leads are negligible.

Hence, the current balance at node A gives  $\frac{v_i}{R} + C \frac{dv}{dt} = 0$ . Integrating this equation for constant  $v_i$ , we get

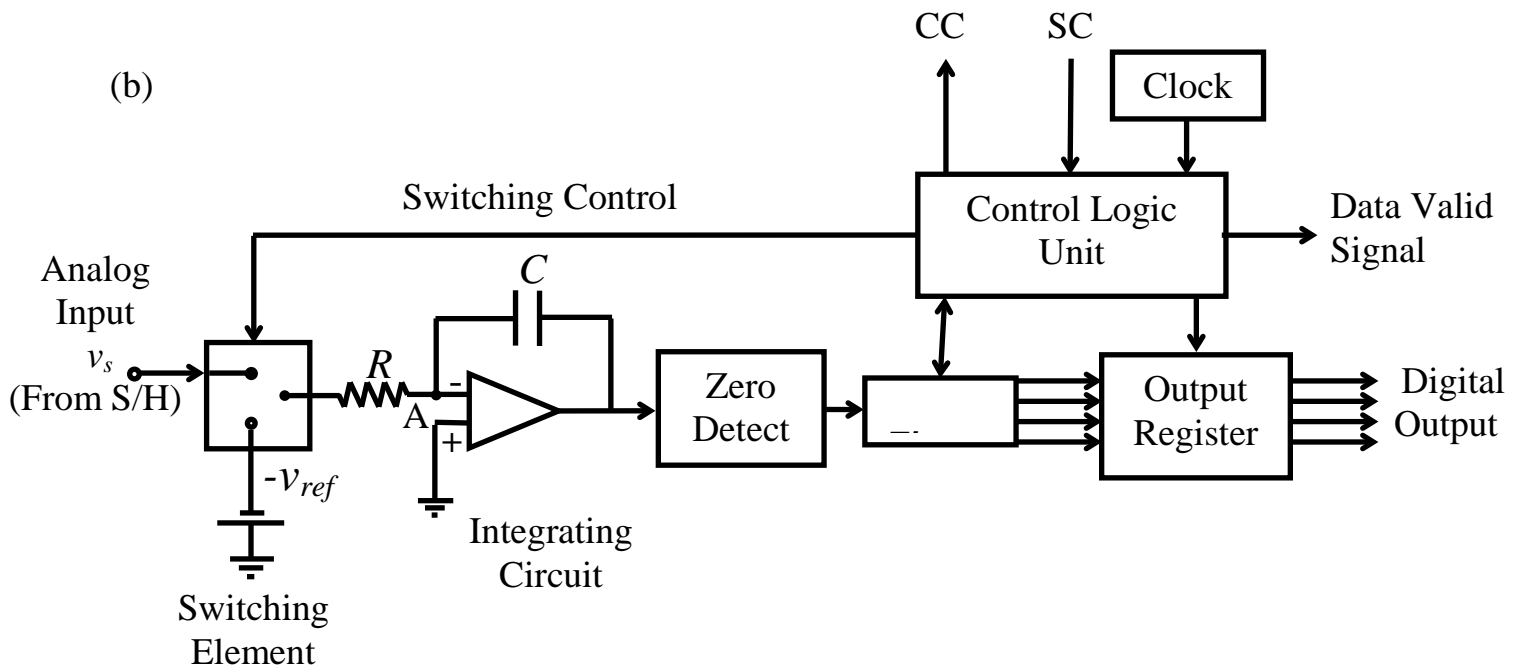
$$v(t) = v(0) - \frac{v_i t}{RC} \quad (i)$$

Equation (i) is used in obtaining a principal result for the dual-slope ADC.

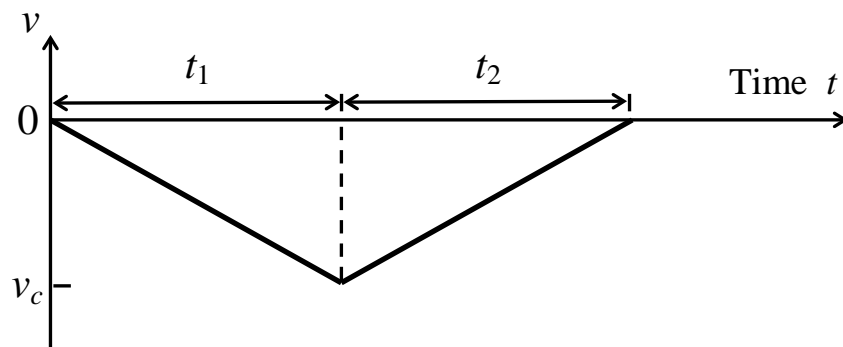
(a)



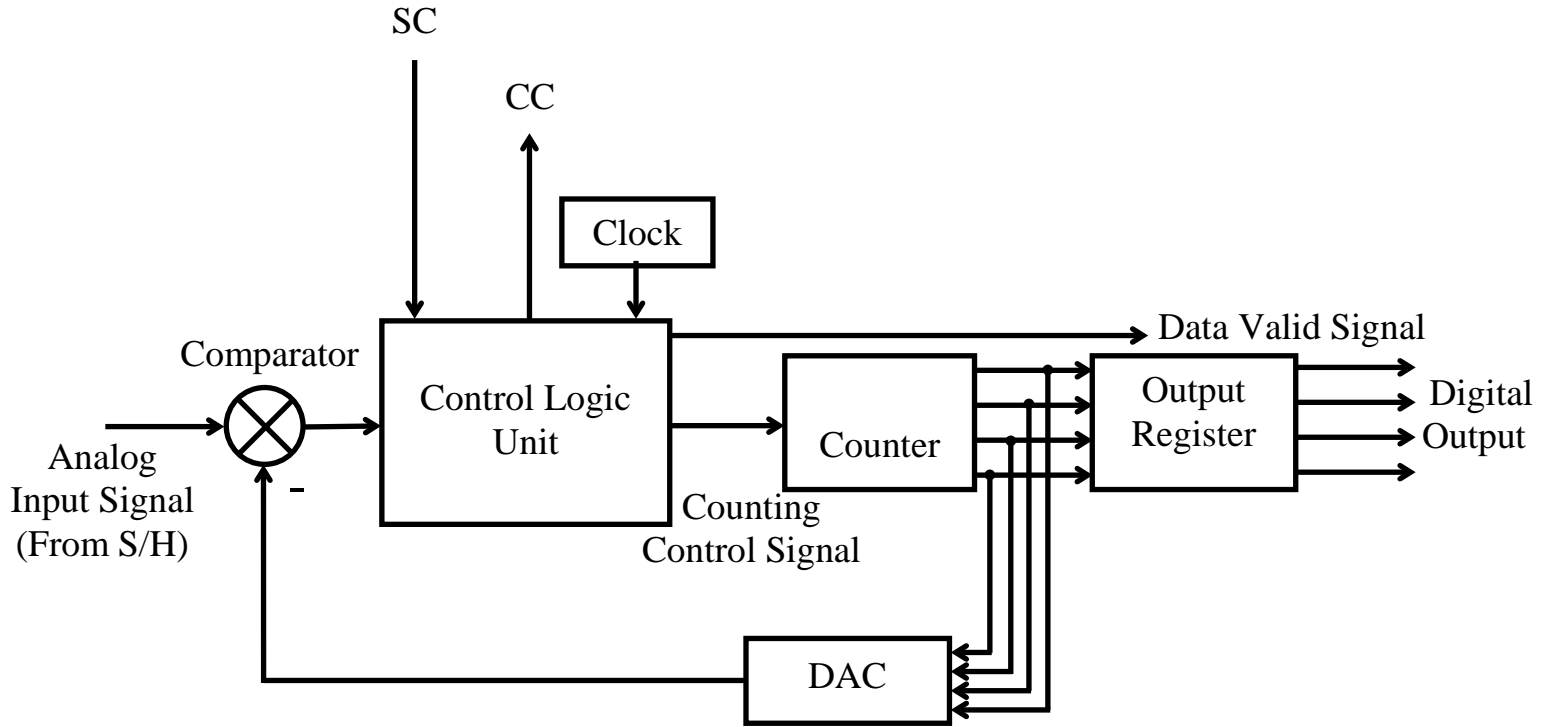
(b)



(c)



(d)



**Figure S2.35:** (a) RC integrating circuit; (b) Dual-slope ADC; (c) Dual-slope charging–discharging curve; (d) Counter ADC.

A schematic diagram for a dual-slope ADC is shown in Figure S2.35(b). Initially, the capacitor  $C$  in the integrating circuit is discharged (zero voltage). Then, the analog signal  $v_s$  is supplied to the switching element and held constant by the S/H. Simultaneously, a control signal “conversion start” is sent to the control logic unit. This clears the timer and the output register (i.e., all bits are set to zero) and sends a pulse to the switching element to connect the input  $v_s$  to the integrating circuit. Also, a signal is sent to the timer to initiate timing (counting). The capacitor  $C$  will begin to charge. Equation (i) is now applicable with input  $v_i = v_s$  and the initial state  $v(0) = 0$ . Suppose that the integrator output  $v$  becomes  $-v_c$  at time  $t = t_1$ . Hence, from Equation (i), we get

$$v_c = \frac{v_s t_1}{RC} \quad (\text{ii})$$

The timer will keep track of the capacitor charging time (as a clock pulse count  $n$ ) and will inform the control logic unit when the elapsed time is  $t_1$  (i.e., when the count is  $n_1$ ). *Note:*  $t_1$  and  $n_1$  are fixed (and known) parameters, but voltage  $v_c$  depends on the value of  $v_s$ , and is unknown.

At this point, the control logic unit sends a signal to the switching unit, which will connect the input lead of the integrator to a reference voltage of opposite polarity (a negative supply voltage  $-v_{ref}$ ). Simultaneously, a signal is sent to the timer to clear its contents and start timing (counting) again. Now the capacitor begins to discharge. The output of the

integrating circuit is monitored by the zero-detect unit. When this output becomes zero, the zero-detect unit sends a signal to the timer to stop counting. A comparator (differential amplifier) with one of the two input leads set at zero potential may serve as the zero-detect unit.

Now suppose that the elapsed time is  $t_2$  (with a corresponding count of  $n_2$ ). It should be clear that Equation (i) is valid for the capacitor discharging process as well. *Note:*  $v_i = -v_{ref}$  and  $v(0) = -v_c$  in this case. Also,  $v(t) = 0$  at  $t = t_2$ . Hence, from Equation (i), we have

$$0 = -v_c + \frac{v_{ref}t_2}{RC}, \text{ or}$$

$$v_c = \frac{v_{ref}t_2}{RC} \quad (\text{iii})$$

On dividing Equation (ii) by Equation (iii), we get  $v_s = v_{ref} \frac{t_2}{t_1}$ . However, the timer pulse

count is proportional to the elapsed time. Hence,  $\frac{t_2}{t_1} = \frac{n_2}{n_1}$ . Now we have

$$v_s = \frac{v_{ref}}{n_1} n_2 \quad (\text{iv})$$

Since  $v_{ref}$  and  $n_1$  are fixed quantities,  $v_{ref}/n_1$  can be interpreted as a scaling factor for the analog input. Then, it follows from Equation (iv) that the second count  $n_2$  is proportional to the analog signal sample  $v_s$ . *Note:* The timer output is available in the digital form. Accordingly, the count  $n_2$  is used as the digital output of the ADC.

At the end of the capacitor discharge period, the count  $n_2$  in the timer is transferred to the output register of the ADC, and the “data valid” signal is set. The contents of the output register are now ready to be read by the interfaced digital system, and the ADC is ready to convert a new sample.

The charging–discharging curve for the capacitor during the conversion process is shown in Figure S2.35(c). The slope of the curve during charging is  $v_s/RC$ , and the slope during discharging is  $+v_{ref}/RC$ . The reason for the use of the term “dual slope” to denote this ADC is therefore clear.

As mentioned before, any variations in  $R$  and  $C$  do not affect the accuracy of the output. But, it should be clear from the foregoing discussion that the conversion time depends on the capacitor discharging time  $t_2$  (*Note:*  $t_1$  is fixed), which in turn depends on  $v_c$  and hence on the input signal value  $v_s$  (see Equation (ii)). It follows that, unlike the successive approximation ADC, the dual-slope ADC has a conversion time that directly depends on the magnitude of the input data sample. This may be considered as a disadvantage because in many applications we prefer to have a constant conversion rate.

The foregoing discussion assumes that the input signal is positive. For a negative signal, the polarity of the supply voltage  $v_{ref}$  has to be changed. Furthermore, the sign has to be properly represented in the contents of the output register as, for example, in the case of successive approximation ADC.

### (b) Counter ADC

The counter-type ADC has several aspects in common with the successive approximation ADC. Both are comparison-type (or closed loop) ADCs. Both use a DAC unit internally to

compare the input signal with the converted signal. The main difference is that in a counter ADC the comparison starts with the LSB and proceeds down. It follows that, in a counter ADC, the conversion time depends on the signal level, because the counting (comparison) stops when a match is made, resulting in shorter conversion times for smaller signal values.

A schematic diagram for a counter ADC is shown in Figure S2.35(d). Note that this is quite similar to Figure 2.38 for the Successive approximation ADC. Initially, all registers are cleared (i.e., all bits and counts are set to zero). As an analog data signal (from the S/H) arrives at the comparator, an SC pulse is sent to the control logic unit. When the ADC is ready for conversion (i.e., when data valid signal is on), the control logic unit initiates the counter. Now, the counter sets its count to 1, and the LSB of the DAC register is set to 1 as well. The resulting DAC output is subtracted from the analog input, by means of the comparator. If the comparator output is positive, the count is incremented by one, and this causes the binary number in the DAC register to be incremented by one LSB. The new (increased) output of the DAC is now compared with the input signal. This cycle of count incrementing and comparison is repeated until the comparator output becomes less than or equal to zero. At that point, the control logic unit sends out a CC signal and transfers the contents of the counter to the output register. Finally, the data valid signal is turned on, indicating that the ADC is ready for a new conversion cycle, and the contents of the output register (the digital output) is available to be read by the interfaced digital system.

The count of the counter is available in the binary form, which is compatible with the output register as well as the DAC register. Hence, the count can be transferred directly to these registers. The count when the analog signal is equal to (or slightly less than) the output of the DAC, is proportional to the analog signal value. Hence, this count represents the digital output. In bipolar operation, the sign of the input signal has to be properly accounted for.

### **Solution 2.36**

Dual-slope (integrating) ADC: In this case, the conversion time is the total time needed to generate the two counts  $n_1$  and  $n_2$  (see Figure S2.35(c). Hence,

$$t_c = (n_1 + n_2)\Delta t \quad (\text{i})$$

Note that  $n_1$  is a fixed count. However,  $n_2$  is a variable count, which represents the digital output, and is proportional to the analog input (signal level). Hence, in this type of ADC, conversion time depends on the analog input level. The largest output for an  $n$  bit converter is  $2^n - 1$ . Hence, the largest conversion time may be given by

$$t_{c\max} = (n_1 + 2^n - 1)\Delta T \quad (\text{ii})$$

Counter ADC: For a counter ADC, the conversion time is proportional to the number of bit transitions (1 LSB/step) from zero to the digital output  $n_o$ . Hence, the conversion time is given by

$$t_c = n_o \Delta t \quad (\text{iii})$$

where  $n_o$  is the digital output value (in decimal). Note that for this ADC as well,  $t_c$  depends on the magnitude of the input data sample. For an  $n$  bit ADC, since the maximum value of  $n_o$  is  $2^n - 1$ , we have the maximum conversion time

$$t_{c\max} = (2^n - 1)\Delta t \quad (\text{iv})$$

By comparing these results with that obtained for a successive-approximation ADC, it can be concluded that the successive-approximation ADC is the fastest of the three types.

### **Solution 2.37**

- (a) **Direct-conversion ADC (Flash ADC):** An  $n$ -bit digital word can represent  $2^n - 1$  digital values, and each additional bit in the word multiplies this by a factor of 2. This ADC, uses a bank of  $2^n - 1$  comparators, each having a reference value corresponding to one of these digital values. The analog signal is sampled into the comparators, in parallel. A comparator fires its digital value if the sampled signal value matches it (to less than half-bit accuracy). This is very fast and capable of gigahertz sampling rates (*Note:* Comparison is done in parallel). Since the bit size increases the number of comparators in an exponential manner, typically, this ADC is limited to 8 bits (to reduce hardware costs and error).
- (b) **Ramp-compare ADC:** This is similar to the dual-slope ADC. The main difference of the present ADC is the use of a ramping signal instead of an integrator. Specifically, it uses a saw-tooth signal that ramps and then quickly returns to zero. Clock counts start as the ramping starts, and stops when the ramp voltage matches (as detected by a comparator) the input signal (analog) value. The clock count gives a digital value, which represents the analog data value. *Note:* The full-scale value (FSV) of the ADC register corresponds to the reference value of the comparator voltage. can be properly scaled using , a comparator fires, and the timer's value is recorded. An oscillator can be used to generate the ramp signal. This ADC is relatively simple with regard to hardware.
- (c) **Wilkinson ADC:** In this ADC, the input voltage (analog data) is compared with that produced by a charging capacitor. The capacitor is allowed to charge until its voltage is equal to the amplitude of the input pulse (a comparator determines when this condition has been reached), and it is discharged to zero. The charge-discharge times are counted using a clock, and used to determine the corresponding digital value. Hence, this approach is basically the same as that of a dual-slope (integrating) ADC.
- (d) **Delta-encoded ADC:** It uses a counter, DAC, and a comparator (compare with other ADCs that use similar hardware). The count is incremented and the resulting digital value is fed into the DAC. The DAC output is compared with the incoming analog data value. The count is stopped when the two values match (up to 1 count increment). The corresponding digital input to the DAC is the output of the ADC. *Note:* Here, “delta” denotes the count “increment,” which is essentially one LSB bit value of the ADC (or DAC). The method is also called “delta-comparison.”
- (e) **Pipeline ADC:** This uses an internal DAC (as with many other ADC methods) and is somewhat similar to the successive-approximation ADC, but is faster. It uses two or more steps of conversion, starting with a “coarse” conversion and ending with a “fine” conversion. After the coarse conversion in the first step, the difference between the analog input value and the DAC output value is then converted using a finer ADC, in the next step. The digital results are combined in a last step.

- (f) **ADC with Intermediate FM Stage:** This uses a voltage-to-frequency converter (VFC or FM) and a frequency counter. First, the VFC is used to convert the analog input signal into an oscillating signal with a frequency proportional to the analog input (i.e., a frequency-modulated or FM signal). Next the frequency counter is used to convert the frequency of the FM signal into a digital value, which is the output of the ADC.

### Solution 2.38

#### **Advantages:**

- Small size
- More functions
- Capability to handle complex tasks
- Higher accuracy
- Faster speed
- Less power
- Less loading problems
- Low cost
- Easy interfacing with digital systems.

### Solution 2.39

Bridge	Equation for $\frac{\delta v_o}{v_{ref}}$	% Nonlinearity
1. Constant voltage bridge	$\frac{\delta R / R}{(4 + 2\delta R / R)}$	$50 \frac{\delta R}{R}$
2. Constant current bridge	$\frac{\delta R / R}{(4 + \delta R / R)}$	$25 \frac{\delta R}{R}$
3. Half bridge	$\frac{R_f}{R} \cdot \frac{\delta R / R}{(1 + \delta R / R)}$	$100 \frac{\delta R}{R}$

	Highest	Intermediate	Lowest
<b>Nonlinearity and Temperature Effects:</b>	Bridge 3	Bridge 1	Bridge 2
<b>Cost:</b>	Bridge 2 (Regulated Current Source)	Bridge 1	Bridge 1 (Simple Circuitry)

Percentage error due to change in supply voltage:

$$\frac{\left[ \frac{R_f (\delta R / R)}{R(1 + \delta R / R)} (v_{ref} + \delta v_{ref}) - \frac{R_f (\delta R / R)}{R (1 + \delta R / R)} v_{ref} \right] \times 100}{\left[ \frac{R_f (\delta R / R)}{R (1 + \delta R / R)} v_{ref} \right]} = \frac{\delta v_{ref}}{v_{ref}} \times 100\%$$

For a 1% change in supply voltage, error = 1%.

---

### **Solution 2.40**

From equation (2.91) we get  $\delta v_o = \frac{[(R + \delta R)R - R(R - \delta R)]}{(R + \delta R + R)(R - \delta R + R)} v_{ref} - 0$

This simplifies to  $\frac{\delta v_o}{v_{ref}} = \frac{2\delta R / R}{4 - (\delta R / R)^2}$  which is nonlinear.

Similarly, it can be shown from equation (2.91) that the pair of changes:  $R_2 \rightarrow R + \delta R$  and  $R_4 \rightarrow R - \delta R$  will result in a nonlinear relation that is the same as before, except for the change in sign:  $\frac{\delta v_o}{v_{ref}} = \frac{-2\delta R / R}{4 - (\delta R / R)^2}$

---

### **Solution 2.41**

From equation (2.98) we get  $\delta v_o = \frac{[(R + \delta R)R - (R - \delta R)R]}{(R + \delta R + R - \delta R + R + R)} i_{ref} - 0$

On simplification we get the linear relation:  $\frac{\delta v_o}{R i_{ref}} = \frac{\delta R / R}{2}$

If  $R_4$  and  $R_3$  are the active elements, with  $R_4$  in tension and  $R_3$  in compression, it is clear from equation (2.98) that we get an identical linear result (not even a sign change).

---

### **Solution 2.42**

Starting with  $\frac{R_1}{R_2} = \frac{R_3}{R_4} = p$ , when  $R_1$  is increased by  $\delta R_1$ , the resulting change in the bridge output (from zero) is given by  $\delta v_o = \frac{(R_1 + \delta R_1) / R_2 - (R_3 / R_4)}{[(R_1 + \delta R_1) / R_2 + 1](R_3 / R_4 + 1)} v_{ref}$  By direct substitution of  $\delta r = \delta R_1 / R_1$  and simplification, we get  $\delta v_o = \frac{p \delta r}{[p(1 + \delta r) + 1](p + 1)} v_{ref}$ . For a given  $\delta r$ , the

maximum value of  $\delta v_o$  occurs when  $\frac{p}{(\alpha p + 1)(p + 1)}$  is a maximum, where  $\alpha = 1 + \delta r$ . The

maximum point of this expression is determined by differentiating it with respect to  $p$  and equating to zero. We get,  $(\alpha p + 1)(p + 1) \times 1 - p[\alpha p + 1 + \alpha(p + 1)] = 0$

On simplification, we have  $1 - \alpha p^2 = 0$ , or  $p = \frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{1 + \delta r}}$

Since  $\delta r$  is very small compared to 1, we note that the maximum sensitivity occurs when  $p$  is almost equal to 1.

### **Solution 2.43**

$$\frac{1}{Z_1} = \frac{1}{R_1} + j\omega C_1, \quad Z_2 = R_2, \quad Z_3 = R_3, \quad Z_4 = R_4 + j\omega L_4$$

Balanced condition:  $Z_1 Z_4 = Z_2 Z_3 \rightarrow Z_4 = Z_2 Z_3 / Z_1$

Substitute:  $(R_4 + j\omega L_4) = R_2 R_3 (\frac{1}{R_1} + j\omega C_1)$  and  $R_4 + j\omega L_4 = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$

Equate real parts and imaginary parts:  $R_4 = \frac{R_2 R_3}{R_1}$  and  $\omega L_4 = \omega C_1 R_2 R_3$

This gives:  $L_4 = C_1 R_2 R_3$

Suppose that  $R_1$  and  $R_2$  are fine-adjustable. Balance the bridge by varying  $R_2$  and then  $R_1$  and again by  $R_2$  and so on, in small steps. Then we will be able to determine  $C_1$  according to:

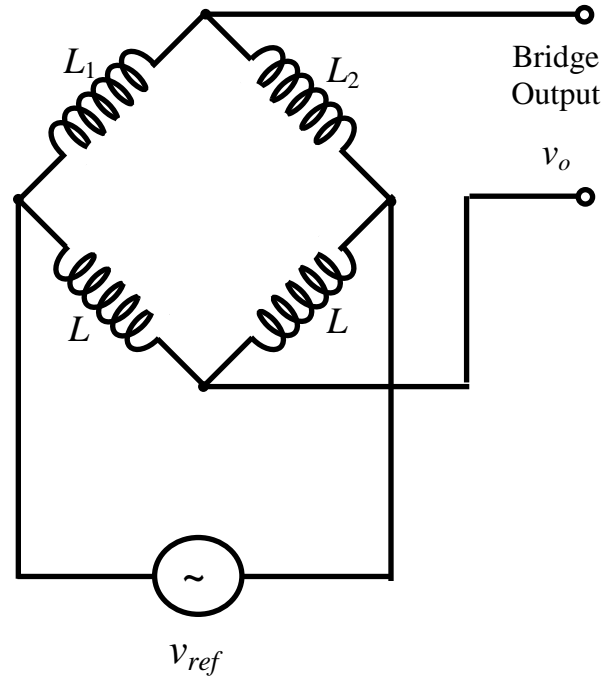
$$C_1 = \frac{R_2 R_3}{L_4}$$

and  $L_4$  according to:  $L_4 = C_1 R_2 R_3$ .

### **Solution 2.44**

When the core is centrally located we have:  $L_1 = L_2$  and  $v_o$  will be sinusoidal with a zero average value. When the core is displaced through  $x$  in one direction, the average output ( $v_o$ ) will increase (+ve) in proportion to  $x$  and when displaced in the opposite direction the average output will decrease (-ve) in proportion to  $x$ . Hence, the average output is a measure of  $x$ . In a full bridge we use  $L_1$  and  $L_2$  in Figure P2.44 as two adjacent arms. Also, use two identical external impedances for the remaining two arms.

This arrangement is shown in Figure S2.44. Then the output  $v_o$  of the bridge will behave as in the half-bridge case.



**Figure S2.44: A full-bridge LVDT circuit.**

**Solution 2.45**

We have the bridge equation  $\frac{\delta v_o}{v_{ref} + \alpha \delta v_o} = \frac{r}{4 + 2r}$  with  $r = \frac{\delta R}{R}$

Then,  $4\delta v_o + 2r\delta v_o = rv_{ref} + \alpha r\delta v_o \rightarrow \delta v_o[4 + 2r - \alpha r] = rv_{ref}$

Now, if  $\alpha = 2$ , we get  $\frac{\delta v_o}{v_{ref}} = \frac{r}{4} = \frac{\delta R}{4R}$

This is linear.

**Solution 2.46**

**(a) Sensitivity**

**Bridge:** Output voltage is zero under balanced conditions. Hence the signal directly gives the “change” in the active strain gauges. Furthermore, we may use 4 active gauges to obtain the maximum sensitivity. Cross-sensitivities will also balance out.

**Pot:** The initial output (i.e., when the strain gauges are not active) is not zero, and the change in resistance of an active gauge is determined by the “change” in the output voltage. This increment in the output can be masked by the comparatively high initial output voltage. Furthermore, only one strain gauge is active. Cross sensitivity directly effects reading.

➔ Bridge circuit has much better sensitivity.

**(b) and (c) Accuracy**

**Bridge:** All four gauges drift by nearly the same amount (due to ambient temperature, humidity, aging, etc.) and they balance out (assuming similar gauges). One resistor can be used for fine adjustment. Also higher sensitivity means higher signal-to-noise ratio (SNR) and better accuracy.

**Pot:** Measurements are affected by drift in the strain gauge and in the circuit itself, and variations in the supply voltage  $v_{ref}$ . Since sensitivity is low, it can be affected more by noise. → Bridge circuit provides better accuracy.

**(d) Complexity and Cost:** Bridge circuit has four resistors and the failure of one will cause malfunction. Pot has only one active gauge and is usually more robust. Reliability of the bridge is low in this sense. However, current through the bridge strain gauges is lower due to the following: (i) Current is divided among two branches; (ii) Due to lower sensitivity in a pot circuit, a higher supply voltage or a higher strain gauge current would be needed.

→ Life expectancy of a bridge circuit would be higher.

**(e) Linearity:** In the operating range  $\delta v_o$  and  $\delta R$  relation is acceptably linear for both bridge and pot circuits. In a pot circuit, however, linearity tends to be less effective because the voltage output under non-active conditions is not zero, and hence the “change” in the output voltage has to be used for the relationship to be linear.

**Solution 2.47**

Bridge equations:  $\frac{v_{ref} - v_A}{R_2} = i + \frac{v_A}{R_1}$  (i);  $\frac{v_{ref} - v_B}{R_4} = \frac{v_B}{R_3} - i$  (ii);  $i = \frac{v_A - v_B}{R_L} = \frac{v_o}{R_L}$  (iii)

(ii) →  $\frac{v_{ref}}{R_2} = v_A \left( \frac{1}{R_1} + \frac{1}{R_2} \right) + i$  and (ii) →  $\frac{v_{ref}}{R_4} = v_B \left( \frac{1}{R_3} + \frac{1}{R_4} \right) - i$

Form  $v_A - v_B$  from the above two equations:

$$v_{ref} \left[ \frac{1}{R_2} \left( \frac{1}{R_3} + \frac{1}{R_4} \right) - \frac{1}{R_4} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \right] = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \left( \frac{1}{R_3} + \frac{1}{R_4} \right) (v_A - v_B) + i \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)$$

Substitute (iii):

$$v_{ref} \left[ \frac{1}{R_2} \left( \frac{1}{R_3} + \frac{1}{R_4} \right) - \frac{1}{R_4} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \right] = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \left( \frac{1}{R_3} + \frac{1}{R_4} \right) v_o + \frac{v_o}{R_L} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)$$

$$\Rightarrow v_o = \frac{\left[ \frac{1}{R_2} \left( \frac{1}{R_3} + \frac{1}{R_4} \right) - \frac{1}{R_4} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \right]}{\left( \frac{1}{R_1} + \frac{1}{R_2} \right) \left( \frac{1}{R_3} + \frac{1}{R_4} \right) + \frac{1}{R_L} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)} v_{ref}$$

Initially:  $R_1 = R_2 = R_3 = R_4 = R \Rightarrow v_o = 0$

Next we have:  $R_1 = R + \delta R \Rightarrow v_o = \delta v_o$

$$\delta v_o = \left[ \frac{\frac{2}{R^2} - \frac{1}{R} \left( \frac{1}{R + \delta R} + \frac{1}{R} \right)}{\left( \frac{1}{R + \delta R} + \frac{1}{R} \right) \frac{2}{R} + \frac{1}{R_L} \left( \frac{1}{R + \delta R} + \frac{3}{R} \right)} \right] v_{ref} \quad (\text{iv})$$

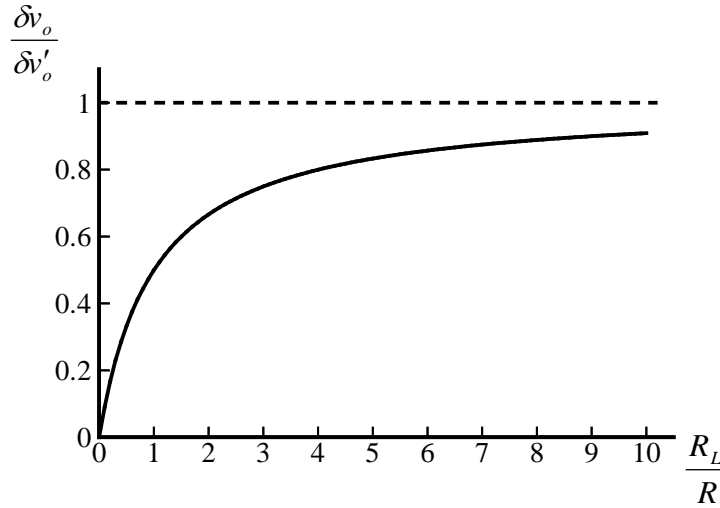
Under open-circuit condition  $R_L \rightarrow \infty$ .

$$\text{Output } \delta v'_o = \left[ \frac{\frac{2}{R^2} - \frac{1}{R} \left( \frac{1}{R + \delta R} + \frac{1}{R} \right)}{\left( \frac{1}{R + \delta R} + \frac{1}{R} \right) \frac{2}{R}} \right] v_{ref} \quad (\text{v})$$

$$\text{From (iv) and (v): } \frac{\delta v_o}{\delta v'_o} = \frac{\left( \frac{1}{R + \delta R} + \frac{1}{R} \right) \frac{2}{R}}{\left( \frac{1}{R + \delta R} + \frac{1}{R} \right) \frac{2}{R} + \frac{1}{R_L} \left( \frac{1}{R + \delta R} + \frac{3}{R} \right)} \rightarrow \frac{\delta v_o}{\delta v'_o} = \frac{1}{(1 + \Delta)}$$

$$\text{Where } \Delta = \frac{\frac{1}{R_L} \left( \frac{1}{R + \delta R} + \frac{3}{R} \right)}{\frac{2}{R} \left( \frac{1}{R + \delta R} + \frac{1}{R} \right)} = \frac{R}{2R_L} \frac{\left( 3 + \frac{1}{1 + \delta R/R} \right)}{\left( 1 + \frac{1}{1 + \delta R/R} \right)}$$

With  $\frac{\delta R}{R} = 0.01$ , we plot  $\frac{\delta v_o}{\delta v'_o}$  against  $\frac{R_L}{R}$ , in Figure S2.47.



**Figure S2.47: Normalized output of a bridge as a function of load resistance.**

### **Solution 2.48**

$$\text{From the bridge circuit we have: } v_o = \left[ \frac{R_1}{(R_1 + R_2)} - \frac{R_3}{(R_3 + R_4)} \right] v_{ref} \quad (\text{neglect load current})$$

If  $R_1$  is changed to  $R_1 + \delta R_1$  we have

$$\begin{aligned}\delta v_o &= \left[ \frac{R_1 + \delta R_1}{(R_1 + \delta R_1 + R_2)} - \frac{R_3}{(R_3 + R_4)} - \frac{R_1}{(R_1 + R_2)} + \frac{R_3}{(R_3 + R_4)} \right] v_{ref} \\ &= \left[ \frac{R_1 + \delta R_1}{(R_1 + R_2 + \delta R_1)} - \frac{R_1}{(R_1 + R_2)} \right] v_{ref} = \left[ \frac{R + \delta R}{2R + \delta R} - \frac{1}{2} \right] v_{ref} \quad \{R_1 = R_2 = R\}\end{aligned}$$

$$\text{Hence, } \delta v_o = \frac{\delta R}{2(2R + \delta R)} v_{ref}$$

If we neglect  $0(\delta R^2)$  terms compared to  $R$  we have:  $\delta v'_o = \frac{\delta R}{4R} v_{ref}$

$$\begin{aligned}\rightarrow \quad \% \text{ error} &= \frac{(\delta v'_o - \delta v_o)}{\delta v_o} \times 100 = \left[ \frac{\frac{\delta R}{4R} - \frac{\delta R}{2(2R + \delta R)}}{\frac{\delta R}{2(2R + \delta R)}} \right] \times 100 \\ &= \left[ \frac{4R + 2\delta R}{4R} - 1 \right] \times 100 = \frac{\delta R}{2R} \times 100\end{aligned}$$

$$\text{For } \frac{\delta R}{R} = 0.05: \% \text{ error} = \frac{0.05}{2} \times 100\% = 2.5\%$$

### **Solution 2.49**

Bridge sensitivity is determined by the output change for a unit change in one of the resistances in the four arms of the bridge. Bridge sensitivity may be increased by increasing the supply voltage ( $v_{ref}$ ) and decreasing the arm resistance (say, by connecting two resistors in parallel for each arm).

Input impedance  $R$  of the bridge (assuming load current is zero) is given by

$$\frac{1}{R} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$$

$$\text{Power dissipation } p = \frac{v_{ref}^2}{R} \rightarrow p = \left[ \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4} \right] v_{ref}^2$$

$$\text{For the rest of the discussion, assume } R_1 = R_2 = R_3 = R_4 = R \rightarrow p = \frac{v_{ref}^2}{R} \quad (i)$$

Bridge sensitivity  $S_b$  may be expressed as  $S_b = \frac{\delta v_o}{\delta R}$

Change  $R_1$  by  $\delta R$ . Then, from the bridge equation  $\frac{\delta v_o}{v_{ref}} = \frac{\delta R}{4R}$  we have:

$$S_b = \frac{v_{ref}}{4R} \quad (ii)$$

Substitute (i) in (ii):  $S_b = \frac{1}{4} \sqrt{\frac{p}{R}}$

Note that  $S_b$  is limited by  $p$ , for a given  $R$ .

### **Solution 2.50**

$$\begin{aligned} \text{Bridge equation: } v_o &= \left[ \frac{1}{1 + R_2/R_1} - \frac{1}{1 + R_4/R_3} \right] v_{ref} \Rightarrow \frac{v_o}{v_{ref}} = \frac{1}{1 + R_2/R_1} - \frac{1}{2} \Rightarrow \\ \frac{1}{1 + R_2/R_1} &= \frac{2v_o + v_{ref}}{2v_{ref}} \Rightarrow \frac{R_2}{R_1} = \frac{2v_{ref}}{2v_o + v_{ref}} - 1 = \frac{v_{ref} - 2v_o}{2v_o + v_{ref}} \Rightarrow R_1 = R_2 \left[ \frac{v_{ref} - 2v_o}{v_{ref} - 2v_o} \right] \end{aligned} \quad (i)$$

It is clear from this result that:  $v_o = 0 \Leftrightarrow R_1 = R_2$

For the circuit with long cables: New  $R_1$  is  $R_1 + R_c$  and new  $R_2$  is  $R_2 + R_c$ .

$$\text{Then, from (i): } R_1 + R_c = (R_2 + R_c) \left[ \frac{v_{ref} + 2v_o}{v_{ref} - 2v_o} \right] \Rightarrow R_1 = R_2 \left[ \frac{v_{ref} + 2v_o}{v_{ref} - 2v_o} \right] + 4R_c \left[ \frac{v_o}{v_{ref} - 2v_o} \right]$$

$$\text{The fractional error: } e = \frac{4R_c \left[ \frac{v_o}{v_{ref} - 2v_o} \right]}{R_2 \left[ \frac{v_{ref} + 2v_o}{v_{ref} - 2v_o} \right]} = \frac{4R_c}{R_2} \left[ \frac{v_o}{v_{ref} + 2v_o} \right]$$

Note that  $e$  decreases with increasing  $R_2$  and  $v_{ref}$ .

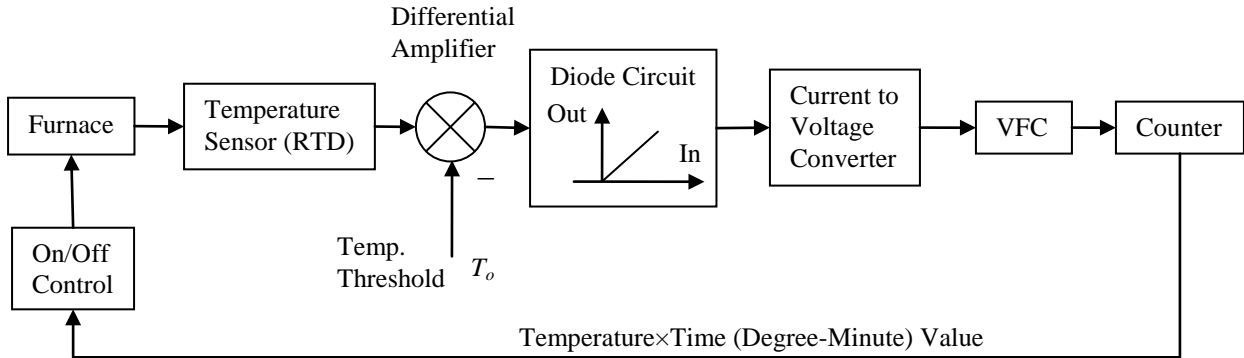
### **Solution 2.51**

See Figure S2.51. Furnace temperature is compared with the threshold by the differential amp. If the difference is negative, the diode circuit provides a zero current. If this difference is positive, the diode circuit provides a proportional current. The current-to-voltage converter converts this current into a voltage. The VFC converts this into a pulse signal whose frequency is proportional to the input voltage. The counter counts the pulses. This count is proportional to the frequency  $\times$  time product and hence the temperature  $\times$  time product. The count, after properly scaled, is used by the on/off controller to turn off the furnace when the specified value of Celsius-Minute reading is reached.

#### **Signal Modification:**

1. RTD circuit converts temperature into voltage

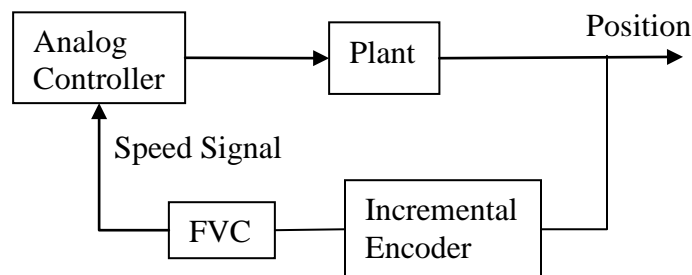
2. Differential amplifier obtains the difference of the furnace temperature and the threshold, and amplifies it
3. The diode circuit converts the output of the difference amplifier into a proportional current, while removing the negative values
4. The current-to-voltage converter converts this current into a voltage
5. The VFC converts voltage into a frequency signal
6. The counter provides a digital count of pulses.



**Figure S2.51: Temperature control system for a furnace.**

### **Solution 2.52**

(a) See Figure S2.52. The pulse sequence from the encoder is fed into a frequency-to-voltage converter (FVC). This generates an analog signal proportional to the pulse frequency (i.e., plant speed).



**Figure S2.52: A position feedback control loop.**

(b) The PWM signal is generated by the digital controller. The pulse amplitude is compatible with the required voltage excitation level for the drive circuit of the dc motor. The average signal is varied simply by varying the pulse width of the controller output.

**Solution 2.53****(a) Conductive Coupling**

This allows through almost all types of noise. Grounding, shielding, and perhaps filtering would be needed to reduce the noise.

**(b) Inductive Coupling**

Allows through ac noise and rejects dc noise (offsets, etc.). Electromagnetic interference noise will pass through. Remove the magnetic flux noise (through a low-reluctance path) and reduce flux linkage (e.g., by reducing the number of coil turns) in order to reduce inductively coupled noise. High-frequency noise effects are usually less in this case.

**(c) Capacitive Coupling**

Allow through ac noise and rejects dc noise. High-frequency noise effects are more serious in this case. Filter out high-frequency noise in order to reduce noise problems.

**(d) Optical Coupling**

Removes all types of noise except optical noise. Reduce optical noise or modulate the source in order to reduce noise problems.

**Method of Eliminating the Effect of Ambient Light**

Modulate the source (by switching, strobing, or shuttering). Demodulate the signal (and filter) at the photoreceiver.

---

**Solution 2.54****Advantages of Optical Coupling**

1. Transmission is one way → No loading of the first circuit by changes in the second circuit
2. Electromagnetic interference does not appear in the optical path
3. Coupling through bundles of optical fiber can be used safely in hazardous (e.g., explosive, chemical) environments
4. Large currents and voltages generated in one circuit (due to short-circuit, open-circuit conditions, malfunctions, etc.) will not damage the other circuit, which might be more delicate, critical, and costly. *Note:* When the signal is large, either the optical source (LED) will saturate or the detector (photosensor) will saturate.

**Advantages of Infrared over Visible Light**

1. Large wave length → Less problems due to dust, dirt, moisture, etc.
2. Not affected by ambient light
3. Can be used for applications in the dark (security systems, etc.)

**Advantages of Pulse Modulation**

1. Frequency spectrum is shifted by high-frequency modulation. Hence it is not distorted by low-frequency noise, which can be filtered out (including ambient light noise)

2. Power usage and thermal effects are reduced.

#### Advantages of Laser-based Systems

1. High intensity
2. Coherent (in phase) beams
3. Single wave length (monochromatic) light
4. Not very sensitive to ambient temperature changes
5. Very good coupling efficiency.

#### Disadvantages

1. Power supplies with high voltages would be needed
2. Short life span and stability
3. Possible eye hazards
4. High dynamic range → other devices (electronics) connected to the laser should have comparable dynamic range → expensive, complex
5. More costly.

Beam vibration can be measured using the arrangement shown in Figure S2.54. The measured frequency will determine the Young's modulus  $E$  through the formula:

$$\omega = \lambda^2 \sqrt{\frac{EI}{\rho A}}$$

where,

$\omega$  = natural frequency of vibration

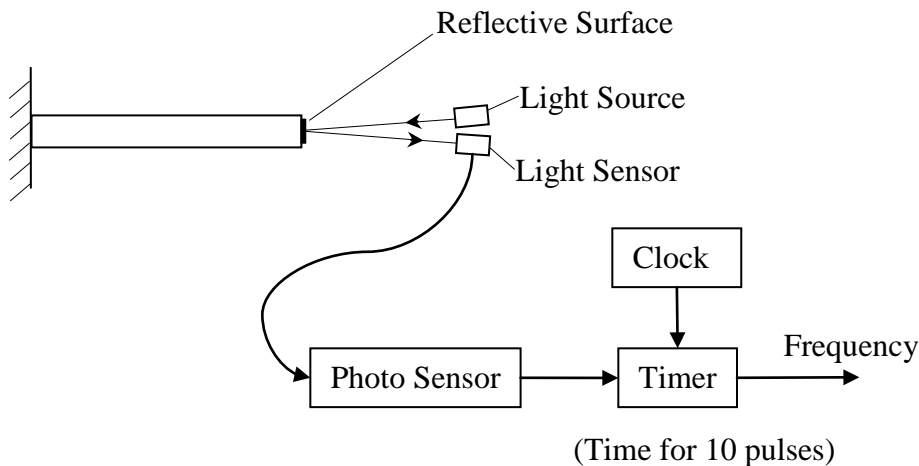
$\lambda$  = modal parameter =  $1.875104/l$  for the first mode of a cantilever beam

$l$  = beam length

$I$  = second moment of area of the beam cross-section about the neutral axis of bending

$A$  = area of cross section of the beam

$\rho$  = mass density of the beam material.



**Figure S2.54: Determination of Young's modulus by measuring beam vibration.**

**Solution 2.55**

Application: Water quality monitoring (e.g., in natural water sources from remote locations, reservoirs, household).

Item	Information
What parameters or variables have to be measured in your application?	pH, turbidity/color, temperature, electrical conductivity
Nature of the information (parameters and variables) needed for the particular application (analog, digital, modulated, demodulated, power level, bandwidth, accuracy, etc.)	Signal conditioning of analog sensors using filters and amplifiers: analog signals. Digital sensors: digital. Transmission: radio signals, modulated signals. Processing in a digital computer for decision making: digital data. Sensing: Hz to kHz; wireless transmission: MHz; sampling: kHz An overall accuracy of 1% would be adequate
List of sensors needed for the application	pH sensor: , turbidity sensor, thermistor, conductivity sensor
Signal provided by each sensor (type— analog, digital, modulated, etc.; power level; frequency range, etc.)	pH sensor: Vernier, digital output (USB), 0-14 pH range with accuracy $\pm 0.2$ pH, time constant 1 s; turbidity sensor (optical): Vernier, 0-200 NTU range with accuracy $\pm 2$ NTU, analog or digital; thermistor: Omega, temperature range 0-100 °C with accuracy $\pm 0.2$ °C, time constant 2.5 s; conductivity sensor: Campbell Scientific, can measure both conductivity and temperature, conductivity range 0.005 to 7.0 mS/cm, temperature range -15to +50 °C, conductivity accuracy $\pm 5\%$ , needs a resistance bridge, analog output
Errors present in the sensor output (SNR, etc.)	Sensor noise, environmental effects (temperature, dust, humidity, etc.), drift and offset (calibration error), line noise, information distortion and loss during transmission, sampling errors, quantization errors
Type of signal conditioning or conversion needed with the sensors (filtering, amplification, modulation, demodulation, ADC, DAC, voltage-frequency conversion, frequency-voltage conversion, etc.)	Filtering, amplification, resistance bridge, modulation, demodulation, S/H, MUX, ADC
Any other comments	SensorDAQ from Vernier, which can be used with NI LabView software and multiple sensor interfaces (analog and digital I/O) for data acquisition and processing by a computer.