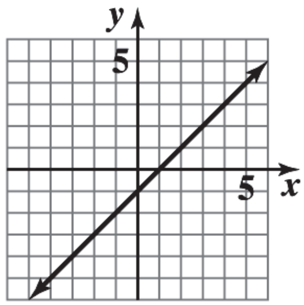
1 FUNCTIONS AND GRAPHS

EXERCISE 1-1

2.



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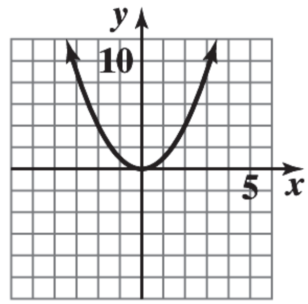

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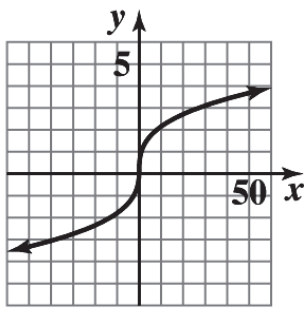
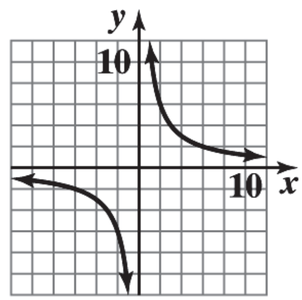


8.



10. The table specifies a function, since for each domain value there corresponds one and only one range value.

12. The table does not specify a function, since more than one range value corresponds to a given domain value.
(Range values 1, 2 correspond to domain value 9.)

14. This is a function.

16. The graph specifies a function; each vertical line in the plane intersects the graph in at most one point.

18. The graph does not specify a function. There are vertical lines which intersect the graph in more than one point. For example, the *y*-axis intersects the graph in two points.

20. The graph does not specify a function.

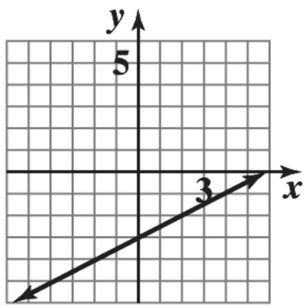
22. $y = 4x + \frac{1}{x}$ is neither linear nor constant.

24. $2x - 4y - 6 = 0$ is linear.

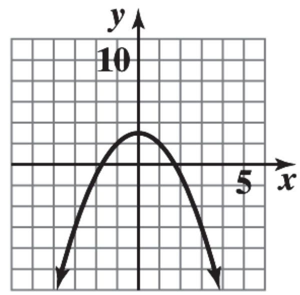
26. $x + xy + 1 = 0$ is neither linear nor constant.

28. $\frac{y-x}{2} + \frac{3+2x}{4} = 1$ simplifies to
$y = \frac{1}{2}$ constant.

30.



32.



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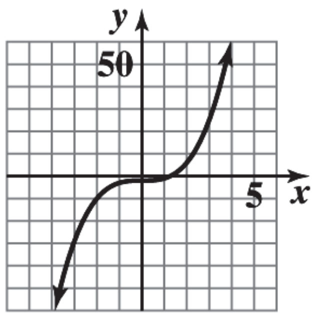
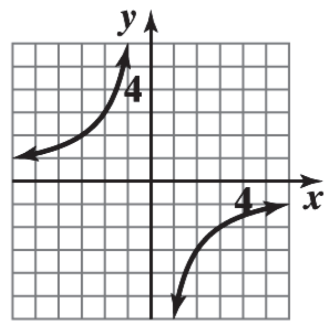


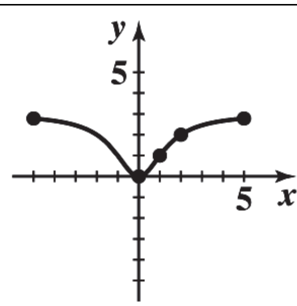
36.



38. $f(x) = \frac{3x^2}{x^2+2}$. Since the denominator is bigger than 1, we note that the values of f are between 0 and 3. Furthermore, the function f has the property that $f(-x) = f(x)$. So, adding points $x = 3$, $x = 4$, $x = 5$, we have:

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$F(x)$	2.78	2.67	2.45	2	1	0	1	2	2.45	2.67	2.78

The sketch is:



40. $y = f(4) = 0$

42. $y = f(-2) = 3$

44. $f(x) = 4$ at $x = 5$.

46. $f(x) = 0$ at $x = -5, 0, 4$.

48. Domain**:** all real numbers.

50. Domain: all real numbers except $x = 2$**.**

52. Domain: $x \geq -5$ or $[-5, \infty)$.

54. Given $6x - 7y = 21$. Solving for y we have: $-7y = 21 - 6x$ and $y = \frac{6}{7}x - 3$.

This equation specifies a function. The domain is R, the set of real numbers.

56. Given $x(x+y)=4$. Solving for y we have: $xy+x^2=4$ and $y=\frac{4-x^2}{x}$.

This equation specifies a function. The domain is all real numbers except 0

58. Given $x^2+y^2=9$. Solving for y we have: $y^2=9-x^2$ and $y=\pm\sqrt{9-x^2}$.

This equation does not define y as a function of x. For example, when $x=0$, $y=\pm 3$.

60. Given $\sqrt{x}-y^3=0.$. Solving for y we have: $y^3=\sqrt{x}$ and $y=x^{1/6}$.

This equation specifies a function. The domain is all nonnegative real numbers, i.e., $x \geq 0$.

62. $f(-3x)=(-3x)^2-4=9x^2-4$

64. $f(x-1)=(x-1)^2-4=x^2-2x+1-4=x^2-2x-3$

66. $f(x^3)=(x^3)^2-4=x^6-4$

68. $f(\sqrt[4]{x})=\left(x^{1/4}\right)^2-4=x^{1/2}-4=\sqrt{x}-4$

70. $f(-3)+f(h)=(-3)^2-4+h^2-4=5+h^2-4=h^2+1$

72. $f(-3+h)=(-3+h)^2-4=9-6h+h^2-4=5-6h+h^2$

74. $f(-3+h)-f(-3)=\left[(-3+h)^2-4\right]-\left[(-3)^2-4\right]=(9-6h+h^2-4)-(9-4)=-6h+h^2$

76. (A) $f(x+h)=-3(x+h)+9=-3x-3h+9$

(B) $f(x+h)-f(x)=(-3x-3h+9)-(-3x+9)=-3h$

(C) $\frac{f(x+h)-f(x)}{h}=\frac{-3h}{h}=-3$

78. (A)
$$\begin{aligned} f(x+h)&=3(x+h)^2+5(x+h)-8\\ &=3(x^2+2xh+h^2)+5x+5h-8\\ &=3x^2+6xh+3h^2+5x+5h-8 \end{aligned}$$

(B)
$$\begin{aligned} f(x+h)-f(x)&=\left(3x^2+6xh+3h^2+5x+5h-8\right)-\left(3x^2+5x-8\right)\\ &=6xh+3h^2+5h \end{aligned}$$

(C) $\frac{f(x+h)-f(x)}{h}=\frac{6xh+3h^2+5h}{h}=6x+3h+5$

80. (A) $f(x+h)=x^2+2xh+h^2+40x+40h$

(B) $f(x+h)-f(x)=2xh+h^2+40h$

(C) $\frac{f(x+h)-f(x)}{h}=2x+h+40$

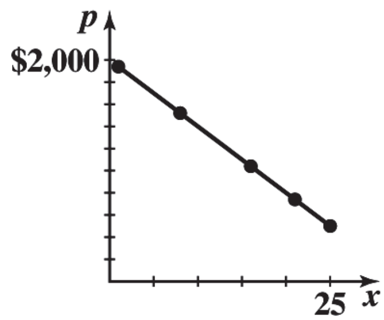
82. Given $A = l\,w = 81$.

Thus, $w = \frac{81}{l}$. Now $P = 2l + 2w = 2l + 2\frac{81}{l} = 2l + \frac{162}{l}$.

The domain is $l > 0$.

84. Given $P = 2\ell + 2w = 160$ or $\ell + w = 80$ and $\ell = 80 - w$.
Now $A = \ell w = (80 - w)w$ and $A = 80w - w^2$.
The domain is $0 \le w \le 80$. [Note: $w \le 80$ since $w > 80$ implies $\ell < 0$.]

86. (A)



(B) $p(11) = 1{,}340$ dollars per computer
$p(18) = 920$ dollars per computer

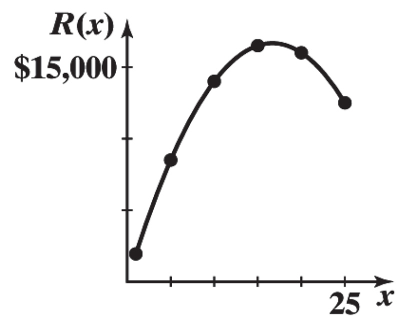
88. (A) $R(x) = xp(x)$
$= x(2{,}000 - 60x)$ thousands of dollars

Domain: $1 \le x \le 25$

(B) Table 11 Revenue

x(thousands)	$R(x)$(thousands)
1	\$1,940
5	8,500
10	14,000
15	16,500
20	16,000
25	12,500

(C)



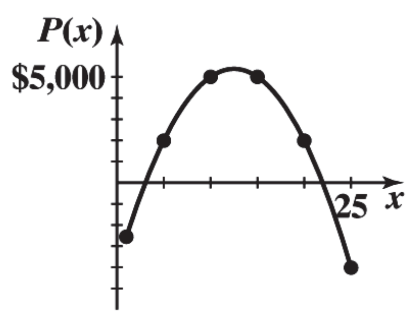
90. (A) $P(x) = R(x) - C(x)$
$= x(2{,}000 - 60x) - (4{,}000 + 500x)$ thousand dollars
$= 1{,}500x - 60x^2 - 4{,}000$

Domain: $1 \le x \le 25$

(B) Table 13 Profit

x (thousands)	$P(x)$ (thousands)
1	–\$2,560
5	2,000
10	5,000
15	5,000
20	2,000
25	–4,000

(C)



92. (A) Given $5v - 2s = 1.4$. Solving for v, we have:
$v = 0.4s + 0.28$.
If $s = 0.51$, then $v = 0.4(0.51) + 0.28 = 0.484$ or 48.4%.

(B) Solving the equation for s, we have:
$s = 2.5v - 0.7$.
If $v = 0.51$, then $s = 2.5(0.51) - 0.7 = 0.575$ or 57.5%.

EXERCISE 1-2

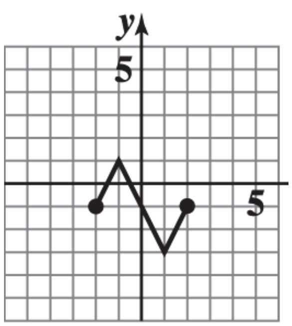
2. $f(x) = 1 + \sqrt{x}$ Domain: $[0, \infty)$; range: $[1, \infty)$.

4. $f(x) = x^2 + 10$ Domain: all real numbers; range: $[10, \infty)$.

6. $f(x) = 5x + 3$ Domain: all real numbers; range: all real numbers.

8. $f(x) = 15 - 20|x|$ Domain: all real numbers; range: $(-\infty, 15]$.

10. $f(x) = -8 + \sqrt[3]{x}$ Domain: all real numbers; range: all real numbers.

12.


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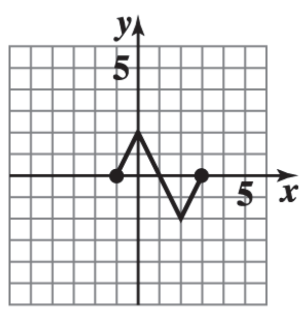

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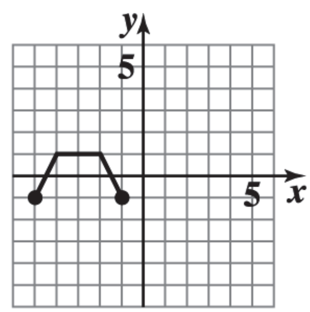

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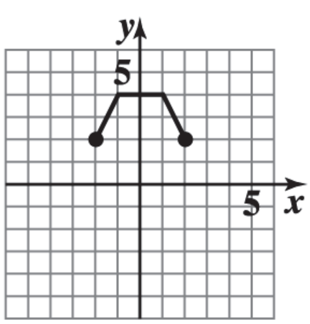

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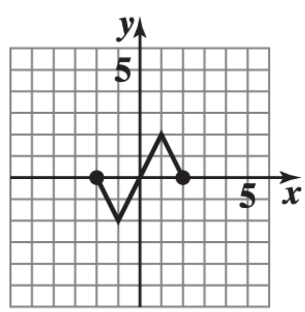

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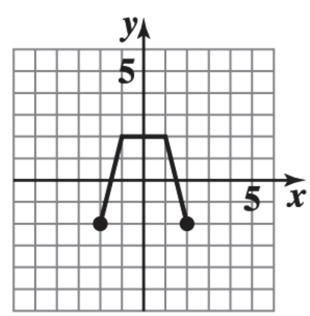

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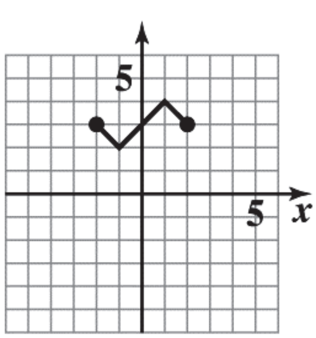

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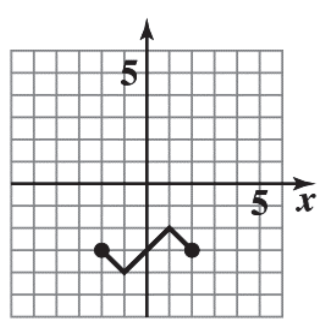
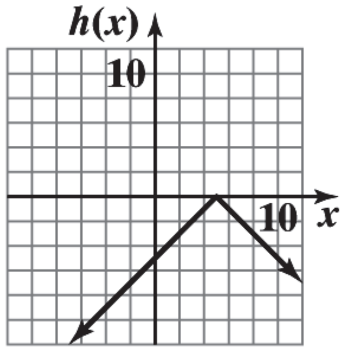
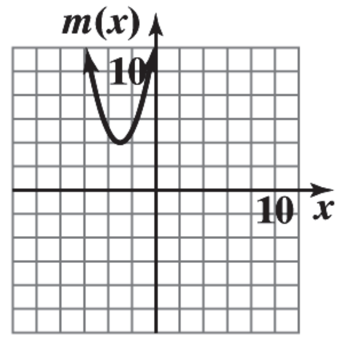


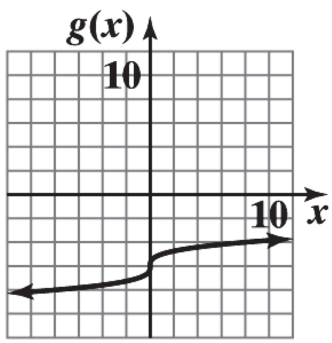
28. The graph of $h(x) = -|x - 5|$ is the graph of $y = |x|$ reflected in the x axis and shifted 5 units to the right.



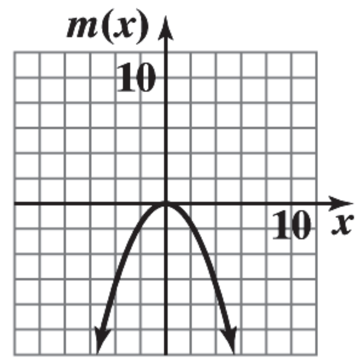
30. The graph of $m(x) = (x + 3)^2 + 4$ is the graph of $y = x^2$ shifted 3 units to the left and 4 units up.



32. The graph of $g(x) = -6 + \sqrt[3]{x}$ is the graph of $y = \sqrt[3]{x}$ shifted 6 units down.



34. The graph of $m(x) = -0.4x^2$ is the graph of $y = x^2$ reflected in the x axis and vertically contracted by a factor of 0.4.



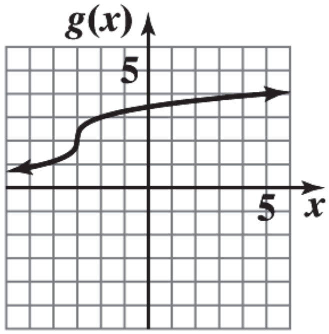
36. The graph of the basic function $y = |x|$ is shifted 3 units to the right and 2 units up. $y = |x - 3| + 2$

38. The graph of the basic function $y = |x|$ is reflected in the x axis, shifted 2 units to the left and 3 units up. Equation: $y = 3 - |x + 2|$

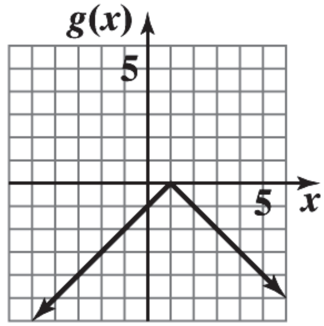
40. The graph of the basic function $\sqrt[3]{x}$ is reflected in the x axis and shifted up 2 units. Equation: $y = 2 - \sqrt[3]{x}$

42. The graph of the basic function $y = x^3$ is reflected in the x axis, shifted to the right 3 units and up 1 unit. Equation: $y = 1 - (x - 3)^3$

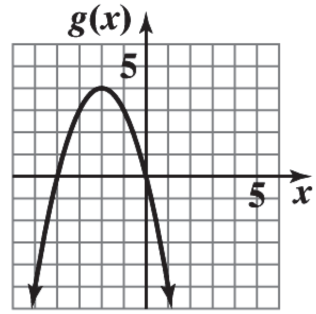
44. $g(x) = \sqrt[3]{x+3} + 2$



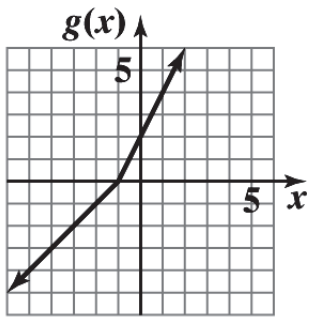
46. $g(x) = -|x - 1|$



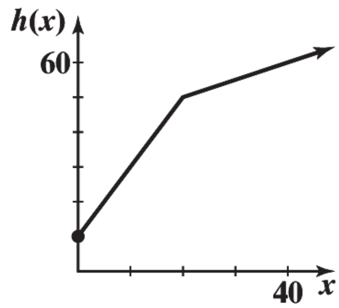
48. $g(x) = 4 - (x + 2)^2$



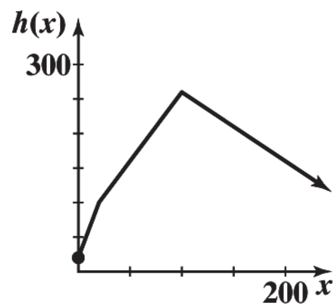
50. $g(x) = \begin{cases} x+1 & \text{if } x < -1 \\ 2+2x & \text{if } x \geq -1 \end{cases}$



52. $h(x) = \begin{cases} 10+2x & \text{if} \quad 0 \leq x \leq 20 \\ 40+0.5x & \text{if} \quad x > 20 \end{cases}$



54. $h(x) = \begin{cases} 4x+20 & \text{if} \quad 0 \leq x \leq 20 \\ 2x+60 & \text{if} \quad 20 < x \leq 100 \\ -x+360 & \text{if} \quad x > 100 \end{cases}$



56. The graph of the basic function $y = x$ is reflected in the x axis and vertically expanded by a factor of 2. Equation: $y = -2x$

58. The graph of the basic function $y = |x|$ is vertically expanded by a factor of 4. Equation: $y = 4|x|$

60. The graph of the basic function $y = x^3$ is vertically contracted by a factor of 0.25. Equation: $y = 0.25x^3$.

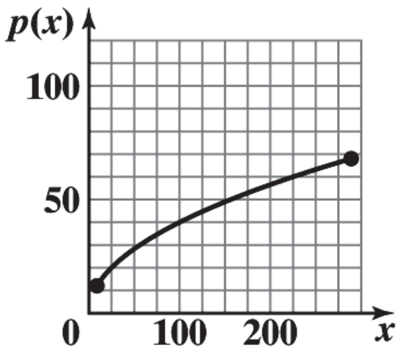
62. Vertical shift, reflection in y axis.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A vertical shift of k units followed by a reflection in y axis moves (a, b) to $(a, b + k)$ and then to $(-a, b + k)$. In the reverse order, a reflection in y axis followed by a vertical shift of k units moves (a, b) to $(-a, b)$ and then to $(-a, b + k)$. The results are the same.

64. Vertical shift, vertical expansion.
Reversing the order can change the result. For example, let (a, b) be a point in the plane. A vertical shift of k units followed by a vertical expansion of h ($h > 1$) moves (a, b) to $(a, b + k)$ and then to $(a, bh + kh)$. In the reverse order, a vertical expansion of h followed by a vertical shift of k units moves (a, b) to (a, bh) and then to $(a, bh + k)$; $(a, bh + kh) \neq (a, bh + k)$.

66. Horizontal shift, vertical contraction.
Reversing the order does not change the result. Consider a point (a, b) in the plane. A horizontal shift of k units followed by a vertical contraction of h $(0 < h < 1)$ moves (a, b) to $(a + k, b)$ and then to $(a + k, bh)$. In the reverse order, a vertical contraction of h followed by a horizontal shift of k units moves (a, b) to (a, bh) and then to $(a + k, bh)$. The results are the same.

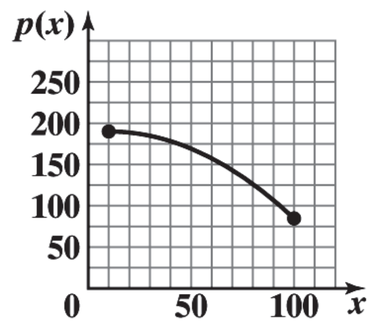
68. (A) The graph of the basic function $y = \sqrt{x}$ is vertically expanded by a factor of 4.

(B)



70. (A) The graph of the basic function $y = x^2$ is reflected in the x axis, vertically contracted by a factor of 0.013, and shifted 10 units to the right and 190 units up.

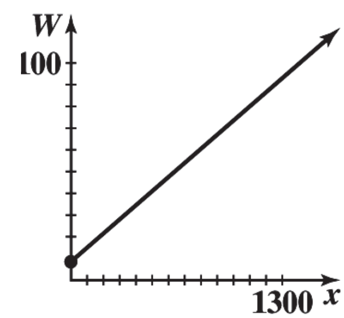
(B)



72. (A) Let x = number of kwh used in a winter month. For $0 \le x \le 700$, the charge is $8.5 + .065x$. At $x = 700$, the charge is \$54. For $x > 700$, the charge is
$54 + .053(x - 700) = 16.9 + 0.053x$.
Thus,

$$W(x) = \begin{cases} 8.5 + .065x & \text{if } 0 \le x \le 700 \\ 16.9 + 0.053x & \text{if } x > 700 \end{cases}$$

(B)

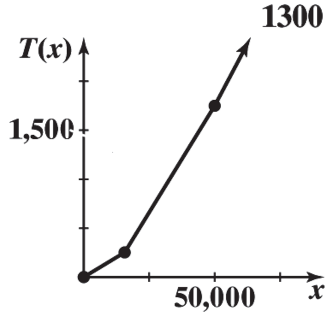


74. (A) Let x = taxable income.
If $0 \le x \le 12{,}500$, the tax due is \$$.02x$. At $x = 12{,}500$, the tax due is \$250. For $12{,}500 < x \le 50{,}000$, the tax due is $250 + .04(x - 12{,}500) = .04x - 250$. For $x > 50{,}000$, the tax due is $1{,}250 + .06(x - 50{,}000) = .06x - 1{,}250$.

Thus,

$$T(x) = \begin{cases} 0.02x & \text{if } 0 \le x \le 12{,}500 \\ 0.04x - 250 & \text{if } 12{,}500 < x \le 50{,}000 \\ 0.06x - 1{,}250 & \text{if } x > 50{,}000 \end{cases}$$

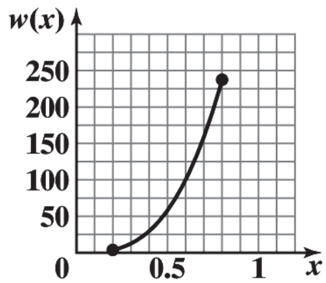
(B)



(C) $T(32{,}000) = \$1{,}030$
$T(64{,}000) = \$2{,}590$

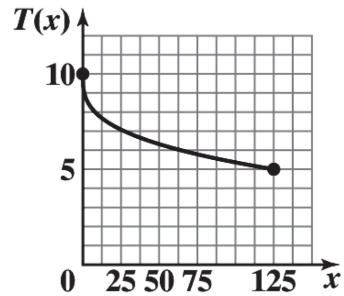
76. (A) The graph of the basic function $y = x^3$ is vertically expanded by a factor of 463.

(B)



78. (A) The graph of the basic function $y = \sqrt[3]{x}$ is reflected in the x axis and shifted up 10 units.

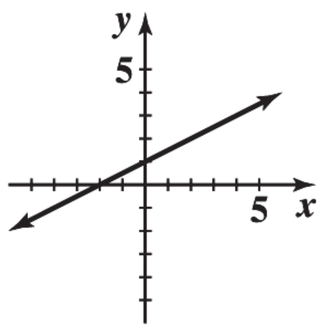
(B)



EXERCISE 1-3

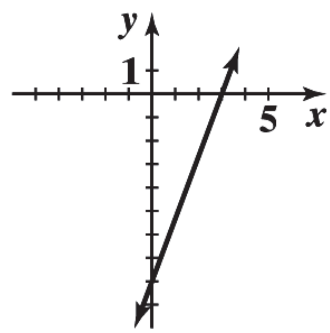
2. $y = \dfrac{x}{2} + 1$

x	y
0	1
2	2
4	3



4. $8x - 3y = 24$

x	y
0	−8
3	0
6	8



6. Slope: $m = 3$

y- intercept: $b = 2$

8. Slope: $m = -\dfrac{10}{3}$

y- intercept: $b = 4$

10. Slope: $m = -4$; x-intercept: 3

12. Slope: $m = -3$; x-intercept: 2

14. Slope: $m = -9/2$; x-intercept: 4/9

16. $y = x + 5$

18. $m = \dfrac{6}{7}$, $b = -\dfrac{9}{2}$; $y = \dfrac{6}{7}x - \dfrac{9}{2}$

20. x- intercept: 1; y- intercept: 3; $y = -3x + 3$

22. x- intercept: 2, y- intercept: -1; $y = \dfrac{1}{2}x - 1$

24. (A) g (B) m (C) n (D) f

26. (A) x-intercepts: -5, -1; y- intercept: -5 (B) Vertex: $(-3, 4)$
(C) Maximum: 4 (D) Range: $y \le 4$ or $(-\infty, 4]$

28. (A) x- intercepts: 1, 5; y- intercept: 5 (B) Vertex: $(3, -4)$
(C) Minimum: -4 (D) Range: $y \ge -4$ or $[-4, \infty)$

30. $g(x) = -(x+2)^2 + 3$

(A) x- intercepts: $-(x+2)^2 + 3 = 0$

$$(x+2)^2 = 3$$
$$x + 2 = \pm\sqrt{3}$$
$$x = -2 - \sqrt{3},\ -2 + \sqrt{3}$$

y- intercept: -1

(B) Vertex: $(-2, 3)$ (C) Maximum: 3 (D) Range: $y \le 3$ or $(-\infty, 3]$

32. $n(x) = (x - 4)^2 - 3$

(A) x- intercepts: $(x-4)^2 - 3 = 0$

$$(x-4)^2 = 3$$
$$x - 4 = \pm\sqrt{3}$$
$$x = 4 - \sqrt{3},\ 4 + \sqrt{3}$$

y- intercept: 13

(B) Vertex: $(4, -3)$ (C) Minimum: -3 (D) Range: $y \ge -3$ or $[-3, \infty)$

34. (A) Slope: $m = \dfrac{5-2}{3-1} = \dfrac{3}{2}$. (B) Point-slope form: $y - 2 = \dfrac{3}{2}(x-1)$.

(C) Slope-intercept form: $y = \dfrac{3}{2}x + \dfrac{1}{2}$. (D) Standard form: $3x - 2y = -1$.

36. (A) Slope: $m = \dfrac{7-3}{-3-2} = -\dfrac{4}{5}$. (B) Point-slope form: $y - 3 = -\dfrac{4}{5}(x-2)$.

(C) Slope-intercept form: $y = -\dfrac{4}{5}x + \dfrac{23}{5}$. (D) Standard form: $4x + 5y = 23$.

38. (A) Slope: $m = \dfrac{4-4}{0-1} = 0$. (B) Point-slope form: $y - 4 = 0$.

(C) Slope-intercept form: $y = 4$. (D) Standard form: $y = 4$.

40. (A) Slope: $m = \dfrac{-3}{0}$ not defined (B) Point-slope form: none.

(C) Slope-intercept form: none. (D) Standard form: $x = 2$.

42. $g(x) = x^2 - 6x + 5 = x^2 - 6x + 9 - 4 = (x-3)^2 - 4$

(A) x- intercepts: $(x-3)^2 - 4 = 0$

$$(x-3)^2 = 4$$
$$x - 3 = \pm 2$$
$$x = 1, 5$$

y- intercept: 5

(B) Vertex: $(3, -4)$ (C) Minimum: -4 (D) Range: $y \ge -4$ or $[-4, \infty)$

44. $s(x) = -4x^2 - 8x - 3 = -4\left[x^2 + 2x + \dfrac{3}{4}\right] = -4\left[x^2 + 2x + 1 - \dfrac{1}{4}\right] = -4\left[(x+1)^2 - \dfrac{1}{4}\right]$

$= -4(x+1)^2 + 1$

(A) x- intercepts: $-4(x+1)^2 + 1 = 0$

$$4(x+1)^2 = 1$$
$$(x+1)^2 = \frac{1}{4}$$
$$x + 1 = \pm\frac{1}{2}$$
$$x = -\frac{3}{2}, -\frac{1}{2}$$

y- intercept: -3

(B) Vertex: $(-1, 1)$ (C) Maximum: 1 (D) Range: $y \le 1$ or $(-\infty, 1]$

46. $v(x) = 0.5x^2 + 4x + 10 = 0.5[x^2 + 8x + 20] = 0.5[x^2 + 8x + 16 + 4] = 0.5[(x+4)^2 + 4] = 0.5(x+4)^2 + 2$

(A) x- intercepts: none
y- intercept: 10

(B) Vertex: $(-4, 2)$ (C) Minimum: 2 (D) Range: $y \ge 2$ or $[2, \infty)$

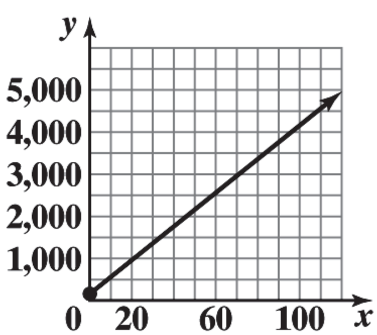
48. $9x + 54 > 0, \quad x > -\frac{54}{6} = -6;$ solution set: $(-6, \infty)$

50. $-4x + 44 \le 0, \quad -4x \le -44, \quad x \ge 11;$ solution set: $[11, \infty)$

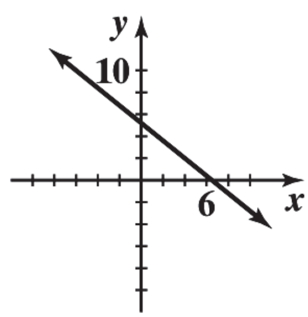
52. $(x+6)(x-3) < 0$
Therefore, either $(x+6) < 0$ and $(x-3) > 0$ or $(x+6) > 0$ and $(x-3) < 0$. The first case is impossible. The second case implies $-6 < x < 3$. Solution set: $(-6, 3)$.

54. $x^2 + 7x + 12 = (x+3)(x+4) \ge 0$
Therefore, either $(x+3) \ge 0$ and $(x+4) \ge 0$ or $(x+3) \le 0$ and $(x+4) \le 0$. The first case implies $x \ge -3$ and the second case implies $x \le -4$. Solution set: $(-\infty, -4] \cup [-3, \infty)$.

56.



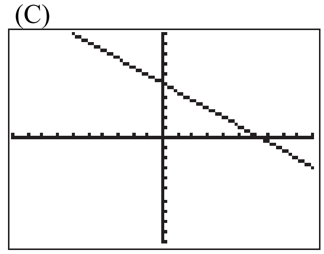
58. (A)



(B) Set $f(x) = 0$,

$-0.8x + 5.2 = 0$, $x = 6.5$.
Set $x = 0$, $y = 5.2$.

(C)



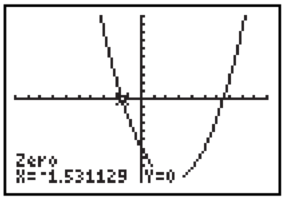
(D) x-intercept: $x = 6.5$,
y-intercept: $y = 5.2$

(E) $-0.8x + 5.2 < 0$
$x > 6.5$

60. $g(x) = -0.6x^2 + 3x + 4$

(A) $g(x) = -2$: $-0.6x^2 + 3x + 4 = -2$

$0.6x^2 - 3x - 6 = 0$

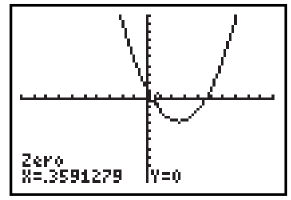


$x = -1.53, 6.53$

(B) $g(x) = 5$: $-0.6x^2 + 3x + 4 = 5$

$-0.6x^2 + 3x - 1 = 0$

$0.6x^2 - 3x + 1 = 0$

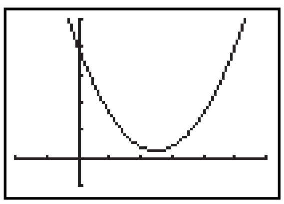


$x = 0.36, 4.64$

(C) $g(x) = 8$: $-0.6x^2 + 3x + 4 = 8$

$-0.6x^2 + 3x - 4 = 0$

$0.6x^2 - 3x + 4 = 0$



No solution

62. Using a graphing utility with $y = 100x - 7x^2 - 10$ and the calculus option with maximum command, we obtain 347.1429 as the maximum value.

64. The slope of the line through (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$; the slope of the line through (x_1, y_1) and (x_3, y_3) is $\frac{y_3 - y_1}{x_3 - x_1}$. By Problem 57, $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-A}{B} = \frac{y_3 - y_1}{x_3 - x_1}$.

66. We have two representations of (d, P): (0, 14.7) and (34, 29.4).

(A) A line relating P to d passes through the above two points.
Its equation is:

$$P - 14.7 = \frac{(29.4 - 14.7)}{(34 - 0)}(d - 0)$$

or $P \approx 0.432d + 14.7$

(B) The rate of change of pressure with respect to depth is approximately 0.432 lbs/in^2 per foot.

(C) For $d = 50$,

$P = 0.432(50) + 14.7 \approx 36.3$ lbs/in^2

(D) For $P = 4$ atmospheres, we have $P = 4(14.7) = 58.8$ lbs/in^2 and hence

$$58.8 = 0.432d + 14.7$$

or $$d = \frac{58.8 - 14.7}{0.432} \approx 102 \text{ ft.}$$

68. We have two representations of (t, a): (0, 2,880) and (180, 0).

(A) The linear model relating altitude a to the time in air t has the following equation:

$$a - 2{,}880 = \frac{(0 - 2{,}880)}{(180 - 0)}(t - 0)$$

or $$a = -16t + 2{,}880$$

(B) The rate of descent for an ATPS system parachute is 16 ft/sec.

(C) It is 16 ft/sec.

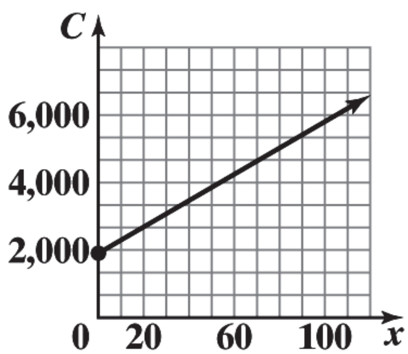
70. Let y be daily cost of producing x tennis rackets. Then we have two points for (x, y): (50, 3,855) and (60, 4,245).

(A) Since x and y are linearly related, then the two points (50, 3,855) and (60, 4,245) will lie on the line expressing the linear relationship between x and y. Therefore

$$y - 3{,}855 = \frac{(4{,}245 - 3{,}855)}{(60 - 50)}(x - 50)$$

or $$y = 39x + 1{,}905$$

(B)



(C) The y intercept, \$1,905, is the fixed cost and the slope, \$39, is the cost per racket.

72. We observe that for (t, V) two points are given: (0, 224,000) and (16, 115,200)

(A) A linear model will be a line passing through the two points (0, 224,000) and (16, 115,200). The equation of this line is:

$$V - 115{,}200 = \frac{(224{,}000 - 115{,}200)}{(0 - 16)}(t - 16) \text{ or}$$

$$V = -6{,}800t + 224{,}000$$

(B) For $t = 10$

$$V = -6{,}800(10) + 224{,}000 = \$156{,}000$$

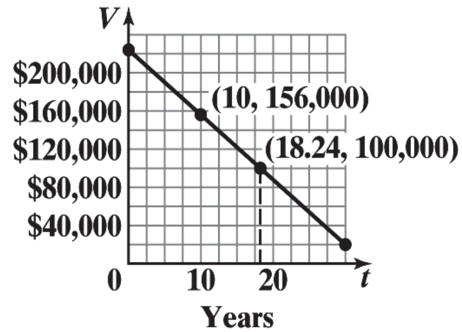
(C) For $V = \$100{,}000$

$100{,}000 = -6{,}800t + 224{,}000$

or $t = \dfrac{(224{,}000 - 100{,}000)}{6{,}800} \approx 18.24$

So, during the 19th year, the depreciated value falls below \$100,000.

(D)



74. Let T be the true airspeed at the altitude A (thousands of feet). Since T increases 1.6%, $T(1000) = 200(1.016) = 203.2.$

(A) A linear relationship between A and T has slope

$m = \dfrac{(203.2 - 200)}{1} = 3.2$. Therefore, $T = 3.2A + 200.$

(B) For $A = 6.5$ (6,500 feet), $T = 3.2(6.5) + 200 = 20.8+200 = 220.8$ mph

76. (A) For (x, p) we have two representations: (9,800, 1.94) and (9,400, 1.82).
The slope is

$$m = \frac{(1.94 - 1.82)}{(9{,}800 - 9{,}400)} = 0.0003$$

Using one of the points, say (9,800, 1.94), we find b:

$1.94 = (0.0003)(9{,}800) + b$

or $b = -1$

So, the desired equation is: $p = 0.0003x - 1$.

(B) Here the two representations of (x, p) are: (9,300, 1.94) and (9,500, 1.82). The slope is

$$m = \frac{(1.94 - 1.82)}{(9{,}300 - 9{,}500)} = -0.0006$$

Using one of the points, say (9,300, 1.94) we find b:

$1.94 = -0.0006(9{,}300) + b$

or $b = 7.52$

So, the desired equation is: $p = -0.0006x + 7.52$.

(C) To find the equilibrium point, we need to solve $0.0003x - 1 = -0.0006x + 7.52$ for x. Observe that

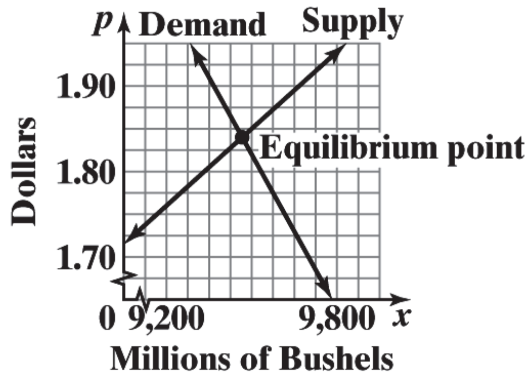
$0.0009x = 8.52$ or

$$x = \frac{8.52}{0.0009} = 9{,}467$$

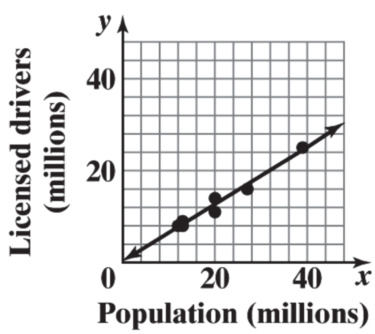
Substituting $x = 9{,}467$ in either of equations in (A) or (B) we obtain $p = .0003(9{,}467) - 1 \approx 1.84$

So, the desired point is (9,467, 1.84).

(D)



78.



(B) At $x = 9.9$, $y = 0.62(9.9) + 0.29 = 6.428$. There were approximately 6,428,000 licensed drivers in Michigan in 2014.

(C) Solve $0.62x + 0.29 = 6.7$ for x:

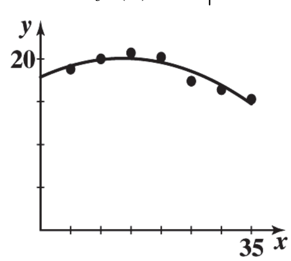
$0.62x = 6.771, \quad x \approx 10.339$

The population of Georgia in 2014 was approximately 10,339,000.

80. Mathematical model: $f(x) = -0.0117x^2 + 0.32x + 17.9$

(A)

x	5	10	15	20	25	30	35
Market Share	18.8	20.0	20.7	20.2	17.4	16.4	15.3
$f(x)$	19.2	19.9	20.0	19.6	18.5	16.9	14.8



(B)

(C) For 2025, $x = 45$ and

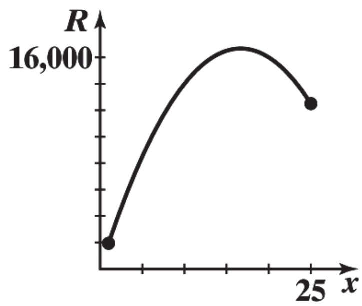
$f(45) = -0.0117(45)^2 + 0.32(45) + 17.9 \approx 8.6$, i.e., 8.6%

For 2028, $x = 48$ and $f(48) = -0.0117(48)^2 + 0.32(48) + 17.9 \approx 6.3$, i.e., 6.3%

(D) Ford's market share rose from 18.8% in 1985 to a high of 20.7% in 1995. After that, Ford's market share decreased each year, reaching a low of 15.3%

82. Verify

84. (A)



(B) $R(x) = 2{,}000x - 60x^2$

$$= -60\left(x^2 - \frac{100}{3}x\right)$$

$$= -60\left[x^2 - \frac{100}{3}x + \frac{2500}{9} - \frac{2500}{9}\right]$$

$$= -60\left[\left(x - \frac{50}{3}\right)^2 - \frac{2500}{9}\right]$$

$$= -60\left(x - \frac{50}{3}\right)^2 + \frac{50{,}000}{3}$$

16.667 thousand computers(16,667 computers);
16,666.667 thousand dollars ($16,666,667)

(C) $1,000

86. (A) The rate of change of height with respect to Dbh is 1.66 ft/in.

(B) One inch increase in Dbh produces a height increase of approximately 1.66 ft.

(C) For $x = 12$, we have: $y = 1.66(12) - 5.14 \approx 15$ ft.

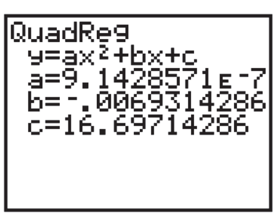
(D) For $y = 25$, we have:

$25 = 1.66x - 5.14$ or $x = \dfrac{25 + 5.14}{1.66} \approx 18$ in.

88. Men: $y = -0.247x + 119.097$
Women: $y = -0.122x + 128.494$

The graphs of these lines indicate that the women will not catch up with the men. To see this algebraically, if we set the equations equal to each other and solve, then we obtain $x = -75.18$, so the lines intersect at a point outside of the domain of our functions. Also, the men's slope is steeper so their times, already lower, are decreasing more rapidly.

90.



For $x = 2{,}300$, the estimated fuel consumption is
$y = a(2{,}300)^2 + b(2{,}300) + c = 5.6$ mpg.

EXERCISE 1-4

2. $f(x) = x^2 - 5x + 6$

(A) Degree: 2

(B) $x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 30$

$x = 2, 3$

x-intercepts: $x = 2, 3$

(C) $f(0) = 0^2 - 5(0) + 6 = 6$
y-intercept: 6

4. $f(x) = 30 - 3x$
(A) Degree: 1
(B)
$$30 - 3x = 0$$
$$3x = 30$$
$$x = 10$$
x-intercept: 10
(C) $f(0) = 30 - 3(0) = 30$
y-intercept: 30

6. $f(x) = 5x^6 + x^4 + x^8 + 10$
(A) Degree: 8
(B) $f(x) \geq 10$ for all x.
No x-intercepts.
(C) $f(0) = 5(0)^6 + (0)^4 + (0)^8 + 10 = 10$
y-intercept: 10

8. $f(x) = (x-5)^2(x+7)^2$
(A) Degree: 4
(B) $(x-5)^2(x+7)^2 = 0$
$x = 5, -7$
x-intercepts: $x = 5, -7$
(C) $f(0) = (0-5)^2(0+7)^2 = (25)(49) = 1,225$
y-intercept: 1,225

10. $f(x) = (2x-5)^2(x^2-9)^4$

(A) Degree: 10

(B) $(2x-5)^2\left(x^2-9\right)^4 = 0$
$x = \frac{5}{2}, -3, 3 \quad x = -3, \frac{1}{2}$
x-intercepts: $-3, 5/2, 3$

(C) $f(0) = [2(0)-5]^2[(0)^2-9)^4 = 5^2 9^4 = 164,025$
y-intercept: 164,025

12. (A) Minimum degree: 2
(B) Negative – it must have even degree, and positive values in the domain are mapped to negative values in the range.

14. (A) Minimum degree: 3

(B) Negative – it must have odd degree, and positive values in the domain are mapped to negative values in the range.

16. (A) Minimum degree: 4

(B) Positive – it must have even degree, and positive values in the domain are mapped to positive values in the range.

18. (A) Minimum degree: 5

(B) Positive – it must have odd degree, and large positive values in the domain are mapped to positive values in the range.

20. A polynomial of degree 7 can have at most 7 x intercepts.

22. A polynomial of degree 6 may have no x intercepts. For example, the polynomial $f(x) = x^6 + 1$ has no x intercepts.

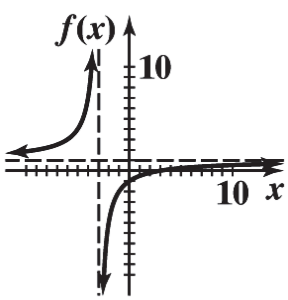
24. (A) Intercepts:

x-intercept(s):	*y*-intercept:
$x - 3 = 0$ $x = 3$ (3, 0)	$f(0) = \dfrac{0-3}{0+3} = -1$ (0, –1)

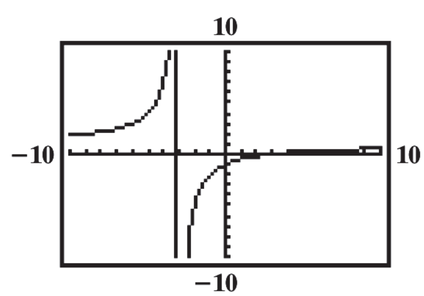
(B) Domain: all real numbers except $x = -3$

(C) Vertical asymptote at $x = -3$ by case 1 of the vertical asymptote procedure on page 57.
Horizontal asymptote at $y = 1$ by case 2 of the horizontal asymptote procedure on page 57.

(D)



(E)



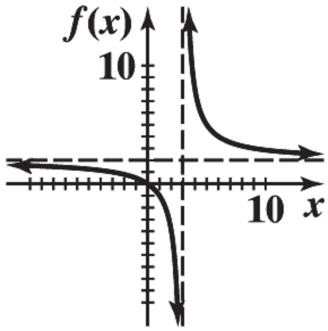
26. (A) Intercepts:

x-intercept(s):	*y*-intercept:
$2x = 0$ $x = 0$ (0, 0)	$f(0) = \dfrac{2(0)}{0-3} = 0$ (0, 0)

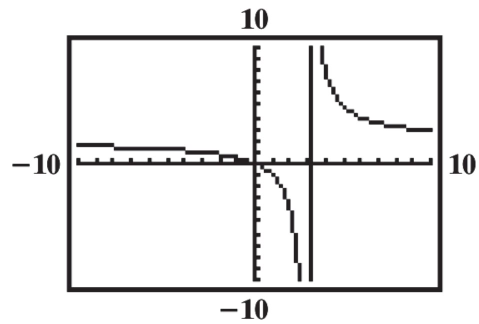
(B) Domain: all real numbers except $x = 3$.

(C) Vertical asymptote at $x = 3$ by case 1 of the vertical asymptote procedure on page 57.
Horizontal asymptote at $y = 2$ by case 2 of the horizontal asymptote procedure on page 57.

(D)



(E)



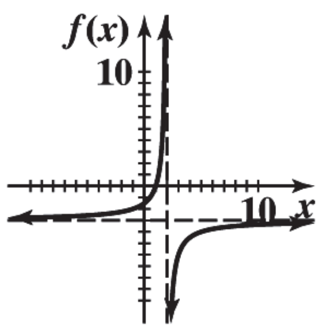
28. (A) Intercepts:

x-intercept:	y-intercept:
$3-3x=0$ $x=1$ $(1, 0)$	$f(0)=\dfrac{3-3(0)}{0-2}=-\dfrac{3}{2}$ $\left(0,-\dfrac{3}{2}\right)$

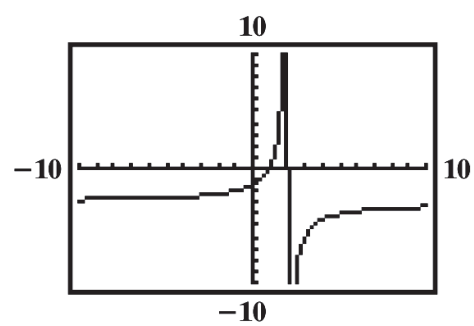
(B) Domain: all real numbers except $x = 2$

(C) Vertical asymptote at $x = 2$ by case 1 of the vertical asymptote procedure on page 57.
Horizontal asymptote at $y = -3$ by case 2 of the horizontal asymptote procedure on page 57.

(D)

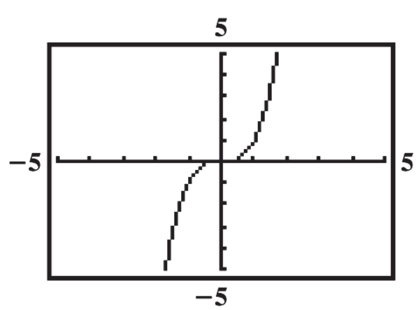


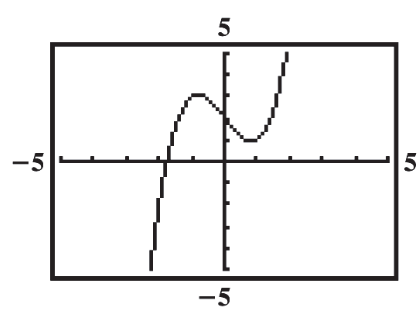
(E)



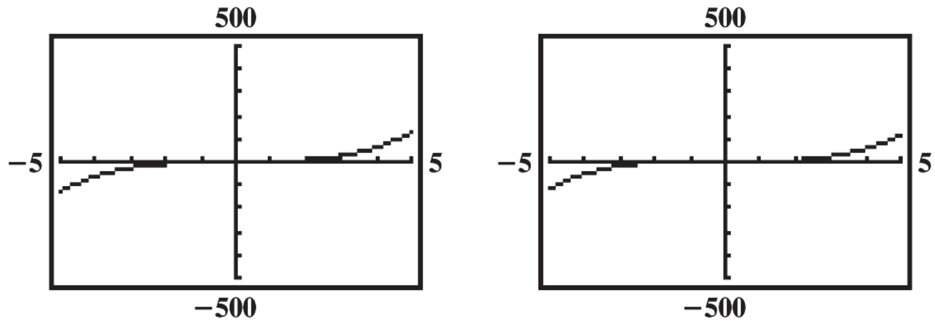
30.

(A)



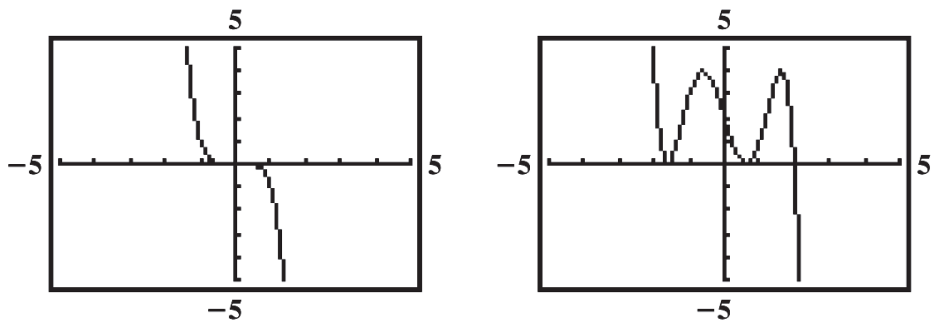


(B)

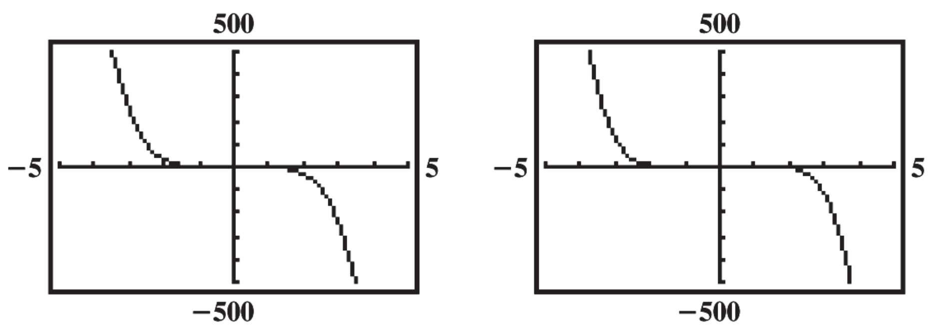


32.

(A)



(B)



34. $y = \frac{6}{4}$, by case 2 for horizontal asymptotes on page 57.

36. $y = -\frac{1}{2}$, by case 2 for horizontal asymptotes on page 57.

38. $y = 0$, by case 1 for horizontal asymptotes on page 57.

40. No horizontal asymptote, by case 3 for horizontal asymptotes on page 57.

42. Here we have denominator $(x^2 - 4)(x^2 - 16) = (x-2)(x+2)(x-4)(x+4)$. Since none of these linear terms are factors of the numerator, the function has vertical asymptotes at $x = 2$, $x = -2$, $x = 4$, and $x = -4$.

44. Here we have denominator $x^2 + 7x - 8 = (x-1)(x+8)$. Also, we have numerator $x^2 - 8x + 7 = (x-1)(x-7)$. By case 2 of the vertical asymptote procedure on page 57, we conclude that the function has a vertical asymptote at $x = -8$.

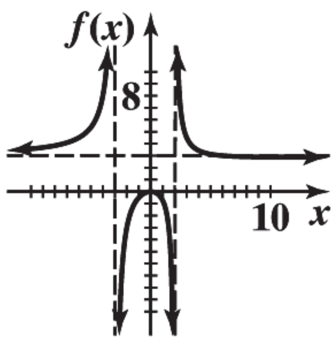
46. Here we have denominator $x^3-3x^2+2x=x(x^2-3x+2)=x(x-2)(x-1)$. We also have numerator $x^2+x-2=(x+2)(x-1)$. By case 2 of the vertical asymptote procedure on page 57, we conclude that the function has vertical asymptotes at $x=0$ and $x=2$.

48. (A) Intercepts:

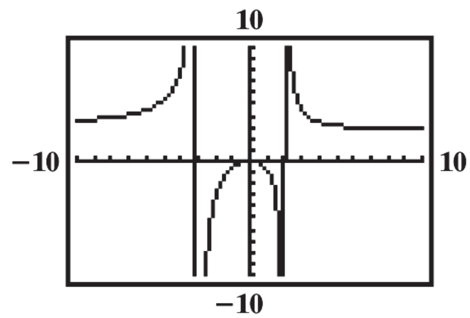
x-intercept(s):	y-intercept:
$3x^2=0$ $x=0$ (0, 0)	$f(0)=0$ (0, 0)

(B) Vertical asymptote when $x^2+x-6=(x-2)(x+3)=0$; so, vertical asymptotes at $x = 2$, $x = –3$. Horizontal asymptote $y=3$.

(C)



(D)

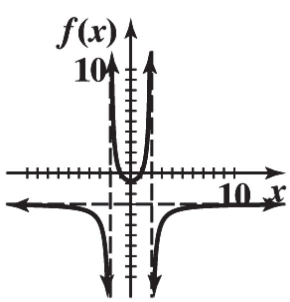


50. (A) Intercepts:

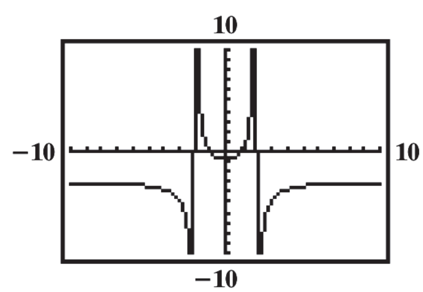
x-intercept(s):	y-intercept:
$3-3x^2=0$ $3x^2=3$ $x=\pm1$ (1,0), (–1,0)	$f(0)=-\frac{3}{4}$ $\left(0,\frac{-3}{4}\right)$

(B) Vertical asymptotes when $x^2-4=0$; i.e. at $x = 2$ and $x = –2$.
Horizontal asymptote at $y=-3$

(C)



(D)

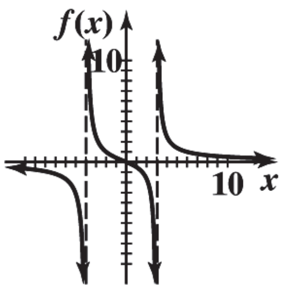


52. (A) Intercepts:

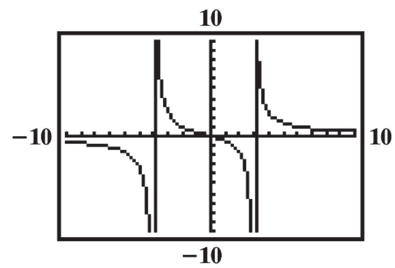
x-intercept(s):	y-intercept:
$5x-10=0$ $x=2$ (2,0)	$f(0)=\frac{-10}{-12}=\frac{5}{6}$ (0,5/6)

(B) Vertical asymptote when $x^2+x-12=(x+4)(x-3)=0$; i.e. when $x=-4$ and when $x=3$. Horizontal asymptote at $y=0$.

(C)



(D)



54. $f(x)=-(x+2)(x-1)=-x^2-x+2$

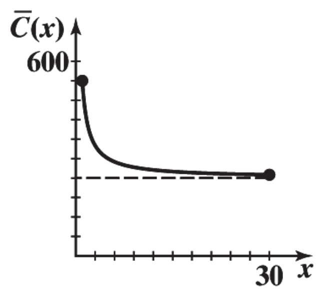
56. $f(x)=x(x+1)(x-1)=x\left(x^2-1\right)=x^3-x$

58. (A) We want $C(x)=mx+b$. Fix costs are $b=\$300$ per day. Given $C(20)=5{,}100$ we have

$$m(20)+300=5{,}100$$
$$20m=4800$$
$$m=240$$
$$C(x)=240x+300$$

(B) $\bar{C}(x)=\frac{C(x)}{x}=\frac{240x+300}{x}=240+\frac{300}{x}$

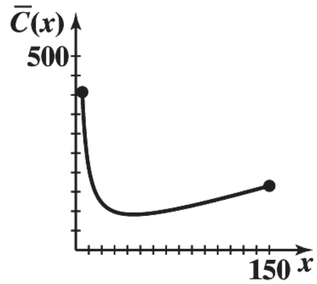
(C)



(D) Average cost tends towards \$240 as production increases.

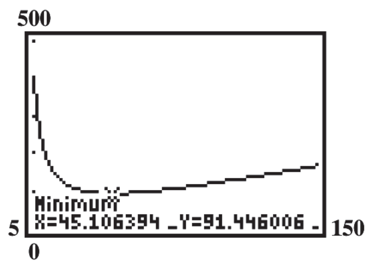
60. (A) $\bar{C}(x)=\frac{x^2+2x+2{,}000}{x}$

(B)



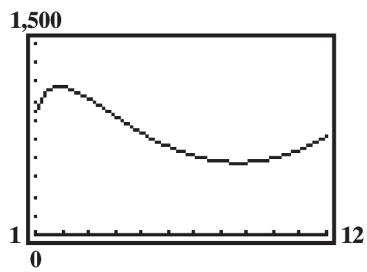
(C) A daily production level of $x = 45$ units per day, results in the lowest average cost of $\overline{C}(45) = \$91.44$ per unit

(D)



62. (A) $\overline{C}(x) = \dfrac{20x^3 - 360x^2 + 2{,}300x - 1{,}000}{x}$

(B)



(C) A minimum average cost of \$566.84 is achieved at a production level of $x = 8.67$ thousand cases per month.

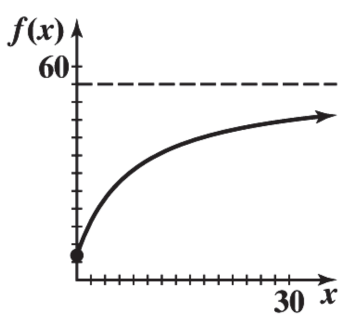
64. (A) Cubic regression model

```
CubicReg
 y=ax³+bx²+cx+d
 a=.0902777778
 b=-1.87202381
 c=10.14484127
 d=241.5714286
```

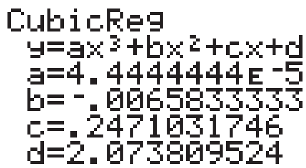
(B) $y(21) = 583$ eggs

66. (A) The horizontal asymptote is $y = 55$.

(B)



68. (A) Cubic regression model



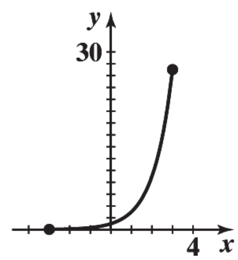
(B) This model gives an estimate of 2.5 divorces per 1,000 marriages.

EXERCISE 1-5

2. A. graph g B. graph f C. graph h D. graph k

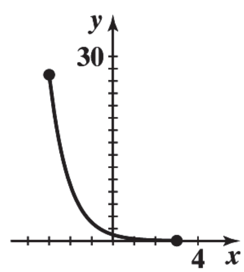
4. $y = 3^x; [-3, 3]$

x	y
–3	$\frac{1}{27}$
–1	$\frac{1}{3}$
0	1
1	3
3	27



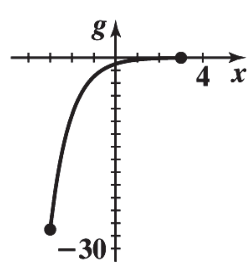
6. $y = 3^{-x}; [-3, 3]$

x	y
–3	27
–1	3
0	1
1	$\frac{1}{3}$
3	$\frac{1}{27}$



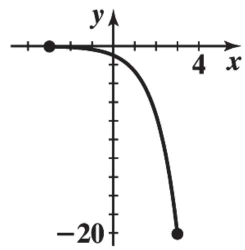
8. $g(x) = -3^{-x}; [-3, 3]$

x	$g(x)$
–3	–27
–1	–3
0	–1
1	$-\frac{1}{3}$
3	$-\frac{1}{27}$



10. $y = -e^x$; [−3, 3]

x	y
−3	≈ −0.05
−1	≈ −0.37
0	−1
1	≈ −2.72
3	≈ −20.09



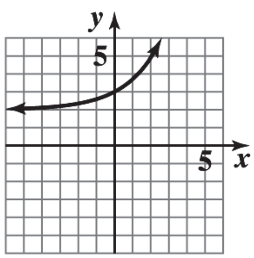
12. The graph of g is the graph of f shifted 2 units to the right.

14. The graph of g is the graph of f reflected in the x axis.

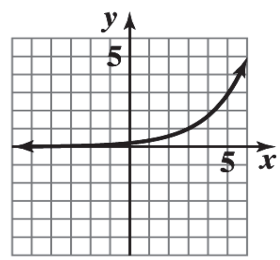
16. The graph of g is the graph of f shifted 2 units down.

18. The graph of g is the graph of f vertically contracted by a factor of 0.5 and shifted 1 unit to the right.

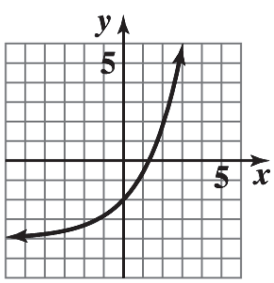
20. (A) $y = f(x) + 2$



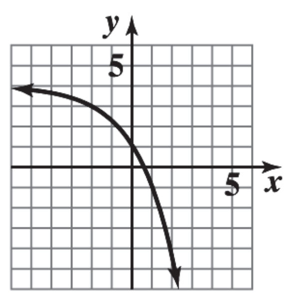
(B) $y = f(x - 3)$



(C) $y = 2f(x) - 4$

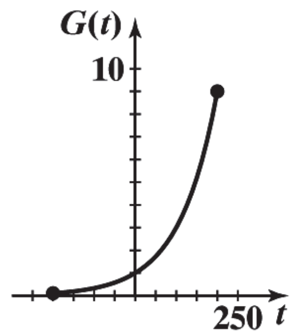


(D) $y = 4 - f(x + 2)$



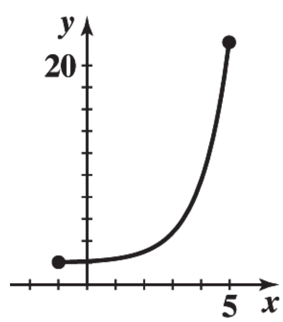
22. $G(t) = 3^{t/100}$; [−200, 200]

x	$G(t)$
−200	$\frac{1}{9}$
−100	$\frac{1}{3}$
0	1
100	3
200	9



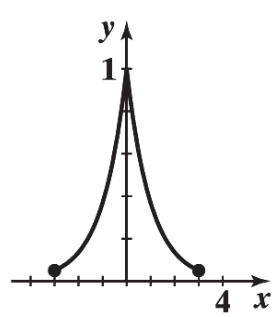
24. $y = 2 + e^{x-2}$; [−1, 5]

x	y
−1	≈ 2.05
0	≈ 2.14
1	≈ 2.37
3	≈ 4.72
5	≈ 22.09



26. $y = e^{-|x|}$; [−3, 3]

x	y
−3	≈ 0.05
−1	≈ 0.37
0	1
1	≈ 0.37
3	≈ 0.05



28. $a = 2$, $b = -2$ for example. The exponential function property: For $x \neq 0$, $a^x = b^x$ if and only if $a = b$ assumes $a > 0$ and $b > 0$.

30.
$$3^{x+4} = 3^{2x-5}$$
$$x + 4 = 2x - 5$$
$$-x = -9$$
$$x = 9$$

32. $5^{x^2-x} = 5^{42}$
$x^2 - x = 42$
$x^2 - x - 42 = 0$
$(x-7)(x+6) = 0$
$x = -6, 7$

34. $(3x+4)^4 = (52)^4$
$3x+4 = 52$
$3x = 48$
$x = 16$

36. $(2x+1)^2 = (3x-1)^2$
$4x^2 + 4x + 1 = 9x^2 - 6x + 1$
$5x^2 - 10x = 0$
$x(x-2) = 0$
$x = 0, 2$

38. $(4x+1)^4 = (5x-10)^4$
$(4x+1)^2 = (5x-10)^2$
$4x+1 = \pm 5(x-2)$
$4x+1 = 5(x-2),\ x = 11$
$4x+1 = -5(x-2),\ x = 1$

40. $10xe^x - 5e^x = 0$
$e^x(10x-5) = 0$
$10x - 5 = 0$ (since $e^x \neq 0$)
$x = 1/2$

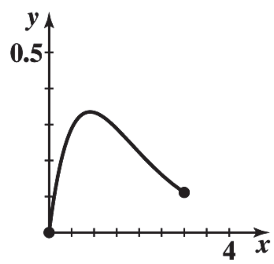
42. $x^2e^{-x} - 9e^{-x} = 0$
$e^{-x}(x^2-9) = 0$
$(x^2-9) = 0$ (since $e^{-x} \neq 0$)
$x = -3, 3$

44. $e^{4x} + e > 0$ for all x;
$e^{4x} + e = 0$ has no solutions.

46. $e^{3x-1} - e = 0$
$e^{3x-1} = e^1$
$3x - 1 = 1$
$x = 2/3$

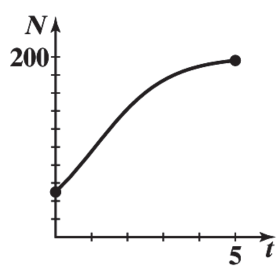
48. $m(x) = x(3^{-x})$; $[0, 3]$

x	$m(x)$
0	0
1	$\frac{1}{3}$
2	$\frac{2}{9}$
3	$\frac{1}{9}$



50. $N = \dfrac{200}{1+3e^{-t}}$; $[0, 5]$

x	N
0	50
1	≈ 95.07
2	≈ 142.25
3	≈ 174.01
4	≈ 184.58
5	≈ 196.04



52. $A = Pe^{rt}$

$A = (24,000)e^{(0.0435)(7)}$

$A = (24,000)e^{0.3045}$

$A = (24,000)(1.35594686)$

$A = \$32,542.72$

54. (A) $A = P(1+\frac{r}{m})^{mt}$

$A = 4000(1+\frac{0.06}{52})^{(52)(0.5)}$

$A = 4000(1.0011538462)^{26}$

$A = 4000(1.030436713)$

$A = \$4121.75$

(B) $A = P(1+\frac{r}{m})^{mt}$

$A = 4000(1+\frac{0.06}{52})^{(52)(10)}$

$A = 4000(1.0011538462)^{520}$

$A = 4000(1.821488661)$

$A = \$7285.95$

56. $A = P(1+\frac{r}{m})^{mt}$

$40,000 = P(1+\frac{0.055}{365})^{(365)(17)}$

$40,000 = P(1.0001506849)^{6205}$

$40,000 = P(2.547034043)$

$P = \$15,705$

58. (A) $A = P(1+\frac{r}{m})^{mt}$

$A = 10,000(1+\frac{0.0135}{4})^{(4)(5)}$

$A = 10,000(1.003375)^{20}$

$A = 10,000(1.069709)$

$A = \$10,697.09$

(B) $A = P(1+\frac{r}{m})^{mt}$

$A = 10,000(1+\frac{0.0130}{12})^{(12)(5)}$

$A = 10,000(1.00108333)^{60}$

$A = 10,000(1.067121479)$

$A = \$10,671.21$

(C) $A = P(1+\frac{r}{m})^{mt}$

$A = 10,000(1+\frac{0.0125}{365})^{(365)(5)}$

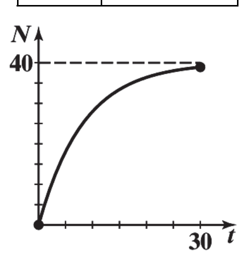
$A = 10,000(1.000034245)^{1825}$

$A = 10,000(1.06449332)$

$A = \$10,644.93$

60. $N = 40(1-e^{-0.12t})$; $[0, 30]$

x	N
0	0
10	≈ 27.95
20	≈ 36.37
30	≈ 38.91



The maximum number of boards an average employee can be expected to produce in 1 day is 40.

62. The exponential regression model (B) $y(10) = 268.8$ exabytes per month

```
ExpReg
 y=a*b^x
 a=3.996184237
 b=1.523286295
```

64. (A) $I(50) = I_o e^{-0.00942(50)} \approx 62\ \%$ (B) $I(100) = I_o e^{-0.00942(100)} \approx 39\ \%$

66. (A) $P = 204e^{0.0077t}$.
(B) Population in 2030: $P(15) = 204e^{0.0077(15)} \approx 229$ million.

68. (A) $P = 7.4e^{0.0113t}$

(B) Population in 2025: $P(10) = 7.4e^{0.0113(10)} \approx 8.29$ billion

Population in 2033: $P(18) = 7.4e^{0.0113(18)} \approx 9.07$ billion

EXERCISE 1-6

2. $\log_2 32 = 5 \Rightarrow 32 = 2^5$

4. $\log_e 1 = 0 \Rightarrow e^0 = 1$

6. $\log_9 27 = \frac{3}{2} \Rightarrow 27 = 9^{3/2}$

8. $36 = 6^2 \Rightarrow \log_6 36 = 2$

10. $9 = 27^{2/3} \Rightarrow \log_{27} 9 = \frac{2}{3}$

12. $M = b^x \Rightarrow \log_b M = x$

14. $\log_{10} \frac{1}{1000} = \log_{10} 10^{-3} = -3$

16. $\log_{10} 10{,}000 = \log_{10} 10^4 = 4$

18. $\log_2 \frac{1}{64} = \log_2 2^{-6} = -6$

20. $\ln(-1)$ is not defined.

22. $\ln(e^{-1}) = -1$

24. $\log_b FG = \log_b F + \log_b G$

26. $\log_b w^{15} = 15 \log_b w$

28. $\frac{\log_3 P}{\log_3 R} = \log_R P$

30. $\log_{10} x = 1$

$x = 10^1 = 10$

32. $\log_b \frac{1}{25} = 2$

$b^2 = \frac{1}{25}$

$b = \frac{1}{5}$

34. $\log_{49} 7 = y$

$$49^y = 7$$
$$y = 1/2$$

36. $\log_b 10,000 = 2$

$$b^2 = 10,000$$
$$b = 100$$

38. $\log_8 x = \frac{5}{3}$

$$x = 8^{5/3} = \left(8^{1/3}\right)^5 = 2^5 = 32$$

40. False; an example of a polynomial function of odd degree that is not one–to–one is $f(x) = x^3 - x$. $f(-1) = f(0) = f(1) = 0$.

42. False; the graph of every function (not necessarily one–to–one) intersects each vertical line at most once.

For example, $f(x) = \dfrac{1}{x-1}$ is a one-to-one function which does not intersect the vertical line $x = 1$.

44. False; $x = -1$ is in the domain of *f*, but cannot be in the range of *g*.

46. True; since *g* is the inverse of *f*, then (*a*, *b*) is on the graph of *f* if and only if (*b*, *a*) is on the graph of *g*. Therefore, *f* is also the inverse of *g*.

48. $\log_b x = \frac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3$

$$\log_b x = \log_b 27^{2/3} + \log_b 2^2 - \log_b 3$$
$$\log_b x = \log_b 9 + \log_b 4 - \log_b 3$$
$$\log_b x = \log_b \frac{(9)(4)}{3}$$
$$\log_b x = \log_b 12$$
$$x = 12$$

50. $\log_b x = 3\log_b 2 + \frac{1}{2}\log_b 25 - \log_b 20$

$$\log_b x = \log_b 2^3 + \log_b 25^{1/2} - \log_b 20$$
$$\log_b x = \log_b 8 + \log_b 5 - \log_b 20$$
$$\log_b x = \log_b \frac{(8)(5)}{20}$$
$$\log_b x = \log_b 2$$
$$x = 2$$

52. $\log_b(x+2)+\log_b x = \log_b 24$

$$\log_b(x+2)x = \log_b 24$$
$$\log_b(x^2+2x) = \log_b 24$$
$$x^2+2x = 24$$
$$x^2+2x-24 = 0$$
$$(x+6)(x-4) = 0$$
$$x = -6, 4$$

Since the domain of $\log_b$ is $(0,\infty)$, omit the negative solution. Therefore, the solution is $x=4$.

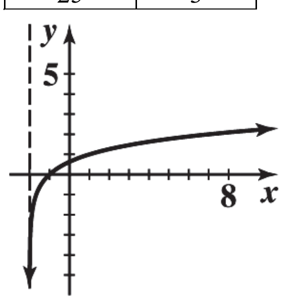
54. $\log_{10}(x+6)-\log_{10}(x-3)=1$

$$\log_{10}\frac{x+6}{x-3}=1$$
$$10^1=\frac{x+6}{x-3}$$
$$10(x-3)=x+6$$
$$10x-30=x+6$$
$$x=4$$

56. $y=\log_3(x+2)$

$$3^y = x+2$$
$$3^y-2=x$$

x	y
$-\frac{53}{27}$	-3
$-\frac{17}{9}$	-2
$-\frac{5}{3}$	-1
-1	0
1	1
7	2
25	3



58. The graph of $y=\log_3(x+2)$ is the graph of $y=\log_3 x$ shifted to the left 2 units.

60. The domain of logarithmic function is defined for positive values only. Therefore, the domain of the function is $x-1>0$ or $x>1$. The range of a logarithmic function is all real numbers. In interval notation the domain is $(1,\infty)$ and the range is $(-\infty,\infty)$.

62. A. $\log 72.604 = 1.86096$

B. $\log 0.033041 = -1.48095$

C. $\ln 40{,}257 = 10.60304$

D. $\ln 0.0059263 = -5.12836$

64. A. $\log x = 2.0832$

$$x = \log^{-1}(2.0832)$$
$$x = 121.1156$$

B. $\log x = -1.1577$

$$x = \log^{-1}(-1.1577)$$
$$x = 0.0696$$

C. $\ln x = 3.1336$

$$x = \ln^{-1}(3.1336)$$
$$x = 22.9565$$

D. $\ln x = -4.3281$

$$x = \ln^{-1}(-4.3281)$$
$$x = 0.0132$$

66.

$$10^x = 153$$
$$\log 10^x = \log 153$$
$$x = 2.1847$$

68.

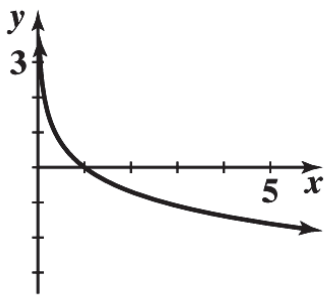
$$e^x = 0.3059$$
$$\ln e^x = \ln 0.3059$$
$$x = -1.1845$$

70.

$$1.02^{4t} = 2$$
$$\ln 1.02^{4t} = \ln 2$$
$$4t \ln 1.02 = \ln 2$$
$$t = \frac{\ln 2}{4 \ln 1.02}$$
$$t = 8.7507$$

72. $y = -\ln x;\ \ x > 0$

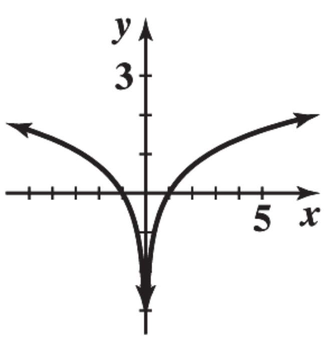
x	y
0.5	≈ 0.69
1	0
2	≈ -0.69
4	≈ -1.39
5	≈ -1.61



Based on the graph above, the function is decreasing on the interval $(0, \infty)$.

74. $y = \ln|x|$

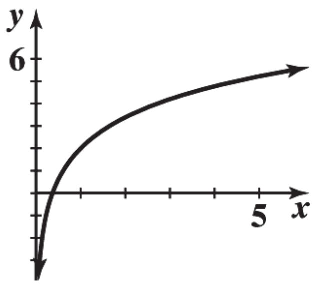
x	y
-5	≈ 1.61
-2	≈ 0.69
1	0
2	≈ 0.69
5	≈ 1.61



Based on the graph above, the function is decreasing on the interval $(-\infty, 0)$ and increasing on the interval $(0, \infty)$.

76. $y = 2\ln x + 2$

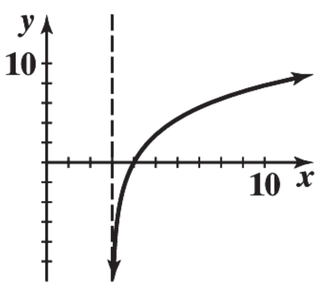
x	y
0.5	≈ 0.61
1	2
2	≈ 3.39
4	≈ 4.77
5	≈ 5.22



Based on the graph above, the function is increasing on the interval $(0, \infty)$.

78. $y = 4\ln(x-3)$

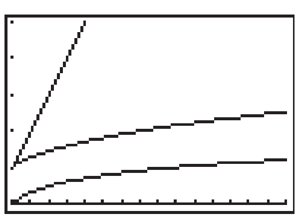
x	y
4	0
6	≈ 4.39
8	≈ 6.44
10	≈ 7.78
12	≈ 8.79



Based on the graph above, the function is increasing on the interval $(3, \infty)$.

80. It is not possible to find a power of 1 that is an arbitrarily selected real number, because 1 raised to any power is 1.

82.



A function *f* is "smaller than" a function *g* on an interval $[a, b]$ if $f(x) < g(x)$ for $a \le x \le b$. Based on the graph above, $\log x < \sqrt[3]{x} < x$ for $1 < x \le 16$.

84. Use the compound interest formula: $A = P(1+r)^t$. The problem is asking for the original amount to double, therefore $A = 2P$.

$$2P = P(1+0.0958)^t$$
$$2 = (1.0958)^t$$
$$\ln 2 = \ln(1.0958)^t$$
$$\ln 2 = t\ln(1.0958)$$
$$\frac{\ln 2}{\ln 1.0958} = t$$
$$7.58 \approx t$$

It will take approximately 8 years for the original amount to double.

86. Use the compound interest formula: $A = P(1+\frac{r}{m})^{mt}$.

(A) $$7500 = 5000(1+\tfrac{0.08}{2})^{2t}$$
$$1.5 = (1.04)^{2t}$$
$$\ln 1.5 = \ln(1.04)^{2t}$$
$$\ln 1.5 = 2t\ln(1.04)$$
$$\frac{\ln 1.5}{2\ln 1.04} = t$$
$$5.17 \approx t$$

It will take approximately 5.17 years for \$5000 to grow to \$7500 if compounded semiannually.

(B) $$7500 = 5000(1+\tfrac{0.08}{12})^{12t}$$
$$1.5 = (1.0066667)^{12t}$$
$$\ln 1.5 = \ln(1.0066667)^{12t}$$
$$\ln 1.5 = 12t\ln(1.0066667)$$
$$\frac{\ln 1.5}{12\ln 1.0066667} = t$$
$$5.09 \approx t$$

It will take approximately 5.09 years for \$5000 to grow to \$7500 if compounded monthly.

88. Use the compound interest formula: $A = Pe^{rt}$.

$$41,000 = 17,000e^{0.0295t}$$
$$\tfrac{41}{17} = e^{0.0295t}$$
$$\ln\tfrac{41}{17} = \ln e^{0.0295t}$$
$$\ln\tfrac{41}{17} = 0.0295t$$
$$\frac{\ln\frac{41}{17}}{0.0295} = t$$
$$29.84 \approx t$$

It will take approximately 29.84 years for \$17,000 to grow to \$41,000 if compounded continuously.

90. Equilibrium occurs when supply and demand are equal. The models from Problem 85 have the demand and supply functions defined by $y = 256.4659159 - 24.03812068\ln x$ and $y = -127.8085281 + 20.01315349\ln x$, respectively. Set both equations equal to each other to yield:

$$256.4659159 - 24.03812068\ln x = -127.8085281 + 20.01315349\ln x$$
$$384.274444 = 44.05127417\ln x$$
$$\frac{384.274444}{44.05127417} = \ln x$$
$$e^{384.274444/44.05127417} = e^{\ln x}$$
$$6145 \approx x$$

Substitute the value above into either equation.

$y = 256.4659159 - 24.03812068\ln x$

$y = 256.4659159 - 24.03812068\ln(6145)$

$y = 256.4659159 - 24.03812068(8.723394022)$

$y = 46.77$

Therefore, equilibrium occurs when 6145 units are produced and sold at a price of \$46.77.

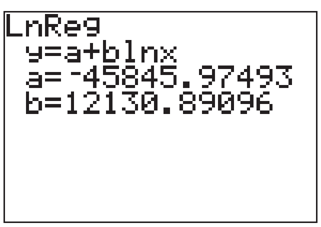
92. (A) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{10^{-13}}{10^{-16}} = 10\log 10^3 = 30$

(B) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{3.16\times10^{-10}}{10^{-16}} = 10\log 3.16\times10^6 \approx 65$

(C) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{10^{-8}}{10^{-16}} = 10\log 10^8 = 80$

(D) $N = 10\log\dfrac{I}{I_0} = 10\log\dfrac{10^{-1}}{10^{-16}} = 10\log 10^{15} = 150$

94.



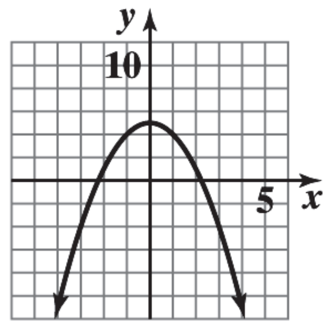
2024: $t = 124$; $y(124) \approx 12{,}628$. Therefore, according to the model, the total production in the year 2024 will be approximately 12,628 million bushels.

96.
$$A = A_0 e^{-0.000124t}$$
$$0.1A_0 = A_0 e^{-0.000124t}$$
$$0.1 = e^{-0.000124t}$$
$$\ln 0.1 = \ln e^{-0.000124t}$$
$$\ln 0.1 = -0.000124t$$
$$18{,}569 \approx t$$

If 10% of the original amount is still remaining, the skull would be approximately 18,569 years old.

CHAPTER 1 REVIEW

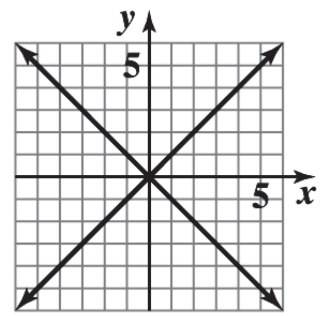
1.



(1-1)

2. $x^2 = y^2$:

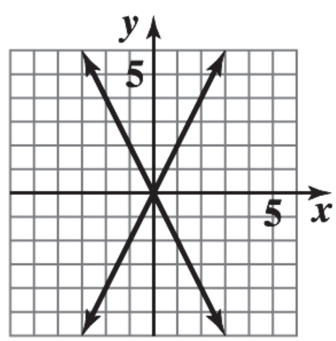
x	-3	-2	-1	0	1	2	3
y	± 3	± 2	± 1	0	± 1	± 2	± 3



(1-1)

3. $y^2 = 4x^2$:

x	-3	-2	-1	0	1	2	3
y	± 6	± 4	± 2	0	± 2	± 4	± 6



(1-1)

4. (A) Not a function; fails vertical line test
(B) A function
(C) A function
(D) Not a function; fails vertical line test (1-1)

5. $f(x) = 2x - 1$, $g(x) = x^2 - 2x$

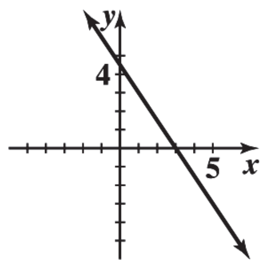
(A) $f(-2) + g(-1) = 2(-2) - 1 + (-1)^2 - 2(-1) = -2$

(B) $f(0)\cdot g(4) = (2\cdot 0 - 1)(4^2 - 2\cdot 4) = -8$

(C) $\dfrac{g(2)}{f(3)} = \dfrac{2^2 - 2\cdot 2}{2\cdot 3 - 1} = 0$

(D) $\dfrac{f(3)}{g(2)}$ not defined because $g(2) = 0$ (1-1)

6. $3x + 2y = 9$



(1-2)

7. The line passes through (6, 0) and (0, 4)

slope $m = \frac{4-0}{0-6} = -\frac{2}{3}$

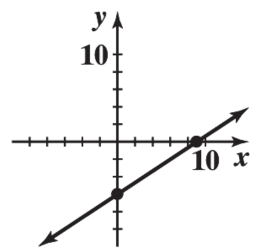
From the slope-intercept form: $y = -\frac{2}{3}x + 4$; multiplying by 3 gives: $3y = -2x + 12$, so

$2x + 3y = 12$ (1-2)

8. x-intercept: $2x = 18$, $x = 9$;
y-intercept: $-3y = 18$, $y = -6$;
slope-intercept form:

$y = \frac{2}{3}x - 6$; slope $= \frac{2}{3}$

Graph:



(1-2)

9. $y = -\frac{2}{3}x + 6$ (1-2)

10. Vertical line: $x = -6$; horizontal line: $y = 5$ (1-2)

11. Use the point-slope form:

(A) $y - 2 = -\frac{2}{3}[x - (-3)]$

$y - 2 = -\frac{2}{3}(x + 3)$

$y = -\frac{2}{3}x$

(B) $y - 3 = 0(x - 3)$

$y = 3$

(1-2)

12. (A) Slope: $\frac{-1-5}{1-(-3)} = -\frac{3}{2}$

$y - 5 = -\frac{3}{2}(x + 3)$

$3x + 2y = 1$

(B) Slope: $\frac{5-5}{4-(-1)} = 0$

$y - 5 = 0(x - 1)$

$y = 5$

(C) Slope: $\frac{-2-7}{-2-(-2)}$ not defined since $-2-(-2) = 0$

$x = -2$ (1-2)

13. $u = e^v$
$v = \ln u$ (1-6)

14. $x = 10^y$
$y = \log x$ (1-6)

15. $\ln M = N$
$M = e^N$ (1-6)

16. $\log u = v$

$u = 10^{v}$ (1-6)

17. $\log_3 x = 2$

$x = 3^2 = 9$ (1-6)

18. $\log_x 36 = 2$

$x^2 = 36$
$x = 6$
(1-6)

19. $\log_2 16 = x$

$2^x = 16$
$x = 4$
(1-6)

20. $10^x = 143.7$

$x = \log 143.7$
$x \approx 2.157$
(1-6)

21. $e^x = 503{,}000$

$x = \ln 503{,}000 \approx 13.128$ (1-6)

22. $\log x = 3.105$

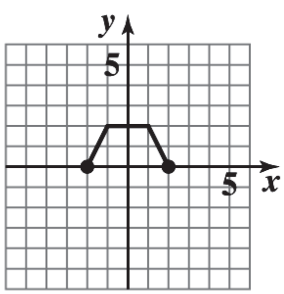
$x = 10^{3.105} \approx 1273.503$ (1-6)

23. $\ln x = -1.147$

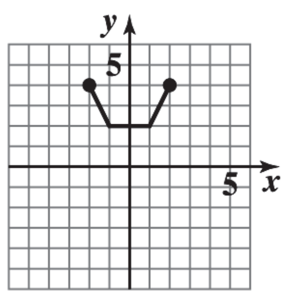
$x = e^{-1.147} \approx 0.318$ (1-6)

24. (A) $y = 4$ (B) $x = 0$ (C) $y = 1$ (D) $x = -1$ or 1
(E) $y = -2$ (F) $x = -5$ or 5 (1-1)

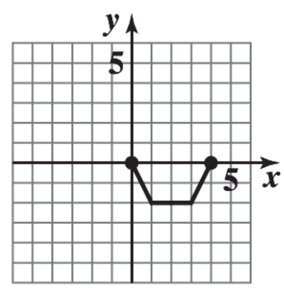
25. (A)



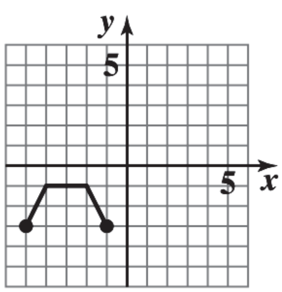
(B)



(C)



(D)



(1-2)

26. $f(x) = -x^2 + 4x = -(x^2 - 4x)$

$= -(x^2 - 4x + 4) + 4$

$= -(x - 2)^2 + 4$ (vertex form)

The graph of $f(x)$ is the graph of $y = x^2$ reflected in the x axis, then shifted right 2 units and up 4 units. (1-3)

27. (A) g (B) m (C) n (D) f (1-2, 1-3)

28. $y = f(x) = (x + 2)^2 - 4$

(A) x intercepts: $(x + 2)^2 - 4 = 0$; y intercept: 0

$$(x + 2)^2 = 4$$
$$x + 2 = -2 \text{ or } 2$$
$$x = -4, 0$$

(B) Vertex: $(-2, -4)$ (C) Minimum: -4 (D) Range: $y \geq -4$ or $[-4, \infty)$ (1-3)

29. $y = 4 - x + 3x^2 = 3x^2 - x + 4$; quadratic function. (1-3)

30. $y = \frac{1+5x}{6} = \frac{5}{6}x + \frac{1}{6}$; linear function. (1-1, 1-3)

31. $y = \frac{7-4x}{2x} = \frac{7}{2x} - 2$; none of these. (1-1), (1-3)

32. $y = 8x + 2(10 - 4x) = 8x + 20 - 8x = 20$; constant function (1-1)

33. $\log(x + 5) = \log(2x - 3)$

$$x + 5 = 2x - 3$$
$$-x = -8$$
$$x = 8$$
(1-6)

34. $2 \ln(x - 1) = \ln(x^2 - 5)$

$$\ln(x - 1)^2 = \ln(x^2 - 5)$$
$$(x - 1)^2 = x^2 - 5$$
$$x^2 - 2x + 1 = x^2 - 5$$
$$-2x = -6$$
$$x = 3$$
(1-6)

35. $2x^2e^x = 3xe^x$

$$2x^2 = 3x$$
$$2x^2 - 3x = 0$$
$$x(2x - 3) = 0$$
$$x = 0,\ 3/2$$
(1-5)

36. $\log_{1/3} 9 = x$

$$\left(\frac{1}{3}\right)^x = 9$$
$$\frac{1}{3^x} = 9$$
$$3^x = \frac{1}{9}$$
$$x = -2$$
(1-6)

37. $35 = 7(3^x)$

$$3^x = 5$$
$$\ln 3^x = \ln 5$$
$$x \ln 3 = \ln 5$$
(1-6)

$$x = \frac{\ln 5}{\ln 3} \approx 1.4650$$
(1-6)

38. $0.01 = e^{-0.05x}$

$$\ln (0.01) = \ln (e^{-0.05x}) = -0.05x$$

Thus, $x = \frac{\ln(0.01)}{-0.05} \approx 92.1034$

39. $8{,}000 = 4{,}000(1.08)^x$

$$(1.08)^x = 2$$
$$\ln(1.08)^x = \ln 2$$
$$x \ln 1.08 = \ln 2$$
$$x = \frac{\ln 2}{\ln 1.08} \approx 9.0065$$
(1-6)

40. $5^{2x-3} = 7.08$

$$\ln(5^{2x-3}) = \ln 7.08$$
$$(2x - 3) \ln 5 = \ln 7.08$$
$$2x \ln 5 - 3 \ln 5 = \ln 7.08$$
$$x = \frac{\ln 7.08 + 3 \ln 5}{2 \ln 5}$$
$$x \approx 2.1081$$
(1-6)

41. (A) $x^2 - x - 6 = 0$ at $x = -2, 3$
Domain: all real numbers except $x = -2, 3$

(B) $5 - x > 0$ for $x < 5$
Domain: $x < 5$ or $(-\infty, 5)$ (1-1)

42. $f(x) = 4x^2 + 4x - 3 = 4(x^2 + x) - 3$

$$= 4\left(x^2 + x + \frac{1}{4}\right) - 3 - 1$$

$$= 4\left(x + \frac{1}{2}\right)^2 - 4 \text{ (vertex form)}$$

Intercepts:
y intercept: $f(0) = 4(0)^2 + 4(0) - 3 = -3$
x intercepts: $f(x) = 0$

$$4\left(x + \frac{1}{2}\right)^2 - 4 = 0$$

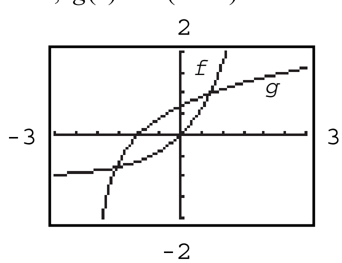
$$\left(x + \frac{1}{2}\right)^2 = 1$$

$$x + \frac{1}{2} = \pm 1$$

$$x = -\frac{1}{2} \pm 1 = -\frac{3}{2}, \frac{1}{2}$$

Vertex: $\left(-\frac{1}{2}, -4\right)$; minimum: -4; range: $y \geq -4$ or $[-4, \infty)$ (1-3)

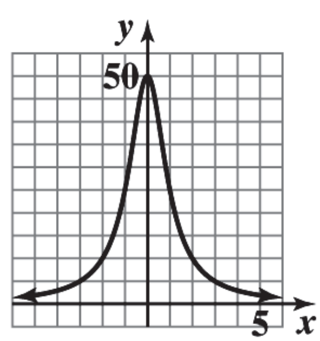
43. $f(x) = e^x - 1$, $g(x) = \ln(x + 2)$



Points of intersection:
$(-1.54, -0.79)$, $(0.69, 0.99)$
(1-5, 1-6)

44. $f(x) = \dfrac{50}{x^2 + 1}$:

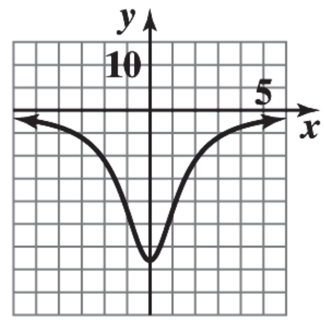
x	-3	-2	-1	0	1	2	3
$f(x)$	5	10	25	50	25	10	5



(1-1)

45. $f(x) = \frac{-66}{2+x^2}$:

x	-3	-2	-1	0	1	2	3
$f(x)$	-6	-11	-22	-66	-22	-11	-6



(1-1)

For Problems 46–50, $f(x) = 5x + 1$.

46. $f(f(0)) = f(5(0) + 1) = f(1) = 5(1) + 1 = 6$ (1-1)

47. $f(f(-1)) = f(5(-1) + 1) = f(-4) = 5(-4) + 1 = -19$ (1-1)

48. $f(2x - 1) = 5(2x - 1) + 1 = 10x - 4$ (1-1)

49. $f(4 - x) = 5(4 - x) + 1 = 20 - 5x + 1 = 21 - 5x$ (1-1)

50. $f(x) = 3 - 2x$

(A) $f(2) = 3 - 2(2) = 3 - 4 = -1$

(B) $f(2 + h) = 3 - 2(2 + h) = 3 - 4 - 2h = -1 - 2h$

(C) $f(2 + h) - f(2) = -1 - 2h - (-1) = -2h$

(D) $\frac{f(2+h)-f(2)}{h} = -\frac{2h}{h} = -2$ (1-1)

51. The graph of m is the graph of $y = |x|$ reflected in the x axis and shifted 4 units to the right. (1–2)

52. The graph of g is the graph of $y = x^3$ vertically contracted by a factor of 0.3 and shifted up 3 units. (1-2)

53. $f(x) = \frac{n(x)}{d(x)} = \frac{5x+4}{x^2-3x+1}$. Since degree $n(x) = 1 < 2 =$ degree $d(x)$, $y = 0$ is the horizontal asymptote. (1-4)

54. $f(x) = \frac{n(x)}{d(x)} = \frac{3x^2+2x-1}{4x^2-5x+3}$. Since degree $n(x) = 2 =$ degree $d(x)$, $y = \frac{3}{4}$ is the horizontal asymptote. (1-4)

55. $f(x) = \frac{n(x)}{d(x)} = \frac{x^2+4}{100x+1}$. Since degree $n(x) = 2 > 1 =$ degree $d(x)$, there is no horizontal asymptote (1-4)

56. $f(x) = \frac{n(x)}{d(x)} = \frac{x^2+100}{x^2-100} = \frac{x^2+100}{(x-10)(x+10)}$. Since $n(x) = x^2 + 100$ has no real zeros and $d(10) = d(-10) = 0$, $x = 10$ and $x = -10$ are the vertical asymptotes of the graph of f. (1-4)

57. $f(x) = \frac{n(x)}{d(x)} = \frac{x^2 + 3x}{x^2 + 2x} = \frac{x(x+3)}{x(x+2)} = \frac{x+3}{x+2}$, $x \neq 0$. $x = -2$ is a vertical asymptote of the graph of f. (1-4)

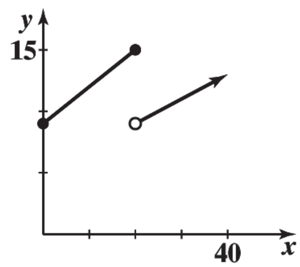
58. True; $p(x) = \frac{p(x)}{1}$ is a rational function for every polynomial p. (1-4)

59. False; $f(x) = \frac{1}{x} = x^{-1}$ is not a polynomial function. (1-4)

60. False; $f(x) = \frac{1}{x^2 + 1}$ has no vertical asymptotes. (1-4)

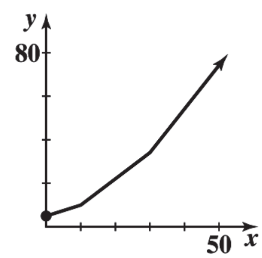
61. True; $f(x) = \frac{x}{x-1}$ has vertical asymptote $x = 1$ and horizontal asymptote $y = 1$. (1-4)

62.



(1-2)

63.



(1-2)

64. $y = -(x-4)^2 + 3$ (1-2, 1-3)

65. $f(x) = -0.4x^2 + 3.2x + 1.2 = -0.4(x^2 - 8x + 16) + 7.6$

$= -0.4(x-4)^2 + 7.6$

(A) y intercept: 1.2

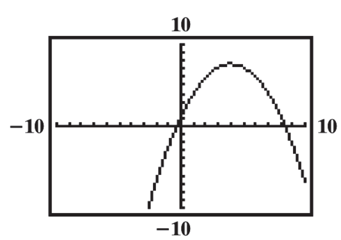
x intercepts: $-0.4(x-4)^2 + 7.6 = 0$

$(x-4)^2 = 19$

$x = 4 + \sqrt{19} \approx 8.359$, $4 - \sqrt{19} \approx -0.359$

(B) Vertex: (4.0, 7.6) (C) Maximum: 7.6 (D) Range: $y \leq 7.6$ or $(-\infty, 7.6]$ (1-3)

66.



(A) y intercept: 1.2

x intercepts: –0.359, 8.359

(B) Vertex: (4.0, 7.6)

(C) Maximum: 7.6

(D) Range: $y \leq 7.6$ or $(-\infty, 7.6]$ (1-3)

67. $\log 10^\pi = \pi \log 10 = \pi$ $10^{\log\sqrt{2}} = y$ is equivalent to $\log y = \log\sqrt{2}$ which implies $y = \sqrt{2}$

Similarly, $\ln e^\pi = \pi \ln e = \pi$ and $e^{\ln\sqrt{2}} = y$ implies $\ln y = \ln\sqrt{2}$ and $y = \sqrt{2}$. (1-6)

68. $\log x - \log 3 = \log 4 - \log(x+4)$

$$\log\frac{x}{3} = \log\frac{4}{x+4}$$
$$\frac{x}{3} = \frac{4}{x+4}$$
$$x(x+4) = 12$$
$$x^2 + 4x - 12 = 0$$
$$(x+6)(x-2) = 0$$
$$x = -6,\ 2$$

Since log(−6) is not defined, −6 is not a solution. Therefore, the solution is $x = 2$. (1-6)

69. $\ln(2x-2) - \ln(x-1) = \ln x$

$$\ln\left(\frac{2x-2}{x-1}\right) = \ln x$$
$$\ln\left[\frac{2(x-1)}{x-1}\right] = \ln x$$
$$\ln 2 = \ln x$$
$$x = 2$$ (1-6)

70. $\ln(x+3) - \ln x = 2\ln 2$

$$\ln\left(\frac{x+3}{x}\right) = \ln(2^2)$$
$$\frac{x+3}{x} = 4$$
$$x + 3 = 4x$$
$$3x = 3$$
$$x = 1$$ (1-6)

71. $\log 3x^2 = 2 + \log 9x$

$$\log 3x^2 - \log 9x = 2$$
$$\log\left(\frac{3x^2}{9x}\right) = 2$$
$$\log\left(\frac{x}{3}\right) = 2$$
$$\frac{x}{3} = 10^2 = 100$$
$$x = 300$$ (1-6)

72. $\ln y = -5t + \ln c$

$$\ln y - \ln c = -5t$$
$$\ln\frac{y}{c} = -5t$$
$$\frac{y}{c} = e^{-5t}$$
$$y = ce^{-5t}$$ (1-6)

73. Let x be *any* positive real number and suppose $\log_1 x = y$. Then $1^y = x$. But, $1^y = 1$, so $x = 1$, i.e., $x = 1$ for all positive real numbers x. This is clearly impossible. (1-6)

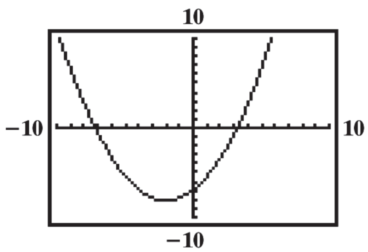
74. The graph of $y = \sqrt[3]{x}$ is vertically expanded by a factor of 2, reflected in the x axis, shifted 1 unit to the left and 1 unit down. Equation: $y = -2\sqrt[3]{x+1} - 1$ (1-2)

75. $G(x) = 0.3x^2 + 1.2x - 6.9 = 0.3(x^2 + 4x + 4) - 8.1$
$$= 0.3(x+2)^2 - 8.1$$

(A) y intercept: −6.9

x intercepts: $0.3(x+2)^2 - 8.1 = 0$
$$(x+2)^2 = 27$$
$$x = -2 + \sqrt{27} \approx 3.196,\quad -2 - \sqrt{27} \approx -7.196$$

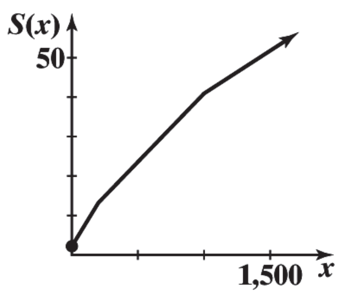
(B) Vertex: (−2, −8.1) (C) Minimum: −8.1 (D) Range: $y \geq -8.1$ or $[-8.1, \infty)$ (1-3)

76. 

(A) y intercept: –6.9
x intercept: –7.196, 3.196
(B) Vertex: (–2, –8.1)
(C) Minimum: –8.1
(D) Range: $y \geq -8.1$ or $[-8.1, \infty)$ (1-3)

77. (A) $S(x) = 3$ if $0 \leq x \leq 20$;
$S(x) = 3 + 0.057(x - 20)$
$= 0.057x + 1.86$ if $20 < x \leq 200$;
$S(200) = 13.26$
$S(x) = 13.26 + 0.0346(x - 200)$
$= 0.0346x + 6.34$ if $200 < x \leq 1000$;
$S(1000) = 40.94$
$S(x) = 40.94 + 0.0217(x - 1000)$
$= 0.0217x + 19.24$ if $x > 1000$

$$\text{Therefore, } S(x) = \begin{cases} 3 & \text{if} \quad 0 \leq x \leq 20 \\ 0.057x + 1.86 & \text{if} \quad 20 < x \leq 200 \\ 0.0346x + 6.34 & \text{if} \quad 200 < x \leq 1000 \\ 0.0217x + 19.24 & \text{if} \quad x > 1000 \end{cases}$$

(B) 

(1-2)

78. $A = P\left(1 + \frac{r}{m}\right)^{mt}$; $P = 5{,}000$, $r = 0.0125$, $m = 4$, $t = 5$.

$$A = 5{,}000\left(1 + \frac{0.0125}{4}\right)^{4(5)} = 5{,}000\left(1 + \frac{0.0125}{4}\right)^{20} \approx 5{,}321.95$$

After 5 years, the CD will be worth \$5,321.95 (1-5)

79. $A = A = P\left(1 + \frac{r}{m}\right)^{mt}$; $P = 5{,}000$, $r = 0.0105$, $m = 365$, $t = 5$

$$A = 5000\ A = 5000\left(1 + \frac{0.0105}{365}\right)^{365(5)} = 5000\left(1 + \frac{0.0105}{365}\right)^{1825} \approx 5269.51$$

After 5 years, the CD will be worth \$5,269.51. (1-5)

80. $A = P\left(1+\frac{r}{m}\right)^{mt}$; $r = 0.0659$, $m = 12$

Solve $P\left(1+\frac{0.0659}{12}\right)^{12t} = 3P$ or $(1.005492)^{12t} = 3$ for t:

$12t \ln(1.005492) = \ln 3,$

$$t = \frac{\ln 3}{12\ln(1.005492)} \approx 16.7 \text{ year.}$$ (1-5)

81. $A = Pe^{rt}$; $r = 0.0739$. Solve $2P = Pe^{0.0739t}$ for t.

$$2P = Pe^{0.0739t}$$
$$e^{0.0739t} = 2$$
$$0.0739t = \ln 2$$ (1-5)
$$t = \frac{\ln 2}{0.0739} \approx 9.38 \text{ years.}$$

82. Let x = person's age in years.

(A) Minimum heart rate: $m = (220 - x)(0.6) = 132 - 0.6x$

(B) Maximum heart rate: $M = (220 - x)(0.85) = 187 - 0.85x$

(C) At $x = 20$, $m = 132 - 0.6(20) = 120$
$M = 187 - 0.85(20) = 170$
range – between 120 and 170 beats per minute.

(D) At $x = 50$, $m = 132 - 0.6(50) = 102$
$M = 187 - 0.85(50) = 144.5$
range – between 102 and 144.5 beats per minute. (1-3)

83. $V = mt + b$

(A) At $t = 0$, $V = 224{,}000$; at $t = 8$, $V = 100{,}000$

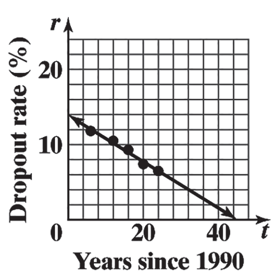
slope $m = \frac{100{,}000 - 224{,}000}{8-0} = -\frac{124{,}000}{8} = -15{,}500$

$V = -15{,}500t + 224{,}000$

(B) At $t = 12$, $V = -15{,}500(12) + 224{,}000 = 38{,}000$.
The bulldozer will be worth $38,000 after 12 years. (1-2)

84. (A) The dropout rate is decreasing at a rate of 0.308 percentage point per year.

(B)



(C) $-0.308t + 13.9 = 3 \Rightarrow t \approx 35.4$; $1990 + 35.4 = 2025.4$; The model predicts that the first year for which the dropout rate is less than 3% is 2026. (1-3)

85. (A) From the given linear regression model, the slope is 4.295. The U.S. CPI is increasing at a rate of 4.295 units per year.

(B) Use the given regression model and let $x = 34$ (the number of years since 1990) to make the prediction: $y = 4.295(34) + 130.59 = 276.62$. The predicted CPI in 2024 is 276.6. (1-3)

86. (A) The area enclosed by the pens is given by

$$A = (2y)x$$

Now, $3x + 4y = 840$

so $$y = 210 - \frac{3}{4}x$$

Thus $$A(x) = 2\left(210 - \frac{3}{4}x\right)x$$
$$= 420x - \frac{3}{2}x^2$$

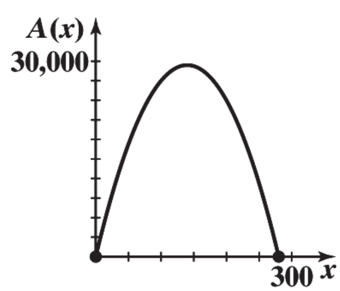
(B) Clearly x and y must be nonnegative; the fact that $y \geq 0$ implies

$$210 - \frac{3}{4}x \geq 0$$

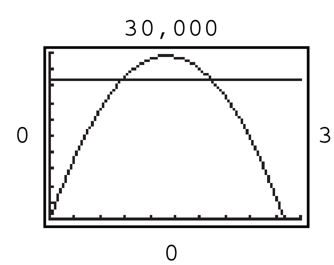
and $$210 \geq \frac{3}{4}x$$
$$840 \geq 3x$$
$$280 \geq x$$

Thus, domain A: $0 \leq x \leq 280$

(C)



(D) Graph $A(x) = 420x - \frac{3}{2}x^2$ and $y = 25{,}000$ together.



There are two values of x that will produce storage areas with a combined area of 25,000 square feet, one near $x = 90$ and the other near $x = 190$.

(E) $x = 86$, $x = 194$

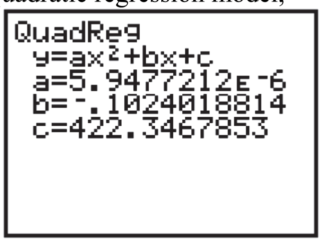
(F) $A(x) = 420x - \frac{3}{2}x^2 = -\frac{3}{2}(x^2 - 280x)$

Completing the square, we have

$$A(x) = -\frac{3}{2}(x^2 - 280x + 19{,}600 - 19{,}600)$$
$$= -\frac{3}{2}[(x - 140)^2 - 19{,}600]$$
$$= -\frac{3}{2}(x - 140)^2 + 29{,}400$$

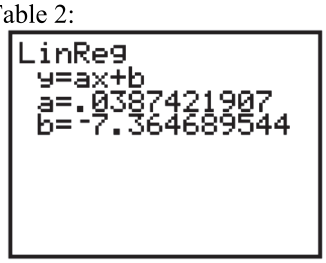
The dimensions that will produce the maximum combined area are:
$x = 140$ ft, $y = 105$ ft. The maximum area is 29,400 sq. ft. (1-3)

87. (A) Quadratic regression model,



To estimate the demand at price level of \$180, we solve the equation $ax^2 + bx + c = 180$ for x.
The result is $x \approx 2{,}833$ sets.

(B) Linear regression model,
Table 2:

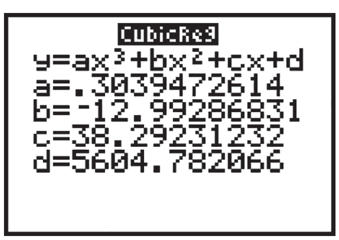


To estimate the supply at a price level of \$180, we solve the equation $ax + b = 180$ for x.
The result is $x \approx 4{,}836$ sets.

(C) The condition is not stable; the price is likely to decrease since the supply at the price level of \$180 exceeds the demand at this level.

(D) Equilibrium price: \$131.59
Equilibrium quantity: 3,587 cookware sets. (1-3)

88. (A)



$$y = 0.30395x^3 - 12.993x^2 + 38.292x + 5{,}604.8$$

(B) $y = 0.30395(38)^3 - 12.993(38)^2 + 38.292(38) + 5{,}604.8 \approx 4{,}976$

The predicted crime index in 2025 is 4,976. (1-4)

89. (A)

$$N(0) = 1$$
$$N\left(\frac{1}{2}\right) = 2$$
$$N(1) = 4 = 2^2$$
$$N\left(\frac{3}{2}\right) = 8 = 2^3$$
$$N(2) = 16 = 2^4$$
$$\vdots$$

Thus, we conclude that

$N(t) = 2^{2t}$ or $N = 4^t$.

(B) We need to solve:

$$2^{2t} = 10^9$$
$$\log 2^{2t} = \log 10^9 = 9$$
$$2t \log 2 = 9$$
$$t = \frac{9}{2\log 2} \approx 14.95$$

Thus, the mouse will die in 15 days. (1-6)

90. Given $I = I_0 e^{-kd}$. When $d = 73.6$, $I = \frac{1}{2} I_0$. Thus, we have:

$$\frac{1}{2} I_0 = I_0 e^{-k(73.6)}$$
$$e^{-k(73.6)} = \frac{1}{2}$$
$$-k(73.6) = \ln \frac{1}{2}$$
$$k = \frac{\ln(0.5)}{-73.6} \approx 0.00942$$

Thus, $k \approx 0.00942$.

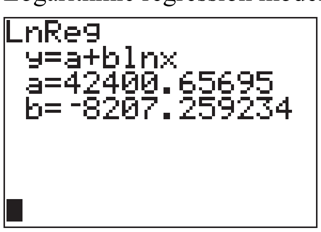
To find the depth at which 1% of the surface light remains, we set $I = 0.01 I_0$ and solve

$0.01 I_0 = I_0 e^{-0.00942d}$ for d:

$$0.01 = e^{-0.00942d}$$
$$-0.00942d = \ln 0.01$$
$$d = \frac{\ln 0.01}{-0.00942} \approx 488.87$$

Thus, 1% of the surface light remains at approximately 489 feet. (1-6)

91. (A) Logarithmic regression model:



Year 2023 corresponds to $x = 83$; $y(83) \approx 6{,}134{,}000$ cows.

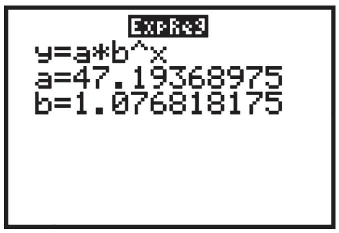
(B) ln (0) is not defined. (1-6)

92. Using the continuous compounding model, we have:

$$2P_0 = P_0 e^{0.03t}$$
$$2 = e^{0.03t}$$
$$0.03t = \ln 2$$
$$t = \frac{\ln 2}{0.03} \approx 23.1$$

Thus, the model predicts that the population will double in approximately 23.1 years. (1-5)

93. (A)



The exponential regression model is $y = 47.194(1.0768)^x$.

To estimate for the year 2025, let $x = 45 \Rightarrow y = 47.19368975(1.076818175)^{45} \approx 1{,}319.140047$. The estimated annual expenditure for Medicare by the U.S. government, rounded to the nearest billion, is approximately $1,319 billion. (This is $1.319 trillion.)

(B) To find the year, solve $47.194(1.0768)^x = 2{,}000$. Note: Use 2,000 because expenditures are in billions of dollars, and 2 trillion is 2,000 billion.

$$47.194(1.0768)^x = 2{,}000$$
$$1.0768^x = \frac{2{,}000}{47.194}$$
$$\ln(1.0768^x) = \ln\left(\frac{2{,}000}{47.194}\right)$$
$$x\ln 1.0768 = \ln\left(\frac{2{,}000}{47.194}\right)$$
$$x = \frac{\ln\left(\frac{2{,}000}{47.194}\right)}{\ln 1.0768} \approx 50.6 \text{ years}$$

$1{,}980 + 50.63 = 2{,}030.63$ Annual expenditures exceed two trillion dollars in the year 2031.

(1-5)