152. $(a, b-3)$

153. $(2x-1)(x^2+x-2) = 2x(x^2+x-2)-1(x^2+x-2)$
$= 2x^3+2x^2-4x-x^2-x+2$
$= 2x^3+2x^2-x^2-4x-x+2$
$= 2x^3+x^2-5x+2$

154. $\left(f(x)\right)^2-2f(x)+6 = (3x-4)^2-2(3x-4)+6$
$= 9x^2-24x+16-6x+8+6$
$= 9x^2-24x-6x+16+8+6$
$= 9x^2-30x+30$

155. $\dfrac{2}{\frac{3}{x}-1} = \dfrac{2x}{\frac{3x}{x}-x} = \dfrac{2x}{3-x}$

Section 1.7

Check Point Exercises

1. **a.** The function $f(x)=x^2+3x-17$ contains neither division nor an even root. The domain of *f* is the set of all real numbers or $(-\infty,\infty)$.

b. The denominator equals zero when $x = 7$ or $x = -7$. These values must be excluded from the domain.
domain of $g = (-\infty,-7)\cup(-7,7)\cup(7,\infty)$.

c. Since $h(x)=\sqrt{9x-27}$ contains an even root; the quantity under the radical must be greater than or equal to 0.
$9x-27\ge 0$
$9x\ge 27$
$x\ge 3$
Thus, the domain of *h* is $\{x|x\ge 3\}$, or the interval $[3,\infty)$.

d. Since the denominator of $j(x)$ contains an even root; the quantity under the radical must be greater than or equal to 0. But that quantity must also not be 0 (because we cannot have division by 0). Thus, $24-3x$ must be strictly greater than 0.
$24-3x>0$
$-3x>-24$
$x<8$
Thus, the domain of *j* is $\{x|x<8\}$, or the interval $(-\infty,8)$.

2. **a.** $(f+g)(x) = f(x)+g(x)$
$= x-5+\left(x^2-1\right)$
$= x-5+x^2-1$
$= -x^2+x-6$
domain: $(-\infty,\infty)$

b. $(f-g)(x) = f(x)-g(x)$
$= x-5-\left(x^2-1\right)$
$= x-5-x^2+1$
$= -x^2+x-4$
domain: $(-\infty,\infty)$

c. $(fg)(x) = (x-5)\left(x^2-1\right)$
$= x\left(x^2-1\right)-5\left(x^2-1\right)$
$= x^3-x-5x^2+5$
$= x^3-5x^2-x+5$
domain: $(-\infty,\infty)$

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$
$= \dfrac{x-5}{x^2-1},\ x\ne\pm 1$
domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

3. **a.** $(f+g)(x) = f(x)+g(x)$
$= \sqrt{x-3}+\sqrt{x+1}$

b. domain of *f*: $x-3\ge 0$
$x\ge 3$
$[3,\infty)$
domain of g: $x+1\ge 0$
$x\ge -1$
$[-1,\infty)$
The domain of $f+g$ is the set of all real numbers that are common to the domain of *f* and the domain of *g*. Thus, the domain of $f+g$ is $[3, \infty)$.

4. **a.** $(B+D)(x)$
$= B(x)+D(x)$
$= (-2.6x^2+49x+3994)+(-0.6x^2+7x+2412)$
$= -2.6x^2+49x+3994-0.6x^2+7x+2412$
$= -3.2x^2+56x+6406$

b. $(B+D)(x)=-3.2x^2+56x+6406$

$(B+D)(5)=-3.2(3)^2+56(3)+6406$

$=6545.2$

The number of births and deaths in the U.S. in 2003 was 6545.2 thousand.

c. $(B+D)(x)$ overestimates the actual number of births and deaths in 2003 by 7.2 thousand.

5. a. $(f\circ g)(x)=f(g(x))$

$=5(2x^2-x-1)+6$

$=10x^2-5x-5+6$

$=10x^2-5x+1$

b. $(g\circ f)(x)=g(f(x))$

$=2(5x+6)^2-(5x+6)-1$

$=2(25x^2+60x+36)-5x-6-1$

$=50x^2+120x+72-5x-6-1$

$=50x^2+115x+65$

c. $(f\circ g)(x)=10x^2-5x+1$

$(f\circ g)(-1)=10(-1)^2-5(-1)+1$

$=10+5+1$

$=16$

6. a. $(f\circ g)(x)=\dfrac{4}{\frac{1}{x}+2}=\dfrac{4x}{1+2x}$

b. domain: $\left\{x \middle| x\neq 0,\ x\neq -\frac{1}{2}\right\}$

or $\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},0\right)\cup(0,\infty)$

7. $h(x)=f\circ g$ where $f(x)=\sqrt{x}$; $g(x)=x^2+5$

Concept and Vocabulary Check 1.7

1. zero

2. negative

3. $f(x)+g(x)$

4. $f(x)-g(x)$

5. $f(x)\cdot g(x)$

6. $\dfrac{f(x)}{g(x)}$; $g(x)$

7. $(-\infty,\infty)$

8. $(2,\infty)$

9. $(0,3)$; $(3,\infty)$

10. composition; $f(g(x))$

11. *f*; $g(x)$

12. composition; $g(f(x))$

13. *g*; $f(x)$

14. false

15. false

16. 2

Exercise Set 1.7

1. The function contains neither division nor an even root. The domain $=(-\infty,\infty)$

2. The function contains neither division nor an even root. The domain $=(-\infty,\infty)$

3. The denominator equals zero when $x=4$. This value must be excluded from the domain.
domain: $(-\infty,4)\cup(4,\infty)$.

4. The denominator equals zero when $x=-5$. This value must be excluded from the domain.
domain: $(-\infty,-5)\cup(-5,\infty)$.

5. The function contains neither division nor an even root. The domain $=(-\infty,\infty)$

6. The function contains neither division nor an even root. The domain $=(-\infty,\infty)$

7. The values that make the denominator equal zero must be excluded from the domain.
domain: $(-\infty,-3)\cup(-3,5)\cup(5,\infty)$

8. The values that make the denominator equal zero must be excluded from the domain.
domain: $(-\infty,-4)\cup(-4,3)\cup(3,\infty)$

9. The values that make the denominators equal zero must be excluded from the domain.
domain: $(-\infty,-7)\cup(-7,9)\cup(9,\infty)$

10. The values that make the denominators equal zero must be excluded from the domain.
domain: $(-\infty,-8)\cup(-8,10)\cup(10,\infty)$

11. The first denominator cannot equal zero. The values that make the second denominator equal zero must be excluded from the domain.
domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

12. The first denominator cannot equal zero. The values that make the second denominator equal zero must be excluded from the domain.
domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

13. Exclude x for $x=0$.
Exclude x for $\frac{3}{x}-1=0$.

$$\frac{3}{x}-1=0$$
$$x\left(\frac{3}{x}-1\right)=x(0)$$
$$3-x=0$$
$$-x=-3$$
$$x=3$$

domain: $(-\infty,0)\cup(0,3)\cup(3,\infty)$

14. Exclude x for $x=0$.
Exclude x for $\frac{4}{x}-1=0$.

$$\frac{4}{x}-1=0$$
$$x\left(\frac{4}{x}-1\right)=x(0)$$
$$4-x=0$$
$$-x=-4$$
$$x=4$$

domain: $(-\infty,0)\cup(0,4)\cup(4,\infty)$

15. Exclude x for $x-1=0$.

$$x-1=0$$
$$x=1$$

Exclude x for $\frac{4}{x-1}-2=0$.

$$\frac{4}{x-1}-2=0$$
$$(x-1)\left(\frac{4}{x-1}-2\right)=(x-1)(0)$$
$$4-2(x-1)=0$$
$$4-2x+2=0$$
$$-2x+6=0$$
$$-2x=-6$$
$$x=3$$

domain: $(-\infty,1)\cup(1,3)\cup(3,\infty)$

16. Exclude x for $x-2=0$.

$$x-2=0$$
$$x=2$$

Exclude x for $\frac{4}{x-2}-3=0$.

$$\frac{4}{x-2}-3=0$$
$$(x-2)\left(\frac{4}{x-2}-3\right)=(x-2)(0)$$
$$4-3(x-2)=0$$
$$4-3x+6=0$$
$$-3x+10=0$$
$$-3x=-10$$
$$x=\frac{10}{3}$$

domain: $(-\infty,2)\cup\left(2,\frac{10}{3}\right)\cup\left(\frac{10}{3},\infty\right)$

17. The expression under the radical must not be negative.

$$x-3\ge 0$$
$$x\ge 3$$

domain: $[3,\infty)$

18. The expression under the radical must not be negative.

$$x+2\ge 0$$
$$x\ge -2$$

domain: $[-2,\infty)$

19. The expression under the radical must be positive.
$x-3>0$
$x>3$
domain: $(3,\infty)$

20. The expression under the radical must be positive.
$x+2>0$
$x>-2$
domain: $(-2,\infty)$

21. The expression under the radical must not be negative.
$5x+35\ge 0$
$5x\ge -35$
$x\ge -7$
domain: $[-7,\infty)$

22. The expression under the radical must not be negative.
$7x-70\ge 0$
$7x\ge 70$
$x\ge 10$
domain: $[10,\infty)$

23. The expression under the radical must not be negative.
$24-2x\ge 0$
$-2x\ge -24$
$\frac{-2x}{-2}\le\frac{-24}{-2}$
$x\le 12$
domain: $(-\infty,12]$

24. The expression under the radical must not be negative.
$84-6x\ge 0$
$-6x\ge -84$
$\frac{-6x}{-6}\le\frac{-84}{-6}$
$x\le 14$
domain: $(-\infty,14]$

25. The expressions under the radicals must not be negative.
$x-2\ge 0$ and $x+3\ge 0$
$x\ge 2$ $\quad$ $x\ge -3$
To make both inequalities true, $x\ge 2$.
domain: $[2,\infty)$

26. The expressions under the radicals must not be negative.
$x-3\ge 0$ and $x+4\ge 0$
$x\ge 3$ $\quad$ $x\ge -4$
To make both inequalities true, $x\ge 3$.
domain: $[3,\infty)$

27. The expression under the radical must not be negative.
$x-2\ge 0$
$x\ge 2$
The denominator equals zero when $x=5$.
domain: $[2,5)\cup(5,\infty)$.

28. The expression under the radical must not be negative.
$x-3\ge 0$
$x\ge 3$
The denominator equals zero when $x=6$.
domain: $[3,6)\cup(6,\infty)$.

29. Find the values that make the denominator equal zero and must be excluded from the domain.
$x^3-5x^2-4x+20$
$=x^2(x-5)-4(x-5)$
$=(x-5)(x^2-4)$
$=(x-5)(x+2)(x-2)$
–2, 2, and 5 must be excluded.
domain: $(-\infty,-2)\cup(-2,2)\cup(2,5)\cup(5,\infty)$

30. Find the values that make the denominator equal zero and must be excluded from the domain.
$x^3-2x^2-9x+18$
$=x^2(x-2)-9(x-2)$
$=(x-2)(x^2-9)$
$=(x-2)(x+3)(x-3)$
–3, 2, and 3 must be excluded.
domain: $(-\infty,-3)\cup(-3,2)\cup(2,3)\cup(3,\infty)$

31. $(f+g)(x) = 3x + 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = f(x) - g(x)$
$= (2x + 3) - (x - 1)$
$= x + 4$
domain: $(-\infty, \infty)$
$(fg)(x) = f(x) \cdot g(x)$
$= (2x+3) \cdot (x-1)$
$= 2x^2 + x - 3$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+3}{x-1}$
domain: $(-\infty,1)\cup(1,\infty)$

32. $(f+g)(x) = 4x - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = (3x - 4) - (x + 2) = 2x - 6$
domain: $(-\infty, \infty)$
$(fg)(x) = (3x - 4)(x + 2) = 3x^2 + 2x - 8$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{3x-4}{x+2}$
domain: $(-\infty,-2)\cup(-2,\infty)$

33. $(f+g)(x) = 3x^2 + x - 5$
domain: $(-\infty, \infty)$
$(f-g)(x) = -3x^2 + x - 5$
domain: $(-\infty, \infty)$
$(fg)(x) = (x-5)(3x^2) = 3x^3 - 15x^2$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{x-5}{3x^2}$
domain: $(-\infty,0)\cup(0,\infty)$

34. $(f+g)(x) = 5x^2 + x - 6$
domain: $(-\infty, \infty)$
$(f-g)(x) = -5x^2 + x - 6$
domain: $(-\infty, \infty)$
$(fg)(x) = (x-6)(5x^2) = 5x^3 - 30x^2$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{x-6}{5x^2}$
domain: $(-\infty,0)\cup(0,\infty)$

35. $(f+g)(x) = 2x^2 - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = 2x^2 - 2x - 4$
domain: $(-\infty, \infty)$
$(fg)(x) = (2x^2 - x - 3)(x+1)$
$= 2x^3 + x^2 - 4x - 3$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{2x^2 - x - 3}{x+1}$
$= \frac{(2x-3)(x+1)}{(x+1)} = 2x - 3$
domain: $(-\infty,-1)\cup(-1,\infty)$

36. $(f+g)(x) = 6x^2 - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = 6x^2 - 2x$
domain: $(-\infty, \infty)$
$(fg)(x) = (6x^2 - x - 1)(x-1) = 6x^3 - 7x^2 + 1$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{6x^2 - x - 1}{x-1}$
domain: $(-\infty,1)\cup(1,\infty)$

37. $(f+g)(x) = (3 - x^2) + (x^2 + 2x - 15)$
$= 2x - 12$
domain: $(-\infty, \infty)$
$(f-g)(x) = (3 - x^2) - (x^2 + 2x - 15)$
$= -2x^2 - 2x + 18$
domain: $(-\infty, \infty)$
$(fg)(x) = (3 - x^2)(x^2 + 2x - 15)$
$= -x^4 - 2x^3 + 18x^2 + 6x - 45$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{3 - x^2}{x^2 + 2x - 15}$
domain: $(-\infty,-5)\cup(-5,3)\cup(3,\infty)$

38. $(f+g)(x) = (5-x^2)+(x^2+4x-12)$
$= 4x-7$
domain: $(-\infty, \infty)$
$(f-g)(x) = (5-x^2)-(x^2+4x-12)$
$= -2x^2-4x+17$
domain: $(-\infty, \infty)$
$(fg)(x) = (5-x^2)(x^2+4x-12)$
$= -x^4-4x^3+17x^2+20x-60$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{5-x^2}{x^2+4x-12}$
domain: $(-\infty,-6)\cup(-6,2)\cup(2,\infty)$

39. $(f+g)(x) = \sqrt{x}+x-4$
domain: $[0,\infty)$
$(f-g)(x) = \sqrt{x}-x+4$
domain: $[0,\infty)$
$(fg)(x) = \sqrt{x}(x-4)$
domain: $[0,\infty)$
$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x-4}$
domain: $[0,4)\cup(4,\infty)$

40. $(f+g)(x) = \sqrt{x}+x-5$
domain: $[0,\infty)$
$(f-g)(x) = \sqrt{x}-x+5$
domain: $[0,\infty)$
$(fg)(x) = \sqrt{x}(x-5)$
domain: $[0,\infty)$
$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x-5}$
domain: $[0,5)\cup(5,\infty)$

41. $(f+g)(x) = 2+\frac{1}{x}+\frac{1}{x} = 2+\frac{2}{x} = \frac{2x+2}{x}$
domain: $(-\infty,0)\cup(0,\infty)$
$(f-g)(x) = 2+\frac{1}{x}-\frac{1}{x} = 2$
domain: $(-\infty,0)\cup(0,\infty)$
$(fg)(x) = \left(2+\frac{1}{x}\right)\cdot\frac{1}{x} = \frac{2}{x}+\frac{1}{x^2} = \frac{2x+1}{x^2}$
domain: $(-\infty,0)\cup(0,\infty)$
$\left(\frac{f}{g}\right)(x) = \frac{2+\frac{1}{x}}{\frac{1}{x}} = \left(2+\frac{1}{x}\right)\cdot x = 2x+1$
domain: $(-\infty,0)\cup(0,\infty)$

42. $(f+g)(x) = 6-\frac{1}{x}+\frac{1}{x} = 6$
domain: $(-\infty,0)\cup(0,\infty)$
$(f-g)(x) = 6-\frac{1}{x}-\frac{1}{x} = 6-\frac{2}{x} = \frac{6x-2}{x}$
domain: $(-\infty,0)\cup(0,\infty)$
$(fg)(x) = \left(6-\frac{1}{x}\right)\cdot\frac{1}{x} = \frac{6}{x}-\frac{1}{x^2} = \frac{6x-1}{x^2}$
domain: $(-\infty,0)\cup(0,\infty)$
$\left(\frac{f}{g}\right)(x) = \frac{6-\frac{1}{x}}{\frac{1}{x}} = \left(6-\frac{1}{x}\right)\cdot x = 6x-1$
domain: $(-\infty,0)\cup(0,\infty)$

43. $(f+g)(x) = f(x)+g(x)$

$$= \frac{5x+1}{x^2-9} + \frac{4x-2}{x^2-9}$$
$$= \frac{9x-1}{x^2-9}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$(f-g)(x) = f(x)-g(x)$

$$= \frac{5x+1}{x^2-9} - \frac{4x-2}{x^2-9}$$
$$= \frac{x+3}{x^2-9}$$
$$= \frac{1}{x-3}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{5x+1}{x^2-9}\cdot\frac{4x-2}{x^2-9}$$
$$= \frac{(5x+1)(4x-2)}{\left(x^2-9\right)^2}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{5x+1}{x^2-9}}{\frac{4x-2}{x^2-9}}$$
$$= \frac{5x+1}{x^2-9}\cdot\frac{x^2-9}{4x-2}$$
$$= \frac{5x+1}{4x-2}$$

The domain must exclude –3, 3, and any values that make $4x-2=0$.

$4x-2=0$

$4x=2$

$x=\frac{1}{2}$

domain: $(-\infty,-3)\cup(-3,\frac{1}{2})\cup(\frac{1}{2},3)\cup(3,\infty)$

44. $(f+g)(x) = f(x)+g(x)$

$$= \frac{3x+1}{x^2-25} + \frac{2x-4}{x^2-25}$$
$$= \frac{5x-3}{x^2-25}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(f-g)(x) = f(x)-g(x)$

$$= \frac{3x+1}{x^2-25} - \frac{2x-4}{x^2-25}$$
$$= \frac{x+5}{x^2-25}$$
$$= \frac{1}{x-5}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{3x+1}{x^2-25}\cdot\frac{2x-4}{x^2-25}$$
$$= \frac{(3x+1)(2x-4)}{\left(x^2-25\right)^2}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{3x+1}{x^2-25}}{\frac{2x-4}{x^2-25}}$$
$$= \frac{3x+1}{x^2-25}\cdot\frac{x^2-25}{2x-4}$$
$$= \frac{3x+1}{2x-4}$$

The domain must exclude –5, 5, and any values that make $2x-4=0$.

$2x-4=0$

$2x=4$

$x=2$

domain: $(-\infty,-5)\cup(-5,2)\cup(2,5)\cup(5,\infty)$

45. $(f+g)(x) = f(x)+g(x)$

$$= \frac{8x}{x-2}+\frac{6}{x+3}$$
$$= \frac{8x(x+3)}{(x-2)(x+3)}+\frac{6(x-2)}{(x-2)(x+3)}$$
$$= \frac{8x^2+24x}{(x-2)(x+3)}+\frac{6x-12}{(x-2)(x+3)}$$
$$= \frac{8x^2+30x-12}{(x-2)(x+3)}$$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

$(f+g)(x) = f(x)-g(x)$

$$= \frac{8x}{x-2}-\frac{6}{x+3}$$
$$= \frac{8x(x+3)}{(x-2)(x+3)}-\frac{6(x-2)}{(x-2)(x+3)}$$
$$= \frac{8x^2+24x}{(x-2)(x+3)}-\frac{6x-12}{(x-2)(x+3)}$$
$$= \frac{8x^2+18x+12}{(x-2)(x+3)}$$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{8x}{x-2}\cdot\frac{6}{x+3}$$
$$= \frac{48x}{(x-2)(x+3)}$$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{8x}{x-2}}{\frac{6}{x+3}}$$
$$= \frac{8x}{x-2}\cdot\frac{x+3}{6}$$
$$= \frac{4x(x+3)}{3(x-2)}$$

The domain must exclude –3, 2, and any values that make $3(x-2)=0$.

$$3(x-2)=0$$
$$3x-6=0$$
$$3x=6$$
$$x=2$$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

46. $(f+g)(x) = f(x)+g(x)$

$$= \frac{9x}{x-4}+\frac{7}{x+8}$$
$$= \frac{9x(x+8)}{(x-4)(x+8)}+\frac{7(x-4)}{(x-4)(x+8)}$$
$$= \frac{9x^2+72x}{(x-4)(x+8)}+\frac{7x-28}{(x-4)(x+8)}$$
$$= \frac{9x^2+79x-28}{(x-4)(x+8)}$$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

$(f+g)(x) = f(x)-g(x)$

$$= \frac{9x}{x-4}-\frac{7}{x+8}$$
$$= \frac{9x(x+8)}{(x-4)(x+8)}-\frac{7(x-4)}{(x-4)(x+8)}$$
$$= \frac{9x^2+72x}{(x-4)(x+8)}-\frac{7x-28}{(x-4)(x+8)}$$
$$= \frac{9x^2+65x+28}{(x-4)(x+8)}$$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{9x}{x-4}\cdot\frac{7}{x+8}$$
$$= \frac{63x}{(x-4)(x+8)}$$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{9x}{x-4}}{\frac{7}{x+8}}$$
$$= \frac{9x}{x-4}\cdot\frac{x+8}{7}$$
$$= \frac{9x(x+8)}{7(x-4)}$$

The domain must exclude –8, 4, and any values that make $7(x-4)=0$.

$$7(x-4)=0$$
$$7x-28=0$$
$$7x=28$$
$$x=4$$

47. $(f+g)(x)=\sqrt{x+4}+\sqrt{x-1}$
domain: $[1,\infty)$
$(f-g)(x)=\sqrt{x+4}-\sqrt{x-1}$
domain: $[1,\infty)$
$(fg)(x)=\sqrt{x+4}\cdot\sqrt{x-1}=\sqrt{x^2+3x-4}$
domain: $[1,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x+4}}{\sqrt{x-1}}$
domain: $(1,\infty)$

48. $(f+g)(x)=\sqrt{x+6}+\sqrt{x-3}$
domain: $[3,\infty)$
$(f-g)(x)=\sqrt{x+6}-\sqrt{x-3}$
domain: $[3,\infty)$
$(fg)(x)=\sqrt{x+6}\cdot\sqrt{x-3}=\sqrt{x^2+3x-18}$
domain: $[3,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x+6}}{\sqrt{x-3}}$
domain: $(3,\infty)$

49. $(f+g)(x)=\sqrt{x-2}+\sqrt{2-x}$
domain: $\{2\}$
$(f-g)(x)=\sqrt{x-2}-\sqrt{2-x}$
domain: $\{2\}$
$(fg)(x)=\sqrt{x-2}\cdot\sqrt{2-x}=\sqrt{-x^2+4x-4}$
domain: $\{2\}$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x-2}}{\sqrt{2-x}}$
domain: $\varnothing$

50. $(f+g)(x)=\sqrt{x-5}+\sqrt{5-x}$
domain: $\{5\}$
$(f-g)(x)=\sqrt{x-5}-\sqrt{5-x}$
domain: $\{5\}$
$(fg)(x)=\sqrt{x-5}\cdot\sqrt{5-x}=\sqrt{-x^2+10x-25}$
domain: $\{5\}$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x-5}}{\sqrt{5-x}}$
domain: $\varnothing$

51. $f(x) = 2x$; $g(x) = x + 7$

a. $(f\circ g)(x)=2(x+7)=2x+14$

b. $(g\circ f)(x)=2x+7$

c. $(f\circ g)(2)=2(2)+14=18$

52. $f(x) = 3x$; $g(x) = x - 5$

a. $(f\circ g)(x)=3(x-5)=3x-15$

b. $(g\circ f)(x)=3x-5$

c. $(f\circ g)(2)=3(2)-15=-9$

53. $f(x) = x + 4$; $g(x) = 2x + 1$

a. $(f\circ g)(x)=(2x+1)+4=2x+5$

b. $(g\circ f)(x)=2(x+4)+1=2x+9$

c. $(f\circ g)(2)=2(2)+5=9$

54. $f(x) = 5x + 2$; $g(x) = 3x - 4$

a. $(f\circ g)(x)=5(3x-4)+2=15x-18$

b. $(g\circ f)(x)=3(5x+2)-4=15x+2$

c. $(f\circ g)(2)=15(2)-18=12$

55. $f(x) = 4x - 3$; $g(x)=5x^2-2$

a. $(f\circ g)(x)=4(5x^2-2)-3$
$=20x^2-11$

b. $(g\circ f)(x)=5(4x-3)^2-2$
$=5(16x^2-24x+9)-2$
$=80x^2-120x+43$

c. $(f\circ g)(2)=20(2)^2-11=69$

56. $f(x)=7x+1;\ g(x)=2x^2-9$

a. $(f\circ g)(x)=7(2x^2-9)+1=14x^2-62$

b. $(g\circ f)(x)=2(7x+1)^2-9$
$=2(49x^2+14x+1)-9$
$=98x^2+28x-7$

c. $(f\circ g)(2)=14(2)^2-62=-6$

57. $f(x)=x^2+2;\ g(x)=x^2-2$

a. $(f\circ g)(x)=(x^2-2)^2+2$
$=x^4-4x^2+4+2$
$=x^4-4x^2+6$

b. $(g\circ f)(x)=(x^2+2)^2-2$
$=x^4+4x^2+4-2$
$=x^4+4x^2+2$

c. $(f\circ g)(2)=2^4-4(2)^2+6=6$

58. $f(x)=x^2+1;\ g(x)=x^2-3$

a. $(f\circ g)(x)=(x^2-3)^2+1$
$=x^4-6x^2+9+1$
$=x^4-6x^2+10$

b. $(g\circ f)(x)=(x^2+1)^2-3$
$=x^4+2x^2+1-3$
$=x^4+2x^2-2$

c. $(f\circ g)(2)=2^4-6(2)^2+10=2$

59. $f(x)=4-x;\quad g(x)=2x^2+x+5$

a. $(f\circ g)(x)=4-\left(2x^2+x+5\right)$
$=4-2x^2-x-5$
$=-2x^2-x-1$

b. $(g\circ f)(x)=2\left(4-x\right)^2+\left(4-x\right)+5$
$=2(16-8x+x^2)+4-x+5$
$=32-16x+2x^2+4-x+5$
$=2x^2-17x+41$

c. $(f\circ g)(2)=-2(2)^2-2-1=-11$

60. $f(x)=5x-2;\quad g(x)=-x^2+4x-1$

a. $(f\circ g)(x)=5\left(-x^2+4x-1\right)-2$
$=-5x^2+20x-5-2$
$=-5x^2+20x-7$

b. $(g\circ f)(x)=-\left(5x-2\right)^2+4\left(5x-2\right)-1$
$=-(25x^2-20x+4)+20x-8-1$
$=-25x^2+20x-4+20x-8-1$
$=-25x^2+40x-13$

c. $(f\circ g)(2)=-5(2)^2+20(2)-7=13$

61. $f(x)=\sqrt{x};\ g(x)=x-1$

a. $(f\circ g)(x)=\sqrt{x-1}$

b. $(g\circ f)(x)=\sqrt{x}-1$

c. $(f\circ g)(2)=\sqrt{2-1}=\sqrt{1}=1$

62. $f(x)=\sqrt{x};\ g(x)=x+2$

a. $(f\circ g)(x)=\sqrt{x+2}$

b. $(g\circ f)(x)=\sqrt{x}+2$

c. $(f\circ g)(2)=\sqrt{2+2}=\sqrt{4}=2$

63. $f(x)=2x-3;\ g(x)=\dfrac{x+3}{2}$

a. $(f\circ g)(x)=2\left(\dfrac{x+3}{2}\right)-3$
$=x+3-3$
$=x$

b. $(g\circ f)(x)=\dfrac{(2x-3)+3}{2}=\dfrac{2x}{2}=x$

c. $(f\circ g)(2)=2$

64. $f(x)=6x-3; g(x)=\frac{x+3}{6}$

a. $(f\circ g)(x)=6\left(\frac{x+3}{6}\right)-3=x+3-3=x$

b. $(g\circ f)(x)=\frac{6x-3+3}{6}=\frac{6x}{6}=x$

c. $(f\circ g)(2)=2$

65. $f(x)=\frac{1}{x};\quad g(x)=\frac{1}{x}$

a. $(f\circ g)(x)=\frac{1}{\frac{1}{x}}=x$

b. $(g\circ f)(x)=\frac{1}{\frac{1}{x}}=x$

c. $(f\circ g)(2)=2$

66. $f(x)=\frac{2}{x};\quad g(x)=\frac{2}{x}$

a. $(f\circ g)(x)=\frac{2}{\frac{2}{x}}=x$

b. $(g\circ f)(x)=\frac{2}{\frac{2}{x}}=x$

c. $(f\circ g)(2)=2$

67. a. $(f\circ g)(x)=f\left(\frac{1}{x}\right)=\frac{2}{\frac{1}{x}+3}, x\neq 0$

$$=\frac{2(x)}{\left(\frac{1}{x}+3\right)(x)}$$

$$=\frac{2x}{1+3x}$$

b. We must exclude 0 because it is excluded from g.

We must exclude $-\frac{1}{3}$ because it causes the denominator of $f\circ g$ to be 0.

domain: $\left(-\infty,-\frac{1}{3}\right)\cup\left(-\frac{1}{3},0\right)\cup(0,\infty)$.

68. a. $f\circ g(x)=f\left(\frac{1}{x}\right)=\frac{5}{\frac{1}{x}+4}=\frac{5x}{1+4x}$

b. We must exclude 0 because it is excluded from g.

We must exclude $-\frac{1}{4}$ because it causes the denominator of $f\circ g$ to be 0.

domain: $\left(-\infty,-\frac{1}{4}\right)\cup\left(-\frac{1}{4},0\right)\cup(0,\infty)$.

69. a. $(f\circ g)(x)=f\left(\frac{4}{x}\right)=\frac{\frac{4}{x}}{\frac{4}{x}+1}$

$$=\frac{\left(\frac{4}{x}\right)(x)}{\left(\frac{4}{x}+1\right)(x)}$$

$$=\frac{4}{4+x}, x\neq -4$$

b. We must exclude 0 because it is excluded from g.

We must exclude -4 because it causes the denominator of $f\circ g$ to be 0.

domain: $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.

70. a. $f\circ g(x)=f\left(\frac{6}{x}\right)=\frac{\frac{6}{x}}{\frac{6}{x}+5}=\frac{6}{6+5x}$

b. We must exclude 0 because it is excluded from g.

We must exclude $-\frac{6}{5}$ because it causes the denominator of $f\circ g$ to be 0.

domain: $\left(-\infty,-\frac{6}{5}\right)\cup\left(-\frac{6}{5},0\right)\cup(0,\infty)$.

71. a. $f\circ g(x)=f(x-2)=\sqrt{x-2}$

b. The expression under the radical in $f\circ g$ must not be negative.

$x-2\geq 0$

$x\geq 2$

domain: $[2,\infty)$.

72. **a.** $f \circ g(x) = f(x-3) = \sqrt{x-3}$

b. The expression under the radical in $f \circ g$ must not be negative.

$x - 3 \geq 0$

$x \geq 3$

domain: $[3, \infty)$.

73. **a.** $(f \circ g)(x) = f(\sqrt{1-x})$

$= \left(\sqrt{1-x}\right)^2 + 4$

$= 1 - x + 4$

$= 5 - x$

b. The domain of $f \circ g$ must exclude any values that are excluded from g.

$1 - x \geq 0$

$-x \geq -1$

$x \leq 1$

domain: (−•, 1].

74. **a.** $(f \circ g)(x) = f(\sqrt{2-x})$

$= \left(\sqrt{2-x}\right)^2 + 1$

$= 2 - x + 1$

$= 3 - x$

b. The domain of $f \circ g$ must exclude any values that are excluded from g.

$2 - x \geq 0$

$-x \geq -2$

$x \leq 2$

domain: (−•, 2].

75. $f(x) = x^4 \quad g(x) = 3x - 1$

76. $f(x) = x^3; g(x) = 2x - 5$

77. $f(x) = \sqrt[3]{x} \quad g(x) = x^2 - 9$

78. $f(x) = \sqrt{x}; g(x) = 5x^2 + 3$

79. $f(x) = |x| \quad g(x) = 2x - 5$

80. $f(x) = |x|; \; g(x) = 3x - 4$

81. $f(x) = \dfrac{1}{x} \quad g(x) = 2x - 3$

82. $f(x) = \dfrac{1}{x}; g(x) = 4x + 5$

83. $(f+g)(-3) = f(-3) + g(-3) = 4 + 1 = 5$

84. $(g-f)(-2) = g(-2) - f(-2) = 2 - 3 = -1$

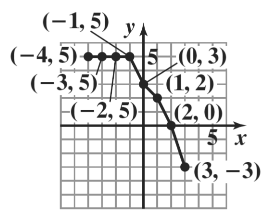
85. $(fg)(2) = f(2)g(2) = (-1)(1) = -1$

86. $\left(\dfrac{g}{f}\right)(3) = \dfrac{g(3)}{f(3)} = \dfrac{0}{-3} = 0$

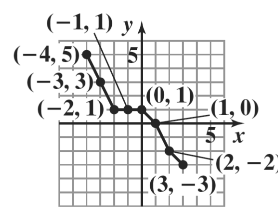
87. The domain of $f + g$ is $[-4, 3]$.

88. The domain of $\dfrac{f}{g}$ is $(-4, 3)$.

89. The graph of $f + g$



90. The graph of $f - g$



91. $(f \circ g)(-1) = f(g(-1)) = f(-3) = 1$

92. $(f \circ g)(1) = f(g(1)) = f(-5) = 3$

93. $(g \circ f)(0) = g(f(0)) = g(2) = -6$

94. $(g \circ f)(-1) = g(f(-1)) = g(1) = -5$

95.
$$(f \circ g)(x) = 7$$
$$2\left(x^2 - 3x + 8\right) - 5 = 7$$
$$2x^2 - 6x + 16 - 5 = 7$$
$$2x^2 - 6x + 11 = 7$$
$$2x^2 - 6x + 4 = 0$$
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0$$
$$x - 1 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = 1 \qquad\qquad x = 2$$

96.
$$(f \circ g)(x) = -5$$
$$1 - 2\left(3x^2 + x - 1\right) = -5$$
$$1 - 6x^2 - 2x + 2 = -5$$
$$-6x^2 - 2x + 3 = -5$$
$$-6x^2 - 2x + 8 = 0$$
$$3x^2 + x - 4 = 0$$
$$(3x+4)(x-1) = 0$$
$$3x + 4 = 0 \quad \text{or} \quad x - 1 = 0$$
$$3x = -4 \qquad\qquad x = 1$$
$$x = -\frac{4}{3}$$

97. **a.** $(M + F)(x) = M(x) + F(x)$
$= (1.53x + 114.8) + (1.46x + 120.7)$
$= 2.99x + 235.5$

b. $(M + F)(x) = 2.99x + 235.5$
$(M + F)(20) = 2.99(20) + 235.5$
$= 295.3$
The total U.S. population in 2005 was 295.3 million.

c. The result in part (b) underestimates the actual total by 2.7 million.

98. **a.** $(F - M)(x) = F(x) - M(x)$
$= (1.46x + 120.7) - (1.53x + 114.8)$
$= -0.07x + 5.9$

b. $(F - M)(x) = -0.07x + 5.9$
$(F - M)(20) = -0.07(20) + 5.9$
$= 4.5$
In 2005 there were 4.5 million more women than men.

c. The result in part (b) overestimates the actual difference by 0.5 million.

99. $(R - C)(20,000)$
$= 65(20,000) - (600,000 + 45(20,000))$
$= -200,000$
The company lost $200,000 since costs exceeded revenues.
$(R - C)(30,000)$
$= 65(30,000) - (600,000 + 45(30,000))$
$= 0$
The company broke even.

100. **a.** The slope for *f* is -0.44 This is the decrease in profits for the first store for each year after 2008.

b. The slope of *g* is 0.51 This is the increase in profits for the second store for each year after 2008.

c. $f + g = -.044x + 13.62 + 0.51x + 11.14$
$= 0.07x + 24.76$
The slope for $f + g$ is 0.07 This is the profit for the two stores combined for each year after 2008.

101. **a.** *f* gives the price of the computer after a $400 discount. *g* gives the price of the computer after a 25% discount.

b. $(f \circ g)(x) = 0.75x - 400$
This models the price of a computer after first a 25% discount and then a $400 discount.

c. $(g \circ f)(x) = 0.75(x - 400)$
This models the price of a computer after first a $400 discount and then a 25% discount.

d. The function $f \circ g$ models the greater discount, since the 25% discount is taken on the regular price first.

102. **a.** *f* gives the cost of a pair of jeans for which a $5 rebate is offered.
g gives the cost of a pair of jeans that has been discounted 40%.

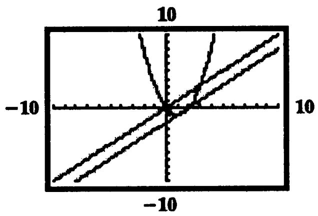
b. $(f \circ g)(x) = 0.6x - 5$
The cost of a pair of jeans is 60% of the regular price minus a $5 rebate.

c. $(g \circ f)(x) = 0.6(x - 5)$
$= 0.6x - 3$
The cost of a pair of jeans is 60% of the regular price minus a $3 rebate.

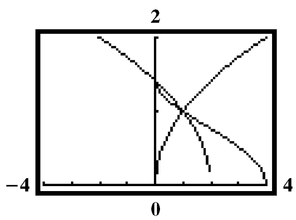
d. $f \circ g$ because of a $5 rebate.

103. – 107. Answers will vary.

108. When your trace reaches $x = 0$, the y value disappears because the function is not defined at $x = 0$.



109.



$(f \circ g)(x) = \sqrt{2-\sqrt{x}}$

The domain of g is $[0,\infty)$.

The expression under the radical in $f \circ g$ must not be negative.

$$2-\sqrt{x} \geq 0$$
$$-\sqrt{x} \geq -2$$
$$\sqrt{x} \leq 2$$
$$x \leq 4$$

domain: $[0,4]$

110. makes sense

111. makes sense

112. does not make sense; Explanations will vary. Sample explanation: It is common that $f \circ g$ and $g \circ f$ are not the same.

113. does not make sense; Explanations will vary. Sample explanation: The diagram illustrates $g(f(x)) = x^2 + 4$.

114. false; Changes to make the statement true will vary. A sample change is: $(f \circ g)(x) = f\left(\sqrt{x^2-4}\right)$

$$= \left(\sqrt{x^2-4}\right)^2 - 4$$
$$= x^2 - 4 - 4$$
$$= x^2 - 8$$

115. false; Changes to make the statement true will vary. A sample change is:

$f(x) = 2x; g(x) = 3x$

$(f \circ g)(x) = f(g(x)) = f(3x) = 2(3x) = 6x$

$(g \circ f)(x) = g(f(x)) = g(f(x)) = 3(2x) = 6x$

116. false; Changes to make the statement true will vary. A sample change is:

$(f \circ g)(4) = f(g(4)) = f(7) = 5$

117. true

118. $(f \circ g)(x) = (f \circ g)(-x)$

$f(g(x)) = f(g(-x))$ since g is even

$f(g(x)) = f(g(x))$ so $f \circ g$ is even

119. Answers will vary.

120. $\{(4,-2),(1,-1),(1,1),(4,2)\}$

The element 1 in the domain corresponds to two elements in the range.

Thus, the relation is not a function.

121.

$$x = \frac{5}{y} + 4$$
$$y(x) = y\left(\frac{5}{y} + 4\right)$$
$$xy = 5 + 4y$$
$$xy - 4y = 5$$
$$y(x-4) = 5$$
$$y = \frac{5}{x-4}$$

122.

$$x = y^2 - 1$$
$$x + 1 = y^2$$
$$\sqrt{x+1} = \sqrt{y^2}$$
$$\sqrt{x+1} = y$$
$$y = \sqrt{x+1}$$

Section 1.8

Check Point Exercises

1. $f\left(g(x)\right) = 4\left(\frac{x+7}{4}\right) - 7$
$= x + 7 - 7$
$= x$

$g\left(f(x)\right) = \frac{(4x-7)+7}{4}$
$= \frac{4x-7+7}{4}$
$= \frac{4x}{4}$
$= x$

$f\left(g(x)\right) = g\left(f(x)\right) = x$

2. $f(x) = 2x + 7$
Replace $f(x)$ with y:
$y = 2x + 7$
Interchange x and y:
$x = 2y + 7$
Solve for y:
$x = 2y + 7$
$x - 7 = 2y$
$\frac{x-7}{2} = y$
Replace y with $f^{-1}(x)$:
$f^{-1}(x) = \frac{x-7}{2}$

3. $f(x) = 4x^3 - 1$
Replace $f(x)$ with y:
$y = 4x^3 - 1$
Interchange x and y:
$x = 4y^3 - 1$
Solve for y:
$x = 4y^3 - 1$
$x + 1 = 4y^3$
$\frac{x+1}{4} = y^3$
$\sqrt[3]{\frac{x+1}{4}} = y$
Replace y with $f^{-1}(x)$:
$f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}$

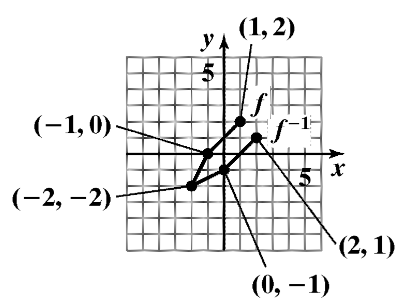
Alternative form for answer:
$f(x)^{-1} = \sqrt[3]{\frac{x+1}{4}} = \frac{\sqrt[3]{x+1}}{\sqrt[3]{4}}$
$= \frac{\sqrt[3]{x+1}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{2x+2}}{\sqrt[3]{8}}$
$= \frac{\sqrt[3]{2x+2}}{2}$

4. $f(x) = \frac{3}{x} - 1$
Replace $f(x)$ with y:
$y = \frac{3}{x} - 1$
Interchange x and y:
$x = \frac{3}{y} - 1$
Solve for y:
$x = \frac{3}{y} - 1$
$xy = 3 - y$
$xy + y = 3$
$y(x+1) = 3$
$y = \frac{3}{x+1}$
Replace y with $f^{-1}(x)$:
$f^{-1}(x) = \frac{3}{x+1}$

5. The graphs of (b) and (c) pass the horizontal line test and thus have an inverse.

6. Find points of f^{-1}.

$f(x)$	$f^{-1}(x)$
(–2,–2)	(–2,–2)
(–1,0)	(0,–1)
(1,2)	(2,1)



7. $f(x) = x^2 + 1$

Replace $f(x)$ with y:

$y = x^2 + 1$

Interchange x and y:

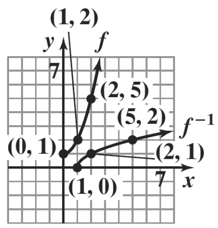
$x = y^2 + 1$

Solve for y:

$$x = y^2 + 1$$
$$x - 1 = y^2$$
$$\sqrt{x-1} = y$$

Replace y with $f^{-1}(x)$:

$f^{-1}(x) = \sqrt{x-1}$



Concept and Vocabulary Check 1.8

1. inverse

2. x; x

3. horizontal; one-to-one

4. $y = x$

Exercise Set 1.8

1. $f(x) = 4x;\ g(x) = \dfrac{x}{4}$

$$f(g(x)) = 4\left(\frac{x}{4}\right) = x$$
$$g(f(x)) = \frac{4x}{4} = x$$

f and *g* are inverses.

2. $f(x) = 6x;\ g(x) = \dfrac{x}{6}$

$$f\left(g(x)\right) = 6\left(\frac{x}{6}\right) = x$$
$$g\left(f(x)\right) = \frac{6x}{6} = x$$

f and *g* are inverses.

3. $f(x) = 3x + 8;\ g(x) = \dfrac{x-8}{3}$

$$f(g(x)) = 3\left(\frac{x-8}{3}\right) + 8 = x - 8 + 8 = x$$
$$g(f(x)) = \frac{(3x+8)-8}{3} = \frac{3x}{3} = x$$

f and *g* are inverses.

4. $f(x) = 4x + 9;\ g(x) = \dfrac{x-9}{4}$

$$f\left(g(x)\right) = 4\left(\frac{x-9}{4}\right) + 9 = x - 9 + 9 = x$$
$$g\left(f(x)\right) = \frac{(4x+9)-9}{4} = \frac{4x}{4} = x$$

f and *g* are inverses.

5. $f(x) = 5x - 9;\ g(x) = \dfrac{x+5}{9}$

$$f(g(x)) = 5\left(\frac{x+5}{9}\right) - 9$$
$$= \frac{5x+25}{9} - 9$$
$$= \frac{5x-56}{9}$$
$$g(f(x)) = \frac{5x-9+5}{9} = \frac{5x-4}{9}$$

f and *g* are not inverses.

6. $f(x) = 3x - 7;\ g(x) = \dfrac{x+3}{7}$

$$f\left(g(x)\right) = 3\left(\frac{x+3}{7}\right) - 7 = \frac{3x+9}{7} - 7 = \frac{3x-40}{7}$$
$$g\left(f(x)\right) = \frac{3x-7+3}{7} = \frac{3x-4}{7}$$

f and *g* are not inverses.

7. $f(x) = \dfrac{3}{x-4};\ g(x) = \dfrac{3}{x} + 4$

$$f(g(x)) = \frac{3}{\frac{3}{x} + 4 - 4} = \frac{3}{\frac{3}{x}} = x$$
$$g(f(x)) = \frac{3}{\frac{3}{x-4}} + 4$$
$$= 3 \cdot \left(\frac{x-4}{3}\right) + 4$$
$$= x - 4 + 4$$
$$= x$$

f and *g* are inverses.

8. $f(x)=\frac{2}{x-5};\ g(x)=\frac{2}{x}+5$

$f(g(x))=\frac{2}{\left(\frac{2}{x}+5\right)-5}=\frac{2x}{2}=x$

$g\left(f(x)\right)=\frac{2}{\frac{2}{x-5}}+5=2\left(\frac{x-5}{2}\right)+5=x-5+5=x$

f and *g* are inverses.

9. $f(x)=-x;\ g(x)=-x$

$f(g(x))=-(-x)=x$

$g(f(x))=-(-x)=x$

f and *g* are inverses.

10. $f(x)=\sqrt[3]{x-4};\ g(x)=x^3+4$

$f\left(g(x)\right)=\sqrt[3]{x^3+4-4}=\sqrt[3]{x^3}=x$

$g\left(f(x)\right)=\left(\sqrt[3]{x-4}\right)^3+4=x-4+4=x$

f and *g* are inverses.

11. a. $f(x)=x+3$

$y=x+3$

$x=y+3$

$y=x-3$

$f^{-1}(x)=x-3$

b. $f(f^{-1}(x))=x-3+3=x$

$f^{-1}(f(x))=x+3-3=x$

12. a. $f(x)=x+5$

$y=x+5$

$x=y+5$

$y=x-5$

$f^{-1}(x)=x-5$

b. $f\left(f^{-1}(x)\right)=x-5+5=x$

$f^{-1}\left(f(x)\right)=x+5-5=x$

13. a. $f(x)=2x$

$y=2x$

$x=2y$

$y=\frac{x}{2}$

$f^{-1}(x)=\frac{x}{2}$

b. $f(f^{-1}(x))=2\left(\frac{x}{2}\right)=x$

$f^{-1}(f(x))=\frac{2x}{2}=x$

14. a. $f(x)=4x$

$y=4x$

$x=4y$

$y=\frac{x}{4}$

$f^{-1}(x)=\frac{x}{4}$

b. $f\left(f^{-1}(x)\right)=4\left(\frac{x}{4}\right)=x$

$f^{-1}\left(f(x)\right)=\frac{4x}{4}=x$

15. a. $f(x)=2x+3$

$y=2x+3$

$x=2y+3$

$x-3=2y$

$y=\frac{x-3}{2}$

$f^{-1}(x)=\frac{x-3}{2}$

b. $f(f^{-1}(x))=2\left(\frac{x-3}{2}\right)+3$

$=x-3+3$

$=x$

$f^{-1}(f(x))=\frac{2x+3-3}{2}=\frac{2x}{2}=x$

16. a. $f(x)=3x-1$

$y=3x-1$

$x=3y-1$

$x+1=3y$

$y=\frac{x+1}{3}$

$f^{-1}(x)=\frac{x+1}{3}$

b. $f\left(f^{-1}(x)\right)=3\left(\frac{x+1}{3}\right)-1=x+1-1=x$

$f^{-1}\left(f(x)\right)=\frac{3x-1+1}{3}=\frac{3x}{3}=x$

17. a.

$$\begin{aligned} f(x) &= x^3+2 \\ y &= x^3+2 \\ x &= y^3+2 \\ x-2 &= y^3 \\ y &= \sqrt[3]{x-2} \\ f^{-1}(x) &= \sqrt[3]{x-2} \end{aligned}$$

b.

$$\begin{aligned} f(f^{-1}(x)) &= \left(\sqrt[3]{x-2}\right)^3+2 \\ &= x-2+2 \\ &= x \end{aligned}$$

$$f^{-1}(f(x)) = \sqrt[3]{x^3+2-2} = \sqrt[3]{x^3} = x$$

18. a.

$$\begin{aligned} f(x) &= x^3-1 \\ y &= x^3-1 \\ x &= y^3-1 \\ x+1 &= y^3 \\ y &= \sqrt[3]{x+1} \\ f^{-1}(x) &= \sqrt[3]{x+1} \end{aligned}$$

b.

$$\begin{aligned} f(f^{-1}(x)) &= \left(\sqrt[3]{x+1}\right)^3-1 \\ &= x+1-1 \\ &= x \end{aligned}$$

$$f^{-1}(f(x)) = \sqrt[3]{x^3-1+1} = \sqrt[3]{x^3} = x$$

19. a.

$$\begin{aligned} f(x) &= (x+2)^3 \\ y &= (x+2)^3 \\ x &= (y+2)^3 \\ \sqrt[3]{x} &= y+2 \\ y &= \sqrt[3]{x}-2 \\ f^{-1}(x) &= \sqrt[3]{x}-2 \end{aligned}$$

b.

$$f(f^{-1}(x)) = \left(\sqrt[3]{x}-2+2\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$$

$$\begin{aligned} f^{-1}(f(x)) &= \sqrt[3]{(x+2)^3}-2 \\ &= x+2-2 \\ &= x \end{aligned}$$

20. a.

$$\begin{aligned} f(x) &= (x-1)^3 \\ y &= (x-1)^3 \\ x &= (y-1)^3 \\ \sqrt[3]{x} &= y-1 \\ y &= \sqrt[3]{x}+1 \end{aligned}$$

b.

$$f\left(f^{-1}(x)\right) = \left(\sqrt[3]{x}+1-1\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$$

$$f^{-1}\left(f(x)\right) = \sqrt[3]{(x-1^3}+1 = x-1+1 = x$$

21. a.

$$\begin{aligned} f(x) &= \frac{1}{x} \\ y &= \frac{1}{x} \\ x &= \frac{1}{y} \\ xy &= 1 \\ y &= \frac{1}{x} \\ f^{-1}(x) &= \frac{1}{x} \end{aligned}$$

b.

$$f(f^{-1}(x)) = \frac{1}{\frac{1}{x}} = x$$

$$f^{-1}(f(x)) = \frac{1}{\frac{1}{x}} = x$$

22. a.

$$\begin{aligned} f(x) &= \frac{2}{x} \\ y &= \frac{2}{x} \\ x &= \frac{2}{y} \\ xy &= 2 \\ y &= \frac{2}{x} \\ f^{-1}(x) &= \frac{2}{x} \end{aligned}$$

b.

$$f(f^{-1}(x)) = \frac{2}{\frac{2}{x}} = 2 \cdot \frac{x}{2} = x$$

$$f^{-1}(f(x)) = \frac{2}{\frac{2}{x}} = 2 \cdot \frac{x}{2} = x$$

23. **a.**

$$f(x) = \sqrt{x}$$
$$y = \sqrt{x}$$
$$x = \sqrt{y}$$
$$y = x^2$$
$$f^{-1}(x) = x^2, x \geq 0$$

b. $f(f^{-1}(x)) = \sqrt{x^2} = |x| = x$ for $x \geq 0$.

$$f^{-1}(f(x)) = (\sqrt{x})^2 = x$$

24. **a.**

$$f(x) = \sqrt[3]{x}$$
$$y = \sqrt[3]{x}$$
$$x = \sqrt[3]{y}$$
$$y = x^3$$
$$f^{-1}(x) = x^3$$

b.

$$f\left(f^{-1}(x)\right) = \sqrt[3]{x^3} = x$$
$$f^{-1}\left(f(x)\right) = \left(\sqrt[3]{x}\right)^3 = x$$

25. **a.**

$$f(x) = \frac{7}{x} - 3$$
$$y = \frac{7}{x} - 3$$
$$x = \frac{7}{y} - 3$$
$$xy = 7 - 3y$$
$$xy + 3y = 7$$
$$y(x+3) = 7$$
$$y = \frac{7}{x+3}$$
$$f^{-1}(x) = \frac{7}{x+3}$$

b.

$$f\left(f^{-1}(x)\right) = \frac{7}{\frac{7}{x+3}} - 3 = x$$
$$f^{-1}\left(f(x)\right) = \frac{7}{\frac{7}{x} - 3 + 3} = x$$

26. **a.**

$$f(x) = \frac{4}{x} + 9$$
$$y = \frac{4}{x} + 9$$
$$x = \frac{4}{y} + 9$$
$$xy = 4 + 9y$$
$$xy - 9y = 4$$
$$y(x-9) = 4$$
$$y = \frac{4}{x-9}$$
$$f^{-1}(x) = \frac{4}{x-9}$$

b.

$$f\left(f^{-1}(x)\right) = \frac{4}{\frac{4}{x-9}} + 9 = x$$
$$f^{-1}\left(f(x)\right) = \frac{4}{\frac{4}{x} + 9 - 9} = x$$

27. **a.**

$$f(x) = \frac{2x+1}{x-3}$$
$$y = \frac{2x+1}{x-3}$$
$$x = \frac{2y+1}{y-3}$$
$$x(y-3) = 2y+1$$
$$xy - 3x = 2y + 1$$
$$xy - 2y = 3x + 1$$
$$y(x-2) = 3x + 1$$
$$y = \frac{3x+1}{x-2}$$
$$f^{-1}(x) = \frac{3x+1}{x-2}$$

b. $f(f^{-1}(x)) = \dfrac{2\left(\dfrac{3x+1}{x-2}\right)+1}{\dfrac{3x+1}{x-2}-3}$

$= \dfrac{2(3x+1)+x-2}{3x+1-3(x-2)} = \dfrac{6x+2+x-2}{3x+1-3x+6}$

$= \dfrac{7x}{7} = x$

$f^{-1}(f(x)) = \dfrac{3\left(\dfrac{2x+1}{x-3}\right)+1}{\dfrac{2x+1}{x-3}-2}$

$= \dfrac{3(2x+1)+x-3}{2x+1-2(x-3)}$

$= \dfrac{6x+3+x-3}{2x+1-2x+6} = \dfrac{7x}{7} = x$

28. **a.** $f(x) = \dfrac{2x-3}{x+1}$

$y = \dfrac{2x-3}{x+1}$

$x = \dfrac{2y-3}{y+1}$

$xy + x = 2y - 3$

$y(x-2) = -x-3$

$y = \dfrac{-x-3}{x-2}$

$f^{-1}(x) = \dfrac{-x-3}{x-2}, \quad x \neq 2$

b. $f\left(f^{-1}(x)\right) = \dfrac{2\left(\dfrac{-x-3}{x-2}\right)-3}{\dfrac{-x-3}{x-2}+1}$

$= \dfrac{-2x-6-3x+6}{-x-3+x-2} = \dfrac{-5x}{-5} = x$

$f^{-1}(f(x)) = \dfrac{-\left(\dfrac{2x-3}{x+1}\right)-3}{\dfrac{2x-3}{x+1}-2}$

$= \dfrac{-2x+3-3x-3}{2x-3-2x-2} = \dfrac{-5x}{-5} = x$

29. The function fails the horizontal line test, so it does not have an inverse function.

30. The function passes the horizontal line test, so it does have an inverse function.

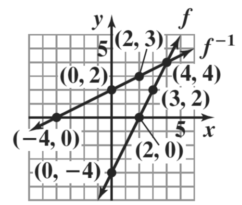
31. The function fails the horizontal line test, so it does not have an inverse function.

32. The function fails the horizontal line test, so it does not have an inverse function.

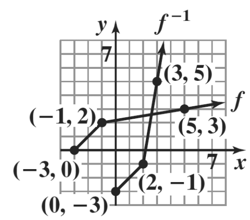
33. The function passes the horizontal line test, so it does have an inverse function.

34. The function passes the horizontal line test, so it does have an inverse function.

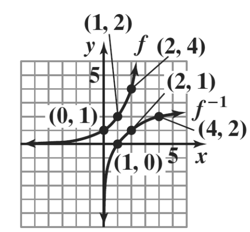
35.



36.



37.



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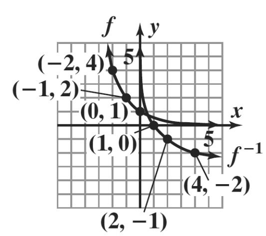


39. **a.** $f(x) = 2x-1$

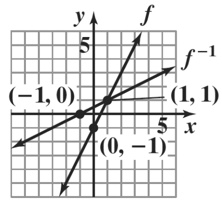
$y = 2x-1$

$x = 2y-1$

$x+1 = 2y$

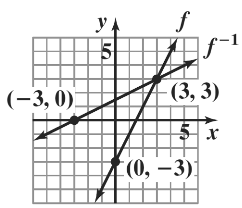
$\dfrac{x+1}{2} = y$

$f^{-1}(x) = \dfrac{x+1}{2}$

b. 

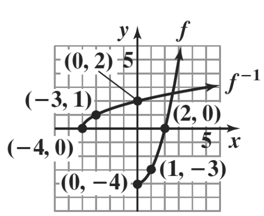
c. domain of f : $(-\infty,\infty)$
range of f : $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

40. **a.**
$$f(x) = 2x-3$$
$$y = 2x-3$$
$$x = 2y-3$$
$$x+3 = 2y$$
$$\frac{x+3}{2} = y$$
$$f^{-1}(x) = \frac{x+3}{2}$$

b. 

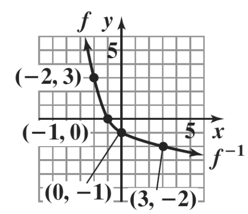
c. domain of f : $(-\infty,\infty)$
range of f : $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

41. **a.**
$$f(x) = x^2-4$$
$$y = x^2-4$$
$$x = y^2-4$$
$$x+4 = y^2$$
$$\sqrt{x+4} = y$$
$$f^{-1}(x) = \sqrt{x+4}$$

b. 

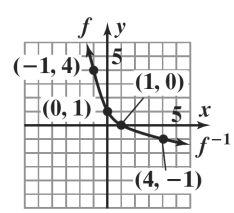
c. domain of f : $[0,\infty)$
range of f : $[-4,\infty)$
domain of f^{-1}: $[-4,\infty)$
range of f^{-1}: $[0,\infty)$

42. **a.**
$$f(x) = x^2-1$$
$$y = x^2-1$$
$$x = y^2-1$$
$$x+1 = y^2$$
$$-\sqrt{x+1} = y$$
$$f^{-1}(x) = -\sqrt{x+1}$$

b. 

c. domain of f : $(-\infty,0]$
range of f : $[-1,\infty)$
domain of f^{-1}: $[-1,\infty)$
range of f^{-1}: $(-\infty,0]$

43. **a.**
$$f(x) = (x-1)^2$$
$$y = (x-1)^2$$
$$x = (y-1)^2$$
$$-\sqrt{x} = y-1$$
$$-\sqrt{x}+1 = y$$
$$f^{-1}(x) = 1-\sqrt{x}$$

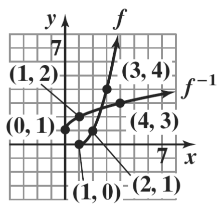
b. 

c. domain of f : $(-\infty,1]$
range of f : $[0,\infty)$
domain of f^{-1}: $[0,\infty)$
range of f^{-1}: $(-\infty,1]$

44. **a.**

$$f(x) = (x-1)^2$$
$$y = (x-1)^2$$
$$x = (y-1)^2$$
$$\sqrt{x} = y-1$$
$$\sqrt{x}+1 = y$$
$$f^{-1}(x) = 1+\sqrt{x}$$

b.



c. domain of f: $[1,\infty)$

range of f: $[0,\infty)$

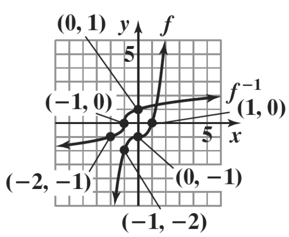
domain of f^{-1}: $[0,\infty)$

range of f^{-1}: $[1,\infty)$

45. **a.**

$$f(x) = x^3 - 1$$
$$y = x^3 - 1$$
$$x = y^3 - 1$$
$$x+1 = y^3$$
$$\sqrt[3]{x+1} = y$$
$$f^{-1}(x) = \sqrt[3]{x+1}$$

b.



c. domain of f: $(-\infty,\infty)$

range of f: $(-\infty,\infty)$

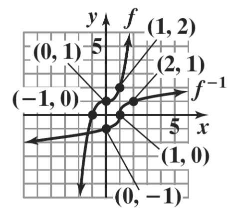
domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

46. **a.**

$$f(x) = x^3 + 1$$
$$y = x^3 + 1$$
$$x = y^3 + 1$$
$$x-1 = y^3$$
$$\sqrt[3]{x-1} = y$$
$$f^{-1}(x) = \sqrt[3]{x-1}$$

b.



c. domain of f: $(-\infty,\infty)$

range of f: $(-\infty,\infty)$

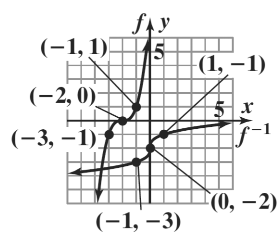
domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

47. **a.**

$$f(x) = (x+2)^3$$
$$y = (x+2)^3$$
$$x = (y+2)^3$$
$$\sqrt[3]{x} = y+2$$
$$\sqrt[3]{x} - 2 = y$$
$$f^{-1}(x) = \sqrt[3]{x} - 2$$

b.



c. domain of f: $(-\infty,\infty)$

range of f: $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

48. a. $f(x) = (x-2)^3$

$y = (x-2)^3$

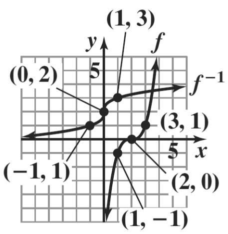
$x = (y-2)^3$

$\sqrt[3]{x} = y - 2$

$\sqrt[3]{x} + 2 = y$

$f^{-1}(x) = \sqrt[3]{x} + 2$

b.



c. domain of f : $(-\infty, \infty)$

range of f : $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

49. a. $f(x) = \sqrt{x-1}$

$y = \sqrt{x-1}$

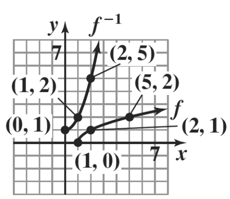
$x = \sqrt{y-1}$

$x^2 = y - 1$

$x^2 + 1 = y$

$f^{-1}(x) = x^2 + 1$

b.



c. domain of f : $[1, \infty)$

range of f : $[0, \infty)$

domain of f^{-1}: $[0, \infty)$

range of f^{-1}: $[1, \infty)$

50. a. $f(x) = \sqrt{x} + 2$

$y = \sqrt{x} + 2$

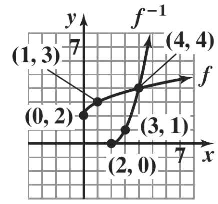
$x = \sqrt{y} + 2$

$x - 2 = \sqrt{y}$

$(x-2)^2 = y$

$f^{-1}(x) = (x-2)^2$

b.



c. domain of f : $[0, \infty)$

range of f : $[2, \infty)$

domain of f^{-1}: $[2, \infty)$

range of f^{-1}: $[0, \infty)$

51. a. $f(x) = \sqrt[3]{x} + 1$

$y = \sqrt[3]{x} + 1$

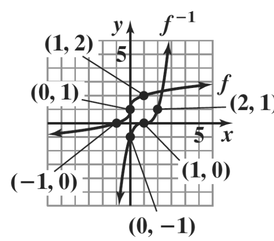
$x = \sqrt[3]{y} + 1$

$x - 1 = \sqrt[3]{y}$

$(x-1)^3 = y$

$f^{-1}(x) = (x-1)^3$

b.



c. domain of f : $(-\infty, \infty)$

range of f : $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

52. a. $f(x) = \sqrt[3]{x-1}$

$y = \sqrt[3]{x-1}$

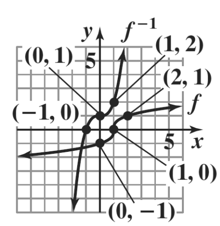
$x = \sqrt[3]{y-1}$

$x^3 = y - 1$

$x^3 + 1 = y$

$f^{-1}(x) = x^3 + 1$

b.



c. domain of f: $(-\infty,\infty)$
range of f: $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

53. $f(g(1)) = f(1) = 5$

54. $f(g(4)) = f(2) = -1$

55. $(g \circ f)(-1) = g(f(-1)) = g(1) = 1$

56. $(g \circ f)(0) = g(f(0)) = g(4) = 2$

57. $f^{-1}(g(10)) = f^{-1}(-1) = 2$, since $f(2) = -1$.

58. $f^{-1}(g(1)) = f^{-1}(1) = -1$, since $f(-1) = 1$.

59.
$$\begin{aligned}(f \circ g)(0) &= f(g(0))\\ &= f(4\cdot 0 - 1)\\ &= f(-1) = 2(-1) - 5 = -7\end{aligned}$$

60.
$$\begin{aligned}(g \circ f)(0) &= g(f(0))\\ &= g(2\cdot 0 - 5)\\ &= g(-5) = 4(-5) - 1 = -21\end{aligned}$$

61. Let $f^{-1}(1) = x$. Then
$$\begin{aligned}f(x) &= 1\\ 2x - 5 &= 1\\ 2x &= 6\\ x &= 3\end{aligned}$$
Thus, $f^{-1}(1) = 3$

62. Let $g^{-1}(7) = x$. Then
$$\begin{aligned}g(x) &= 7\\ 4x - 1 &= 7\\ 4x &= 8\\ x &= 2\end{aligned}$$
Thus, $g^{-1}(7) = 2$

63.
$$\begin{aligned}g(f[h(1)]) &= g(f[1^2 + 1 + 2])\\ &= g(f(4))\\ &= g(2\cdot 4 - 5)\\ &= g(3)\\ &= 4\cdot 3 - 1 = 11\end{aligned}$$

64.
$$\begin{aligned}f(g[h(1)]) &= f(g[1^2 + 1 + 2])\\ &= f(g(4))\\ &= f(4\cdot 4 - 1)\\ &= f(15)\\ &= 2\cdot 15 - 5 = 25\end{aligned}$$

65. **a.** {(Zambia, 4.2), (Colombia, 4.5), (Poland, 3.3), (Italy, 3.3), (United States, 2.5)}

b. {(4.2, Zambia), (4.5 , Colombia), (3.3 , Poland), (3.3, Italy), (2.5, United States)}
f is not a one-to-one function because the inverse of *f* is not a function.

66. **a.** {(Zambia,- 7.3), (Colombia, - 4.5), (Poland, - 2.8), (Italy, - 2.8), (United States, - 1.9)}

b. { (- 7.3, Zambia), (- 4.5, Colombia), (- 2.8, Po land), (- 2.8, Italy), (- 1.9, United States)}
g is not a one-to-one function because the inverse of *g* is not a function.

67. **a.** It passes the horizontal line test and is one-to-one.

b. $f^{-1}(0.25) = 15$ If there are 15 people in the room, the probability that 2 of them have the same birthday is 0.25.
$f^{-1}(0.5) = 21$ If there are 21 people in the room, the probability that 2 of them have the same birthday is 0.5.
$f^{-1}(0.7) = 30$ If there are 30 people in the room, the probability that 2 of them have the same birthday is 0.7.

68. **a.** This function fails the horizontal line test. Thus, this function does not have an inverse.

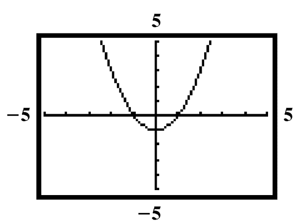
b. The average happiness level is 3 at 12 noon and at 7 p.m. These values can be represented as $(12, 3)$ and $(19, 3)$.

c. The graph does not represent a one-to-one function. $(12, 3)$ and $(19, 3)$ are an example of two *x*-values that correspond to the same *y*-value.

69.
$$\begin{aligned} f(g(x)) &= \frac{9}{5}\left[\frac{5}{9}(x-32)\right]+32 \\ &= x-32+32 \\ &= x \end{aligned}$$
$$\begin{aligned} g(f(x)) &= \frac{5}{9}\left[\left(\frac{9}{5}x+32\right)-32\right] \\ &= x+32-32 \\ &= x \end{aligned}$$
f and *g* are inverses.

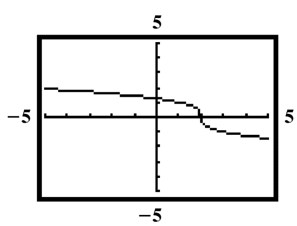
70. – 75. Answers will vary.

76.



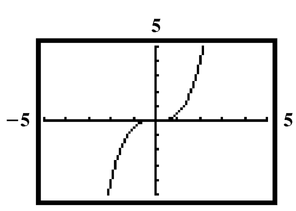
not one-to-one

77.



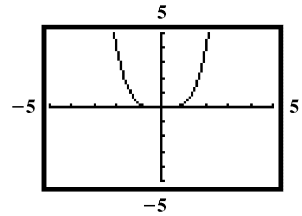
one-to-one

78.



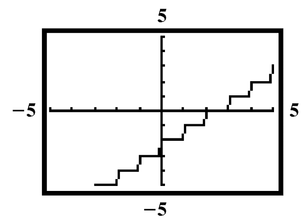
one-to-one

79.



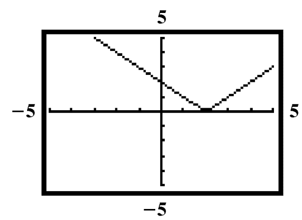
not one-to-one

80.



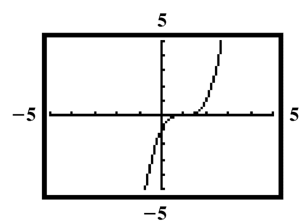
not one-to-one

81.



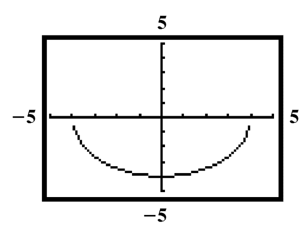
not one-to-one

82.



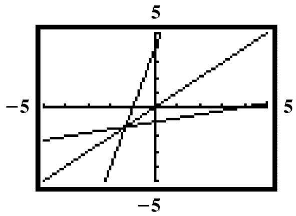
one-to-one

83.



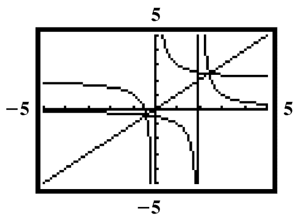
not one-to-one

84.



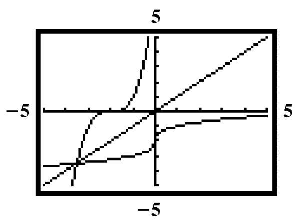
f and g are inverses

85.



f and g are inverses

86.



f and g are inverses

87. makes sense

88. makes sense

89. makes sense

90. makes sense

91. false; Changes to make the statement true will vary. A sample change is: The inverse is $\{(4,1), (7,2)\}$.

92. false; Changes to make the statement true will vary. A sample change is: $f(x) = 5$ is a horizontal line, so it does not pass the horizontal line test.

93. false; Changes to make the statement true will vary. A sample change is: $f^{-1}(x) = \frac{x}{3}$.

94. true

95. $(f \circ g)(x) = 3(x+5) = 3x+15.$

$$y = 3x+15$$
$$x = 3y+15$$
$$y = \frac{x-15}{3}$$
$$(f \circ g)^{-1}(x) = \frac{x-15}{3}$$
$$g(x) = x+5$$
$$y = x+5$$
$$x = y+5$$
$$y = x-5$$
$$g^{-1}(x) = x-5$$
$$f(x) = 3x$$
$$y = 3x$$
$$x = 3y$$
$$y = \frac{x}{3}$$
$$f^{-1}(x) = \frac{x}{3}$$
$$(g^{-1} \circ f^{-1})(x) = \frac{x}{3} - 5 = \frac{x-15}{3}$$

96.

$$f(x) = \frac{3x-2}{5x-3}$$
$$y = \frac{3x-2}{5x-3}$$
$$x = \frac{3y-2}{5y-3}$$
$$x(5y-3) = 3y-2$$
$$5xy-3x = 3y-2$$
$$5xy-3y = 3x-2$$
$$y(5x-3) = 3x-2$$
$$y = \frac{3x-2}{5x-3}$$
$$f^{-1}(x) = \frac{3x-2}{5x-3}$$

Note: An alternative approach is to show that $(f \circ f)(x) = x.$

97. No, there will be 2 times when the spacecraft is at the same height, when it is going up and when it is coming down.

98. $8 + f^{-1}(x-1) = 10$

$$f^{-1}(x-1) = 2$$
$$f(2) = x-1$$
$$6 = x-1$$
$$7 = x$$
$$x = 7$$

99. Answers will vary.