

Chapter 2: Motion in a Straight Line

Concept Checks

2.1. d 2.2. b 2.3. b 2.4. c 2.5. a) 3 b) 1 c) 4 d) 2 2.6. c 2.7. d 2.8. c 2.9. d

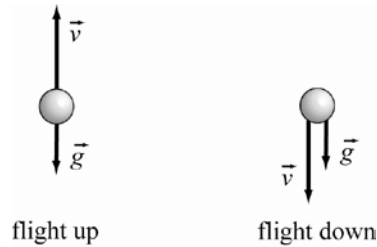
Multiple-Choice Questions

2.1. e 2.2. c 2.3. c 2.4. b 2.5. e 2.6. a 2.7. d 2.8. c 2.9. a 2.10. b 2.11. b 2.12. d 2.13. c 2.14. d 2.15. a 2.16. c

Conceptual Questions

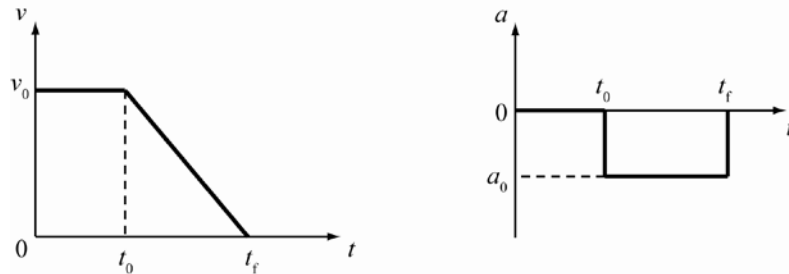
- 2.17. Velocity and speed are defined differently. The magnitude of average velocity and average speed are the same only when the direction of movement does not change. If the direction changes during movement, it is known that the net displacement is smaller than the net distance. Using the definition of average velocity and speed, it can be said that the magnitude of average velocity is less than the average speed when the direction changes during movement. Here, only Christine changes direction during her movement. Therefore, only Christine has a magnitude of average velocity which is smaller than her average speed.

- 2.18. The acceleration due to gravity is always pointing downward to the center of the Earth.



It can be seen that the direction of velocity is opposite to the direction of acceleration when the ball is in flight upward. The direction of velocity is the same as the direction of acceleration when the ball is in flight downward.

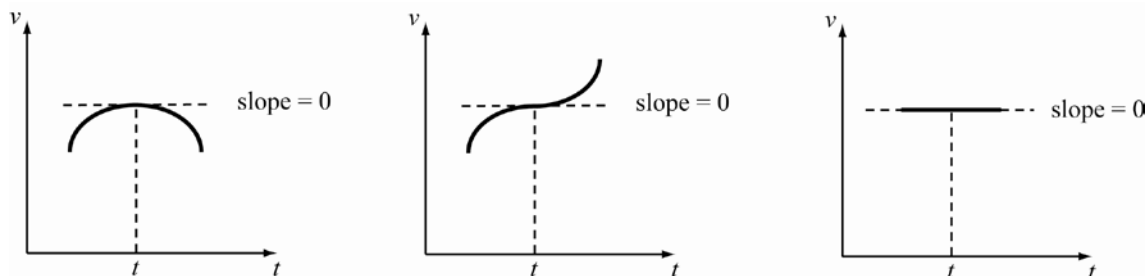
- 2.19. The car, before the brakes are applied, has a constant velocity, v_0 , and zero acceleration. After the brakes are applied, the acceleration is constant and in the direction opposite to the velocity. In velocity versus time and acceleration versus time graphs, the motion is described in the figures below.



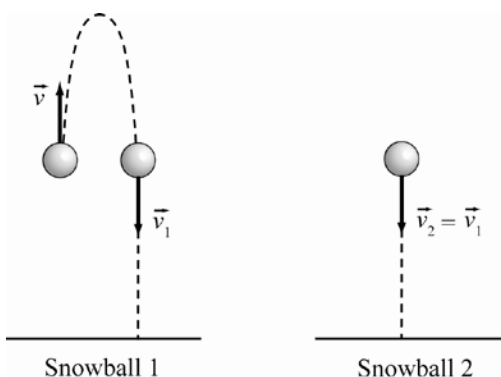
- 2.20. There are two cars, car 1 and car 2. The decelerations are $a_1 = 2a_2 = -a_0$ after applying the brakes. Before applying the brakes, the velocities of both cars are the same, $v_1 = v_2 = v_0$. When the cars have completely stopped, the final velocities are zero, $v_f = 0$. $v_f = v_0 + at = 0 \Rightarrow t = -\frac{v_0}{a}$. Therefore, the ratio of time taken

$$\text{to stop is Ratio} = \frac{\text{time of car 1}}{\text{time of car 2}} = \frac{-v_0 / -a_0}{-v_0 / \left(-\frac{1}{2}a_0\right)} = \frac{1}{2}. \text{ So the ratio is one half.}$$

- 2.21. Here a and v are instantaneous acceleration and velocity. If $a = 0$ and $v \neq 0$ at time t , then at that moment the object is moving at a constant velocity. In other words, the slope of a curve in a velocity versus time plot is zero at time t . See the plots below.

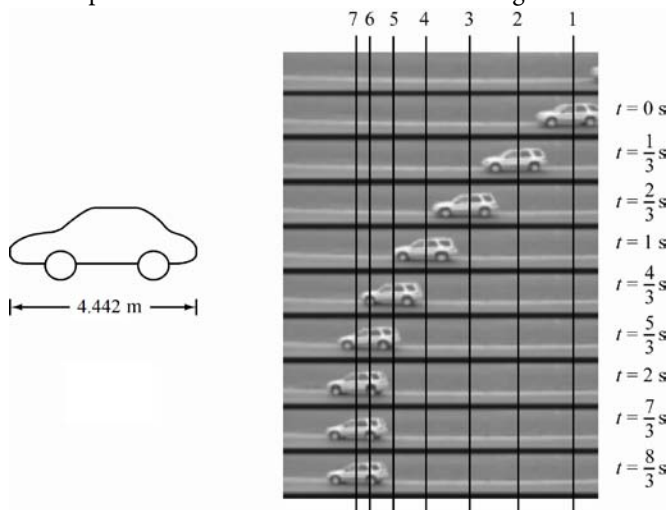


- 2.22. The direction of motion is determined by the direction of velocity. Acceleration is defined as a change in velocity per change in time. The change in velocity, Δv , can be positive or negative depending on the values of initial and final velocities, $\Delta v = v_f - v_i$. If the acceleration is in the opposite direction to the motion, it means that the magnitude of the objects velocity is decreasing. This occurs when an object is slowing down.
- 2.23. If there is no air resistance, then the acceleration does not depend on the mass of an object. Therefore, both snowballs have the same acceleration. Since initial velocities are zero, and the snowballs will cover the same distance, both snowballs will hit the ground at the same time. They will both have the same speed.
- 2.24. Acceleration is independent of the mass of an object if there is no air resistance.

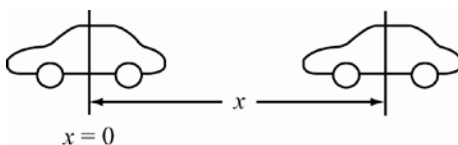


Snowball 1 will return to its original position after Δt , and then it falls in the same way as snowball 2. Therefore snowball 2 will hit the ground first since it has a shorter path. However, both snowballs have the same speed when they hit the ground.

- 2.25. Make sure the scale for the displacements of the car is correct. The length of the car is 174.9 in = 4.442 m.



Measuring the length of the car in the figure above with a ruler, the car in this scale is 0.80 ± 0.05 cm. Draw vertical lines at the center of the car as shown in the figure above. Assume line 7 is the origin ($x = 0$).



Assume a constant acceleration $a = a_0$. Use the equations $v = v_0 + at$ and $x = x_0 + v_0t + (1/2)at^2$. When the car has completely stopped, $v = 0$ at $t = t_0$.

$$0 = v_0 + at_0 \Rightarrow v_0 = -at_0$$

Use the final stopping position as the origin, $x = 0$ at $t = t_0$.

$$0 = x_0 + v_0t_0 + \frac{1}{2}at_0^2$$

Substituting $v_0 = -at_0$ and simplifying gives

$$x_0 - at_0^2 + \frac{1}{2}at_0^2 = 0 \Rightarrow x_0 - \frac{1}{2}at_0^2 = 0 \Rightarrow a = \frac{2x_0}{t_0^2}$$

Note that time t_0 is the time required to stop from a distance x_0 . First measure the length of the car. The length of the car is 0.80 cm. The actual length of the car is 4.442 m, therefore the scale is $\frac{4.442 \text{ m}}{0.80 \text{ cm}} = 5.5 \text{ m/cm}$. The error in measurement is $(0.05 \text{ cm}) 5.5 \text{ m/cm} \approx 0.275 \text{ m}$ (round at the end).

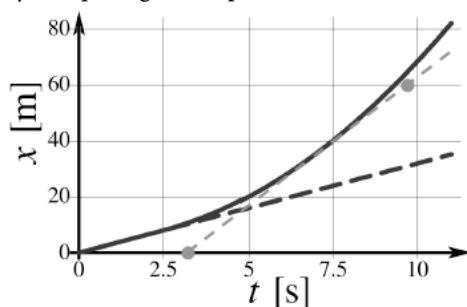
So the scale is $5.5 \pm 0.275 \text{ m/cm}$. The farthest distance of the car from the origin is $2.9 \pm 0.05 \text{ cm}$. Multiplying by the scale, 15.95 m, $t_0 = (0.333)(6 \text{ s}) = 1.998 \text{ s}$. The acceleration can be found using

$$a = 2x_0 / t_0^2: a = \frac{2(15.95 \text{ m})}{(1.998 \text{ s})^2} = 7.991 \text{ m/s}^2. \text{ Because the scale has two significant digits, round the result to}$$

two significant digits: $a = 8.0 \text{ m/s}^2$. Since the error in the measurement is $\Delta x_0 = 0.275 \text{ m}$, the error of the acceleration is

$$\Delta a = \frac{2\Delta x_0}{t_0^2} = \frac{2(0.275 \text{ m})}{(1.998 \text{ s})^2} \approx 0.1 \text{ m/s}^2.$$

- 2.26. Velocity can be estimated by computing the slope of a curve in a distance versus time plot.



Velocity is defined by $v = \Delta x / \Delta t$. If acceleration is constant, then $a = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$. (a) Estimate the slope of the dashed blue line. Pick two points: it is more accurate to pick a point that coincides with horizontal lines of the grid. Choosing points $t = 0$ s, $x = 0$ m and $t = 6.25$ s, $x = 20$ m:

$$v = \frac{20. \text{ m} - 0 \text{ m}}{6.25 \text{ s} - 0 \text{ s}} = 3.2 \text{ m/s}$$

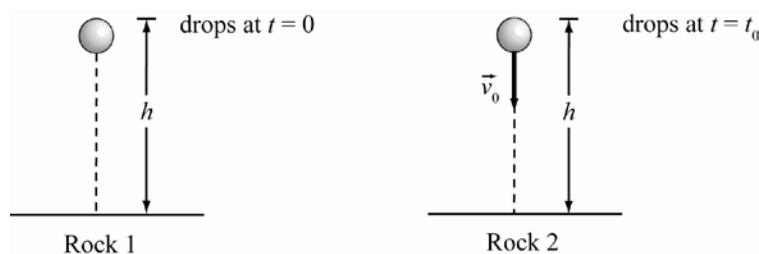
(b) Examine the sketch. There is a tangent to the curve at $t = 7.5$ s. Pick two points on the line. Choosing points: $t = 3.4$ s, $x = 0$ m and $t = 9.8$ s, $x = 60$ m:

$$v = \frac{60. \text{ m} - 0 \text{ m}}{9.8 \text{ s} - 3.4 \text{ s}} = 9.4 \text{ m/s}$$

(c) From (a), $v = 3.2$ m/s at $t = 2.5$ s and from (b), $v = 9.4$ m/s at $t = 7.5$ s. From the definition of constant acceleration,

$$a = \frac{9.4 \text{ m/s} - 3.2 \text{ m/s}}{7.5 \text{ s} - 2.5 \text{ s}} = \frac{6.2 \text{ m/s}}{5.0 \text{ s}} = 1.2 \text{ m/s}^2.$$

- 2.27. There are two rocks, rock 1 and rock 2. Both rocks are dropped from height h . Rock 1 has initial velocity $v = 0$ and rock 2 has $v = v_0$ and is thrown at $t = t_0$.



Rock 1: $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

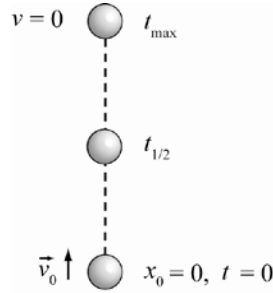
Rock 2: $h = v_0(t - t_0) + \frac{1}{2}g(t - t_0)^2 \Rightarrow \frac{1}{2}g(t - t_0)^2 + v_0(t - t_0) - h = 0$

This equation has roots $t - t_0 = \frac{-v_0 \pm \sqrt{v_0^2 + 2gh}}{g}$. Choose the positive root since $(t - t_0) > 0$. Therefore

$t_0 = t + \frac{v_0 - \sqrt{v_0^2 + 2gh}}{g}$. Substituting $t = \sqrt{\frac{2h}{g}}$ gives:

$$t_0 = \sqrt{\frac{2h}{g}} + \frac{v_0}{g} - \frac{\sqrt{v_0^2 + 2gh}}{g} \text{ or } \sqrt{\frac{2h}{g}} + \frac{v_0}{g} - \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}.$$

- 2.28. I want to know when the object is at half its maximum height. The wrench is thrown upwards with an initial velocity $v(t=0) = v_0$, $x = x_0 + v_0 t - \frac{1}{2} g t^2$, $v = v_0 - g t$, and $g = 9.81 \text{ m/s}^2$.



At maximum height, $v = 0$. $v = v_0 - g t \Rightarrow 0 = v_0 - g t_{\max} \Rightarrow v_0 = g t_{\max}$. Substitute $t_{\max} = v_0 / g$ into $x = x_0 + v_0 t - (1/2) g t^2$.

$$x_{\max} = v_0 \left(\frac{v_0}{g} \right) - \frac{1}{2} g \left(\frac{v_0}{g} \right)^2 = \frac{v_0^2}{g} - \frac{1}{2} \left(\frac{v_0^2}{g} \right) = \frac{v_0^2}{g} \left(1 - \frac{1}{2} \right) = \frac{v_0^2}{2g}$$

Therefore, half of the maximum height is $x_{1/2} = \frac{v_0^2}{4g}$. Substitute this into the equation for x .

$$x_{1/2} = \frac{v_0^2}{4g} = v_0 t_{1/2} - \frac{1}{2} g t_{1/2}^2 \Rightarrow \frac{1}{2} g t_{1/2}^2 - v_0 t_{1/2} + \frac{v_0^2}{4g} = 0$$

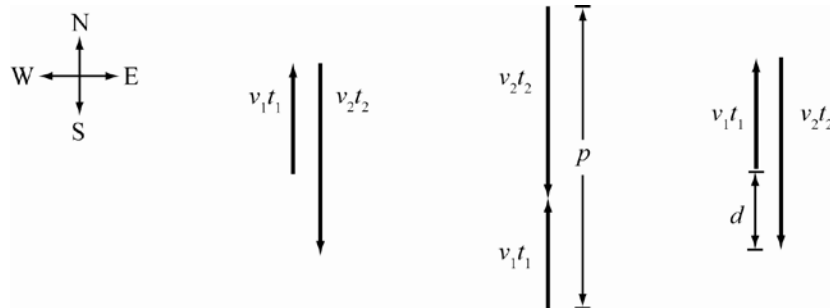
This is a quadratic equation with respect to $t_{1/2}$. The solutions to this equation are:

$$t_{1/2} = \frac{v_0 \pm \sqrt{v_0^2 - 4 \left(\frac{1}{2} g \right) \left(\frac{v_0^2}{4g} \right)}}{2 \left(\frac{1}{2} g \right)} = \frac{v_0 \pm \sqrt{v_0^2 - \frac{1}{2} v_0^2}}{g} = \frac{v_0 \pm v_0 \left(\frac{1}{\sqrt{2}} \right)}{g} = \frac{v_0}{g} \left(1 \pm \frac{1}{\sqrt{2}} \right)$$

Exercises

- 2.29. **THINK:** What is the distance traveled, p , and the displacement d if $v_1 = 30.0 \text{ m/s}$ due north for $t_1 = 10.0 \text{ min}$ and $v_2 = 40.0 \text{ m/s}$ due south for $t_2 = 20.0 \text{ min}$? Times should be in SI units: $t_1 = 10.0 \text{ min} (60 \text{ s/min}) = 6.00 \cdot 10^2 \text{ s}$, $t_2 = 20.0 \text{ min} (60 \text{ s/min}) = 1.20 \cdot 10^3 \text{ s}$.

SKETCH:



RESEARCH: The distance is equal to the product of velocity and time. The distance traveled is $p = v_1 t_1 + v_2 t_2$ and the displacement is the distance between where you start and where you finish, $d = v_1 t_1 - v_2 t_2$.

SIMPLIFY: There is no need to simplify.

CALCULATE: $p = v_1 t_1 + v_2 t_2 = (30. \text{ m/s})(6.00 \cdot 10^2 \text{ s}) + (40. \text{ m/s})(1.20 \cdot 10^3 \text{ s}) = 66,000. \text{ m}$

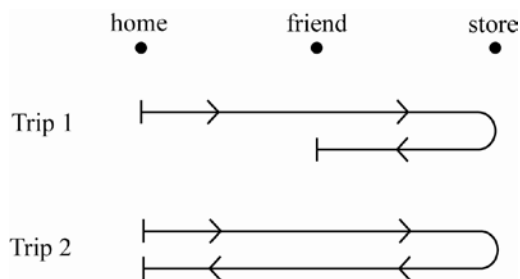
$$d = v_1 t_1 - v_2 t_2 = (30. \text{ m/s})(6.00 \cdot 10^2 \text{ s}) - (40. \text{ m/s})(1.20 \cdot 10^3 \text{ s}) = -30,000. \text{ m}$$

ROUND: The total distance traveled is 66.0 km, and the displacement is 30.0 km in southern direction.

DOUBLE-CHECK: The distance traveled is larger than the displacement as expected. Displacement is also expected to be towards the south since the second part of the trip going south is faster and has a longer duration.

- 2.30. **THINK:** I want to find the displacement and the distance traveled for a trip to the store, which is 1000. m away, and back. Let $l = 1000. \text{ m}$.

SKETCH:



RESEARCH: displacement (d) = final position – initial position
distance traveled = distance of path taken

SIMPLIFY:

(a) $d = \frac{1}{2}l - 0 = \frac{1}{2}l$

(b) $p = l + \frac{1}{2}l = \frac{3}{2}l$

(c) $d = 0 - 0 = 0$

(d) $p = l + l = 2l$

CALCULATE:

(a) $d = \frac{1}{2}l = \frac{1}{2}(1000. \text{ m}) = 500.0 \text{ m}$

(b) $p = \frac{3}{2}l = \frac{3}{2}(1000. \text{ m}) = 1500. \text{ m}$

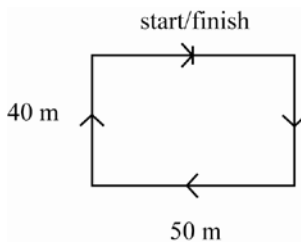
(c) $d = 0 \text{ m}$

(d) $p = 2l = 2(1000. \text{ m}) = 2000. \text{ m}$

ROUND: No rounding is necessary.

DOUBLE-CHECK: These values are reasonable: they are of the order of the distance to the store.

- 2.31. **THINK:** I want to find the average velocity when I run around a rectangular 50 m by 40 m track in 100 s.
SKETCH:



RESEARCH: average velocity = $\frac{\text{final position} - \text{initial position}}{\text{time}}$

SIMPLIFY: $\bar{v} = \frac{x_f - x_i}{t}$

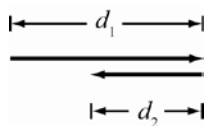
CALCULATE: $\bar{v} = \frac{0 \text{ m} - 0 \text{ m}}{100 \text{ s}} = 0 \text{ m/s}$

ROUND: Rounding is not necessary, because the result of 0 m/s is exact.

DOUBLE-CHECK: Since the final and initial positions are the same point, the average velocity will be zero. The answer may be displeasing at first since someone ran around a track and had no average velocity. Note that the speed would not be zero.

- 2.32. THINK:** I want to find the average velocity and the average speed of the electron that travels $d_1 = 2.42 \text{ m}$ in $t_1 = 2.91 \cdot 10^{-8} \text{ s}$ in the positive x -direction then $d_2 = 1.69 \text{ m}$ in $t_2 = 3.43 \cdot 10^{-8} \text{ s}$ in the opposite direction.

SKETCH:



RESEARCH:

(a) average velocity = $\frac{\text{final position} - \text{initial position}}{\text{time}}$

(b) speed = $\frac{\text{total distance traveled}}{\text{time}}$

SIMPLIFY:

(a) $\bar{v} = \frac{d_1 - d_2}{t_1 + t_2}$

(b) $s = \frac{d_1 + d_2}{t_1 + t_2}$

CALCULATE:

(a) $\bar{v} = \frac{d_1 - d_2}{t_1 + t_2} = \frac{2.42 \text{ m} - 1.69 \text{ m}}{2.91 \cdot 10^{-8} \text{ s} + 3.43 \cdot 10^{-8} \text{ s}} = 11,514,195 \text{ m/s}$

(b) $s = \frac{d_1 + d_2}{t_1 + t_2} = \frac{2.42 \text{ m} + 1.69 \text{ m}}{2.91 \cdot 10^{-8} \text{ s} + 3.43 \cdot 10^{-8} \text{ s}} = 64,826,498 \text{ m/s}$

ROUND:

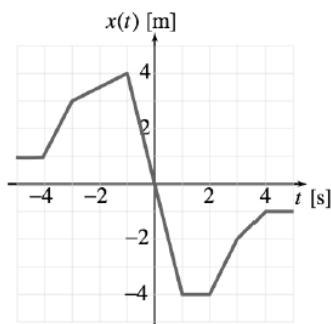
(a) $\bar{v} = 1.15 \cdot 10^7 \text{ m/s}$

(b) $s = 6.48 \cdot 10^7 \text{ m/s}$

DOUBLE-CHECK: The average velocity is less than the speed, which makes sense since the electron changes direction.

- 2.33. **THINK:** The provided graph must be used to answer several questions about the speed and velocity of a particle. Questions about velocity are equivalent to questions about the slope of the position function.

SKETCH:



RESEARCH: The velocity is given by the slope on a distance versus time graph. A steeper slope means a greater speed.

$$\text{average velocity} = \frac{\text{final position} - \text{initial position}}{\text{time}}, \quad \text{speed} = \frac{\text{total distance traveled}}{\text{time}}$$

- (a) The largest speed is where the slope is the steepest.
- (b) The average velocity is the total displacement over the time interval.
- (c) The average speed is the total distance traveled over the time interval.
- (d) The ratio of the velocities is $v_1 : v_2$.
- (e) A velocity of zero is indicated by a slope that is horizontal.

SIMPLIFY:

- (a) The largest speed is given by the steepest slope occurring between -1 s and $+1$ s.

$$s = \frac{|x(t_2) - x(t_1)|}{t_2 - t_1}, \quad \text{with } t_2 = 1 \text{ s and } t_1 = -1 \text{ s}.$$

- (b) The average velocity is given by the total displacement over the time interval.

$$\bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}, \quad \text{with } t_2 = 5 \text{ s and } t_1 = -5 \text{ s}.$$

- (c) In order to calculate the speed in the interval -5 s to 5 s, the path must first be determined. The path is given by starting at 1 m, going to 4 m, then turning around to move to -4 m and finishing at -1 m. So the total distance traveled is

$$\begin{aligned} p &= |(4 \text{ m} - 1 \text{ m})| + |((-4 \text{ m}) - 4 \text{ m})| + |(-1 \text{ m} - (-4 \text{ m}))| \\ &= 3 \text{ m} + 8 \text{ m} + 3 \text{ m} \\ &= 14 \text{ m} \end{aligned}$$

This path can be used to find the speed of the particle in this time interval.

$$s = \frac{p}{t_2 - t_1}, \quad \text{with } t_2 = 5 \text{ s and } t_1 = -5 \text{ s}.$$

- (d) The first velocity is given by $v_1 = \frac{x(t_3) - x(t_2)}{t_3 - t_2}$ and the second by $v_2 = \frac{x(t_4) - x(t_3)}{t_4 - t_3}$,

- (e) The velocity is zero in the regions 1 s to 2 s, -5 s to -4 s, and 4 s to 5 s.

CALCULATE:

(a) $s = \frac{|-4 \text{ m} - 4 \text{ m}|}{1 \text{ s} - (-1 \text{ s})} = 4.0 \text{ m/s}$

(b) $\bar{v} = \frac{-1 \text{ m} - 1 \text{ m}}{5 \text{ s} - (-5 \text{ s})} = -0.20 \text{ m/s}$

$$(c) \bar{s} = \frac{14 \text{ m}}{5 \text{ s} - (-5 \text{ s})} = 1.4 \text{ m/s}$$

$$(d) v_1 = \frac{(-2 \text{ m}) - (-4 \text{ m})}{3 \text{ s} - 2 \text{ s}} = 2.0 \text{ m/s}, v_2 = \frac{(-1 \text{ m}) - (-2 \text{ m})}{4 \text{ s} - 3 \text{ s}} = 1.0 \text{ m/s}, \quad \text{so } v_1 : v_2 = 2 : 1.$$

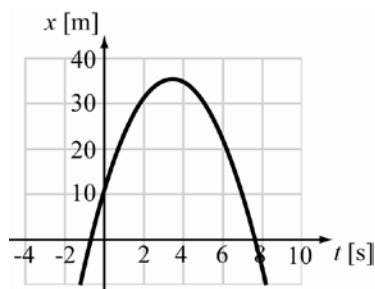
(e) There is nothing to calculate.

ROUND: Rounding is not necessary in this case, because we can read the values of the positions and times off the graph to at least 2 digit precision.

DOUBLE-CHECK: The values are reasonable for a range of positions between -4 m and 4 m with times on the order of seconds. Each calculation has the expected units.

- 2.34. **THINK:** I want to find the average velocity of a particle whose position is given by the equation $x(t) = 11 + 14t - 2.0t^2$ during the time interval $t = 1.0 \text{ s}$ to $t = 4.0 \text{ s}$.

SKETCH:



RESEARCH: The average velocity is given by the total displacement over the time interval.

$$\bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}, \text{ with } t_2 = 4.0 \text{ s and } t_1 = 1.0 \text{ s}.$$

$$\text{SIMPLIFY: } \bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{(11 + 14t_2 - 2.0t_2^2) - (11 + 14t_1 - 2.0t_1^2)}{t_2 - t_1} = \frac{14(t_2 - t_1) - 2.0(t_2^2 - t_1^2)}{t_2 - t_1}$$

$$\text{CALCULATE: } \bar{v} = \frac{14(4.0 \text{ s} - 1.0 \text{ s}) - 2.0((4.0 \text{ s})^2 - (1.0 \text{ s})^2)}{4.0 \text{ s} - 1.0 \text{ s}} = 4.0 \text{ m/s}$$

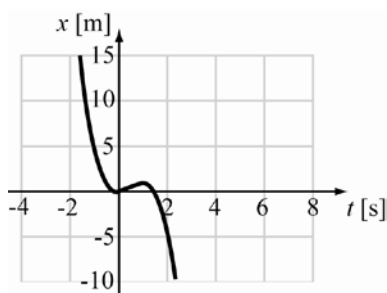
ROUND: The values given are all accurate to two significant digits, so the answer is given by two significant digits: $v = 4.0 \text{ m/s}$.

DOUBLE-CHECK: A reasonable approximation of the average velocity from $t = 1$ to $t = 4$ is to look at the instantaneous velocity at the midpoint. The instantaneous velocity is given by the derivative of the position, which is

$$v = \frac{d}{dt}(11 + 14t - 2.0t^2) = 0 + 1(14) - 2(2.0t) = 14 - 4.0t.$$

The value of the instantaneous velocity at $t = 2.5 \text{ s}$ is $14 - 4.0(2.5) = 4.0 \text{ m/s}$. The fact that the calculated average value matches the instantaneous velocity at the midpoint lends support to the answer.

- 2.35. **THINK:** I want to find the position of a particle when it reaches its maximum speed. I know the equation for the position as a function of time: $x = 3.0t^2 - 2.0t^3$. I will need to find the expression for the velocity and the acceleration to determine when the speed will be at its maximum. The maximum speed in the x -direction will occur at a point where the acceleration is zero.

SKETCH:

RESEARCH: The velocity is the derivative of the position function with respect to time. In turn, the acceleration is given by derivative of the velocity function with respect to time. The expressions can be found using the formulas:

$$v(t) = \frac{d}{dt}x(t), \quad a(t) = \frac{d}{dt}v(t).$$

Find the places where the acceleration is zero. The maximum speed will be the maximum of the speeds at the places where the acceleration is zero.

SIMPLIFY: $v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}(3.0t^2 - 2.0t^3) = 2 \cdot 3.0t^{2-1} - 3 \cdot 2.0t^{3-1} = 6.0t - 6.0t^2$

$$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}(6.0t - 6.0t^2) = 6.0t^{1-1} - 2 \cdot 6.0t^{2-1} = 6.0 - 12t$$

CALCULATE: Solving for the value of t where a is zero:

$$0 = 6.0 - 12t \Rightarrow 6.0 = 12t \Rightarrow t = 0.50 \text{ s}$$

This time can now be used to solve for the position:

$$x(0.50) = 3.0(0.50)^2 - 2.0(0.50)^3 = 0.500 \text{ m}$$

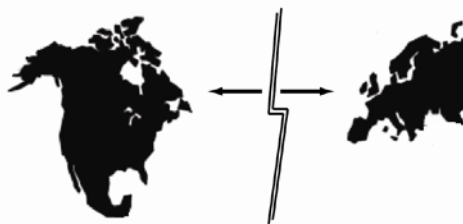
Since there is only one place where the acceleration is zero, the maximum speed in the positive x -direction must occur here.

ROUND: Since all variables and parameters are accurate to 2 significant digits, the answer should be too: $x = 0.50 \text{ m}$.

DOUBLE-CHECK: The validity of the answer can be confirmed by checking the velocity at $t = 0.50 \text{ s}$ and times around this point. At $t = 0.49 \text{ s}$, the velocity is 1.4994 m/s , and at $t = 0.51 \text{ s}$ the velocity is also 1.4994 m/s . Since these are both smaller than the velocity at 0.50 s ($v = 1.5 \text{ m/s}$), the answer is valid.

- 2.36. **THINK:** I want to find the time it took for the North American and European continents to reach a separation of 3000 mi if they are traveling at a speed of 10 mm/yr. First convert units:

$$d = (3000 \text{ mi})(1609 \text{ m/mi}) = 4827000 \text{ m}, \quad v = (10 \text{ mm/yr})(10^{-3} \text{ m/mm}) = 0.01 \text{ m/yr}.$$

SKETCH:

RESEARCH: The time can be found using the familiar equation: $d = vt$.

SIMPLIFY: The equation becomes $t = d/v$.

CALCULATE: $t = \frac{4827000 \text{ m}}{0.01 \text{ m/yr}} = 482,700,000 \text{ yr}$

ROUND: The values given in the question are given to one significant digit, thus the answer also should only have one significant digit: $t = 5 \cdot 10^8 \text{ yr}$.

DOUBLE-CHECK: The super continent Pangea existed about 250 million years ago or $2.5 \cdot 10^8$ years. Thus, this approximation is in the ballpark.

2.37.

THINK:

(a) I want to find the velocity at $t = 10.0 \text{ s}$ of a particle whose position is given by the function $x(t) = At^3 + Bt^2 + Ct + D$, where $A = 2.10 \text{ m/s}^3$, $B = 1.00 \text{ m/s}^2$, $C = -4.10 \text{ m/s}$, and $D = 3.00 \text{ m}$. I can differentiate the position function to derive the velocity function.

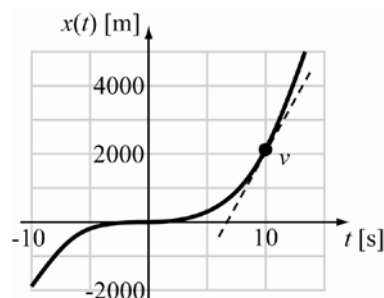
(b) I want to find the time(s) when the object is at rest. The object is at rest when the velocity is zero. I'll solve the velocity function I obtain in (a) equal to zero.

(c) I want to find the acceleration of the object at $t = 0.50 \text{ s}$. I can differentiate the velocity function found in part (a) to derive the acceleration function, and then calculate the acceleration at $t = 0.50 \text{ s}$.

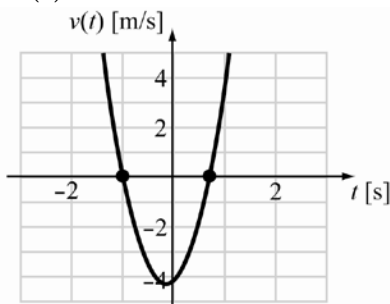
(d) I want to plot the function for the acceleration found in part (c) between the time range of -10.0 s to 10.0 s .

SKETCH:

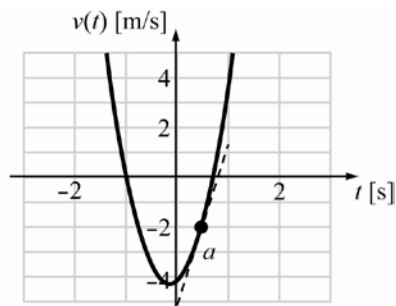
(a)



(b)



(c)



(d) The plot is part of CALCULATE.

RESEARCH:

(a) The velocity is given by the time derivative of the position function $v(t) = \frac{d}{dt}x(t)$.

(b) To find the time when the object is at rest, set the velocity to zero, and solve for t . This is a quadratic equation of the form $ax^2 + bx + c = 0$, whose solution is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

(c) The acceleration is given by the time derivative of the velocity: $a(t) = \frac{d}{dt}v(t)$.

(d) The equation for acceleration found in part (c) can be used to plot the graph of the function.

SIMPLIFY:

$$(a) \quad v(t) = \frac{d}{dt}x(t) = \frac{d}{dt}(At^3 + Bt^2 + Ct + D) = 3At^2 + 2Bt + C$$

(b) Set the velocity equal to zero and solve for t using the quadratic formula:

$$t = \frac{-2B \pm \sqrt{4B^2 - 4(3A)(C)}}{2(3A)} = \frac{-2B \pm \sqrt{4B^2 - 12AC}}{6A}$$

$$(c) \quad a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}(3At^2 + 2Bt + C) = 6At + 2B$$

(d) There is no need to simplify this equation.

CALCULATE:

$$(a) \quad v(t = 10.0 \text{ s}) = 3(2.10 \text{ m/s}^3)(10.0 \text{ s})^2 + 2(1.00 \text{ m/s}^2)(10.0 \text{ s}) - 4.10 \text{ m/s} = 645.9 \text{ m/s}$$

$$(b) \quad t = \frac{-2(1.00 \text{ m/s}^2) \pm \sqrt{4(1.00 \text{ m/s}^2)^2 - 12(2.10 \text{ m/s}^3)(-4.10 \text{ m/s})}}{6(2.10 \text{ m/s}^3)}$$

$$= 0.6634553 \text{ s}, -0.9809156 \text{ s}$$

$$(c) \quad a(t = 0.50 \text{ s}) = 6(2.10 \text{ m/s}^3)(0.50 \text{ s}) + 2(1.00 \text{ m/s}^2) = 8.30 \text{ m/s}^2$$

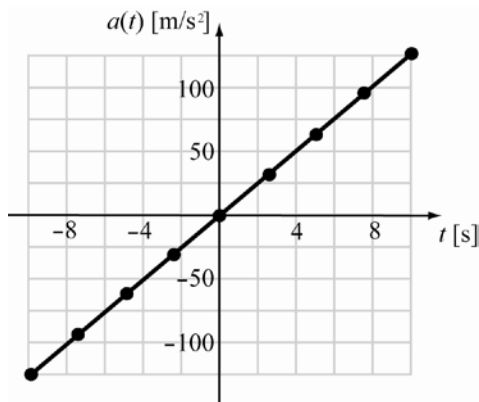
(d) The acceleration function, $a(t) = 6At + 2B$, can be used to compute the acceleration for time steps of 2.5 s. For example:

$$a(t = -2.5 \text{ s}) = 6(2.10 \text{ m/s}^3)(-2.5 \text{ s}) + 2(1.00 \text{ m/s}^2) = -29.5 \text{ m/s}^2$$

The result is given in the following table.

t [s]	-10.0	-7.5	-5.0	-2.5	0.0	2.5	5.0	7.5	10.0
a [m/s^2]	-124.0	-92.5	-61.0	-29.5	2.0	33.5	65.0	96.5	128.0

These values are used to plot the function.



ROUND:

(a) The accuracy will be determined by the factor $3(2.10 \text{ m/s}^3)(10.0 \text{ s})^2$, which only has two significant digits. Thus the velocity at 10.0 s is 646 m/s.

(b) The parameters are accurate to two significant digits, thus the solutions will also have three significant digits: $t = 0.663 \text{ s}$ and -0.981 s

(c) The accuracy is limited by the values with the smallest number of significant figures. This requires three significant figures. The acceleration is then $a = 8.30 \text{ m/s}^2$.

(d) No rounding is necessary.

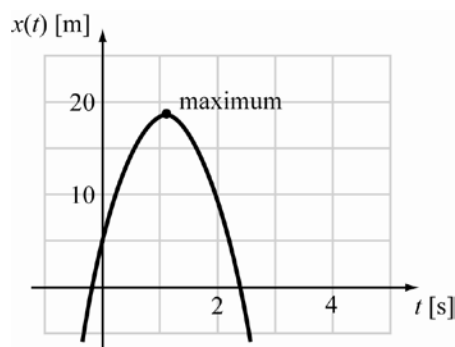
DOUBLE-CHECK:

- (a) This result is reasonable given the parameters. For example, $t^2 = (10.0 \text{ s})^2 = 100. \text{ s}$, so the velocity should be in the hundreds of meters per second.
- (b) Since the function is quadratic, there should be two solutions. The negative solution means that the object was at rest 0.98 seconds before the time designated $t = 0 \text{ s}$.
- (c) These values are consistent with the parameters.
- (d) The function for the acceleration is linear which the graph reflects.

- 2.38. THINK:** I want to determine the time when a particle will reach its maximum displacement and what the displacement will be at that time. The equation of the object's displacement is given as:

$$x(t) = 4.35 \text{ m} + (25.9 \text{ m/s})t - (11.79 \text{ m/s}^2)t^2$$

Differentiating x with respect to t gives the equation for the velocity. This is important since the time at which the velocity is zero is the moment at which the object has reached its maximum displacement.

SKETCH:

RESEARCH: The velocity is the derivative: $v = \frac{d}{dt}x(t)$. Find the value of t that makes the velocity zero.

Then, for part (b), substitute that value of t back into $x(t)$.

SIMPLIFY:
$$v = \frac{d}{dt}[4.35 \text{ m} + (25.9 \text{ m/s})t - (11.79 \text{ m/s}^2)t^2]$$

$$= 25.9 \text{ m/s} - 2(11.79 \text{ m/s}^2)t$$

Time for the maximum displacement is found by solving for t in the equation:
 $25.9 \text{ m/s} - 2(11.79 \text{ m/s}^2)t = 0$.

CALCULATE:

(a)
$$t = \frac{25.9 \text{ m/s}}{2(11.79 \text{ m/s}^2)} = 1.0984 \text{ s}$$

(b)
$$x(t) = 4.35 \text{ m} + (25.9 \text{ m/s})t - (11.79 \text{ m/s}^2)t^2$$

$$= 4.35 \text{ m} + (25.9 \text{ m/s})(1.10 \text{ s}) - (11.79 \text{ m/s}^2)(1.10 \text{ s})^2$$

$$= 18.5741 \text{ m}$$

ROUND:

- (a) The accuracy of this time is limited by the parameter 25.9 m/s, thus the time is $t = 1.10 \text{ s}$.
- (b) The least accurate term in the expression for $x(t)$ is accurate to the nearest tenth, so $x_{\text{max}} = 18.6 \text{ m}$.

DOUBLE-CHECK: Consider the positions just before and after the time $t = 1.10 \text{ s}$. $x = 18.5 \text{ m}$ for $t = 1.00 \text{ s}$, and $x = 18.5 \text{ m}$ for $t = 1.20 \text{ s}$. These values are less than the value calculated for x_{max} , which confirms the accuracy of the result.

- 2.39. THINK:** I want to calculate the average acceleration of the bank robbers getaway car. He starts with an initial speed of 45 mph and reaches a speed of 22.5 mph in the opposite direction in 12.4 s. First convert the velocities to SI units:

$$v_i = (45 \text{ mph}) \left(0.447 \frac{\text{m/s}}{\text{mph}} \right) = 20.115 \text{ m/s}$$

$$v_f = (-22.5 \text{ mph}) \left(0.447 \frac{\text{m/s}}{\text{mph}} \right) = -10.0575 \text{ m/s}$$

SKETCH:



RESEARCH: average acceleration = $\frac{\text{change in velocity}}{\text{change in time}}$

SIMPLIFY: $\bar{a} = \frac{v_f - v_i}{t}$

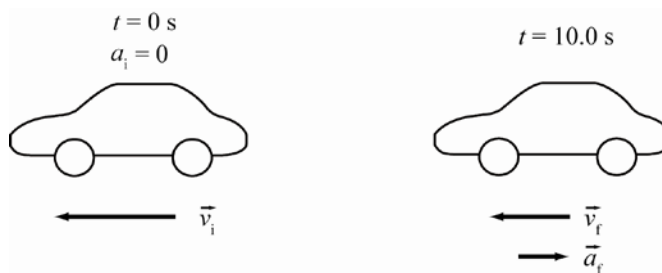
CALCULATE: $\bar{a} = \frac{(-10.0575 \text{ m/s}) - (20.115 \text{ m/s})}{12.4 \text{ s}} = -2.433 \text{ m/s}^2$

ROUND: The least precise of the velocities given in the question had two significant figures. Therefore, the final answer should also have two significant figures. The acceleration is $\bar{a} = -2.4 \text{ m/s}^2$, or 2.4 m/s^2 in the backward direction.

DOUBLE-CHECK: A top-of-the-line car can accelerate from 0 to 60 mph in 3 s. This corresponds to an acceleration of 8.94 m/s^2 . It is reasonable for a getaway car to be able to accelerate at a fraction of this value.

- 2.40. THINK:** I want to find the magnitude and direction of average acceleration of a car which goes from 22.0 m/s in the west direction to 17.0 m/s in the west direction in 10.0 s: $v_f = 17.0 \text{ m/s}$, $v_i = 22.0 \text{ m/s}$, $t = 10.0 \text{ s}$.

SKETCH:



RESEARCH: $\bar{a} = \frac{v_f - v_i}{t}$

SIMPLIFY: There is no need to simplify the above equation.

CALCULATE: $\bar{a} = \frac{17.0 \text{ m/s} - 22.0 \text{ m/s}}{10.0 \text{ s}} = -0.5000 \text{ m/s}^2$. The negative indicates the acceleration is east.

ROUND: The average acceleration is $\bar{a} = 0.500 \text{ m/s}^2$ east.

DOUBLE-CHECK: An acceleration of -0.500 m/s^2 is reasonable since a high performance car can accelerate at about 9 m/s^2 .

- 2.41. **THINK:** I want to find the magnitude of the constant acceleration of a car that goes 0.500 km in 10.0 s: $d = 0.500 \text{ km}$, $t = 10.0 \text{ s}$.

SKETCH:



RESEARCH: The position of the car under constant acceleration is given by $d = \frac{1}{2}at^2$.

SIMPLIFY: Solving for acceleration gives $a = \frac{2d}{t^2}$.

CALCULATE: $a = \frac{2(0.500 \text{ km})}{(10.0 \text{ s})^2} = 0.0100 \text{ km/s}^2$

ROUND: The values all have three significant figures. Thus, the average acceleration is $a = 0.0100 \text{ km/s}^2$, which is 10.0 m/s^2 .

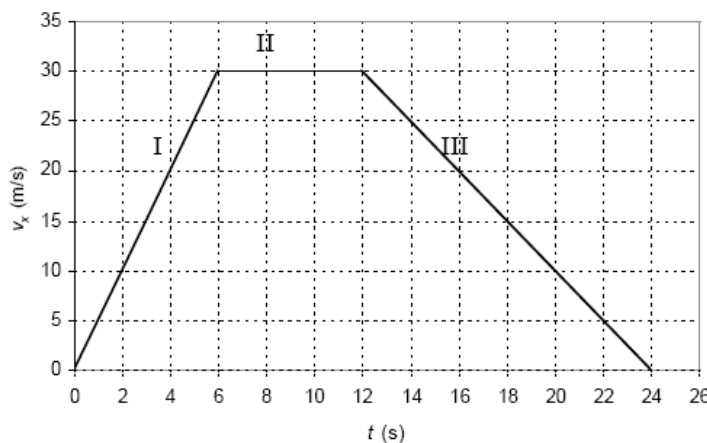
DOUBLE-CHECK: This acceleration is on the order of a high performance car which can accelerate from 0 to 60 mph in 3 seconds, or 9 m/s^2 .

- 2.42. **THINK:**

(a) I want to find the average acceleration of a car and the distance it travels by analyzing a velocity versus time graph. Each segment has a linear graph. Therefore, the acceleration is constant in each segment.

(b) The displacement is the area under the curve of a velocity versus time graph.

SKETCH:



RESEARCH:

(a) The acceleration is given by the slope of a velocity versus time graph.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

(b) The displacement is the sum of the areas of two triangles and one rectangle. Recall the area formulas:

$$\text{area of a triangle} = \frac{\text{base} \times \text{height}}{2}$$

$$\text{area of a rectangle} = \text{base} \times \text{height}$$

SIMPLIFY:

$$(a) \quad a_I = \frac{v_{I_2} - v_{I_1}}{t_{I_2} - t_{I_1}}, \quad a_{II} = \frac{v_{II_2} - v_{II_1}}{t_{II_2} - t_{II_1}}, \quad a_{III} = \frac{v_{III_2} - v_{III_1}}{t_{III_2} - t_{III_1}}$$

$$(b) \quad x = \frac{1}{2}v_{I_2}(t_{I_2} - t_{I_1}) + v_{II_2}(t_{II_2} - t_{II_1}) + \frac{1}{2}v_{III_2}(t_{III_2} - t_{III_1})$$

CALCULATE:

$$(a) \quad a_I = \frac{30.0 \text{ m/s} - 0 \text{ m/s}}{6.0 \text{ s} - 0 \text{ s}} = 5.0 \text{ m/s}^2, \quad a_{II} = \frac{30.0 \text{ m/s} - 30.0 \text{ m/s}}{12.0 \text{ s} - 6.0 \text{ s}} = 0.0 \text{ m/s}^2,$$

$$a_{III} = \frac{0.0 \text{ m/s} - 30.0 \text{ m/s}}{24.0 \text{ s} - 12.0 \text{ s}} = -2.50 \text{ m/s}^2$$

$$(b) \quad x = \frac{1}{2}(30.0 \text{ m/s})(6.0 \text{ s} - 0.0 \text{ s}) + (30.0 \text{ m/s})(12.0 \text{ s} - 6.0 \text{ s}) + \frac{1}{2}(30.0 \text{ m/s})(24.0 \text{ s} - 12.0 \text{ s}) = 450.0 \text{ m}$$

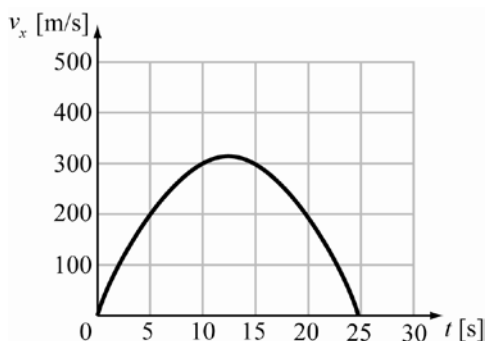
ROUND:

(a) Rounding is not necessary in this case, because the values of the velocities and times can be read off the graph to at least two digit precision.

(b) The answer is limited by the value 6.0 s, giving $x = 450 \text{ m}$.

DOUBLE-CHECK: The accelerations calculated in part (a) are similar to those of cars. The distance of 450 m is reasonable. The acceleration in I should be -2 times the acceleration in III, since the change in velocities are opposites, and the time in III for the change in velocity is twice the change in time that occurs in I.

- 2.43. **THINK:** I want to find the acceleration of a particle when it reaches its maximum displacement. The velocity of the particle is given by the equation $v_x = 50.0t - 2.0t^3$. The maximum displacement must occur when the velocity is zero. The expression for the acceleration can be found by differentiating the velocity with respect to time.

SKETCH:

RESEARCH: The acceleration is the derivative of the velocity: $a = \frac{d}{dt}v_x$. The maximum displacement will occur at a point where the velocity is zero. So, I can find the time at which the displacement is maximal by solving $v_x = 50.0t - 2.0t^3 = 0$ for t . The question says to consider after $t = 0$, so I will reject zero and negative roots. Then differentiate v with respect to t to obtain a formula for the acceleration. Evaluate the acceleration at the time where the displacement is maximized (which is when the velocity is zero).

SIMPLIFY: No simplification is required.

CALCULATE: Solving $v_x = 50.0t - 2.0t^3 = 0$ for t : $0 = 2.0t(25 - t^2)$, so $t = 0, \pm 5.0$. So, take $t = 5$. Now, differentiate v with respect to t to find the expression for the acceleration.

$$\begin{aligned} a &= \frac{d}{dt}(50.0t - 2.0t^3) \\ &= 50.0 - 6.0t^2 \end{aligned}$$

Substitute $t = 5.0$ s into the expression for acceleration:

$$a = 50.0 - 6.0t^2 = 50.0 - 6.0(5.0 \text{ s})^2 = -100 \text{ m/s}^2$$

ROUND: The solution is limited by the accuracy of $6.0t^2$, where $t = 5.0$ s, so it must be significant to two digits. This gives $50.0 - 150 = -100 \text{ m/s}^2$, which is also accurate to two significant figures. Therefore, the acceleration must be accurate to two significant figures: $a = -1.0 \cdot 10^2 \text{ m/s}^2$.

DOUBLE-CHECK: The acceleration must be negative at this point, since the displacement would continue to increase if a was positive.

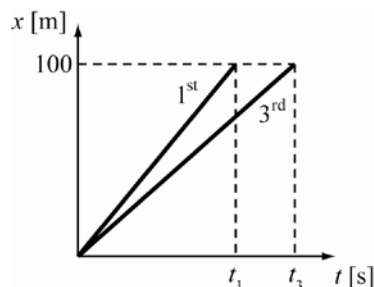
2.44. THINK:

(a) I want to know the distance between the first and third place runner when the first crosses the finish line, assuming they run at their average speeds throughout the race. The race is 100. m and the first place runner completes the race in 9.77 s while the third place runner takes 10.07 s to reach the finish line: $d = 100. \text{ m}$, $t_1 = 9.77 \text{ s}$, and $t_3 = 10.07 \text{ s}$.

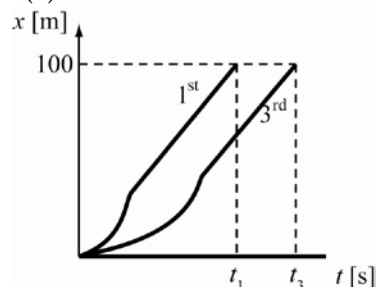
(b) I want to know the distance between the two runners when the first crosses the finish line, assuming they both accelerate to a top speed of 12 m/s: $d = 100. \text{ m}$, $t_1 = 9.77 \text{ s}$, $t_3 = 10.07 \text{ s}$, and $v = 12 \text{ m/s}$.

SKETCH:

(a)



(b)



RESEARCH:

(a) First the average speed of each runner must be calculated: $\bar{s} = d/t$. From this the distance between the two runners can be found: $\Delta d = d_1 - d_2$, where d_1 is 100. m and d_2 is the position of the third place runner at 9.77 s.

(b) Since both runners are running at 12 m/s at the end of the race, the distance between the runners will be the distance the 3rd place runner runs after the first place runner crosses the line: $\Delta d = v\Delta t$.

SIMPLIFY:

$$(a) \Delta d = d_1 - d_2 = d_1 - \bar{s}_3 t_1 = d_1 - \left(\frac{d_1}{t_3}\right) t_1 = d_1 - d_1 \left(\frac{t_1}{t_3}\right) = d_1 \left(1 - \frac{t_1}{t_3}\right)$$

$$(b) \Delta d = v(t_3 - t_1)$$

CALCULATE:

$$(a) \Delta d = d_1 \left(1 - \frac{t_1}{t_3}\right) = (100. \text{ m}) \left(1 - \frac{9.77 \text{ s}}{10.07 \text{ s}}\right) = 2.9791 \text{ m}$$

$$(b) \Delta d = (12 \text{ m/s})(10.07 \text{ s} - 9.77 \text{ s}) = 3.6 \text{ m}$$

ROUND:

(a) The answer is limited to 3 significant figures from 9.77 s so $\Delta d = 2.98 \text{ m}$.

(b) The distance then is 3.60 m between the first and third place runners.

DOUBLE-CHECK:

The two calculated distances are a small fraction (about 3%) of the race. It is reasonable for the third place runner to finish a small fraction of the track behind the first place finisher.

2.45. THINK:

(a) Since the motion is all in one direction, the average speed equals the distance covered divided by the time taken. I want to know the distance between the place where the ball was caught and midfield. I also want to know the time taken to cover this distance. The average speed will be the quotient of those two quantities.

(b) Same as in (a), but now I need to know the distance between midfield and the place where the run ended.

(c) I do not need to calculate the acceleration over each small time interval, since all that matters is the velocity at the start of the run and at the end. The average acceleration is the difference between those two quantities, divided by the time taken.

SKETCH:

In this case a sketch is not needed, since the only relevant quantities are those describing the runner at the start and end of the run, and at midfield.

RESEARCH:

The distance between two positions can be represented as $\Delta d = d_f - d_i$, where d_i is the initial position and d_f is the final one. The corresponding time difference is $\Delta t = t_f - t_i$. The average speed is $\Delta d / \Delta t$.

(a) Midfield is the 50-yard line, so $d_i = -1$ yd, $d_f = 50$ yd, $t_i = 0.00$ s, and $t_f = 5.73$ s.

(b) The end of the run is 1 yard past $d = 100$ yd, so $d_i = 50$ yd, $d_f = 101$ yd, $t_i = 5.73$ s, and $t_f = 12.01$ s.

(c) The average velocity is $\Delta v / \Delta t = (v_f - v_i) / (t_f - t_i)$. For this calculation, $t_i = 0$, $t_f = 12.01$ s, and $v_i = v_f = 0$ m/s, since the runner starts and finishes the run at a standstill.

SIMPLIFY:

$$(a), (b) \quad \frac{\Delta d}{\Delta t} = \frac{d_f - d_i}{t_f - t_i}$$

(c) No simplification needed

CALCULATE:

$$(a) \quad \frac{\Delta d}{\Delta t} = \frac{(50 \text{ yd}) - (-1 \text{ yd})}{(5.73 \text{ s}) - (0.00 \text{ s})} = 8.900522356 \text{ yd/s} \left(\frac{3 \text{ ft}}{1 \text{ yd}} \right) \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 8.138638743 \text{ m/s}$$

$$(b) \quad \frac{\Delta d}{\Delta t} = \frac{(101 \text{ yd}) - (50 \text{ yd})}{(12.01 \text{ s}) - (5.73 \text{ s})} = 8.121019108 \text{ yd/s} \left(\frac{3 \text{ ft}}{1 \text{ yd}} \right) \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 7.425859873 \text{ m/s}$$

$$(c) \quad \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{0 \text{ m/s} - 0 \text{ m/s}}{12.01 \text{ s} - 0.00 \text{ s}} = 0 \text{ m/s}^2$$

ROUND:

(a) We assume that the yard lines are exact, but the answer is limited to 3 significant figures by the time data. So the average speed is 8.14 m/s.

(b) The average speed is 7.43 m/s.

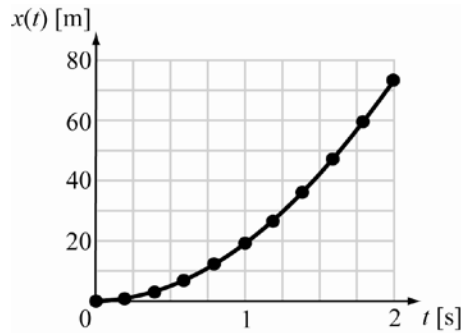
(c) The average velocity is 0 m/s².

DOUBLE-CHECK:

The average speeds in parts (a) and (b) are reasonable speeds (8.9 ft/s is about 18 mph), and it makes sense that the average speed during the second half of the run would be slightly less than during the first half, due to fatigue. In part (c) it is logical that average acceleration would be zero, since the net change in velocity is zero.

- 2.46. **THINK:** Use the difference formula to find the average velocity, and then the average acceleration of the jet given its position at several times, and determine whether the acceleration is constant.

SKETCH:



RESEARCH: The difference formula $m = \frac{\text{final point} - \text{initial point}}{\text{final time} - \text{initial time}}$.

SIMPLIFY: For velocity the difference formula is $v = (x_f - x_i) / (t_f - t_i)$ and the corresponding difference formula for the acceleration is $a = (v_f - v_i) / (t_f - t_i)$.

CALCULATE: As an example, $v = (6.6 \text{ m} - 3.0 \text{ m}) / (0.60 \text{ s} - 0.40 \text{ s}) = 18.0 \text{ m/s}$, and the acceleration is $a = (26.0 \text{ m/s} - 18.0 \text{ m/s}) / (0.80 \text{ s} - 0.60 \text{ s}) = 40.0 \text{ m/s}^2$.

t [s]	x [m]	v [m/s]	a [m/s ²]
0.00	0	0.0	
0.20	0.70	3.5	17.5
0.40	3.0	11.5	40
0.60	6.6	18	32.5
0.80	11.8	26	40
1.00	18.5	33.5	37.5
1.20	26.6	40.5	35
1.40	36.2	48	37.5
1.60	47.3	55.5	37.5
1.80	59.9	63	37.5
2.00	73.9	70	35

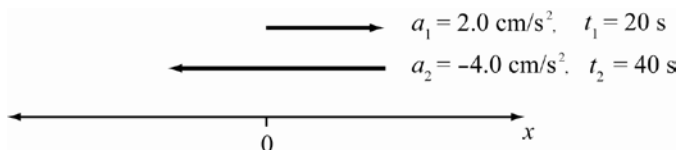
ROUND: The position measurements are given to the nearest tenth of a meter, and the time measurements are given to two significant figures. Therefore each of the stated results for velocity and acceleration should be rounded to two significant figures.

t [s]	x [m]	v [m/s]	a [m/s ²]
0.00	0.0	0.0	
0.20	0.70	3.5	18
0.40	3.0	12	40.
0.60	6.6	18	33
0.80	11.8	26	40.
1.00	18.5	34	38
1.20	26.6	41	35
1.40	36.2	48	38
1.60	47.3	56	38
1.80	59.9	63	38
2.00	73.9	70.	35

DOUBLE-CHECK: The final speed of the jet is 70. m/s, which is equivalent to 250 km/hr, the typical take-off speed of a commercial jet airliner.

- 2.47. **THINK:** I want to find the position of a particle after it accelerates from rest at $a_1 = 2.00 \text{ cm/s}^2$ for $t_1 = 20.0 \text{ s}$ then accelerates at $a_2 = -4.00 \text{ cm/s}^2$ for $t_2 = 40.0 \text{ s}$.

SKETCH:



RESEARCH: The position of a particle undergoing constant acceleration is given by the formula $x = x_0 + v_0 t + \frac{1}{2} a t^2$. The same particle's velocity is given by $v = v_0 + a t$. The final speed at the end of the first segment is the initial speed for the second segment.

SIMPLIFY: For the first 20 s the particle's position is $x_1 = \frac{1}{2} a_1 t_1^2$. This is the initial position for the second segment of the particle's trip. For the second segment, the particle is no longer at rest but has a speed of $v = a_1 t_1$.

$$x = x_1 + v_0 t_2 + \frac{1}{2} a_2 t_2^2 = \frac{1}{2} a_1 t_1^2 + a_1 t_1 t_2 + \frac{1}{2} a_2 t_2^2$$

CALCULATE:

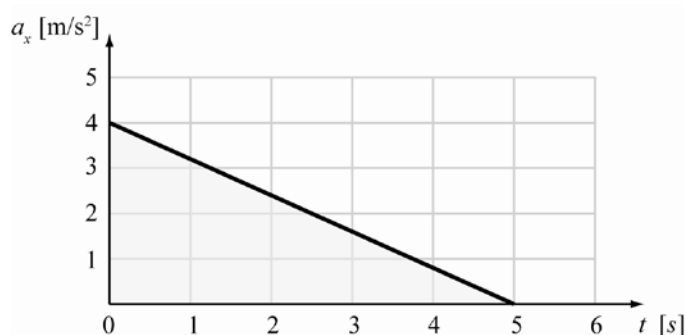
$$x = \frac{1}{2} (2.00 \text{ cm/s}^2) (20.0 \text{ s})^2 + (2.00 \text{ cm/s}^2) (20.0 \text{ s}) (40.0 \text{ s}) + \frac{1}{2} (-4.00 \text{ cm/s}^2) (40.0 \text{ s})^2 = -1200 \text{ cm}$$

ROUND: The variables are given with three significant figures. Therefore, the particle is $-1.20 \cdot 10^3 \text{ cm}$ from its original position.

DOUBLE-CHECK: Note that the second phase of the trip has a greater magnitude of acceleration than the first part. The duration of the second phase is longer; thus the final position is expected to be negative.

- 2.48. **THINK:** The car has a velocity of +6 m/s and a position of +12 m at $t = 0$. What is its velocity at $t = 5.0$ s? The change in the velocity is given by the area under the curve in an acceleration versus time graph.

SKETCH:



RESEARCH: $v = v_0 + \text{area}$, area of triangle = $\frac{\text{base} \cdot \text{height}}{2}$

SIMPLIFY: $v = v_0 + \frac{\Delta a \Delta t}{2}$

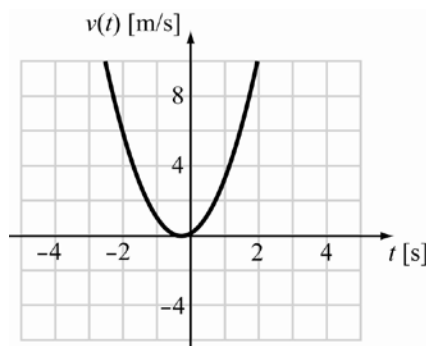
CALCULATE: $v = 6 \text{ m/s} + \frac{(4.0 \text{ m/s}^2)(5.0 \text{ s})}{2} = 16 \text{ m/s}$

ROUND: The function can only be accurate to the first digit before the decimal point. Thus $v = 16 \text{ m/s}$.

DOUBLE-CHECK: 16 m/s is approximately 58 km/h, which is a reasonable speed for a car.

- 2.49. **THINK:** I want to find the position of a car at $t_f = 3.0$ s if the velocity is given by the equation $v = At^2 + Bt$ with $A = 2.0 \text{ m/s}^3$ and $B = 1.0 \text{ m/s}^2$.

SKETCH:



RESEARCH: The position is given by the integral of the velocity function: $x = x_0 + \int_0^{t_f} v(t) dt$.

SIMPLIFY: Since the car starts at the origin, $x_0 = 0$ m.

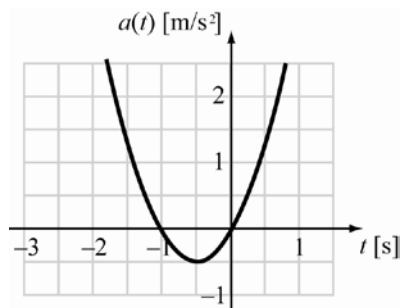
$$x = \int_0^{t_f} v(t) dt = \int_0^{t_f} (At^2 + Bt) dt = \frac{1}{3} At_f^3 + \frac{1}{2} Bt_f^2$$

CALCULATE: $x = \frac{1}{3}(2.0 \text{ m/s}^3)(3.0 \text{ s})^3 + \frac{1}{2}(1.0 \text{ m/s}^2)(3.0 \text{ s})^2 = 22.5 \text{ m}$

ROUND: The parameters are given to two significant digits, and so the answer must also contain two significant digits: $x = 23 \text{ m}$.

DOUBLE-CHECK: This is a reasonable distance for a car to travel in 3.0 s.

- 2.50. **THINK:** An object starts at rest (so $v_0 = 0 \text{ m/s}$) and has an acceleration defined by $a(t) = Bt^2 - (1/2)Ct$, where $B = 2.0 \text{ m/s}^4$ and $C = -4.0 \text{ m/s}^3$. I want to find its velocity and distance traveled after 5.0 s. Measure the position from the starting point $x_0 = 0$ m.

SKETCH:**RESEARCH:**(a) The velocity is given by integrating the acceleration with respect to time: $v = \int a(t) dt$.(b) The position is given by integrating the velocity with respect to time: $x = \int v(t) dt$.**SIMPLIFY:**

$$v = \int a(t) dt = \int \left(Bt^2 - \frac{1}{2} Ct \right) dt = \frac{1}{3} Bt^3 - \frac{1}{4} Ct^2 + v_0, \text{ and}$$

$$x = \int v dt = \int \left(\frac{1}{3} Bt^3 - \frac{1}{4} Ct^2 + v_0 \right) dt = \frac{1}{12} Bt^4 - \frac{1}{12} Ct^3 + v_0 t + x_0$$

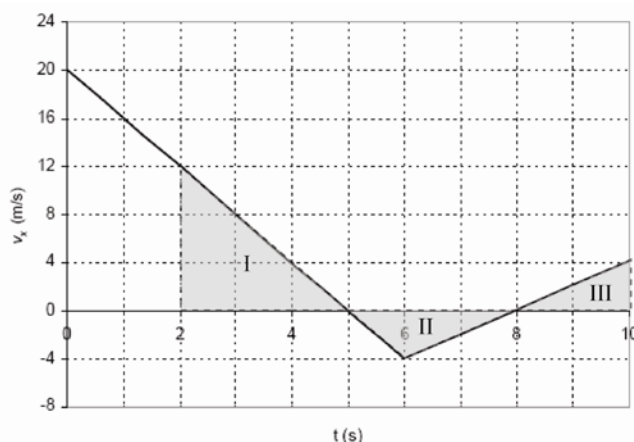
CALCULATE:

$$v = \frac{1}{3} Bt^3 - \frac{1}{4} Ct^2 + v_0 = \frac{1}{3} (2.0 \text{ m/s}^4) (5.0 \text{ s})^3 - \frac{1}{4} (-4.0 \text{ m/s}^3) (5.0 \text{ s})^2 + 0 \text{ m/s} = 108.33 \text{ m/s}$$

$$x = \frac{1}{12} (2.0 \text{ m/s}^4) (5.0 \text{ s})^4 - \frac{1}{12} (-4.0 \text{ m/s}^3) (5.0 \text{ s})^3 + (0 \text{ m/s}) (5.0 \text{ s}) + (0 \text{ m}) = 145.83 \text{ m}$$

ROUND: All parameters have two significant digits. Thus the answers should also have two significant figures: at $t = 5.0 \text{ s}$, $v = 110 \text{ m/s}$ and $x = 150 \text{ m}$.**DOUBLE-CHECK:** The distance traveled has units of meters, and the velocity has units of meters per second. These are appropriate units for a distance and velocity, respectively.

- 2.51. **THINK:** A car is accelerating as shown in the graph. At $t_0 = 2.0 \text{ s}$, its position is $x_0 = 2.0 \text{ m}$. I want to determine its position at $t = 10.0 \text{ s}$.

SKETCH:

RESEARCH: The change in position is given by the area under the curve of the velocity versus time graph plus the initial displacement: $x = x_0 + \text{area}$. Note that region II is under the t -axis will give a negative area. Let A_1 be the area of region I, let A_2 be the area of region II, and let A_3 be the area of region III.

SIMPLIFY: $x = x_0 + A_I + A_{II} + A_{III}$

CALCULATE:

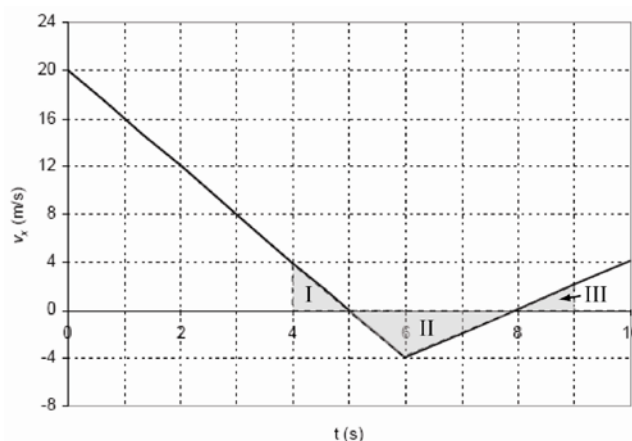
$$x = 2.0 \text{ m} + \frac{1}{2}(12.0 \text{ m/s})(5.0 \text{ s} - 2.0 \text{ s}) + \frac{1}{2}(-4.0 \text{ m/s})(8.0 \text{ s} - 5.0 \text{ s}) + \frac{1}{2}(4.0 \text{ m/s})(10.0 \text{ s} - 8.0 \text{ s}) = 18 \text{ m}$$

ROUND: The answer should be given to the least accurate calculated area. These are all accurate to the meter, thus the position is $x = 18 \text{ m}$.

DOUBLE-CHECK: The maximum velocity is 12 m/s. If this were sustained over the 8 second interval, the distance traveled would be $2.0 \text{ m} + (12 \text{ m/s})(8.0 \text{ s}) = 98 \text{ m}$. Since there was a deceleration and then an acceleration, we expect that the actual distance will be much less than the value 98 m.

- 2.52. **THINK:** A car is accelerating as shown in the graph. I want to determine its displacement between $t = 4 \text{ s}$ and $t = 9 \text{ s}$.

SKETCH:



RESEARCH: The change in position is given by the area under the curve of a velocity versus time graph. Note that it is hard to read the value of the velocity at $t = 9.0 \text{ s}$. This difficulty can be overcome by finding the slope of the line for this section. Using the slope, the velocity during this time can be determined:

$\Delta x = \text{Area}$, $m = \frac{\text{rise}}{\text{run}}$. Let A_I be the area of region I, let A_{II} be the area of region II, and let A_{III} be the area

of region III.

SIMPLIFY: $\Delta x = A_I + A_{II} + A_{III}$

CALCULATE: $m = \frac{4.0 \text{ m/s} - (-4.0 \text{ m/s})}{10.0 \text{ s} - 6.0 \text{ s}} = 2.0 \text{ m/s}^2$

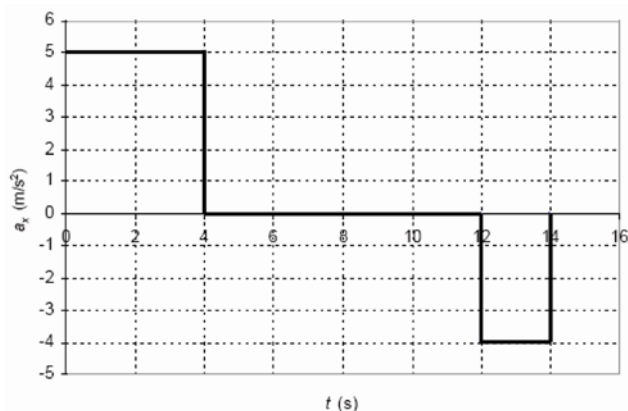
$$\Delta x = \frac{1}{2}(4.0 \text{ m/s})(5.0 \text{ s} - 4.0 \text{ s}) + \frac{1}{2}(-4.0 \text{ m/s})(8.0 \text{ s} - 5.0 \text{ s}) + \frac{1}{2}(2.0 \text{ m/s})(9.0 \text{ s} - 8.0 \text{ s}) = -3.0 \text{ m}$$

ROUND: $\Delta x = -3.0 \text{ m}$

DOUBLE-CHECK: The car will end up with a negative displacement since the area of region II is larger than the combined areas of regions I and III. The overall displacement is less than if the car had traveled constantly at its maximum velocity of 4 m/s (when the displacement would have been 20 m).

- 2.53. **THINK:** A motorcycle is accelerating at different rates as shown in the graph. I want to determine (a) its speed at $t = 4.00$ s and $t = 14.0$ s, and (b) its total displacement between $t = 0$ and $t = 14.0$ s.

SKETCH:



RESEARCH:

(a) The velocity of the motorcycle is defined by the area under the curve of the acceleration versus time graph. This area can be found by counting the blocks under the curve then multiply by the area of one block: 1 block = $(2 \text{ s}) (1 \text{ m/s}^2) = 2 \text{ m/s}$.

(b) The displacement can be found by separating the acceleration into three parts: The first phase has an acceleration of $a_1 = 5 \text{ m/s}^2$ for times between 0 to 4 seconds. The second phase has no acceleration, thus the motorcycle has a constant speed. The third phase has a constant acceleration of $a_3 = -4 \text{ m/s}^2$. Recall the position and velocity of an object under constant acceleration is $x = x_0 + v_0 t + (1/2)at^2$ and $v = v_0 + at$, respectively.

SIMPLIFY: At $t = 4.00$ s and 14.0 s, there are 10 blocks and 6 blocks respectively. Recall that blocks under the time axis are negative. In the first phase the position is given by $x = (1/2)a_1(\Delta t_1)^2$ where Δt is the duration of the phase. The velocity at the end of this phase is $v = a_1 \Delta t_1$. The position and velocity of the first phase gives the initial position and velocity for the second phase.

$$x = x_0 + v_0 \Delta t_2 + \frac{1}{2} a_1 (\Delta t_1)^2 + a_1 \Delta t_1 \Delta t_2$$

Since the velocity is constant in the second phase, this value is also the initial velocity of the third phase.

$$x = x_0 + v_0 \Delta t_3 + \frac{1}{2} a_3 (\Delta t_3)^2 = \frac{1}{2} a_1 (\Delta t_1)^2 + a_1 \Delta t_1 \Delta t_2 + a_1 \Delta t_1 \Delta t_3 + \frac{1}{2} a_3 (\Delta t_3)^2$$

CALCULATE:

(a) $v(t = 4.00 \text{ s}) = 10(2.00 \text{ m/s}) = 20.0 \text{ m/s}$, $v(t = 14.0 \text{ s}) = 6(2.00 \text{ m/s}) = 12.0 \text{ m/s}$

(b)

$$\begin{aligned} x &= \frac{1}{2}(5.0 \text{ m/s}^2)(4.00 \text{ s} - 0 \text{ s})^2 + (5.0 \text{ m/s}^2)(4.00 \text{ s} - 0 \text{ s})(12.0 \text{ s} - 4.0 \text{ s}) + (5.0 \text{ m/s}^2)(4.00 \text{ s} - 0 \text{ s})(14.0 \text{ s} - 12.0 \text{ s}) \\ &\quad + \frac{1}{2}(-4.0 \text{ m/s}^2)(14.0 \text{ s} - 12.0 \text{ s})^2 \\ &= 232 \text{ m} \end{aligned}$$

ROUND:

(a) Rounding is not necessary in this case, because the values of the accelerations and times can be read off the graph to at least two digit precision.

(b) The motorcycle has traveled 232 m in 14.0 s.

DOUBLE-CHECK: The velocity of the motorcycle at $t = 14$ s is less than the speed at $t = 4$ s, which makes sense since the bike decelerated in the third phase. Since the bike was traveling at a maximum speed of 20 m/s, the most distance it could cover in 14 seconds would be 280 m. The calculated value is less than this, which makes sense since the bike decelerated in the third phase.

- 2.54. **THINK:** I want to find the time it takes the car to accelerate from rest to a speed of $v = 22.2$ m/s. I know that $v_0 = 0$ m/s, $v = 22.2$ m/s, distance = 243 m, and a is constant.

SKETCH:



RESEARCH: Recall that given constant acceleration, $d = (1/2)(v_0 + v)t$.

SIMPLIFY: $t = \frac{2d}{v_0 + v}$

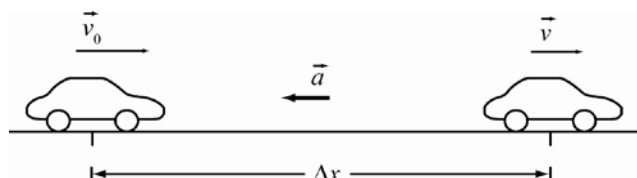
CALCULATE: $t = \frac{2(243 \text{ m})}{0.0 \text{ m/s} + 22.2 \text{ m/s}} = 21.8919 \text{ s}$

ROUND: Therefore, $t = 21.9$ s since each value used in the calculation has three significant digits.

DOUBLE-CHECK: The units of the solution are units of time, and the calculated time is a reasonable amount of time for a car to cover 243 m.

- 2.55. **THINK:** I want to determine (a) how long it takes for a car to decelerate from $v_0 = 31.0$ m/s to $v = 12.0$ m/s over a distance of 380. m and (b) the value of the acceleration.

SKETCH:



RESEARCH: Since the acceleration is constant, the time can be determined using the equation:

$$\Delta x = (1/2)(v_0 + v)t, \text{ and the acceleration can be found using } v^2 = v_0^2 + 2a\Delta x.$$

SIMPLIFY:

(a) $\Delta x = \frac{1}{2}(v_0 + v)t \Rightarrow (v_0 + v)t = 2\Delta x \Rightarrow t = \frac{2\Delta x}{v_0 + v}$

(b) $v^2 = v_0^2 + 2a\Delta x \Rightarrow 2a\Delta x = v^2 - v_0^2 \Rightarrow a = \frac{v^2 - v_0^2}{2\Delta x}$

CALCULATE:

(a) $t = \frac{2\Delta x}{v_0 + v} = \frac{2(380. \text{ m})}{(31.0 \text{ m/s} + 12.0 \text{ m/s})} = 17.674 \text{ s}$

(b) $a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{(12.0 \text{ m/s})^2 - (31.0 \text{ m/s})^2}{2(380. \text{ m})} = -1.075 \text{ m/s}^2$

ROUND: Each result is limited to three significant figures as the values used in the calculations each have three significant figures.

(a) $t = 17.7 \text{ s}$

(b) $a = -1.08 \text{ m/s}^2$

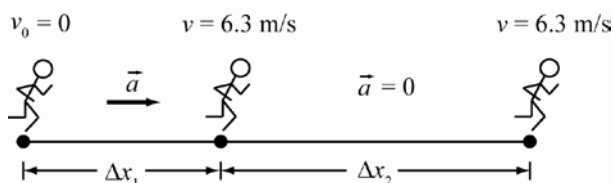
DOUBLE-CHECK:

(a) The resulting time has appropriate units and is reasonable for the car to slow down.

(b) The acceleration is negative, indicating that it opposes the initial velocity, causing the car to slow down.

- 2.56. THINK:** I want to find (a) the total distance covered in time $t = 59.7$ s, and (b) the velocity of the runner at $t = 59.7$ s. It will be useful to know the time taken to accelerate, t_1 , and the time taken to run at the achieved constant velocity, t_2 . Note that the mass of the runner is irrelevant.

SKETCH:



RESEARCH: The runner accelerates from rest to some velocity, v , then continues to run at this constant velocity. The total distance covered, Δx , will be the sum of the distance covered while accelerating, Δx_1 , and the distance covered while at constant velocity, Δx_2 : $\Delta x = \Delta x_1 + \Delta x_2$. The distance Δx_1 is determined by $v^2 = v_0^2 + 2a\Delta x_1$. The distance Δx_2 is determined by $\Delta x_2 = vt_2$. It will be necessary to know the time taken to run this distance Δx_2 . This time, t_2 , and the time taken to cover the distance Δx_1 , t_1 , must sum to the given total time of 59.7 s: $t_{\text{total}} = t_1 + t_2$. The time t_1 can be determined using the equation: $v = v_0 + at_1$.

SIMPLIFY:

$$v^2 = v_0^2 + 2a\Delta x_1 \Rightarrow \Delta x_1 = \frac{v^2 - v_0^2}{2a}, \quad v = v_0 + at_1 \Rightarrow t_1 = \frac{v - v_0}{a}$$

$$\Delta x_2 = vt_2 \Rightarrow \Delta x_2 = v(t_{\text{total}} - t_1) \Rightarrow \Delta x_2 = v\left(t_{\text{total}} - \frac{v - v_0}{a}\right)$$

$$\text{Finally, the total distance covered is } \Delta x = \Delta x_1 + \Delta x_2 = \frac{v^2 - v_0^2}{2a} + v\left(t_{\text{total}} - \frac{v - v_0}{a}\right) = \frac{v^2}{2a} + v\left(t_{\text{total}} - \frac{v}{a}\right).$$

CALCULATE:

$$\begin{aligned} \text{(a) } \Delta x &= \frac{(6.3 \text{ m/s})^2}{2(1.25 \text{ m/s}^2)} + (6.3 \text{ m/s})\left(59.7 \text{ s} - \frac{6.3 \text{ m/s}}{1.25 \text{ m/s}^2}\right) \\ &= 360.234 \text{ m} \end{aligned}$$

$$\text{(b) Since } t_1 = \frac{6.3 \text{ m/s} - 0 \text{ m/s}}{1.25 \text{ m/s}^2} = 5.0 \text{ s} \text{ is the time taken to reach the final velocity, the velocity of the runner}$$

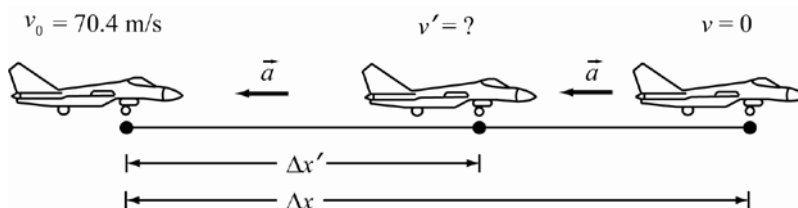
at $t_{\text{total}} = 59.7$ s is 6.3 m/s.

ROUND: Since v has only two significant digits, $\Delta x = 360$ m, or $3.6 \cdot 10^2$ m.

DOUBLE-CHECK: This seems like a reasonable distance to cover in the total time, given most of the distance is covered at the constant velocity 6.3 m/s. Since the runner stops accelerating after 5.0 s, the velocity of the runner is still 6.3 m/s at 59.7 s.

- 2.57. THINK:** I am given $v_0 = 70.4$ m/s, $v = 0$, $\Delta x = 197.4$ m, and constant acceleration. I am asked to find the velocity v' when the jet is 44.2 m from its stopping position. This means the jet has traveled $\Delta x' = 197.4 \text{ m} - 44.2 \text{ m} = 153.2$ m on the aircraft carrier.

SKETCH:



RESEARCH: The initial and final velocities are known, as is the total distance traveled. Therefore the equation $v^2 = v_0^2 + 2a\Delta x$ can be used to find the acceleration of the jet. Once the acceleration is known, the intermediate velocity v' can be determined using $(v')^2 = v_0^2 + 2a\Delta x'$.

SIMPLIFY: First find the constant acceleration using the total distance traveled, Δx , the initial velocity, v_0 , and the final velocity, v : $a = \frac{v^2 - v_0^2}{2\Delta x} = -\frac{v_0^2}{2\Delta x}$ (since $v = 0$ m/s). Next, find the requested intermediate velocity, v' :

$$(v')^2 = v_0^2 + 2a\Delta x' \Rightarrow (v')^2 = v_0^2 + 2\left(-\frac{v_0^2}{2\Delta x}\right)\Delta x' \Rightarrow v' = \sqrt{v_0^2 - \frac{v_0^2}{\Delta x}\Delta x'}$$

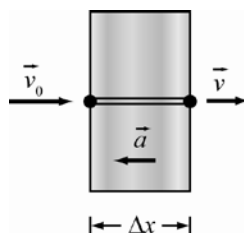
CALCULATE: $v' = \sqrt{(70.4 \text{ m/s})^2 - \frac{(70.4 \text{ m/s})^2}{(197.4 \text{ m})}(153.2 \text{ m})} = 33.313 \text{ m/s}$

ROUND: At $\Delta x' = 153.2 \text{ m}$, the velocity is $v' = 33.3 \text{ m/s}$.

DOUBLE-CHECK: This v' is less than v_0 , but greater than v , and therefore makes sense.

- 2.58. **THINK:** I want to find the acceleration of a bullet passing through a board, given that $\Delta x = 10.0 \text{ cm} = 0.100 \text{ m}$, $v_0 = 400. \text{ m/s}$, and $v = 200. \text{ m/s}$. I expect the acceleration to be negative, since the bullet is slowing down.

SKETCH:



RESEARCH: $v^2 = v_0^2 + 2a\Delta x$

SIMPLIFY: $a = \frac{v^2 - v_0^2}{2\Delta x}$

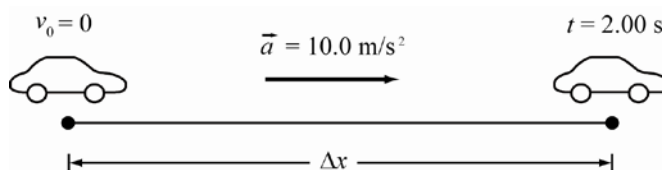
CALCULATE: $a = \frac{(200. \text{ m/s})^2 - (400. \text{ m/s})^2}{2(0.100 \text{ m})} = -600,000. \text{ m/s}^2$

ROUND: Since each velocity is given to three significant digits, $a = -6.00 \cdot 10^5 \text{ m/s}^2$.

DOUBLE-CHECK: That a is negative indicates it is in the opposite direction of the initial velocity, so the bullet slows down. The speed of the bullet decreases by 200 m/s in 0.1 m, so I am not surprised to get such a large value for the acceleration.

- 2.59. **THINK:** A car accelerates from rest with $a = 10.0 \text{ m/s}^2$. I want to know how far it travels in 2.00 s .

SKETCH:



RESEARCH: $\Delta x = v_0 t + \frac{1}{2}at^2$

SIMPLIFY: Since $v_0 = 0 \text{ m/s}$, $\Delta x = \frac{1}{2}at^2$.

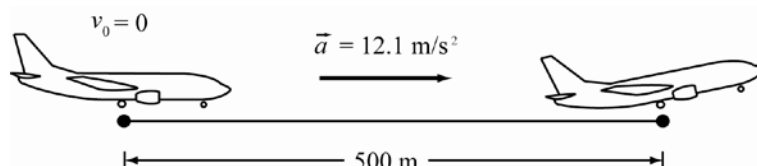
CALCULATE: $\Delta x = \frac{1}{2}(10.0 \text{ m/s}^2)(2.00 \text{ s})^2 = 20.0 \text{ m}$

ROUND: $\Delta x = 20.0 \text{ m}$

DOUBLE-CHECK: This seems like a reasonable distance to cover within 2.00 s given $a = 10.0 \text{ m/s}^2$.

2.60. THINK: A airplane accelerates from rest at $a = 12.1 \text{ m/s}^2$. I want to know its velocity at 500. m.

SKETCH:



RESEARCH: $v^2 = v_0^2 + 2a\Delta x$; $v_0 = 0$, $a = 12.1 \text{ m/s}^2$, $\Delta x = 500 \text{ m}$

SIMPLIFY: $v = \sqrt{v_0^2 + 2a\Delta x}$
 $= \sqrt{2a\Delta x}$

CALCULATE: $v = \sqrt{2(12.1 \text{ m/s}^2)(500. \text{ m})} = 110.00 \text{ m/s}$

ROUND: $v = 110. \text{ m/s}$

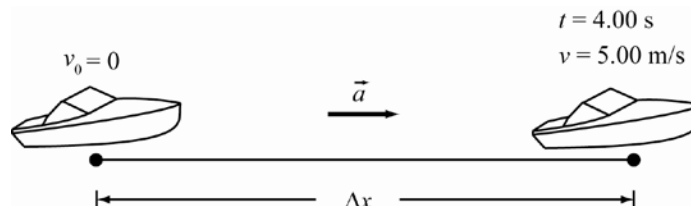
DOUBLE-CHECK: This take-off speed is about 400 kph, which is reasonable for a small plane.

2.61. THINK:

(a) I know that $v_0 = 0 \text{ m/s}$, $v = 5.00 \text{ m/s}$, and a is constant. I want to find v_{avg} .

(b) $t = 4.00 \text{ s}$ is given. I want to find Δx .

SKETCH:



RESEARCH:

(a) $v_{\text{avg}} = \frac{v_0 + v}{2}$

(b) a is unknown, so use $\Delta x = \frac{1}{2}(v_0 + v)t$

SIMPLIFY: It is not necessary to simplify the equations above.

CALCULATE:

(a) $v_{\text{avg}} = \frac{5.00 \text{ m/s} + 0 \text{ m/s}}{2} = 2.50 \text{ m/s}$

(b) $\Delta x = \frac{1}{2}(5.00 \text{ m/s} + 0 \text{ m/s})(4.00 \text{ s}) = 10.00 \text{ m}$

ROUND:

(a) v is precise to three significant digits, so $v_{\text{avg}} = 2.50 \text{ m/s}$.

(b) Each v and t have three significant digits, so $\Delta x = 10.0 \text{ m}$.

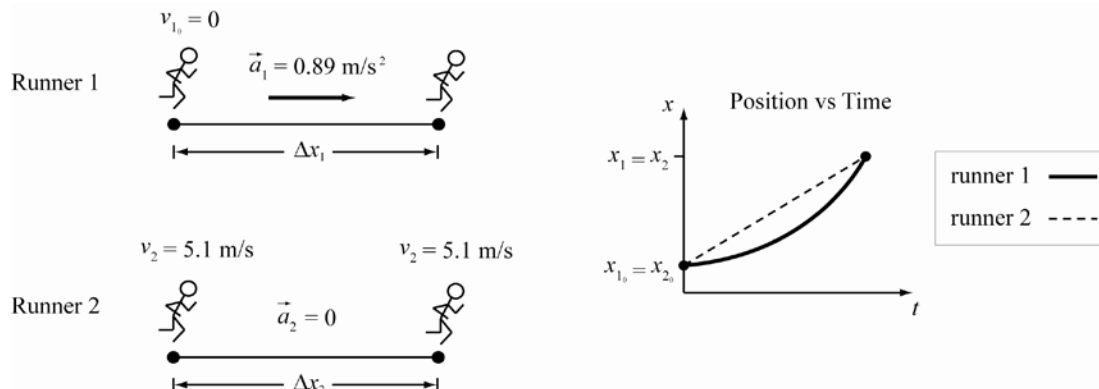
DOUBLE-CHECK:

(a) This v_{avg} is between the given v_0 and v , and therefore makes sense.

(b) This is a reasonable distance to cover in 4.00 s when $v_{\text{avg}} = 2.50 \text{ m/s}$.

- 2.62. THINK:** I have been given information on two runners. Runner 1 has an initial velocity $v_{1_0} = 0$ and an acceleration $a_1 = 0.89 \text{ m/s}^2$. Runner 2 has a constant velocity of $v_2 = 5.1 \text{ m/s}$. I want to know the distance traveled by runner 1 before he catches up to runner 2. Note that both runners cover the same distance, that is, $\Delta x_1 = \Delta x_2$, in the same time, t .

SKETCH:



RESEARCH: For runner 1, $\Delta x_1 = v_{1_0} t + (1/2) a_1 t^2$. For runner 2, $\Delta x_2 = v_2 t$.

SIMPLIFY: Since the time is not given, substitute the equation for runner 2 for the value of t : $t = \Delta x_2 / v_2$. Then for runner 1:

$$\Delta x_1 = v_{1_0} t + \frac{1}{2} a_1 t^2 \Rightarrow \Delta x_1 = \frac{1}{2} a_1 t^2 \Rightarrow \Delta x_1 = \frac{1}{2} a_1 \left(\frac{\Delta x_2}{v_2} \right)^2$$

Since $\Delta x_1 = \Delta x_2$, I write:

$$\Delta x_1 = \frac{1}{2} a_1 \left(\frac{\Delta x_1}{v_2} \right)^2 \Rightarrow \frac{1}{2} a_1 \frac{\Delta x_1^2}{v_2^2} - \Delta x_1 = 0 \Rightarrow \Delta x_1 \left(\frac{1}{2} a_1 \frac{\Delta x_1}{v_2^2} - 1 \right) = 0$$

Observe that one solution is $\Delta x_1 = 0$. This is true when runner 2 first passes runner 1. The other solution occurs when runner 1 catches up to runner 2: $\frac{1}{2} a_1 \frac{\Delta x_1}{v_2^2} - 1 = 0$. Then $\Delta x_1 = \frac{2v_2^2}{a_1}$.

CALCULATE: $\Delta x_1 = \frac{2(5.1 \text{ m/s})^2}{(0.89 \text{ m/s}^2)} = 58.449 \text{ m}$

ROUND: $\Delta x_1 = 58 \text{ m}$

DOUBLE-CHECK: A runner might catch up to another runner on a race track in 58 m.

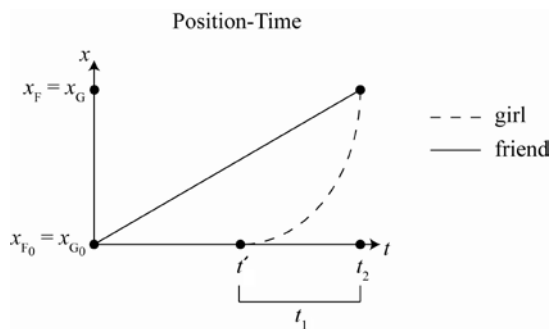
- 2.63. THINK:**

(a) The girl is initially at rest, so $v_{1_0} = 0$, and then she waits $t' = 20 \text{ s}$ before accelerating at $a_1 = 2.2 \text{ m/s}^2$. Her friend has constant velocity $v_2 = 8.0 \text{ m/s}$. I want to know the time required for the girl to catch up with her friend, t_1 . Note that both people travel the same distance: $\Delta x_1 = \Delta x_2$. The time the girl spends riding her bike is t_1 . The friend, however, has a t' head-start; the friend travels for a total time of $t_2 = t' + t_1$.

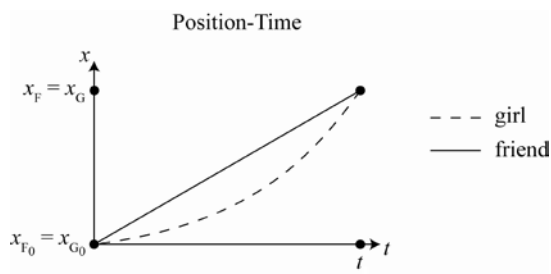
(b) The initial conditions of the girl have changed. Now $v_{1_0} = 1.2 \text{ m/s}$. The initial conditions of the friend are the same: $v_2 = 8.0 \text{ m/s}$. Now there is no time delay between when the friend passes the girl and when the girl begins to accelerate. The time taken to catch up is that found in part a), $t = 20 \text{ s}$. I will use $t = 16.2 \text{ s}$ for my calculations, keeping in mind that t has only two significant figures. I want to know the acceleration of the girl, a_1 , required to catch her friend in time t .

SKETCH:

(a)



(b)

**RESEARCH:**

(a) The distance the girl travels is $\Delta x_1 = v_{i_0} t_1 + \frac{1}{2} a_1 t_1^2$. The distance her friend travels is $\Delta x_2 = v_2 t_2$.

(b) $\Delta x_1 = v_{i_0} t + \frac{1}{2} a_1 t^2$, $\Delta x_2 = v_2 t$

SIMPLIFY:

(a) Since $v_{i_0} = 0$, $\Delta x_1 = \frac{1}{2} a_1 t_1^2$. Also, since $t_2 = t' + t_1$, $\Delta x_2 = v_2 (t' + t_1)$. Recall that $\Delta x_1 = \Delta x_2$. This leads to $\frac{1}{2} a_1 t_1^2 = v_2 (t' + t_1)$. Now solve for t_1 : $\frac{1}{2} a_1 t_1^2 = v_2 t' + v_2 t_1 \Rightarrow \frac{1}{2} a_1 t_1^2 - v_2 t_1 - v_2 t' = 0$.

The quadratic formula gives:

$$t_1 = \frac{v_2 \pm \sqrt{v_2^2 - 4\left(\frac{1}{2}a_1\right)(-v_2 t')}}{2\left(\frac{1}{2}a_1\right)} = \frac{v_2 \pm \sqrt{v_2^2 + 2a_1 v_2 t'}}{a_1}$$

(b) As in part (a), $\Delta x_1 = \Delta x_2$, and so $v_{i_0} t + \frac{1}{2} a_1 t^2 = v_2 t$. Solving for a_1 gives:

$$\frac{1}{2} a_1 t^2 = v_2 t - v_{i_0} t \Rightarrow a_1 = \frac{2(v_2 - v_{i_0})}{t}$$

CALCULATE:

$$\begin{aligned}
 \text{(a) } t_1 &= \frac{(8.0 \text{ m/s}) \pm \sqrt{(-8.0 \text{ m/s})^2 + 2(2.2 \text{ m/s}^2)(8.0 \text{ m/s})(20 \text{ s})}}{2.2 \text{ m/s}^2} \\
 &= \frac{(8.0 \text{ m/s}) \pm \sqrt{64 \text{ m}^2/\text{s}^2 + 704 \text{ m}^2/\text{s}^2}}{2.2 \text{ m/s}^2} \\
 &= \frac{(8.0 \text{ m/s}) \pm 27.7 \text{ m/s}}{2.2 \text{ m/s}^2} \\
 &= 16.2272, -8.9545 \\
 \text{(b) } a_1 &= \frac{2(8.0 \text{ m/s} - 1.2 \text{ m/s})}{16.2 \text{ s}} = 0.840 \text{ m/s}^2
 \end{aligned}$$

ROUND:

(a) Time must be positive, so take the positive solution, $t_1 = 16 \text{ s}$.

(b) $a_1 = 0.84 \text{ m/s}^2$

DOUBLE-CHECK:

(a) The units of the result are those of time. This is a reasonable amount of time to catch up to the friend who is traveling at $v_2 = 8.0 \text{ m/s}$.

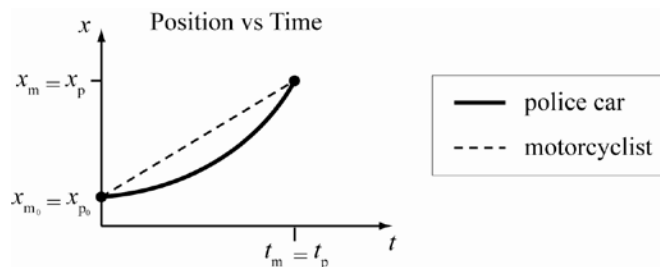
(b) This acceleration is less than that in part (a). Without the 20 s head-start, the friend does not travel as far, and so the acceleration of the girl should be less in part (b) than in part a), given the same time.

2.64. THINK: The motorcyclist is moving with a constant velocity $v_m = 36.0 \text{ m/s}$. The police car has an initial velocity $v_{p_0} = 0$, and acceleration $a_p = 4.0 \text{ m/s}^2$.

(a) I want to find the time required for the police car to catch up to the motorcycle. Note both the police car and the motorcycle travel for the same amount of time: $t_p = t_m$.

(b) I want to find the final speed of the police car, v_p .

(c) I want to find the distance traveled by the police car at the moment when it catches up to the motorcycle. Note the motorcyclist and the police car will have both traveled the same distance from the police car's initial position, once the police car catches up to the motorcycle. That is, $\Delta x_m = \Delta x_p$.

SKETCH:**RESEARCH:**

(a) To find t_p , use $\Delta x_p = v_{p_0} t_p + (1/2) a_p t_p^2$ for the police car and $\Delta x_m = v_m t_m$ for the motorcycle.

(b) To find v_p , use $v_p = v_{p_0} + a_p t_p$ for the police car.

(c) Since $\Delta x_p = \Delta x_m$, $\Delta x_p = v_m t_m$ for the police car.

SIMPLIFY:(a) Since $\Delta x_p = \Delta x_m$:

$$v_{p0} t_p + \frac{1}{2} a_p t_p^2 = v_m t_m$$

$$v_{p0} t_p + \frac{1}{2} a_p t_p^2 = v_m t_p \quad \text{Since } t_m = t_p,$$

$$\frac{1}{2} a_p t_p^2 = v_m t_p \quad \text{Since } v_{p0} = 0,$$

$$\frac{1}{2} a_p t_p^2 - v_m t_p = 0$$

$$t_p \left(\frac{1}{2} a_p t_p - v_m \right) = 0$$

There are two solutions for t_p here: $t_p = 0$ or $\left(\frac{1}{2} a_p t_p - v_m \right) = 0$. The first solution corresponds to the time when the motorcycle first passes the stationary police car. The second solution gives the time when the police car catches up to the motorcycle. Rearranging gives: $t_p = 2v_m / a_p$.

(b) $v_p = v_{p0} + a_p t_p \Rightarrow v_p = a_p t_p$, since $v_{p0} = 0$. Substituting $t_p = 2v_m / a_p$ into this equation gives:

$$v_p = a_p (2v_m / a_p) = 2v_m.$$

(c) No simplification is necessary.

CALCULATE:

$$(a) \quad t_p = \frac{2(36.0 \text{ m/s})}{4.0 \text{ m/s}^2} = 18.0 \text{ s}$$

$$(b) \quad v_p = 2(36.0 \text{ m/s}) = 72.0 \text{ m/s}$$

$$(c) \quad \Delta x_p = (36.0 \text{ m/s})(18.0 \text{ s}) = 648 \text{ m}$$

ROUND:

(a) a_p has only two significant figures, so $t_p = 18 \text{ s}$.

(b) v_m has three significant digits, so $v_p = 72.0 \text{ m/s}$.

(c) a_p has only two significant digits, so $\Delta x_p = 650 \text{ m}$.

DOUBLE-CHECK:

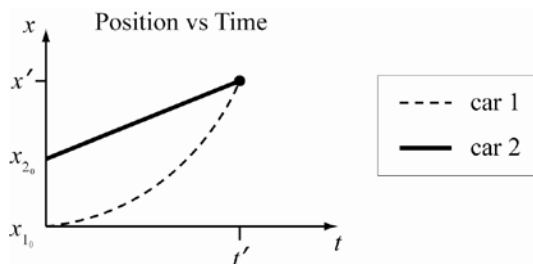
(a) The calculated time is reasonable for the police car to catch the motorcyclist.

(b) The calculated speed is fast, but it is a realistic speed for a police car to achieve while chasing a speeding vehicle.

(c) The distance is a reasonable distance to cover in 18 s given that the average speed of the police car is

$$v_{\text{avg}} = (1/2)(v_{p0} + v_p) = (1/2)v_p = 36.0 \text{ m/s}.$$

- 2.65. THINK:** Since no information is given about the direction of the second car, it is assumed that both cars travel in the same direction. The first car accelerates from rest with $a_1 = 2.00 \text{ m/s}^2$. The second car moves with constant velocity $v_2 = 4.00 \text{ m/s}$. The cars are 30.0 m apart. Take the initial position of car 1 to be $x_{10} = 0$. Then the initial position of car 2 is $x_{20} = 30.0 \text{ m}$. Both cars will have the same final position: $x_1 = x_2 = x'$. Both cars will travel for the same amount of time: $t_1 = t_2 = t'$.
- (a) I want to know the position of the collision, x' .
- (b) I want to know the time at which the collision occurs, t' .

SKETCH:

RESEARCH: car 1: $\Delta x_1 = v_{i_0} t_1 + \frac{1}{2} a_1 t_1^2$; car 2: $\Delta x_2 = v_2 t_2$

SIMPLIFY:

(a) To solve for x' , use $\Delta x_1 = v_{i_0} t_1 + (1/2) a_1 t_1^2 \Rightarrow x' - x_{i_0} = v_{i_0} t_1 + (1/2) a_1 t_1^2$. Since $x_{i_0} = 0$ and $v_{i_0} = 0$, $x' = (1/2) a_1 t_1^2$. Time t_1 is not known, but $t_1 = t_2$ and $\Delta x_2 = v_2 t_2$, therefore, $t_1 = t_2 = \Delta x_2 / v_2 = (x' - x_{2_0}) / v_2$. Inserting this t_1 into the first equation yields

$$x' = \frac{1}{2} a_1 \left(\frac{x' - x_{2_0}}{v_2} \right)^2 = \frac{a_1}{2 v_2^2} \left((x')^2 - 2 x_{2_0} x' + x_{2_0}^2 \right)$$

Rearranging gives:

$$\frac{a_1}{2 v_2^2} (x')^2 - \left(\frac{a_1 x_{2_0}}{v_2^2} + 1 \right) x' + \frac{a_1 x_{2_0}^2}{2 v_2^2} = 0 \Rightarrow (x')^2 - \left(2 x_{2_0} + \frac{2 v_2^2}{a_1} \right) x' + x_{2_0}^2 = 0.$$

This is a quadratic equation. Solving for x' :

$$x' = \frac{\left(2 x_{2_0} + \frac{2 v_2^2}{a_1} \right) \pm \sqrt{\left(2 x_{2_0} + \frac{2 v_2^2}{a_1} \right)^2 - 4 x_{2_0}^2}}{2}$$

(b) To solve for t' , use $t' = t_2 = \frac{x' - x_{2_0}}{v_2}$, from above.

CALCULATE:

$$(a) \quad x' = \frac{\left(2(30.0 \text{ m}) + \frac{2(4.00 \text{ m/s})^2}{2.00 \text{ m/s}^2} \right) \pm \sqrt{\left(2(30.0 \text{ m}) + \frac{2(4.00 \text{ m/s})^2}{2.00 \text{ m/s}^2} \right)^2 - 4(30.0 \text{ m})^2}}{2}$$

$$= 14.68 \text{ m}, 61.32 \text{ m}$$

The first solution may be disregarded; with both cars moving in the same direction, the position of the collision cannot be between their two initial positions. That is, x' cannot be between $x_{i_0} = 0$ and $x_{2_0} = 30 \text{ m}$.

$$(b) \quad t' = \frac{x' - x_{2_0}}{v_2} = \frac{61.32 \text{ m} - 30.0 \text{ m}}{4.00 \text{ m/s}} = 7.830 \text{ s}$$

ROUND:

$$(a) \quad x' = 61.3 \text{ m}$$

$$(b) \quad t' = 7.83 \text{ s}$$

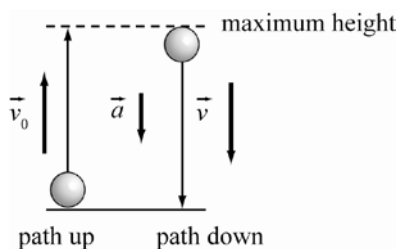
DOUBLE-CHECK:

(a) This collision position has units of distance, and is greater than the initial positions of both cars, as it should be.

(b) The time is reasonable since this is the time required for car 2 to travel $\Delta x_2 = x' - x_{2_0} = 30 \text{ m}$ at a speed of $v_2 = 4.0 \text{ m/s}$.

- 2.66. THINK:** I know that $v_0 = 26.4 \text{ m/s}$ and $a = -g = -9.81 \text{ m/s}^2$. I want to find t_{total} . Note that once the ball gets back to the starting point, $v = -26.4 \text{ m/s}$, or $v = -v_0$.

SKETCH:



RESEARCH: $v = v_0 + at$

SIMPLIFY: $t = \frac{v - v_0}{a} = \frac{-v_0 - v_0}{-g} = \frac{2v_0}{g}$

CALCULATE: $t = \frac{2(26.4 \text{ m/s})}{9.81 \text{ m/s}^2} = 5.38226 \text{ s}$

ROUND: Since all the values given have three significant digits, $t = 5.38 \text{ s}$.

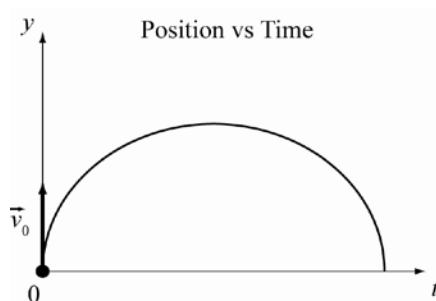
DOUBLE-CHECK: This seems like a reasonable amount of time for the ball to be up in the air.

- 2.67. THINK:** I know that $v_0 = 10.0 \text{ m/s}$, $a = -g = -9.81 \text{ m/s}^2$, and $y_0 = 0 \text{ m}$.

(a) I want to find the velocity v at $t = 0.50 \text{ s}$.

(b) I want to find the height h of the stone at $t = 0.50 \text{ s}$.

SKETCH:



RESEARCH:

(a) $v = v_0 + at$

(b) $\Delta y = v_0 t + \frac{1}{2}at^2$ and $\Delta y = h$

SIMPLIFY:

(a) $v = v_0 - gt$

(b) $h = v_0 t + \frac{1}{2}at^2 = v_0 t - \frac{1}{2}gt^2$

CALCULATE:

(a) $v = 10.0 \text{ m/s} - (9.81 \text{ m/s}^2)(0.50 \text{ s})$
 $= 10.0 \text{ m/s} - 4.905 \text{ m/s}$
 $= 5.095 \text{ m/s}$

(b) $h = (10.0 \text{ m/s})(0.50 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.50 \text{ s})^2$
 $= 5.0 \text{ m} - 1.226 \text{ m}$
 $= 3.774 \text{ m}$

ROUND:

(a) Subtracting two numbers is precise to the least precise decimal place of the numbers. Therefore, $v = 5.1 \text{ m/s}$.

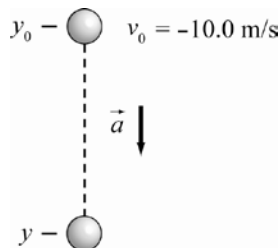
(b) $h = 3.8 \text{ m}$

DOUBLE-CHECK:

(a) $v < v_0$, and this makes sense since speed decreases as the object rises.

(b) This is a reasonable height for a ball to achieve in 0.50 s after it is thrown upward.

2.68. **THINK:** I know that $v_0 = -10.0 \text{ m/s}$, and $a = -g = -9.81 \text{ m/s}^2$. I want to find v at $t = 0.500 \text{ s}$.

SKETCH:

RESEARCH: $v = v_0 + at$

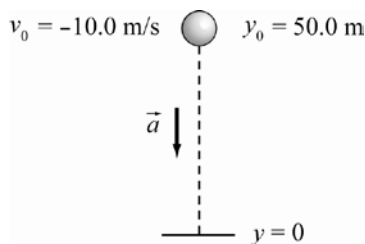
SIMPLIFY: $v = v_0 - gt$

CALCULATE: $v = -10.0 \text{ m/s} - (9.81 \text{ m/s}^2)(0.500 \text{ s}) = -10.0 \text{ m/s} - 4.905 \text{ m/s} = -14.905 \text{ m/s}$

ROUND: Subtracting two numbers is precise to the least precise decimal place of the numbers. Therefore, $v = -14.9 \text{ m/s}$.

DOUBLE-CHECK: A negative v indicates that the stone is (still) falling downward. This makes sense, since the stone was thrown downward.

2.69. **THINK:** Take “downward” to be along the negative y -axis. I know that $v_0 = -10.0 \text{ m/s}$, $\Delta y = -50.0 \text{ m}$, and $a = -g = -9.81 \text{ m/s}^2$. I want to find t , the time when the ball reaches the ground.

SKETCH:

RESEARCH: $\Delta y = v_0 t + \frac{1}{2}at^2$

SIMPLIFY: $\frac{1}{2}at^2 + v_0 t - \Delta y = 0$. This is a quadratic equation. Solving for t :

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}a\right)(-\Delta y)}}{2\left(\frac{1}{2}a\right)} = \frac{-v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{-g}$$

CALCULATE: $t = \frac{-(-10.0 \text{ m/s}) \pm \sqrt{(-10.0 \text{ m/s})^2 - 2(9.81 \text{ m/s}^2)(-50.0 \text{ m})}}{-9.81 \text{ m/s}^2}$
 $= -4.3709 \text{ s}, 2.3322 \text{ s}$

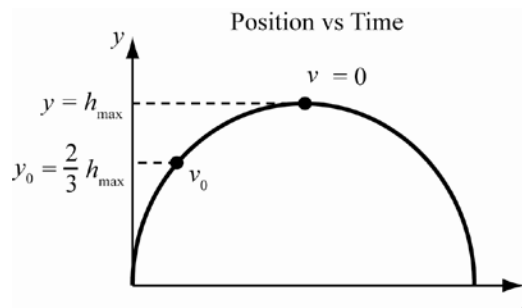
The time interval has to be positive, so $t = 2.3322 \text{ s}$.

ROUND: All original quantities are precise to three significant digits, therefore $t = 2.33 \text{ s}$.

DOUBLE-CHECK: A negative v indicates that the stone is (still) falling downward. This makes sense, since the stone was thrown downward. The velocity is even more negative after 0.500 s than it was initially, which is consistent with the downward acceleration.

- 2.70. **THINK:** I know that $v_0 = 20.0 \text{ m/s}$, $y_0 = (2/3)h_{\text{max}}$, and $a = -g = -9.81 \text{ m/s}^2$. I want to find h_{max} . Note that when $y = h_{\text{max}}$, the velocity is $v = 0$.

SKETCH:



RESEARCH: $v^2 = v_0^2 + 2a(y - y_0)$

SIMPLIFY: $v^2 = v_0^2 - 2g\left(h_{\text{max}} - \frac{2}{3}h_{\text{max}}\right) \Rightarrow v^2 - v_0^2 = -2g\left(\frac{1}{3}h_{\text{max}}\right) \Rightarrow h_{\text{max}} = -\frac{3(v^2 - v_0^2)}{2g} \Rightarrow h_{\text{max}} = \frac{3v_0^2}{2g}$

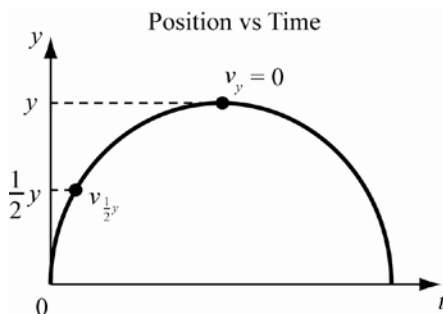
CALCULATE: $h_{\text{max}} = \frac{3(20.0 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 61.16 \text{ m}$

ROUND: $h_{\text{max}} = 61.2 \text{ m}$

DOUBLE-CHECK: h_{max} is positive which is consistent with the sketch. This seems like a reasonable height to achieve by throwing the ball upward.

- 2.71. **THINK:** I know the final height is y and the initial velocity is v_0 . The velocity at this height is zero: $v_y = 0$. Also, $a_y = -g$. I want to know the velocity at half of the final height, $v_{\frac{1}{2}y}$. Assume $y_0 = 0$.

SKETCH:



RESEARCH: $v_y^2 = v_0^2 + 2a(y - y_0)$

SIMPLIFY: The initial velocity, v_0 , is $v_0 = \sqrt{v_y^2 - 2a(y - y_0)} = \sqrt{2gy}$. Then $v_{\frac{1}{2}y}$, in terms of the maximum height y , is

$$\left(v_{\frac{1}{2}y}\right)^2 = v_0^2 + 2a\left(\left(\frac{1}{2}y\right) - y_0\right) \Rightarrow v_{\frac{1}{2}y}^2 = (\sqrt{2gy})^2 - 2g\left(\frac{1}{2}y\right) \Rightarrow v_{\frac{1}{2}y}^2 = 2gy - gy \Rightarrow v_{\frac{1}{2}y} = \sqrt{gy}$$

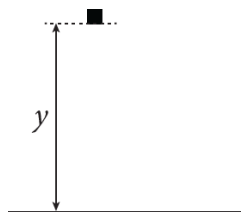
CALCULATE: This step is not necessary.

ROUND: This step is not necessary.

DOUBLE-CHECK: The units of $v_{\frac{1}{2}y}$ are: $\left[v_{\frac{1}{2}y}\right] = \sqrt{\left(\frac{\text{m}}{\text{s}^2}\right)(\text{m})} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \text{m/s}$, which is a unit of velocity.

- 2.72. **THINK:** The acceleration of an object due to gravity on the surface of the Moon is independent of the mass of the object.

SKETCH:



RESEARCH: We can use $y = \frac{1}{2}gt^2$, where y is the distance the objects fall, t is the time it takes for the objects to fall, and g is the acceleration of gravity on the Moon.

SIMPLIFY: We can solve our equation for g : $y = \frac{1}{2}gt^2 \Rightarrow g = \frac{2y}{t^2}$.

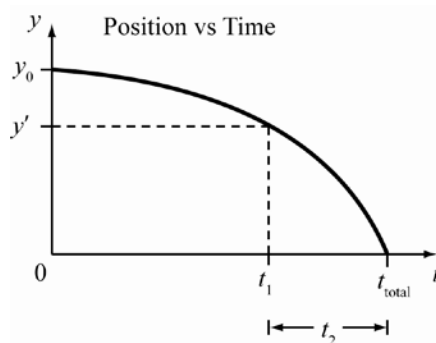
CALCULATE: $g = \frac{2y}{t^2} = \frac{2(1.6 \text{ m})}{(1.4 \text{ s})^2} = 1.6327 \text{ m/s}^2$.

ROUND: The values given are all accurate to two significant digits, so the answer is given two by two significant digits: $g = 1.6 \text{ m/s}^2$.

DOUBLE-CHECK: The Moon is smaller and less dense than the Earth, so it makes sense that the acceleration of gravity on the surface of the Moon is about 6 times less than the acceleration of gravity on the surface of the Earth.

- 2.73. **THINK:** The bowling ball is released from rest. In such a case we have already studied the relationship between vertical distance fallen and time in Example 2.5, “Reaction Time”, in the book. With this result in our arsenal, all we have to do here is to compute the time t_{total} it takes the ball to fall from Bill’s apartment down to the ground and subtract from it the time t_1 it takes the ball to fall from Bill’s apartment down to John’s apartment.

SKETCH:



RESEARCH: We will use the formula $t = \sqrt{2h/g}$ from Example 2.5. If you look at the sketch, you see that $t_{\text{total}} = \sqrt{2h_{\text{total}}/g} = \sqrt{2y_0/g}$ and that $t_1 = \sqrt{2h_1/g} = \sqrt{2(y_0 - y')/g}$.

SIMPLIFY: Solving for the time difference gives:

$$t_2 = t_{\text{total}} - t_1 = \sqrt{2y_0/g} - \sqrt{2(y_0 - y')/g}$$

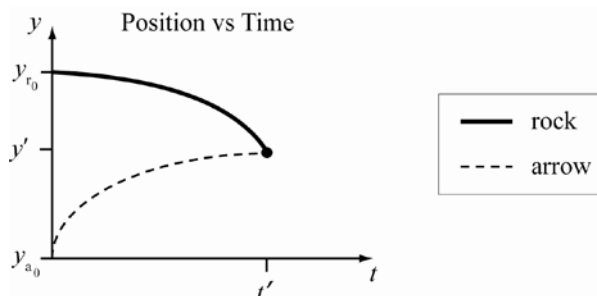
CALCULATE: $t_2 = \sqrt{2(63.17 \text{ m}) / (9.81 \text{ m/s}^2)} - \sqrt{2(63.17 \text{ m} - 40.95 \text{ m}) / (9.81 \text{ m/s}^2)}$
 $= 1.4603 \text{ s}$

ROUND: We round to $t_2 = 1.46 \text{ s}$, because g has three significant figures.

DOUBLE-CHECK: The units of the solution are those of time, which is already a good minimum requirement for a valid solution. But we can do better! If we compute the time it takes an object to fall 40.95 m from rest, we find from again using $t = \sqrt{2h/g}$ that this time is 2.89 s. In the problem here the bowling ball clearly already has a significant downward velocity as it passes the height of 40.95 m, and so we expect a time t_2 shorter than 2.89 s, which is clearly fulfilled for our solution.

- 2.74. **THINK:** The information known for the rock is the initial velocity, $v_{r_0} = 0$ and the initial height, $y_{r_0} = 18.35 \text{ m}$. The information known for the arrow is the initial velocity, $v_{a_0} = 47.4 \text{ m/s}$ and the initial height, $y_{a_0} = 0$. For both, $a = -g = -9.81 \text{ m/s}^2$. Note that both the rock and the arrow will have the same final position, y' , and both travel for the same time, t' . I want to find t' .

SKETCH:



RESEARCH: $\Delta y = v_0 t + \frac{1}{2} a t^2$

SIMPLIFY: For the rock, $y_r - y_{r_0} = v_{r_0} t' + (1/2) a (t')^2 \Rightarrow y_r = -\frac{1}{2} g (t')^2 + y_{r_0}$. For the arrow, $y_a - y_{a_0} = v_{a_0} t' + (1/2) a (t')^2 \Rightarrow y_a = v_{a_0} t' - (1/2) g (t')^2$. As the final positions for each are the same, we know $y_r = y_a \Rightarrow -\frac{1}{2} g (t')^2 + y_{r_0} = v_{a_0} t' - \frac{1}{2} g (t')^2 \Rightarrow y_{r_0} = v_{a_0} t' \Rightarrow t' = \frac{y_{r_0}}{v_{a_0}}$.

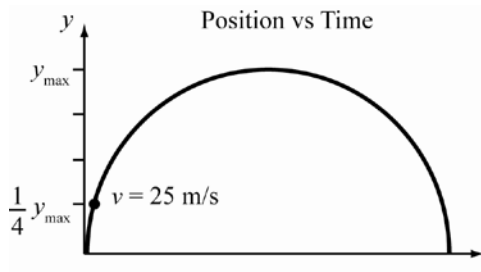
CALCULATE: $t' = \frac{18.35 \text{ m}}{47.4 \text{ m/s}} = 0.38713 \text{ s}$

ROUND: v_{a_0} is given to three significant figures, so $t' = 0.387 \text{ s}$.

DOUBLE-CHECK: This is a reasonable time for an arrow of initial velocity 47.4 m/s to rise to a height less than 18.35 m (the height from which the rock was dropped).

- 2.75. **THINK:** At $y = (1/4)y_{\max}$, $v = 25$ m/s. Also, $a = -g = -9.81$ m/s² and $y_0 = 0$. I want to find v_0 . It will be useful to know y_{\max} . At y_{\max} , $v' = 0$.

SKETCH:



RESEARCH: $v^2 = v_0^2 + 2a(y - y_0)$

SIMPLIFY: $v^2 = v_0^2 + 2a(y - y_0) = v_0^2 - 2g\left(\frac{1}{4}y_{\max}\right)$

$$\Rightarrow v_0^2 = v^2 + 2g\left(\frac{1}{4}y_{\max}\right) = v^2 + \frac{1}{2}gy_{\max}$$

Now I must find y_{\max} . When y_{\max} is achieved, the velocity v' is zero. Then

$$(v')^2 = v_0^2 + 2a(y_{\max} - y_0) \Rightarrow 0 = v_0^2 - 2gy_{\max} \Rightarrow y_{\max} = \frac{v_0^2}{2g}.$$

Inserting this into the equation above gives

$$v_0^2 = v^2 + \frac{1}{2}g\left(\frac{v_0^2}{2g}\right) \Rightarrow v_0^2 = v^2 + \frac{1}{4}v_0^2 \Rightarrow \frac{3}{4}v_0^2 = v^2 \Rightarrow v_0^2 = \frac{4}{3}v^2 \Rightarrow v_0 = \frac{2}{\sqrt{3}}v.$$

CALCULATE: $v_0 = \frac{2}{\sqrt{3}}(25 \text{ m/s}) = 28.87 \text{ m/s}$

ROUND: The value for v limits the calculation to two significant figures. So $v_0 = 29$ m/s.

DOUBLE-CHECK: v_0 is greater than $v = 25$ m/s, as it should be.

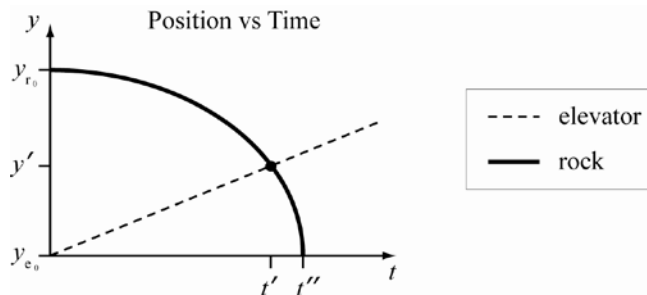
- 2.76. **THINK:** For the elevator, the velocity is $v_e = 1.75$ m/s, the acceleration is $a_e = 0$, and the initial height is $y_{e_0} = 0$. For the rock, the initial velocity is $v_{r_0} = 0$, the acceleration is $a_r = -g = -9.81$ m/s², and the initial height is $y_{r_0} = 80.0$ m.

(a) I need to find the time it takes the rock to intercept the elevator, t' .

(b) I need to find the time it takes the rock to hit the ground at $y_r = 0$, t'' .

When the rock intercepts the elevator, both are at the same position y' , and have taken the same time, t' , to arrive there.

SKETCH:



RESEARCH: The elevator position is determined from $\Delta y_e = v_e t$. For the rock, $\Delta y_r = v_{r_0} t + (1/2)a_r t^2$.

SIMPLIFY: For the elevator, $y_e - y_{e_0} = v_e t \Rightarrow y_e = v_e t$. For the rock, $y_r - y_{r_0} = v_{r_0} t + (1/2)a_r t^2 \Rightarrow y_r - y_{r_0} = -(1/2)gt^2$.

(a) Both objects take the same time to intercept each other, and both have the same position at interception:

$$y_e = y_r \Rightarrow v_e t' = y_{r_0} - \frac{1}{2}g(t')^2 \Rightarrow \frac{1}{2}g(t')^2 + v_e t' - y_{r_0} = 0.$$

Solving for t' in the quadratic equation gives:

$$t' = \frac{-v_e \pm \sqrt{v_e^2 - 4\left(\frac{1}{2}g\right)(-y_{r_0})}}{2\left(\frac{1}{2}g\right)} = \frac{-v_e \pm \sqrt{v_e^2 + 2gy_{r_0}}}{g}.$$

(b) The total fall time, t'' , for the rock is $\Delta y_r = y_r - y_{r_0} = v_{r_0} t'' + (1/2)a_r (t'')^2$. The final position is $y_r = 0$. With $v_{r_0} = 0$, $-y_{r_0} = -(1/2)g(t'')^2 \Rightarrow t'' = \sqrt{2y_{r_0}/g}$.

CALCULATE:

$$(a) \quad t' = \frac{-(1.75 \text{ m/s}) \pm \sqrt{(1.75 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(80.0 \text{ m})}}{9.81 \text{ m/s}^2} = 3.8641 \text{ s}, -4.2209 \text{ s}$$

$$(b) \quad t'' = \sqrt{\frac{2(80.0 \text{ m})}{9.81 \text{ m/s}^2}} = 4.0386 \text{ s}$$

ROUND: The values given have three significant figures, so the final answers will also have three significant figures.

(a) Taking the positive solution for time, $t' = 3.86 \text{ s}$.

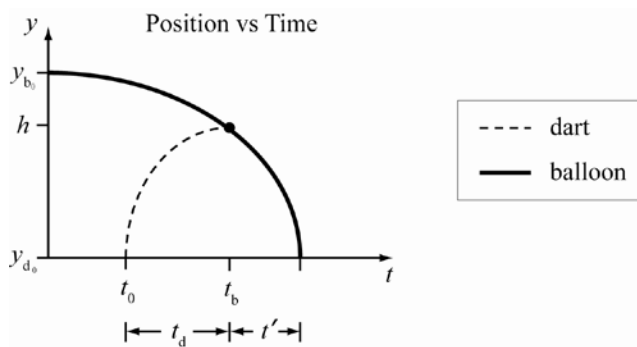
(b) $t'' = 4.04 \text{ s}$

DOUBLE-CHECK: The total time to fall is greater than the intercept time, as it should be.

2.77. THINK: It is probably a good idea to read through the solution of the “Melon Drop” problem, Solved Problem 2.5 in the textbook before getting started with the present problem. The present problem has the additional complication that the water balloon gets dropped some time before the dart get fired, whereas in the “Melon Drop” problem both projectiles get launched simultaneously. For the first 2 seconds, only our water balloon is in free fall, and we can calculate its position y_{b_0} and velocity v_{b_0} at the end of this time interval.

a) After the initial two seconds the dart also gets launched, and then both objects (water balloon and dart) are in free-fall. Their initial distance is y_{b_0} , and their relative velocity is the difference between the initial velocity of the dart and v_{b_0} . The time until the two objects meet is then simply the ratio of the initial distance and the relative velocity.

b) For this part we simply calculate the time it takes for the balloon to free-fall the entire height h and subtract our answer from part a).

SKETCH:**RESEARCH:**

(a) The position and velocity of the balloon after the time $t_0 = 2$ s are

$$y_{b_0} = h - \frac{1}{2}gt_0^2$$

$$v_{b_0} = -gt_0$$

The time it takes then for the balloon and the dart to meet is the ratio of their initial distance to their initial relative velocity:

$$t_d = y_{b_0} / (v_{d_0} - v_{b_0})$$

Our answer for part a) is the sum of the time t_0 , during which the balloon was in free-fall alone, and the time t_1 , $t_b = t_d + t_0$.

(b) The total time it takes for the balloon to fall all the way to the ground is

$$t_{\text{total}} = \sqrt{2h/g}$$

We get our answer for part b) by subtracting the result of part a) from this total time:

$$t' = t_{\text{total}} - t_d$$

SIMPLIFY:

(a) If we insert the expressions for the initial distance and relative speed into $t_d = y_{b_0} / (v_{d_0} - v_{b_0})$, we find

$t_d = y_{b_0} / (v_{d_0} - v_{b_0}) = (h - \frac{1}{2}gt_0^2) / (v_{d_0} + gt_0)$. Adding t_0 then gives us our final answer:

$$t_b = t_0 + (h - \frac{1}{2}gt_0^2) / (v_{d_0} + gt_0)$$

(b) For the time between the balloon being hit by the dart and the water reaching the ground we find by

inserting $t_{\text{total}} = \sqrt{2h/g}$ into $t' = t_{\text{total}} - t_b$:

$$t' = \sqrt{2h/g} - t_b$$

CALCULATE:

$$(a) \quad t_b = 2.00 \text{ s} + \frac{80.0 \text{ m} - \frac{1}{2}(9.81 \text{ m/s}^2)(2.00 \text{ s})^2}{20.0 \text{ m/s} + (9.81 \text{ m/s}^2)(2.00 \text{ s})} = 3.524 \text{ s}$$

$$(b) \quad t' = \sqrt{2(80.0 \text{ m}) / (9.81 \text{ m/s}^2)} - 3.524 \text{ s} = 0.515 \text{ s}.$$

ROUND:

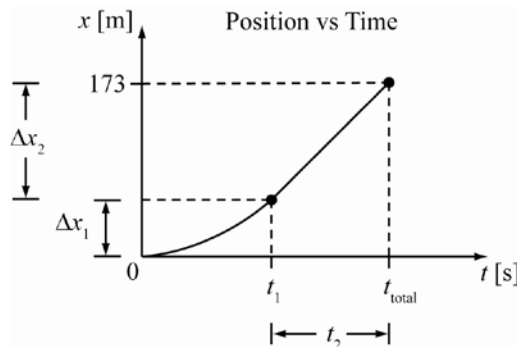
$$(a) \quad t_b = 3.52 \text{ s}$$

$$(b) \quad t' = 0.515 \text{ s}$$

DOUBLE-CHECK: The solution we showed in this problem is basically the double-check step in Solved Problem 2.5. Conversely, we can use the solution method of Solved Problem 2.5 as a double-check for what we have done here. This is left as an exercise for the reader.

- 2.78. THINK:** I know the runner's initial velocity, $v_0 = 0$, her acceleration, $a = 1.23 \text{ m/s}^2$, her final velocity, $v = 5.10 \text{ m/s}$, and the distance she traveled, $\Delta x = 173 \text{ m}$. I want to know the total time t_{total} . Note that Δx is composed of a displacement Δx_1 which occurs while accelerating and a displacement Δx_2 which occurs at a constant velocity. That is, $\Delta x = \Delta x_1 + \Delta x_2$. Mass is irrelevant.

SKETCH:



RESEARCH: The total time is the sum of the times for each displacement. Let $t_{\text{total}} = t_1 + t_2$ with t_1 the time for displacement Δx_1 and t_2 the time for displacement Δx_2 . For t_1 , use $v = v_0 + at_1$. For Δx_1 , use $\Delta x_1 = (1/2)(v + v_0)t_1$. For t_2 , use $\Delta x_2 = vt_2$.

SIMPLIFY: Note that

$$\Delta x_1 = \frac{1}{2}(v + v_0)t_1 = \frac{1}{2}vt_1 = \frac{1}{2}v \frac{v}{a} = \frac{v^2}{2a}.$$

To compute the value of t_{total} , first simplify expressions for t_1 and t_2 :

$$t_1 = \frac{v - v_0}{a} = \frac{v}{a} \quad \text{and} \quad t_2 = \frac{\Delta x_2}{v} = \frac{\Delta x - \Delta x_1}{v} = \frac{\Delta x - \frac{v^2}{2a}}{v} = \frac{\Delta x}{v} - \frac{v}{2a}.$$

Using the last two equations t_{total} can be calculated as follows:

$$t_{\text{total}} = t_1 + t_2 = \frac{v}{a} + \frac{\Delta x}{v} - \frac{v}{2a} = \frac{v}{2a} + \frac{\Delta x}{v}.$$

CALCULATE: $t_{\text{total}} = \frac{5.10 \text{ m/s}}{2(1.23 \text{ m/s}^2)} + \frac{173 \text{ m}}{5.10 \text{ m/s}} = 35.995 \text{ s}$

ROUND: Each initial value has three significant figures, so $t_{\text{total}} = 36.0 \text{ s}$

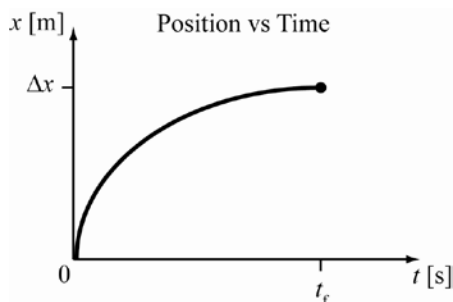
DOUBLE-CHECK: This is a reasonable amount of time required to run 173 m.

- 2.79. THINK:** Let the moment the jet touches down correspond to the time $t = 0$. The initial velocity is

$$v_0 = \left(\frac{142.4 \text{ mi}}{1 \text{ hr}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) \left(\frac{1609.3 \text{ m}}{1 \text{ mi}} \right) = 63.66 \text{ m/s}.$$

The jet comes to rest in a time of $t_f = 12.4 \text{ s}$. The jet comes to a complete stop, which makes the final velocity zero, so $v_f = 0$. I want to compute the distance the jet travels after it touches down, Δx .

SKETCH:



RESEARCH: To determine the distance traveled, the following equation can be used: $\Delta x = \frac{1}{2}(v_0 + v)t$.

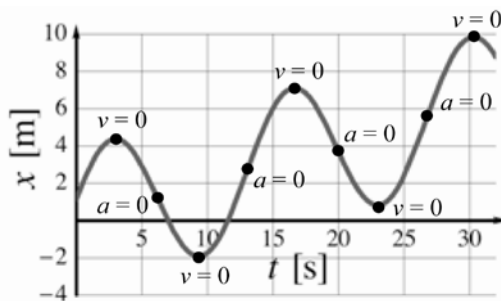
SIMPLIFY: With $v = 0$, the equation becomes $\Delta x = (v_0 t) / 2$.

CALCULATE: $\Delta x = \frac{1}{2}(63.66 \text{ m/s})(12.4 \text{ s}) = 394.7 \text{ m}$.

ROUND: Since t has three significant digits, the result should be rounded to $\Delta x = 395 \text{ m}$.

DOUBLE-CHECK: This is a reasonable distance to decelerate from 63.66 m/s in 12.4 s.

- 2.80. Velocity is the slope of the position versus time graph. Therefore, $v = 0$ at the local maxima and minima. Acceleration is the slope of the velocity versus time graph. On a position versus time graph, acceleration, a is zero at inflection points on the curve that are not maxima or minima, i.e. $a = 0$ as the slope of x vs. t approaches a constant value over some non-zero time interval, Δt :



- 2.81. **THINK:** The acceleration is the derivative of the velocity with respect to time, that is, the instantaneous change in velocity. Since the car is stopped and then accelerates to 60.0 miles per hour, we can infer that the acceleration is positive in the direction of the car's motion.

SKETCH: Sketch the motion of the car at 0 s and time 4.20 s. Since the acceleration is unknown, use the variable a to represent the size of the acceleration.

$$t = 0$$

$$|\vec{v}_x| = 0$$

$$|\vec{a}| = a$$

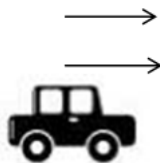


$$x_0 = 0$$

$$0 < t < 4.2$$

$$0 < |\vec{v}_x| < 60.0$$

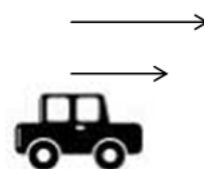
$$|\vec{a}| = a$$



$$t = 4.20$$

$$|\vec{v}_x| = 60.0$$

$$|\vec{a}| = a$$



$$x = ?$$

RESEARCH: Since this problem involves motion with constant acceleration, use equation (2.23). (a) The velocity in the positive x direction at time t is equal to the velocity in the positive x direction at time 0 plus the acceleration in the x direction multiplied by the time, $v_x = v_{x0} + a_x t$. (b) Having found the acceleration in the positive x direction, use the equation $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$ to find the position x at time $t = 4.20$ s. To make the calculations simple and straightforward, take the position of the car at time $t = 0$ s to be the zero of our coordinate system, so $x_0 = 0$ miles.

SIMPLIFY: (a) Use the velocity at times $t = 0$ and $t = 4.2$, solve the equation $v_x = v_{x0} + a_x t$ for a_x to get:

$$\begin{aligned} v_x &= v_{x0} + a_x t \\ -v_{x0} + v_x &= -v_{x0} + v_{x0} + a_x t = a_x t \\ (v_x - v_{x0})/t &= (a_x t)/t \\ \frac{v_x - v_{x0}}{t} &= a_x \end{aligned}$$

(b) Using the expression for a_x and algebra to find an expression for the total distance traveled:

$$\begin{aligned} x &= x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \\ &= x_0 + v_{x0}t + \frac{1}{2}\left(\frac{v_x - v_{x0}}{t}\right)t^2 \\ &= x_0 + v_{x0}t + \frac{1}{2}(v_x - v_{x0})t \end{aligned}$$

CALCULATE: (a) Since the car starts at rest, $v_{x0} = 0$ mph. Also, the velocity at time $t = 4.20$ s is $v_x = 60.0$ mph in the positive x direction. Using these values gives $a_x = \frac{v_x - v_{x0}}{t} = \frac{60.0 - 0}{4.2} = \frac{100}{7} \text{ mph} \cdot \text{s}^{-1}$.

Since time t is given in miles per hour, it is necessary to convert this to miles per second to make the units consistent. Convert this to a more convenient set of units, such as $\text{mi} \cdot \text{s}^{-2}$ to make future calculations easier:

$$\frac{100}{7} \cdot \frac{\text{mi}}{\text{hour} \cdot \text{sec}} \cdot \frac{1 \text{ hour}}{3600 \text{ sec}} = \frac{1}{252} \cdot \frac{\text{mi}}{\text{sec}^2} \quad \text{(b) Plug in values to find } x, \text{ the location of the car at time } t = 4.20 \text{ s.}$$

This gives:

$$\begin{aligned} x &= x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \\ &= 0 + 0 + \frac{1}{2} \cdot \frac{1}{252} \cdot (4.20)^2 \\ &= \frac{7}{200} = 0.035 \text{ mi} \end{aligned}$$

ROUND: Since the measured values have 3 significant figures, the answers in both parts will have 3 significant figures. (a) For the acceleration, $\frac{1}{252} \frac{\text{mi}}{\text{sec}^2} \approx 0.00397 \frac{\text{mi}}{\text{sec}^2}$ or $3.97 \times 10^{-3} \frac{\text{mi}}{\text{sec}^2}$. (b) Using

scientific notation, $0.035 = 3.50 \times 10^{-2} \text{ mi}$. Note also that if we convert to SI units, we obtain (a) 6.39 m/s^2 for the acceleration and (b) 56.3 m for the distance.

DOUBLE-CHECK:

(a) Accelerating at a constant rate of $3.97 \times 10^{-3} \frac{\text{mi}}{\text{sec}^2}$ for 4.20 seconds from a standing start means that the car will be going $3.97 \times 10^{-3} \frac{\text{mi}}{\text{sec}^2} \cdot 4.2 \text{ sec} \cdot \frac{3600 \text{ sec}}{\text{hour}}$ or 60.0 mph after 4.20 seconds. This agrees with the question statement.

(b) Since the car is at position $x = \frac{1}{2}a_x t^2 = \frac{1}{504}t^2$ miles at time t seconds, the derivative with respect to time gives the velocity as a function of time: $\frac{dx}{dt} = \frac{d\left(\frac{1}{504}t^2\right)}{dt} = \frac{1}{504} \cdot 2t = \frac{1}{252}t$ miles per second, or $\frac{1}{252}t \frac{\text{mi}}{\text{sec}} \cdot \frac{3600 \text{ sec}}{\text{hour}} = \frac{100}{7}t \frac{\text{mi}}{\text{hour}}$ at time t . At time $t = 4.20 \text{ s}$, this gives a velocity of $\frac{100}{7}(4.2) = 60.0 \text{ mph}$, which agrees with the setup for this problem.

2.82. THINK: It is known for a car that when the initial velocity is

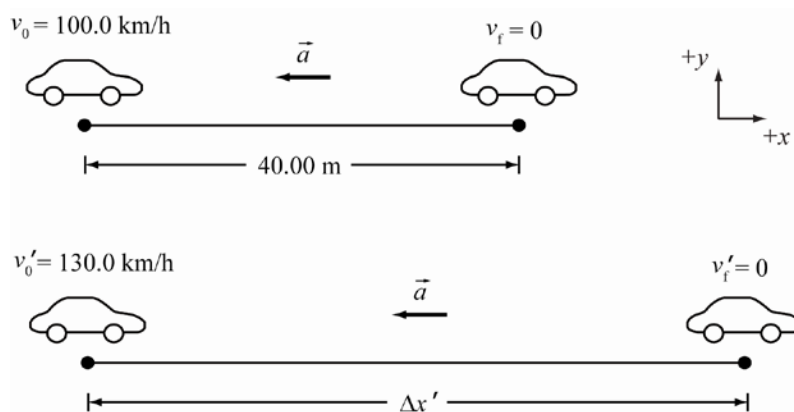
$$v_0 = 100.0 \frac{\text{km}}{\text{h}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 27.7778 \text{ m/s},$$

and the final velocity is $v_f = 0$, the stopping distance is $\Delta x = 40 \text{ m}$. Determine the stopping distance, $\Delta x'$ when the initial velocity is

$$v_0' = 130.0 \frac{\text{km}}{\text{h}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 36.1111 \text{ m/s},$$

and the final velocity is $v_f' = 0$. The road conditions are the same in each case, so it can be assumed that the acceleration does not change.

SKETCH:



RESEARCH: The acceleration, a can be determined from the original conditions with $v^2 = v_0^2 + 2a\Delta x$. Substitute the value of the acceleration computed from the first set of conditions as the acceleration in the second conditions to determine $\Delta x'$.

SIMPLIFY: With $v = 0$, $0 = v_0^2 + 2a\Delta x \Rightarrow a = -v_0^2 / (2\Delta x)$. Then, $v_f'^2 = v_0'^2 + 2a\Delta x'$. With $v_f' = 0$,

$$\Delta x' = -\frac{v_0'^2}{2a} = -\frac{v_0'^2}{2\left(-\frac{v_0^2}{2\Delta x}\right)} = \frac{v_0'^2}{v_0^2} \Delta x.$$

CALCULATE: $\Delta x' = \frac{(36.1111 \text{ m/s})^2}{(27.7778 \text{ m/s})^2} (40.00 \text{ m}) = 67.5999 \text{ m}$

Note that the unit conversion from km/h to m/s was not necessary as the units of velocity cancel each other in the ratio.

ROUND: $\Delta x' = 67.60 \text{ m}$

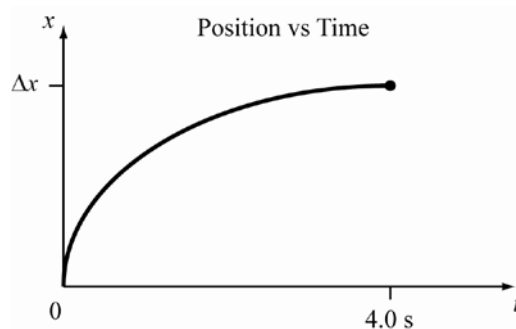
DOUBLE-CHECK: The stopping distance for the larger initial velocity is greater than the stopping distance for the small initial velocity, as it should be.

2.83. THINK: The initial velocity is

$$v_0 = 60.0 \frac{\text{km}}{\text{h}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 16.67 \text{ m/s}.$$

The final velocity is $v = 0$. The stop time is $t = 4.00 \text{ s}$. The deceleration is uniform. Determine (a) the distance traveled while stopping, Δx and (b) the deceleration, a . I expect $a < 0$.

SKETCH:



RESEARCH:

(a) To determine the stopping distance, use $\Delta x = t(v_0 + v)/2$.

(b) To determine a , use $v = v_0 + at$.

SIMPLIFY:

(a) With $v = 0$, $\Delta x = v_0 t / 2$.

(b) With $v = 0$, $0 = v_0 + at \Rightarrow a = -v_0 / t$.

CALCULATE:

$$(a) \Delta x = \frac{(16.67 \text{ m/s})(4.00 \text{ s})}{2} = 33.34 \text{ m}$$

$$(b) a = -\frac{16.67 \text{ m/s}}{4.00 \text{ s}} = -4.167 \text{ m/s}^2$$

ROUND:

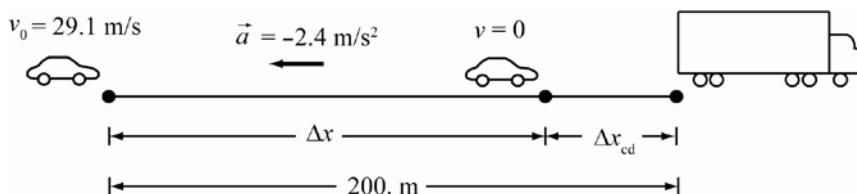
$$(a) \Delta x = 33.3 \text{ m}$$

$$(b) a = -4.17 \text{ m/s}^2$$

DOUBLE-CHECK: The distance traveled while stopping is of an appropriate order of magnitude. A car can reasonably stop from 60 km/h in a distance of about 30 m. The acceleration is negative, indicating that the car is slowing down from its initial velocity.

2.84. THINK: The car's initial velocity is $v_0 = 29.1 \text{ m/s}$. The deceleration is $a = -2.4 \text{ m/s}^2$. Assume that the final velocity is $v = 0$, that is the car does not hit the truck. The truck is a distance $d = 200.0 \text{ m}$ when the car begins to decelerate. Determine (a) the final distance between the car and the truck, Δx_{cd} and (b) the time it takes to stop, t .

SKETCH:



RESEARCH:

(a) The distance to the truck is the difference between the initial distance d and the stopping distance Δx :

$$\Delta x_{cd} = d - \Delta x. \Delta x \text{ can be determined from } v^2 = v_0^2 + 2a\Delta x.$$

(b) The stop time is determined from $v = v_0 + at$.

SIMPLIFY:

(a) With $v = 0$, $v^2 = v_0^2 + 2a\Delta x \Rightarrow 0 = v_0^2 + 2a\Delta x \Rightarrow \Delta x = -v_0^2 / 2a$. Then, $\Delta x_{cd} = d + (v_0^2 / 2a)$.

(b) With $v = 0$, $t = -v_0 / a$.

CALCULATE:

$$(a) \Delta x_{cd} = 200.0 \text{ m} + \frac{(29.1 \text{ m/s})^2}{2(-2.4 \text{ m/s}^2)} = 200.0 \text{ m} - 176.4 \text{ m} = 23.6 \text{ m}$$

$$(b) t = -\frac{(29.1 \text{ m/s})}{(-2.4 \text{ m/s}^2)} = 12.13 \text{ s}$$

ROUND:

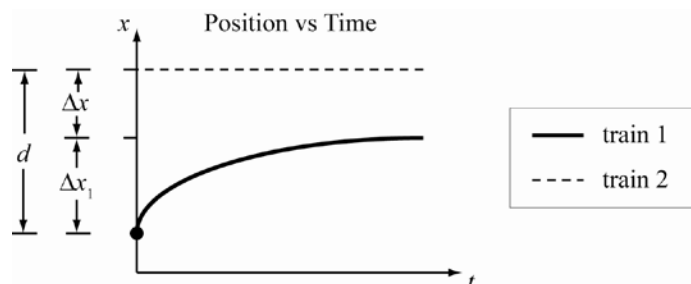
(a) Since the acceleration has two significant figures, $\Delta x_{cd} = 24 \text{ m}$

(b) Rounding to two significant figures, $t = 12 \text{ s}$.

DOUBLE-CHECK: The stopping time and distance are realistic for a car decelerating from 29.1 m/s.

- 2.85. **THINK:** For train 1, it is known that $v_{1,0} = 40.0 \text{ m/s}$, $a_1 = -6.0 \text{ m/s}^2$ and $v_1 = 0$. For train 2, it is known that $v_2 = 0$ and $a_2 = 0$. The distance between the trains is $d = 100.0 \text{ m}$. Determine the distance between the trains after train 1 stops, Δx .

SKETCH:



RESEARCH: The final distance between the trains, Δx is the difference between the initial distance, d and the stopping distance of train 1, Δx_1 : $\Delta x = d - \Delta x_1$.

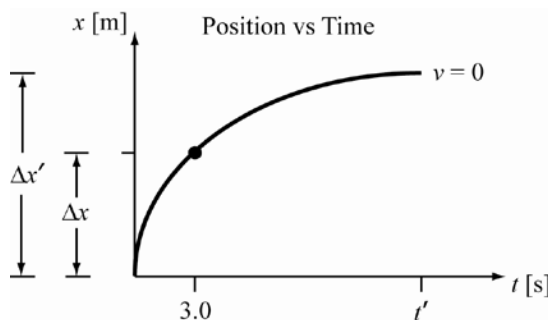
SIMPLIFY: With $v_1 = 0$, $v_1^2 = v_{1,0}^2 + 2a_1\Delta x_1 \Rightarrow 0 = v_{1,0}^2 + 2a_1\Delta x_1 \Rightarrow \Delta x_1 = -\frac{v_{1,0}^2}{2a_1}$. Then, $\Delta x = d + \frac{v_{1,0}^2}{2a_1}$.

$$\text{CALCULATE: } \Delta x = 100.0 \text{ m} + \frac{(40.0 \text{ m/s})^2}{2(-6.0 \text{ m/s}^2)} = 100.0 \text{ m} - 133.3 \text{ m} = -33.3 \text{ m}$$

ROUND: Note that Δx is determined to be a negative value. This is due to the stopping distance being greater than the initial distance between the trains. This implies that train 1 actually collides with train 2. Then the distance between the two trains is zero.

DOUBLE-CHECK: It is reasonable for train 1 to collide with train 2 given the initial separation of only 100.0 m and an initial velocity for train 1 of 40.0 m/s.

- 2.86. **THINK:** The initial velocity is $v_0 = 25.0 \text{ m/s}$. The acceleration is $a = -1.2 \text{ m/s}^2$. Determine (a) the distance Δx traveled in $t = 3.0 \text{ s}$, (b) the velocity, v after traveling this distance, (c) the stopping time, t' and (d) the stopping distance, $\Delta x'$. Note when the car is stopped, $v' = 0$.

SKETCH:**RESEARCH:**

(a) To determine Δx , use $\Delta x = v_0 t + (at^2)/2$.

(b) To determine v , use $v = v_0 + at$.

(c) To determine t' , use $v = v_0 + at$.

(c) To determine $\Delta x'$, use $v^2 = v_0^2 + 2a\Delta x$.

SIMPLIFY:

(a) It is not necessary to simplify.

(b) It is not necessary to simplify.

(c) With $v' = 0$, $v' = v_0 + at' \Rightarrow t' = -v_0 / a$.

(d) With $v' = 0$, $v'^2 = v_0^2 + 2a\Delta x' \Rightarrow \Delta x' = -v_0^2 / 2a$.

CALCULATE:

$$(a) \Delta x = (25.0 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}(-1.2 \text{ m/s}^2)(3.0 \text{ s})^2 = 69.6 \text{ m}$$

$$(b) v = 25.0 \text{ m/s} + (-1.2 \text{ m/s}^2)(3.0 \text{ s}) = 21.4 \text{ m/s}$$

$$(c) t' = -\frac{(25.0 \text{ m/s})}{(-1.2 \text{ m/s}^2)} = 20.83 \text{ s}$$

$$(d) \Delta x' = -\frac{(25.0 \text{ m/s})^2}{2(-1.2 \text{ m/s}^2)} = 260.4 \text{ m}$$

ROUND: Both the acceleration and the time have two significant figures, so the results should be rounded to $\Delta x = 70. \text{ m}$, $v = 21 \text{ m/s}$, $t' = 21 \text{ s}$ and $\Delta x' = 260 \text{ m}$.

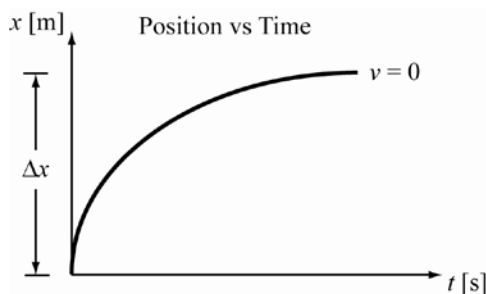
DOUBLE-CHECK: The car travels 70 m while decelerating, which is less than the 75 m it would have traveled in the same time if it had not been decelerating. The velocity after decelerating is less than the initial velocity. The stopping distance is greater than the distance traveled in 3.0 s, and the stopping time is greater than the intermediate time of 3.0 s. All of these facts support the calculated values.

2.87. THINK: The initial velocity is

$$v_0 = 212.809 \text{ mph} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1609.3 \text{ m}}{\text{mile}} \right) = 95.1315 \text{ m/s.}$$

The acceleration is $a = -8.0 \text{ m/s}^2$. The final speed is $v = 0$. Determine the stopping distance, Δx .

SKETCH:



RESEARCH: Use $v^2 = v_0^2 + 2a\Delta x$.

SIMPLIFY: With $v = 0$, $0 = v_0^2 + 2a\Delta x \Rightarrow \Delta x = -v_0^2 / 2a$.

CALCULATE: $\Delta x = -\frac{(95.1315 \text{ m/s})^2}{2(-8.0 \text{ m/s}^2)} = 565.6 \text{ m}$

ROUND: The acceleration has two significant figures, so the result should be rounded to $\Delta x = 570 \text{ m}$.

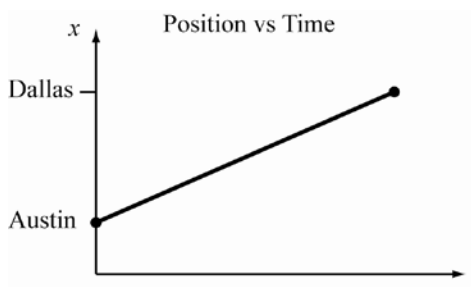
DOUBLE-CHECK: The initial velocity is large and the deceleration has a magnitude close to that of gravity. A stopping distance greater than half of a kilometer is reasonable.

2.88. THINK: The velocity can be converted to SI units as follows:

$$v_0 = 245 \text{ mph} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1609.3 \text{ m}}{\text{mile}} \right) = 109.5 \text{ m/s}.$$

The distance is $\Delta x = 362 \text{ km} = 3.62 \cdot 10^5 \text{ m}$. Determine the time, t to travel the distance, Δx . Note the acceleration is $a = 0$.

SKETCH:



RESEARCH: For $a = 0$, use $\Delta x = vt$.

SIMPLIFY: $t = \frac{\Delta x}{v}$

CALCULATE: $t = \frac{3.62 \cdot 10^5 \text{ m}}{109.5 \text{ m/s}} = 3306 \text{ s}$

ROUND: The distance Δx has three significant figures, so the result should be rounded to $t = 3310 \text{ s}$.

DOUBLE-CHECK: The time in hours is

$$3310 \text{ s} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 0.919 \text{ h}.$$

An hour is a reasonable amount of time to fly a distance of 362 km.

2.89. The position is given by $x = at^3 + bt^2 + c$, where $a = 2.0 \text{ m/s}^3$, $b = 2.0 \text{ m/s}^2$ and $c = 3.0 \text{ m}$.

(a) Determine the sled's position between $t_1 = 4.0 \text{ s}$ and $t_2 = 9.0 \text{ s}$.

$$x(4.0 \text{ s}) = (2.0 \text{ m/s}^3)(4.0 \text{ s})^3 + (2.0 \text{ m/s}^2)(4.0 \text{ s})^2 + 3.0 \text{ m} = 163 \text{ m} \approx 160 \text{ m}$$

$$x(9.0 \text{ s}) = (2.0 \text{ m/s}^3)(9.0 \text{ s})^3 + (2.0 \text{ m/s}^2)(9.0 \text{ s})^2 + 3.0 \text{ m} = 1623 \text{ m} \approx 1600 \text{ m}$$

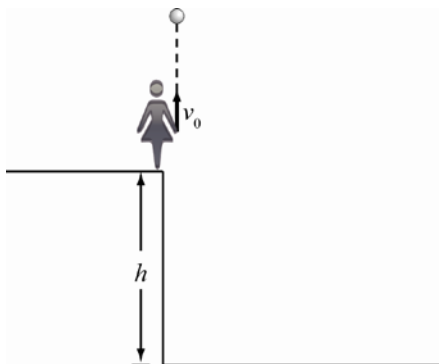
The sled is between $x = 160 \text{ m}$ and $x = 1600 \text{ m}$.

(b) Determine the sled's average speed over this interval.

$$V_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{1623 \text{ m} - 163 \text{ m}}{9.0 \text{ s} - 4.0 \text{ s}} = \frac{1460 \text{ m}}{5.0 \text{ s}} \approx 292 \text{ m/s} = 290 \text{ m/s}$$

- 2.90. THINK:** The cliff has a height of $h = 100. \text{ m}$ above the ground. The girl throws a rock straight up with a speed of $v_0 = 8.00 \text{ m/s}$. Determine how long it takes for the rock to hit the ground and find the speed, v of the rock just before it hits the ground. The acceleration due to gravity is $a = -g = -9.81 \text{ m/s}^2$.

SKETCH:



RESEARCH: The total displacement in the vertical direction is given by $\Delta y = y_f - y_i$. If the top of the cliff is taken to be the origin of the system, then $y_i = 0$ and $y_f = -h = -100. \text{ m}$. Therefore, $\Delta y = -h$.

$$(a) \Delta y = v_0 t + \frac{1}{2} a t^2$$

$$(b) v^2 = v_0^2 + 2a\Delta y$$

SIMPLIFY:

(a) The quadratic equation can be used to solve for t from the equation $gt^2/2 - v_0 t + \Delta y = 0$:

$$t = \frac{v_0 \pm \sqrt{v_0^2 - 4\left(\frac{g}{2}\right)(-h)}}{2\left(\frac{g}{2}\right)} = \frac{v_0 \pm \sqrt{v_0^2 + 2gh}}{g}$$

$$(b) v = \sqrt{v_0^2 + 2gh}$$

CALCULATE:

$$(a) t = \frac{8.00 \text{ m/s} \pm \sqrt{(8.00 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(100. \text{ m})}}{(9.81 \text{ m/s}^2)}$$

$$= 5.40378 \text{ s or } -3.77 \text{ s}$$

The negative time is impossible.

$$(b) v = \sqrt{(8.00 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(100. \text{ m})} = 45.011 \text{ m/s}$$

ROUND:

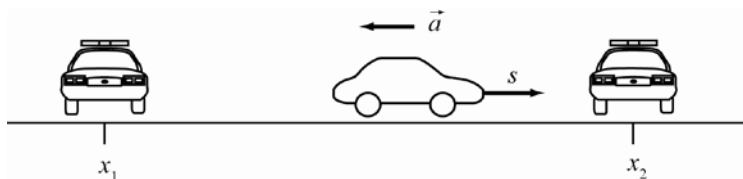
$$(a) t = 5.40 \text{ s}$$

$$(b) v = 45.0 \text{ m/s}$$

DOUBLE-CHECK: The calculated time and speed for the rock are reasonable considering the height of the cliff. Also, the units are correct units for time and speed.

- 2.91. THINK:** The police have a double speed trap set up. A sedan passes the first speed trap at a speed of $s_1 = 105.9$ mph. The sedan decelerates and after a time, $t = 7.05$ s it passes the second speed trap at a speed of $s_2 = 67.1$ mph. Determine the sedan's deceleration and the distance between the police cruisers.

SKETCH:



RESEARCH:

(a) Convert the speeds to SI units as follows:

$$s_1 = 105.9 \text{ mph} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1609.3 \text{ m}}{\text{mile}} \right) = 47.34 \text{ m/s}$$

$$s_2 = 67.1 \text{ mph} \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{1609.3 \text{ m}}{\text{mile}} \right) = 29.996 \text{ m/s}.$$

The sedan's velocity, v can be written in terms of its initial velocity, v_0 the time t , and its acceleration a :

$$v = v_0 + at. \text{ Substitute } s_1 \text{ for } v_0 \text{ and } s_2 \text{ for } v.$$

(b) The distance between the cruisers is given by: $\Delta x = x_2 - x_1 = v_0 t + (1/2)at^2$.

SIMPLIFY:

$$(a) \ a = \frac{v - v_0}{t} = \frac{s_2 - s_1}{t}$$

(b) Substitute s_1 for v_0 and the expression from part (a) for a : $\Delta x = s_1 t + (1/2)at^2$

CALCULATE:

$$(a) \ a = \frac{29.996 \text{ m/s} - 47.34 \text{ m/s}}{7.05 \text{ s}} = -2.4602 \text{ m/s}^2$$

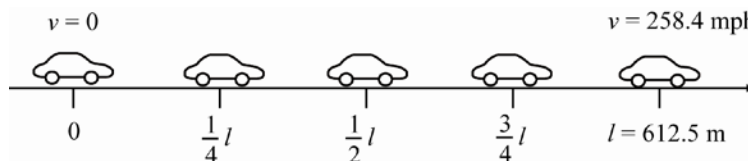
$$(b) \ \Delta x = (47.34 \text{ m/s})(7.05 \text{ s}) + \frac{1}{2}(-2.4602 \text{ m/s}^2)(7.05 \text{ s})^2 = 272.6079 \text{ m}$$

ROUND: The least number of significant figures provided in the problem are three, so the results should be rounded to $a = -2.46 \text{ m/s}^2$ and $\Delta x = 273 \text{ m}$.

DOUBLE-CHECK: The sedan did not have its brakes applied, so the values calculated are reasonable for the situation. The acceleration would have been larger, and the distance would have been much smaller, if the brakes had been used. The results also have the proper units.

- 2.92. THINK:** The initial speed of a new racecar is $v_0 = 0$ (standing start). The car accelerates with a constant acceleration and reaches a speed of $v = 258.4$ mph at a distance of $l = 612.5$ m. Determine a relationship between the speed and distance.

SKETCH:



RESEARCH: The acceleration is constant, so there are two expressions for velocity and distance:

$$v = v_0 + at, \quad x = x_0 + v_0 t + (1/2)at^2.$$

SIMPLIFY: It is given that $v_0 = 0$ and $x_0 = 0$, so the above expressions simplify to $v = at$, $x = \frac{1}{2}at^2$.

Thus, $t = \sqrt{2x/a}$. Substituting this expression into $v = at$,

$$v = a\sqrt{\frac{2x}{a}} = \sqrt{2ax}.$$

CALCULATE: (1) The speed at a distance of $x = l/4$ is given by:

$$v_{l/4} = \sqrt{2a\frac{l}{4}} = \sqrt{\frac{1}{4}}(\sqrt{2al}) = \frac{1}{2}\sqrt{2al}.$$

Note that $v = \sqrt{2al}$, therefore, $v_{l/4} = v/2$.

$$v_{l/4} = \frac{1}{2}(258.4 \text{ mph}) = 129.2 \text{ mph}$$

(2) Similarly, substituting $x = l/2$ into $v = \sqrt{2ax}$,

$$v_{l/2} = \sqrt{\frac{1}{2}}v = \sqrt{\frac{1}{2}}(258.4 \text{ mph}) = 182.716 \text{ mph}.$$

(3) Substituting $x = 3l/4$ into $v = \sqrt{2ax}$,

$$v_{3l/4} = \sqrt{\frac{3}{4}}v = \sqrt{\frac{3}{4}}(258.4 \text{ mph}) = 223.781 \text{ mph}.$$

ROUND: Initially there are four significant figures, so the results should be rounded to $v_{l/4} = 129.2 \text{ mph}$, $v_{l/2} = 182.7 \text{ mph}$ and $v_{3l/4} = 223.8 \text{ mph}$.

DOUBLE-CHECK: Note that $v_{l/4} < v_{l/2} < v_{3l/4} < v$ as expected.

2.93. THINK: An expression of y as a function of t is given. Determine the speed and acceleration from this function, $y(t)$. The first derivative of $y(t)$ yields speed as a function of time, $v = dy/dt$, and the second derivative yields acceleration as a function of time, $a = dv/dt$.

SKETCH: A sketch is not needed to solve the problem.

RESEARCH: From a table of common derivatives: $\frac{d \sin(\alpha t + \beta)}{dt} = \alpha \cos(\alpha t + \beta)$, and

$$\frac{d \cos(\alpha t + \beta)}{dt} = -\alpha \sin(\alpha t + \beta).$$

SIMPLIFY: It is not necessary to simplify.

CALCULATE:

$$(a) \quad v = \frac{d}{dt}(3.8 \sin(0.46t/s - 0.31) \text{ m} - 0.2t \text{ m/s} + 5.0 \text{ m})$$

$$= 3.8(0.46) \cos(0.46t/s - 0.31) \text{ m/s} - 0.2 \text{ m/s}$$

$$= 1.748 \cos(0.46t/s - 0.31) \text{ m/s} - 0.2 \text{ m/s}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(1.748 \cos(0.46t/s - 0.31) \text{ m/s} - 0.2 \text{ m/s})$$

$$= -1.748(0.46) \sin(0.46t/s - 0.31) \text{ m/s}^2$$

$$= -0.80408 \sin(0.46t/s - 0.31) \text{ m/s}^2$$

(b) Set $a = 0$: $0 = -0.80408 \sin(0.46t/s - 0.31) \text{ m/s}^2 \Rightarrow \sin(0.46t/s - 0.31) = 0$. It is known that $\sin \alpha = 0$ when $\alpha = n\pi$ and n is an integer. Therefore,

$$0.46t/s - 0.31 = n\pi \Rightarrow t = \frac{n\pi + 0.31}{0.46} \text{ s}.$$

The times between 0 and 30 s that satisfy $a = 0$ are:

$$\begin{aligned} t &= 6.8295n + 0.6739 \text{ s} \\ &= 0.6739 \text{ s for } n = 0 \\ &= 7.5034 \text{ s for } n = 1 \\ &= 14.3329 \text{ s for } n = 2 \\ &= 21.1624 \text{ s for } n = 3 \\ &= 27.9919 \text{ s for } n = 4. \end{aligned}$$

ROUND: Rounding to two significant figures,

(a) $v = 1.7 \cos(0.46t / \text{s} - 0.31) \text{ m/s} - 0.2 \text{ m/s}$, $a = -0.80 \sin(0.46t / \text{s} - 0.31) \text{ m/s}^2$

(b) $t = 0.67 \text{ s}$, 7.5 s , 14 s , 21 s and 28 s .

DOUBLE-CHECK: For oscillatory motion, where the position is expressed in terms of a sinusoidal function, the velocity is always out of phase with respect to the position. Out of phase means if $x = \sin t$, then $v = \cos t = \sin(t + \pi/2)$. The acceleration is proportional to the position function. For example, if $x = A \sin t$, $a = -A \sin t$.

2.94. THINK: An expression for position as a function of time is given as $x(t) = 4t^2$.

SKETCH: A sketch is not needed to solve the problem.

RESEARCH: $(a + b)^2 = a^2 + 2ab + b^2$

SIMPLIFY: Simplification is not necessary.

CALCULATE:

(a) $x(2.00) = 4(2.00)^2 \text{ m} = 16.00 \text{ m}$

(b) $x(2.00 + \Delta t) = 4(2.00 + \Delta t)^2 \text{ m} = 4(4.00 + 4.00\Delta t + \Delta t^2) \text{ m}$
 $= (16.00 + 16.00\Delta t + 4\Delta t^2) \text{ m}$

(c) $\frac{\Delta x}{\Delta t} = \frac{x(2.00 + \Delta t) - x(2.00)}{\Delta t}$
 $= \frac{16.00 + 16.00\Delta t + 4\Delta t^2 - 16.00 \text{ m}}{\Delta t \text{ s}}$
 $= (16.00 + 4\Delta t) \text{ m/s}$

Taking the limit as $\Delta t \rightarrow 0$: $\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \rightarrow 0} 16.00 + \lim_{\Delta t \rightarrow 0} 4\Delta t = 16.00 \text{ m/s}$.

ROUND: Rounding to three significant figures,

(a) $x(2.00) = 16.0 \text{ m}$

(b) $x(2.00 + \Delta t) = (16.0 + 16.0\Delta t + 4\Delta t^2) \text{ m}$

(c) $\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = 16.0 \text{ m/s}$

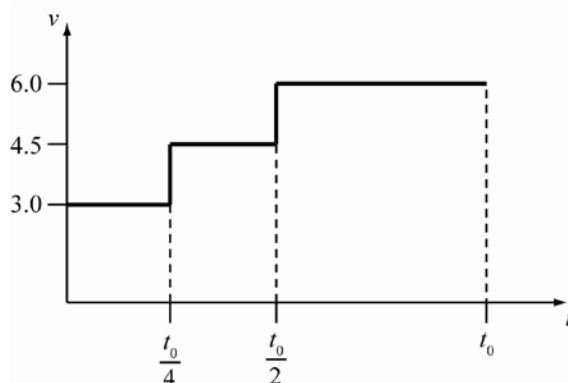
DOUBLE-CHECK: The value of the position function near $t = 2.00 \text{ s}$ coincides with its value at $t = 2.00 \text{ s}$. This should be the case, since the position function is continuous. The value of the velocity can also be

found from the derivative: $\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}(4t^2) = 8t$. Substitute $t = 2.00 \text{ s}$,

$\left. \frac{dx}{dt} \right|_{t=2.00} = 8(2.00) = 16.00 \text{ m/s}$. This value agrees with what was calculated in part (c).

- 2.95. THINK:** The distance to the destination is 199 miles or 320 km. To solve the problem it is easiest to draw a velocity versus time graph. The distance is then given by the area under the curve.

SKETCH:



RESEARCH: For a constant speed, the distance is given by $x = vt$.

SIMPLIFY: To simplify, divide the distance into three parts.

Part 1: from $t = 0$ to $t = t_0 / 4$.

Part 2: from $t = t_0 / 4$ to $t = t_0 / 2$.

Part 3: from $t = t_0 / 2$ to $t = t_0$.

CALCULATE:

(a) The distances are $x_1 = 3.0t_0 / 4$, $x_2 = 4.5t_0 / 4$ and $x_3 = 6.0t_0 / 2$. The total distance is given by

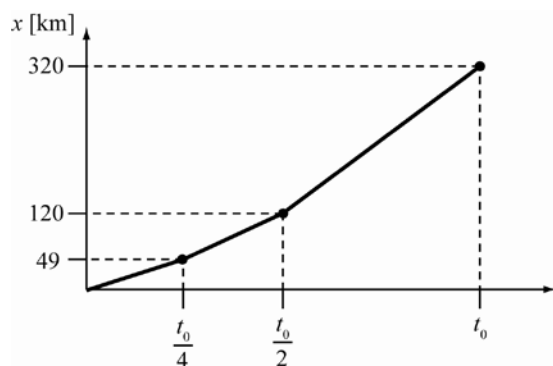
$$x = x_1 + x_2 + x_3 = \frac{(3.0 + 4.5 + 12)t_0}{4} \text{ m} = \frac{19.5t_0}{4} \text{ m} \Rightarrow t_0 = \frac{4x}{19.5} \text{ s.}$$

$$t_0 = \frac{4(320 \cdot 10^3)}{19.5} \text{ s} = 65.6410 \cdot 10^3 \text{ s} = 65641 \text{ s} \Rightarrow t_0 = 18.2336 \text{ h}$$

(b) The distances are:

$$x_1 = 3.0 \left(\frac{65641}{4} \right) \text{ m} = 49.23 \text{ km}, \quad x_2 = 4.5 \left(\frac{65641}{4} \right) \text{ m} = 73.85 \text{ km}, \quad x_3 = 6.0 \left(\frac{65641}{2} \right) \text{ m} = 196.92 \text{ km}.$$

ROUND: Since the speeds are given to two significant figures, the results should be rounded to $x_1 = 49 \text{ km}$, $x_2 = 74 \text{ km}$ and $x_3 = 2.0 \cdot 10^2 \text{ km}$. $x_1 + x_2 = 123 \text{ km} \approx 120 \text{ km}$, and then $x = x_1 + x_2 + x_3 = 323 \text{ km} \approx 320 \text{ km}$.

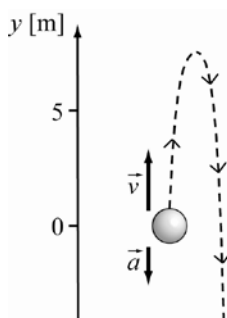


DOUBLE-CHECK: The sum of the distances x_1 , x_2 and x_3 must be equal to the total distance of 320 km:

$x_1 + x_2 + x_3 = 49.23 + 73.85 + 196.92 = 320 \text{ km}$ as expected. Also, note that $x_1 < x_2 < x_3$ since $v_1 < v_2 < v_3$.

- 2.96. THINK:** The initial speed is $v_0 = 15.0$ m/s. Assume there is no air resistance. The acceleration due to gravity is given by $g = 9.81$ m/s². t_1 is the time taken from the original position to the 5.00 m position on the way up. The time it takes from the initial position to 5.00 m on its way down is t_2 .

SKETCH:



RESEARCH: For motion with a constant acceleration, the expressions for speed and distances are

$$v = v_0 + at, \quad y = y_0 + v_0 t + \frac{1}{2}at^2. \quad \text{The acceleration due to gravity is } a = -g.$$

SIMPLIFY:

- (a) At the maximum height, the velocity is $v = 0$. Using $y_0 = 0$:

$$0 = v_0 - gt \Rightarrow t = \frac{v_0}{g},$$

$$y = y_{\max} = v_0 t - \frac{1}{2}gt^2.$$

Substituting $t = v_0 / g$,

$$y_{\max} = v_0 \left(\frac{v_0}{g} \right) - \frac{1}{2}g \left(\frac{v_0}{g} \right)^2 = \frac{v_0^2}{2g}.$$

- (b) If the motion of the ball starts from the maximum height, there is free fall motion with $v_0 = 0$.

$$v = -gt \Rightarrow t = -\frac{v}{g}$$

$$y = y_{\max} + v_0 t - \frac{1}{2}gt^2 = y_{\max} - \frac{1}{2}gt^2$$

Substituting $t = v/g$:

$$y = y_{\max} - \frac{v^2}{2g} \Rightarrow v = \sqrt{(y_{\max} - y)2g}.$$

CALCULATE:

$$(a) \quad y_{\max} = \frac{(15.0 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 11.468 \text{ m}$$

$$(b) \quad v = \sqrt{(11.468 - 5.00)(2)(9.81 \text{ m/s}^2)} = 11.265 \text{ m/s. Thus the speed at this point is 11.265 m/s.}$$

(c,d) Using $y = y_0 + v_0 t + (at^2)/2$, $y = v_0 t - (1/2)gt^2$. Using $v_0 = 15.0$ m/s, $g = 9.81$ m/s² and $y = 5.00$ m, the quadratic equation is $(1/2)(9.81 \text{ m/s}^2)t^2 - 15.0t + 5.00 \text{ m} = 0$. Solving the quadratic equation:

$$t = \frac{15.0 \pm \sqrt{(15.0)^2 - 2(9.81)5}}{9.81} \text{ s} = \frac{15.0 \pm 11.265}{9.81} \text{ s} = 1.529 \pm 1.1483 = 2.6773 \text{ s and } 0.3807 \text{ s}$$

ROUND:

- (a) Rounding to three significant figures, $y_{\max} = 11.5$ m.

(b) All the numerical values have three significant figures, so the result is rounded to $v = 11.3$ m/s. Note the speed on the way up is the same as the speed on the way down.

(c) Rounding the values to three significant figures, $t_1 = 0.381$ s.

(d) $t_2 = 2.68$ s.

DOUBLE-CHECK: The speed at $t_1 = 0.381$ s and $t_2 = 2.68$ s must be the same and it is equal to the speed determined in part (b).

$$v = v_0 - gt$$

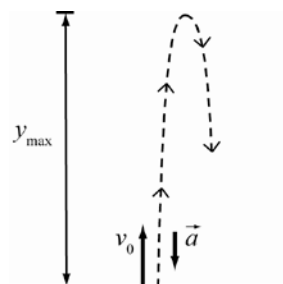
$$v_1 = 15.0 - (9.81)(0.381) = 11.2624 \text{ m/s} \approx 11.3 \text{ m/s}$$

$$v_2 = 15.0 - (9.81)(2.68) = -11.2908 \text{ m/s} \approx -11.3 \text{ m/s}$$

As can be seen, $|v_1| = |v_2|$ is equal to the result in part (b).

2.97. THINK: The maximum height is $y_{\max} = 240$ ft = 73.152 m. The acceleration due to gravity is given by $g = 9.81$ m/s².

SKETCH:



RESEARCH: To solve this constant acceleration problem, use $v = v_0 - gt$ and $y = y_0 + v_0 t - (gt^2/2)$.
 $y_0 = 0$.

SIMPLIFY:

(a) At a maximum height, the velocity v is zero.

$$v_0 - gt = 0 \Rightarrow t = \frac{v_0}{g}$$

$$y_{\max} = v_0 \left(\frac{v_0}{g} \right) - \frac{1}{2} g \left(\frac{v_0}{g} \right)^2 = \frac{v_0^2}{2g} \Rightarrow v_0 = \sqrt{2gy_{\max}}$$

(b) If the motion is considering as starting from the maximum height y_{\max} , there is free fall motion with $v_0 = 0$.

$$v = -gt \Rightarrow t = \frac{v}{g}$$

$$y = y_{\max} - \frac{1}{2} gt^2 = y_{\max} - \frac{1}{2} g \left(\frac{v}{g} \right)^2 = y_{\max} - \frac{v^2}{2g} \Rightarrow v = \sqrt{(y_{\max} - y)2g}$$

(c) Note that v_0 is equal to the speed in part (b), $v_0 = -26.788$ m/s and v is equal to the original speed but in the opposite direction, $v = -37.884$ m/s.

$$t = \frac{v_0 - v}{g}$$

CALCULATE:

$$(a) \quad v_0 = \sqrt{2(9.81)(73.152)} = 37.885 \text{ m/s}$$

(b) $y = \frac{y_{\max}}{2}$, so $v = \sqrt{\left(y_{\max} - \frac{y_{\max}}{2}\right)2g} = \sqrt{gy_{\max}} = \sqrt{(9.81)73.152} = 26.788 \text{ m/s}$. Choose the positive root

because the problem asks for the speed, which is never negative.

(c) $t = \frac{37.884 \text{ m/s} - 26.788 \text{ m/s}}{(9.81 \text{ m/s}^2)} = 1.131 \text{ s}$

ROUND:

(a) Rounding to three significant figures, $v_0 = 37.9 \text{ m/s}$.

(b) Rounding to three significant figures, $v = 26.8 \text{ m/s}$.

(c) Rounding to three significant figures, $t = 1.13 \text{ s}$.

DOUBLE-CHECK: It is known that $v = \sqrt{2gy}$. This means that the ratio of two speeds is:

$$\frac{v_1}{v_2} = \frac{\sqrt{2gy_1}}{\sqrt{2gy_2}} = \sqrt{\frac{y_1}{y_2}}.$$

The result in part (b) is for $y = y_{\max}/2$, so the ratio is

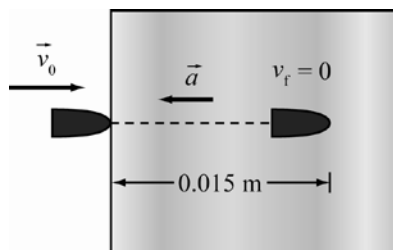
$$\frac{v_{1/2}}{v_0} = \sqrt{\frac{\frac{1}{2}y_{\max}}{y_{\max}}} = \sqrt{\frac{1}{2}} = 0.7071.$$

Using the results in parts (a) and (b):

$$\frac{v_{1/2}}{v_0} = \frac{26.8 \text{ m/s}}{37.9 \text{ m/s}} = 0.7071 \text{ as expected.}$$

2.98. THINK: The initial velocity is $v_0 = 200. \text{ m/s}$. There is constant acceleration and the maximum distance is $x_{\max} = 1.5 \text{ cm} = 0.015 \text{ m}$.

SKETCH:



RESEARCH: To solve a constant acceleration motion, use $v = v_0 + at$. There is a deceleration of a .

$$x = v_0 t + \frac{1}{2}at^2$$

SIMPLIFY: At the final position, $v = 0$.

$$v_0 - at = 0 \Rightarrow a = \frac{v_0}{t}$$

Substituting $a = v_0/t$ into $x_{\max} = v_0 t - (1/2)at^2$ gives:

$$x_{\max} = v_0 t - \frac{1}{2} \frac{v_0}{t} t^2 = \frac{1}{2} v_0 t \Rightarrow t = \frac{2x_{\max}}{v_0}$$

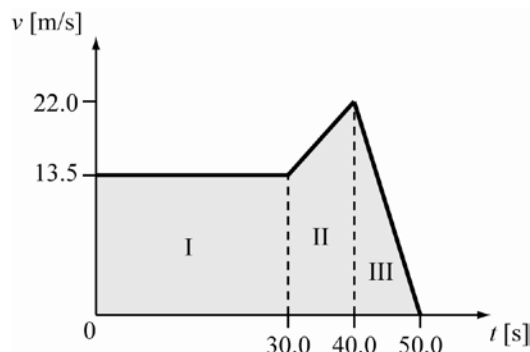
CALCULATE: $t = \frac{2(0.015 \text{ m})}{200. \text{ m/s}} = 1.5 \cdot 10^{-4} \text{ s}$

ROUND: Rounding to two significant figures yields the same result, $t = 1.5 \cdot 10^{-4} \text{ s}$

DOUBLE-CHECK: It is expected the resulting time is small for the bullet to stop at a short distance.

- 2.99. THINK:** $v_1 = 13.5 \text{ m/s}$ for $\Delta t = 30.0 \text{ s}$. $v_2 = 22.0 \text{ m/s}$ after $\Delta t = 10.0 \text{ s}$ (at $t = 40.0 \text{ s}$). $v_3 = 0$ after $\Delta t = 10.0 \text{ s}$ (at $t = 50.0 \text{ s}$). It will be easier to determine the distance from the area under the curve of the velocity versus time graph.

SKETCH:



RESEARCH: Divide and label the graph into three parts as shown above.

SIMPLIFY: The total distance, d is the sum of the areas under the graph, $d = A_1 + A_2 + A_3$.

CALCULATE:
$$d = (13.5 \text{ m/s})(30.0 \text{ s}) + \frac{1}{2}(13.5 \text{ m/s} + 22.0 \text{ m/s})(10.0 \text{ s}) + \frac{1}{2}(22.0 \text{ m/s})(10.0 \text{ s})$$

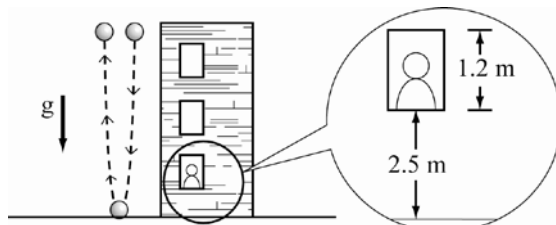
$$= 405 \text{ m} + 177.5 \text{ m} + 110 \text{ m} = 692.5 \text{ m}$$

ROUND: The speeds are given in three significant figures, so the result should be rounded to $d = 693 \text{ m}$.

DOUBLE-CHECK: From the velocity versus time plot, the distance can be estimated by assuming the speed is constant for all time, t : $d = (13.5 \text{ m/s})(50.0 \text{ s}) = 675 \text{ m}$. This estimate is in agreement with the previous result.

- 2.100. THINK:** It is given that the initial velocity is $v_0 = 0$. The time for the round trip is $t = 5.0 \text{ s}$.

SKETCH:



RESEARCH: $a = -g$. Using two expressions for velocity and distance:

(a) $v = v_0 + at$

(b) $y = y_0 + v_0 t + \frac{1}{2}at^2$

SIMPLIFY:

(a) $y_0 = y_{\max}$, $v = -gt$

(b) $y = y_{\max} - \frac{1}{2}gt^2$

(c) The distance from the top of the window to the ground is $1.2 + 2.5 = 3.7 \text{ m}$. From part (b),

$$y = y_{\max} - \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2(y_{\max} - y)}{g}}.$$

CALCULATE: The time taken from the roof to the ground is half the time of the round trip, $t = 5.0/2 = 2.5 \text{ s}$.

(a) The velocity before the ball hits the ground is $v = -(9.81)(2.5) = -24.525 \text{ m/s}$. So the speed is 24.525 m/s .

(b) $y = 0$ (ground), and t is the time from the roof to the ground.

$$0 = y_{\max} - \frac{1}{2}gt^2 \Rightarrow y_{\max} = \frac{1}{2}gt^2 \Rightarrow y_{\max} = \frac{1}{2}(9.81 \text{ m/s}^2)(2.5 \text{ s})^2 = 30.656 \text{ m}$$

$$(c) \quad t = \sqrt{\frac{2(30.656 - 3.7)}{(9.81)}} = 2.3443 \text{ s}$$

ROUND: Rounding to two significant figures, $|v| = 25 \text{ m/s}$, $y_{\max} = 31 \text{ m}$ and $t = 2.3 \text{ s}$.

DOUBLE-CHECK: The speed in part (a) is consistent with an object accelerating uniformly due to gravity. The distance in (b) is a reasonable height for a building. For the result of part (c), the time must be less than 2.5 s, which it is.

2.101. From a mathematical table: $\frac{d}{dt}e^{\alpha t} = \alpha e^{\alpha t}$.

$$(a) \quad x(t) = 2x_0 = \frac{1}{4}x_0 e^{3\alpha t} \Rightarrow e^{3\alpha t} = 8 \Rightarrow 3\alpha t = \ln 8 \Rightarrow t = \frac{1}{3\alpha} \ln 8$$

$$(b) \quad v(t) = \frac{dx}{dt} = \frac{3\alpha}{4}x_0 e^{3\alpha t}$$

$$(c) \quad a(t) = \frac{dv}{dt} = \frac{(3\alpha)^2}{4}x_0 e^{3\alpha t} = \frac{9\alpha^2}{4}x_0 e^{3\alpha t}$$

(d) αt must be dimensionless. Since the units of t are s, the units of α are s^{-1} .

2.102. Note that $\frac{d}{dt}(t^n) = nt^{n-1}$.

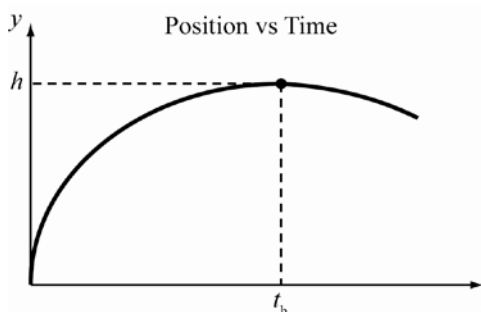
$$(a) \quad v = \frac{dx}{dt} = 4At^3 - 3Bt^2$$

$$(b) \quad a = \frac{dv}{dt} = 12At^2 - 6Bt$$

Multi-Version Exercises

2.103. THINK: The initial velocity is $v_0 = 28.0 \text{ m/s}$. The acceleration is $a = -g = -9.81 \text{ m/s}^2$. The velocity, v is zero at the maximum height. Determine the time, t to achieve the maximum height.

SKETCH:



RESEARCH: To determine the velocity use $v = v_0 + at_h$.

$$\text{SIMPLIFY: } at_h = v - v_0 \Rightarrow t_h = \frac{v - v_0}{a} = \frac{-v_0}{-g} = \frac{v_0}{g}$$

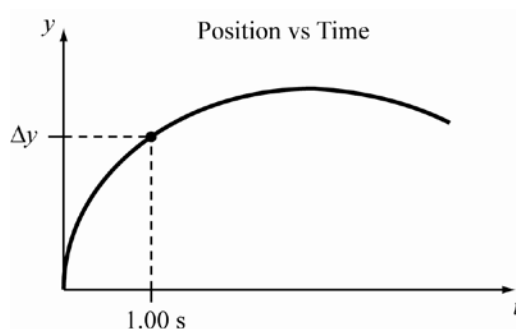
$$\text{CALCULATE: } t_h = \frac{28.0 \text{ m/s}}{9.81 \text{ m/s}^2} = 2.8542 \text{ s}$$

ROUND: The initial values have three significant figures, so the result should be rounded to $t_h = 2.85 \text{ s}$.

DOUBLE-CHECK: The initial velocity of the object is about 30 m/s, and gravity will cause the velocity to decrease about 10 m/s per second. It should take roughly three seconds for the object to reach its maximum height.

- 2.104. THINK:** The initial velocity is $v_0 = 28.0$ m/s. The time is $t = 1.00$ s. The acceleration is $a = -g = -9.81$ m/s². Determine the height above the initial position, Δy .

SKETCH:



RESEARCH: To determine the height use $\Delta y = v_0 t + (at^2)/2$.

SIMPLIFY: $\Delta y = v_0 t - \frac{1}{2}(gt^2)$

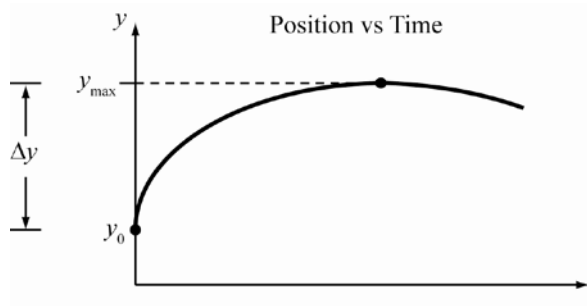
CALCULATE: $\Delta y = (28.0 \text{ m/s})(1.00 \text{ s}) - \frac{1}{2}((9.81 \text{ m/s}^2)(1.00 \text{ s})^2) = 23.095 \text{ m}$

ROUND: As all initial values have three significant figures, the result should be rounded to $\Delta y = 23.1$ m.

DOUBLE-CHECK: The displacement Δy is positive, indicating that the final position is higher than the initial position. This is consistent with the positive initial velocity.

- 2.105. THINK:** The initial velocity is $v_0 = 28.0$ m/s. The acceleration is $a = -g = -9.81$ m/s². The velocity, v is zero at the maximum height. Determine the maximum height, Δy above the projection point.

SKETCH:



RESEARCH: The maximum height can be determined from the following equation: $v^2 = v_0^2 + 2a\Delta y$.

SIMPLIFY: With $v = 0$, $0 = v_0^2 - 2g\Delta y \Rightarrow \Delta y = \frac{v_0^2}{2g}$.

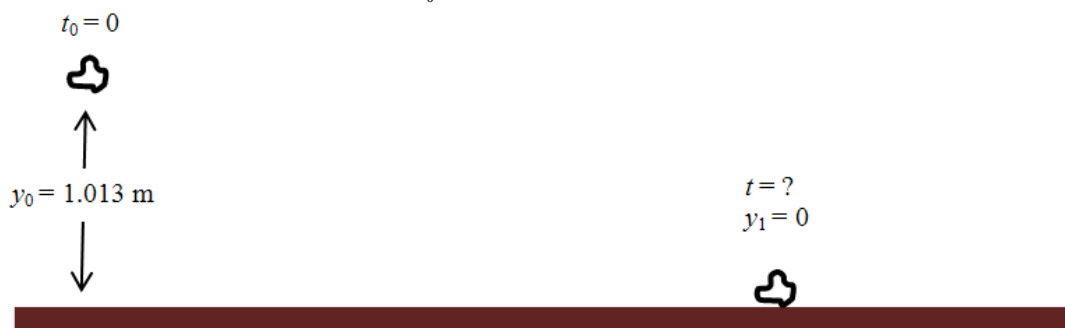
CALCULATE: $\Delta y = \frac{(28.0 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 39.96 \text{ m}$

ROUND: $v_0 = 28.0$ m/s has three significant figures, so the result should be rounded to $\Delta y = 40.0$ m.

DOUBLE-CHECK: The height has units of meters, which are an appropriate unit of distance. The calculated value is a reasonable maximum height for an object launched with a velocity of 28 m/s to achieve.

- 2.106. THINK:** Since the rock is dropped from a fixed height and allowed to fall to the surface of Mars, this question involves free fall. It is necessary to impose a coordinate system. Choose $y = 0$ to represent the surface of Mars and $t_0 = 0$ to be the time at which the rock is released.

SKETCH: Sketch the situation at time $t_0 = 0$ and time t , when the rock hits the surface.



RESEARCH: For objects in free fall, equations 2.25 can be used to compute velocity and position. In particular, the equation $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ can be used. In this case, $y_0 = 1.013$ m. Since the object is not thrown but dropped with no initial velocity, $v_{y0} = 0$ m/s, and $g = 3.699$ m/s² on the surface of Mars.

SIMPLIFY: The starting position and velocity ($y_0 = 1.013$ m and $v_{y0} = 0$ m/s), final position ($y = 0$ m) and gravitational acceleration are known. Using the fact that $v_{y0} = 0$ m/s and solving the equation for t gives:

$$\begin{aligned} 0 &= y_0 + 0t - \frac{1}{2}(g)t^2 = \\ \frac{g}{2}t^2 &= y_0 \Rightarrow \\ t^2 &= \frac{2 \cdot y_0}{g} \\ t &= \sqrt{\frac{2 \cdot y_0}{g}} \end{aligned}$$

CALCULATE: On Mars, the gravitational acceleration $g = 3.699$ m/s². Since the rock is dropped from a height of 1.013 m, $y_0 = 1.013$ m. Plugging these numbers into our formula gives a time $t = \sqrt{\frac{2.026}{3.699}}$ s.

ROUND: In this case, all measured values are given to four significant figures, so our final answer has four significant digits. Using the calculator to find the square root gives a time $t = 0.7401$ s.

DOUBLE-CHECK: First note that the answer seems reasonable. The rock is not dropped from an extreme height, so it makes sense that it would take less than one second to fall to the Martian surface. To check the answer by working backwards, first note that the velocity of the rock at time t is given by the equation $v_y = v_{y0} - gt = 0 - gt = -gt$ in this problem. Plug this and the value $v_{y0} = 0$ into the equation to find the average velocity $\bar{v}_y = \frac{1}{2}(v_y + 0) = \frac{1}{2}(-gt)$. Combining this with the expression for position gives:

$$\begin{aligned} y &= y_0 + \bar{v}_y t \\ &= y_0 + \left(\frac{1}{2}(-gt)\right)t \end{aligned}$$

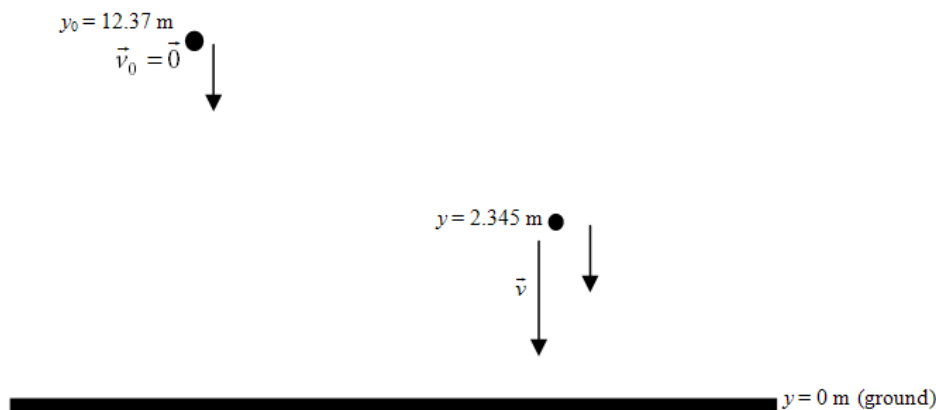
Using the fact that the rock was dropped from a height of $y_0 = 1.013$ m and that the gravitational acceleration on Mars is $g = 3.699$ m/s², it is possible to confirm that the height of the rock at time $t = 0.7401$ s is $y = 1.013 + \frac{1}{2}(-3.699)(0.7401^2) = 0$, which confirms the answer.

- 2.107.** The time that the rock takes to fall is related to the distance it falls by $y = \frac{1}{2}gt^2$.

$$y = \frac{1}{2}gt^2 = \frac{1}{2}(3.699 \text{ m/s}^2)(0.8198 \text{ s})^2 = 1.243 \text{ m}$$

- 2.108. THINK:** Since the ball is dropped from a fixed height with no initial velocity and allowed to fall freely, this question involves free fall. It is necessary to impose a coordinate system. Choose $y = 0$ to represent the ground. Let $t_0 = 0$ be the time when the ball is released from height $y_0 = 12.37 \text{ m}$ and t_1 be the time the ball reaches height $y_1 = 2.345 \text{ m}$.

SKETCH: Sketch the ball when it is dropped and when it is at height 2.345 m.



RESEARCH: Equations (2.25) are used for objects in free fall. Since the ball is released with no initial velocity, we know that $v_{0y} = 0$. We also know that on Earth, the gravitational acceleration is 9.81 m/s^2 . In this problem, it is necessary to find the time that the ball reaches 2.345 m and find the velocity at that time. This can be done using equations (2.25) part (i) and (iii):

$$(i) \quad y = y_0 - \frac{1}{2}gt^2$$

$$(iii) \quad v_y = -gt$$

SIMPLIFY: We use algebra to find the time t_1 at which the ball will reach height $y_1 = 2.345 \text{ m}$ in terms of the initial height y_0 and gravitational acceleration g :

$$\begin{aligned} y_1 &= y_0 - \frac{1}{2}g(t_1)^2 \Rightarrow \\ \frac{1}{2}g(t_1)^2 &= y_0 - y_1 \Rightarrow \\ (t_1)^2 &= \frac{2}{g}(y_0 - y_1) \Rightarrow \\ t_1 &= \sqrt{\frac{2}{g}(y_0 - y_1)} \end{aligned}$$

Combining this with the equation for velocity gives $v_{y1} = -gt_1 = -g\sqrt{\frac{2}{g}(y_0 - y_1)}$.

CALCULATE: The ball is dropped from an initial height of 12.37 m above the ground, and we want to know the speed when it reaches 2.345 m above the ground, so the ball is dropped from an initial height of 12.37 m above the ground, and we want to know the speed when it reaches 2.345 m above the ground, so $y_0 = 12.37$ and $y_1 = 2.345 \text{ m}$. Use this to calculate $v_{y1} = -9.81\sqrt{\frac{2}{9.81}(12.37 - 2.345)} \text{ m/s}$.

ROUND: The heights above ground (12.37 and 2.345) have four significant figures, so the final answer should be rounded to four significant figures. The speed of the ball at time t_1 is then $-9.81\sqrt{\frac{2}{9.81}(12.37 - 2.345)} = -14.02 \text{ m/s}$. The velocity of the ball when it reaches a height of 2.345 m above the ground is 14.02 m/s towards the ground.

DOUBLE-CHECK: To double check that the ball is going 14.02 m/s towards the ground, we use equation (2.25) (v) to work backwards and find the ball's height when the velocity is 14.02 m/s. We know that:

$$\begin{aligned}
 v_y^2 &= v_{y0}^2 - 2g(y - y_0) \Rightarrow \\
 v_y^2 &= 0^2 - 2g(y - y_0) = -2g(y - y_0) \Rightarrow \\
 \frac{v_y^2}{-2g} &= y - y_0 \Rightarrow \\
 \frac{v_y^2}{-2g} + y_0 &= y
 \end{aligned}$$

We take the gravitational acceleration $g = 9.81 \text{ m/s}^2$ and the initial height $y_0 = 12.37 \text{ m}$, and solve for y

when $v_y = -14.02 \text{ m/s}$. Then $y = \frac{v_y^2}{-2g} + y_0 = \frac{(-14.02)^2}{-2(9.81)} + 12.37 = 2.352 \text{ m}$ above the ground. Though this

doesn't match the question exactly, it is off by less than 4 mm, so we are very close to the given value. In fact, if we keep the full accuracy of our calculation without rounding, we get that the ball reaches a velocity of $14.0246\dots \text{ m/s}$ towards the ground at a height of 2.345 m above the ground.

- 2.109.** Using the results noted in the double-check step of the preceding problem,

$$\begin{aligned}
 y_0 - y &= \frac{v^2}{2g} \\
 y &= y_0 - \frac{v^2}{2g} = 13.51 \text{ m} - \frac{(14.787 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 2.37 \text{ m}.
 \end{aligned}$$

By the rule for subtraction, the result is significant to the hundredths place.

- 2.110.** Again using the results from the double-check step of the earlier problem,

$$\begin{aligned}
 y_0 - y &= \frac{v^2}{2g} \\
 y_0 &= y + \frac{v^2}{2g} = 2.387 \text{ m} + \frac{(15.524 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 14.670 \text{ m}.
 \end{aligned}$$

Note that if the value of g is treated as exact, by the addition rule the result has five significant figures.