Chapter 1

INTRODUCTION

PROBLEMS AND SOLUTIONS

Problem 1.1 *Derive the least-squares equations for finding a and b which provide the best fit to the three equations*

$$\begin{aligned} 2 &= a + b \\ 4 &= 3a + b \\ 1 &= 2a + b. \end{aligned}$$

(a) *Express the system of equations in matrix form as*

$$z = A \begin{bmatrix} a \\ b \end{bmatrix},$$

where z is a column vector with two rows, and the matrix A is 2×2.

(b) *Take the matrix product $A^T A$ for A derived in (a).*

(c) *Take the 2×2 matrix inverse*

$$\left[A^{\mathrm{T}} A\right]^{-1}$$

for the A derived in (a). [Hint: Use the general formula

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix}^{-1} = \frac{1}{m_{11}m_{22} - m_{12}^2} \begin{bmatrix} m_{22} & -m_{12} \\ -m_{12} & m_{11} \end{bmatrix}$$

for inverting symmetric 2×2 matrices.]

(d) *Take the matrix product*

$$A^T \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

for the A derived in (a).

(e) *Calculate the least squares solution*

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \left[A^{\mathrm{T}} A\right]^{-1} A^{\mathrm{T}} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

for the $\left[A^{\mathrm{T}} A\right]^{-1}$ derived in (c) and

$$A^T \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

derived in (d).