

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 1) Find a constant solution of $y' = t(y - 1)$. 1) _____
Enter just an integer.
- 2) Find a constant solution of $y' = 10y - 7$. 2) _____
Enter just a reduced fraction of form $\frac{a}{b}$.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 3) Consider the differential equation $y' = t^3(y + 3)$. Which of the following statements is/are true? 3) _____
(I) $f(t) = -3$ is a constant solution to this differential equation.
(II) $f(t) = 0$ is a constant solution to this differential equation.
(III) If $f(t)$ is a solution to the differential equation with initial conditions $y(1) = 0$, then $f'(1) = 3$.
A) I only
B) I and III
C) III only
D) I, II, and III
E) II only

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 4) Find $f'(1)$ if $f(t)$ is a solution to the initial value problem: $y' = ty^2 + 5$, $y(1) = 1$. 4) _____
Enter just an integer.
- 5) Find $f'(0)$ if $f(t)$ is a solution to the initial value problem: $y' = e^{2t} + y$, $y(0) = -1$. 5) _____
Enter just an integer.
- 6) Find $f'(1)$ if $f(t)$ is a solution to the initial value problem: $y' = e^{2t} - y$, $y(1) = 0$. 6) _____
Enter just a real number (no approximations).

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 7) Consider the differential equation $y' = y - y^2$. Which of the following statements is/are true? 7) _____
- A) If $f(t)$ is a solution to the differential equation satisfying the initial condition $y(0) = 0$, then $f'(0) = 0$.
- B) The constant function $f(t) = 1$ is a solution to this differential equation.
- C) The function $f(t) = \frac{1}{(1 + e^{-t})}$ is a solution to this differential equation with initial condition $y(0) = \frac{1}{2}$.
- D) This differential equation has infinitely many solutions.
- E) All of these statements are true.

Solve the problem.

- 8) Write a differential equation that expresses the following description of a rate: When ice cream is removed from the freezer, it warms up at a rate proportional to the difference between the temperature of the ice cream and the room temperature of 76° . (Use y for the temperature of the ice cream, t for the time, and k for an unknown constant.) 8) _____
- A) $y' = 76 - ky$ B) $y' = k(76 - t)$ C) $y' = k(76 - y)$ D) $y' = 76t - ky$
- 9) The growth rate of a certain stock is modeled by $\frac{dV}{dt} = k(44 - V)$, $V = \$21$ when $t = 0$, where $V =$ 9) _____
the value of the stock, per share, after time t (in months), and $k =$ a constant. Find the solution to the differential equation in terms of t and k .
- A) $V = 44 - 23e^{kt}$ B) $V = 21 - 23e^{-kt}$ C) $V = 44 - 44e^{-kt}$ D) $V = 44 - 23e^{-kt}$
- 10) Solve the differential equation model of radioactive decay: 10) _____
 $\frac{dQ}{dt} = -0.3Q$.
- A) $Q(t) = Q_0e^{-t}$ B) $Q(t) = \frac{-1}{0.3t} + Q_0$
- C) $Q(t) = Q_0e^{-0.3t}$ D) $Q(t) = -Q_0 \ln 0.3t + c$
- 11) Sales (in thousands) of a certain product are declining at a rate proportional to the amount of sales, with a decay constant of 11% per year. Write a differential equation to express the rate of sales decline. 11) _____
- A) $dy/dt = -0.89y$ B) $dy/dt = -0.11y$ C) $dy/dt = e^{-0.11t}$ D) $dy/dt = -0.11t$
- 12) Which of the following functions solves the differential equation: $y' = -4y$? 12) _____
- A) $y = \ln 4t$ B) $y = e^{-4t}$ C) $y = -e^{-4t}$ D) none of these

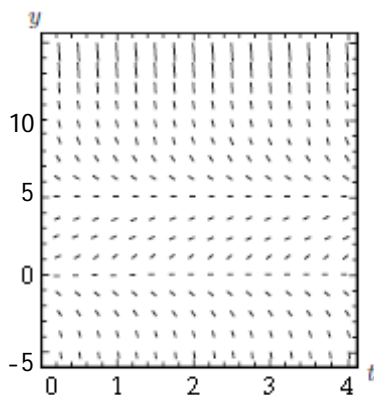
- 13) Which of the following functions solves the differential equation: $y' = e^{-2x} + 3$? 13) _____
- A) $y = e^{-2x} + 3$ B) $y = \frac{1}{2}e^{-2x} + 3$
- C) $y = -\frac{1}{2}e^{-2x} + 3x$ D) none of these

- 14) Which of the following functions solves the differential equation: $y' = -6xy$? 14) _____
- A) $y = e^{-3x^2}$ B) $y = 7e^{-3x^2}$ C) $y = 7e^{-x^2}$ D) $y = e^{-x^2}$

- 15) Which of the following functions solves the differential equation: $y' = y^2$? 15) _____
- A) $y = \ln|1 + x|$ B) $y = -\frac{1}{x^2}$ C) $y = -\frac{1}{x+1}$ D) $y = \frac{1}{3}x^3$

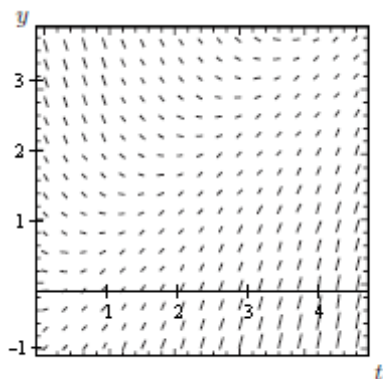
Use the figure to answer the question.

- 16) The figure shows a slope field of the differential equation $y' = 5y(5 - y)$. Use the figure to determine the constant solutions (if any) of the differential equation. 16) _____

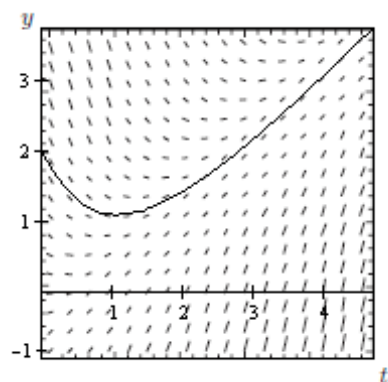


- A) None B) $y = -5, y = 5$ C) $y = 0, y = 5$ D) $y = 0$

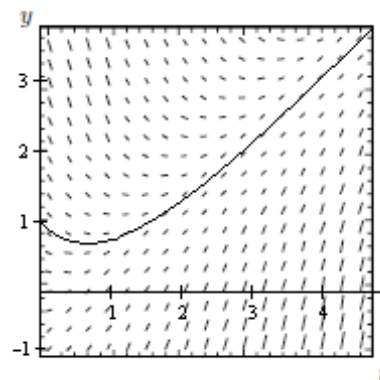
- 17) The figure shows a slope field for the differential equation $y' = t - y$. Draw an approximation of a portion of the solution curve for $y' = t - y$ that goes through the point (0, 2). Based on the slope field, can this solution pass through the point (1.1, 0.4)? 17) _____



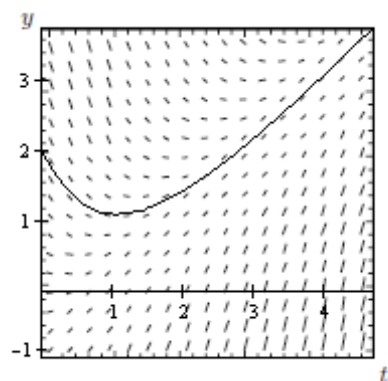
A) No



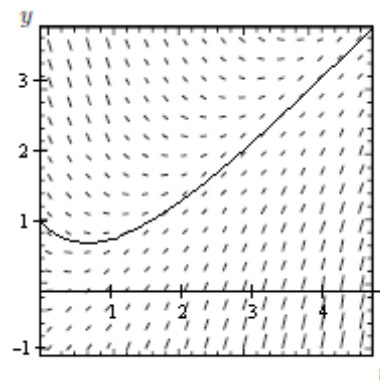
B) No



C) Yes



D) Yes



Find the general solution for the differential equation.

18) $y \frac{dy}{dx} = x^2 - 2x$

18) _____

A) $y = \frac{1}{3}x^3 - 2x + C$

B) $y^2 = \frac{1}{3}x^3 - 2x^2 + C$

C) $y = \frac{2}{3}x^3 - 2x^2 + C$

D) $y^2 = \frac{2}{3}x^3 - 2x^2 + C$

19) $\frac{dy}{dx} = y^2 e^{2x}$

19) _____

A) $y = -\frac{1}{\frac{e^{2x}}{2}} + C$

B) $y = -\frac{e^{2x}}{2} + C$

C) $y = \frac{1}{\frac{e^{2x}}{2}} + C$

D) $y = -\frac{1}{2e^{2x}} + C$

20) Solve the differential equation: $y' = e^y \sin t$.

20) _____

A) $y = e^{-\cos t} + C$

B) $y = \ln(-\sin t) + C$

C) $y = \cos(\ln t) + C$

D) $y = -\ln(\cos t + C)$

E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

21) Given the differential equation: $y' = y^2 - t^2y^2$, is this the solution: $y = \frac{1}{\frac{1}{3}t^3 - t + C}$? 21) _____

Enter your answer as just "yes" or "no".

22) Given the differential equation: $y' = 3t^2(y - 7)$, is this the solution $y = 7 + ce^{t^3}$? 22) _____
Enter just "yes" or "no"

23) Given the differential equation: $y' = e^{2t} - 1$, is this the solution $y = \frac{e^{2t}}{2} - t + C$? 23) _____
Enter "yes" or "no".

24) Given the differential equation: $(t^2 + 1)y' = yt$, is this the solution $y = c\sqrt{t^2 + 1}$? 24) _____
Enter "yes" or "no"

25) Given the differential equation: $y' = \frac{1}{ty}$ is this the solution $y = \pm\sqrt{2 \ln|t| + C}$? 25) _____
Enter "yes" or "no".

26) Given the differential equation: $ty' = \ln t$, is this the solution $y = \frac{(\ln t)^2}{2} + C$? 26) _____
Enter "yes" or "no".

27) Given the differential equation: $(t + 1)y' = yt^2 - y$, is this the solution $y = ce^{t^2/2}$? 27) _____
Enter just "yes" or "no".

28) Given the differential equation: $\frac{dy}{dt} = \frac{t + e^t}{y}$, is this the solution $y = \pm\sqrt{t^2 + 2e^t + C}$? 28) _____
Enter "yes" or "no".

29) Given the differential equation: $y' = \frac{t \sin t^2}{y}$, is this the solution $y = \pm\sqrt{C - \cos t^2}$? 29) _____
Enter "yes" or "no"

30) Find the constant solutions to the differential equation: $y' = y^2e^t - 2ye^t$. 30) _____
Enter just one integer or two separated by a comma (no label).

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the differential equation with the given initial condition.

31) $\frac{dy}{dx} + 2x = 3x^2$; $y(0) = 13$ 31) _____

A) $y = x^3 + 2x^2 + 13$

B) $y = 3x^3 + x^2 + 13$

C) $y = 3x^3 + 2x^2 + 13$

D) $y = x^3 - x^2 + 13$

32) $\frac{dy}{dx} = \frac{x^2}{y}$; $y(0) = 6$ 32) _____

A) $y^2 = \frac{2x^3}{3} + 36$

B) $y^2 = \frac{x^3}{3} + 6$

C) $y^2 = \frac{x^3}{3} + 36$

D) $y^2 = \frac{2x^3}{3} + 6$

33) $(6x + 5)y = \frac{dy}{dx}$; $y(0) = 1$ 33) _____

A) $y = e^{6x^2 + 5x}$

B) $y = e^{3x^2 + 5x + 1}$

C) $y = e^{3x^2 + 5x}$

D) $y = e^{6x^2 + 5x + 1}$

34) $x \frac{dy}{dx} - 6y\sqrt{x} = 0$; $y(0) = 1$ 34) _____

A) $y = e^{12x^{1/2}}$

B) $y = e^{12x^{-1/2}}$

C) $y = e^{12x^{1/2} + 1}$

D) $y = e^{6x^{-1/2} + 1}$

35) $\frac{dy}{dx} = \frac{2 - x^2}{3y + 6}$; $y(0) = 1$ 35) _____

A) $\frac{3}{2}y^2 + 6y = 2x - \frac{1}{3}x^3 + 1$

B) $\frac{3}{2}y^2 + 6y = 2x - \frac{1}{3}x^3 + \frac{15}{2}$

C) $\frac{3}{2}y^2 + 6y = 2x - x^3 + \frac{15}{2}$

D) $\frac{3}{2}y^2 + 6y = 2x - \frac{1}{2}x^3 + 9$

36) $y' = y(t - 2)$; $y(0) = 1$ 36) _____

A) $y = 0$

B) $y = \frac{t^2}{2} - 2t$

C) $y = e^{[(t^2/2) - 2t]} - 2t + 1$

D) $y = e^{[(t^2/2) - 2t]}$

E) none of these

37) $y' = \tan t \sec^2 t$; $y(0) = 1$

37) _____

A) $y = \ln|\tan t| + 1$

B) $y = \tan t + 1$

C) $y = \frac{\sec^2 t}{3} + \frac{2}{3}$

D) $y = \frac{\tan^2 t}{2} + 1$

38) $y' = y^2$, $y(0) = 1$

38) _____

A) $t + 1$

B) $\frac{1}{1-t}$

C) $t - 1$

D) $\frac{1}{t+1}$

39) $y' = \frac{x}{y}$, $y(3) = 5$

39) _____

A) $y = \sqrt{x^2 + 9}$

B) $y = \pm\sqrt{x^2 + 9}$

C) $y = \pm\sqrt{x^2 + 16}$

D) $y = \sqrt{x^2 + 16}$

40) $y' = y + 1$, $y(0) = 1$

40) _____

A) $y = t^2 + 1$

B) $y = t + 1$

C) $y = e^t - 2$

D) $y = 2e^t - 1$

41) $y' = ty$, $y(0) = -1$

41) _____

A) $y = e^t + 1$

B) $y = \frac{t^2}{2}$

C) $y = -1 + e^{t^2/2}$

D) $y = -e^{t^2/2}$

42) $y' = \sin t \cos^3 t$, $y\left(\frac{\pi}{3}\right) = 0$

42) _____

A) $y = \frac{1}{64} - \frac{\cos^4 t}{4}$

B) $y = 64 - \cos^4 t$

C) $y = \frac{\cos^4 t}{4}$

D) $y = 16 + \frac{\cos^4 t}{4}$

43) $y' = 3t^2(4 - y)^2$, $y(0) = 2$

43) _____

A) $y = 4 + t^3$

B) $y = 4 - \frac{1}{t^3 + \frac{1}{2}}$

C) $y = \frac{1}{t^3 + 2}$

D) $y = t^3 + \frac{1}{2}$

44) $y' = -e^{-y}$, $y(0) = 0$

44) _____

A) $y = e^{-1}$

B) $y = -\ln|t + 1|$

C) $y = 0$

D) $y = e^t$

E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

45) Given the differential equation with the given initial condition: $y' = \frac{3t^2 + 1}{2y}$; $y(1) = -5$ 45) _____

is this the solution $y = -\sqrt{t^3 + t + 2}$?
Enter "yes" or "no".

46) Given the differential equation with the given initial condition: $\frac{dy}{dt} = y^2 \ln t$; $y(1) = \frac{1}{3}$ 46) _____

is this the solution $y = \frac{1}{2 - t \ln t + t}$?
Enter "yes" or "no".

47) Given the differential equation with the given initial condition: $y' = \sqrt{\frac{t+1}{y}}$; $y(0) = 4$ 47) _____

is this the solution $y = ((t+1)^{3/2} + 4)^{2/3}$?
Enter "yes" or "no".

48) Given the differential equation with the given initial condition: $y' = t \cos t$; $y(0) = 0$ 48) _____

is this the solution $y = t \sin t + \cos t - 1$?
Enter "yes" or "no".

49) Given the differential equation with the given initial condition: $yy' = te^{t^2}$; $y(0) = 1$ 49) _____

is this the solution $y = e^{t^2/2}$?
Enter "yes" or "no".

50) Given the differential equation with the given initial condition: $y' = e^{-y}$; $y(0) = 0$ 50) _____

is this the solution $y = \ln |t + 1|$?
Enter "yes" or "no".

51) Given the differential equation with the given initial condition: $\frac{dy}{dt} = 3t^2 + \sin t$; $y(0) = 2$ 51) _____

is this the solution $y = t^3 - \cos t + 1$?
Enter "yes" or "no".

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 52) Let t represent the number of hours that a packing machine is operated and $y(t)$ represent the probability that the machine breaks down at least once during the t hours of operation. It has been observed that the rate of increase of the probability of a breakdown is proportional to the probability of not having a breakdown. Find a differential equation describing this situation. 52) _____
- A) $y' = k(1 - y)$; $y(0) = 0$
B) $y' = ky$; $y(0) = 0$
C) $y' = ky$; There is not enough information given to determine initial conditions.
D) $y' = (1 + y)$; $y(0) = 0$
E) $y' = k(1 - y)$; There is not enough information given to determine initial conditions.
- 53) Suppose water is seeping from an underground storage facility at a rate that is proportional to the square amount of water present. If $f(t) = y$ is the amount of water present at time t , find a differential equation describing the situation. 53) _____
- A) $y' = ky^2$, $k < 0$
B) $y' = ky^2$, $k < 0$; $y(0) = 0$
C) $y' = ky^2$, $k > 0$; $y(0) = 0$
D) $y' = ky^2$, $k > 0$
E) none of these
- 54) Suppose the relationship between the price p , of a product and the weekly sales, s , of the product is given by the differential equation $\frac{dp}{ds} = -\frac{1}{10}(s + 3)$. Then 54) _____
- A) as the price increases the rate of change of the price also increases.
B) as sales increase, the price increases.
C) the rate of the decrease of the price is proportional to the sales.
D) $s = 0$ is a constant solution to this differential equation.
E) all of these
- 55) The annual sales y (in millions of dollars) of a company satisfy the differential equation $\frac{dy}{dt} = 0.2y$; $y(0) = 2$. Which of the following is a verbal description of the rate of change of annual sales? 55) _____
- A) The annual sales are increasing at \$0.2 million (\$200,000) per year.
B) The annual sales are increasing at a rate proportional to \$0.2 million (\$200,000) per year.
C) The annual sales are increasing at a rate proportional to the annual sales.
D) The annual sales are decreasing at a rate proportional to the annual sales.

56) The annual sales y (in millions of dollars) of a company satisfy the differential equation $\frac{dy}{dt} = 0.2(10 - y)$; $y(0) = 2$. Which of the following is a verbal description of the rate of change of annual sales.

56) _____

- A) The annual sales are increasing at \$0.2 million (\$200,000) per year to an upper limit of \$5 million.
- B) The annual sales are increasing at a rate proportional to \$0.2 million (\$200,000) per year.
- C) The annual sales are decreasing at a rate proportional to the annual sales.
- D) The annual sales are increasing at a rate proportional to the difference between the annual sales and an upper limit of \$10 million.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

57) A cool object is to be heated to a maximum temperature $M = M^{\circ}\text{C}$. At any time t , the rate at which the temperature rises is proportional to the difference between the actual temperature and the maximal temperature. If the object is originally 0°C , find and solve a differential equation describing this situation. Is this the solution: $y(t) = M - Me^{-kt}$? Enter "yes" or "no".

57) _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

58) When a dead body is discovered, one of the first steps in the ensuing investigation is for a medical examiner to determine the time of death as closely as possible. If the temperature of the medium has been fairly constant and less than 48 hours have passed since death, Newton's law of cooling can be used. Newton's law of cooling states, $\frac{dT}{dt} = -k(T - T_M)$, where k is a constant, T is the temperature of the object after t hours, and T_M is the (constant) temperature of the surrounding medium. Assuming the temperature of a body at death is 98.6°F , the temperature of the surrounding air is 70°F , and at the end of one hour the body temperature is 89°F , what is the temperature of the body after 4 hours? Round to the nearest tenth of a degree.

58) _____

- A) 5.6°F
- B) 70.5°F
- C) 89°F
- D) 75.6°F

59) When a dead body is discovered, one of the first steps in the ensuing investigation is for a medical examiner to determine the time of death as closely as possible. If the temperature of the medium has been fairly constant and less than 48 hours have passed since death, Newton's law of cooling can be used. Newton's law of cooling states, $\frac{dT}{dt} = -k(T - T_M)$, where k is a constant, T is the temperature of the object after t hours, and T_M is the (constant) temperature of the surrounding medium. Assuming the temperature of a body at death is 98.6°F , the temperature of the surrounding air is 69°F , and at the end of one hour the body temperature is 90°F , when will the temperature of the body be 73°F ? Round to the nearest tenth of an hour.

59) _____

- A) 5.8 hr
- B) 2.8 hr
- C) 0.4 hr
- D) 0.2 hr

- 60) Earth's atmospheric pressure p is often modeled by assuming that the rate dp/dh at which p changes with the altitude h above sea level is proportional to p . Suppose that the pressure at sea level is 1013 millibars and that the pressure at an altitude of 9 km is 341 millibars.

60) _____

Solve the initial value problem

Differential equation: $\frac{dp}{dh} = kp$,

Initial condition: $p = p_0$ when $h = 0$

to express p in terms of h . Determine the values of p_0 and k from the given altitude-pressure data.

A) $p = 1013e^{-0.128h}$

B) $p = 1013e^{-0.094h}$

C) $p = 1013e^{-0.046h}$

D) $p = 1013e^{-0.121h}$

- 61) Earth's atmospheric pressure p is often modeled by assuming that the rate dp/dh at which p changes with the altitude h above sea level is proportional to p . Suppose that the pressure at sea level is 1013 millibars and that the pressure at an altitude of 12 km is 237 millibars.

61) _____

What is the atmospheric pressure at an altitude of 18 km? Round to the nearest millibar.

(You will first need to solve the initial value problem

Differential equation: $\frac{dp}{dh} = kp$,

Initial condition: $p = p_0$ when $h = 0$

and determine the values of p_0 and k from the given altitude-pressure data).

A) ≈ 118 millibars

B) ≈ 111 millibars

C) ≈ 121 millibars

D) ≈ 115 millibars

- 62) Find the integrating factor, the general solution, and the particular solution satisfying the initial condition.

62) _____

$2ty' - y = \frac{6}{t}; y(1) = -1, t > 0$

A) integrating factor: $t^{-1/2}$

general solution: $y = -4 + Ct^{1/2}$

particular solution: $y = -4 + 3t^{1/2}$

B) integrating factor: $2t$

general solution: $y = -\frac{3}{4t^2} + \frac{C}{2t}$

particular solution: $y = -\frac{3}{4t^2} + \frac{1}{4t}$

C) integrating factor: $t^{-1/2}$

general solution: $y = -\frac{2}{t} + Ct^{1/2}$

particular solution: $y = -\frac{2}{t} + t^{1/2}$

D) integrating factor: $2t$

general solution: $y = -\frac{3}{2t^2} + \frac{C}{2t}$

particular solution: $y = -\frac{3}{2t^2} + \frac{1}{2t}$

63) Find the integrating factor, the general solution, and the particular solution satisfying the initial condition.

63) _____

$$y' - 4y = -2e^{2t}; y(0) = -1$$

A) integrating factor: e^{-4t}

$$\text{general solution: } y = e^{2t} + Ce^{4t}$$

$$\text{particular solution: } y = e^{2t} - 2e^{4t}$$

B) integrating factor: e^{4t}

$$\text{general solution: } y = -2t + Ce^{4t}$$

$$\text{particular solution: } y = -2t - e^{4t}$$

C) integrating factor: e^{4t}

$$\text{general solution: } y = \frac{1}{3}e^{6t} + Ce^{4t}$$

$$\text{particular solution: } y = \frac{1}{3}e^{6t} - \frac{4}{3}e^{4t}$$

D) integrating factor: e^{-4t}

$$\text{general solution: } y = -e^{-2t} + Ce^{4t}$$

$$\text{particular solution: } y = -e^{-2t} - 2e^{4t}$$

Solve the equation using an integrating factor.

64) $y' + 2y = 15, t > 0$

64) _____

A) $y = \frac{15}{2} + Ce^{-2t}$

B) $y = \frac{15}{2} + e^{2t} + Ce^{-2t}$

C) $y = 15 + Ce^{2t}$

D) $y = \frac{15}{5} + Ce^{2t}$

65) $y' + 4y = 12, t > 0$

65) _____

A) $y = 12 + Ce^{-4t}$

B) $y = \frac{1}{3} + Ce^{4t}$

C) $y = 3 + Ce^{-4t}$

D) $y = 3 + Ce^{-12t}$

66) $y' + 2ty = 19t, t > 0$

66) _____

A) $y = \frac{19}{2} + 2x + Ce^{-t^2}$

B) $y = 19 + Ce^{t^2}$

C) $y = \frac{19}{2} + Ce^{-t^2}$

D) $y = \frac{19}{5} + Ce^{t^2}$

67) $ty' - 2y - 7t = 0, t > 0$

67) _____

A) $y = -\frac{7}{t} + C$

B) $y = -7 + Ct$

C) $y = -7t^2 + Ct^3$

D) $y = -7t + Ct^2$

68) $ty' + 5ty - t^2 = 0, t > 0$

68) _____

A) $y = \frac{t}{5} - \frac{1}{5} + Ce^{-5t}$

B) $y = \frac{t}{5} - \frac{1}{25} + Ce^{-5t}$

C) $y = \frac{t}{5} - \frac{1}{25} + Ce^{5t}$

D) $y = \frac{t}{5} + \frac{1}{5} + Ce^{5t}$

- 69) $y' + t^2y = 4t^2, t > 0$ 69) _____
 A) $y = 4x + Ce^{-t^2/2}$ B) $y = 4 + Ce^{-t^3/3}$
 C) $y = 4 + Ce^{-t^2/2}$ D) $y = 4x + Ce^{-t^3/3}$
- 70) $y' - 2y = -2t, t > 0$ 70) _____
 A) $y = Ct + e^{2t}$ B) $y = 1/2 + Ce^{-2t}$
 C) $y = t - Ce^{2t}$ D) $y = t + 1/2 + Ce^{2t}$
- 71) $y' + \frac{3}{x}y = 6x^2, t > 0$ 71) _____
 A) $y = Cx^{-3}$ B) $y = x^3 + C$ C) $y = x^3 + Cx^{-3}$ D) $y = Cx^3 + x^{-3}$
- 72) $y' - 3y = 6, t > 0$ 72) _____
 A) $y = Ce^{3t} - 2$ B) $y = Ce^{-3t}$ C) $y = Ce^{3t}$ D) $y = Ce^{3t} + C$
- 73) $y' + \frac{4}{t}y = t^4, t > 0$ 73) _____
 A) $y = t^{-4} + t^5$ B) $y = -t^{-4} + t^3 + C$
 C) $y = Ct^{-4} + \frac{1}{9}t^5$ D) $y = Ct^4 + t^5$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 74) Combine the terms y and y' into the derivative of a product: $(3t + 7)y' + 3y = 4t$. 74) _____
 Is this derivative correct: $\frac{d}{dt}[(3t + 7)y] = 4t$?
 Enter "yes" or "no".
- 75) Combine the terms y and y' into the derivative of a product: $y' \tan t + y \sec^2 t = 1$. 75) _____
 Is this derivative correct: $\frac{d}{dt}[y \tan t] = 1$?
 Enter "yes" or "no".
- 76) Combine the terms y and y' into the derivative of a product, then solve the equation. 76) _____
 $e^{3t^2}y' + 6te^{3t^2}y = \frac{5\sqrt{t}}{4}$. Is this the solution: $y = \frac{5}{6}t^{3/2}e^{-3t^2} + Ce^{-3t^2}$?
 Enter "yes" or "no".

77) Combine the terms y and y' into the derivative of a product, then solve the equation.

77) _____

$$\frac{y'}{t^3} - \frac{3y}{t^4} = t \quad \text{Is this the solution: } y = \frac{t^5}{2} + Ct^3?$$

Enter "yes" or "no".

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the initial value problem.

78) $y' + y = 3$; $y(0) = 0$.

78) _____

A) $y = 3t$

B) $y = e^{-t}(3t - e^{-t})$

C) $y = \sqrt{6t}$

D) $y = 3 - 3e^{-t}$

79) $ty' + 3y = 5t$; $y(1) = 1$, $t > 0$

79) _____

A) $y = \frac{5}{4}t - \frac{1}{4t^3}$

B) $y = \sqrt{\frac{4}{3}t^2 + 1} - \sqrt{\frac{4}{3}}$

C) $y = 5t^2 - 4t$

D) $y = 5t^3 - 4t^2$

80) $y' + y = 2e^t$; $y(0) = 14$

80) _____

A) $y = 4e^2 + 20e^{-t}$

B) $y = 14e^t$

C) $y = e^t + 13e^{-t}$

D) $y = 2e^t + 11e^{-t}$

81) $y' + 5y = 3$; $y(0) = 1$

81) _____

A) $y = \frac{3}{5}e^{5t} + \frac{2}{5}$

B) $y = \frac{2}{5}e^{5t} + \frac{3}{5}$

C) $y = \frac{2}{5}e^{-5t} + \frac{3}{5}$

D) $y = \frac{3}{5}e^{-5t} + \frac{2}{5}$

82) $y' + ty = 2t$; $y(0) = 5$

82) _____

A) $y = 3e^{t^2/2} + 2$

B) $y = 2e^{t^2/2} + 3$

C) $y = 3e^{-t^2/2} + 2$

D) $y = 2e^{-t^2/2} + 3$

83) $2y' - 4ty = 8t$; $y(0) = 11$

83) _____

A) $y = 2 + 11e^{t^2}$

B) $y = -1 + 12e^{t^2}$

C) $y = -2 + 13e^{t^2}$

D) $y = -2 + 13e^{-t^2}$

84) $ty' + (1 + t)y = 2$; $y(4) = 2$

84) _____

A) $y = \frac{2 + 6e^{-4 - t}}{t}$

B) $y = \frac{2 + 8e^{6 - t}}{t}$

C) $y = \frac{2 + 6e^{4 - t}}{t}$

D) $y = \frac{2 + 8e^{4 - t}}{t}$

85) $y' + 10ty - e^{-5t^2} = 0$; $y(0) = 2$

85) _____

A) $y = (t + 2)e^{-5t^2}$

B) $y = te^{-5t^2} + 2$

C) $y = te^{-5t} + 2$

D) $y = (t + 2)e^{-5t}$

Solve the problem.

- 86) An initial deposit of \$24,000 is made into an account that earns 5% compounded continuously. Money is then withdrawn at a constant rate of \$4000 a year until the amount in the account is 0. Find the equation for the amount in the account at any time t . When is the amount 0? 86) _____
- A) $A = 60,000 - 36,000e^{0.05t}$
10.017 years
- B) $A = 80,000 - 56,000e^{0.05t}$
8.352 years
- C) $A = 60,000 - 36,000e^{0.05t}$
8.352 years
- D) $A = 80,000 - 56,000e^{0.05t}$
7.134 years
- 87) An initial deposit of \$8,000 is made into an account earning 6.5% compounded continuously. Thereafter, money is deposited into the account at a constant rate of \$2600 per year. Find the amount in this account at any time t . How much is in this account after 5 years? 87) _____
- A) $A = 52,000e^{0.065t} - 44,000$
\$27,969.59
- B) $A = 44,000e^{0.065t} - 36,000$
\$24,897.34
- C) $A = 60,000e^{0.065t} - 52,000$
\$31,041.84
- D) $A = 48,000e^{0.065t} - 40,000$
\$26,433.47

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 88) Suppose that \$1000 is deposited in a savings account that pays 6% annual interest compounded continuously. At what rate (in dollars per year) is it earning interest after 5 years? Enter just an integer representing the amount to the nearest dollar (no units). 88) _____
- 89) How much would you need to invest per month - in effect, continuously - in an investment account that pays an annual interest rate of 9%, compounded continuously, in order for the account to be worth \$100,000 after 20 years? Enter just an integer representing dollars to the nearest dollar (no units) 89) _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 90) Richard deposits \$2000 in an IRA at 10% interest compounded continuously for his retirement in 25 years. He intends to make continuous deposits at the rate of \$2500 a year. How much will he have accumulated in 20 years? Round your answer to the nearest dollar. 90) _____
- A) \$174,505 B) \$177,505 C) \$180,505 D) \$170,505
- 91) A tank contains 2000 L of a solution consisting of 50 kg of salt dissolved in water. Pure water is pumped into the tank at the rate of 10L/s, and the mixture (kept uniform by stirring) is pumped out at the same rate. How long will it be until only 5 kg of salt remain in the tank? 91) _____
- A) approximately 276 seconds B) approximately 703 seconds
- C) approximately 460 seconds D) approximately 689 seconds

- 92) A nutritionist proposes the following model for weight loss on a program she is developing: 92) _____

$$\frac{dw}{dt} + 0.006w = \frac{1}{3600}C$$

where $w(t)$ is a person's weight (in pounds) after t days of consuming exactly C calories per day. A person weighing 180 pounds goes on this diet program consuming 2400 calories per day. Use the above model to predict how long will it take this person to lose 15 pounds.

- A) 41 days B) 39 days C) 35 days D) 37 days

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 93) There is a differential equation that is a mathematical model of the situation in which the time rate of change in the population of a certain organism is proportional to the product of the current population and the difference between the current population and the limiting factor of 100,000. Is this the equation $\frac{dP}{dt} = kP(100,000 - P)$? Enter "yes" or "no". 93) _____

- 94) A fly population increases at a rate proportional to the amount present. After two years the population has doubled. After three years it is 20,000. Find the number of flies initially present. Enter just an integer. 94) _____

- 95) A jug of milk at 50° is placed outdoors at a temperature of 100° . If after 5 minutes the temperature of the milk is 60° , write the equation giving the temperature of the milk as a function of time. Enter your answer exactly as: $T = ae^{bt} + c$ 95) _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 96) Suppose an isolated island has a native population of 8000 and a person from a visiting ship introduces a disease which has an infection rate of 0.00005. Assume that the rate of spread of the disease satisfies the following logistic equation: 96) _____

$$\frac{dy}{dt} = k \left(1 - \frac{y}{N} \right) y,$$

where N is the size of the population and y is the number infected at time t .

Write an equation for the number of infected natives after t days.

A) $y = \frac{8000}{1 + 7999e^{-0.4t}}$

B) $y = \frac{8000}{1 + 7999e^{-0.00005t}}$

C) $y = \frac{7999}{1 + 8000e^{-0.4t}}$

D) $y = \frac{7999}{1 + 8000e^{-0.00005t}}$

- 97) Suppose an isolated island has a native population of 10,000 and a person from a visiting ship introduces a disease which has an infection rate of 0.00005. Assume that the rate of spread of the disease satisfies the following logistic equation:

97) _____

$$\frac{dy}{dt} = k \left(1 - \frac{y}{N} \right) y,$$

where N is the size of the population and y is the number infected at time t .

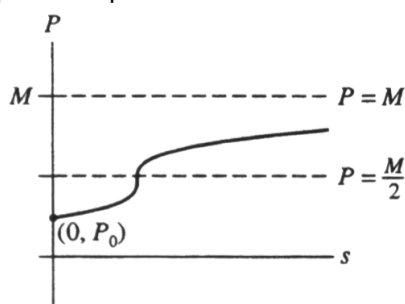
How many individuals are infected after 15 days?

- A) 1481 B) 1331 C) 1531 D) 1409

- 98) Suppose the graph below gives a solution to the differential equation $\frac{dP}{ds} = g(P)$ where P is the

98) _____

price of a product and s is the weekly sales.



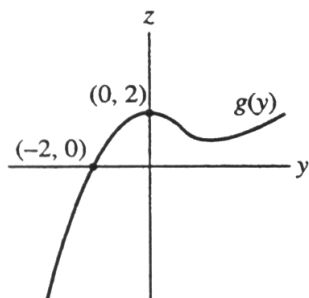
Which of the following statements is/are true?

- (I) $g(M) = 0$
 (II) $g'(M) = 0$
 (III) $g\left(\frac{M}{2}\right) = 0$
 (IV) $g(P_0) > 0$

- A) I and IV
 B) IV only
 C) I, III, and IV
 D) I, II, and IV
 E) I only

99) Suppose the following is a graph of $z = g(y)$.

99) _____



Which of the following can then be said about the solution $y = f(t)$ to the initial value problem

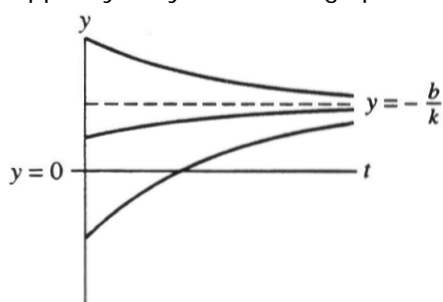
$$y' = g(y); \quad y(0) = -1?$$

- (I) $f(t)$ is an increasing function
- (II) $f(t)$ is always positive
- (III) $f(t)$ has an inflection point when $y = 2$.

- A) I only
- B) I and III
- C) I, II, and III
- D) III only
- E) II only

100) Suppose $y' = ky + b$ and the graphs of several solutions of the differential equation are as below:

100) _____

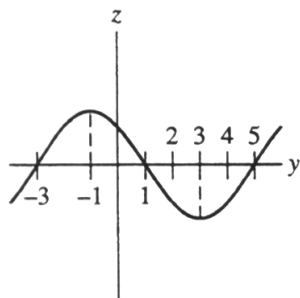


Then

- (I) k is negative.
- (II) k is positive.
- (III) b is positive.
- (IV) b is negative.

- A) II and III
- B) I and IV
- C) II and IV
- D) I and III
- E) not enough information given

Consider the differential equation $y' = g(y)$ where $g(y)$ is the function whose graph is shown below:



Indicate whether the following statements are true or false.

- 101) $y = -3$, $y = 1$, and $y = 5$ are the constant solutions to $y' = g(y)$. 101) _____
 A) True B) False
- 102) $y = 2$ is the only constant solution of $y' = g(y)$. 102) _____
 A) True B) False
- 103) If the initial value of $y(0)$ is greater than 6, then the corresponding solution will be an increasing function. 103) _____
 A) True B) False
- 104) If the initial value of $y(0)$ is 3, then the corresponding solution has an inflection point. 104) _____
 A) True B) False
- 105) If the initial value of $y(0)$ is 2, then the corresponding solution has an inflection point. 105) _____
 A) True B) False
- 106) For what y value(s) does a solution of $y' = y^2 - 3y + 2$ have inflection points? 106) _____
 A) $y = 2$ and $y = 1$
 B) $y = \frac{3}{2}$
 C) $y = 2$
 D) $y = 0$
 E) none of these
- 107) Let $y' = 2 - y$. Which of the following properties hold for the solution $y = f(t)$ determined by the initial condition $y(0) = 1$? 107) _____
 (I) It is always concave down.
 (II) It is a constant solution.
 (III) It is always decreasing.
 A) III only
 B) I only
 C) II only
 D) I and III
 E) none of these

108) Let $y' = y^3$. Which of the following properties hold for the solution $y = f(t)$ determined by the initial condition $y(0) = -2$?

108) _____

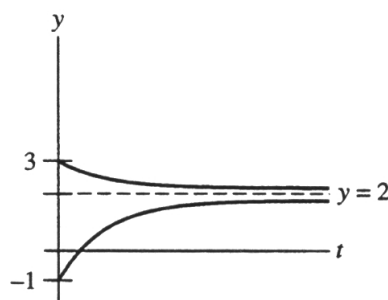
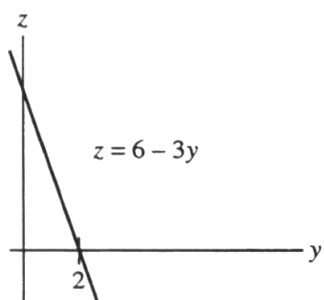
- (I) It is always increasing.
- (II) It has an inflection point.
- (III) It is always concave down.

- A) I only
- B) III only
- C) I and II
- D) II only
- E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

109) One or more initial conditions are given for the differential equation. Use the qualitative theory of autonomous differential equations to sketch the graphs of the corresponding solution. Include a yz -graph as well as a ty -graph. Do these graphs represent:
 $y' = 6 - 3y$; $y(0) = -1$; $y(0) = 3$?

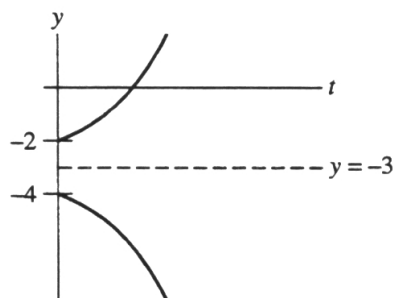
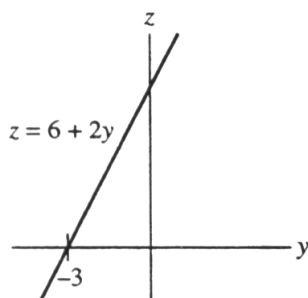
109) _____



Enter just "yes" or "no".

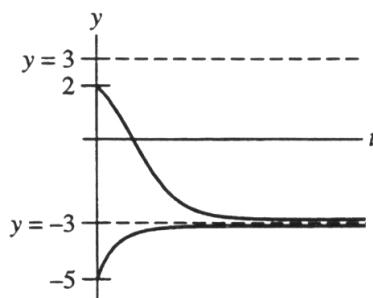
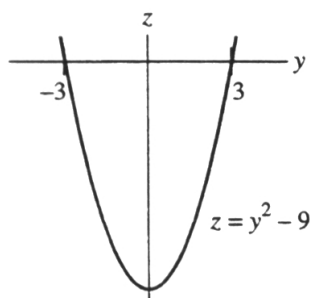
110) One or more initial conditions are given for the differential equation. Use the qualitative theory of autonomous differential equations to sketch the graphs of the corresponding solution. Include a yz -graph as well as a ty -graph. $y' = 6 + 2y$; $y(0) = -4$; $y(0) = -2$
 Do these graphs represent the situation?

110) _____



Enter just "yes" or "no".

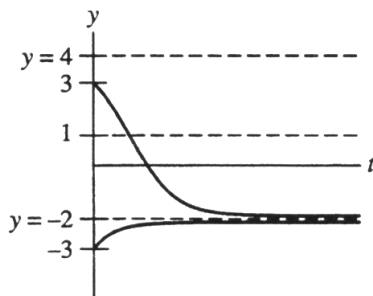
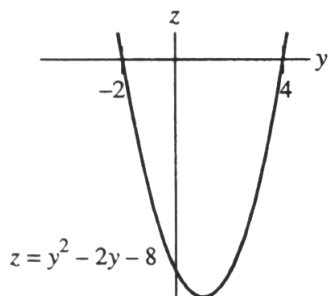
- 111) One or more initial conditions are given for the differential equation. Use the qualitative theory of autonomous differential equations to sketch the graphs of the corresponding solution. Include a yz -graph as well as a ty -graph. $y' = y^2 - 9$; $y(0) = -5$; $y(0) = 2$
Do these graphs represent the situation?



Enter just "yes" or "no".

111) _____

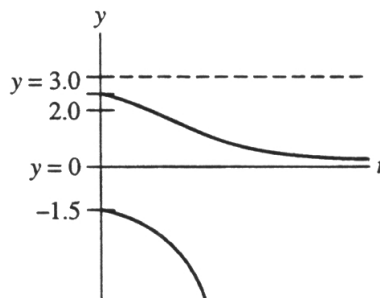
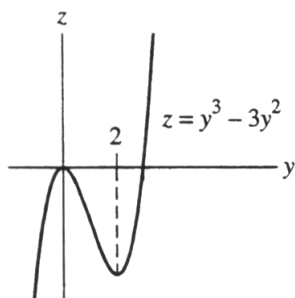
- 112) One or more initial conditions are given for the differential equation. Use the qualitative theory of autonomous differential equations to sketch the graphs of the corresponding solution. Include a yz -graph as well as a ty -graph. $y' = y^2 - 2y - 8$; $y(0) = -3$; $y(0) = 3$
Do these graphs represent the situation?



Enter just "yes" or "no".

112) _____

- 113) One or more initial conditions are given for the differential equation. Use the qualitative theory of autonomous differential equations to sketch the graphs of the corresponding solution. Include a yz -graph as well as a ty -graph. $y' = y^3 - 3y^2$; $y(0) = -1.5$; $y(0) = 2.5$
Do these graphs represent the situation?

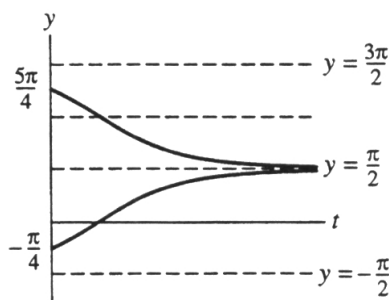
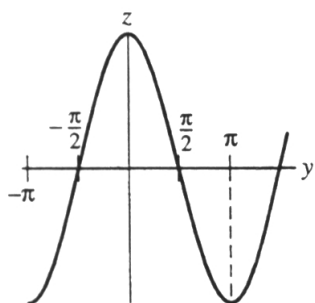


Enter just "yes" or "no".

113) _____

- 114) One or more initial conditions are given for the differential equation. Use the qualitative theory of autonomous differential equations to sketch the graphs of the corresponding solution. Include a yz -graph as well as a ty -graph. $y' = \cos y$; $y(0) = -\frac{\pi}{4}$; $y(0) = \frac{5\pi}{4}$

Do these graphs represent the situation?

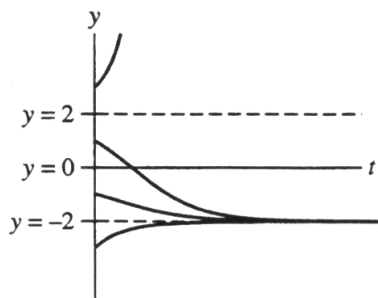
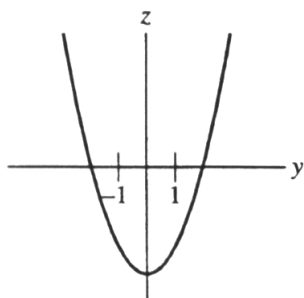


Enter just "yes" or "no".

- 115) One or more initial conditions are given for the differential equation. Use the qualitative theory of autonomous differential equations to sketch the graphs of the corresponding solution. Include a yz -graph as well as a ty -graph.

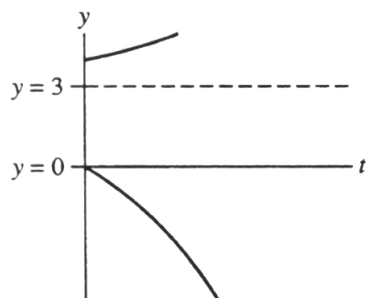
$$y' = y^2 - 4; y(0) = -3; y(0) = -1; y(0) = 1; y(0) = 3$$

Do these graphs represent the situation?



Enter just "yes" or "no".

- 116) Given $y' = y - 3$. On a ty -coordinate system sketch the solutions corresponding to the initial conditions $y(0) = 0$ and $y(0) = 4$. Does this graph represent the situation?



Enter "yes" or "no".

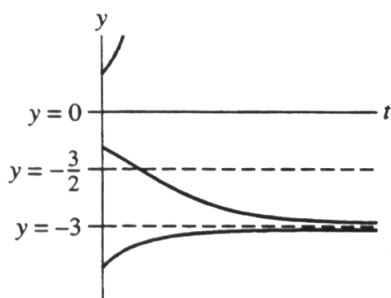
114) _____

115) _____

116) _____

- 117) Given $y' = y(y + 3)$. On a ty -coordinate system sketch the solutions corresponding to the initial conditions $y(0) = -4$, $y(0) = -1$, and $y(0) = 1$. Does this graph represent the situation?

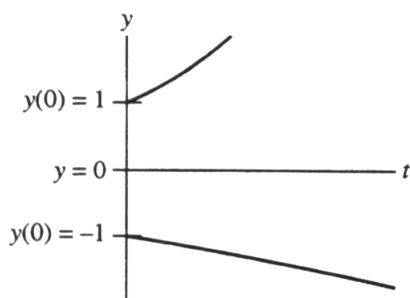
117) _____



Enter "yes" or "no".

- 118) Given $y' = e^y - 1$. On a ty -coordinate system sketch the solutions corresponding to the initial conditions $y(0) = -1$ and $y(0) = 1$. Does this graph represent the situation?

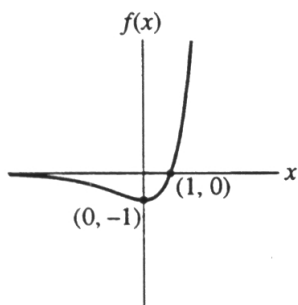
118) _____



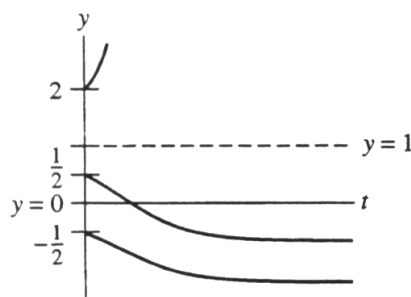
Enter "yes" or "no".

- 119) Below is a sketch of $f(x) = (x - 1)e^x$.

119) _____



On a ty -coordinate system, sketch the solutions to the differential equation $y' = (y - 1)e^y$ corresponding to the initial conditions $y(0) = 2$, $y(0) = \frac{1}{2}$, and $y(0) = -\frac{1}{2}$. Does the



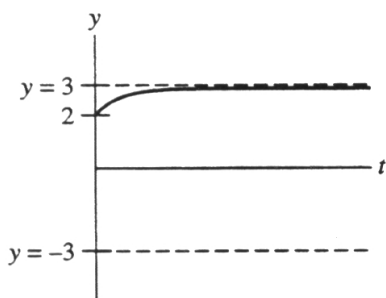
following graph represent this situation?
Enter "yes" or "no".

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

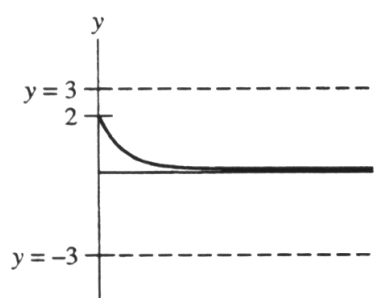
120) Which of the following is a sketch of the solution of $y' = y^2 - 9$; $y(0) = 2$?

120) _____

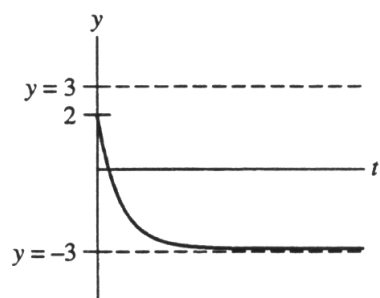
A)



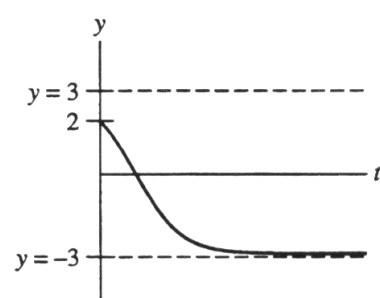
B)



C)



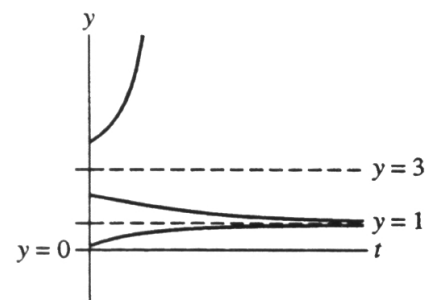
D)



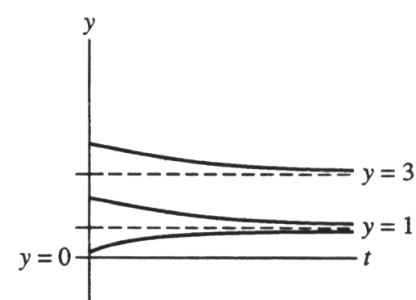
121) Consider the differential equation $y' = y^2 - 4y + 3$. Which of the following could be a graph of solutions to this differential equation?

121) _____

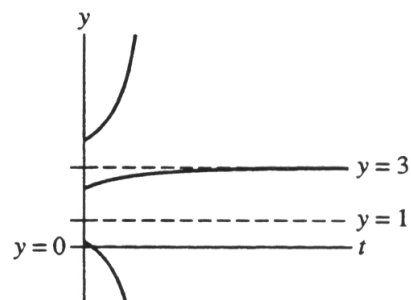
A)



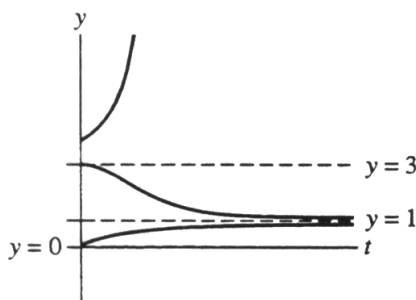
B)



C)

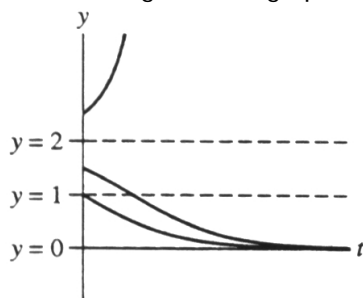


D)



122) The following could be graphs of solutions to which of the following differential equations?

122) _____



- A) $y' = y(y + 2)$
- B) $y' = y^2 + 2$
- C) $y' = 3y(y - 2)$
- D) $y' = (y - 2)e^y$
- E) none of these

Solve the problem.

123) A skydiver's terminal velocity is 46 meters per second. That is, no matter how long the skydiver falls, his or her speed will not exceed 46 meters per second but will get arbitrarily close to that value. The velocity in meters per second, $v(t)$, after t seconds satisfies the differential equation $v'(t) = \frac{49}{5} - kv(t)$. What is the value of k ?

123) _____

- A) $\frac{49}{115}$
- B) $\frac{230}{49}$
- C) $\frac{98}{5}$
- D) $\frac{49}{230}$

124) A large boulder is placed in a river to help divert the water. Suppose the rate at which the boulder erodes is proportional to the product of its current size and the difference between its original size, B , and 10 times its current size. Give a differential equation that is satisfied by $f(t)$, the height at time t .

124) _____

- A) $y' = ky(B - 10y)$
- B) $y' = 10ky(B - y)$
- C) $y' = kBy(-10y)$
- D) $y' = ky(10y - B)$

125) The logistic differential equation

125) _____

$$\frac{dP}{dt} = 0.09P(800 - P)$$

describes the growth of a population P , where t is measured in years. Find the carrying capacity of the population.

- A) 800
- B) 400
- C) 1600
- D) 7.2

126) The logistic differential equation

126) _____

$$\frac{dP}{dt} = 0.0005P(1400 - P)$$

describes the growth of a population P , where t is measured in years. Find the carrying capacity of the population.

- A) 1400
- B) 700
- C) 0.7
- D) 2800

127) The logistic differential equation

127) _____

$$\frac{dN}{dt} = 0.044N(1000 - N)$$

describes the growth of a population N , where t is measured in years.

Find the intrinsic rate, r .

A) $r = 44$

B) $r = 44,000$

C) $r = 1000$

D) $r = 0.044$

128) The logistic differential equation

128) _____

$$\frac{dN}{dt} = -0.02N^2 + 2N$$

describes the growth of a population N , where t is measured in years.

Find the intrinsic rate, r .

A) $r = -0.02$

B) $r = 2$

C) $r = 100$

D) $r = 0.02$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

129) Suppose an infectious disease spreads through an elementary school at a rate

129) _____

proportional to the product of the percentage of pupils who have the disease and the percentage of pupils who have not yet contracted the disease. Suppose that at the beginning of the epidemic 5% of the pupils have the disease. Let $f(t)$ be the percentage of pupils who have the disease at time t ; give the differential equation satisfied by $f(t)$. Does the following accurately describe this situation: $y' = ky(100 - y)$; $y(0) = 0.05$, where k is a positive constant?

Enter "yes" or "no".

130) A savings account earns 6% annual interest, compounded continuously. An initial deposit of \$8500 is made, and thereafter money is withdrawn continuously at the rate of \$480 per year. Does the following accurately represent this situation:

130) _____

$$y' = 0.06y - 480; y(0) = 8500$$

Enter "yes" or "no".

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

131) A man opens a savings account that earns interest at an annual rate of 6% compounded continuously. He plans to make continuous withdrawals at a rate of \$300 per year. What will happen if his initial deposit is \$5000? [Hint: Let $f(x)$ be the savings account balance at time t , and determine the differential equation satisfied by $f(t)$.]

131) _____

A) The balance will increase indefinitely.

B) The balance will decrease until it runs out.

C) The balance will increase at an increasing rate until it reaches \$18,000, at which point it will increase at a decreasing rate.

D) The balance will remain at \$5000 as long as the interest and withdrawals remain the same.

E) none of these

- 132) A population of algae consists of 4000 algae at time $t = 0$. Conditions will support at most 600,000 algae. Assume that the rate of growth of algae is proportional both to the number present (in thousands) and to the difference between 600,000 and the number present (in thousands). Write a differential equation using 0.03 for the constant of proportionality. 132) _____
- A) $dy/dt = 0.03(600 - y)$ B) $dy/dt = 4000y(600 - 0.03y)$
 C) $dy/dt = 0.03y(y - 600)$ D) $dy/dt = 0.03y(600 - y)$
- 133) A certain developing country has a population of 500,000. The yearly rate of increase of literacy among the people is proportional to the number of illiterate people in the population. Letting $f(t)$ represent the number of literate people, determine the differential equation that $f(t)$ satisfies. (Let k represent a positive constant.) 133) _____
- A) $f'(t) = 500,000(1 - f(t))$
 B) $f'(t) = 500,000 - kf(t)$
 C) $f'(t) = \frac{kf(t)}{500,000}$
 D) $f'(t) = k(500,000 - f(t))$
 E) none of these
- 134) Suppose that a substance A is converted to substance B at a rate that is proportional to the cube of the amount of B present. The amount of A and B together is always constant, say M . If $f(t) = y$ is the amount of A present at time t , then which of the following differential equation describes the situation? 134) _____
- A) $y' = ky^3; k < 0$
 B) $y' = k(M - y)^3; k < 0$
 C) $y' = ky^3; k > 0$
 D) $y' = k(M - y)^3; k > 0$
 E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 135) A certain drug is introduced into a person's bloodstream. Suppose that the rate of decrease of the concentration of the drug in the blood is directly proportional to the product of two quantities: (a) the amount of time elapsed since the drug was introduced, and (b) the square of the concentration. Let $y = f(t)$ denote the concentration of the drug in the blood at time t . Set up a differential equation satisfied by $f(t)$. Does the following accurately describe this situation: $y' = kty^2$, where k is a negative constant? Enter "yes" or "no". 135) _____
- 136) A certain chemical vaporizes when exposed to the air. Suppose $f(t)$ is the amount of chemical present. It is found that the rate of vaporization of the chemical is proportional to the amount of chemical present squared. Write a differential equation satisfied by $f(t)$. Does this equation accurately represent this situation: $y' = ky^2; k < 0$? Enter "yes" or "no". 136) _____

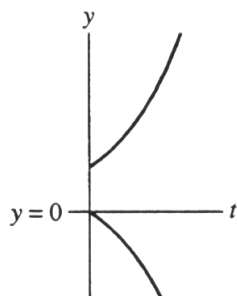
- 137) Depending on the type of soil there is a constant M that represents the maximum amount of water the soil can absorb per cubic ft. If the rate of absorption is proportional to the difference between the maximum amount of water that could be absorbed and the amount of water that has been absorbed, write a differential equation satisfied by $y = f(t)$, the amount of water in the soil at time t . Does this equation accurately describe this situation: $y' = k(M - y)$; $k > 0$?
Enter "yes" or "no". 137) _____
- 138) A sports enthusiast drinks 2 liters of water per hour. Water is eliminated from the body at a rate proportional to the amount of water in the body (due to perspiration). Write a differential equation satisfied by $f(t)$, the amount of water in the body. Does this equation accurately describe this situation: $y' = 2 - ky$?
Enter "yes" or "no". 138) _____
- 139) A patient is receiving a steady infusion of glucose. Let y denote the concentration of glucose in the blood at time t , measured in milligrams of glucose per 100 cubic centimeters of blood, and suppose that y satisfies the differential equation $y' = 48 - 0.4y$. What will be the approximate concentration of glucose in the blood after a long period of time, provided the glucose infusion is continued at the same rate?
Enter just an integer representing the number of mg glucose per 100cc blood (no units) 139) _____
- 140) After a baby whale is born, its weight gain at any time is proportional to the product of its weight and the difference between its weight and its weight at maturity. Give a differential equation satisfied by $f(t)$, its weight at time t . Does the following accurately describe this situation?
 $y' = ky(M - y)$;
 M = weight at maturity;
 $k > 0$
Enter "yes" or "no". 140) _____
- 141) The birth rate in a certain city is 2% per year and the death rate is 2.5% per year. Also, there is a net movement of population into the city at the rate of 4000 people per year. Let $N = f(t)$ be the city's population at time t . Write the differential equation satisfied by $f(t)$. Does this equation accurately represent this situation: $y' = 4000 - 0.005y$?
Enter "yes" or "no". 141) _____
- 142) An investment earns 25% interest per year. Every year \$10,000 is withdrawn in order to pay dividends to the investors. Set up a differential equation satisfied by $f(t)$, the amount of money invested at time t . Does this equation accurately describe this situation:
 $y' = 0.25y - 10,000$?
Enter "yes" or "no". 142) _____
- 143) A millionaire wants to set up a trust for her grandchild. She wants to put a lump sum of money into an account earning 10% interest. She'd like her grandchild to be able to withdraw \$100 every month for the rest of the child's life. Write a differential equation satisfied by $f(t)$, the amount of money in the account at time t . Does the equation,
 $y' = 0.1y - 100$, accurately describe this situation?
Enter "yes" or "no". 143) _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

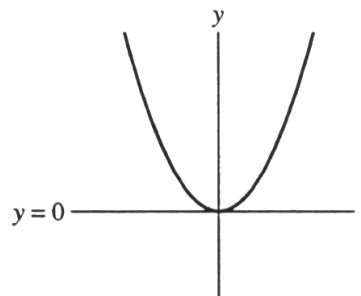
- 144) Suppose that an epidemic is spreading at a rate proportional to the square of the infected population. Let $f(t)$ be the number of infected people at time t , and suppose $y = f(t)$ satisfies a differential equation $y' = g(y)$. Which of the following sets of curves could represent solutions of $y' = g(y)$? [Hint: First determine the differential equation $y' = g(y)$.]

144) _____

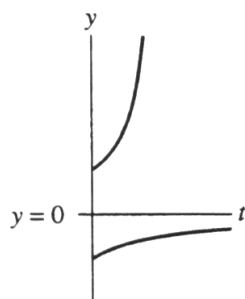
A)



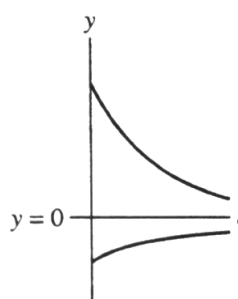
B)



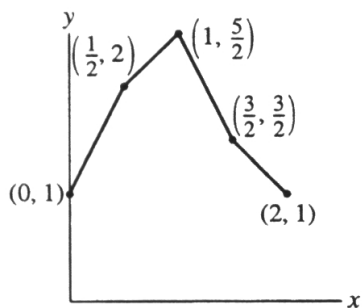
C)



D)



The following is a polygonal path obtained from Euler's method with $n = 4$ to approximate a solution $f(t)$ of a differential equation. Indicate whether the following statements are true or false:



- 145) $f'(0) = 2$

A) True

B) False

145) _____

- 146) $f(1) \approx \frac{5}{2}$

A) True

B) False

146) _____

147) $f'\left(\frac{3}{2}\right) \approx \frac{3}{2}$ 147) _____
 A) True B) False

148) $f'\left(\frac{5}{2}\right) = 1$ 148) _____
 A) True B) False

149) $f'\left(\frac{1}{2}\right) \approx 1$ 149) _____
 A) True B) False

150) $f(0) = 1$ 150) _____
 A) True B) False

151) Let $f(t)$ be the solution to of $y' = y^2t - y$, $y(0) = 1$. Which of the following statements is true? 151) _____
 A) $f'(1) = 0$
 B) $f(t)$ will be a constant solution of the differential equation.
 C) f is increasing at the origin.
 D) f is decreasing at the origin.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

152) Use Euler's method with $n = 5$ on the interval $0 \leq t \leq \frac{1}{2}$ to approximate the solution $f(t)$ to 152) _____

$y' = 3y$, $y(0) = 1$. Is the following the correct answer?

$t_0 = 0$; $y_0 = 1$

$t_1 = 0.1$; $y_1 = 1.3$

$t_2 = 0.2$; $y_2 = 1.69$

$t_3 = 0.3$; $y_3 = 2.197$

$t_4 = 0.4$; $y_4 = 2.8561$

$t_5 = 0.5$; $y_5 = 3.71293$

Enter just "yes" or "no".

153) Use Euler's method with $n = 5$ on the interval $0 \leq t \leq \frac{1}{2}$ to approximate the solution

153) _____

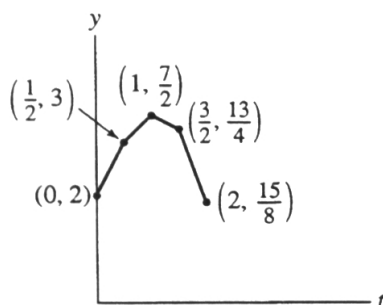
$f(t)$ to $y' = -(y + t)$, $y(0) = -1$. Is the following the correct answer?

$t_0 = 0;$ $y_0 = -1$
 $t_1 = 0.1;$ $y_1 = -0.9$
 $t_2 = 0.2;$ $y_2 = -0.82$
 $t_3 = 0.3;$ $y_3 = -0.758$
 $t_4 = 0.4;$ $y_4 = -0.7122$
 $t_5 = 0.5;$ $y_5 = -0.68098$

Enter "yes" or "no".

154) Use Euler's method with $n = 4$ on the interval $0 \leq t \leq 2$ to approximate the solution $f(t)$ to $y' = y - 4t$, $y(0) = 2$. Is the following graph accurate?

154) _____



Enter "yes" or "no".

155) Use Euler's method with $n = 5$ to approximate $f(1)$ if $y = f(t)$ satisfies the differential equation $y' = y$, $y(0) = 1$. Compare this answer with the exact value of $f(1)$. Is the following the correct answer?

155) _____

Euler's method: $f(1) \approx 2.488$; Actual value: $f(t) = e \approx 2.718$

Enter "yes" or "no".

156) Use Euler's method with $n = 4$ to approximate the solution $f(t)$ to $y' = t + y - 1$, $y(0) = 2$ for $0 \leq t \leq 2$. Estimate $f(2)$.

156) _____

Enter just a reduced fraction of form $\frac{a}{b}$.

157) Use Euler's method with $n = 4$ to approximate the solution $f(t)$ to $y' = y^2 + ty - 3$, $y(1) = 2$ for $1 \leq t \leq 3$. Estimate $f(3)$.

157) _____

Enter just a real number rounded off to two decimal places.

158) Use Euler's method with $n = 5$ to approximate the solution $f(t)$ to $y' = 5 - y$, $y(0) = 1$ for $0 \leq t \leq 1$. Estimate $f(1)$.

158) _____

Enter just a real number rounded off to two decimal places.

- 159) Use Euler's method with $n = 2$ to approximate the solution $f(t)$ to $y' = 2y - t$, $y(0) = 1$. Estimate $f(1)$. 159) _____
 Enter just a reduced fraction of form $\frac{a}{b}$.
- 160) Use Euler's method with $n = 3$ to approximate the solution $f(t)$ to $y' = 9t + y^2$, $y(0) = 0$. Estimate $f(1)$. 160) _____
 Enter just a reduced fraction of form $\frac{a}{b}$.
- 161) Let $f(t)$ be the solution of $y' = ty + 0.2$, $y(0) = 3$. Use Euler's method on $0 \leq t \leq 1$ with $n = 2$ to estimate $f(1)$. Is $f(1) \approx 3.875$ the correct answer? 161) _____
 Enter "yes" or "no".

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 162) Suppose that $y = f(t)$ satisfies the differential equation $y' = t^2 + 3y$, $f(0) = 2$. If Euler's method with $n = 10$ is used to construct an approximation $p(t)$ to $f(t)$ for $0 \leq t \leq 1$, find $p(0.1)$. 162) _____
 A) 2.6
 B) 3.1
 C) 6.0
 D) 2.2
 E) none of these
- 163) Let $f(t)$ be the solution of $y' = y^2t + y + e^t$, $f(0) = 2$. If Euler's method with $n = 4$ is used to approximate $f(t)$ for $0 \leq t \leq 2$, find $f\left(\frac{1}{2}\right)$. 163) _____
 A) $3 + e^2$
 B) $\frac{y^2}{2} + y + e^{1/2}$
 C) $2(4 + \sqrt{e})$
 D) $\frac{7}{2}$
 E) none of these

Solve the problem.

- 164) A population of ants, y , living in a colony grows at a rate $\frac{dy}{dt} = 0.05y - 0.1y^{1/2}$, 164) _____
 where t is time in weeks. The initial population is 1000 insects. Using Euler's method with $h = 1$, what is the number of ants after 4 weeks?
 A) 1169 ants B) 1212 ants C) 1174 ants D) 1201 ants

Answer Key

Testname: UNTITLED10

- 1) 1
- 2) $\frac{7}{10}$
- 3) B
- 4) 6
- 5) 0
- 6) e^2
- 7) E
- 8) C
- 9) D
- 10) C
- 11) B
- 12) B
- 13) C
- 14) B
- 15) C
- 16) C
- 17) A
- 18) D
- 19) A
- 20) D
- 21) yes
- 22) yes
- 23) yes
- 24) yes
- 25) yes
- 26) yes
- 27) no
- 28) yes
- 29) yes
- 30) 0, 2
- 31) D
- 32) A
- 33) C
- 34) A
- 35) B
- 36) D
- 37) D
- 38) B
- 39) D
- 40) D
- 41) D
- 42) A

Answer Key

Testname: UNTITLED10

- 43) B
- 44) B
- 45) no
- 46) yes
- 47) no
- 48) yes
- 49) yes
- 50) yes
- 51) no
- 52) A
- 53) A
- 54) C
- 55) C
- 56) D
- 57) yes
- 58) D
- 59) A
- 60) D
- 61) D
- 62) C
- 63) A
- 64) A
- 65) C
- 66) C
- 67) D
- 68) B
- 69) B
- 70) D
- 71) C
- 72) A
- 73) C
- 74) yes
- 75) yes
- 76) yes
- 77) yes
- 78) D
- 79) A
- 80) C
- 81) C
- 82) C
- 83) C
- 84) C
- 85) A

Answer Key

Testname: UNTITLED10

- 86) D
- 87) D
- 88) 81
- 89) 149
- 90) A
- 91) C
- 92) A
- 93) yes
- 94) 7062
- 95) $T = -50e^{-0.045} + 100$
- 96) A
- 97) C
- 98) C
- 99) B
- 100) D
- 101) A
- 102) B
- 103) A
- 104) A
- 105) B
- 106) B
- 107) B
- 108) B
- 109) yes
- 110) yes
- 111) yes
- 112) yes
- 113) yes
- 114) yes
- 115) yes
- 116) yes
- 117) yes
- 118) yes
- 119) yes
- 120) D
- 121) A
- 122) C
- 123) D
- 124) A
- 125) A
- 126) A
- 127) A
- 128) B

Answer Key

Testname: UNTITLED10

129) yes

130) yes

131) D

132) D

133) D

134) B

135) yes

136) yes

137) yes

138) yes

139) 120

140) yes

141) yes

142) yes

143) yes

144) C

145) A

146) A

147) B

148) B

149) A

150) A

151) D

152) yes

153) yes

154) yes

155) yes

156) $\frac{65}{8}$

157) 3198.47

158) 3.69

159) $\frac{15}{4}$

160) $\frac{10}{3}$

161) yes

162) A

163) D

164) D