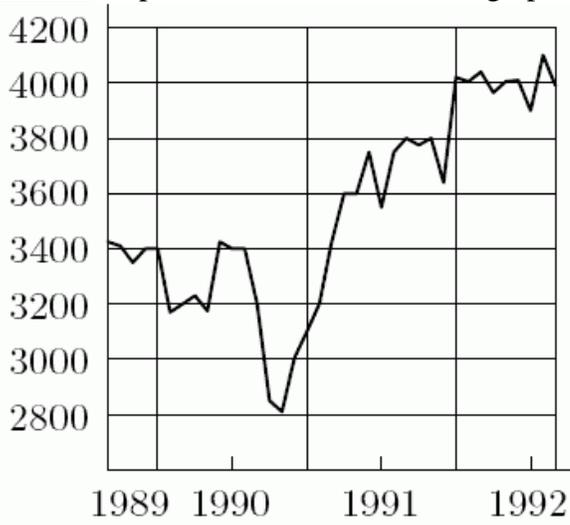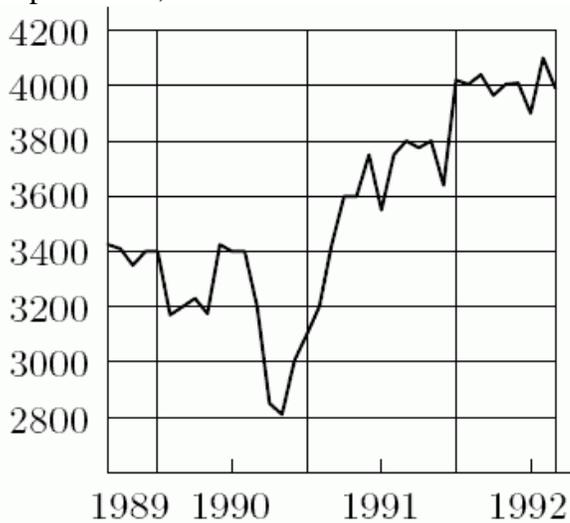1. The empirical function $W = f(t)$, given in the graph below, comes from the <u>Wall Street Journal</u>, September 4, 1992. From the graph, determine the domain of the function.

A)   September 1989 to August 1992   C)   0 to 4200
B)   2810 to 4090   D)   January 1988 to January 1993
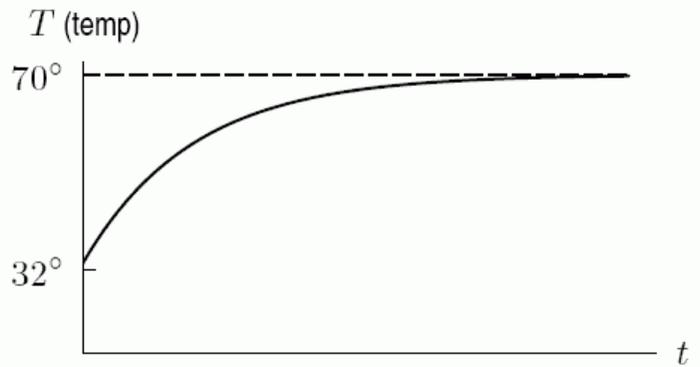Ans: A   difficulty: easy   section: 1.1

2. The curve $W = f(t)$, given in the graph below, comes from the <u>Wall Street Journal</u>, September 4, 1992. W is a function.

Ans: True   difficulty: easy   section: 1.1

3. Draw a graph which accurately represents the temperature of the contents of a cup left overnight in a room. Assume the room is at $70°$ and the cup is originally filled with water slightly above the freezing point.
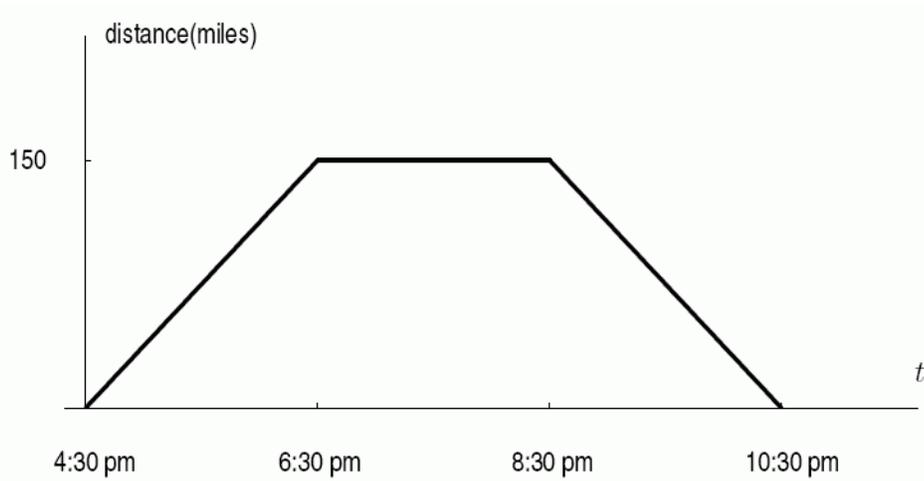   Ans:

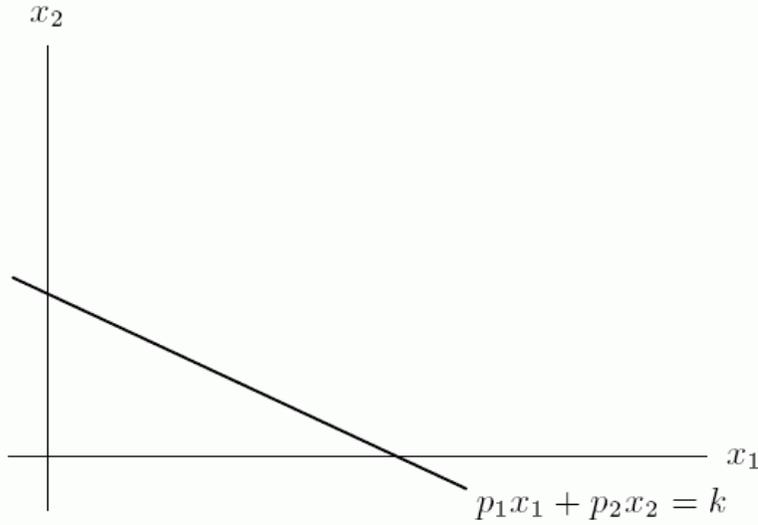
   difficulty: easy   section: 1.1

4. Suppose the Long Island Railroad train from East Hampton to Manhattan leaves at 4:30 pm and takes two hours to reach Manhattan.   It waits two hours at the station and then returns, arriving back in East Hampton at 10:30 pm. Draw a graph representing the distance of the train from the Farmingdale station in East Hampton as a function of time from 4:30 pm to 10:30 pm. The distance from East Hampton to Manhattan is 150 miles.
   Ans:

   difficulty: medium   section: 1.1

5. Suppose we buy quantities x_1 and x_2, respectively, of two goods. The following graph shows the budget constraint $p_1 x_1 + p_2 x_2 = k$, where p_1 and p_2 are the prices of the two goods and k is the available budget. If the budget is doubled, but prices remain the same, what is the x_2 – intercept of the new budget constraint?

x_2

x_1

$p_1 x_1 + p_2 x_2 = k$

A) $\dfrac{2k}{p_1}$ B) $\dfrac{2k}{p_2}$ C) $\dfrac{k}{2p_1}$ D) $\dfrac{k}{2p_2}$
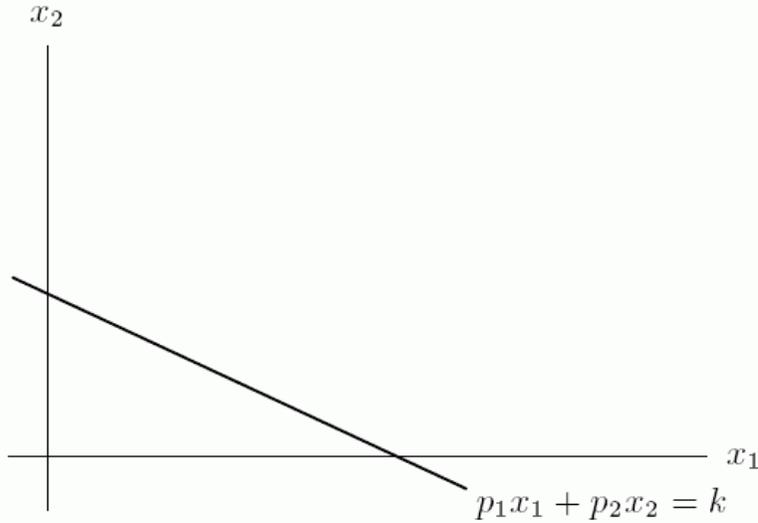
Ans: B   difficulty: medium   section: 1.1

6. Suppose we buy quantities x_1 and x_2, respectively, of two goods. The following graph shows the budget constraint $p_1 x_1 + p_2 x_2 = k$, where p_1 and p_2 are the prices of the two goods and k is the available budget. If the price of the first good is doubled, but the other values are unchanged, what is the x_2 – intercept of the new budget constraint?

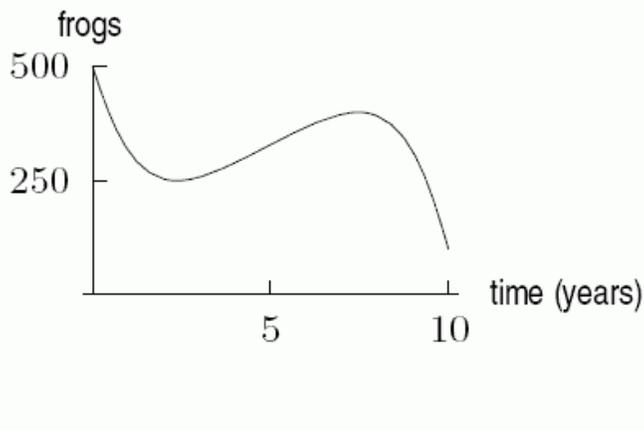A) $\dfrac{2k}{p_2}$ B) $\dfrac{2k}{p_1}$ C) $\dfrac{k}{p_2}$ D) $\dfrac{k}{p_1}$

Ans: C   difficulty: medium   section: 1.1

7. A function is linear for $x \le 2$ and also linear for $x \ge 2$. This function has the following values: $f(-4) = 8$; $f(2) = 5$; $f(4) = 11$. Find formula(s) (or equation(s)) which describe this function.

A) $f(x) = \begin{cases} \dfrac{1}{2}x+4, & x \le 2 \\ -3x+11, & x \ge 2 \end{cases}$

C) $f(x) = \begin{cases} -2x+9, & x \le 2 \\ 3x-1, & x \ge 2 \end{cases}$

B) $f(x) = \begin{cases} 2x+6, & x \le 2 \\ \dfrac{1}{3}x-11, & x \ge 2 \end{cases}$

D) $f(x) = \begin{cases} -\dfrac{1}{2}x+6, & x \le 2 \\ 3x-1, & x \ge 2 \end{cases}$

Ans: D   difficulty: medium   section: 1.1

8. A pond has a population of 500 frogs. Over a ten-year period of time the number of frogs drops quickly by 20%, then increases slowly for 5 years before dropping to almost zero. Does the following graph accurately represent the number of frogs in the pond over the ten-year period of time?

Ans: no
difficulty: easy   section: 1.1

9. Suppose a 40 $^{\circ}F$ container of water is placed in the freezer overnight. The next morning, it is put on the counter in a 70 $^{\circ}F$ room and then at the end of the day heated to the boiling point.   What is the range of the function?
A)   From 40 $^{\circ}F$ to 70 $^{\circ}F$.
B)   From the middle of the night until the middle of the next day.
C)   From the first evening until the end of the next day.
D)   From 32 degrees F to 212 degrees F.
Ans: D   difficulty: easy   section: 1.1

10. A school library opened in 1980. In January, 2000 they had 25,000 books. One year later, they had 25,420 books. Assuming they acquire the same number of books at the start of each month, how many books did they have in January, 2003?
Ans: 26,260
difficulty: easy   section: 1.1

11. A school library opened in January of 1980. In January, 2000 they had 35,000 books. One year later, they had 35,490 books. Assuming they acquire the same number of books at the start of each month, how many books did they have in July of 1980?
Ans: 25,445
difficulty: medium   section: 1.1

12. A school library opened in 1980. In January, 2000 they had 25,000 books. One year later, they had 25,470 books. Assuming they acquire the same number of books at the start of each month, the linear formula for the number of books, N, in the library as a function of the number of years t the library has been open is given by $N =$ _____ + _____ t.
Part A: 15,600
Part B: 470
difficulty: easy   section: 1.1

13. A school library opened in 1980. In January, 2000 they had 40,000 books. One year later, they had 40,410 books. Assume that they acquire the same number of books at the start of each month.   If you graph the function with domain 1980-2010, describe the *y*-intercept of the graph in the context of the problem.
A)   The number of books the library had in 1980
B)   The number of books the library will have in 2010
C)   The year the library had 40,000 books
D)   The year the library had no books
Ans: A   difficulty: easy   section: 1.1

14. Write a formula representing the function that says: The circumference of a circle is proportional to the diameter of the circle.
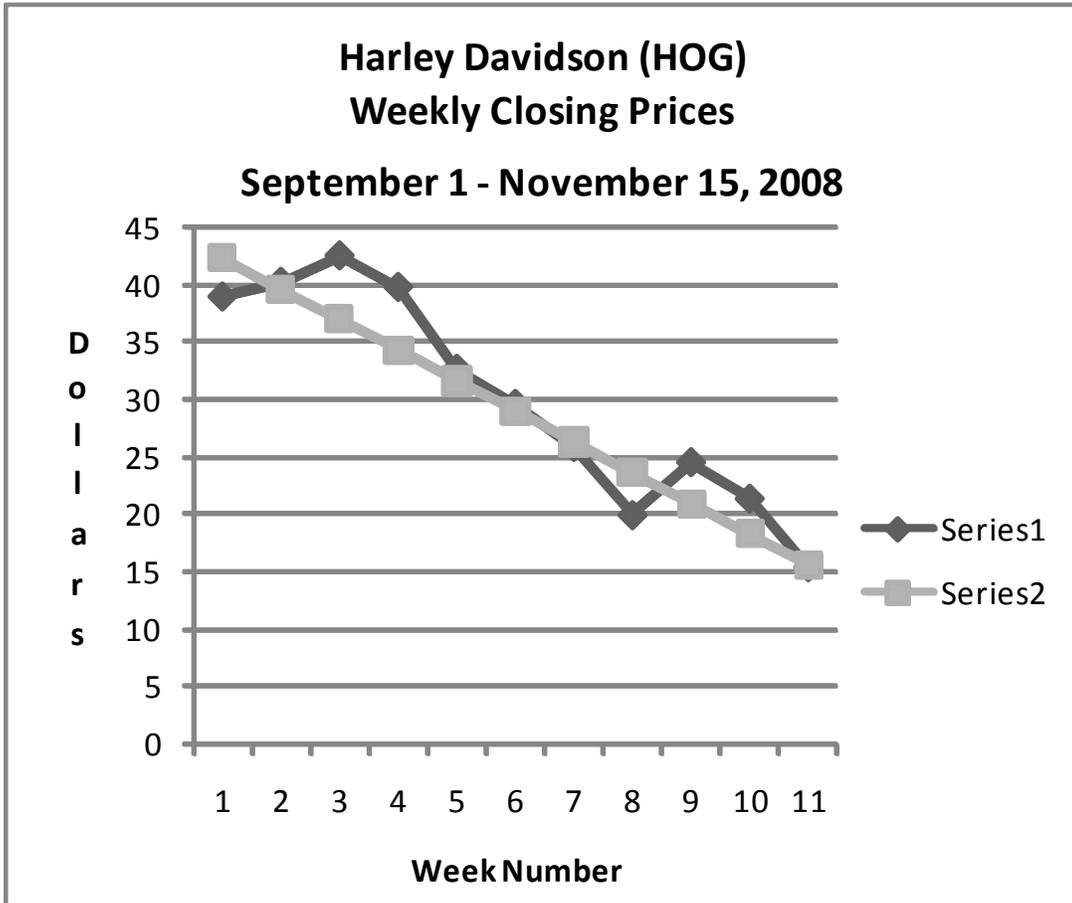Ans: C=(pi)d
difficulty: easy   section: 1.1

15. The illumination, I, of a candle is inversely proportional to the square of its distance, d, from the object it illuminates.   Write a formula that expresses this relationship.
Ans: $I = \dfrac{k}{D^2}$
difficulty: easy   section: 1.1

16. Harley Davidson (ticker symbol HOG) stock prices dropped sharply in late 2008. Series 1 in the graph below shows the actual prices at the end of each week. The trend over time is approximately linear; and the graph of a possible linear model is given by Series 2. Based on the data given, find the linear model and use it to approximate the stock's price on November 30, 2008, assuming the current trend continued.

Harley Davidson (HOG)
Weekly Closing Prices

September 1 - November 15, 2008

Ans: $y = -2.5x + 45$ and $12.00, but answers will vary slightly.
difficulty: easy   section: 1.1

17. If $f(x) = e^{x+2}$, find and simplify the difference quotient $\dfrac{f(6+h) - f(6)}{h}$.

A) 1   B) $e^8 - e^6$ C) $\dfrac{e^{x+h}}{h}$ D) $\dfrac{e^8(e^h - 1)}{h}$

Ans: D   difficulty: medium   section: 1.1

18. The following table defines three functions for $0 \le x \le 4$. The function y_1 is most likely

x	y_1	y_2	y_3
0	4.65	4.65	4.65
1	7.44	5.11	3.79
2	11.90	5.97	2.93
3	19.05	9.55	2.07
4	30.47	15.28	1.21

A) exponential   B) neither   C) linear
Ans: A   difficulty: medium   section: 1.2

19. The following table defines three functions for $0 \le x \le 4$. One is linear, one is exponential, and one is neither. The equation for the linear function is $y =$ _____ $x +$ _____ .

x	y_1	y_2	y_3
0	4.35	4.35	4.35
1	6.96	5.11	3.59
2	11.14	5.97	2.83
3	17.82	9.55	2.07
4	28.51	15.28	1.31

Part A: –0.76
Part B: 4.35
difficulty: medium   section: 1.2

20. The following table defines three functions for $0 \le x \le 4$. One is linear, one is exponential, and one is neither. The equation for the exponential function is $y =$ __A__ (__B__)^x.

x	y_1	y_2	y_3
0	4.65	4.65	4.65
1	7.90	5.11	3.79
2	13.44	5.97	2.93
3	22.85	9.55	2.07
4	38.84	15.28	1.21

Part A: 4.65
Part B: 1.7
difficulty: medium   section: 1.2

21. The following table of data is either linear or exponential. If it is linear, give values of a and b such that $y = ax + b$. If it is exponential, give values of a and b such that $y = a(b)^x$.

x	0	0.50	1.00	1.50	2.00
y	0.94	2.42	3.90	5.38	6.86

$a = $ _____; $b = $ _____
Part A: 2.96
Part B: 0.94
difficulty: medium   section: 1.2

22. On the $x-$interval between E and F, the function graphed below is:

Mark all that apply.
A) increasing   B) decreasing   C) concave up   D) concave down
Ans: B, C   difficulty: easy   section: 1.2

23. A bar of soap starts out at 175 grams. What is the formula for the quantity S grams of soap remaining after t days if the decrease is 10 grams per day?
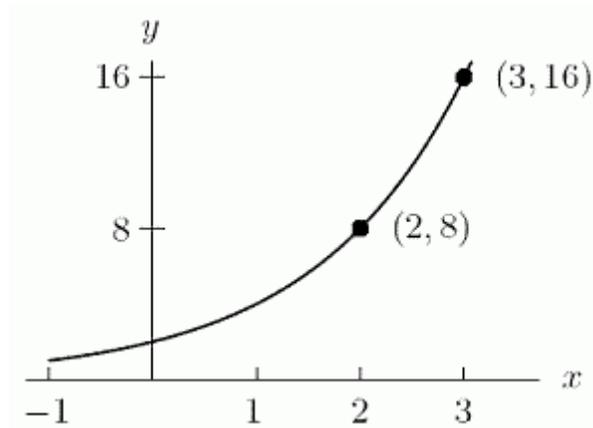Ans: $S = 175 - 10t$
difficulty: easy   section: 1.2

24. A bar of soap starts out at 100 grams. What is the formula for the quantity S grams of soap remaining after t days if the decrease is 15% per day?
Ans: $S = 100(0.85)^t$
difficulty: easy   section: 1.2

25. Give a possible formula for the function in the following figure:

Ans: $y = 2(2^x)$
difficulty: medium   section: 1.2

26. The following functions represent exponential growth or exponential decay.   Mark the one(s) that represent exponential growth.
    A) $P = 10(1.1)^t$ B) $Q = 4.4e^{0.05t}$ C) $S = 5e^{-0.25t}$ D) $R = 12(0.8)^t$
    Ans: A, B   difficulty: easy   section: 1.2

27. Joe invested $20,000 in the stock market, and Sam invested $20,000. Joe's investment increased in value by 5% per year for 10 years. Sam's investment decreased in value by 10% for 5 years and then increased by 10% for the next 5 years.   At the end of the 10 years, whose investment was worth more, Joe's or Sam's?
    Ans: Joe's
    difficulty: medium   section: 1.2

28. A bakery has 300 lbs of flour. If they use 10% of the available flour each day, how much do they have after 20 days?   Round to the nearest pound.
    Ans: 36
    difficulty: easy   section: 1.2

29. In the book <u>One Grain of Rice</u>, a girl receives a reward that starts with one grain of rice on day one, two grains on day two, four on day three and eight on day four.   Each day, she receives double the number of grains of rice.   How many grains of rice does she receive on the 30th day?
    A) 900   B) 1,073,741,824   C) 4,640,650,289   D) 60
    Ans: B   difficulty: easy   section: 1.2

30. Lisinopril is an ACE inhibitor derived from the venom of a Brazilian pit viper frequently used in the treatment of hypertension. Because of Lisinopril's relatively long half life of 12 hours, patients need to take a dose just once per day. A patient takes his first dose, 20 mg, at 6 pm on Saturday.
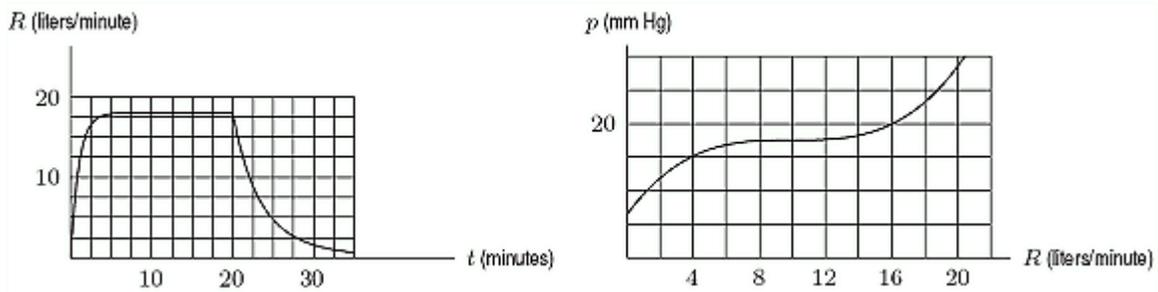   a) How many hours does it take for the amount of Lisinopril in the patient's body to decrease to 16 mg? Round to two decimal places.
    b) How many milligrams remain in the patient's body right before he takes his next 20 mg at 6 pm on Sunday? Round to two decimal places
    Ans: a) 3.86 hours
          b) 5.00 milligrams
    difficulty: medium   section: 1.2

31. One of the graphs below shows the rate of flow, R, of blood from the heart in a man who bicycles for twenty minutes, starting at $t = 0$ minutes. The other graph shows the pressure, p, in the artery leading to a man's lungs as a function of the rate of flow of blood from the heart. Estimate $p(R(15))$.

    A) 26.5   B) 23   C) 17.5   D) 13
    Ans: B   difficulty: easy   section: 1.3

32. Given the function $f(x) = e^{-x^2/3}$ find $g(x)$ and $h(x)$ such that $f(x) = g(h(x))$

    A) $g(x) = e^{-x^2}$ and $h(x) = x/3$ C) $g(x) = e^x$ and $h(x) = -x^2/3$

    B) $g(x) = x/3$ and $h(x) = e^{-x^2}$ D) $g(x) = -x^2/3$ and $h(x) = e^x$
    Ans: C   difficulty: easy   section: 1.3

33. Given the function $m(z) = z^2$, find and simplify $m(z-h) - m(z)$.
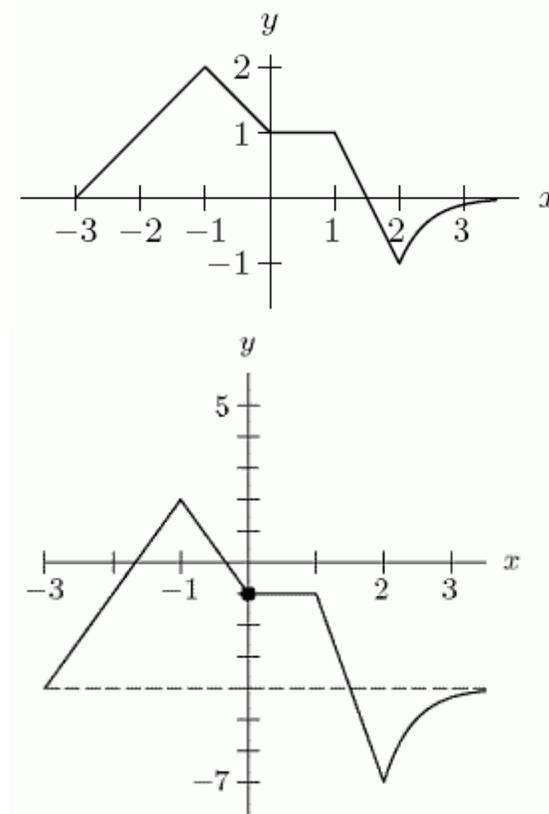
    Ans: $-2zh + h^2$
    difficulty: easy   section: 1.3

34. Given the function $y = f(x) = \dfrac{8}{5x-8}$, give a formula for the inverse function of $f(x)$.

    Ans: $\dfrac{8}{5y} + \dfrac{8}{5}$ or $\dfrac{8+8y}{5}$

    difficulty: medium   section: 1.3

35. The graph of $y = f(x)$ is shown in the first figure. What graph is shown in the second figure?

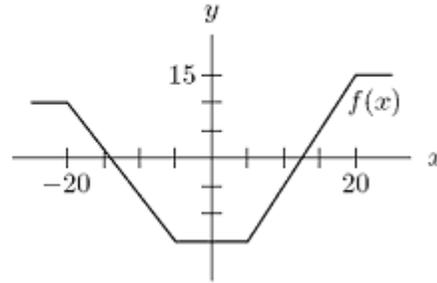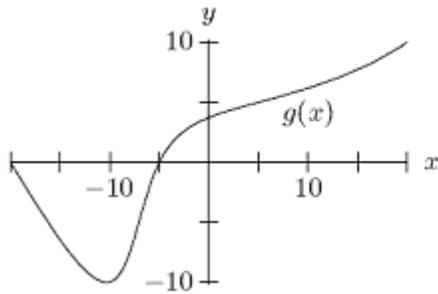   A) $y = 3f(x) - 4$ C) $y = 3f(x) + 4$
    B) $y = -3f(x) - 4$ D) $y = -3f(x) + 4$
    Ans: A   difficulty: medium   section: 1.3

36. Given the graphs of $y = g(x)$ and $y = f(x)$ in the following figure, estimate $g(f(5))$.

Ans: –7
difficulty: easy   section: 1.3

37. Write an equation for the graph obtained by shifting the graph of $y = x^3$ vertically upward by 3 units, followed by vertically stretching the graph by a factor of 2 and reflecting the graph across the $x-$ axis.
Ans: $y = -2x^3 - 6$ or $y = -2(x^3 + 3)$
difficulty: medium   section: 1.3

38. If the graph of $y = f(x)$ is shrunk vertically by a factor of 1/2, then shifted vertically by 4 units, then stretched vertically by a factor of 2, the resulting graph the same as the original graph.
Ans: False   difficulty: medium   section: 1.3

39. Given the information in the table and the following conditions, find $h(1)$.
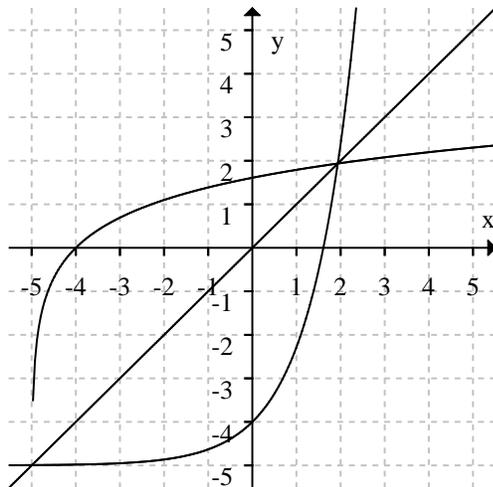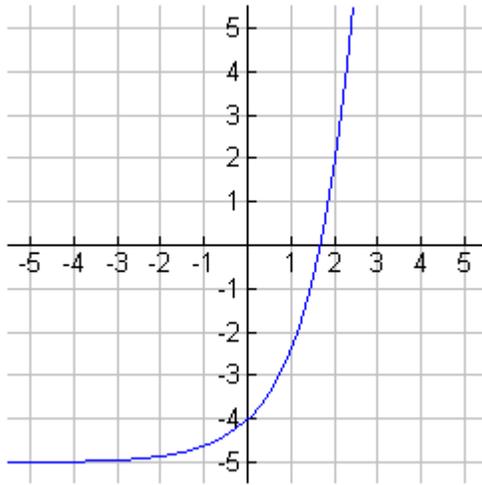- $f(x)$ is even.
- $g(x)$ is odd.
- $h(x) = g(x)^2 - f(x)$

x	$f(x)$	$g(x)$
-3	7	22
-2	2	10
-1	−1	1
0	−2	2

Ans: 2
difficulty: medium   section: 1.3

40. Is the function graphed in the following figure invertible?

Ans: no
difficulty: easy   section: 1.3

41. Given the function $q(x) = x^3$, which of the following is equivalent to $q(2x+a)+q(x)$?

A) $8x^3 + a^3$
C) $8x^3 + 12x^2a + 6xa^2 + a^3$
B) $9x^3 + a^3$
D) $9x^3 + 12x^2a + 6xa^2 + a^3$
Ans: D   difficulty: easy   section: 1.3

42. Given the function $q(x) = x^3$, which of the following is equivalent to $q(x^5)+q(x+a)$?

A) $x^8 + x^3 + 3x^2a + 3xa^2 + a^3$
C) $x^8 + x^3 + a^3$
B) $x^{15} + x^3 + 3x^2a + 3xa^2 + a^3$
D) $x^{15} + x^3 + a^3$
Ans: B   difficulty: easy   section: 1.3

43. Is the function $f(x) = x^5 + x^8 + x^9$ odd, even , or neither?
Ans: neither
difficulty: easy   section: 1.3

44. The cost of shipping r kilograms of material is given by the function $C = f(r) = 175 + 6r$.
Find a formula for the inverse function.
Ans: $r=(C - 175)/6$
difficulty: easy   section: 1.3

45. For $g(x) = 3x^2 + 7x$ and $h(x) = 3x - 1$, $h(g(x)) = $ __A__ $x^2 + $ __B__ $x + $ __C__.
Part A: 9
Part B: 21
Part C: –1
difficulty: easy   section: 1.3

46. The graph of a function y = f(x) is given below.   Sketch a graph of it's inverse function on the same axes.

Ans:
difficulty: medium   section: 1.3

47. The following describes a function that must be invertible: $R(t)$ is the revenue earned by a shoe shop on the tth th day of the year.
Ans: False   difficulty: easy   section: 1.3

48. The following describes a function that must be invertible: $N(p)$ is the number of words on page p of a science fiction novel.
Ans: False   difficulty: easy   section: 1.3

49. The following describes a function that must be invertible: A woman takes a tablet of 10 mg of a common allergy medicine with a half life of approximately 8 hours. $L(t)$ gives the amount of medicine in her system at time t where $0 < t < 24$.
    Ans: True   difficulty: easy   section: 1.3

50. Suppose $f(x)$ and $g(x)$ are both increasing functions.
    a) Must $f(g(x))$ be an increasing function? If so, explain why. If not, give an example.
    b) Must $f(g(x))$ be invertible? If so explain why. If not, give an example.
    Ans: Answers will vary. One example is given below.
         a) Yes. The function $g(x)$ is increasing, so the inputs to $f(x)$ are increasing. Since f is an increasing function for inputs that are increasing, $f(g(x))$ must also be increasing.
         b) Yes. Since $f(g(x))$ is increasing, it is one-to-one and hence invertible.
    difficulty: hard   section: 1.3

51. A one-to-one function is described in words as follows. Take x and multiply by 4. Then add 3 and cube the result. Describe the inverse function in words.
    Ans: Take the cube root of x, then subtract 3, and finally divide by 4.
    difficulty: medium   section: 1.3

52. In 1909, the Danish biochemist Sören Peter Lauritz Sörensen (1868-1939) introduced the pH function as a measure of the acidity of a chemical substance: $\mathrm{pH} = f([H+]) = -\log_{10}[H^+]$, where $[H+]$ is the molecular concentration of hydrogen ions (moles per liter, M). Sörensen determined that, for $0 < \mathrm{pH} < 7$, the substance is an acid; when $\mathrm{pH} = 7$, the substance is neutral; and for $\mathrm{pH} > 7$, the substance is a base or is said to be alkaline. The $[H+]$ for the worst known instance of acid rain is 3.98×10^{-3} M. Find the pH of the worst known instance of acid rain. Round to 1 decimal place.
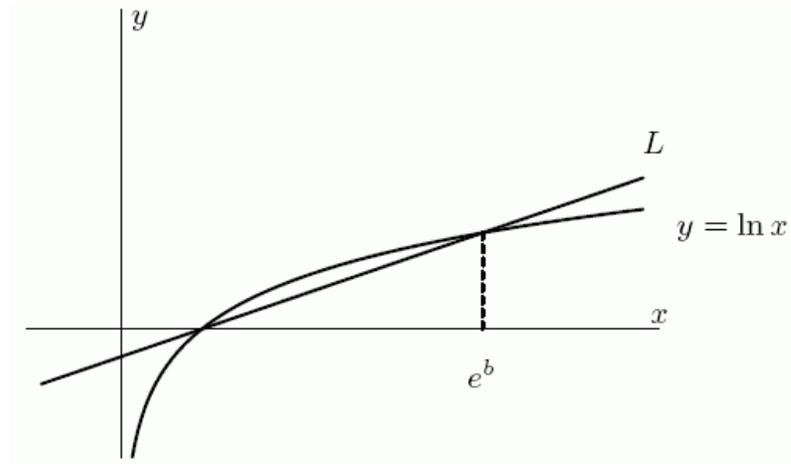   Ans: 2.4
    difficulty: easy   section: 1.4

53. Suppose that $N(t) = 100{,}000{,}000 \cdot 2^{t/10}$ gives the population of a certain country t years after a census was taken. A historian has a collection of documents that are not dated, but do refer to the population of this country at several times. In order to help the historian date these documents, find the inverse function for the function N. Round to whole numbers.
    Ans: $t = 14 \log N - 266$
    difficulty: medium   section: 1.4

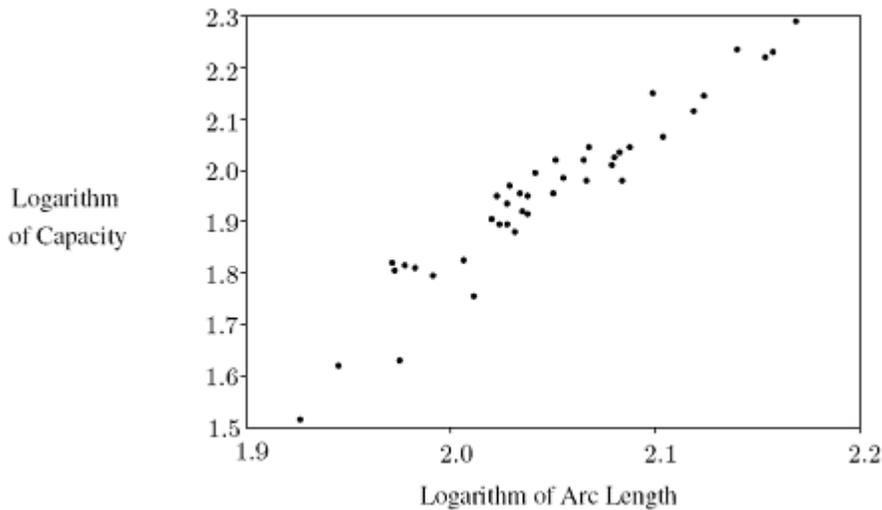54. Find the equation for the line L shown below.

A) $y = b(x+1-e^b)$

C) $y = \dfrac{b}{b-e^b}(x-e^b)$

B) $y = \dfrac{b}{e^b -1}(x-1)$

D) $y = \dfrac{b}{e^b -e}(x-e)$

Ans: B   difficulty: medium   section: 1.4

55. Here are some data from a Scientific American article on Old World monkeys. From the data presented, give an approximate formula for C = cranial capacity (in cm^3) as a function of A = arc length of skull (in cm). Round to one decimal place.

Cranial Capacity of contemporary Old World monkeys is related to arc length of skull as shown.

Ans: $C = A^{2.3}(501)$

difficulty: hard   section: 1.4

56. Suppose there is an initial population of 100 rabbits on *Prosperity Island*. Assuming that the rabbits have more than enough of everything they need to live prosperously, we might expect the population to grow exponentially. If so, find a formula for $P(t)$, the number of rabbits on *Prosperity Island* at time t, given that after one year there are 140 rabbits on the island. (Assume t is in years.) Use this formula to determine how many years it will be before there are 400 rabbits on *Prosperity Island*. Round to 1 decimal place.

    Ans: $P(t) = 100(1.4)^t$, $\quad P(400) = 4.1$

    difficulty: medium   section: 1.4

57. *Cramped Quarters Island* is a tiny island which, although able to support a *limited* population of rabbits, doesn't have enough space or food supplies to support unlimited exponential growth. It is suggested that if $Q(t)$ = population of rabbits on *Cramped Quarters Island* at time t, then the quantity $(800 - Q(t))$ will be an exponentially decaying function of t. Given that there were 400 rabbits at time $t = 0$, and 500 rabbits one year later, find the general formula for $Q(t)$, the population of rabbits on *Cramped Quarters Island* at time t and use it to determine the rabbit population after 3 years (to the nearest rabbit).

    Ans: $Q(t) = 400(1.25)^t$, $\quad Q(3) = 631$

    difficulty: medium   section: 1.4

58. An exponentially decaying substance was weighed every hour and the results are given below. If $Q = Q_0 e^{-kt}$ gives the weight of the substance Q, at time t hours since 9:00 am, then $Q_0 = \_\_\_\_\_$ and $k = \_\_\_\_\_$. Round k to 2 decimal places.

Time	Weight (in grams)
9am	12
10am	10.643
11am	9.440
12 noon	8.372
1 pm	7.425

    Part A: 12
    Part B: 0.12
    difficulty: medium   section: 1.4

59. An exponentially decaying substance was weighed every hour and the results are given below.   What is the approximate half-life, in hours, of the substance?   Round to 1 decimal place.

Time	Weight (in grams)
9am	16
10am	14.623
11am	13.364
12 noon	12.214
1 pm	11.163

Ans: 7.7 hours
difficulty: medium   section: 1.4

60. The number of bacteria in milk grows at a rate of 10% per day once the milk has been bottled. When the milk is put in the bottles, it has an average bacteria count of 500 million per bottle.   Suppose milk cannot be safely consumed if the bacteria count is greater than 2.9 billion per bottle. How many days will the milk be safe to drink once it has been bottled?   Round to the nearest integer.
Ans: 18
difficulty: medium   section: 1.4

61. *Cramped Quarters Island* is a tiny island which, although able to support a *limited* population of rabbits, doesn't have enough space or food supplies to support unlimited exponential growth. It is suggested that if $Q(t)$ = population of rabbits on *Cramped Quarters Island* at time t, then the quantity $(800 - Q(t))$ will be an exponentially decaying function of t.   If at time $t = 0$ there were 250 rabbits, and the population was increasing at an instantaneous rate of   100 rabbits per year, give the general formula for $Q(t)$.

Ans: $Q(t) = 800 - 250e^{-0.4t}$
difficulty: medium   section: 1.4

62. In 1992, the Population Crisis Committee wrote:
Large cities in developing countries are growing much faster than cities in the industrialized world ever have. London, which in 1810 became the first industrial city to top 1 million, now has a population of 11 million. By contrast, Mexico City's population stood at only a million just 50 years ago and now is 20 million.
Assume that the instantaneous percentage growth rates of London and Mexico City were constant over the last two centuries.   How many times greater is Mexico City's percentage growth rate than London's (to one decimal place)?
Ans: 4.5
difficulty: hard   section: 1.4

63. $6e^{\ln(a^3)}$ is equivalent to $6a^3$
Ans: True   difficulty: easy   section: 1.4

64. $8 \ln b^e$ is equivalent to $8b$
    Ans: False   difficulty: easy   section: 1.4

65. Solve $6^x = 2$ for x. Round to 2 decimal places.
    Ans: 0.39
    difficulty: easy   section: 1.4

66. Solve $6e^{3x} = 11e^{8x}$ for x. Round to 3 decimal places.
    Ans: –0.121
    difficulty: easy   section: 1.4

67. What is the doubling time in years of prices which are increasing by 8% a year? Round to two decimal places.
    Ans: 9.01
    difficulty: medium   section: 1.4

68. If the size of a bacteria colony doubles in 8 hours, how many hours will it take for the number of bacteria to be 11 times the original amount? Round to 2 decimal places.
    Ans: 27.68
    difficulty: medium   section: 1.4

69. Tornados are classified in several ways. A tornado's classification on the Fujita Scale as F1 through F5 is most commonly cited. Another classification of tornados is by path length, given by the formula $Pl = 2\log(L) + 1$ where L is the length of the tornado's path in miles. The Binger, Oklahoma tornado of 1981 was an F4 (originally thought to be an F5) whose path was 16 miles in length. What was its Pl classification?
    A) P1   B) P2   C) P3   D) P4   E) P5
    Ans: C   difficulty: easy   section: 1.4

70. What is the period of the function $c(x) = \cos x + 0.5\cos 5x$?
    A) 2π B) π C) $\dfrac{\pi}{5}$ D) The function is not periodic
    Ans: A   difficulty: medium   section: 1.5

71. At the point $x = -\pi$, the function $c(x) = \cos 2x + 0.4\cos 5x$ has
    A) A local maximum   B) A local minimum   C) Neither
    Ans: A   difficulty: medium   section: 1.5

72. At high tide, the water level is 14 feet below a certain pier. At low tide the water level is 26 feet below the pier. Assuming sinusoidal behavior, sketch a graph of $y = f(t) =$ the water level, relative to the pier, at time t (in hours) if at $t = 0$ the water level is -20 feet and falling, until it reaches the first low tide at $t = 3$. Based on your sketch and the information provided above, the formula for $f(t)$ can be written

$$f(t) = \underline{\quad\quad} \sin(\underline{\quad\quad} \pi t) + \underline{\quad\quad} .$$

Part A: –6
Part B: 1/6
Part C: –20
difficulty: medium   section: 1.5

73. In nature, the population of two animals, one of which preys on the other (such as foxes and rabbits) are observed to oscillate with time, and are found to be well approximated by trigonometric functions. The population of foxes is given by the graph below.   What is the amplitude?

Ans: 300
difficulty: easy   section: 1.5

74. One of the functions below is a quadratic, one is a cubic, and one is a periodic function. Which one is periodic?

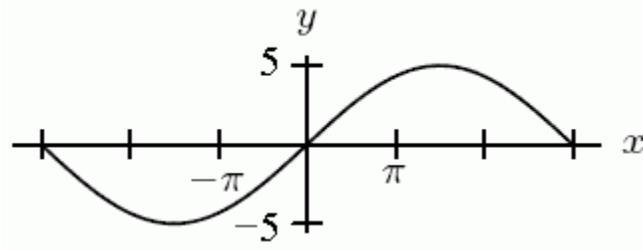| x	$f(x)$
0.2	−0.42
0.4	−0.65
0.6	0.96
0.8	−0.15
1.2	0.84

x	$g(x)$
1.3	0.41
1.7	0.81
2.5	0.65
3.0	−0.10
3.5	−1.35

x	$h(x)$
0.5	−1.13
1.2	0.13
1.8	0.03
2.0	0.00
2.2	0.05

A) $g(x)$ B) $f(x)$ C) $h(x)$
Ans: B   difficulty: medium   section: 1.5

75. Give a formula for the following sinusoidal function as a transformation of $f(x) = \sin(x)$

Ans: $y = 5\sin\left(\dfrac{x}{3}\right)$

difficulty: medium   section: 1.5

76. Temperatures in Town A oscillate daily between 30° F at 4am and 60° F at 4pm. The temperature in Town B is consistently 10° F colder than in Town A.   What does the

formula $H = -15\cos\left(\dfrac{\pi}{12}t\right) + 45$ represent?

A)   Temperature in Town A, in terms of time where time is measured in hours from 4am.
B)   Temperature in Town B, in terms of time where time is measured in hours from 4am.
C)   Temperature in Town A, in terms of time where time is measured in hours from 4pm.
D)   Temperature in Town B, in terms of time where time is measured in hours from 4pm.
Ans: A   difficulty: medium   section: 1.5

77. What is the period of the function $g(t) = 9 + 8\cos \pi t$
Ans: 2
difficulty: easy   section: 1.5

78. Consider the functions $f(x) = 5 + \sin 3x$ and $g(x) = 3\sin x$. Which has a larger vertical intercept?
    A) $f(x)$ B) $g(x)$
    Ans: A   difficulty: easy   section: 1.5

79. Sketch a well-labeled graph of a periodic function such that :
    -- $f(0) = 850$
    -- the period is 12
    -- the amplitude is 550.
    Then write a few sentences illustrating how such a function might apply to a scorpion population.
    Ans: Answers will vary. The graph of $f(t) = 850 + 550\sin(\frac{\pi}{6}t)$ is one such periodic
        function. One story: At the start of a 12-month experiment about scorpion
        populations, there are 850 scorpions.   Over the first three months, the scorpion
        population increases to a maximum of 1400, then returns to its initial population at
        mid-year.   The low point of the population is after 9 months and is 300.
    difficulty: medium   section: 1.5

80. Consider the function $f(\theta) = 50 + 75\cos(28.274\theta)$.   Select all true statements about
    this function from the list below.   Answers are rounded to two decimal places.
    A)   The period of the function is 0.22.   D)   The period of the function is 0.38.
    B)   The amplitude of the function is 75.   E)   The amplitude of the function is 50.
    C)   The vertical shift of the function is 50. F)   The vertical shift of the function is 75.
    Ans: A, B, C   difficulty: easy   section: 1.5

81. Write an equation of a periodic function with amplitude 20, vertical shift 30, period 2π,
    and horizontal translation (phase shift) 6 units to the right.
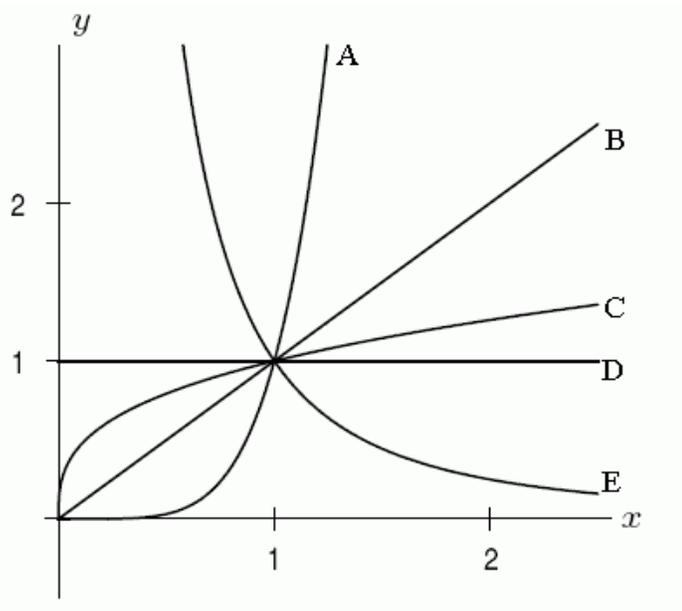   A) $f(t) = 30 + 20\cos(t - 6)$ D) $f(t) = 30\sin(6t - 20)$
    B) $f(t) = 20 + 30\sin(t + 6)$ E)   None of the above.
    C) $f(t) = 20\cos(30t + 6)$
    Ans: A   difficulty: easy   section: 1.5

82. The following figure shows the graphs for $x \geq 0$ of the functions $y = x$, $y = x^{-2}$, $y = x^0$, $y = x^{1/3}$, and $y = x^5$. Which one is the graph of $y = x^0$?

Ans: D
difficulty: easy   section: 1.6

83. Graphically find a solution to the equation $6^x = x^6$ in the interval $[-1 \leq x \leq 1]$. Give your answer accurate to 2 decimal places.
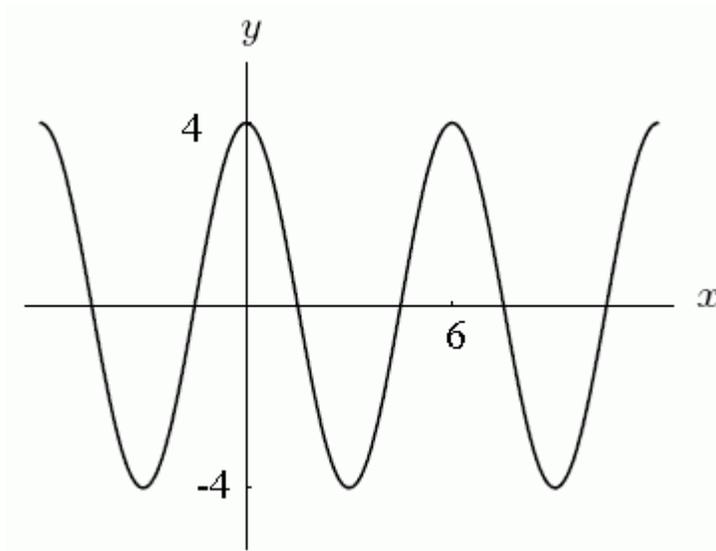Ans: –0.79
difficulty: medium   section: 1.6

84. A spherical cell takes in nutrients through its cell wall at a rate proportional to the area of the cell wall. The rate at which the cell uses nutrients is proportional to its volume. Which of the following expresses the rate at which the cell uses nutrients as a function of its radius, r. (A is a constant.)
A) $\dfrac{A}{r^2}$ B) $\dfrac{A}{r^3}$ C) Ar^2 D) Ar^3
Ans: D   difficulty: medium   section: 1.6

85. A possible equation for the following function is $y = \underline{\quad} + \underline{\quad} \cos\left(\underline{\quad}\pi x\right)$.
    Enter fractional coefficients as decimals, accurate to 2 decimal places.

Part A: 0
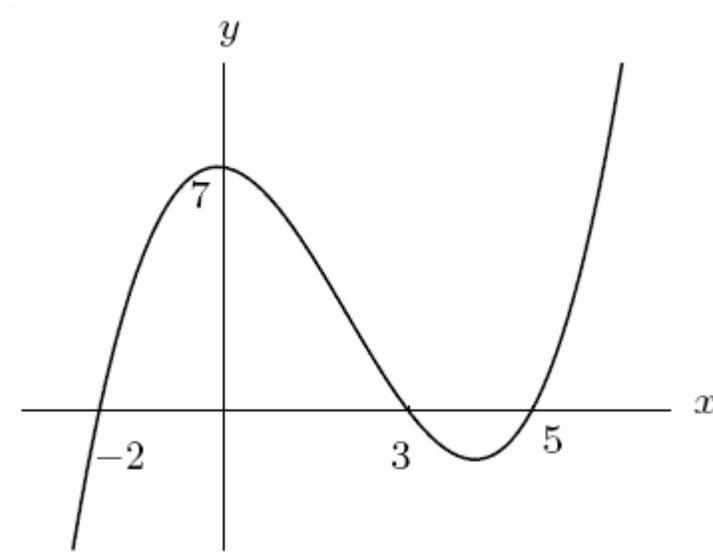Part B: 4
Part C: 0.33
difficulty: medium   section: 1.6

86. A possible equation for the following function is $y = \underline{\quad}\left(\underline{\quad}\right)^x$. Enter fractions
    as decimals, accurate to 3 decimal places.

Part A: 5
Part B: 0.858
difficulty: medium   section: 1.6

87. A possible equation for the following function is
    $y = $ _____$(x - $_____$)(x - $_____$)(x - $_____$)$.   Enter fractions as decimals, accurate to
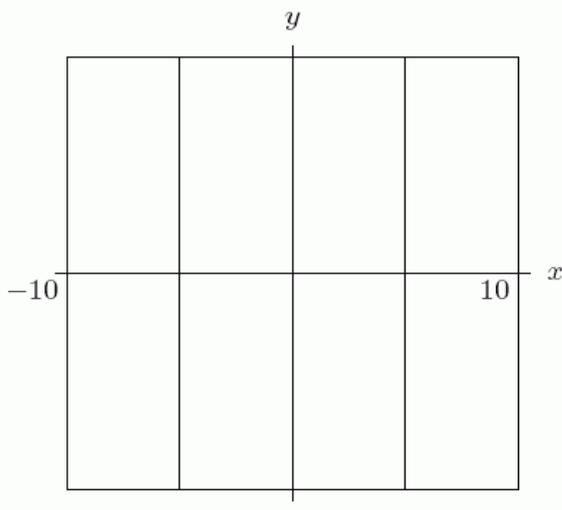   2 decimal places.   The three roots should be entered in increasing order.

Part A: 0.23
Part B: –2
Part C: 3
Part D: 5
difficulty: medium   section: 1.6

88. Using the standard viewing rectangle (-10 ≤ x ≤ 10, - 10 ≤ y ≤ 10), I graphed a cubic
    polynomial and saw two vertical lines, as shown.   There must be another root outside of
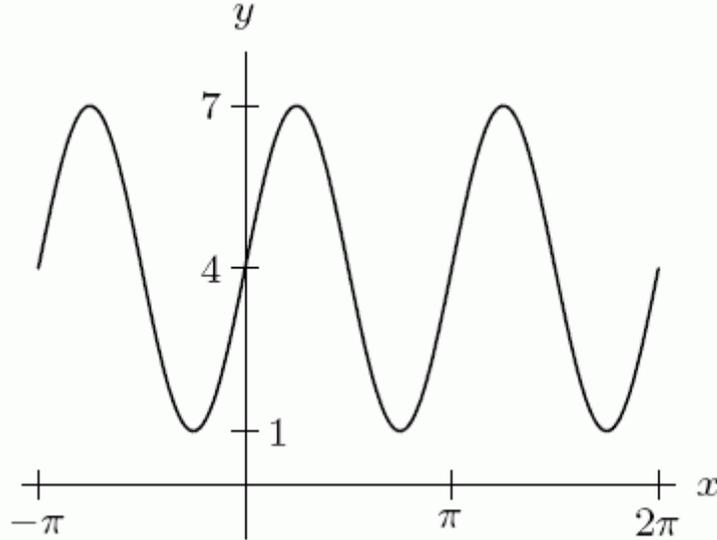   the window.

Ans: True   difficulty: medium   section: 1.6

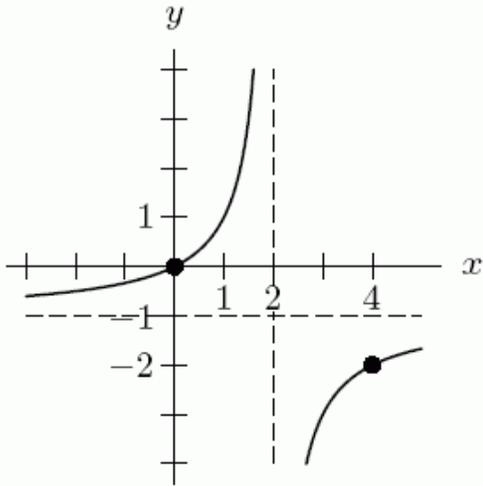89. Make a graphical sketch of the following function. Include a scale on the *x*- and *y*-axis.

$$y = 4 + 3\sin 2x$$

Ans:

difficulty: medium   section: 1.6

90. Find a possible equation for the following function given a root at (0,0), a vertical asymptote at *y*=-1 and horizontal asymptote at *x*=2.

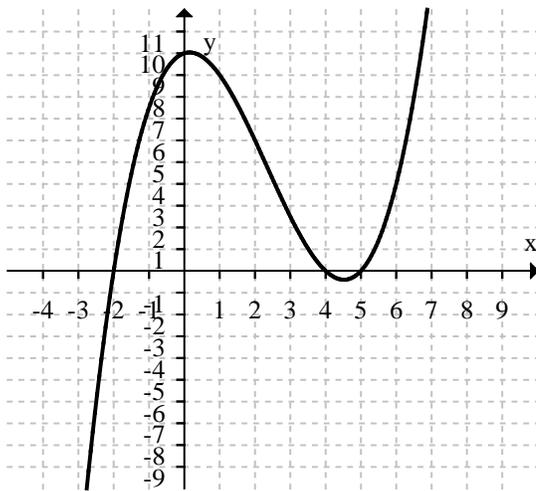Ans: $y = \dfrac{-x}{x-2}$

difficulty: medium   section: 1.6

91. Coulomb's law says that the electrical force between two charged objects is directly proportional to the product of the quantity of charge on the objects and inversely proportional to the square of the distance between the objects. Let q_1 and q_2 be the charge on the two objects.   Let d be the distance between the objects and F be the electrical force between them.   Translate Coulomb's Law into mathematical symbols.

A) $F = \dfrac{q_1 q_2}{d^2}$ B) $F = \dfrac{kd}{(q_1 - q_2)^2}$ C) $F = \dfrac{k(q_2 - q_1)^2}{d^2}$ D) $F = \dfrac{kq_1 q_2}{d^2}$

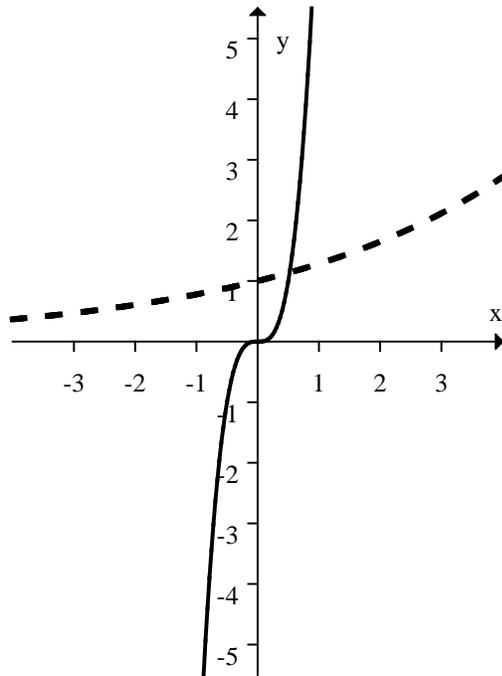Ans: D   difficulty: easy   section: 1.6

92. Write in factored form the equation of the polynomial graphed below.   All key features are shown.

Ans: $p(x) = \frac{1}{4}(x+2)(x-4)(x-5)$
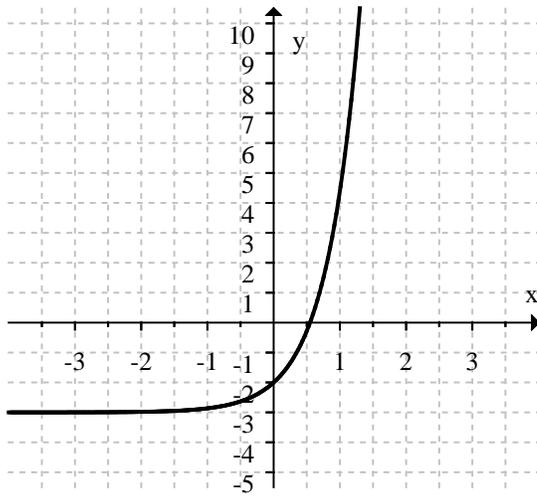
difficulty: medium   section: 1.6

93. The solid curve below is a portion of the graph of $f(x) = 8x^3$ and the dashed curve is a portion of the graph of $g(x) = e^{0.25x}$. The domain of both functions is all real numbers.Which of the following statements are true. Check all that apply.

A)   At x = 1, $f(x) > g(x)$.
B)   For all x > 1, $f(x) > g(x)$.
C) $\lim_{x \to -\infty} g(x) = 0$.
D)   There is only one value of x for which $f(x) = g(x)$.
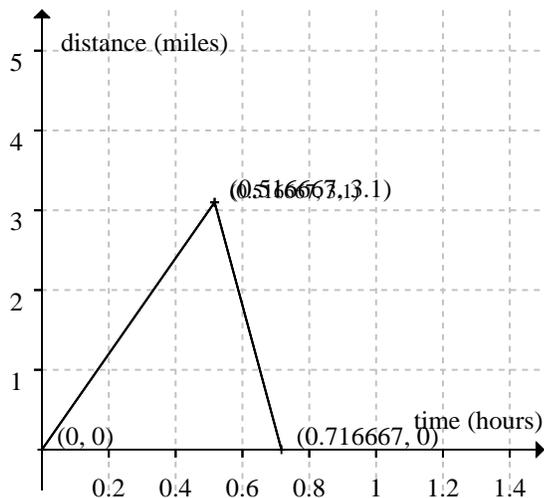Ans: A, C, D   difficulty: medium   section: 1.6

94. Using base e and transformations, find a formula for the exponential function shown in the graph below.

Ans: The answer should be close to $f(x) = e^{2x} - 3$.

difficulty: medium   section: 1.6

95. Delia runs the 3.1 miles from home to the park at 6 mph, jumps on her bike and returns home in 12 minutes.

    a) Sketch a well-labeled graph of Delia's distance from home as a function of time.
    b) Find the slope of each segment of the graph and interpret their meaning.
    c) What does it mean to say that Delia's velocity is inversely proportional to the time she takes for her run/ride?

Ans:

The slope of the first segment is 6 miles per hour; her velocity on the way to the park.   The slope of the second segment is -15.5 miles per hour; her velocity on the way home.

The faster Delia goes, the less time it takes for her to get back home. In formula form,   rate = d/t..

difficulty: hard   section: 1.6

96. Is the function $\dfrac{1}{\sin x}$ continuous on the interval $\dfrac{3\pi}{4} \le x \le \dfrac{5\pi}{4}$?

    Ans: no
    difficulty: easy   section: 1.7

97. Is the function $\dfrac{1}{\sqrt{x^2 - 9}}$ continuous on the interval $0 \le x \le 4$?

    Ans: no
    difficulty: easy   section: 1.7

98. What are the point(s) of discontinuity of the function $\dfrac{1}{x(x-6)^2}$?Select all that apply.
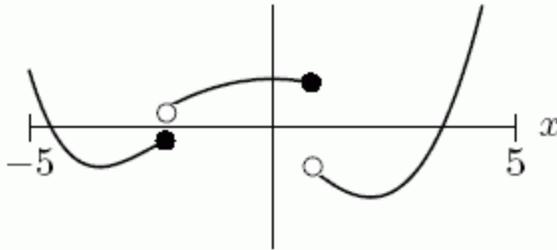
    A) 0   B) 1   C) 6   D) 12   E) 36
    Ans: A, C   difficulty: easy   section: 1.7

99. If $f(x)$ and $g(x)$ are both continuous functions, what can you say about $f(x) + g(x)$?
    A) It cannot be determined if it is continuous or not.
    B) It is not continuous.
    C) It is continuous.
    Ans: C   difficulty: easy   section: 1.7

100. If $f(x)$ and $g(x)$ are both continuous functions, what can you say about $f(x)/8$?
    A) It is continuous.
    B) It is not continuous.
    C) It cannot be determined if it is continuous or not.
    Ans: A   difficulty: easy   section: 1.7

101. Sketch a graph of a function on the interval [-5, 5] with exactly 2 zeros and at least 2 places where the function is not continuous.
    Ans: Many answers possible.   One possible answer:
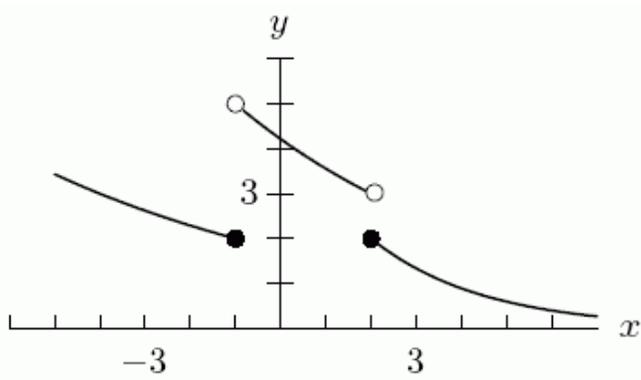

   difficulty: medium   section: 1.7

102. Is the function $g(x) = \begin{cases} |x+1|, -10 < x \le 2 \\ -x^2 + 3x, \ 2 < x < 10 \end{cases}$ continuous at $x = 2$?

    Ans: no
    difficulty: medium   section: 1.7

103. For what value of a is the function $f(t) = \begin{cases} t - a, \ -10 < t \le 3 \\ t^2 + 10t, \ 3 < t < 10 \end{cases}$ continuous?

    A) 10   B) –36   C) 30   D) –18
    Ans: B   difficulty: medium   section: 1.7

104. Use the following figure to give the approximate value (to the nearest integer) of $\lim_{x \to -4} f(x)$. If the limit does not exist, enter "DNE".

Ans: 3
difficulty: easy   section: 1.8

105. Use algebra to evaluate the exact value of $\lim_{h \to 0} \dfrac{(16+h)^{1/2} - 4}{h}$.
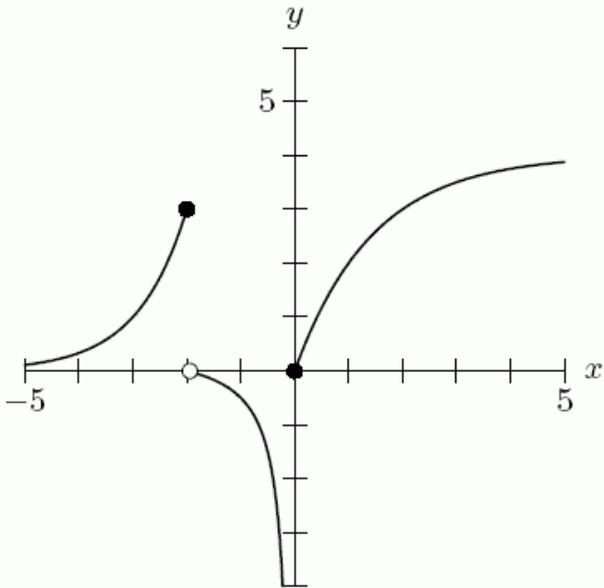
Ans: 1/8
difficulty: medium   section: 1.8

106. Does $\lim_{x \to -8} \dfrac{|x+8|}{x+8}$ exist?

Ans: no
difficulty: medium   section: 1.8

107. Use algebra to evaluate the right-hand limit of $f(x) = \dfrac{|x-8|}{2x-16}$ as $x \to 8$.

A) 1/2   B) -1/2   C) 1/8   D) -1/8
Ans: A   difficulty: medium   section: 1.8

108. Use the following figure to give the value of $\lim\limits_{x \to 0^-} f(x)$. If the limit does not exist, enter "DNE".

Ans: DNE
difficulty: easy   section: 1.8

109. If $f(x) = x^2 - 25$ and $g(x) = 3x - 4$, evaluate $\lim\limits_{x \to 0} [f(x) + g(x)]$. If the limit does not exist, enter "DNE".
Ans: –29
difficulty: easy   section: 1.8

110. Let $f(x) = e^x$ and $g(x) = x^2 - 2$. Find $\lim\limits_{x \to 1}(8f(x) + 6g(2x))$. Round to two decimal places, if necessary.
A) –21.75   B) 71.11   C) 20   D) 33.75
Ans: D   difficulty: medium   section: 1.8

111. Find, to two decimal places, $\lim\limits_{\theta \to 0} \dfrac{8\sin(6\theta)}{\theta}$
A) 48.00   B) 1.33   C) 1   D) 0   E) DNE
Ans: A   difficulty: easy   section: 1.8

112. Find, $\lim\limits_{x\to\infty} \dfrac{4x^7 - 3x + 2}{11x^7 + 100x - 5}$ if it exists.

   A) 1   B) $\dfrac{4}{11}$ C) $\dfrac{7}{11}$ D) DNE   E) $\dfrac{-2}{5}$

   Ans: B   difficulty: medium   section: 1.8

113. Find $\lim\limits_{x\to 0} \dfrac{1}{6x^7}$ if it exists.

   A) $\dfrac{1}{6}$ B) 42   C) 1   D) $\dfrac{1}{7}$ E) DNE

   Ans: E   difficulty: easy   section: 1.8

114. Suppose $\lim\limits_{x\to 0} f(x) = 7$ and $\lim\limits_{x\to 0} g(x) = 11$. Find $\lim\limits_{x\to 0} \dfrac{7g(x)}{f(x) + 7}$. Round to two decimal

   places.
   A) 5.50   B) 1.57   C) 7.00   D) 1.27   E) None of the above
   Ans: A   difficulty: easy   section: 1.8

115. A population of bacteria is introduced into a nutrient solution in a dish. The nutrient solution is replenished by the exact same amount each day. At first the bacteria population grows very quickly. Soon, population growth must slow down, as the nutrients become more scarce given the number of bacteria. Eventually the bacteria population stabilizes. A model for the bacteria population growth is given by

   $P(t) = \dfrac{5000}{6 + e^{-5t}}$ thousand bacteria. What is $\lim\limits_{t\to\infty} P(t)$?

   A) 6000   B) 1000   C) 833   D) 5000   E) None of the above
   Ans: C   difficulty: medium   section: 1.8

116. Could the function described by the following table of values be exponential?

x	5.2	5.3	5.4	5.5	5.6
$f(x)$	26	28.60	31.46	34.61	38.07

   Ans: yes
   difficulty: easy   section: 1 review

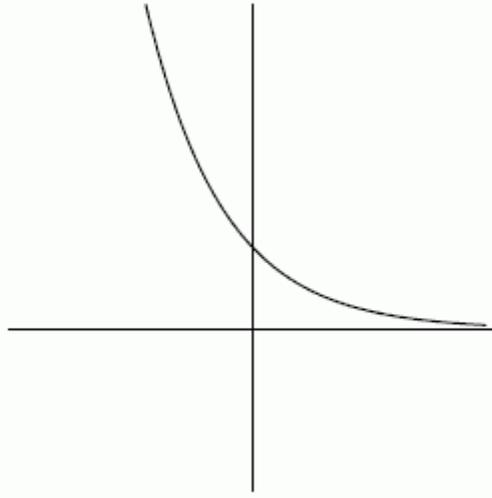117. What is the doubling time of quantity Q growing exponentially according to the formula $Q(t) = Q_0 5^t$? Round to 2 decimal places.
   Ans: 0.43
   difficulty: medium   section: 1 review

118. Which function is graphed in the following figure?

    A. $\ln(e^x)+1$ B. $-2\ln x$ C. e^{-x} D.

$x^5 + 2x^4 - x^3 + 2x^2 + 5$ E. $\dfrac{1}{x+1}$

Ans: C
difficulty: medium   section: 1 review

119. Give an expression for $h(x)$ which agrees with the following table of values.

x	$f(x)$	$g(x)$	$h(x)$
0	−7	0	-
1	−3	2	5
2	1	8	2.50
3	5	18	1.67
4	9	32	1.25
5	13	50	1

Ans: $h(x) = 5/x$
difficulty: medium   section: 1 review

120. You are offered two jobs starting on July 1st of 2008. Firm A offers you $50,000 a year to start and you can expect an annual raise of 4% every July 1st. At firm B you would start at $40,000 but can expect an annual 6% increase every July 1st. After how many years would the job at firm B first pay more than the job at firm A?
Ans: 12
difficulty: medium   section: 1 review

121. You have $500 invested in a bank account earning 8.5% compounded annually.   How many years will it take to triple your money?   Round to the nearest whole year.
Ans: 13
difficulty: medium   section: 1 review

122. You have $500 invested in a bank account earning 6.4% compounded monthly, earning $\frac{6.4}{12}$% interest each month. How much interest do you earn the first year? Round to the nearest cent.
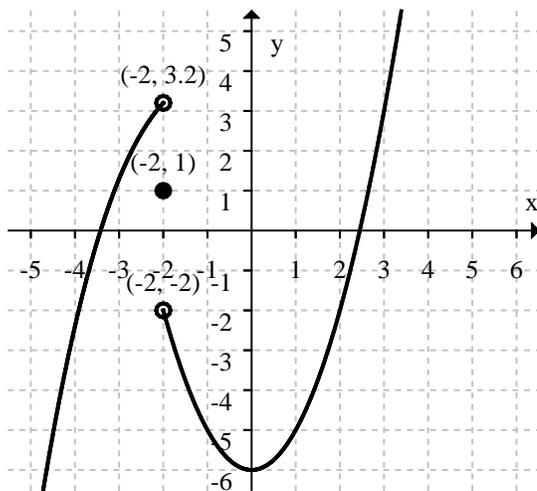Ans: $32.96
difficulty: medium   section: 1 review

123. The elimination half-life of aspirin in plasma is estimated to be between 15 and 20 minutes.   Assuming the upper estimate of 20 minutes, how long will it take a dose of 81 mg to decrease to $^2/_3$ the original amount (54 mg)   in the plasma?
A)   1.4 minutes   B)   9.5 minutes   C)   12 minutes   D)   11.7 minutes
Ans: D   difficulty: hard   section: 1 review

124. Consider the graph of the function $f(x)$ given below.   What is $\lim\limits_{x \to -2} f(x)$?

A) 3.2   B) 1   C) -2   D) DNE
Ans: D   difficulty: easy   section: 1 review

125. The data in the table describe the percentage of fissures that developed in turbines after h hours of running time.   There is moderate linear relationship (r = 0.93)between the number of hours the turbines ran and the percentage of turbines that developed fissures. Which of the formulas below best models the linear relationship?

Hours Run	Number of Turbines Run	Number of Fissures	% of Turbines with Fissures
400	39	0	0
1000	53	4	0.08
1400	33	2	0.06
1800	73	7	0.1
2200	30	5	0.17
2600	39	9	0.23
3000	42	9	0.21
3400	13	6	0.46
3800	34	22	0.65
4200	40	21	0.53
4600	36	21	0.58

A) $f = 0.00016h - 0.134$ C) $f = -0.002h + 0.132$

B) $f = 1.6h - 0.133$ D) $f = 0.93h - 0.13$

Ans: A   difficulty: medium   section: 1 review