1

MODELS, MEASUREMENTS, AND VECTORS

Answers to Multiple-Choice Problems

1. B **2.** A **3.** C **4.** A **5.** B **6.** A **7.** B **8.** D **9.** C **10.** C **11.** B **12.** B **13.** D

Solutions to Problems

1.1. Set Up: We know the following equalities: $1\text{ mg} = 10^{-3}\text{ g}$; $1\ \mu\text{g} = 10^{-6}\text{ g}$; $1\text{ kilohms} = 1000\text{ ohms}$; and $1\text{ milliamp} = 10^{-3}\text{ amp}$.
Solve: In each case multiply the quantity to be converted by unity, expressed in different units. Construct an expression for unity so that the units to be changed cancel and we are left with the new desired units.

(a) $(2400\text{ mg/day})\left(\dfrac{10^{-3}\text{ g}}{1\text{ mg}}\right) = 2.40\text{ g/day}$

(b) $(120\ \mu\text{g/day})\left(\dfrac{10^{-6}\text{ g}}{1\ \mu\text{g}}\right) = 1.20 \times 10^{-4}\text{ g/day}$

(c) $(500\text{ mg/day})\left(\dfrac{10^{-3}\text{ g}}{1\text{ mg}}\right) = 0.500\text{ g/day}$

(d) $(1500\text{ ohms})\left(\dfrac{1\text{ kilohm}}{10^{3}\text{ ohms}}\right) = 1.50\text{ kilohms}$

(e) $(0.020\text{ amp}) = \left(\dfrac{1\text{ milliamp}}{10^{-3}\text{ amp}}\right) = 20\text{ milliamps}$

Reflect: In each case, the number representing the quantity is larger when expressed in the smaller unit. For example, it takes more milligrams to express a mass than to express the mass in grams.

1.2. Set Up: From the meaning of the metric prefixes we know: $1\text{ megaohm} = 1\text{ M}\Omega = 10^{6}\text{ ohms}$; $1\text{ picofarad} = 1\text{ pf} = 10^{-12}\text{ farad}$; $1\text{ gigameter} = 1\text{ Gm} = 10^{9}\text{ m}$; $1\text{ nanometer} = 1\text{ nm} = 10^{-9}\text{ m}$; and $1\text{ femtometer} = 1\text{ fm} = 10^{-15}\text{ m}$.

Solve: (a) $(7.85\text{ megohms})\left(\dfrac{10^{6}\text{ ohms}}{1\text{ megaohm}}\right) = 7.85 \times 10^{6}\text{ ohms}$

(b) $(5\text{ picofarads})\left(\dfrac{10^{-12}\text{ farad}}{1\text{ picofarad}}\right) = 5 \times 10^{-12}\text{ picofarad}$

(c) $(3.00 \times 10^{8}\text{ m/s})\left(\dfrac{1\text{ gigameter}}{10^{9}\text{ m}}\right) = 0.300\text{ gigameter/s}$

(d) $(400\text{ nm})\left(\dfrac{10^{-9}\text{ meter}}{1\text{ nm}}\right) = 4.00 \times 10^{-7}\text{ meter}$; $(700\text{ nm})\left(\dfrac{10^{-9}\text{ meter}}{1\text{ nm}}\right) = 7.00 \times 10^{-7}\text{ meter}$

(e) $(2\text{ femtometer})\left(\dfrac{10^{-15}\text{ meter}}{1\text{ femtometer}}\right) = 2 \times 10^{-15}\text{ meter}$

1.3. Set Up: We know the equalities: $1\text{ mg} = 10^{-3}\text{ g}$, $1\ \mu\text{g} = 10^{-6}\text{ g}$ and $1\text{ kg} = 10^{3}\text{ g}$.

Solve: (a) $(410\text{ mg/day})\left(\dfrac{10^{-3}\text{ g}}{1\text{ mg}}\right)\left(\dfrac{1\ \mu\text{g}}{10^{-6}\text{ g}}\right) = 4.10 \times 10^{5}\ \mu\text{g/day}$

(b) $(12\text{ mg/kg})(75\text{ kg}) = (900\text{ mg})\left(\dfrac{10^{-3}\text{ g}}{1\text{ mg}}\right) = 0.900\text{ g}$

(c) The mass of each tablet is

$$(2.0\text{ mg})\left(\frac{10^{-3}\text{ g}}{1\text{ mg}}\right) = 2.0 \times 10^{-3}\text{ g}.$$

The number of tablets required each day is the number of grams recommended per day divided by the number of grams per tablet:

$$\frac{0.0030\text{ g/day}}{2.0 \times 10^{-3}\text{ g/tablet}} = 1.5\text{ tablet/day}.$$

Take 2 tablets each day.

(d) $(0.000070\text{ g/day})\left(\dfrac{1\text{ mg}}{10^{-3}\text{ g}}\right) = 0.070\text{ mg/day}$

1.4. Set Up: In part (a), we need to solve an equation of the form

$$\left(\frac{2.54\text{ cm}}{1.00\text{ in.}}\right)x(1.0\text{ mi}) = y,$$

where x is a series of conversion equalities and y is the unknown number of kilometers. For part (b), we must prove $(1.00\text{ mL})x = 1.00\text{ cm}^3$ using the equalities $1\text{ L} = 1000\text{ cm}^3$ and $10^3\text{ mL} = 1\text{ L}$. Part (c) requires the application of the equality $1\text{ mL} = 1\text{ cm}^3$.

Solve: (a) $\left[\left(\dfrac{2.54\text{ cm}}{1.00\text{ in.}}\right)\left(\dfrac{12.0\text{ in.}}{1.00\text{ ft}}\right)\left(\dfrac{5280\text{ ft}}{1.00\text{ mi}}\right)\right](1.00\text{ mi})\left[\left(\dfrac{1.00\text{ m}}{100\text{ cm}}\right)\left(\dfrac{1.00\text{ km}}{10^3\text{ m}}\right)\right] = y;\ y = 1.61\text{ km}$

(b) $(1.00\text{ mL})\left(\dfrac{1.00\text{ L}}{10^3\text{ mL}}\right)\left(\dfrac{1000\text{ cm}^3}{1.00\text{ L}}\right) = 1.00\text{ cm}^3$

(c) $(1.00\text{ L})\left(\dfrac{10^3\text{ mL}}{1.00\text{ L}}\right)\left(\dfrac{1.00\text{ cm}^3}{1.00\text{ mL}}\right) = 1000\text{ cm}^3$

1.5. Set Up: We need to apply the following conversion equalities: $1000\text{ g} = 1.00\text{ kg}$, $100\text{ cm} = 1.00\text{ m}$, and $1.00\ L = 1000\text{ cm}^3$.

Solve: (a) $(1.00\text{ g/cm}^3)\left(\dfrac{1.00\text{ kg}}{1000\text{ g}}\right)\left(\dfrac{100\text{ cm}}{1.00\text{ m}}\right)^3 = 1000\text{ kg/m}^3$

(b) $(1050\text{ kg/m}^3)\left(\dfrac{1000\text{ g}}{1.00\text{ kg}}\right)\left(\dfrac{1.00\text{ m}}{1000\text{ cm}}\right)^3 = 1.05\text{ g/cm}^3$

(c) $(1.00\text{ L})\left(\dfrac{1000\text{ cm}^3}{1.00\text{ L}}\right)\left(\dfrac{1.00\text{ g}}{1.00\text{ cm}^3}\right)\left(\dfrac{1.00\text{ kg}}{1000\text{ g}}\right) = 1.00\text{ kg};\ (1.00\text{ kg})\left(\dfrac{2.205\text{ lb}}{1.00\text{ kg}}\right) = 2.20\text{ lb}$

Reflect: We could express the density of water as 1.00 kg/L.

1.6. Set Up: We know that $1\text{ km} = 0.6214\text{ mi}$ and that mph means miles per hour.

Solve: (a) $(150\text{ km/h})\left(\dfrac{0.6214\text{ mi}}{1\text{ km}}\right) = 93\text{ mi/h} = 93\text{ mph}$

(b) $(65\text{ mph})\left(\dfrac{1\text{ km}}{0.6214\text{ mi}}\right) = 105\text{ km/h}$

1.7. Set Up: Apply the equalities of $1\ \text{mi/h} = 1.609\ \text{km/h}$ and $1.0\ \text{mi/h} = 0.4470\ \text{m/s}$.
Solve: (a) $(1450\ \text{mi/h})[(1.609\ \text{km/h})/(1.00\ \text{mi/h})] = 2330\ \text{km/h}$
(b) $(1450\ \text{mi/h})[(0.4470\ \text{m/s})/(1.00\ \text{mi/h})] = 648\ \text{m/s}$

1.8. Set Up: We know: $1\ \text{in.} = 2.54\ \text{cm}$; $1\ \text{cm} = 10^{-2}\ \text{m}$; and $1\ \text{mm} = 10^{-3}\ \text{m}$.
Solve: (a) $(\frac{3}{8}\ \text{in.})\left(\frac{2.54\ \text{cm}}{1\ \text{in.}}\right)\left(\frac{10^{-2}\ \text{m}}{1\ \text{cm}}\right)\left(\frac{1\ \text{mm}}{10^{-3}\ \text{m}}\right) = 9.5\ \text{mm}$
(b) $(12\ \text{mm})\left(\frac{10^{-3}\ \text{m}}{1\ \text{mm}}\right)\left(\frac{1\ \text{cm}}{10^{-2}\ \text{m}}\right)\left(\frac{1\ \text{in.}}{2.54\ \text{cm}}\right) = 0.47\ \text{in.}$
(c) Express and compare the English wrench sizes as decimal values: $\frac{3}{8}'' = 0.375$ in., $\frac{4}{8}'' = 0.500$ in. and $\frac{5}{8}'' = 0.625$ in. The $\frac{1}{2}$-inch wrench is closest to 12 mm.

1.9. Set Up: We apply the equalities of $1\ \text{km} = 0.6214\ \text{mi}$ and $1\ \text{gal} = 3.788\ \text{L}$.
Solve: $(37.5\ \text{mi/gal})\left(\frac{1\ \text{km}}{0.6214\ \text{mi}}\right)\left(\frac{1\ \text{gal}}{3.788\ \text{L}}\right) = 15.9\ \text{km/L}$
Reflect: Note how the unit conversion strategy, of cancellation of units, automatically tells us whether to multiply or divide by the conversion factor.

1.10. Set Up: Apply the given conversion factors, $1\ \text{furlong} = 0.1250\ \text{mi}$ and $1\ \text{fortnight} = 14\ \text{days}$, along with $1\ \text{day} = 24\ \text{h}$.
Solve: $(180{,}000\ \text{furlongs/fortnight})\left(\frac{0.125\ \text{mi}}{1\ \text{furlong}}\right)\left(\frac{1\ \text{fortnight}}{14\ \text{days}}\right)\left(\frac{1\ \text{day}}{24\ \text{h}}\right) = 67\ \text{mi/h}$

1.11. Set Up: We know: $1\ \text{euro} = \$1.25$; and $1\ \text{gal} = 3.788\ \text{L}$.
Solve: $(1.35\ \text{euros/L})\left(\frac{\$1.25}{1\ \text{euro}}\right)\left(\frac{3.788\ \text{L}}{1\ \text{gal}}\right) = \$6.39$ per gallon. Currently, in 2005, gasoline in the U.S. costs about \$2 per gallon so the price in Europe is about three times higher.

1.12. Set Up: From Appendix A, the volume V of a sphere is given in terms of its radius as $V = \frac{4}{3}\pi r^3$ while its surface area A is given as $A = 4\pi r^2$. Also, by definition, the radius is one-half the diameter or $r = d/2 = 1.0\ \mu\text{m}$. Finally, the necessary equalities for this problem are: $1\ \mu\text{m} = 10^{-6}\ \text{m}$; $1\ \text{cm} = 10^{-2}\ \text{m}$; and $1\ \text{mm} = 10^{-3}\ \text{m}$.
Solve: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0\ \mu\text{m})^3\left(\frac{10^{-6}\ \text{m}}{1\ \mu\text{m}}\right)^3\left(\frac{1\ \text{cm}}{10^{-2}\ \text{m}}\right)^3 = 4.2 \times 10^{-12}\ \text{cm}^3$
and

$$A = 4\pi r^2 = 4\pi(1.0\ \mu\text{m})^2\left(\frac{10^{-6}\ \text{m}}{1\ \mu\text{m}}\right)^2\left(\frac{1\ \text{mm}}{10^{-3}\ \text{m}}\right)^2 = 1.3 \times 10^{-5}\ \text{mm}^2$$

1.13. Set Up: We apply the basic time relations of $1\ \text{h} = 60\ \text{min}$ and $1\ \text{min} = 60\ \text{s}$ in part (a) and use the result of (a) in part (b). Similarly, apply the result of part (b) in solving part (c).
Solve: (a) $(1\ \text{h})\left(\frac{60\ \text{min}}{1\ \text{h}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 3600\ \text{s}$
(b) $(24\ \text{h/day})[(3600\ \text{s})/(1\ \text{h})] = 86.4 \times 10^3\ \text{s/day}$
(c) $(365\ \text{day/yr})[(86.4 \times 10^3\ \text{s})/(1\ \text{day})] = 31{,}536{,}000\ \text{s/yr} = 3.15 \times 10^7\ \text{s/yr}$

1.14. Set Up: (a) The appropriate equalities required are: $1\ \text{mi} = 1609\ \text{m}$; $1\ \text{h} = 3600\ \text{s}$; $1\ \text{mph} = 0.4470\ \text{m/s}$; $1\ \text{ft} = 0.3048\ \text{m}$; and $1\ \text{mi} = 5280\ \text{ft}$.
Solve: (a) $(3.00 \times 10^8\ \text{m/s})\left(\frac{1\ \text{mi}}{1609\ \text{m}}\right) = 1.86 \times 10^5\ \text{mi/s}$;

$$(3.00 \times 10^8\ \text{m/s})\left(\frac{1\ \text{mi}}{1609\ \text{m}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 6.71 \times 10^8\ \text{mph}$$

(b) $(1100\ \text{ft/s})\left(\dfrac{0.3048\ \text{m}}{1\ \text{ft}}\right) = 335\ \text{m/s}$; $(1100\ \text{ft/s})\left(\dfrac{1\ \text{mi}}{5280\ \text{ft}}\right)\left(\dfrac{3600\ \text{s}}{1\ \text{h}}\right) = 750\ \text{mph}$

(c) $(60\ \text{mi/h})\left(\dfrac{5280\ \text{ft}}{1\ \text{mi}}\right)\left(\dfrac{1\ \text{h}}{3600\text{s}}\right) = 88\ \text{ft/s}$

(d) $(9.8\ \text{m/s}^2)\left(\dfrac{1\ \text{ft}}{0.3048\ \text{m}}\right) = 32\ \text{ft/s}^2$

1.15. Set Up: In each case, round the last significant figure.
Solve: (a) 3.14, 3.1416, 3.1415927 **(b)** 2.72, 2.7183, 2.7182818 **(c)** 3.61, 3.6056, 3.6055513
Reflect: All of these representations of the quantities are imprecise, but become more precise as additional significant figures are retained.

1.16. Set Up: Use a calculator to calculate the decimal equivalent of each fraction and then round the numeral to the specified number of significant figures. Compare to π rounded to the same number of significant figures.
Solve: (a) 3.14286 **(b)** 3.14159 **(c)** The exact value of π rounded to six significant figures is 3.14159. Since the fraction $22/7$ differs in the fourth significant figure, it is accurate to only three significant figures. The fraction $355/113$ and π agree when expressed to six figures and thus agree to this precision.

1.17. Set Up: Calculate the trig function with the maximum and minimum possible values of the angle.
Solve: (a) $\cos 3.5° = 0.9981$ and $\cos 4.5° = 0.9969$, so the range is 0.9969 to 0.9981.
(b) $\sin 3.5° = 0.061$ and $\sin 4.5° = 0.078$, so the range is 0.061 to 0.078.
(c) $\tan 3.5° = 0.061$ and $\tan 4.5° = 0.079$, so the range is 0.061 to 0.079.

1.18. Set Up: We know the relations: mass = density × volume; and $1.00\ \text{L} = 1.00 \times 10^3\ \text{cm}^3$.
Solve: ***Water*** $m = (1.00\ \text{g/cm}^3)(1.00 \times 10^3\ \text{cm}^3) = 1.00 \times 10^3\ \text{g}$;
Blood: $m = (1.05\ \text{g/cm}^3)(1.00 \times 10^3\ \text{cm}^3) = 1.05 \times 10^3\ \text{g}$;
Seawater: $m = (1.03\ \text{g/cm}^3)(1.00 \times 10^3\ \text{cm}^3) = 1.03 \times 10^3\ \text{g}$.

1.19. Set Up: We are given the relation density = mass/volume = m/V where $V = \frac{4}{3}\pi r^3$ for a sphere. From Appendix F, the earth has mass of $m = 5.97 \times 10^{24}$ kg and a radius of $r = 6.38 \times 10^6$ m whereas for the sun at the end of its lifetime, $m = 1.99 \times 10^{30}$ kg and $r = 7500\ \text{km} = 7.5 \times 10^6$ m. The star possesses a radius of $r = 10\ \text{km} = 1.0 \times 10^4$ m and a mass of $m = 1.99 \times 10^{30}$ kg.
Solve: (a) The earth has volume $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6.38 \times 10^6\ \text{m})^3 = 1.088 \times 10^{21}\ \text{m}^3$.

$$\text{density} = \frac{m}{V} = \frac{5.97 \times 10^{24}\ \text{kg}}{1.088 \times 10^{21}\ \text{m}^3} = (5.49 \times 10^3\ \text{kg/m}^3)\left(\frac{10^3\ \text{g}}{1\ \text{kg}}\right)\left(\frac{1\ \text{m}}{10^2\ \text{cm}}\right)^3 = 5.49\ \text{g/cm}^3$$

(b) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7.5 \times 10^6\ \text{m})^3 = 1.77 \times 10^{21}\ \text{m}^3$

$$\text{density} = \frac{m}{V} = \frac{1.99 \times 10^{30}\ \text{kg}}{1.77 \times 10^{21}\ \text{m}^3} = (1.1 \times 10^9\ \text{kg/m}^3)\left(\frac{1\ \text{g/cm}^3}{1000\ \text{kg/m}^3}\right) = 1.1 \times 10^6\ \text{g/cm}^3$$

(c) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0 \times 10^4\ \text{m})^3 = 4.19 \times 10^{12}\ \text{m}^3$

$$\text{density} = \frac{m}{V} = \frac{1.99 \times 10^{30}\ \text{kg}}{4.19 \times 10^{12}\ \text{m}^3} = (4.7 \times 10^{17}\ \text{kg/m}^3)\left(\frac{1\ \text{g/cm}^3}{1000\ \text{kg/m}^3}\right) = 4.7 \times 10^{14}\ \text{g/cm}^3$$

Reflect: For a fixed mass, the density scales as $1/r^3$. Thus, the answer to (c) can also be obtained from (b) as

$$(1.1 \times 10^6\ \text{g/cm}^3)\left(\frac{7.50 \times 10^6\ \text{m}}{1.0 \times 10^4\ \text{m}}\right)^3 = 4.7 \times 10^{14}\ \text{g/cm}^3.$$

1.20. Set Up: To calculate the densities, we need to find the spherical volume, $V = \frac{4}{3}\pi r^3$, and the mass of the atom or nucleus as the sum of the masses of its constituent particles. For the atom, $m = 2(m_p + m_n + m_e)$ while for the

nucleus $m = 2(m_p + m_n)$. We thus need mass data from Appendix F: $m_p = 1.673 \times 10^{-27}$ kg; $m_n = 1.675 \times 10^{-27}$ kg; and $m_e = 9.109 \times 10^{-31}$ kg. The unit conversion factor 1 g/cm^3 = 1000 kg/m^3 is also needed.
Solve: (a) Given $r = 0.050\text{ nm} = 0.050 \times 10^{-9}\text{ m}$, $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.050 \times 10^{-9}\text{ m})^3 = 5.24 \times 10^{-31}\text{ m}^3$. $m = 2(m_p + m_n + m_e) = 6.70 \times 10^{-27}$ kg

$$\text{density} = \frac{m}{V} = \frac{6.70 \times 10^{-27}\text{ kg}}{5.24 \times 10^{-31}\text{ m}^3} = 1.3 \times 10^4\text{ kg/m}^3 = 13\text{ g/cm}^3$$

The density of the helium atom is 13 times larger than the density of pure water.
(b) Given $r = 1.0\text{ fm} = 1.0 \times 10^{-15}\text{ m}$, $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0 \times 10^{-15}\text{ m})^3 = 4.19 \times 10^{-45}\text{ m}^3$.

$$m = 2(m_p + m_n) = 6.70 \times 10^{-27}\text{ kg}$$

$$\text{density} = \frac{m}{V} = \frac{6.70 \times 10^{-27}\text{ kg}}{4.19 \times 10^{-45}\text{ m}^3} = 1.6 \times 10^{18}\text{ kg/m}^3 = 1.6 \times 10^{15}\text{ g/cm}^3$$

In Problem 1.19 we found the density of a neutron star to be 4.7×10^{14} g/cm^3. By comparison, the density of the helium nucleus is 3 times larger than the density of a neutron star.

1.21. Set Up: We know the density and mass; thus we can find the volume using the relation density = mass/volume = m/V and the given data, density = 19.5 g/cm^3 and $m_{\text{critical}} = 60.0$ kg. The radius is then found from the volume equation for a sphere, $V = \frac{4}{3}\pi r^3$, and the result for volume.
Solve: $V = m_{\text{critical}}/\text{density} = \left(\frac{60.0\text{ kg}}{19.5\text{ g/cm}^3}\right)\left(\frac{1000\text{ g}}{1.0\text{ kg}}\right) = 3080\text{ cm}^3$

$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3}{4\pi}(3080\text{ cm}^3)} = 9.0\text{ cm}$$

1.22. Set Up: From Appendix A, a thin spherical shell has volume $V = At$, where $A = 4\pi r^2$ is the surface area of the shell and t is its thickness. We are given $r = 1.0\ \mu\text{m} = 1.0 \times 10^{-6}$ m and $t = 50.0\text{ nm} = 50.0 \times 10^{-9}$ m, and we know the unit conversions 1 m^3 = 10^6 cm^3 and 1 mg = 10^{-3} g.
Solve: $V = 4\pi r^2 t = 4\pi(1.0 \times 10^{-6}\text{ m})^2(50.0 \times 10^{-9}\text{ m}) = 6.28 \times 10^{-19}\text{ m}^3 = 6.28 \times 10^{-13}\text{ cm}^3$

$$\text{mass} = (\text{density})(\text{volume}) = (1.0\text{ g/cm}^3)(6.28 \times 10^{-13}\text{ cm}^3) = 6.28 \times 10^{-13}\text{ g} = 6.28 \times 10^{-10}\text{ mg}$$

1.23. Set Up: The mass can be calculated as the product of the density and volume. The volume of the washer is the volume V_d of a solid disk of radius r_d minus the volume V_h of the disk-shaped hole of radius r_h. In general, the volume of a disk of radius r and thickness t is $\pi r^2 t$. We also need to apply the unit conversions 1 m^3 = 10^6 cm^3 and 1 g/cm^3 = 10^3 kg/m^3.
Solve: The volume of the washer is:

$$V = V_d - V_h = \pi(r_d^2 - r_h^2)t = \pi[(2.25\text{ cm})^2 - (0.625\text{ cm})^2](0.150\text{ cm}) = 2.20\text{ cm}^3$$

The density of the washer material is 8600 kg/m^3[(1 g/cm^3)/(10^3 kg/m^3)] = 8.60 g/cm^3. Finally, the mass of the washer is: mass = (density)(volume) = (8.60 g/cm^3)(2.20 cm^3) = 18.9 g.
Reflect: This mass corresponds to a weight of about 0.7 oz, a reasonable value for a washer.

1.24. Set Up: The world population when this solution is being written is about 6.4×10^9 people. Estimate an average mass of a person to be about 50 kg which corresponds to a weight of about 110 lbs.
Solve: The total mass of all the people is about (50 kg/person)(6.4×10^9 persons) = 3×10^{11} kg

1.25. Set Up: For this estimate, assume a highway speed of 65 mi/h and a tunnel length of 0.5 mile and apply the relation $v = d/t$ to determine the time for a single car to traverse the tunnel.
Solve: The time for a single car to pass through the tunnel is: $t_{\text{car}} = (0.5\text{ mi})/(65\text{ mi/h}) = 7.7 \times 10^{-3}$ h. However, if we assume an average car length of 12 ft and a two-car length between each car, the number of cars in one lane of the tunnel at any given instant is:

$$(0.5\text{ mi})\left(\frac{5280\text{ ft}}{1.0\text{ mi}}\right)\left(\frac{1\text{ car}}{36\text{ ft}}\right) = 73\text{ cars.}$$

Considering this string of cars as a single unit passing through the tunnel, the total number of cars that can pass in one lane in time $t = 1.0$ h is:

$$N_{\text{cars}} = (73 \text{ cars/unit})[(1.0 \text{ h})/(7.7 \times 10^{-3} \text{ h/unit})] = 9480 \text{ cars}$$

Since there are two lanes, the final total is roughly 19,000.

1.26. Set Up: Assume that the density of a cell is 1000 kg/m^3, the same as for water. Also assume a mass of 70 kg (a weight of about 150 lbs) for a typical prson. The volume of a sphere is $V = \frac{4}{3}\pi r^3$.
Solve: The volume is: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.0 \times 10^{-6} \text{ m})^3 = 4.2 \times 10^{-18} \text{ m}^3$. The mass of a cell is:

$$\text{mass} = (\text{density})(\text{volume}) = (1000 \text{ kg/m}^3)(4.2 \times 10^{-18} \text{ m}^3) = 4 \times 10^{-15} \text{ kg}$$

The number of cells in the typical person is then $(70 \text{ kg})/(4 \times 10^{-15} \text{ kg/cell}) = 2 \times 10^{16}$ cells.

1.27. Set Up: The number of kernels can be calculated as $N = V_{\text{bottle}}/V_{\text{kernel}}$. Based on an Internet search, Iowan corn farmers use a sieve having a hole size of 0.3125 in. $\cong$ 8 mm to remove kernel fragments. Therefore estimate the average kernel length as 10 mm, the width as 6 mm and the depth as 3 mm. We must also apply the conversion factors 1 L = 1000 cm^3 and 1 cm = 10 mm.
Solve: The volume of the kernel is: $V_{\text{kernel}} = (10 \text{ mm})(6 \text{ mm})(3 \text{ mm}) = 180 \text{ mm}^3$. The bottle's volume is: $V_{\text{bottle}} = (2.0 \text{ L})[(1000 \text{ cm}^3)/(1.0 \text{ L})][(10 \text{ mm})^3/(1.0 \text{ cm})^3] = 2.0 \times 10^6 \text{ mm}^3$. The number of kernels is then $N_{\text{kernels}} = V_{\text{bottle}}/V_{\text{kernels}} \approx (2.0 \times 10^6 \text{ mm}^3)/(180 \text{ mm}^3) = 11{,}000$ kernels.
Reflect: This estimate is highly dependent upon your estimate of the kernel dimensions. And since these dimensions vary amongst the different available types of corn, acceptable answers could range from 6,500 to 20,000.

1.28. Set Up: The number of drops is $N_{\text{drops}} = V_{\text{ocean}}/V_{\text{drop}}$ where $V_{\text{drop}} = \frac{4}{3}\pi(r_{\text{drop}})^3$. Since about 70% of the earth's surface is covered by an ocean, $V_{\text{ocean}} = 0.70A_{\text{surface}}d_{\text{ocean}} = 0.70[4\pi(r_{\text{earth}})^2]d_{\text{ocean}}$. A drop's size is dependent on the orifice from which it originates. If we base our estimate using a simple eye dropper, $r_{\text{drop}} \approx 1.0$ mm. Finally, from Appendix F, the radius of the earth is 6.38×10^6 m.
Solve: $V_{\text{ocean}} = (0.70)(4\pi)(6.38 \times 10^6 \text{ m})^2(4000 \text{ m}) = 1.43 \times 10^{18} \text{ m}^3 = 1.43 \times 10^9 \text{ km}^3$

$$V_{\text{drop}} = \tfrac{4}{3}\pi(1.0 \text{ mm})^3[(1.0 \text{ m})/(1000 \text{ mm})]^3 = 4.2 \times 10^{-9} \text{ m}^3$$

$$N_{\text{drops}} = V_{\text{ocean}}/V_{\text{drop}} = (1.43 \times 10^{18} \text{ m}^3)/(4.2 \times 10^{-9} \text{ m}^3) \approx 3 \times 10^{26} \text{ drops}$$

1.29. Set Up: Estimate the thickness of a dollar bill by measuring a short stack, say ten, and dividing the measurement by the total number of bills. I obtain a thickness of roughly 1 mm. From Appendix F, the distance from the earth to the moon is 3.8×10^8 m. The number of bills is simply this distance divided by the thickness of one bill.
Solve: $N_{\text{bills}} = \left(\dfrac{3.8 \times 10^8 \text{ m}}{0.1 \text{ mm/bill}}\right)\left(\dfrac{10^3 \text{ mm}}{1 \text{ m}}\right) = 3.8 \times 10^{12} \text{ bills} \approx 4 \times 10^{12} \text{ bills}$
Reflect: This answer represents 4 trillion dollars! The cost of a single space shuttle mission in 2005 is significantly less—roughly 1 billion dollars.

1.30. Set Up: For part (a), estimate that a person takes 12 breaths per minute. In part (b), use the relation for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, to calculate the radius required as $r = [3V/(4\pi)]^{1/3}$.
Solve: (a) The estimated number of breaths in two weeks equals:

$$(12 \text{ breaths/min})\left(\frac{60 \text{ min}}{1 \text{ h}}\right)\left(\frac{24 \text{ h}}{1 \text{ day}}\right)\left(\frac{7 \text{ days}}{1 \text{ week}}\right)(2 \text{ weeks}) = 2.4 \times 10^5 \text{ breaths}$$

The total amount of air breathed by one person in two weeks is:

$$V = (\tfrac{1}{2} \text{ L/breath})(2.4 \times 10^5 \text{ breaths})\left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right) = 1 \times 10^2 \text{ m}^3$$

(b) For a spherical vehicle, the radius required is:

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = \left[\frac{3(1 \times 10^2 \text{ m}^3)}{4\pi}\right]^{1/3} = 3 \text{ m.}$$

The diameter of the space station needs to be about 6 m to contain the air for one person for two weeks.

1.31. Set Up: An average middle-aged (40 year-old) adult at rest has a heart rate of roughly 75 beats per minute. To calculate the number of beats in a lifetime, use the current average lifespan of 80 years. The volume of blood pumped during this interval is then the volume per beat multiplied by the total beats.

Solve: $N_{\text{beats}} = (75\text{ beats/min})\left(\dfrac{60\text{ min}}{1\text{ h}}\right)\left(\dfrac{24\text{ h}}{1\text{ day}}\right)\left(\dfrac{365\text{ days}}{\text{yr}}\right)\left(\dfrac{80\text{ yr}}{\text{lifespan}}\right) = 3 \times 10^9\text{ beats/lifespan}$

$$V_{\text{blood}} = (50\text{ cm}^3\text{/beat})\left(\frac{1\text{ L}}{1000\text{ cm}^3}\right)\left(\frac{1\text{ gal}}{3.788\text{ L}}\right)\left(\frac{3 \times 10^9\text{ beats}}{\text{lifespan}}\right) = 4 \times 10^7\text{ gal/lifespan}$$

1.32. Set Up: Estimate that one step is $\frac{1}{3}$ m. Estimate that you walk 100 m in 1 minute. The distance to the Moon is 3.8×10^8 m.

Solve: The time it takes is

$$t = \left(\frac{1\text{ minute}}{100\text{ m}}\right)(3.8 \times 10^8\text{ m}) = 3.8 \times 10^6\text{ minutes,}$$

which is about 7 years. The number of steps would be:

$$\frac{3.8 \times 10^8\text{ m}}{\frac{1}{3}\text{ m/step}} = 1 \times 10^9\text{ steps.}$$

1.33. Set Up: The cost would equal the number of dollar bills required; the surface area of the U.S. divided by the surface area of a single dollar bill. By drawing a rectangle on a map of the U.S., the approximate area is 2600 mi by 1300 mi or 3,380,000 mi^2. This estimate is within 10 percent of the actual area, 3,794,083 mi^2. The population is roughly 3.0×10^8 while the area of a dollar bill, as measured with a ruler, is approximately $6\frac{1}{8}$ in. by $2\frac{5}{8}$ in.

Solve: $A_{\text{U.S.}} = (3{,}380{,}000\text{ mi}^2)[(5280\text{ ft})/(1\text{ mi})]^2[(12\text{ in.})/(1\text{ ft})]^2 = 1.4 \times 10^{16}\text{ in.}^2$

$$A_{\text{bill}} = (6.125\text{ in.})(2.625\text{ in.}) = 16.1\text{ in.}^2$$

$$\text{Total cost} = N_{\text{bills}} = A_{\text{U.S.}}/A_{\text{bill}} = (1.4 \times 10^{16}\text{ in.}^2)/(16.1\text{ in.}^2\text{/bill}) = 9 \times 10^{14}\text{ bills}$$

$$\text{Cost per person} = (9 \times 10^{14}\text{ dollars})/(3.0 \times 10^8\text{ persons}) = 3 \times 10^6\text{ dollars/person}$$

Reflect: The actual cost would be somewhat larger, because the land isn't flat.

1.34. Set Up: Convert the radian values to degrees as:

$$\frac{\pi}{3}\text{ rad} = 60°;\text{ and }\frac{\pi}{6}\text{ rad} = 30°.$$

Solve: The vectors are shown in Figure 1.34.

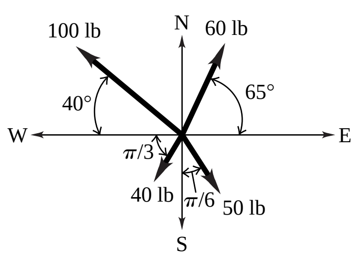


Figure 1.34

1.35. Set Up: The displacement vector $\vec{d}$ is directed from the initial position of the object to the final position. In each case, its magnitude d is the length of the line that connects points 1 and 2.

Solve: **(a)** The initial position (point 1), the final position (point 2), the displacement vector $\vec{d}$ and its direction ϕ is shown in Figure 1.35 for each case.

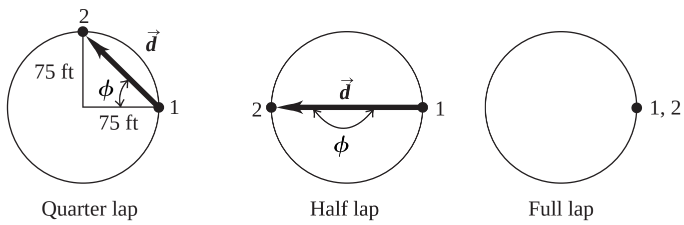


Figure 1.35

(b) ***Quarter lap:*** The magnitude is $d = \sqrt{r^2 + r^2} = \sqrt{2}r = \sqrt{2}(75\ \text{ft}) = 106$ ft. From Figure 1.35, $\tan\phi = r/r = 1.0$ or $\phi = 45°$. ***Half lap:*** The magnitude is $d = 2r = 150$ ft. The direction is $\phi = 180°$. ***Full lap:*** The final position equals the initial position. The displacement is therefore equal to zero and the direction is undefined.
Reflect: In each case, the magnitude of the displacement is less than the distance traveled.

1.36. Set Up: The displacement vector is directed from the initial position of the object to the final position.
Solve: (a) The initial and final positions of the bug are shown in Figure 1.36a. The curved path of the bug is shown in Figure 1.36b and the bug's displacement vector is shown in Figure 1.36c.

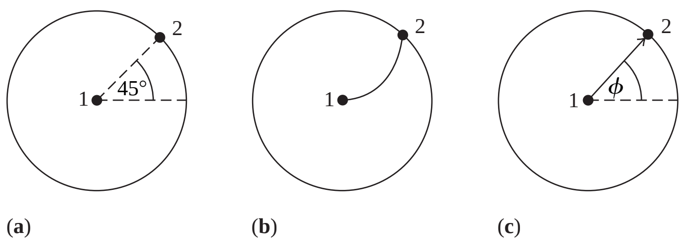


Figure 1.36

(b) The magnitude of the displacement vector is the length of the line that connects points 1 and 2 and is equal to the radius of the turntable, 6 inches. The direction of the displacement vector is shown in Figure 1.36c, with $\phi = 45°$.

1.37. Set Up: Draw the vectors to scale on graph paper, using the tip to tail addition method. For part (a), simply draw $\vec{B}$ so that its tail lies at the tip of $\vec{A}$. Then draw the vector $\vec{R}$ from the tail of $\vec{A}$ to the tip of $\vec{B}$. For (b), add $-\vec{B}$ to $\vec{A}$ by drawing $-\vec{B}$ in the opposite direction to $\vec{B}$. For (c), add $-\vec{A}$ to $-\vec{B}$. For (d), add $-\vec{A}$ to $\vec{B}$.
Solve: The vector sums and differences are shown in Figure. 1.37a-d.

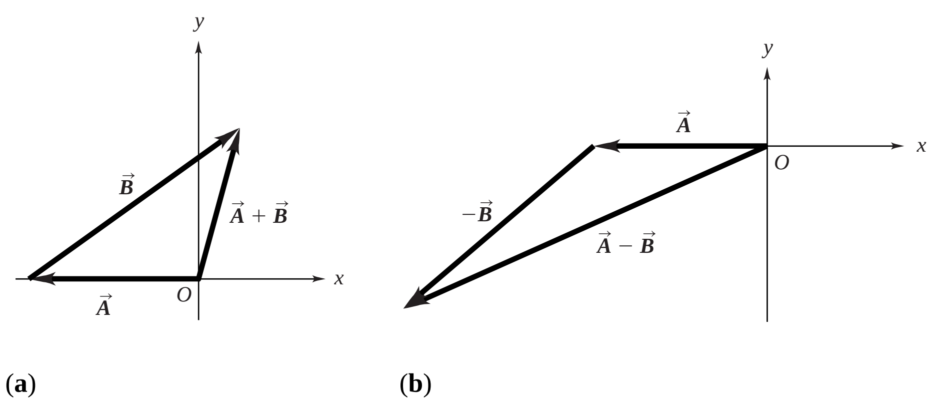


Figure 1.37

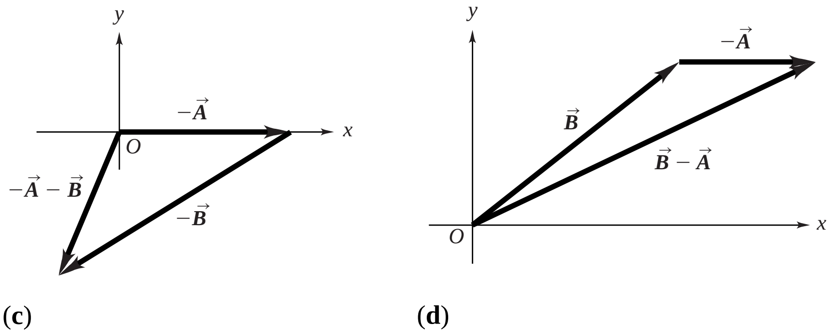


Figure 1.37

Reflect: $-\vec{A}-\vec{B}=-(\vec{A}+\vec{B})$ so it has the same magnitude and opposite direction as $\vec{A}+\vec{B}$. Similarly, $\vec{B}-\vec{A}=-(\vec{A}-\vec{B})$ and $\vec{B}-\vec{A}$ and $\vec{A}-\vec{B}$ have equal magnitudes and opposite directions.

1.38. Set Up: The two forces applied by the cables add to give a resultant force that is vertical and has magnitude 2.25 N. Draw the vector addition diagram with the two forces head to tail.
Solve: (a) The vector addition diagram is drawn in Figure 1.38.

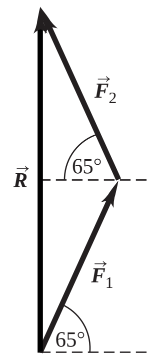


Figure 1.38

(b) Careful measurement of the length of F_1 and F_2 gives $F_1 = F_2 = 1.24$ N.

1.39. Set Up: Use a ruler and protractor to draw the vectors described. Then draw the corresponding horizontal and vertical components.
Solve: (a) Figure 1.39 gives components 4.7 m, 8.1 m.

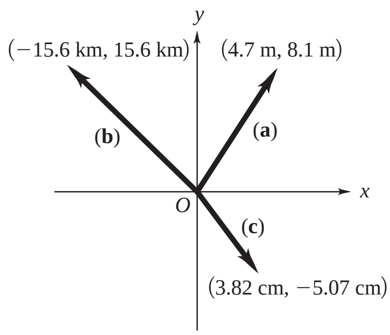


Figure 1.39

(b) Figure 1.39 gives components −15.6 km, 15.6 km.
(c) Figure 1.39 gives components 3.82 cm, −5.07 cm.

1.40. Set Up: In each case the direction of the vector is specified by the angle θ measured counterclockwise from the $+x$ axis. The components are calculated as: $A_x = A\cos\theta$; $A_y = A\sin\theta$.

Solve: (a) $A_x = (50.0\ \text{N})(\cos 60°) = +25.0\ \text{N}$; $A_y = (50.0\ \text{N})(\sin 60°) = +43.3\ \text{N}$. The sketch of the vector and its components in Figure 1.40a shows that both components are positive and that $|A_y| > |A_x|$.

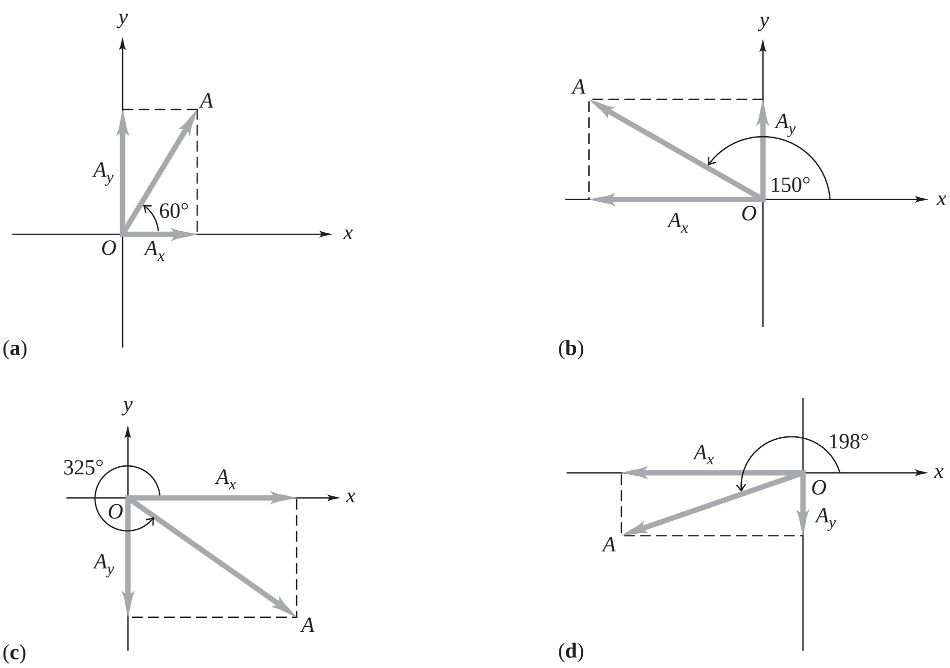


Figure 1.40

(b) $\theta = (5\pi/6\ \text{rad})(180°/\pi\ \text{rad}) = 150°$; $A_x = (75\ \text{m/s})(\cos 150°) = -65.0\ \text{m/s}$; $A_y = (75\ \text{m/s})(\sin 150°) = +37.5\ \text{m/s}$. Figure 1.40b shows that A_x is negative, A_y is positive and $|A_x| > |A_y|$.

(c) $A_x = (254\ \text{lb})(\cos 325°) = 208\ \text{lb}$; $A_y = (254\ \text{lb})(\sin 325°) = -146\ \text{lb}$. Figure 1.40c shows that A_x is positive, A_y is negative and $|A_x| > |A_y|$.

(d) $\theta = (1.1\pi\ \text{rad})(180°/\pi\ \text{rad}) = 198°$; $A_x = (69\ \text{km})(\cos 198°) = -66\ \text{km}$; $A_y = (69\ \text{km})(\sin 198°) = -21\ \text{km}$. Figure 1.40d shows that both A_x and A_y are negative and $|A_x| > |A_y|$.

1.41. Set Up: In each case, create a sketch (Figure 1.41) showing the components and the resultant to determine the quadrant in which the resultant vector $\vec{A}$ lies. The component vectors add to give the resultant.

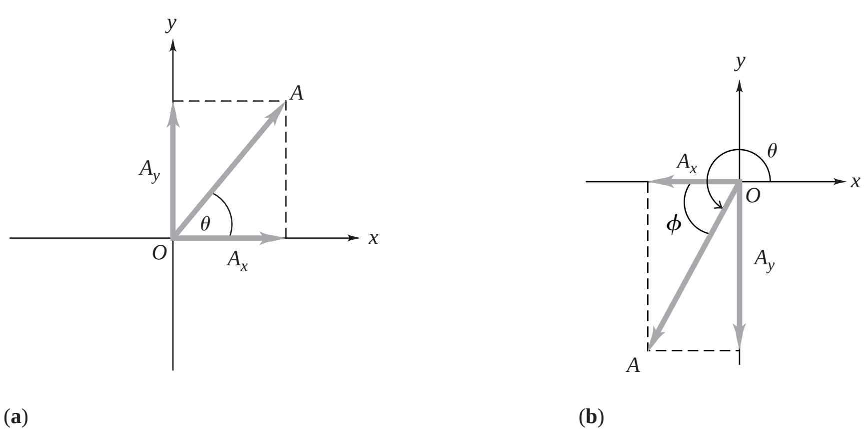


Figure 1.41

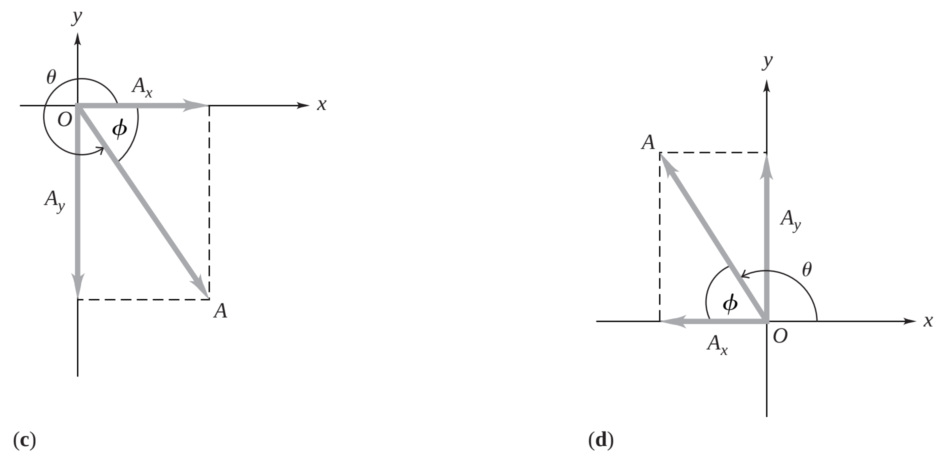


Figure 1.41

Solve: (a) $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(4.0\text{ m})^2 + (5.0\text{ m})^2} = 6.4\text{ m}$; $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{5.0\text{ m}}{4.0\text{ m}}$ and $\theta = 51°$.

(b) $A = \sqrt{(-3.0\text{ km})^2 + (-6.0\text{ km})^2} = 6.7\text{ km}$; $\tan\theta = \dfrac{-6.0\text{ m}}{-3.0\text{ m}}$ and $\theta = 243°$.

(My calculator gives $\phi = 63°$ and $\theta = \phi + 180°$.)

(c) $A = \sqrt{(9.0\text{ m/s})^2 + (-17\text{ m/s})^2} = 19\text{ m/s}$; $\tan\theta = \dfrac{-17\text{ m/s}}{9\text{ m/s}}$ and $\theta = 298°$.

(My calculator gives $\phi = -62°$ and $\theta = \phi + 360°$.)

(d) $A = \sqrt{(-8.0\text{ N})^2 + (12\text{ N})^2} = 14\text{ N}$. $\tan\theta = \dfrac{12\text{ N}}{-8.0\text{ N}}$ and $\theta = 124°$

(My calculator gives $\phi = -56°$ and $\theta = \phi + 180°$.)

Reflect: The signs of the components determine the quadrant in which the resultant lies.

1.42. Set Up: Use coordinates for which the $+x$ axis is horizontal and the $+y$ direction is upward. The force $\vec{F}$ and its x and y components are shown in Figure 1.42.

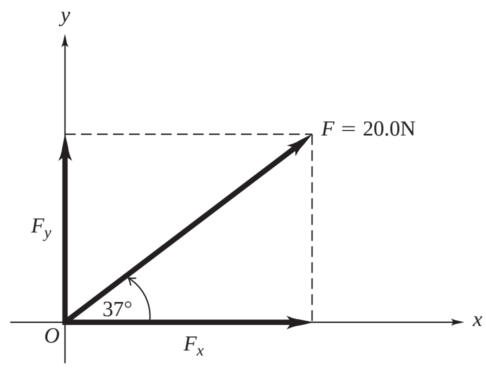


Figure 1.42

Solve: (a) $F_x = F(\cos 37°) = (20.0\text{ N})(\cos 37°) = 16.0\text{ N}$

(b) $F_y = F\sin 37° = (20.0\text{ N})(\sin 37°) = 12.0\text{ N}$

1.43. Set Up: In each case, use a sketch (Figure 1.43) showing the components and the resultant in order to determine the quadrant in which the resultant vector $\vec{A}$ lies. The component vectors add to give the resultant.

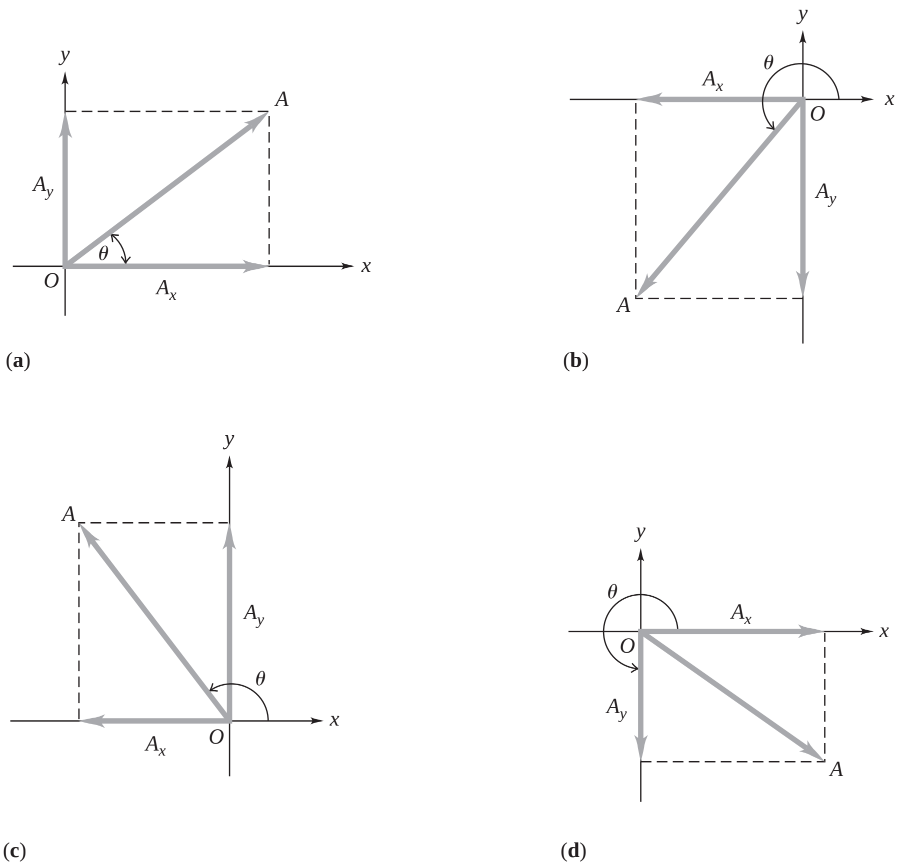


Figure 1.43

Solve: (a) $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(8.0\text{ lb})^2 + (6.0\text{ lb})^2} = 10.0\text{ lb}$; $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{6.0\text{ lb}}{8.0\text{ lb}}$ and $\theta = 37°$.

(b) $A = \sqrt{(-24\text{ m/s})^2 + (-31\text{ m/s})^2} = 39\text{ m/s}$; $\tan\theta = \dfrac{-31\text{ m/s}}{-24\text{ m/s}}$ and $\theta = 232°$

(My calculator gives $\phi = 52°$ and $\theta = \phi + 180°$.)

(c) $A = \sqrt{(-1500\text{ km})^2 + (2000\text{ km})^2} = 2500\text{ km}$; $\tan\theta = \dfrac{2000\text{ km}}{-1500\text{ km}}$ and $\theta = 127°$

(My calculator gives $\phi = -53°$ and $\theta = \phi + 180°$.)

(d) $A = \sqrt{(71.3\text{ N})^2 + (-54.7\text{ N})^2} = 89.9\text{ N}$; $\tan\theta = \dfrac{-54.7\text{ N}}{71.3\text{ N}}$ and $\theta = 323°$

(My calculator gives $\phi = -37°$ and $\theta = \phi + 360°$.)

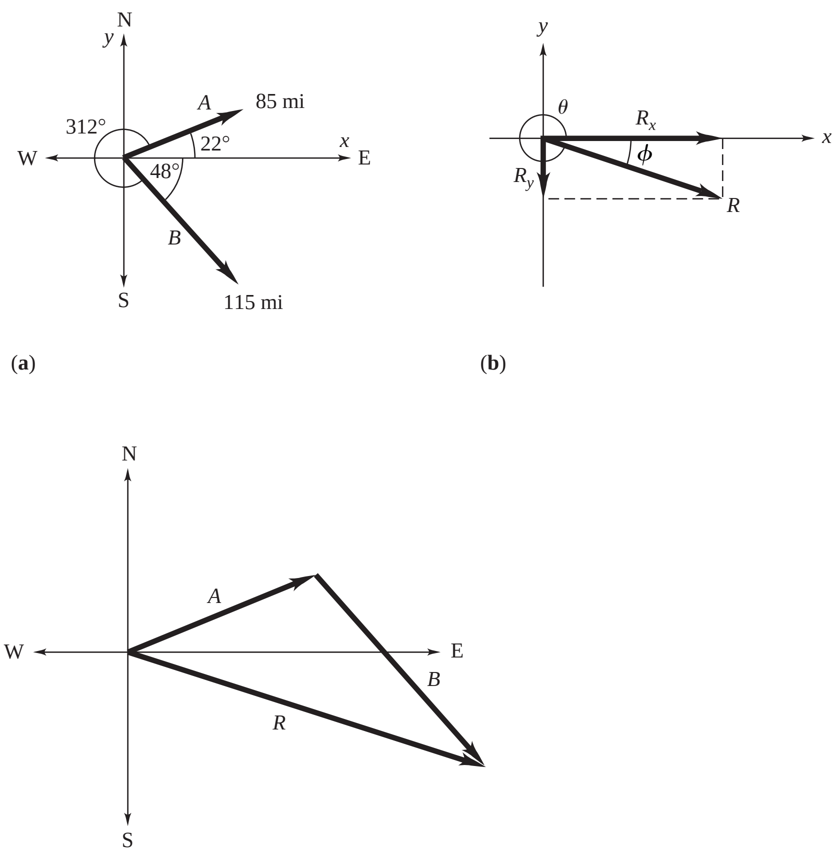
1.44. Set Up: For each vector, use the relations $R_x = R\cos\theta$ and $R_y = R\sin\theta$.

Solve: For vector $\vec{A}$: $A_x = (12.0\text{ m})\cos(90° - 37°) = 7.2\text{ m}$; $A_y = (12.0\text{ m})\sin(90° - 37°) = 9.6\text{ m}$. For vector $\vec{B}$: $B_x = (15.0\text{ m})\cos(320°) = 11.5\text{ m}$; $B_y = (15.0\text{ m})\sin(320°) = -9.6\text{ m}$. For vector $\vec{C}$: $C_x = (6.0\text{ m})\cos(240°) = -3.0\text{ m}$; $C_y = (6.0\text{ m})\sin(240°) = -5.2\text{ m}$.

1.45. Set Up: For parts (a) and (c), apply the appropriate signs to the relations $R_x = A_x + B_x$ and $R_y = A_y + B_y$. For (b) and (d), find the magnitude as $R = \sqrt{R_x^2 + R_y^2}$ and the direction as $\theta = \tan^{-1}(R_y/R_x)$.

Solve: (a) $R_x = A_x + B_x = 1.30\text{ cm} + 4.10\text{ cm} = 5.40\text{ cm}$; $R_y = A_y + B_y = 2.25\text{ cm} + (-3.75\text{ cm}) = -1.50\text{ cm}$
(b) $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(5.40\text{ cm})^2 + (-1.50\text{ cm})^2} = 5.60\text{ cm}$; $\theta = \tan^{-1}[(-1.50\text{ cm})/(5.40\text{ cm})] = -15.5°$. The resultant vector thus makes an angle of 344.5° counterclockwise from the $+x$ axis.
(c) $R_x = B_x + (-A_x) = 4.10\text{ cm} + (-1.30\text{ cm}) = 2.80\text{ cm}$; $R_y = B_y + (-A_y) = -3.75\text{ cm} + (-2.25\text{ cm}) = -6.00\text{ cm}$
(d) $R = \sqrt{(2.80\text{ cm})^2 + (-6.00\text{ cm})^2} = 6.62\text{ cm}$; $\theta = \tan^{-1}[(-6.00\text{ cm})/(2.80\text{ cm})] = -65.0°$. The resultant vector thus makes an angle of 295.0° counterclockwise from the $+x$ axis.
Reflect: Note that $\vec{B} - \vec{A}$ has a larger magnitude than $\vec{B} + \vec{A}$. Vector addition is very different from addition of scalars.

1.46. Set Up: Use coordinates where $+x$ is east and $+y$ is north. $\vec{A}$ and $\vec{B}$ are the two given displacements (Figure 1.46a). Displacement $\vec{B}$ makes an angle of $\theta = 312°$ counterclockwise from the $+x$ axis. Calculate $\vec{R} = \vec{A} + \vec{B}$.



(c)

Figure 1.46

Solve: (a) $A_x = A\cos 22° = (85\text{ mi})(\cos 22°) = 78.8\text{ mi}$; $A_y = A\sin 22° = (85\text{ mi})(\sin 22°) = 31.8\text{ mi}$.
$B_x = B\cos 312° = (115\text{ mi})(\cos 312°) = 77.0\text{ mi}$; $B_y = B\sin 312° = (115\text{ mi})(\sin 312°) = -85.5\text{ mi}$
$R_x = A_x + B_x = 78.8\text{ mi} + 77.0\text{ mi} = 155.8\text{ mi}$; $R_y = A_y + B_y = 31.8\text{ mi} + (-85.5\text{ mi}) = -53.7\text{ mi}$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(155.8\text{ mi})^2 + (-53.7\text{ mi})^2} = 165\text{ mi};\ \tan\theta = \frac{-53.7\text{ mi}}{155.8\text{ mi}}\text{ and }\theta = 341°;$$

or $\phi = 19°$ south of east, as shown in Figure 1.46b.
(b) The vector addition diagram for $\vec{R} = \vec{A} + \vec{B}$ is given in Figure 1.46c. The magnitude and direction of $\vec{R}$ in this diagram agrees with our calculation using components.

1.47. Set Up: Use coordinates for which the $+y$ axis is downward. Let the two vectors be $\vec{A}$ and $\vec{B}$ and let the angles they make on either side of the $+y$ axis be ϕ_A and ϕ_B. Since the resultant, $\vec{R} = \vec{A} + \vec{B}$, is in the $+y$ direction, $R_x = 0$. The two vectors and their components are shown in Figure 1.47a.

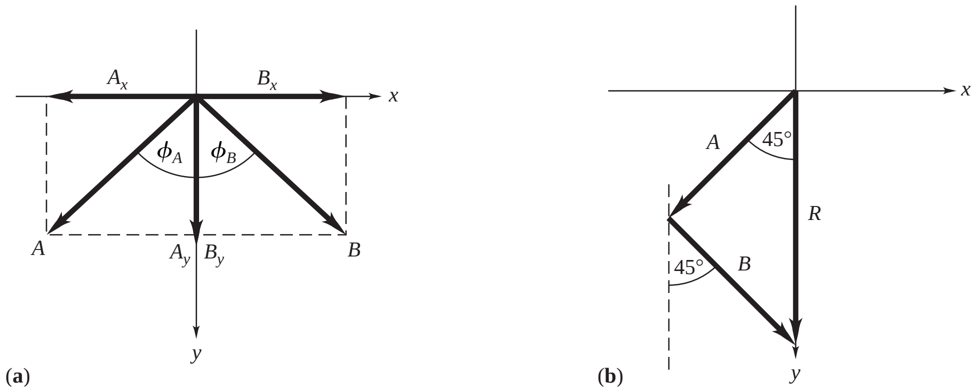


Figure 1.47

Solve: (a) $R_x = A_x + B_x = -A\sin\phi_A + B\sin\phi_B$. Since $A = B$ and $R_x = 0$, we can conclude that $\sin\phi_A = \sin\phi_B$ and $\phi_A = \phi_B$. Similarly, since $\phi_A + \phi_B = 90°$, $\phi_A = \phi_B = 45°$. The vector addition diagram for $\vec{R} = \vec{A} + \vec{B}$ is given in Figure 1.47b.

(b) $A_y + B_y = R_y$ and $R_y = 75$ N. $A\cos\phi_A + B\cos\phi_B = 75$ N. $A = B$ and $\phi_A = \phi_B = 45°$, so $2A(\cos 45°) = 75$ N and $A = 53$ N.

Reflect: $A + B = 106$ N. The magnitude of the vector sum $\vec{A} + \vec{B}$ is less than the sum of the magnitudes $A + B$.

1.48. Set Up: The counterclockwise angles each vector makes with the $+x$ axis are: $\theta_A = 30°$, $\theta_B = 120°$ and $\theta_C = 233°$. The components of each vector are shown in Figure 1.48a.

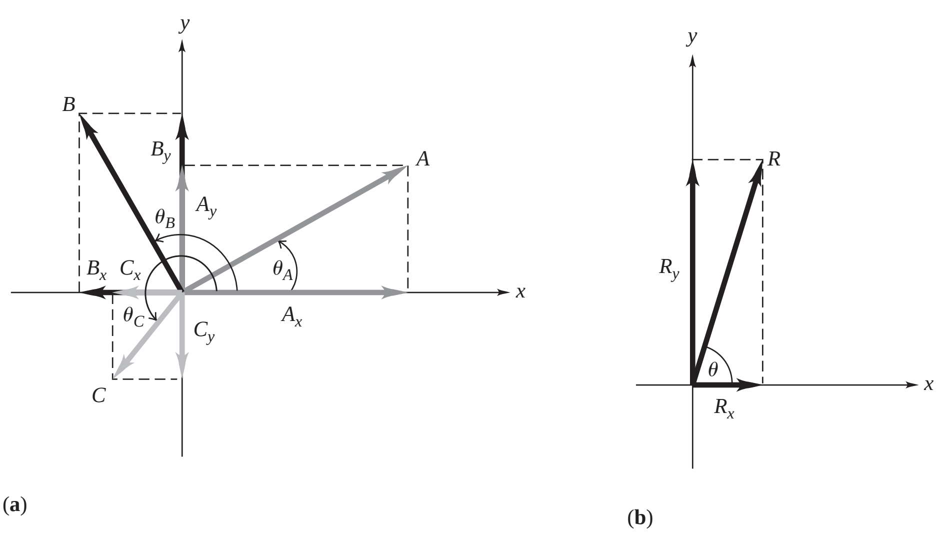
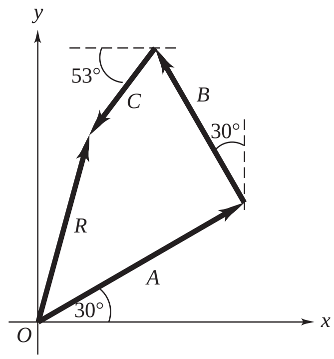


Figure 1.48



(c)

Figure 1.48

Solve: (a) $A_x = A\cos 30° = 87\text{ N}$; $A_y = A\sin 30° = 50\text{ N}$; $B_x = B\cos 120° = -40\text{ N}$; $B_y = B\sin 120° = 69\text{ N}$; $C_x = C\cos 233° = -24\text{ N}$; $C_y = C\sin 233° = -32\text{ N}$.

(b) $\vec{R} = \vec{A} + \vec{B} + \vec{C}$ is the resultant pull.

$$R_x = A_x + B_x + C_x = 87\text{ N} + (-40\text{ N}) + (-24\text{ N}) = +23\text{ N}$$

$$R_y = A_y + B_y + C_y = 50\text{ N} + 69\text{ N} + (-32\text{ N}) = +87\text{ N}$$

(c) R_x, R_y and $\vec{R}$ are shown in Figure 1.48b.

$$R = \sqrt{R_x^2 + R_y^2} = 90\text{ N and } \tan\theta = \frac{87\text{ N}}{23\text{ N}} \text{ so } \theta = 75°$$

(d) The vector addition diagram is given in Figure 1.48c. Careful measurement gives an $\vec{R}$ value that agrees with our results using components.

1.49. Set Up: Use coordinates for which $+x$ is east and $+y$ is north. Each of the professor's displacement vectors make an angle of 0° or 180° with one of these axes. The components of his total displacement can thus be calculated directly from $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$.

Solve: (a) $R_x = A_x + B_x + C_x = 0 + (-4.75\text{ km}) + 0 = -4.75\text{ km} = 4.75\text{ km west}$; $R_y = A_y + B_y + C_y = 3.25\text{ km} + 0 + (-1.50\text{ km}) = 1.75\text{ km} = 1.75\text{ km north}$; $R = \sqrt{R_x^2 + R_y^2} = 5.06\text{ km}$; $\theta = \tan^{-1}(R_y/R_x) = \tan^{-1}[(+1.75)/(-4.75)] = -20.2°$; $\phi = 180° - 20.2° = 69.8°$ west of north

(b) From the scaled sketch in Figure 1.49, the graphical sum agrees with the calculated values.

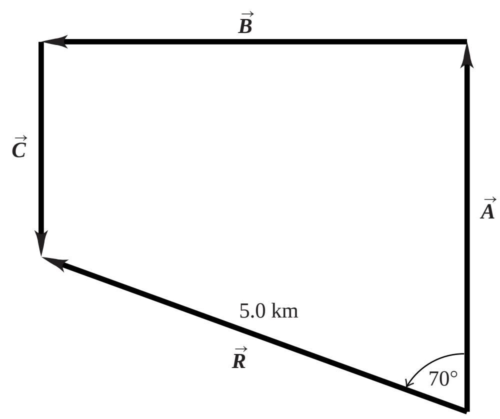


Figure 1.49

Reflect: The magnitude of his resultant displacement is very different from the distance he traveled, which is 159.50 km.

1.50. Set Up: Use coordinates for which $+x$ is east and $+y$ is north. The driver's vector displacements are: $\vec{A} = 2.6$ km, 0° of north; $\vec{B} = 4.0$ km, 0° of east; $\vec{C} = 3.1$ km, 45° north of east.

Solve: $R_x = A_x + B_x + C_x = 0 + 4.0\text{ km} + (3.1\text{ km})\cos(45°) = 6.2\text{ km}$; $R_y = A_y + B_y + C_y = 2.6\text{ km} + 0 + (3.1\text{ km})(\sin 45°) = 4.8\text{ km}$; $R = \sqrt{R_x^2 + R_y^2} = 7.8\text{ km}$; $\theta = \tan^{-1}[(4.8\text{ km})/(6.2\text{ km})] = 38°$; $\vec{R} = 7.8$ km, 38° north of east. This result is confirmed by the sketch in Figure 1.50.

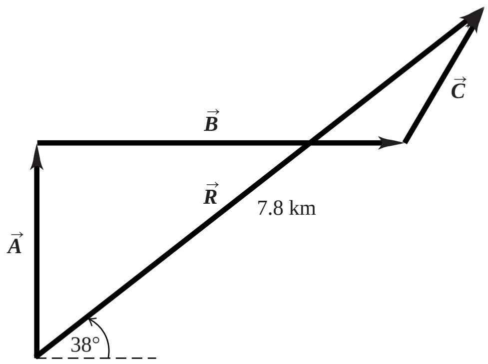


Figure 1.50

1.51. Set Up: We know that an object of mass 1000 g weighs 2.205 lbs and 16 oz = 1 lb.

Solve: $(5.00\text{ oz})\left(\dfrac{1\text{ lb}}{16\text{ oz}}\right)\left(\dfrac{1000\text{ g}}{2.205\text{ lb}}\right) = 142\text{ g}$ and $(5.25\text{ oz})\left(\dfrac{1\text{ lb}}{16\text{ oz}}\right)\left(\dfrac{1000\text{ g}}{2.205\text{ lb}}\right) = 149\text{ g}$

The acceptable limits are 142 g to 149 g.

Reflect: The range in acceptable weight is $\frac{1}{4}$ oz. Since $\frac{1}{4}$ oz = 7 g, the range in mass of 7 g is consistent. As we will see in a later chapter, mass has units of kg or g and is a different physical quantity than weight, which can have units of ounces. But for objects close to the surface of the earth, mass and weight are proportional; we can therefore say that a certain weight is equivalent to a certain mass.

1.52. Set Up: Use your calculator to display $\pi \times 10^7$. We know that 1 yr = 365.24 days, 1 day = 24 h, and 1 h = 3600 s.

Solve: $(365.24\text{ days}/1\text{ yr})\left(\dfrac{24\text{ h}}{1\text{ day}}\right)\left(\dfrac{3600\text{ s}}{1\text{ h}}\right) = 3.15567\ldots \times 10^7\text{ s}$; $\pi \times 10^7\text{ s} = 3.14159\ldots \times 10^7\text{ s}$

The approximate expression is accurate to two significant figures.

1.53. Set Up: Estimate 12 breaths per minute. We know 1 day = 24 h, 1 h = 60 min and 1000 L = 1 m^3.

Solve: (a) $(12\text{ breaths/min})\left(\dfrac{60\text{ min}}{1\text{ h}}\right)\left(\dfrac{24\text{ h}}{1\text{ day}}\right) = 17{,}280\text{ breaths/day}$. The volume of air breathed in one day is $(\frac{1}{2}\text{ L/breath})(17{,}280\text{ breaths/day}) = 8640\text{ L} = 8.64\text{ m}^3$.

The mass of air breathed in one day is the density of air times the volume of air breathed: $m = (1.29\text{ kg/m}^3)(8.64\text{ m}^3) = 11.1\text{ kg}$. As 20% of this quantity is oxygen, the mass of oxygen breathed in 1 day is $(0.20)(11.1\text{ kg}) = 2.2\text{ kg} = 2200\text{ g}$.

(b) 8.64 m^3 and $V = l^3$, so $l = V^{1/3} = 2.1\text{ m}$.

Reflect: A person could not survive one day in a closed tank of this size because the exhaled air is breathed back into the tank and thus reduces the percent of oxygen in the air in the tank. That is, a person cannot extract all of the oxygen from the air in an enclosed space.

1.54. Set Up: Make the following estimates: a person takes 10 breaths per minute; the jumbo jet has 30 rows, each 10 passengers wide; each row, including adjacent aisles, is 40 ft wide and has a width of 4.0 ft; the cabin height is 8 ft. Use the equality 1 ft^3 = 0.02832 m^3.

Solve: The total air breathed in by each person is: $(\frac{1}{2}\text{ L/breath})(10\text{ breaths/min})(60\text{ min/h})(17\text{ h}) = 5100\text{ L}$
Then 300 people breathe: $(300\text{ persons})(5100\text{ L/person}) = 1.53 \times 10^6\text{ L} = 1530\text{ m}^3$.
The volume of the airplane cabin is approximately:

$$(8\text{ ft})(40\text{ ft})(30\text{ rows})(4.0\text{ ft/row}) = (38{,}400\text{ ft}^3)\left(\frac{0.02832\text{ m}^3}{1\text{ ft}^3}\right) = 1100\text{ m}^3$$

The volume of air breathed and the volume of the cabin of the airliner are comparable.

1.55. Set Up: The volume V of blood pumped during each heartbeat is the total volume of blood in the body divided by the number of heartbeats in 1.0 min. We will need to apply $1\text{ L} = 1000\text{ cm}^3$.
Solve: The number of heartbeats in 1.0 min is 75. The volume of blood is thus:

$$V = \frac{5.0\text{ L}}{75\text{ heartbeats}} = 6.7 \times 10^{-2}\text{ L} = 67\text{ cm}^3.$$

1.56. Set Up: The x axis is shown to lie along the axis of the bone. Let $F = 2.75\text{ N}$ be the magnitude of the force applied by each tendon. The angles that the tendons make with the x axis are 20.0°, 30.0°, 40.0°, 50.0°, and 60.0°.
Solve: (a) $R_x = F_{1x} + F_{2x} + F_{3x} + F_{4x} + F_{5x}$

$$R_x = (2.75\text{ N})(\cos 20.0° + \cos 30.0° + \cos 40.0° + \cos 50.0° + \cos 60.0°) = 10.2\text{ N}$$

$$R_y = F_{1y} + F_{2y} + F_{3y} + F_{4y} + F_{5y}$$

$$R_y = (2.75\text{ N})(\sin 20.0° + \sin 30.0° + \sin 40.0° + \sin 50.0° + \sin 60.0°) = 8.57\text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = 13.3\text{ N};\ \tan\phi = \frac{8.57\text{ N}}{10.2\text{ N}}\text{ so } \phi = 40.0°$$

The resultant $\vec{R}$ and its components are shown in Figure 1.56a.

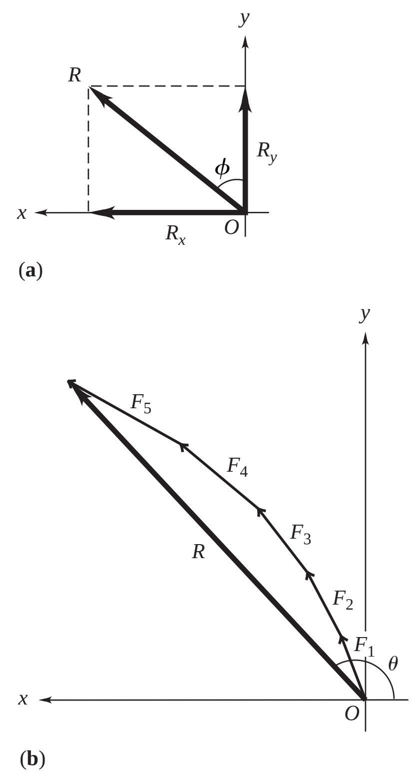


Figure 1.56

(b) The vector addition diagram is shown in Figure 1.56b. Careful measurements in the diagram give results that agree with what we calculated using components.

1.57. Set Up: Estimate 2 ft per step and 100 steps per minute. Use: 1 mi = 5280 ft.
Solve: (a) The distance is $(2160\text{ mi})(5280\text{ ft/mi}) = 1.14 \times 10^7$ ft. The number of steps is calculated as this distance divided by the distance per step:

$$\frac{1.14 \times 10^7\text{ ft}}{2\text{ ft/step}} = 5.7 \times 10^6\text{ steps.}$$

(b) The time in minutes is the number of steps required divided by the number of steps per minute

$$\frac{5.7 \times 10^6\text{ steps}}{100\text{ steps/min}} = 5.7 \times 10^4\text{ min} = 950\text{ h.}$$

At 8 h per day it takes about 120 days.

1.58. Set Up: Assume you walk 1 mile each way.
Solve: (a) In 15 years, there are 780 weeks. If 45 weeks are vacation, you walk 735 weeks which represents 3675 working days. Since you walk 2 miles per working day, $(2\text{ miles/day})(3675\text{ days}) = 7400$ mi.
(b) In the U.S., this is approximately the equivalent of two coast-to-coast trips.

1.59. Set Up: Calculate the volume of liquid in liters using 128 oz = 1 gal = 3.788 L. Use the result to calculate the mass of arsenic using the given arsenic per liter quantity 10 μg/L and the unit conversion 1 μg $= 1 \times 10^{-6}$ g.
Solve: You drink 64 oz = 2 qt = 0.5 gal or $(0.5\text{ gal})[(3.788\text{ L})/(1\text{ gal})] = 1.894$ L. The mass of arsenic received per day is $(10\ \mu\text{g/L})(1.894\text{ L}) = 19\ \mu\text{g} = 1.9 \times 10^{-5}$ g.
Reflect: While this is a very small amount of arsenic, concern about the accumulation in the body over a long period of time may exist.

1.60. Set Up: Assume a weight of 190 lbs, which corresponds to a mass of 86 kg. I estimate the volume V of my body by approximating it as a rectangular solid with dimensions: 67 in. × 12 in. × 8 in.
Solve: (a) $V = (67\text{ in.})(12\text{ in.})(8\text{ in.}) = (6.4 \times 10^3\text{ in.}^3)\left(\dfrac{10^{-3}\text{ m}^3}{61.02\text{ in.}^3}\right) = 0.10\text{ m}^3$

$$\text{average density} = \frac{\text{mass}}{\text{volume}} = \frac{86\text{ kg}}{0.10\text{ m}^3} = 860\text{ kg/m}^3$$

(b) According to this estimate, my average density is somewhat less than the density of water. This is sensible; I know I float in water because my body contains air cavities.

1.61. Set Up: Use coordinates for which $+x$ is east and $+y$ is north. The spelunker's vector displacements are: $\vec{A}$ = 180 m, 0° of west; $\vec{B}$ = 210 m, 45° east of south; $\vec{C}$ = 280 m, 30° east of north; and the unknown displacement $\vec{D}$. The vector sum of these four displacements is zero.
Solve: $A_x + B_x + C_x + D_x = -180\text{ m} + (210\text{ m})(\sin 45°) + (280\text{ m})(\sin 30°) + D_x = 0$ and $D_x = -108$ m. $A_y + B_y + C_y + D_y = (-210\text{ m})(\cos 45°) + (280\text{ m})(\cos 30°) + D_y = 0$ and $D_y = -94$ m.

$$D = \sqrt{D_x^2 + D_y^2} = 143\text{ m};\ \theta = \tan^{-1}[(-94\text{ m})/(-108)] = 41°;\ \vec{D} = 143\text{ m, } 41° \text{ south of west.}$$

This result is confirmed by the sketch in Figure 1.61.

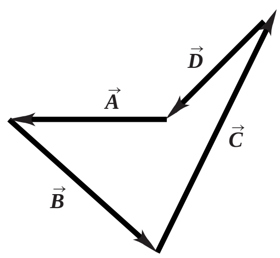


Figure 1.61

Reflect: We always add vectors by separately adding their x and y components.

1.62. Set Up: Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are: $\vec{A} = 2.00$ km, 0° of east; $\vec{B} = 3.50$ m, 45° south of east; and $\vec{R} = 5.80$ m, 0° east

Solve: $C_x = R_x - A_x - B_x = 5.80 \text{ km} - (2.00 \text{ km}) - (3.50 \text{ km})(\cos 45°) = 1.33 \text{ km}$; $C_y = R_y - A_y - B_y = 0 \text{ km} - 0 \text{ km} - (-3.50 \text{ km})(\sin 45°) = 2.47 \text{ km}$;

$$C = \sqrt{(1.33 \text{ km})^2 + (2.47 \text{ km})^2} = 2.81 \text{ km};\ \theta = \tan^{-1}[(2.47 \text{ km})/(1.33 \text{ km})] = 61.7° \text{ north of east.}$$

The vector addition diagram in Figure 1.62 shows good qualitative agreement with these values.

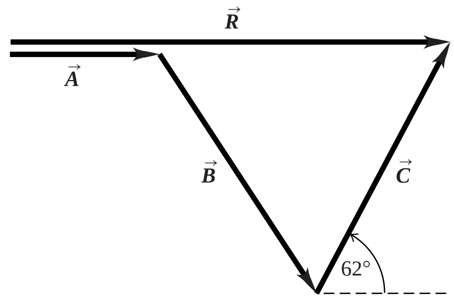


Figure 1.62

1.63. Set Up: Use coordinates having a horizontal $+x$ axis and an upward $+y$ axis. Then $R_x = 5.60$ N.

Solve: $A_x + B_x = R_x$ and $A\cos 32° + B\sin 32° = R_x$. Since $A = B$, $2A\cos 32° = R_x$ and $A = R_x/[(2)(\cos 32°)] = 3.30$ N.

1.64. Set Up: For parts (a) and (b), draw a vector diagram of the forces on the boulder with the $+x$ direction along the hillside and the $+y$ direction in the downward direction and perpendicular to the hillside. For part (c), draw a similar diagram with $\alpha = 35.0°$ and $w = 550$ N.

(a) $w_x = w\sin\alpha$ **(b)** $w_y = w\cos\alpha$ **(c)** The maximum allowable weight is $w = w_x/(\sin\alpha) = (550 \text{ N})/(\sin 35.0°) = 959$ N.

1.65. Set Up: Referring to the vector diagram in Figure 1.65, the resultant weight of the forearm and the weight, $\vec{R}$, is 132.5 N upward while the biceps' force, $\vec{B}$, acts at 43° from the $+y$ axis. The force of the elbow is thus found as $\vec{E} = \vec{R} - \vec{B}$.

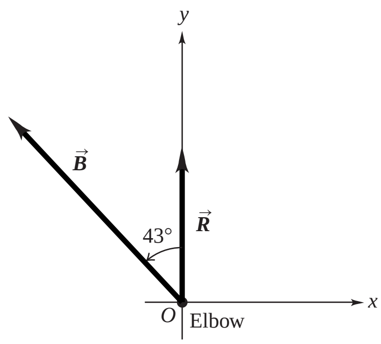


Figure 1.65

Solve: The components are: $E_x = R_x - B_x = 0 - (-232 \text{ N})(\sin 43°) = 158 \text{ N}$; $E_y = R_y - B_y = 132.5 \text{ N} - (232 \text{ N})(\cos 43°) = -37$ N. The elbow force is thus

$$E = \sqrt{(158 \text{ N})^2 + (-37 \text{ N})^2} = 162 \text{ N}$$

and acts at $\theta = \tan^{-1}[(-37)/158] = -13°$ or 13° below the horizontal.

Reflect: The force exerted by the elbow is larger that the total weight of the arm and the object it is carrying.

1.66. Set Up: Approximate the mass of an atom by the mass m_p of a proton, where $m_p = 1.67 \times 10^{-27}$ kg. The mass of the sun is 1.99×10^{30} kg.
Solve: (a) 10^{100} represents the numeral "1" followed by one hundred zeroes.

(b) The number of atoms in the sun is about $\frac{1.99 \times 10^{30}\text{ kg}}{1.67 \times 10^{-27}\text{ kg}} = 1 \times 10^{57}$ atoms.

$$1 \times 10^{57}\left(\frac{1\text{ googol}}{10^{100}}\right) = 1 \times 10^{-43}\text{ googol}$$

(c) $1.0 \times 10^{1....0}$, where 1. . . .0 is the numeral "1" followed by one hundred zeroes. In standard notation, a goololplex is "1" followed by a googol of zeroes.