CHAPTER ONE

Solutions for Section 1.1

1. **(a)** The story in (a) matches Graph (IV), in which the person forgot her books and had to return home.
(b) The story in (b) matches Graph (II), the flat tire story. Note the long period of time during which the distance from home did not change (the horizontal part).
(c) The story in (c) matches Graph (III), in which the person started calmly but sped up later.
The first graph (I) does not match any of the given stories. In this picture, the person keeps going away from home, but his speed decreases as time passes. So a story for this might be: *I started walking to school at a good pace, but since I stayed up all night studying calculus, I got more and more tired the farther I walked.*

2. The height is going down as time goes on. A possible graph is shown in Figure 1.1. The graph is decreasing.

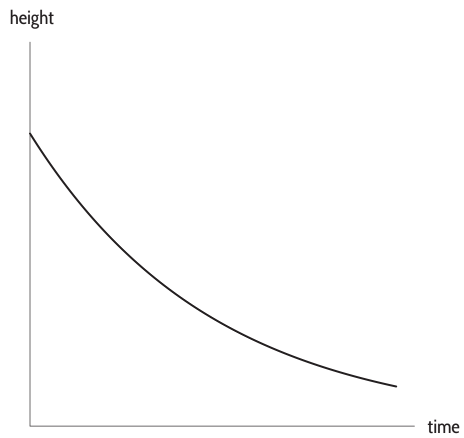


Figure 1.1

3. The amount of carbon dioxide is going up as time goes on. A possible graph is shown in Figure 1.2. The graph is increasing.

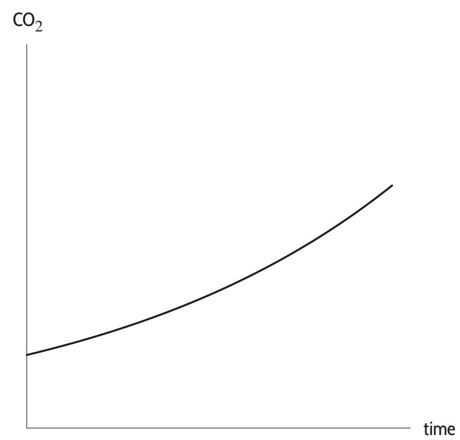


Figure 1.2

4. The number of air conditioning units sold is going up as temperature goes up. A possible graph is shown in Figure 1.3. The graph is increasing.

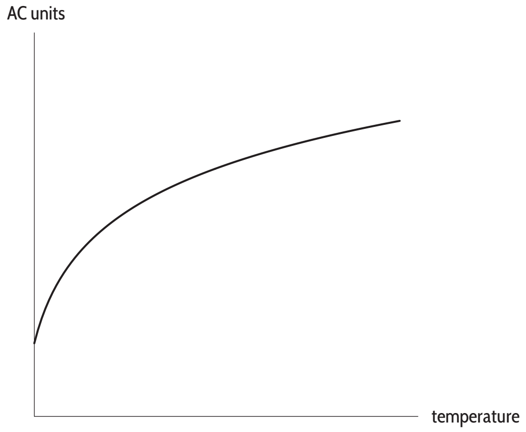


Figure 1.3

5. The noise level is going down as distance goes up. A possible graph is shown in Figure 1.4. The graph is decreasing.

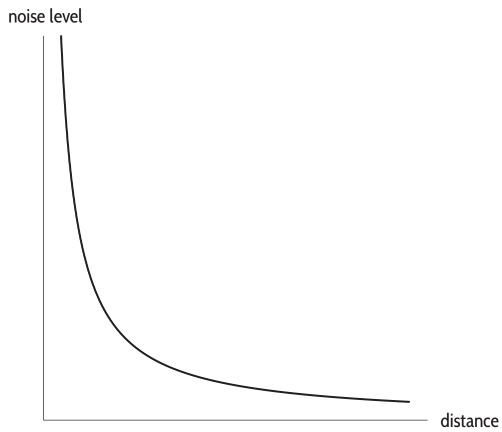


Figure 1.4

6. If we let t represent the number of years since 1900, then the population increased between $t = 0$ and $t = 50$, stayed approximately constant between $t = 50$ and $t = 60$, and decreased between $t = 60$ and $t = 105$. Figure 1.5 shows one possible graph. Many other answers are also possible.

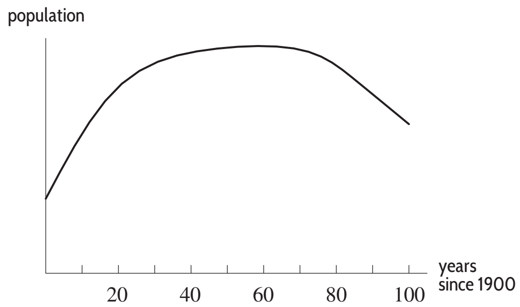


Figure 1.5

7. The statement $f(5) = 49.2$ tells us that $W = 49.2$ when $t = 5$. In other words, in 2015, Argentina produced 49.2 million metric tons of wheat.

8. (a) If we consider the equation

$$C = 4T - 160$$

simply as a mathematical relationship between two variables C and T, any T value is possible. However, if we think of it as a relationship between cricket chirps and temperature, then C cannot be less than 0. Since $C = 0$ leads to $0 = 4T - 160$, and so $T = 40°\text{F}$, we see that T cannot be less than 40°F. In addition, we are told that the function is not defined for temperatures above 134°. Thus, for the function $C = f(T)$ we have

$$\begin{aligned}\text{Domain} &= \text{All } T \text{ values between } 40°\text{F and } 134°\text{F}\\ &= \text{All } T \text{ values with } 40 \le T \le 134\\ &= [40, 134].\end{aligned}$$

(b) Again, if we consider $C = 4T - 160$ simply as a mathematical relationship, its range is all real C values. However, when thinking of the meaning of $C = f(T)$ for crickets, we see that the function predicts cricket chirps per minute between 0 (at $T = 40°\text{F}$) and 376 (at $T = 134°\text{F}$). Hence,

$$\begin{aligned}\text{Range} &= \text{All } C \text{ values from 0 to 376}\\ &= \text{All } C \text{ values with } 0 \le C \le 376\\ &= [0, 376].\end{aligned}$$

9. (a) The statement $f(15) = 400$ means that $C = 400$ when $t = 15$. In other words, in the year 2015, the concentration of carbon dioxide in the atmosphere was 400 ppm.

(b) The expression $f(20)$ represents the concentration of carbon dioxide in the year 2020.

10. (a) At $p = 0$, we see $r = 8$. At $p = 3$, we see $r = 7$.

(b) When $p = 2$, we see $r = 10$. Thus, $f(2) = 10$.

11. Substituting $x = 5$ into $f(x) = 2x + 3$ gives

$$f(5) = 2(5) + 3 = 10 + 3 = 13.$$

12. Substituting $x = 5$ into $f(x) = 10x - x^2$ gives

$$f(5) = 10(5) - (5)^2 = 50 - 25 = 25.$$

13. We want the y-coordinate of the graph at the point where its x-coordinate is 5. Looking at the graph, we see that the y-coordinate of this point is 3. Thus

$$f(5) = 3.$$

14. Looking at the graph, we see that the point on the graph with an x-coordinate of 5 has a y-coordinate of 2. Thus

$$f(5) = 2.$$

15. In the table, we must find the value of $f(x)$ when $x = 5$. Looking at the table, we see that when $x = 5$ we have

$$f(5) = 4.1$$

16. (a) We are asked for the value of y when x is zero. That is, we are asked for $f(0)$. Plugging in we get

$$f(0) = (0)^2 + 2 = 0 + 2 = 2.$$

(b) Substituting we get

$$f(3) = (3)^2 + 2 = 9 + 2 = 11.$$

(c) Asking what values of x give a y-value of 11 is the same as solving

$$\begin{aligned} y = 11 &= x^2 + 2 \\ x^2 &= 9 \\ x &= \pm\sqrt{9} = \pm 3. \end{aligned}$$

We can also solve this problem graphically. Looking at Figure 1.6, we see that the graph of $f(x)$ intersects the line $y = 11$ at $x = 3$ and $x = -3$. Thus, when x equals 3 or x equals -3 we have $f(x) = 11$.

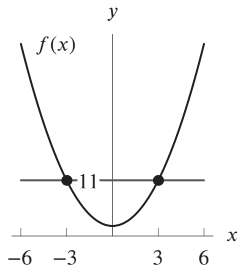


Figure 1.6

(d) No. No matter what, x^2 is greater than or equal to 0, so $y = x^2 + 2$ is greater than or equal to 2.

17. The year 2000 was 12 years before 2012 so 2000 corresponds to $t = 12$. Thus, an expression that represents the statement is:

$$f(12) = 7.049.$$

18. The year 2012 was 0 years before 2012 so 2012 corresponds to $t = 0$. Thus, an expression that represents the statement is:

$$f(0) \text{ meters.}$$

19. The year 1949 was $2012 - 1949 = 63$ years before 2012 so 1949 corresponds to $t = 63$. Similarly, we see that the year 2000 corresponds to $t = 12$. Thus, an expression that represents the statement is:

$$f(63) = f(12).$$

20. Since $t = 1$ means one year before 2012, then $t = 1$ corresponds to the year 2011. Similarly, $t = 0$ corresponds to the year 2012. Thus, $f(1)$ and $f(0)$ are the average annual sea level values, in meters, in 2011 and 2012, respectively. Because 8 millimeters is the same as 0.008 meters, the average sea level in 2012, $f(0)$, is 0.008 less than the sea level in 2011 which is $f(1)$. An expression that represents the statement is:

$$f(0) = f(1) - 0.008.$$

Note that there are other possible equivalent expressions, such as: $f(0) - f(1) = -0.008$.

21. **(a)** Since the potato is getting hotter, the temperature is increasing. The temperature of the potato eventually levels off at the temperature of the oven, so the best answer is graph (III).

(b) The vertical intercept represents the temperature of the potato at time $t = 0$ or just before it is put in the oven.

22. **(a)** $f(30) = 10$ means that the value of f at $t = 30$ was 10. In other words, the temperature at time $t = 30$ minutes was 10°C. So, 30 minutes after the object was placed outside, it had cooled to 10 °C.

(b) The intercept a measures the value of $f(t)$ when $t = 0$. In other words, when the object was initially put outside, it had a temperature of a°C. The intercept b measures the value of t when $f(t) = 0$. In other words, at time b the object's temperature is 0 °C.

23. **(a)** Since $N = f(h)$, the statement $f(500) = 100$, means that if $h = 500$, then $N = 100$. This tells us that at an elevation of 500 feet above sea level, there are 100 species of bats.

(b) The vertical intercept k is on the N-axis, so it represents the value of N when $h = 0$. The intercept, k, is the number of bat species at sea level. The horizontal intercept, c, represents the value of h when $N = 0$. The intercept, c, is the lowest elevation above which no bats are found.

24. The number of species of algae is low when there are few snails or lots of snails. The greatest number of species of algae (about 10) occurs when the number of snails is at a medium level (around 125 snails per square meter.) The graph supports the statement that diversity peaks at intermediate predation levels, when there are an intermediate number of snails.

25. **(a)** From the graph, we estimate $f(3) = 0.14$. This means that after 3 hours, the level of nicotine is about 0.14 mg.
(b) About 4 hours.
(c) The vertical intercept is 0.4. It represents the level of nicotine in the blood right after the cigarette is smoked.
(d) A horizontal intercept would represent the value of t when $N = 0$, or the number of hours until all nicotine is gone from the body.

26. **(a)** The original deposit is the balance, B, when $t = 0$, which is the vertical intercept. The original deposit was \$1000.
(b) It appears that $f(10) \approx 2200$. The balance in the account after 10 years is about \$2200.
(c) When $B = 5000$, it appears that $t \approx 20$. It takes about 20 years for the balance in the account to reach \$5000.

27. **(a)** The statement $f(10) = 0.2$ means that $C = 0.2$ when $t = 10$. In other words, CFC consumption was 0.2 million tons in 1997.
(b) The vertical intercept is the value of C when $t = 0$. It represents CFC consumption in 1987.
(c) The horizontal intercept is the value of t when $C = 0$. It represents the year when CFC consumption is expected to be zero.

28. **(a)** We see that the deficit is decreasing during this time period.
(b) The statement $f(4) = 485$ means that the deficit was 485 billion dollars in 2014.
(c) We can estimate the location of 4 on the horizontal axis, since the graph covers 2011–2015, corresponding to $1 \le t \le 5$. The value 485 on the vertical axis is then the corresponding y-coordinate. These values, and a dot at the corresponding point, are shown in Figure 1.7.

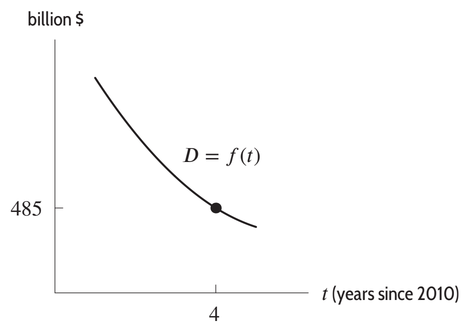


Figure 1.7

(d) A vertical intercept represents the value of D when $t = 0$, which is the US deficit (in billions of dollars) in the year 2010.
(e) A horizontal intercept represents the value of t when $D = 0$, or the number of years after 2010 when the deficit is zero: the horizontal intercept occurs at a moment when the deficit is zero.

29. See Figure 1.8.

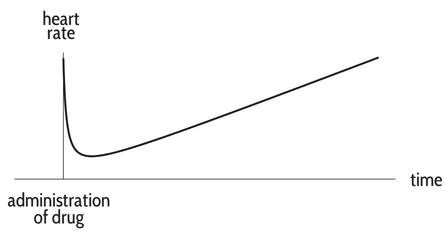


Figure 1.8

30. Figure 1.9 shows one possible graph of gas mileage increasing to a high at a speed of 45 mph and then decreasing again. Other graphs are also possible.

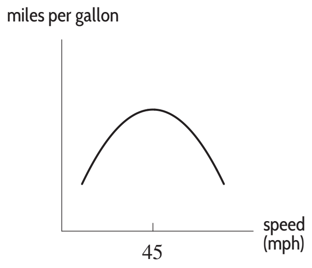


Figure 1.9

31. One possible graph is shown in Figure 1.10. Since the patient has none of the drug in the body before the injection, the vertical intercept is 0. The peak concentration is labeled on the concentration (vertical) axis and the time until peak concentration is labeled on the time (horizontal) axis. See Figure 1.10.

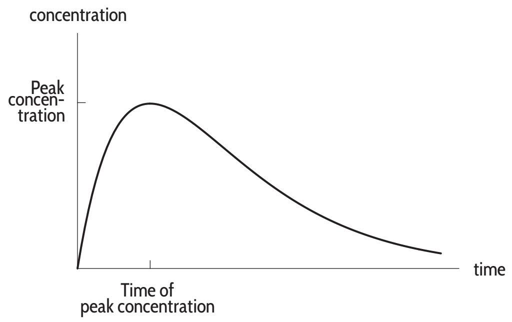


Figure 1.10

32. **(a)** We want a graph of expected return as a function of risk, so expected return is on the vertical axis and risk is on the horizontal axis. The return increases as the risk increases, so the graph might look like Figure 1.11. However, the graph may not be a straight line; many other answers are also possible.

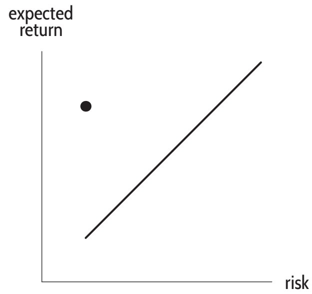


Figure 1.11

(b) High expected return and low risk is in the top left corner. See Figure 1.11.

33. **(a)** The statement $f(1000) = 3500$ means that when $a = 1000$, we have $S = 3500$. In other words, when \$1000 is spent on advertising, the number of sales per month is 3500.
(b) Graph I, because we expect that as advertising expenditures go up, sales will go up (not down).
(c) The vertical intercept represents the value of S when $a = 0$, or the sales per month if no money is spent on advertising.

34. **(a)** In 1970, fertilizer use in the US was about 13 million tons, in India about 2 million tons, and in the former Soviet Union about 10 million tons.
(b) Fertilizer use in the US rose steadily between 1950 and 1980 and has stayed relatively constant at about 18 million tons since then. Fertilizer use in India has risen steadily throughout this 50-year period. Fertilizer use in the former Soviet Union rose rapidly between 1950 and 1985 and then declined very rapidly between 1985 and 2000.

35. **(a)** (I) The incidence of cancer increase with age, but the rate of increase slows down slightly. The graph is nearly linear. This type of cancer is closely related to the aging process.

(II) In this case a peak is reached at about age 55, after which the incidence decreases.

(III) This type of cancer has an increased incidence until the age of about 48, then a slight decrease, followed by a gradual increase.

(IV) In this case the incidence rises steeply until the age of 30, after which it levels out completely.

(V) This type of cancer is relatively frequent in young children, and its incidence increases gradually from about the age of 20.

(VI) This type of cancer is not age-related – all age-groups are equally vulnerable, although the overall incidence is low (assuming each graph has the same vertical scale).

(b) Graph (V) shows a relatively high incidence rate for children. Leukemia behaves in this way.

(c) Graph (III) could represent cancer in women with menopause as a significant factor. Breast cancer is a possibility here.

(d) Graph (I) shows a cancer which might be caused by toxins building up in the body. Lung cancer is a good example of this.

36. **(a)** Since 2008 corresponds to $t = 0$, the average annual sea level in Aberdeen in 2008 was 7.094 meters.

(b) Looking at the table, we see that the average annual sea level was 7.019 twenty five years before 2008, or in the year 1983. Similar reasoning shows that the average sea level was 6.957 meters 125 years before 2008, or in 1883.

(c) Because 125 years before 2008 the year was 1883, we see that the sea level value corresponding to the year 1883 is 6.957 (this is the sea level value corresponding to $t = 125$). Similar reasoning yields the table:

Year	1883	1908	1933	1958	1983	2008
S	6.957	6.938	6.965	6.992	7.019	7.094

37. See Figure 1.12.

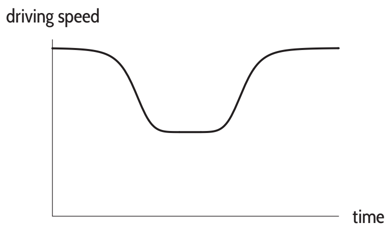


Figure 1.12

38. See Figure 1.13.

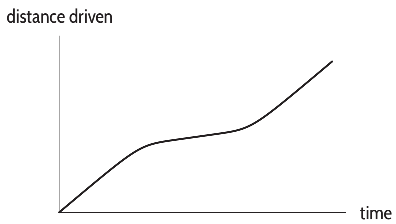


Figure 1.13

39. See Figure 1.14.

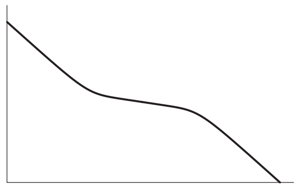


Figure 1.14

40. See Figure 1.15.

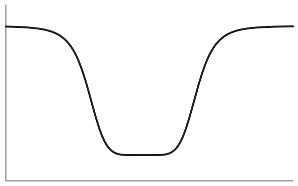


Figure 1.15

Solutions for Section 1.2

1. The slope is $(3-2)/(2-0) = 1/2$. So the equation of the line is $y = (1/2)x + 2$.

2. The slope is $(1-0)/(1-0) = 1$. So the equation of the line is $y = x$.

3. The slope is

$$\text{Slope} = \frac{3-1}{2-(-2)} = \frac{2}{4} = \frac{1}{2}.$$

Now we know that $y = (1/2)x + b$. Using the point $(-2, 1)$, we have $1 = -2/2 + b$, which yields $b = 2$. Thus, the equation of the line is $y = (1/2)x + 2$.

4. The slope is

$$\text{Slope} = \frac{-1-5}{2-4} = \frac{-6}{-2} = 3.$$

Substituting $m = 3$ and the point $(4, 5)$ into the equation of the line, $y = b + mx$, we have

$$\begin{aligned} 5 &= b + 3 \cdot 4 \\ 5 &= b + 12 \\ b &= -7. \end{aligned}$$

The equation of the line is $y = -7 + 3x$.

5. Rewriting the equation as

$$y = -\frac{12}{7}x + \frac{2}{7}$$

shows that the line has slope $-12/7$ and vertical intercept $2/7$.

6. Rewriting the equation as

$$y = 4 - \frac{3}{2}x$$

shows that the line has slope $-3/2$ and vertical intercept 4.

7. Rewriting the equation of the line as

$$y = \frac{12}{6}x - \frac{4}{6}$$
$$y = 2x - \frac{2}{3},$$

we see that the line has slope 2 and vertical intercept $-2/3$.
s

8. Rewriting the equation of the line as

$$-y = \frac{-2}{4}x - 2$$
$$y = \frac{1}{2}x + 2,$$

we see the line has slope $1/2$ and vertical intercept 2.

9. The slope is positive for lines l_1 and l_2 and negative for lines l_3 and l_4. The y-intercept is positive for lines l_1 and l_3 and negative for lines l_2 and l_4. Thus, the lines match up as follows:

(a) l_1
(b) l_3
(c) l_2
(d) l_4

10. **(a)** Lines l_2 and l_3 are parallel and thus have the same slope. Of these two, line l_2 has a larger y-intercept.
(b) Lines l_1 and l_3 have the same y-intercept. Of these two, line l_1 has a larger slope.

11. **(a)** The slope is constant at 750 people per year so this is a linear function. The vertical intercept (at $t = 0$) is 28,600. We have

$$P = 28{,}600 + 750t.$$

(b) The population in 2020, when $t = 5$, is predicted to be

$$P = 28{,}600 + 750 \cdot 5 = 32{,}350.$$

(c) We use $P = 35{,}000$ and solve for t:

$$P = 28{,}600 + 750t$$
$$35{,}000 = 28{,}600 + 750t$$
$$6{,}400 = 750t$$
$$t = 8.53.$$

The population will reach 35,000 people in about 8.53 years, or during the year 2023.

12. This is a linear function with vertical intercept 35 and slope 0.20. The formula for the monthly charge is

$$C = 35 + 0.20m.$$

13. **(a)** The first company's price for a day's rental with m miles on it is $C_1(m) = 40 + 0.15m$. Its competitor's price for a day's rental with m miles on it is $C_2(m) = 50 + 0.10m$.
(b) See Figure 1.16.

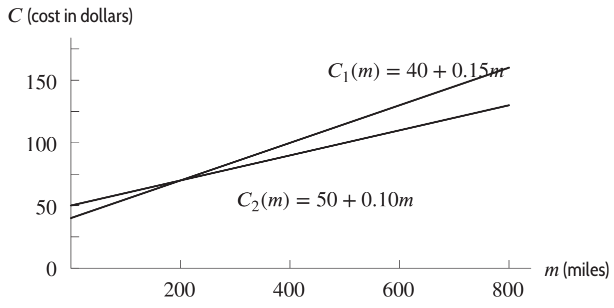


Figure 1.16

(c) To find which company is cheaper, we need to determine where the two lines intersect. We let $C_1 = C_2$, and thus

$$\begin{aligned} 40 + 0.15m &= 50 + 0.10m \\ 0.05m &= 10 \\ m &= 200. \end{aligned}$$

If you are going more than 200 miles a day, the competitor is cheaper. If you are going less than 200 miles a day, the first company is cheaper.

14. This is a linear function with two parts to the formula:
For $0 \leq m \leq 120$, we pay only the \$14.99, so

$$P = 14.99.$$

For $120 < m$, the 10-cent charge is only on the minutes above 120, that is $m - 120$ minutes:

$$P = 14.99 + 0.1(m - 120).$$

15. (a) The vertical intercept appears to be about 300 miles. When this trip started (at $t = 0$) the person was 300 miles from home.

(b) The slope appears to be about $(550 - 300)/5 = 50$ miles per hour. This person is moving away from home at a rate of about 50 miles per hour.

(c) We have $D = 300 + 50t$.

16. (a) On the interval from 0 to 1 the value of y decreases by 2. On the interval from 1 to 2 the value of y decreases by 2. And on the interval from 2 to 3 the value of y decreases by 2. Thus, the function has a constant rate of change and it could therefore be linear.

(b) On the interval from 15 to 20 the value of s increases by 10. On the interval from 20 to 25 the value of s increases by 10. And on the interval from 25 to 30 the value of s increases by 10. Thus, the function has a constant rate of change and could be linear.

(c) On the interval from 1 to 2 the value of w increases by 5. On the interval from 2 to 3 the value of w increases by 8. Thus, we see that the slope of the function is not constant and so the function is not linear.

17. For the function given by table (a), we know that the slope is

$$\text{slope} = \frac{27 - 25}{0 - 1} = -2.$$

We also know that at $x = 0$ we have $y = 27$. Thus we know that the vertical intercept is 27. The formula for the function is

$$y = -2x + 27.$$

For the function in table (b), we know that the slope is

$$\text{slope} = \frac{72 - 62}{20 - 15} = \frac{10}{5} = 2.$$

Thus, we know that the function will take on the form

$$s = 2t + b.$$

Substituting in the coordinates (15, 62) we get

$$\begin{aligned} s &= 2t + b \\ 62 &= 2(15) + b \\ &= 30 + b \\ 32 &= b. \end{aligned}$$

Thus, a formula for the function would be

$$s = 2t + 32.$$

18. **(a)** The slope is −1.35 billion dollars per year. McDonald's revenue is decreasing at a rate of 1.35 billion dollars per year.
(b) The vertical intercept is 28.1 billion dollars. In 2013, McDonald's revenue was 28.1 billion dollars.
(c) Substituting $t = 5$, we have $R = 28.1 - 1.35 \cdot 5 = 21.35$ billion dollars.
(d) We substitute $R = 22$ and solve for t:

$$\begin{aligned} 28.1 - 1.35t &= 22 \\ -1.35t &= -6.1 \\ t &= \frac{6.1}{1.35} \approx 4.52 \text{ years.} \end{aligned}$$

The function predicts that annual revenue will fall to 22 billion dollars in 2017.

19. **(a)** Reading coordinates from the graph, we see that rainfall $r = 100$ mm corresponds to about $Q = 600$ kg/hectare, and $r = 600$ mm corresponds to about $Q = 5800$ kg/hectare. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta Q}{\Delta r} = \frac{5800 - 600}{600 - 100} = 10.4 \text{ kg/hectare per mm.}$$

(b) Every additional 1 mm of annual rainfall corresponds to an additional 10.4 kg of grass per hectare.
(c) Using the slope, we see that the equation has the form

$$Q = b + 10.4r.$$

Substituting $r = 100$ and $Q = 600$ we can solve for b.

$$\begin{aligned} b + 10.4(100) &= 600 \\ b &= -440. \end{aligned}$$

The equation of the line is

$$Q = -440 + 10.4r.$$

20. **(a)** Reading coordinates from the graph, we see that rainfall $r = 100$ mm corresponds to about $Q = 300$ kg/hectare, and $r = 600$ mm corresponds to about $Q = 3200$ kg/hectare. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta Q}{\Delta r} = \frac{3200 - 300}{600 - 100} = 5.8 \text{ kg/hectare per mm.}$$

(b) Every additional 1 mm of annual rainfall corresponds to an additional 5.8 kg of grass per hectare.
(c) Using the slope, we see that the equation has the form

$$Q = b + 5.8r.$$

Substituting $r = 100$ and $Q = 300$ we can solve for b.

$$\begin{aligned} b + 5.8(100) &= 300 \\ b &= -280. \end{aligned}$$

The equation of the line is

$$Q = -280 + 5.8r.$$

21. The difference quotient $\Delta Q/\Delta r$ equals the slope of the line and represents the increase in the quantity of grass per millimeter of rainfall. We see from the graph that the slope of the line for 1939 is larger than the slope of the line for 1997. Thus, each additional 1 mm of rainfall in 1939 led to a larger increase in the quantity of grass than in 1997.

22. **(a)** We select two points on the line. Using the values (1992, 6.75) and (2006, 12.25) we have

$$\text{Slope} = \frac{12.25 - 6.75}{2006 - 1992 \text{ years}} = 0.4 \ \frac{\text{million barrels per day}}{\text{year}}.$$

(b) With t as the year and $f(t)$ as the quantity of imports in millions of barrels per day, we have

$$f(t) = 6.75 + 0.4(t - 1992).$$

Alternatively, if t as the number of years since 1992 and if $f(t)$ is the quantity of imports in millions of barrels per day

$$f(t) = 6.75 + 0.4t.$$

(c) We have

$$\begin{aligned} 6.75 + 0.4t &= 18 \\ t &= 28.125. \end{aligned}$$

The model predicts imports will reach 18 million barrels a day in the year $1992 + 28.125 = 2020.125$.

This prediction could serve as a guideline, but it is very risky to put too much reliance on a prediction more than ten years into the future, especially since there has been a drop in imports since 2008. Many unexpected events could drastically change the economic environment during that time. This prediction should be used with caution.

23. **(a)** We know that the function for q in terms of p will take on the form

$$q = mp + b.$$

We know that the slope will represent the change in q over the corresponding change in p. Thus

$$m = \text{slope} = \frac{4-3}{12-15} = \frac{1}{-3} = -\frac{1}{3}.$$

Thus, the function will take on the form

$$q = -\frac{1}{3}p + b.$$

Substituting the values $q = 3$, $p = 15$, we get

$$\begin{aligned} 3 &= -\frac{1}{3}(15) + b \\ 3 &= -5 + b \\ b &= 8. \end{aligned}$$

Thus, the formula for q in terms of p is

$$q = -\frac{1}{3}p + 8.$$

(b) We know that the function for p in terms of q will take on the form

$$p = mq + b.$$

We know that the slope will represent the change in p over the corresponding change in q. Thus

$$m = \text{slope} = \frac{12-15}{4-3} = -3.$$

Thus, the function will take on the form

$$p = -3q + b.$$

Substituting the values $q = 3$, $p = 15$ again, we get

$$\begin{aligned} 15 &= (-3)(3) + b \\ 15 &= -9 + b \\ b &= 24. \end{aligned}$$

Thus, a formula for p in terms of q is

$$p = -3q + 24.$$

24. **(a)** The vertical intercept appears to be about 480. In 2000, world milk production was about 480 million tons of milk.

(b) World milk production appears to be about 480 at $t = 0$ and about 600 at $t = 12$. We estimate

$$\text{Slope} = \frac{600 - 480}{12 - 0} = 10 \text{ million tons/year.}$$

World milk production increased at a rate of about 10 million tons per year during this 12-year period.

(c) We have $M = 480 + 10t$.

25. **(a)** Let x be the average minimum daily temperature (°C), and let y be the date the marmot is first sighted. Reading coordinates from the graph, we see that temperature $x = 12$°C corresponds to date $y = 137$ days after Jan 1, and $x = 22$°C corresponds to $y = 109$ days. Using a difference quotient, we have

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{109 - 137}{22 - 12} = -2.8 \frac{\text{days}}{°\text{C}}.$$

(b) The slope is negative. An increase in the temperatures corresponds to an earlier sighting of a marmot. Marmots come out of hibernation earlier in years with warmer average daily minimum temperature.

(c) We have

$$\Delta y = \text{Slope} \times \Delta x = (-2.8)(6) = 16.8 \text{ days.}$$

If temperatures are 6°C higher, then marmots come out of hibernations about 17 days earlier.

(d) Using the slope, we see that the equation has the form

$$y = b - 2.8x.$$

Substituting $x = 12$ and $y = 137$ we solve for b.

$$\begin{aligned} b - 2.8(12) &= 137 \\ b &= 171. \end{aligned}$$

The equation of the line is

$$y = 171 - 2.8x.$$

26. **(a)** If the first date of bare ground is 140, then, according to the figure, the first bluebell flower is sighted about day 150, that is $150 - 140 = 10$ days later.

(b) Let x be the first date of bare ground, and let y be the date the first bluebell flower is sighted. Reading coordinates from the graph, we see that date $x = 130$ days after Jan 1 corresponds to date $y = 142$ days after Jan 1, and $x = 170$ days corresponds to $y = 173$ days. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{173 - 142}{170 - 130} = 0.775 \text{ days per day.}$$

(c) The slope is positive, so an increase in the x-variable corresponds to an increase in the y-variable. These variables represents days in the year, and larger values indicate days that are later in the year. Thus if bare ground first occurs later in the year, then bluebells first flower later in the year. Positive slope means that bluebells flower later when the snow cover lasts longer.

(d) Using the slope, we see that the equation has the form

$$y = b + 0.775x.$$

Substituting $x = 130$ and $y = 142$ we can solve for b.

$$\begin{aligned} b + 0.775(130) &= 142 \\ b &= 41.25. \end{aligned}$$

The equation of the line is

$$y = 41.25 + 0.775x.$$

27. **(a)** We have $m = \dfrac{256}{18} = 14.222$ cm per hour. When the snow started, there were 100 cm on the ground, so

$$f(t) = 100 + 14.222t.$$

(b) The domain of f is $0 \le t \le 18$ hours. The range is $100 \le f(t) \le 356$ cm.

28. **(a)** A linear function has the form

$$P = b + mt$$

where t is in years since 2007. The slope is

$$\text{Slope } = \frac{\Delta P}{\Delta t} = 0.9.$$

The function takes the value 12.5 in the year 2007 (when $t = 0$). Thus, the formula is

$$P = 12.5 + 0.9t.$$

(b) In 2013, we have $t = 6$. We predict $P = 12.5 + 0.9 \cdot 6 = 17.9$, that is, there are 17.9% below the poverty level in 2013.

(c) The difference is $17.9 - 14.5 = 3.4\%$; the prediction is too high.

29. **(a)** We want $P = b + mt$, where t is in years since 1975. The slope is

$$\text{Slope } = m = \frac{\Delta P}{\Delta t} = \frac{2048 - 1241}{30 - 0} = 26.9.$$

Since $P = 1241$ when $t = 0$, the vertical intercept is 1241 so

$$P = 1241 + 26.9t.$$

(b) The slope tells us that world grain production has been increasing at a rate of 26.9 million tons per year.

(c) The vertical intercept tells us that world grain production was 1241 million tons in 1975 (when $t = 0$.)

(d) We substitute $t = 40$ to find $P = 1241 + 26.9 \cdot 40 = 2317$ million tons of grain.

(e) Substituting $P = 2500$ and solving for t, we have

$$\begin{aligned} 2500 &= 1241 + 26.9t \\ 1259 &= 26.9t \\ t &= 46.803. \end{aligned}$$

Grain production is predicted to reach 2500 million tons during the year 2021.

30. **(a)** We are given two points: $S = 942.5$ when $t = 0$ and $S = 384.7$ when $t = 8$. We use these two points to find the slope. Since $S = f(t)$, we know the slope is $\Delta S/\Delta t$:

$$m = \frac{\Delta S}{\Delta t} = \frac{384.7 - 942.5}{8 - 0} = -69.7.$$

The vertical intercept (when $t = 0$) is 942.5, so the linear equation is

$$S = 942.5 - 69.7t.$$

(b) The slope is -69.7 million CDs per year. Sales of music CDs were decreasing at a rate of 69.7 million CDs a year. The vertical intercept is 942.5 million CDs and represents the sales in the year 2000.

(c) In the year 2012, we have $t = 12$ and we predict

$$\text{Sales } = 942.5 - 69.7 \cdot 12 = 106.1 \text{ million CDs}.$$

31. **(a)** Since the initial value is \$25,000 and the slope is -2000, the value of the vehicle at time t is

$$V(t) = 25000 - 2000t.$$

The cost of repairs has initial value 0 and slope 1500, so

$$C(t) = 1500t.$$

Figure 1.17 shows the graphs of these two functions.

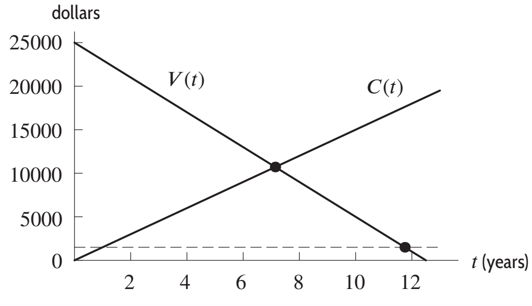


Figure 1.17

(b) The vehicle value equals the repair cost, when $V(t) = C(t)$.

$$\begin{aligned} 25000 - 2000t &= 1500t \\ 25000 &= 3500t \\ 7.143 &= t. \end{aligned}$$

Thus, replace during the eighth year (because the first year ends at $t = 1$). Since 0.143 years is $0.143(12) = 1.7$ months, replacement should take place in the second month of the eighth year.

(c) Since 6% of the orginal value is 1500, the vehicle should be replaced when $V(t) = 1500$.

$$\begin{aligned} 25000 - 2000t &= 1500 \\ 25000 &= 1500 + 2000t \\ 23500 &= 2000t \\ 11.750 &= t. \end{aligned}$$

Thus, in the twelfth year (because the first year ends at $t = 1$). Since 0.75 years is 9 months, replacement should take place at the end of the ninth month of the twelfth year.

32. **(a)** It appears that P is a linear function of d since P decreases by 10 on each interval as d increases by 20.

(b) The slope of the line is

$$m = \frac{\Delta P}{\Delta d} = \frac{-10}{20} = -0.5.$$

We use this slope and the first point in the table to find the equation of the line. We have

$$P = 100 - 0.5d.$$

(c) The slope is -0.5 %/ft. The percent found goes down by 0.5% for every additional foot separating the rescuers.

(d) The vertical intercept is 100%. If the distance separating the rescuers is zero, the percent found is 100%. This makes sense: if the rescuers walk with no distance between them (touching shoulders), they will find everyone.

The horizontal intercept is the value of d when $P = 0$. Substituting $P = 0$, we have

$$\begin{aligned} P &= 100 - 0.5d \\ 0 &= 100 - 0.5d \\ d &= 200. \end{aligned}$$

The horizontal intercept is 200 feet. This is the separation distance between rescuers at which, according to the model, no one is found. This is unreasonable, because even when the searchers are far apart, the search will sometimes be successful. This suggests that at some point, the linear relationship ceases to hold. We have extrapolated too far beyond the given data.

33. **(a)** We find the slope m and intercept b in the linear equation $C = b + mw$. To find the slope m, we use

$$m = \frac{\Delta C}{\Delta w} = \frac{1098.32 - 694.55}{8 - 5} = 134.59 \text{ dollars per ton.}$$

We substitute to find b:

$$\begin{aligned} C &= b + mw \\ 694.55 &= b + (134.59)(5) \\ b &= 21.6 \text{ dollars.} \end{aligned}$$

The linear formula is $C = 21.6 + 134.59w$.

(b) The slope is 134.59 dollars per ton. Each additional ton of waste collected costs \$134.59.

(c) The intercept is \$21.6. The flat fee for waste collection is \$21.6. This is the amount charged even if there is no waste.

34. **(a)** We are given two points: $N = 34$ when $l = 11$ and $N = 26$ when $l = 44$. We use these two points to find the slope. Since $N = f(l)$, we know the slope is $\Delta N/\Delta l$:

$$m = \frac{\Delta N}{\Delta l} = \frac{26 - 34}{44 - 11} = -0.2424.$$

We use the point $l = 11$, $N = 34$ and the slope $m = -0.2424$ to find the vertical intercept:

$$\begin{aligned} N &= b + ml \\ 34 &= b - 0.2424 \cdot 11 \\ 34 &= b - 2.67 \\ b &= 36.67. \end{aligned}$$

The equation of the line is

$$N = 36.67 - 0.2424l.$$

(b) The slope is -0.2424 species per degree latitude. In other words, the number of coastal dune plant species decreases by about 0.2424 for every additional degree South in latitude. The vertical intercept is 36.67 species, which represents the number of coastal dune plant species at the equator, if the model (and Australia) extended that far.

(c) See Figure 1.18.

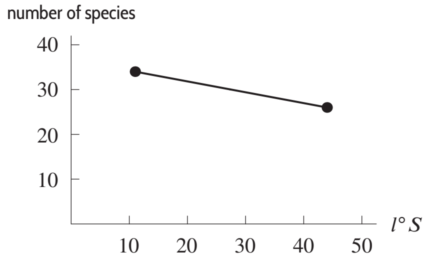


Figure 1.18

35. **(a)** This could be a linear function because w increases by 2.7 as h increases by 4.

(b) We find the slope m and the intercept b in the linear equation $w = b + mh$. We first find the slope m using the first two points in the table. Since we want w to be a function of h, we take

$$m = \frac{\Delta w}{\Delta h} = \frac{82.4 - 79.7}{172 - 168} = 0.68.$$

Substituting the first point and the slope $m = 0.68$ into the linear equation $w = b + mh$, we have $79.7 = b + (0.68)(168)$, so $b = -34.54$. The linear function is

$$w = 0.68h - 34.54.$$

The slope, $m = 0.68$, is in units of kg per cm.

(c) We find the slope and intercept in the linear function $h = b + mw$ using $m = \Delta h / \Delta w$ to obtain the linear function

$$h = 1.47w + 50.79.$$

Alternatively, we could solve the linear equation found in part (b) for h. The slope, $m = 1.47$, has units cm per kg.

36. Both formulas are linear, with slope -1, which means that in one year there is a decrease of one beat per minute, so (c) is correct.

37. If M is the male's maximum heart rate and F is the female's, then

$$\begin{aligned} M &= 220 - a \\ F &= 226 - a, \end{aligned}$$

so F is 6 larger than M. Thus, female's maximum heart rate exceeds the male's if they are the same age.

38. If f is the age of the female, and m the age of the male, then if they have the same MHR,

$$226 - f = 220 - m,$$

so

$$f - m = 6.$$

Thus the female is 6 years older than the male.

39. (a) If old and new formulas give the same MHR for females, we have

$$226 - a = 208 - 0.7a,$$

so

$$a = 60 \text{ years.}$$

For males we need to solve the equation

$$220 - a = 208 - 0.7a,$$

so

$$a = 40 \text{ years.}$$

(b) The old formula starts at either 226 or 220 and decreases. The new formula starts at 208 and decreases slower than the old formula. Thus, the new formula predicts a lower MHR for young people and a higher MHR for older people, so the answer is (ii).

(c) Under the old formula,

$$\text{Heart rate reached } = 0.85(220 - 65) = 131.75 \text{ beats/minute,}$$

whereas under the new formula,

$$\text{Heart rate reached } = 0.85(208 - 0.7 \cdot 65) = 138.125 \text{ beats/minute.}$$

The difference is $138.125 - 131.75 = 6.375$ beats/minute more under the new formula.

40. If the MHR is linear with age then the rate of change, in beats per minute per year, is approximately the same at any age. Between 0 and 21 years,

$$\text{Rate of change } = -\frac{12}{21} = -0.571 \text{ beats per minute per year,}$$

whereas between 0 and 33 years,

$$\text{Rate of change } = -\frac{19}{33} = -0.576 \text{ beats per minute per year.}$$

These are approximately the same, so they are consistent with MHR being approximately linear with age.

41. If the MHR is linear with age then the slope, in beats per minute per year, would be approximately the same at any age. Between 0 and 21 years,

$$\text{Rate of change } = -\frac{9}{21} = -0.429 \text{ beats per minute per year,}$$

whereas between 0 and 33 years,

$$\text{Rate of change } = -\frac{26}{33} = -0.788 \text{ beats per minute per year.}$$

These are not approximately the same, so they are not consistent with MHR being approximately linear with age.

42. (a) We have $y = 3.5$ when $u = 0$, which occurs when the unemployment rate does not change. When the unemployment rate is constant, Okun'a law states that US production increases by 3.5% annually.

(b) When unemployment rises from 5% to 8% we have $u = 3$ and therefore

$$y = 3.5 - 2u = 3.5 - 2 \cdot 3 = -2.5.$$

National production for the year decreases by 2.5%.

(c) If annual production does not change, then $y = 0$. Hence $0 = 3.5 - 2u$ and so $u = 1.75$. There is no change in annual production if the unemployment rate goes up 1.75%.

(d) The coefficient -2 is the slope $\Delta y/\Delta u$. Every 1% increase in the unemployment rate during the course of a year results in an additional 2% decrease in annual production for the year.

43. **(a)** The average hourly wage for men is AU\$29.40 and the height premium is 3%, so

$$\text{Slope of men's line } = \frac{(3\%)\text{ AU\$29.40}}{10\text{ cm}} = 0.0882\text{ AU\$/cm}.$$

Since the wage is AU\$29.40 for height $x = 178$, we have

$$\text{Men's wage } = 0.0882(x - 178) + 29.40$$

so

$$\begin{aligned}\text{Men's wage } &= 29.40 + 0.0882x - 0.0882 \cdot 178\\ \text{Men's wage } &= 13.70 + 0.0882x.\end{aligned}$$

(b) The average hourly wage for women is AU\$24.78 and the height premium is 2%, so

$$\text{Slope of women's line } = \frac{(2\%)\text{ AU\$24.78}}{10\text{ cm}} = 0.0496\text{ AU\$/cm}.$$

Since the wage is AU\$24.78 for height $y = 164$, we have

$$\text{Women's wage } = 0.0496(y - 164) + 24.78$$

so

$$\begin{aligned}\text{Women's wage } &= 24.78 + 0.0496y - 0.0496 \cdot 164\\ \text{Women's wage } &= 16.65 + 0.0496y.\end{aligned}$$

(c) For $x = 178$, we are given that

$$\text{Men's average wage } = \text{AU\$29.40}.$$

For $y = 178$, we find

$$\text{Women's average wage } = 16.65 + 0.0496 \cdot 178 = \text{AU\$25.48}.$$

Thus the wage difference is 29.40 − 25.48 =AU\$3.92.

(d) The line representing men's wages has vertical intercept 13.70 and slope 0.0082; the line representing women's wages has vertical intercept 16.65 and slope 0.0496. Since the vertical intercept of men's wages is lower but the slope is larger, the two lines intersect. Thus, the model predicts there is a height making the two average hourly wages equal. For men and women of the same height, $x = y$. If the average hourly wages are the same

$$\begin{aligned}\text{Men's wage } &= \text{ Women's wage}\\ 13.70 + 0.0882x &= 16.65 + 0.0496x\\ x &= \frac{16.65 - 13.70}{0.0882 - 0.0496} = 76.425\text{ cm}.\end{aligned}$$

However, this height 76.425 cm, or about 30 inches, or 2 foot 6 inches, is an unrealistic height. Thus for all realistic heights, men's average hourly wage is predicted to be larger than women's average hourly wage.

Solutions for Section 1.3

1. The graph shows a concave down function.
2. The graph shows a concave up function.
3. The graph is concave up.
4. This graph is neither concave up or down.
5. As t increases w decreases, so the function is decreasing. The rate at which w is decreasing is itself decreasing: as t goes from 0 to 4, w decreases by 42, but as t goes from 4 to 8, w decreases by 36. Thus, the function is concave up.

6. One possible answer is shown in Figure 1.19.

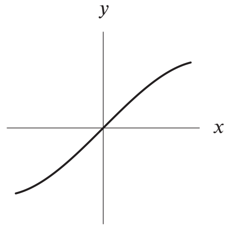


Figure 1.19

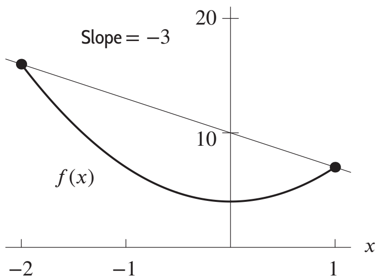
7. The function is increasing and concave up between D and E, and between H and I. It is increasing and concave down between A and B, and between E and F. It is decreasing and concave up between C and D, and between G and H. Finally, it is decreasing and concave down between B and C, and between F and G

8. The average rate of change R between $x = 1$ and $x = 3$ is

$$\begin{aligned} R &= \frac{f(3) - f(1)}{3 - 1} \\ &= \frac{18 - 2}{2} \\ &= \frac{16}{2} \\ &= 8. \end{aligned}$$

9. The average rate of change R between $x = -2$ and $x = 1$ is:

$$R = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{3(1)^2 + 4 - (3(-2)^2 + 4)}{1 + 2} = \frac{7 - 16}{3} = -3.$$



10. When $t = 0$, we have $B = 1000(1.02)^0 = 1000$. When $t = 5$, we have $B = 1000(1.02)^5 = 1104.08$. We have

$$\text{Average rate of change } = \frac{\Delta B}{\Delta t} = \frac{1104.08 - 1000}{5 - 0} = 20.82 \text{ dollars/year.}$$

During this five year period, the amount of money in the account increased at an average rate of \$20.82 per year.

11. In February of 2008, there were 15 million square kilometers of Arctic Sea covered by ice.

12. In February of 1983, there were 16 million square kilometers of Arctic Sea covered by ice.

13. Over the 25 year period from February 1983 to February 2008, the area of Arctic Sea covered by ice, shrunk, on average, by 40,000 square kilometers per year.

14. On February 1st, 2015, there were almost 14 million square kilometers of Arctic Sea covered by ice.

15. Over the month of February 2015 (between February 1 and February 28), the area of Arctic Sea covered by ice, grew, on average, by about 15,900 square kilometers per day.

16. **(a)** Between 2013 and 2015,

$$\begin{aligned}\text{Change in net sales} &= \text{Net sales in 2015} - \text{Net sales in 2013}\\ &= 15.80 - 16.15\\ &= -0.35 \text{ billion dollars.}\end{aligned}$$

(b) Between 2011 and 2015,

$$\begin{aligned}\text{Average rate of change in net sales} &= \frac{\text{Change in net sales}}{\text{Change in time}}\\ &= \frac{\text{Net sales in 2015} - \text{Net sales in 2011}}{2015 - 2011}\\ &= \frac{15.80 - 14.55}{2015 - 2011}\\ &= 0.313 \text{ billion dollars per year.}\end{aligned}$$

This means that the Gap's net sales increased on average by 0.313 billion (313 million) dollars per year between 2011 and 2015.

(c) The average rate of change was negative from 2014 to 2015, when sales decreased.

17. **(a)** Between 1950 and 2000

$$\begin{aligned}\text{Change in production} &= \text{Production in 2000} - \text{Production in 1950}\\ &= 95 - 11 \quad \text{(in millions)}\\ &= 84 \text{ million bicycles.}\end{aligned}$$

(b) The average rate of change R is the change in amounts (from part (a)) divided by the change in time.

$$\begin{aligned}R &= \frac{95 - 11}{2000 - 1950}\\ &= \frac{84}{50}\\ &= 1.68 \quad \text{million bicycles per year.}\end{aligned}$$

This means that production of bicycles has increased on average by 1.68 million bicycles per year between 1950 and 2000.

18. **(a)** The average rate of change is the change in average attendance divided by the change in time. Between 2011 and 2015,

$$\begin{aligned}\text{Average rate of change} = \frac{17.26 - 17.12}{2015 - 2011} &= 0.035 \text{ million people per year}\\ &= 35{,}000 \text{ people per year.}\end{aligned}$$

(b) For each of the years from 2011–2015, the annual increase in the average attendance was:

$$\begin{aligned}\text{2011 to 2012} &: 17.18 - 17.12 = 0.06\\ \text{2012 to 2013} &: 0.12\\ \text{2013 to 2014} &: 0.06\\ \text{2014 to 2015} &: -0.1.\end{aligned}$$

(c) We average the four yearly rates of change:

$$\begin{aligned}\text{Average of the four figures in part (b)} &= \frac{0.06 + 0.12 + 0.06 - 0.1}{4}\\ &= \frac{0.14}{4} = 0.035, \text{ which is the same as part (a).}\end{aligned}$$

19. **(a)** Since

$$\text{Average velocity} = \frac{f(20) - f(0)}{20 - 0} = \frac{3.6 - 0}{20} = 0.18,$$

the average velocity of the plane is about 0.18 thousand feet per second, or 180 feet per second.

(b) Since

$$\text{Average velocity} = \frac{f(40) - f(20)}{40 - 20} = \frac{5.6 - 3.6}{40 - 20} = 0.1,$$

the average velocity of the plane is about 0.1 thousand feet per second, or 100 feet per second.

20. **(a)** The value of imports was higher in 2015 than in 2001. The 2001 figure is about 1.4 and the 2015 figure is about 2.8, making the 2015 figure 1.4 trillion dollars higher than the 2001 figure.

(b) The average rate of change R is the change in values divided by the change in time.

$$\begin{aligned} R &= \frac{2.8 - 1.4}{2015 - 2001} \\ &= \frac{1.4}{14} \\ &= 0.1 \text{ trillion dollars per year.} \end{aligned}$$

Between 2001 and 2015, the value of US imports has increased by an average of about 100 billion dollars per year.

21. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the distance of the particle from a point, we read off the graph that $s(0) = 1$ and $s(3) = 4$. Thus,

$$\text{Average velocity } = \frac{\Delta s(t)}{\Delta t} = \frac{s(3) - s(0)}{3 - 0} = \frac{4 - 1}{3} = 1 \text{ meter/sec.}$$

22. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the distance of the particle from a point, we read off the graph that $s(1) = 2$ and $s(3) = 6$. Thus,

$$\text{Average velocity } = \frac{\Delta s(t)}{\Delta t} = \frac{s(3) - s(1)}{3 - 1} = \frac{6 - 2}{2} = 2 \text{ meters/sec.}$$

23. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the distance of the particle from a point, we find the values of $s(3) = 72$ and $s(10) = 144$. Using these values, we find

$$\text{Average velocity } = \frac{\Delta s(t)}{\Delta t} = \frac{s(10) - s(3)}{10 - 3} = \frac{144 - 72}{7} = \frac{72}{7} = 10.286 \text{ cm/sec.}$$

24. **(a)** We have

$$\begin{aligned} \text{Average rate of change of } P &= \frac{6.45 - 5.68}{2005 - 1995} = 0.077 \text{ billion people per year.} \\ \text{Average rate of change of } A &= \frac{45.9 - 36.1}{2005 - 1995} = 0.980 \text{ million cars per year.} \\ \text{Average rate of change of } C &= \frac{2168 - 91}{2005 - 1995} = 207.7 \text{ million subscribers per year.} \end{aligned}$$

(b) (i) The number of people is increasing faster since the population is increasing at 77 million per year and car production is increasing at less than 1 million per year.

(ii) The number of cell phone subscribers is increasing faster since the population is increasing at 77 million per year and the number of cell phone subscribers is increasing at about 208 million per year.

25. **(a)** Between 2003 and 2015, we have

$$\begin{aligned} \text{Change in sales} &= \text{Sales in 2015} - \text{Sales in 2003} \\ &= 63.06 - 26.97 \\ &= 36.09 \text{ billion dollars.} \end{aligned}$$

(b) Over the same period, we have

$$\begin{aligned} \begin{array}{c}\text{Average rate of change in sales} \\ \text{between 2003 and 2015}\end{array} &= \frac{\text{Change in sales}}{\text{Change in time}} \\ &= \frac{\text{Sales in 2015} - \text{Sales in 2003}}{2015 - 2003} \\ &= \frac{63.06 - 26.97}{2015 - 2003} \\ &= 3.0075 \text{ billion dollars per year.} \end{aligned}$$

This means that Pepsico's sales increased on average by 3.0075 billion dollars per year between 2003 and 2015.

26. **(a)** Negative. Rain forests are being continually destroyed to make way for housing, industry and other uses.
(b) Positive. Virtually every country has a population which is increasing, so the world's population must also be increasing overall.
(c) Negative. Since a vaccine for polio was found, the number of cases has dropped every year to almost zero today.
(d) Negative. As time passes and more sand is eroded, the height of the sand dune decreases.
(e) Positive. As time passes the price of just about everything tends to increase.

27. **(a)** The average rate of change R is the difference in amounts divided by the change in time.

$$\begin{aligned} R &= \frac{711 - 803}{2015 - 2003} \\ &= \frac{-92}{12} \\ &\approx -7.667 \quad \text{million pounds/yr.} \end{aligned}$$

This means that in the years between 2003 and 2015, the production of tobacco decreased at a rate of approximately 7.667 million pounds per year.

(b) To have a positive rate of change, the production has to increase during one of these 1 year intervals. Looking at the data, we can see that between 2003 and 2004, production of tobacco increased by 79 million pounds. Thus, the average rate of change is positive between 2003 and 2004.

Likewise, the annual average rate of change is positive between 2005 and 2009, between 2011 and 2012, and between 2013 and 2014. Nevertheless, the average rate of change over the period 2003 to 2015 is negative.

28. **(a)** This function is increasing and the graph is concave down.
(b) From the graph, we estimate that when $t = 5$, we have $L = 70$ and when $t = 15$, we have $L = 130$. Thus,

$$\text{Average rate of change of length} = \frac{\Delta L}{\Delta t} = \frac{130 - 70}{15 - 5} = 6 \text{ cm/year.}$$

During this ten year period, the sturgeon's length increased at an average rate of 6 centimeters per year.

29. Between 1940 and 2000:

$$\begin{aligned} \text{Average rate of change} &= \frac{136{,}891 - 47{,}520}{60} \\ &= \frac{89{,}371}{60} \\ &= 1490 \quad \text{thousand people/year.} \end{aligned}$$

This means that, between 1940 and 2000, the labor force increased by an average of 1,490,000 workers per year. Between 1940 and 1960:

$$\begin{aligned} \text{Average rate of change} &= \frac{65{,}778 - 47{,}520}{20} \\ &= \frac{18{,}258}{20} \\ &= 912.9 \quad \text{thousand people/year.} \end{aligned}$$

This means that, between 1940 and 1960, the labor force increased by an average of 912,900 workers per year. Between 1980 and 2000:

$$\begin{aligned} \text{Average rate of change} &= \frac{136{,}891 - 99{,}303}{20} \\ &= \frac{37{,}588}{20} \\ &= 1879 \quad \text{thousand people/year.} \end{aligned}$$

This means that, between 1980 and 2000, the labor force increased by an average of about 1,879,000 workers per year.

30. Between 1950 and 2001, we have

$$\text{Average rate of change} = \frac{\text{Change in marine catch}}{\text{Change in years}} = \frac{99 - 17}{2001 - 1950} = 1.61 \text{ million tons/year.}$$

Between 1950 and 2001, marine catch increased at an average rate of 1.61 million tons each year.

31. **(a)** The change is given by

$$\begin{aligned}\text{Change between 2003 and 2015} &= \text{Revenues in 2015} - \text{Revenues in 2003}\\ &= 152.2 - 184.0\\ &= -31.8 \text{ billion dollars.}\end{aligned}$$

(b) The rate of change is given by

$$\begin{aligned}\begin{matrix}\text{Average rate of change}\\ \text{between 2003 and 2015}\end{matrix} &= \frac{\text{Change in revenues}}{\text{Change in time}}\\ &= \frac{\text{Revenues in 2015} - \text{Revenues in 2003}}{2015 - 2003}\\ &= \frac{152.2 - 184.0}{2015 - 2003}\\ &= -2.65 \text{ billion dollars per year.}\end{aligned}$$

This means that General Motors' revenues decreased on average by 2.65 billion dollars per year between 2003 and 2015.

(c) To have a negative rate of change, the revenue has to decrease during one of these 1 year intervals. Looking at the data, we can see that between 2006 and 2007, revenues decreased by 24.5 billion dollars. Thus the average rate of change is negative between 2006 and 2007.

Likewise, the annual average rate of change is negative in all 1 year intervals between 2006 and 2010, and also between 2014 and 2015.

32. We have

$$\text{Average rate of change} = \frac{73{,}365{,}880 - 12{,}168{,}450}{2003 - 1977} = \frac{61{,}197{,}430}{26} = 2{,}353{,}747.3 \text{ households/year.}$$

During this 26-year period, the number of US households with cable television increased, on average, by 2,353,747.3 households per year.

33. **(a)** As time passes after a cigarette is smoked, the nicotine level in the body decreases, so the average rate of change is negative. The slope of a secant line between $t = 0$ and $t = 3$ is negative.

(b) The nicotine level at $t = 0$ is $N = 0.4$ and the nicotine level at $t = 3$ is about $N = 0.14$. We have

$$\text{Average rate of change} = \frac{f(3) - f(0)}{3 - 0} = \frac{0.14 - 0.4}{3 - 0} = -0.087 \text{ mg/hour.}$$

During this three hour period, nicotine decreased at an average rate of 0.087 mg of nicotine per hour.

34. **(a)** (i) Between the 6^{th} and 8^{th} minutes the concentration changes from 0.336 to 0.298 so we have

$$\text{Average rate of change} = \frac{0.298 - 0.336}{8 - 6} = -0.019 \text{ (mg/ml)/min.}$$

(ii) Between the 8^{th} and 10^{th} minutes we have

$$\text{Average rate of change} = \frac{0.266 - 0.298}{10 - 8} = -0.016 \text{ (mg/ml)/min.}$$

(b) The signs are negative because the concentration is decreasing. The magnitude of the decrease is decreasing because, as the concentration falls, the rate at which it falls decreases.

35. Between 1804 and 1927, the world's population increased 1 billion people in 123 years, for an average rate of change of $1/123$ billion people per year. We convert this to people per minute:

$$\frac{1,000,000,000}{123} \text{ people/year} \cdot \frac{1}{60 \cdot 24 \cdot 365} \text{ years/minute} = 15.468 \text{ people/minute.}$$

Between 1804 and 1927, the population of the world increased at an average rate of 15.468 people per minute. Similarly, we find the following:
Between 1927 and 1960, the increase was 57.654 people per minute.
Between 1960 and 1974, the increase was 135.899 people per minute.
Between 1974 and 1987, the increase was 146.353 people per minute.
Between 1987 and 1999, the increase was 158.549 people per minute.

36. **(a)** The muscle contracts at 30 cm/sec when it is not pulling any load at all.
(b) If the load is 4 kg, the contraction velocity is zero. Thus, the maximum load the muscle can move is 4 kg.

37. **(a)** The contraction velocity for 1 kg is about 13 cm/sec and for 3 kg it is about 2 cm/sec. The change is $2 - 13 = -11$ cm/sec.
(b) Using the answer to part (a), we have

$$\text{Average rate of change} = \frac{2-13}{3-1} = -5.5 \text{ (cm/sec)/kg}.$$

38. **(a)** Between 2000 and 2015

$$\begin{aligned}\text{Change in sales} &= \text{Sales in 2015} - \text{Sales in 2000}\\ &= 55.36 - 33.7\\ &= 21.66 \text{ billion dollars.}\end{aligned}$$

(b) Between 2000 and 2015

$$\begin{aligned}\begin{array}{c}\text{Average rate of change}\\ \text{in sales}\end{array} &= \frac{\text{Change in sales}}{\text{Change in time}}\\ &= \frac{\text{Sales in 2015} - \text{Sales in 2000}}{2015-2000}\\ &= \frac{55.36-33.7}{15}\\ &= 1.44 \text{ billion dollars per year.}\end{aligned}$$

This means that Intel's sales increased on average by 1.44 billion dollars per year between 2000 and 2015.

39. **(a)** (i) $f(2001) = 272$
(ii) $f(2014) = 525$
(b) The average yearly increase is the rate of change.

$$\text{Yearly increase} = \frac{f(2014) - f(2001)}{2014-2001} = \frac{525-272}{13} = 19.46 \text{ billionaires per year.}$$

(c) Since we assume the rate of increase remains constant, we use a linear function with slope 19.46 billionaires per year. The equation is

$$f(t) = b + 19.46t$$

where $f(2001) = 272$, so

$$\begin{aligned}272 &= b + 19.46(2001)\\ b &= -38{,}667.5.\end{aligned}$$

Thus, $f(t) = 19.46t - 38{,}667.5$.

40. **(a)** See Figure 1.20.
(b) The average velocity between two times can be represented by the slope of the line joining these points on the position curve. In Figure 1.21, the average velocity between $t = 0$ and $t = 3$ is equal to the slope of l_1, while the average velocity between $t = 3$ and $t = 6$ is equal to the slope of l_2. We can see that the slope of l_2 is greater than the slope of l_1 and so the average velocity between $t = 3$ and $t = 6$ is greater.

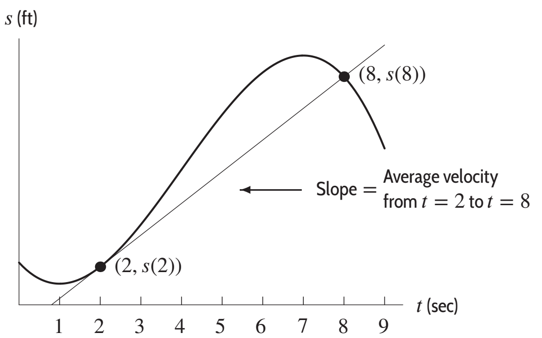


Figure 1.20

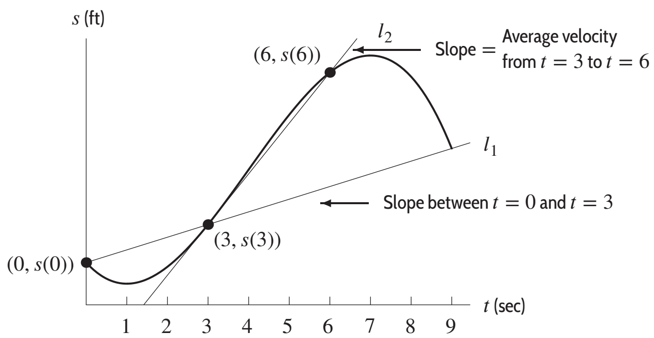


Figure 1.21

(c) Since the average velocity can be represented as the slope of the line between the two values given, we just have to look at the slope of the line passing through $(6, s(6))$ and $(9, s(9))$. We notice that the distance traveled at $t = 6$ is greater than the distance traveled at $t = 9$. Thus, the line between those two points will have negative slope, and thus, the average velocity will be negative.

41. **(a)** If the lizard were running faster and faster, the graph would be concave up. In fact, the graph looks linear; it is not concave up or concave down. The lizard is running at a relatively constant velocity during this short time interval.

(b) During this 0.8 seconds, the lizard goes about 2.1 meters, so the average velocity is about $2.1/0.8 = 2.6$ meters per second.

42. We know that a function is linear if its slope is constant. A function is concave up if the slope increases as t gets larger. And a function is concave down if the slope decreases as t gets larger. Looking at the points $t = 10$, $t = 20$ and $t = 30$ we see that the slope of $F(t)$ decreases since

$$\frac{F(20) - F(10)}{20 - 10} = \frac{22 - 15}{10} = 0.7$$

while

$$\frac{F(30) - F(20)}{30 - 20} = \frac{28 - 22}{10} = 0.6.$$

Looking at the points $t = 10$, $t = 20$ and $t = 30$ we see that the slope of $G(t)$ is constant

$$\frac{G(20) - G(10)}{20 - 10} = \frac{18 - 15}{10} = 0.3$$

and

$$\frac{G(30) - G(20)}{30 - 20} = \frac{21 - 18}{10} = 0.3.$$

Also note that the slope of $G(t)$ is constant everywhere.

Looking at the points $t = 10$, $t = 20$ and $t = 30$ we see that the slope of $H(t)$ increases since

$$\frac{H(20) - H(10)}{20 - 10} = \frac{17 - 15}{10} = 0.2$$

while

$$\frac{H(30) - H(20)}{30 - 20} = \frac{20 - 17}{10} = 0.3.$$

Thus $F(t)$ is concave down, $G(t)$ is linear, and $H(t)$ is concave up.

43. Between 0 and 21 years,

$$\text{Average rate of change} = -\frac{9}{21} = -0.429 \text{ beats per minute per year,}$$

whereas between 0 and 33 years,

$$\text{Average rate of change} = -\frac{26}{33} = -0.788 \text{ beats per minute per year.}$$

Because the average rate of change is negative, a decreasing function is suggested. Also, as age increases, the average rate of change decreases, suggesting the graph of the function is concave down. (Since the average rate of change is negative and increasing in absolute value, this rate is decreasing.)

44. Velocity is the slope of the graph of distance against time, so we draw a graph with a slope that starts out small and increases (so the graph gets steeper.) Then the slope decreases until the car stops. When the car is stopped, the distance stays constant and the slope is zero. See Figure 1.22.

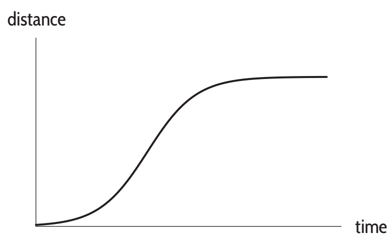


Figure 1.22

45. We have

$$\text{Relative change } = \frac{15{,}000 - 12{,}000}{12{,}000} = 0.25.$$

The quantity B increases by 25%.

46. We have

$$\text{Relative change } = \frac{450 - 400}{400} = 0.125.$$

The quantity S increases by 12.5%.

47. We have

$$\text{Relative change } = \frac{0.05 - 0.3}{0.3} = -0.833.$$

The quantity W decreases by 83.3%.

48. We have

$$\text{Relative change } = \frac{47 - 50}{50} = -0.06.$$

The quantity R decreases by 6%.

49. The changes are 86.7 in 1931 and 4485.4 in 2008, but the relative changes are

$$\text{Relative change, 1931, } = -\frac{86.7}{164.6} = 52.7\%$$

$$\text{Relative change, 2008, } = -\frac{4485.4}{13{,}261.8} = 33.8\%.$$

The 1931 change is relatively bigger.

50. The changes are 2.0 million for 1800-1810 and 28.0 million for 1950-1960. The relative changes are

$$\frac{2.0}{5.2} = 38.5\% \quad \text{and} \quad \frac{28.0}{151.3} = 18.5\%.$$

The relative change is bigger for 1800-1810. On the other hand, the change in 1950-1960 represents the baby boom that has had various important consequences for the US in the last 50 years so many would consider the implications of that change greater.

51. The changes are 5 students for the small class and 20 students for the larger class. The relative changes are

$$\frac{5}{5} = 100\% \quad \text{and} \quad \frac{20}{30} = 66.667\%.$$

The relative change for the small class is bigger than the relative change for the large class, but the extra 20 students added to a class of 30 has a potentially larger effect on the class.

52. The changes are 400,000 and 500,000. The relative changes are

$$\frac{400{,}000}{100{,}000} = 400\% \quad \text{and} \quad \frac{500{,}000}{20{,}000{,}000} = 2.5\%.$$

The relative change for the first sales is much larger and would have a much larger effect on the company.

53. **(a)** We have

$$\text{Relative change } = \frac{2000 - 1000}{1000} = 1 = 100\%.$$

(b) We have

$$\text{Relative change } = \frac{1000 - 2000}{2000} = -0.5 = -50\%.$$

(c) We have

$$\text{Relative change } = \frac{1{,}001{,}000 - 1{,}000{,}000}{1{,}000{,}000} = 0.001 = 0.1\%.$$

54. We have

$$\text{Relative change } = \frac{260.64 - 298.97}{298.97} = -0.128 = -12.8\%.$$

The Dow Jones average dropped by 12.8%.

55. We have

$$\text{Relative change} = \frac{\text{Change in cost}}{\text{Initial cost}} = \frac{49 - 47}{47} = 0.043.$$

The cost to mail a letter increased by 4.3%.

56. Let $f(t)$ be the CPI in January of year t. Since the inflation rate is the relative rate of change with $\Delta t = 1$, we have

$$\text{Inflation rate} = \frac{1}{f}\frac{\Delta f}{\Delta t} = \frac{1}{f(t)}\frac{f(t+1) - f(t)}{1} = \frac{f(t+1) - f(t)}{f(t)}$$

we have

$$\begin{aligned}
\text{2007 inflation} &= \frac{211.08 - 202.416}{202.416} = 0.0428 = 4.28\% \text{ per year}\\
\text{2008 inflation} &= \frac{211.143 - 211.08}{211.08} = 0.0003 = 0.03\% \text{ per year}\\
\text{2009 inflation} &= \frac{216.687 - 211.143}{211.143} = 0.0263 = 2.63\% \text{ per year}\\
\text{2010 inflation} &= \frac{220.223 - 216.687}{216.687} = 0.0163 = 1.63\% \text{ per year}\\
\text{2011 inflation} &= \frac{226.655 - 220.223}{220.223} = 0.0292 = 2.92\% \text{ per year}
\end{aligned}$$

Rounding to two decimal places, we have

Table 1.1

Year	2007	2008	2009	2010	2011
Inflation (% per year)	4.28%	0.03%	2.63%	1.63%	2.92%

57. **(a)** Let $f(t)$ be the GDP at time t. We are using $\Delta t = 0.25$, so

$$\text{Relative growth rate} = \frac{(\Delta f(t))/\Delta t}{f(t)} \approx \frac{(f(t+0.25) - f(t))/0.25}{f(t)}$$

we have

$$\begin{aligned}
\text{Relative growth rate at 0} &\approx \frac{(14.81 - 14.67)/0.25}{14.67} = 0.038 = 3.8\% \text{ per year}\\
\text{Relative growth rate at 0.25} &\approx \frac{(14.84 - 14.81)/0.25}{14.81} = 0.008 = 0.8\% \text{ per year}\\
\text{Relative growth rate at 0.5} &\approx \frac{(14.55 - 14.84)/0.25}{14.84} = -0.078 = -7.8\% \text{ per year}\\
\text{Relative growth rate at 0.75} &\approx \frac{(14.38 - 14.55)/0.25}{14.55} = -0.047 = -4.7\% \text{ per year.}
\end{aligned}$$

We have

Table 1.2

t (years since 2008)	0	0.25	0.5	0.75
GDP growth rate (% per year)	3.8	0.8	−7.8	−4.7

(b) Yes, since the relative growth rate was negative in the last two quarters of 2008, the GDP decreased then, and the economy was in recession.

58. **(a)** The largest time interval was 2010–2012 since the percentage growth rate increased from −60.5 to 69.9 from 2010 to 2012. This means the US exports of biofuels grew relatively more from 2010 to 2012 than from 2012 to 2013. (Note that the percentage growth rate was a decreasing function of time over 2012–2014.)

(b) The largest time interval was 2012–2013 since the percentage growth rates were positive for each of these two consecutive years. This means that the amount of biofuels exported from the US steadily increased during the two years from 2012 to 2013, then decreased in 2014.

59. (a) The largest time interval was 2007–2009 since the percentage growth rate increased from −14.6 in 2007 to 6.3 in 2009. This means that from 2007 to 2009 the US consumption of hydroelectric power grew relatively more with each successive year.

(b) The largest time interval was 2012–2014 since the percentage growth rates were negative for each of these three consecutive years. This means that the amount of hydroelectric power consumed by the US industrial sector steadily decreased during the three year span from 2012 to 2014.

60. (a) The largest time interval was 2008–2010 since the percentage growth rate decreased from 3.6 in 2008 to −29.7 in 2010. This means that from 2008 to 2010 the US price per watt of a solar panel fell relatively more with each successive year.

(b) The largest time interval was 2009–2010 since the percentage growth rates were negative for each of these two consecutive years. This means that the US price per watt of a solar panel decreased during the two year span from 2009 to 2010, after an increase in the previous year.

61. (a) We have

$$\text{Relative change in price of candy} = \frac{\text{Change in price}}{\text{Initial price}} = \frac{1.25 - 1}{1} = 0.25.$$

The price of the candy has gone up by 25%.

(b) We have

$$\text{Relative change in quantity of sold candy} = \frac{\text{Change in quantity}}{\text{Initial quantity}} = \frac{2440 - 2765}{2765} = -0.12.$$

The number of candy sold dropped down by 12%.

(c) We have

$$\left|\frac{\text{Relative change in quantity}}{\text{Relative change in price}}\right| = \left|\frac{-0.12}{0.25}\right| = 0.48.$$

So for a 1% increase in a \$1 candy the quantity sold drops by 0.48%.

Solutions for Section 1.4

1. See Figure 1.23

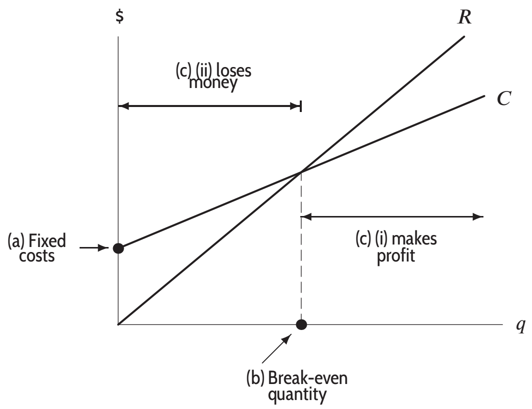


Figure 1.23

2. (a) The company makes a profit when the revenue is greater than the cost of production. Looking at the figure in the problem, we see that this occurs whenever more than roughly 335 items are produced and sold.

(b) If $q = 600$, revenue ≈ 2400 and cost ≈ 1750 so profit is about \$650.

3. (a) The fixed costs are the price of producing zero units, or $C(0)$, which is the vertical intercept. Thus, the fixed costs are roughly \$75. The marginal cost is the slope of the line. We know that

$$C(0) = 75$$

and looking at the graph we can also tell that

$$C(30) = 300.$$

Thus, the slope or the marginal cost is

$$\text{Marginal cost} = \frac{300 - 75}{30 - 0} = \frac{225}{30} = 7.50 \text{ dollars per unit.}$$

(b) Looking at the graph it seems that

$$C(10) \approx 150.$$

Alternatively, using what we know from parts (a) and (b) we know that the cost function is

$$C(q) = 7.5q + 75.$$

Thus,

$$C(10) = 7.5(10) + 75 = 75 + 75 = \$150.$$

The total cost of producing 10 items is \$150.

4. We know that the fixed cost is the cost that the company would have to pay if no items were produced. Thus

$$\text{Fixed cost} = C(0) = \$5000.$$

We know that the cost function is linear and we know that the slope of the function is exactly the marginal cost. Thus

$$\text{Slope} = \text{Marginal cost} = \frac{5020 - 5000}{5 - 0} = \frac{20}{5} = 4 \text{ dollars per unit produced.}$$

Thus, the marginal cost is 4 dollars per unit produced. We know that since $C(q)$ is linear

$$C(q) = m \cdot q + b,$$

where m is the slope and b is the value of $C(0)$, the vertical intercept. Or in other words, m is equal to the marginal cost and b is equal to the fixed cost. Thus

$$C(q) = 4q + 5000.$$

5. The 5.7 represents the fixed cost of \$5.7 million.

The 0.002 represents the variable cost per unit in millions of dollars. Thus, each additional unit costs an additional \$0.002 million = \$2000 to produce.

6. (a) A company with little or no fixed costs would be one that does not need much start-up capital and whose costs are mainly on a per unit basis. An example of such a company is a consulting company, whose major expense is the time of its consultants. Such a company would have little fixed costs to worry about.

(b) A company with marginal cost of zero (or almost) would be one that can produce a product with little or no additional costs per unit. An example is a computer software company. The major expense of such a company is software development, a fixed cost. Additional copies of its software can be very easily made. Thus, its marginal costs are rather small.

7. (a) The statement $f(12) = 60$ says that when $p = 12$, we have $q = 60$. When the price is \$12, we expect to sell 60 units.

(b) Decreasing, because as price increases, we expect less to be sold.

8. (a) The cost of producing 500 units is

$$C(500) = 6000 + 10(500) = 6000 + 5000 = \$11{,}000.$$

The revenue the company makes by selling 500 units is

$$R(500) = 12(500) = \$6000.$$

Thus, the cost of making 500 units is greater than the money the company will make by selling the 500 units, so the company does not make a profit.

The cost of producing 5000 units is

$$C(5000) = 6000 + 10(5000) = 6000 + 50000 = \$56{,}000.$$

The revenue the company makes by selling 5000 units is

$$R(5000) = 12(5000) = \$60{,}000.$$

Thus, the cost of making 5000 units is less than the money the company will make by selling the 5000 units, so the company does make a profit.

(b) The break-even point is the number of units that the company has to produce so that in selling those units, it makes as much as it spent on producing them. That is, we are looking for q such that

$$C(q) = R(q).$$

Solving we get

$$\begin{aligned} C(q) &= R(q) \\ 6000 + 10q &= 12q \\ 2q &= 6000 \\ q &= 3000. \end{aligned}$$

Thus, if the company produces and sells 3000 units, it will break even.
Graphically, the break-even point, which occurs at (3000, \$36,000), is the point at which the graphs of the cost and the revenue functions intersect. (See Figure 1.24.)

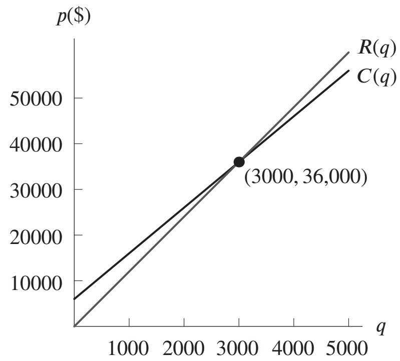


Figure 1.24

9. The 5500 tells us the quantity that would be demanded if the price were \$0. The 100 tells us the change in the quantity demanded if the price increases by \$1. Thus, a price increase of \$1 causes demand to drop by 100 units.

10. Since the price p is on the vertical axis and the quantity q is on the horizontal axis we would like a formula which expresses p in terms of q. Thus

$$\begin{aligned} 75p + 50q &= 300 \\ 75p &= 300 - 50q \\ p &= 4 - \frac{2}{3}q. \end{aligned}$$

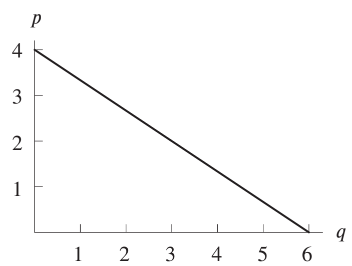


Figure 1.25

From the graph in Figure 1.25 we see that the vertical intercept occurs at $p = 4$ dollars and the horizontal intercept occurs at $q = 6$. This tells us that at a price of \$4 or more nobody would buy the product. On the other hand even if the product were given out for free, no more than 6 units would be demanded.

11. The cost function $C(q) = b + mq$ satisfies $C(0) = 500$, so $b = 500$, and $MC = m = 6$. So

$$C(q) = 500 + 6q.$$

The revenue function is $R(q) = 12q$, so the profit function is

$$\pi(q) = R(q) - C(q) = 12q - 500 - 6q = 6q - 500.$$

12. The cost function $C(q) = b + mq$ satisfies $C(0) = 35{,}000$, so $b = 35{,}000$, and $MC = m = 10$. So

$$C(q) = 35{,}000 + 10q.$$

The revenue function is $R(q) = 15q$, so the profit function is

$$\pi(q) = R(q) - C(q) = 15q - 35{,}000 - 10q = 5q - 35{,}000.$$

13. The cost function $C(q) = b + mq$ satisfies $C(0) = 5000$, so $b = 5000$, and $MC = m = 15$. So

$$C(q) = 5000 + 15q.$$

The revenue function is $R(q) = 60q$, so the profit function is

$$\pi(q) = R(q) - C(q) = 60q - 5000 - 15q = 45q - 5000.$$

14. The cost function $C(q) = b + mq$ satisfies $C(0) = 0$, so $b = 0$. There are 32 ounces in a quart, so 160 in 5 quarts, or 20 cups at 8 ounces per cup. Thus each cup costs the operator 20 cents $= 0.20$ dollars, so $MC = m = 0.20$. So

$$C(q) = 0.20q.$$

The revenue function is $R(q) = 0.25q$. So the profit function is

$$\pi(q) = R(q) - C(q) = 0.25q - 0.20q = 0.05q.$$

15. **(a)** We know that the cost function will be of the form

$$C(q) = b + m \cdot q$$

where m is the slope of the graph and b is the vertical intercept. We also know that the fixed cost is the vertical intercept and the variable cost is the slope. Thus, we have

$$C(q) = 5000 + 30q.$$

We know that the revenue function will take on the form

$$R(q) = pq$$

where p is the price charged per unit. In our case the company sells the chairs at \$50 a piece so

$$R(q) = 50q.$$

(b) Marginal cost is \$30 per chair. Marginal revenue is \$50 per chair.

(c)

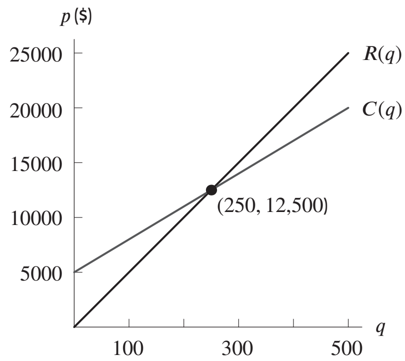


Figure 1.26

(d) We know that the break-even point is the number of chairs that the company has to sell so that the revenue will equal the cost of producing these chairs. In other words, we are looking for q such that

$$C(q) = R(q).$$

Solving we get

$$\begin{aligned} C(q) &= R(q) \\ 5000 + 30q &= 50q \\ 20q &= 5000 \\ q &= 250. \end{aligned}$$

Thus, the break-even point is 250 chairs and \$12,500. Graphically, it is the point in Figure 1.26 where the cost function intersects the revenue function.

16. (a) Regardless of the number of rides taken, the visitor pays \$21 to get in. In addition, the visitor pays \$4.50 per ride, so the function $R(n)$ is

$$R(n) = 21 + 4.5n.$$

(b) Substituting in the values $n = 2$ and $n = 8$ into the formula for $R(n)$, we get

$$R(2) = 21 + 4.5(2) = 21 + 9 = \$30.$$

This means that admission and 2 rides costs \$30.

$$R(8) = 21 + 4.5(8) = 21 + 36 = \$57.$$

This means that admission and 8 rides costs \$57.

17. (a) We know that the fixed cost of the first price list is \$100 and the variable cost is \$0.03. Thus, the cost of making q copies under the first option is

$$C_1(q) = 100 + 0.03q.$$

We know that the fixed cost of the second price list is \$200 and the variable cost is \$0.02. Thus, the cost of making q copies under the second option is

$$C_2(q) = 200 + 0.02q.$$

(b) At 5000 copies, the first price list gives the cost

$$C_1(5000) = 100 + 5000(0.03) = 100 + 150 = \$250.$$

At 5000 copies, the second price list gives the cost

$$C_2(5000) = 200 + 5000(0.02) = 200 + 100 = \$300.$$

Thus, for 5000 copies, the first price list is cheaper.

(c) We are asked to find the point q at which

$$C_1(q) = C_2(q).$$

Solving we get

$$\begin{aligned} C_1(q) &= C_2(q) \\ 100 + 0.03q &= 200 + 0.02q \\ 100 &= 0.01q \\ q &= 10{,}000. \end{aligned}$$

Thus, if one needs to make ten thousand copies, the cost under both price lists will be the same.

18. (a) The fixed cost is the cost that would have to be paid even if nothing was produced. That is, the fixed cost is

$$C(0) = 4000 + 2(0) = \$4000.$$

(b) We know that for every additional item produced, the company must pay another \$2. Thus \$2 is the marginal cost.

(c) We know that the revenue takes on the form

$$R(q) = p \cdot q$$

where q is the quantity produced and p is the price the company is charging for an item. Thus in our case,

$$p = \frac{R(q)}{q} = \frac{10q}{q} = \$10.$$

So the company is charging \$10 per item.

(d)

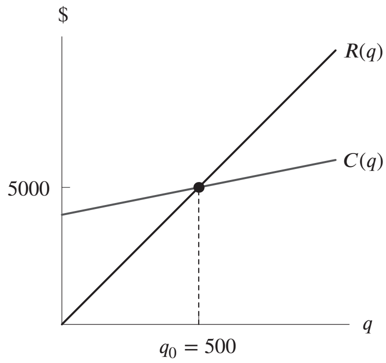


Figure 1.27

We know that the company will make a profit for $q > q_0$ since the line $R(q)$ lies above the line $C(q)$ in that region, so revenues are greater than costs.

(e) Looking at Figure 1.27 we see that the two graphs intersect at the point where

$$q_0 = 500.$$

Thus, the company will break even if it produces 500 units. Algebraically, we know that the company will break even for q_0 such that the cost function is equal to the revenue function at q_0. That is, when

$$C(q_0) = R(q_0).$$

Solving we get

$$\begin{aligned} C(q_0) &= R(q_0) \\ 4000 + 2q_0 &= 10q_0 \\ 8q_0 &= 4000 \\ q_0 &= 500. \end{aligned}$$

And

$$C(500) = R(500) = \$5{,}000.$$

19. (a) We know that the function for the cost of running the theater is of the form

$$C = mq + b$$

where q is the number of customers, m is the variable cost and b is the fixed cost. Thus, the function for the cost is

$$C = 6q + 5000.$$

We know that the revenue function is of the form

$$R = pq$$

where p is the price charged per customer. Thus, the revenue function is

$$R = 11q.$$

The theater makes a profit when the revenue is greater than the cost, that is when

$$R > C.$$

Substituting $R = 11q$ and $C = 6q + 5000$, we get

$$\begin{aligned} R &> C \\ 11q &> 6q + 5000 \\ 5q &> 5000 \\ q &> 1000. \end{aligned}$$

Thus, the theater makes a profit when it has more than 1000 customers.

(b) The graph of the two functions is shown in Figure 1.28.

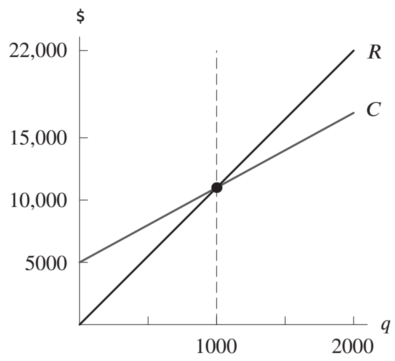


Figure 1.28

20. (a) We have $C(q) = 6000 + 2q$ and $R(q) = 5q$ and so

$$\pi(q) = R(q) - C(q) = 5q - (6000 + 2q) = -6000 + 3q.$$

(b) See Figure 1.29. We find the break-even point, q_0, by setting the revenue equal to the cost and solving for q:

$$\begin{aligned} \text{Revenue} &= \text{Cost} \\ 5q &= 6000 + 2q \\ 3q &= 6000 \\ q &= 2000. \end{aligned}$$

The break-even point is $q_0 = 2000$ puzzles. Notice that this is the same answer we get if we set the profit function equal to zero.

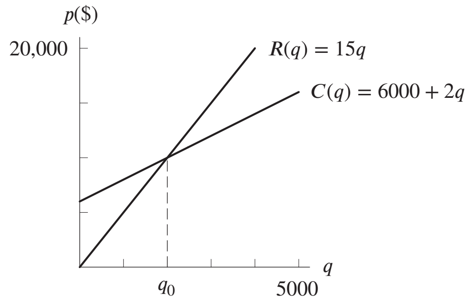


Figure 1.29: Cost and revenue functions for the jigsaw puzzle company

21. **(a)** The cost function is of the form

$$C(q) = b + m \cdot q$$

where m is the variable cost and b is the fixed cost. Since the variable cost is \$20 and the fixed cost is \$650,000, we get

$$C(q) = 650{,}000 + 20q.$$

The revenue function is of the form

$$R(q) = pq$$

where p is the price that the company is charging the buyer for one pair. In our case the company charges \$70 a pair so we get

$$R(q) = 70q.$$

The profit function is the difference between revenue and cost, so

$$\pi(q) = R(q) - C(q) = 70q - (650{,}000 + 20q) = 70q - 650{,}000 - 20q = 50q - 650{,}000.$$

(b) Marginal cost is \$20 per pair. Marginal revenue is \$70 per pair. Marginal profit is \$50 per pair.

(c) We are asked for the number of pairs of shoes that need to be produced and sold so that the profit is larger than zero. That is, we are trying to find q such that

$$\pi(q) > 0.$$

Solving we get

$$\begin{aligned} \pi(q) &> 0 \\ 50q - 650{,}000 &> 0 \\ 50q &> 650{,}000 \\ q &> 13{,}000. \end{aligned}$$

Thus, if the company produces and sells more than 13,000 pairs of shoes, it will make a profit.

22. **(a)** We know that the function for the value of the robot at time t will be of the form

$$V(t) = m \cdot t + b.$$

We know that at time $t = 0$ the value of the robot is \$15,000. Thus the vertical intercept b is

$$b = 15{,}000.$$

We know that m is the slope of the line. Also at time $t = 10$ the value is \$0. Thus

$$m = \frac{0 - 15{,}000}{10 - 0} = \frac{-15{,}000}{10} = -1500.$$

Thus we get

$$V(t) = -1500t + 15{,}000 \text{ dollars.}$$

(b) The value of the robot in three years is

$$V(3) = -1500(3) + 15{,}000 = -4500 + 15{,}000 = \$10{,}500.$$

23. **(a)** We know that the function for the value of the tractor will be of the form

$$V(t) = m \cdot t + b$$

where m is the slope and b is the vertical intercept. We know that the vertical intercept is simply the value of the function at time $t = 0$, which is \$50,000. Thus,

$$b = \$50{,}000.$$

Since we know the value of the tractor at time $t = 20$ we know that the slope is

$$m = \frac{V(20) - V(0)}{20 - 0} = \frac{10{,}000 - 50{,}000}{20} = \frac{-40{,}000}{20} = -2000.$$

Thus we get

$$V(t) = -2000t + 50{,}000 \text{ dollars.}$$

(b)

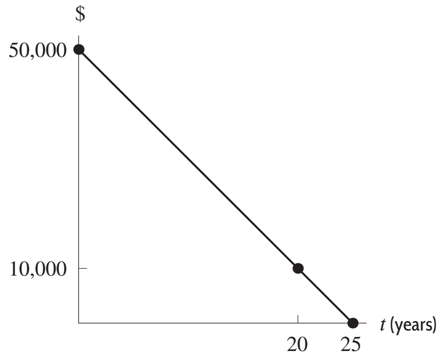


Figure 1.30

(c) Looking at Figure 1.30 we see that the vertical intercept occurs at the point $(0, 50{,}000)$ and the horizontal intercept occurs at $(25, 0)$. The vertical intercept tells us the value of the tractor at time $t = 0$, namely, when it was brand new. The horizontal intercept tells us at what time t the value of the tractor will be \$0. Thus the tractor is worth \$50,000 when it is new, and it is worth nothing after 25 years.

24. **(a)** The slope of the linear function is

$$\text{Slope} = \frac{\Delta V}{\Delta t} = \frac{25{,}000 - 100{,}000}{15 - 0} = -5000.$$

The vertical intercept is $V = 100{,}000$ so the formula is

$$V = 100{,}000 - 5000t.$$

(b) At $t = 5$, we have $V = 100{,}000 - 5000 \cdot 5 = 75{,}000$. In 2020, the bus is worth \$75,000.

(c) The vertical intercept is 100,000 and represents the value of the bus in dollars in 2015. The horizontal intercept is the value of t when $V = 0$. Solving $0 = 100{,}000 - 5000t$ for t, we find $t = 20$. The bus is worth nothing when $t = 20$ years, that is, in the year 2035.

(d) The domain of the function is $0 \leq t \leq 20$. The bus cannot have negative value.

25. We know that at the point where the price is \$1 per scoop the quantity must be 240. Thus we can fill in the graph as follows:

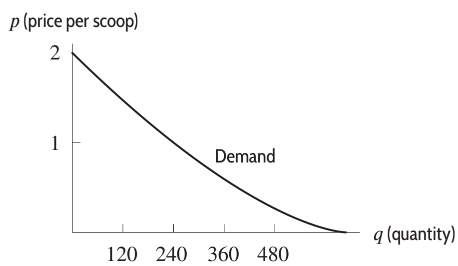


Figure 1.31

(a) Looking at Figure 1.31 we see that when the price per scoop is half a dollar, the quantity given by the demand curve is roughly 360 scoops.

(b) Looking at Figure 1.31 we see that when the price per scoop is \$1.50, the quantity given by the demand curve is roughly 120 scoops.

26. (a) Since cost is less than revenue for quantities in the table between 20 and 60 units, production appears to be profitable between these values.

(b) Profit = Revenue − Cost is show in Table 1.3. The maximum profit is obtained at a production level of about 40 units.

Table 1.3

Quantity	0	10	20	30	40	50	60	70	80
Cost (\$)	120	400	600	780	1000	1320	1800	2500	3400
Revenue (\$)	0	300	600	900	1200	1500	1800	2100	2400
Profit	−120	−100	0	120	200	180	0	−400	−1000

27. (a) We know that as the price per unit increases, the quantity supplied increases, while the quantity demanded decreases. So Table 1.28 of the text is the demand curve (since as the price increases the quantity decreases), while Table 1.29 of the text is the supply curve (since as the price increases the quantity increases.)

(b) Looking at the demand curve data in Table 1.28 we see that a price of \$155 gives a quantity of roughly 14.

(c) Looking at the supply curve data in Table 1.29 we see that a price of \$155 gives a quantity of roughly 24.

(d) Since supply exceeds demand at a price of \$155, the shift would be to a lower price.

(e) Looking at the demand curve data in Table 1.28 we see that if the price is less than or equal to \$143 the consumers would buy at least 20 items.

(f) Looking at the data for the supply curve (Table 1.29) we see that if the price is greater than or equal to \$110 the supplier will produce at least 20 items.

28. (a) We know that the equilibrium point is the point where the supply and demand curves intersect. Looking at the figure in the problem, we see that the price at which they intersect is \$10 per unit and the corresponding quantity is 3000 units.

(b) We know that the supply curve climbs upward while the demand curve slopes downward. Thus we see from the figure that at the price of \$12 per unit the suppliers will be willing to produce 3500 units while the consumers will be ready to buy 2500 units. Thus we see that when the price is above the equilibrium point, more items would be produced than the consumers will be willing to buy. Thus the producers end up wasting money by producing that which will not be bought, so the producers are better off lowering the price.

(c) Looking at the point on the rising curve where the price is \$8 per unit, we see that the suppliers will be willing to produce 2500 units, whereas looking at the point on the downward sloping curve where the price is \$8 per unit, we see that the consumers will be willing to buy 3500 units. Thus we see that when the price is less than the equilibrium price, the consumers are willing to buy more products than the suppliers would make and the suppliers can thus make more money by producing more units and raising the price.

29. (a) We know that the cost function will be of the form

$$C = mq + b$$

where m is the variable cost and b is the fixed cost. In this case this gives

$$C = 5q + 7000.$$

We know that the revenue function is of the form

$$R = pq$$

where p is the price per shirt. Thus in this case we have

$$R = 12q.$$

(b) We are given

$$q = 2000 - 40p.$$

We are asked to find the demand when the price is \$12. Plugging in $p = 12$ we get

$$q = 2000 - 40(12) = 2000 - 480 = 1520.$$

Given this demand we know that the cost of producing $q = 1520$ shirts is

$$C = 5(1520) + 7000 = 7600 + 7000 = \$14{,}600.$$

The revenue from selling $q = 1520$ shirts is

$$R = 12(1520) = \$18{,}240.$$

Thus the profit is

$$\pi(12) = R - C$$

or in other words

$$\pi(12) = 18{,}240 - 14{,}600 = \$3640.$$

(c) Since we know that

$$q = 2000 - 40p,$$
$$C = 5q + 7000,$$

and

$$R = pq,$$

we can write

$$C = 5q + 7000 = 5(2000 - 40p) + 7000 = 10{,}000 - 200p + 7000 = 17{,}000 - 200p$$

and

$$R = pq = p(2000 - 40p) = 2000p - 40p^2.$$

We also know that the profit is the difference between the revenue and the cost so

$$\pi(p) = R - C = 2000p - 40p^2 - (17{,}000 - 200p) = -40p^2 + 2200p - 17{,}000.$$

(d) Looking at Figure 1.32 we see that the maximum profit occurs when the company charges about \$27.50 per shirt. At this price, the profit is about \$13,250.

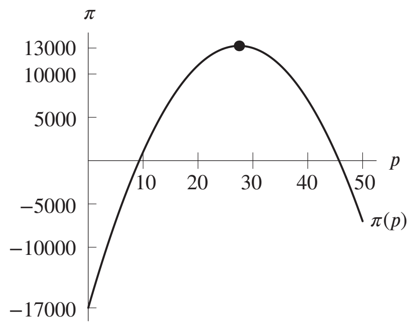


Figure 1.32

30. We know that a formula for passengers versus price will take the form

$$N = mp + b$$

where N is the number of passengers on the boat when the price of a tour is p dollars. We know two points on the line thus we know that the slope is

$$\text{slope} = \frac{650 - 500}{20 - 25} = \frac{150}{-5} = -30.$$

Thus the function will look like

$$N = -30p + b.$$

Plugging in the point (20, 650) we get

$$\begin{aligned} N &= -30p + b \\ 650 &= (-30)(20) + b \\ &= -600 + b \\ b &= 1250. \end{aligned}$$

Thus a formula for the number of passengers as a function of tour price is

$$N = -30p + 1250.$$

31. **(a)** If we think of q as a linear function of p, then q is the dependent variable, p is the independent variable, and the slope $m = \Delta q/\Delta p$. We can use any two points to find the slope. If we use the first two points, we get

$$\text{Slope} = m = \frac{\Delta q}{\Delta p} = \frac{460 - 500}{18 - 16} = \frac{-40}{2} = -20.$$

The units are the units of q over the units of p, or tons per dollar. The slope tells us that, for every dollar increase in price, the number of tons sold every month will decrease by 20.

To write q as a linear function of p, we need to find the vertical intercept, b. Since q is a linear function of p, we have $q = b + mp$. We know that $m = -20$ and we can use any of the points in the table, such as $p = 16$, $q = 500$, to find b. Substituting gives

$$\begin{aligned} q &= b + mp \\ 500 &= b + (-20)(16) \\ 500 &= b - 320 \\ 820 &= b. \end{aligned}$$

Therefore, the vertical intercept is 820 and the equation of the line is

$$q = 820 - 20p.$$

(b) If we now consider p as a linear function of q, we have

$$\text{Slope} = m = \frac{\Delta p}{\Delta q} = \frac{18 - 16}{460 - 500} = \frac{2}{-40} = -\frac{1}{20} = -0.05.$$

The units of the slope are dollars per ton. The slope tells us that, if we want to sell one more ton of the product every month, we should reduce the price by \$0.05.

Since p is a linear function of q, we have $p = b + mq$ and $m = -0.05$. To find b, we substitute any point from the table such as $p = 16$, $q = 500$ into this equation:

$$\begin{aligned} p &= b + mq \\ 16 &= b + (-0.05)(500) \\ 16 &= b - 25 \\ 41 &= b. \end{aligned}$$

The equation of the line is

$$p = 41 - 0.05q.$$

Alternatively, notice that we could have taken our answer to part (a), that is $q = 820 - 20p$, and solved for p.

32. **(a)** The quantity demanded at a price of \$100 is $q = 120{,}000 - 500(100) = 70{,}000$ units. The quantity supplied at a price of \$100 is $q = 1000(100) = 100{,}000$ units. At a price of \$100, the supply is larger than the demand, so some goods remain unsold and we expect the market to push prices down.

(b) The supply and demand curves are shown in Figure 1.33. The equilibrium price is about $p^* = \$80$ and the equilibrium quantity is about $q^* = 80{,}000$ units. The market will push prices downward from \$100, toward the equilibrium price of \$80. This agrees with the conclusion to part (a) which says that prices will drop.

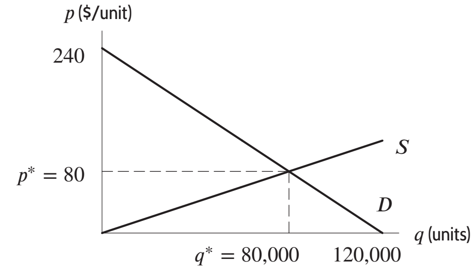


Figure 1.33: Demand and supply curves for a product

33. One possible supply curve is shown in Figure 1.34. Many other answers are possible.

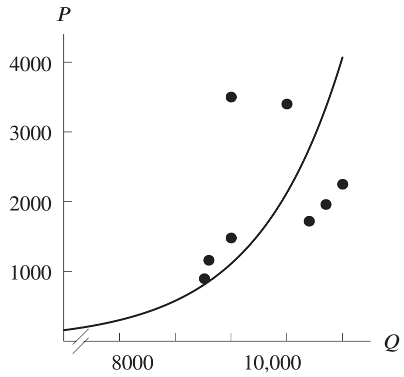


Figure 1.34

34. **(a)** The total amount spent on drivers is $30{,}000d$ and the total amount spent on cars is $20{,}000c$, so the budget constraint is

$$30{,}000d + 20{,}000c = 720{,}000.$$

(b) When $c = 0$, we have $30{,}000d = 720{,}000$ which gives $d = 24$ drivers. If the company does not replace any cars, it can pay 24 drivers.

When $d = 0$, we have $20{,}000c = 720{,}000$ which gives $c = 36$ cars. If the company hires no drivers, it can replace 36 cars.

35. **(a)** We know that the total amount the company will spend on raw materials will be

$$\text{price for raw materials} = \$100 \cdot m$$

where m is the number of units of raw materials the company will buy. We know that the total amount the company will spend on paying employees will be

$$\text{total employee expenditure} = \$25{,}000 \cdot r$$

where r is the number of employees the company will hire. Since the total amount the company spends is \$500,000, we get

$$25{,}000r + 100m = 500{,}000.$$

(b) Solving for m we get

$$\begin{aligned} 25{,}000r + 100m &= 500{,}000 \\ 100m &= 500{,}000 - 25{,}000r \\ m &= 5000 - 250r. \end{aligned}$$

(c) Solving for r we get

$$\begin{aligned} 25{,}000r + 100m &= 500{,}000 \\ 25{,}000r &= 500{,}000 - 100m \\ r &= 20 - \frac{100}{25{,}000}m \\ &= 20 - \frac{1}{250}m. \end{aligned}$$

36. (a) The amount spent on books will be

$$\text{Amount for books} = \$80 \cdot b$$

where b is the number of books bought. The amount of money spent on outings is

$$\text{money spent on outings} = \$20 \cdot s$$

where s is the number of social outings. Since we want to spend all of the \$2000 budget we end up with

$$80b + 20s = 2000.$$

(b)

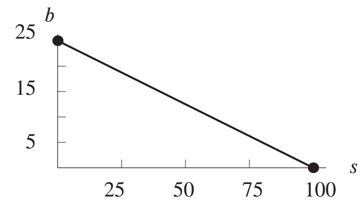


Figure 1.35

(c) Looking at Figure 1.35 we see that the vertical intercept occurs at the point

$$(0, 25)$$

and the horizontal intercept occurs at

$$(100, 0).$$

The vertical intercept tells us how many books we would be able to buy if we wanted to spend all of the budget on books. That is, we could buy at most 25 books. The horizontal intercept tells how many social outings we could afford if we wanted to spend all of the budget on outings. That is, we would be able to go on at most 100 outings.

37. (a) If the bakery owner decreases the price, the customers want to buy more. Thus, the slope of $d(q)$ is negative. If the owner increases the price, she is willing to make more cakes. Thus the slope of $s(q)$ is positive.

(b) To determine whether an ordered pair (q, p) is a solution to the inequality, we substitute the values of q and p into the inequality and see whether the resulting statement is true. Substituting the two ordered pairs gives

(60, 18): $18 \le 20 - 60/20$, or $18 \le 17$. This is false, so (60, 18) is not a solution.

(120, 12): $12 \le 20 - 120/20$, or $12 \le 14$. This is true, so (120, 12) is a solution.

The pair (60, 18) is not a solution to the inequality $p \le 20 - q/20$. This means that the price \$18 is higher than the unit price at which customers would be willing to buy a total of 60 cakes. So customers are not willing to buy 60 cakes at \$18. The pair (120, 12) is a solution, meaning that \$12 is not more than the price at which customers would be willing to buy 120 cakes. Thus customers are willing to buy a total of 120 (and more) cakes at \$12. Each solution (q, p) represents a quantity of cakes q that customers would be willing to buy at the unit price p.

(c) In order to be a solution to both of the given inequalities, a point (q, p) must lie on or below the line $p = 20 - q/20$ and on or above the line $p = 11 + q/40$. Thus, the solution set of the given system of inequalities is the region shaded in Figure 1.36.

A point (q, p) in this region represents a quantity q of cakes that customers would be willing to buy, and that the bakery-owner would be willing to make and sell, at the price p.

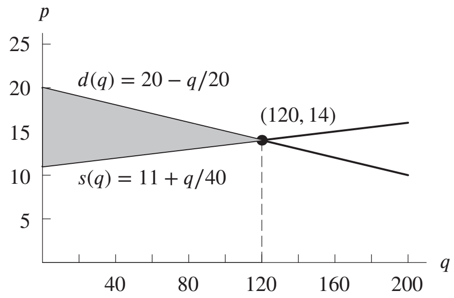


Figure 1.36: Possible cake sales at different prices and quantities

(d) To find the rightmost point of this region, we need to find the intersection point of the lines $p = 20 - q/20$ and $p = 11 + q/40$. At this point, p is equal to both $20 - q/20$ and $11 + q/40$, so these two expressions are equal to each other:

$$\begin{aligned} 20 - \frac{q}{20} &= 11 + \frac{q}{40} \\ 9 &= \frac{q}{20} + \frac{q}{40} \\ 9 &= \frac{3q}{40} \\ q &= 120. \end{aligned}$$

Therefore, $q = 120$ is the maximum number of cakes that can be sold at a price at which customers are willing to buy them all, and the owner of the bakery is willing to make them all. The price at this point is $p = 20 - 120/20 = 14$ dollars. (In economics, this price is called the *equilibrium price*, since at this point there is no incentive for the owner of the bakery to raise or lower the price of the cakes.)

38. (a) See Figure 1.37.

(b) If the slope of the supply curve increases then the supply curve will intersect the demand curve sooner, resulting in a higher equilibrium price p_1 and lower equilibrium quantity q_1. Intuitively, this makes sense since if the slope of the supply curve increases. The amount produced at a given price decreases. See Figure 1.38.

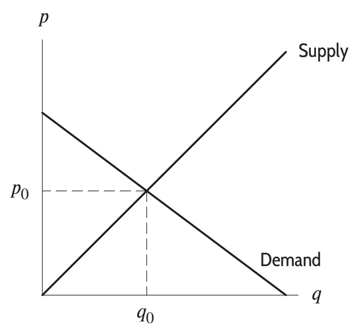


Figure 1.37

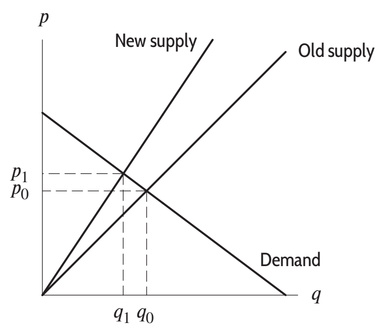


Figure 1.38

(c) When the slope of the demand curve becomes more negative, the demand function will decrease more rapidly and will intersect the supply curve at a lower value of q_1. This will also result in a lower value of p_1 and so the equilibrium price p_1 and equilibrium quantity q_1 will decrease. This follows our intuition, since if demand for a product lessens, the price and quantity purchased of the product will go down. See Figure 1.39.

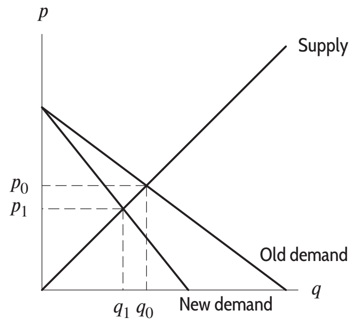


Figure 1.39

39. The quantity demanded for a product is $q = 4$ when the price is $p = 50$. The demand for the product is $q = 3.9$ when the price is $p = 51$. The relative change in demand is

$$\frac{3.9 - 4}{4} = -0.025$$

and the relative change in price is

$$\frac{51 - 50}{50} = 0.02.$$

Hence

$$\left|\frac{\text{Relative change in demand}}{\text{Relative change in price}}\right| = \left|\frac{-0.025}{0.02}\right| = 1.25.$$

A 1% increase in the price resulted in a 1.25% decrease in the demand.

40. The original demand equation, $q = 100 - 5p$, tells us that

$$\text{Quantity demanded } = 100 - 5\left(\begin{array}{c}\text{Amount per unit}\\ \text{paid by consumers}\end{array}\right).$$

The consumers pay $p + 2$ dollars per unit because they pay the price p plus \$2 tax. Thus, the new demand equation is

$$q = 100 - 5(p + 2) = 90 - 5p.$$

See Figure 1.40.

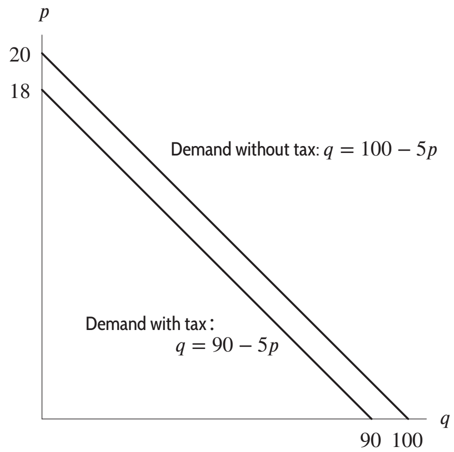


Figure 1.40

41. The original supply equation, $q = 4p - 20$, tells us that

$$\text{Quantity supplied } = 4\left(\begin{array}{c}\text{Amount per unit}\\ \text{received by suppliers}\end{array}\right) - 20.$$

The suppliers receive only $p-2$ dollars per unit because \$2 goes to the government as taxes. Thus, the new supply equation is

$$q = 4(p - 2) - 20 = 4p - 28.$$

See Figure 1.41.

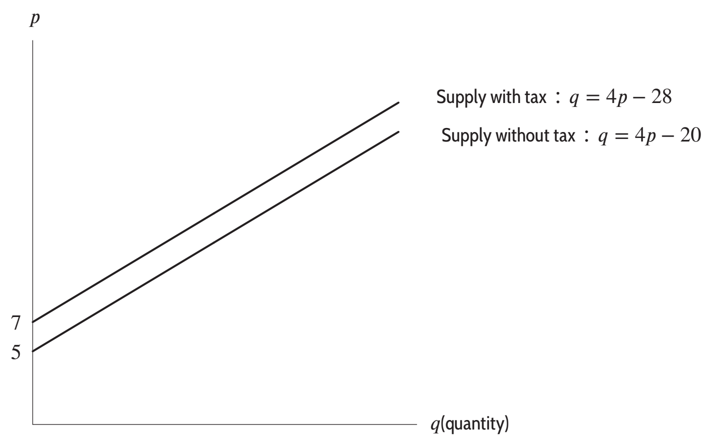


Figure 1.41

42. Before the tax is imposed, the equilibrium is found by solving the equations $q = 0.5p - 25$ and $q = 165 - 0.5p$. Setting the values of q equal, we have

$$\begin{aligned} 0.5p - 25 &= 165 - 0.5p \\ p &= 190 \text{ dollars} \end{aligned}$$

Substituting into one of the equation for q, we find $q = 0.5(190) - 25 = 70$ units. Thus, the pre-tax equilibrium is

$$p = \$190, \quad q = 70 \text{ units}.$$

The original supply equation, $q = 0.5p - 25$, tells us that

$$\text{Quantity supplied } = 0.5\left(\begin{array}{c}\text{Amount per unit}\\ \text{received by suppliers}\end{array}\right) - 25.$$

When the tax is imposed, the suppliers receive only $p - 8$ dollars per unit because \$8 goes to the government as taxes. Thus, the new supply curve is

$$q = 0.5(p - 8) - 25 = 0.5p - 29.$$

The demand curve is still

$$q = 165 - 0.5p.$$

To find the equilibrium, we solve the equations $q = 0.5p - 29$ and $q = 165 - 0.5p$. Setting the values of q equal, we have

$$\begin{aligned} 0.5p - 29 &= 165 - 0.5p \\ p &= 194 \text{ dollars} \end{aligned}$$

Substituting into one of the equations for q, we find that $q = 0.5(194) - 29 = 68$ units. Thus, the post-tax equilibrium is

$$p = \$194, q = 68 \text{ units}.$$

43. (a) The equilibrium price and quantity occur when demand equals supply. If we graph these functions on the same axes, we get Figure 1.42.

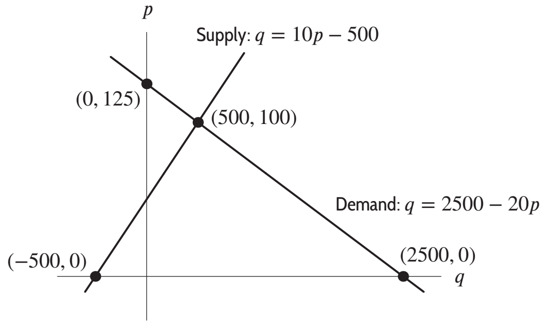


Figure 1.42

We can see from the graph in Figure 1.42 that the supply and demand curves intersect at the point (500, 100). The equilibrium price is \$100 and the equilibrium quantity is 500. This answer can also be obtained algebraically, by setting the values of q equal and solving

$$\begin{aligned} 2500 - 20p &= 10p - 500 \\ 3000 &= 30p \\ p &= 100. \\ q &= 2500 - 20(100) = 500. \end{aligned}$$

(b) The \$6 tax imposed on suppliers has the effect of moving the supply curve upward by \$6.

The original supply equation, $q = 10p - 500$, tells us that

$$\text{Quantity supplied } = 10\left(\begin{array}{c}\text{Amount received per}\\ \text{unit by suppliers}\end{array}\right) - 500.$$

The suppliers receive only $p - 6$ dollars per unit because \$6 goes to the government as taxes. Thus, the new supply equation is

$$q = 10(p - 6) - 500 = 10p - 560.$$

The demand equation $q = 2500 - 20p$ is unchanged.

We can find the new equilibrium price and quantity by graphing the new supply equation with the demand equation.

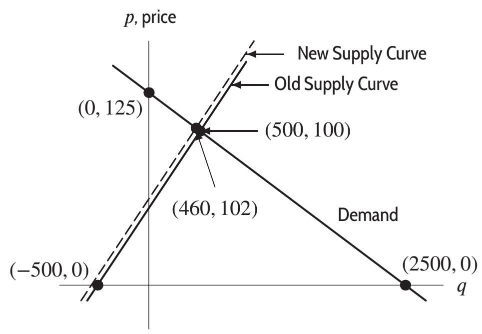


Figure 1.43: Specific tax shifts supply curve

From Figure 1.43, we see that the new equilibrium price (including tax) is \$102 and the new equilibrium quantity is 460 units. We can also obtain these results algebraically:

$$\begin{aligned} 2500 - 20p &= 10p - 560 \\ 3060 &= 30p \\ p &= 102, \end{aligned}$$

so $q = 10(102) - 560 = 460$.

(c) The tax paid by the consumer is \$2, since the new equilibrium price of \$102 is \$2 more than the old equilibrium price of \$100. Since the tax is \$6, the producer pays \$4 of the tax and receives \$102 − \$6 = \$96 per item after taxes.

(d) The tax received by the government per unit product is \$6. Thus, the total revenue received by the government is equal to the tax per unit times the number of units sold, which is just the equilibrium quantity. Thus,

$$\text{Government revenue} = \text{Tax} \cdot \text{Quantity} = 6 \cdot 460 = \$2760.$$

44. (a) The original demand equation, $q = 100 - 2p$, tells us that

$$\text{Quantity demanded} = 100 - 2\left(\begin{array}{c}\text{Amount per unit}\\ \text{paid by consumers}\end{array}\right).$$

The consumers pay $p+0.05p = 1.05p$ dollars per unit because they pay the price p plus 5% tax. Thus, the new demand equation is

$$q = 100 - 2(1.05p) = 100 - 2.1p.$$

The supply equation remains the same.

(b) To find the new equilibrium price and quantity, we find the intersection of the demand curve and the supply curve $q = 3p - 50$.

$$\begin{aligned} 100 - 2.1p &= 3p - 50 \\ 150 &= 5.1p \\ p &= \$29.41. \end{aligned}$$

The equilibrium price is \$29.41.

The equilibrium quantity q is given by

$$q = 3(29.41) - 50 = 88.23 - 50 = 38.23 \text{ units}.$$

(c) Since the pre-tax price was \$30 and the suppliers' new price is \$29.41 per unit,

$$\text{Tax paid by supplier} = \$30 - \$29.41 = \$0.59.$$

The consumers' new price is $1.05p = 1.05(29.41) = \$30.88$ per unit and the pre-tax price was \$30, so

$$\text{Tax paid by consumer} = 30.88 - 30 = \$0.88.$$

The total tax paid per unit by suppliers and consumers together is $0.59 + 0.88 = \$1.47$ per unit.

(d) The government receives \$1.47 per unit on 38.23 units. The total tax collected is $(1.47)(38.23) = \$56.20$.

45. (a) The original supply equation, $q = 3p - 50$, tells us that

$$\text{Quantity supplied} = 3\left(\begin{array}{c}\text{Amount received per}\\ \text{unit by suppliers}\end{array}\right) - 50.$$

The suppliers receive only $p - 0.05p = 0.95p$ dollars per unit because 5% of the price goes to the government. Thus, the new supply equation is

$$q = 3(0.95p) - 50 = 2.85p - 50.$$

The demand equation, $q = 100 - 2p$ is unchanged.

(b) At the equilibrium, supply equals demand:

$$\begin{aligned} 2.85p - 50 &= 100 - 2p \\ 4.85p &= 150 \\ p &= \$30.93. \end{aligned}$$

The equilibrium price is \$30.93. The equilibrium quantity q is the quantity demanded at the equilibrium price:

$$q = 100 - 2(30.93) = 38.14 \text{ units}.$$

(c) Since the pretax price was \$30 and consumers' new price is \$30.93,

$$\text{Tax paid by consumers} = 30.93 - 30 = \$0.93$$

The supplier keeps $0.95p = 0.95(30.93) = \$29.38$ per unit, so

$$\text{Tax paid by suppliers} = 30 - 29.38 = \$0.62.$$

The total tax per unit paid by consumers and suppliers together is $0.93 + 0.62 = \$1.55$ per unit.

(d) The government receives \$1.55 per unit on 38.14 units. The total tax collected is $(1.55)(38.14) = \$59.12$.

Solutions for Section 1.5

1. **(a)** Town (i) has the largest percent growth rate, at 12%.
(b) Town (ii) has the largest initial population, at 1000.
(c) Yes, town (iv) is decreasing in size, since the decay factor is 0.9, which is less than 1.

2. **(a)** Initial amount = 100; exponential growth; growth rate = 7% = 0.07.
(b) Initial amount = 5.3; exponential growth; growth rate = 5.4% = 0.054.
(c) Initial amount = 3500; exponential decay; decay rate = −7% = −0.07.
(d) Initial amount = 12; exponential decay; decay rate = −12% = −0.12.

3. Graph (I) and (II) are of increasing functions, therefore the growth factor for both functions is greater than 1. Since graph (I) increases faster than graph (II), graph (I) corresponds to the larger growth factor. Therefore graph (I) matches $Q = 50(1.4)^t$ and graph (II) is $Q = 50(1.2)^t$.

Similarly, since graphs (III) and (IV) are both of decreasing functions, they have growth factors between 0 and 1. Since graph (IV) decreases faster than graph (III), graph (IV) has the smaller decay factor. Thus graph (III) is $Q = 50(0.8)^t$ and graph (IV) is $Q = 50(0.6)^t$.

4. **(a)** The city's population is increasing at 5% per year. It is growing exponentially. Thus, it must be either I or II. We notice that part (b) has a higher rate of growth and thus we choose the exponential curve that grows more slowly, II.
(b) Using the same reasoning as part (a), we know that the graph of the population is an exponential growth curve. Comparing it to the previous city, part (b) has a faster rate of growth and thus we choose the faster growing exponential curve, I.
(c) The population is growing by a constant number of people each year. Thus, the graph of the population of the city is a line. Since the population is growing, we know that the line must have positive slope. Thus, the city's population is best described by graph III.
(d) The city's population does not change. That means that the population is a constant. It is thus a line with 0 slope, or graph V.

The remaining curves are IV and VI. Curve IV shows a population decreasing at an exponential rate. Comparison of the slope of curve IV at the points where it intersects curves I and II shows that the slope of IV is less steep than the slope of I or II. From this we know that the city's population is decreasing at a rate less than I or II's is increasing. It is decreasing at a rate of perhaps 4% per year.

Graph VI shows a population decreasing linearly—by the same number of people each year. By noticing that graph VI is more downward sloping than III is upward sloping, we can determine that the annual population decrease of a city described by VI is greater than the annual population increase of a city described by graph III. Thus, the population of city VI is decreasing by more than 5000 people per year.

5. **(a)** Since the y-intercept of each graph is above the x-axis, we know that a, c, and p must all be positive.
(b) Since the graph of $y = ab^x$ is increasing and goes through $(1, 1)$, the y-intercept, a, must be less than 1. Since the y-intercept is positive, we know that $0 < a < 1$. Since $y = cd^x$ and $y = pq^x$ are both decreasing, both d and q must be between 0 and 1.
(c) The graphs of $y = cd^x$ and $y = pq^x$ have the same y-intercepts, therefore we have $c = p$.
(d) Since the point $(1, 1)$ lies on the graph of $y = ab^x$, we know that $1 = ab^1$. Therefore, a and b are reciprocals of each other. Similarly, p and q are reciprocals of each other.

6. This looks like an exponential function $y = Ca^t$. The y-intercept is 500 so we have $y = 500a^t$. We use the point $(3, 2000)$ to find a:

$$\begin{aligned} y &= 500a^t \\ 2000 &= 500a^3 \\ 4 &= a^3 \\ a &= 4^{1/3} = 1.59. \end{aligned}$$

The formula is $y = 500(1.59)^t$.

7. This looks like an exponential decay function $y = Ca^t$. The y-intercept is 30 so we have $y = 30a^t$. We use the point $(25, 6)$ to find a:

$$\begin{aligned} y &= 30a^t \\ 6 &= 30a^{25} \\ 0.2 &= a^{25} \\ a &= (0.2)^{1/25} = 0.94. \end{aligned}$$

The formula is $y = 30(0.94)^t$.

8. We look for an equation of the form $y = y_0 a^x$ since the graph looks exponential. The points $(0, 3)$ and $(2, 12)$ are on the graph, so

$$3 = y_0 a^0 = y_0$$

and

$$12 = y_0 \cdot a^2 = 3 \cdot a^2, \quad \text{giving} \quad a = \pm 2.$$

Since $a > 0$, our equation is $y = 3(2^x)$.

9. We look for an equation of the form $y = y_0 a^x$ since the graph looks exponential. The points $(0, 18)$ and $(2, 8)$ are on the graph, so

$$18 = y_0 a^0 = y_0$$

and

$$8 = y_0 \cdot a^2 = 18 \cdot a^2, \quad \text{giving} \quad a = \pm\sqrt{\frac{4}{9}}.$$

Since $a > 0$, our equation is

$$y = 18\left(\frac{2}{3}\right)^x.$$

10. **(a)** If G is increasing at a constant percent rate, G is an exponential function of t. The formula is $G = 685.4(1.02)^t$.
(b) If G is increasing at a constant rate, G is a linear function of t. The formula is $G = 685.4 + 7t$.

11. **(a)** The function is linear with initial population of 1000 and slope of 50, so $P = 1000 + 50t$.
(b) This function is exponential with initial population of 1000 and growth rate of 5%, so $P = 1000(1.05)^t$.

12. **(a)** Since the price is decreasing at a constant absolute rate, the price of the product is a linear function of t. In t days, the product will cost $80 - 4t$ dollars.
(b) Since the price is decreasing at a constant relative rate, the price of the product is an exponential function of t. In t days, the product will cost $80(0.95)^t$.

13. **(a)** This is a linear function with slope -2 grams per day and intercept 30 grams. The function is $Q = 30 - 2t$, and the graph is shown in Figure 1.44.

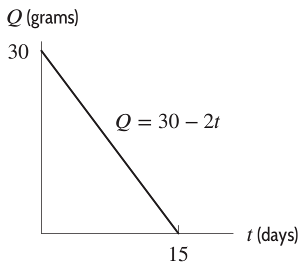


Figure 1.44

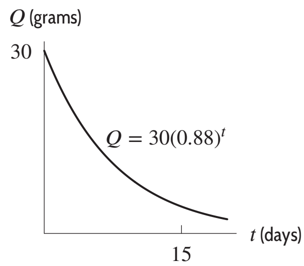


Figure 1.45

(b) Since the quantity is decreasing by a constant percent change, this is an exponential function with base $1-0.12 = 0.88$. The function is $Q = 30(0.88)^t$, and the graph is shown in Figure 1.45.

14. **(a)** We see from the base 1.0109 that the percent rate of growth is 1.09% per year.

(b) The initial population (in the year 2015) is 7.32 billion people. When $t = 5$, we have $P = 7.32(1.0109)^5 = 7.73$. The predicted world population in the year 2020 is 7.73 billion people.

(c) We have

$$\text{Average rate of change} = \frac{\Delta P}{\Delta t} = \frac{7.73 - 7.32}{2020 - 2015} = 0.082 \text{ billion people per year.}$$

The population of the world is projected to increase by about 82 million people per year from 2015 to 2020.

15. **(a)** Since the percent rate of change is constant, A is an exponential function of t. The initial quantity is 50 and the base is $1 - 0.06 = 0.94$. The formula is

$$A = 50(0.94)^t.$$

(b) When $t = 24$, we have $A = 50(0.94)^{24} = 11.33$. After one day, 11.33 mg of quinine remains.

(c) See Figure 1.46. The graph is decreasing and concave up, as we would expect with an exponential decay function.

(d) In Figure 1.46, it appears that when $A = 5$, we have $t \approx 37$. It takes about 37 hours until 5 mg remains.

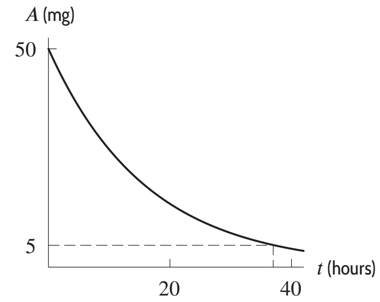


Figure 1.46

16. This function is exponential with initial value 234 and growth rate of 1.8%, so $\text{CPI} = 234(1.018)^t$.

17. Since the decay rate is 0.48%, the base of the exponential function is $1 - 0.0048 = 0.9952$. If P represents the percent of forested land in 1990 remaining t years after 1990, when $t = 0$ we have $P = 100$. The formula is $P = 100(0.9952)^t$. The percent remaining in 2030 will be $P = 100(0.9952)^{40} = 82.493$. About 82.493% of the land will be covered in forests 40 years later if the rate continues.

18. See Figure 1.47. The graph is decreasing and concave up. It looks like an exponential decay function with y-intercept 100.

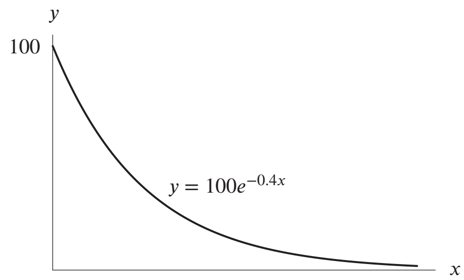


Figure 1.47

19. **(a)** See Table 1.4.

Table 1.4

x	0	1	2	3
e^x	1	2.72	7.39	20.09

(b) See Figure 1.48. The function $y = e^x$ is an exponential growth function.

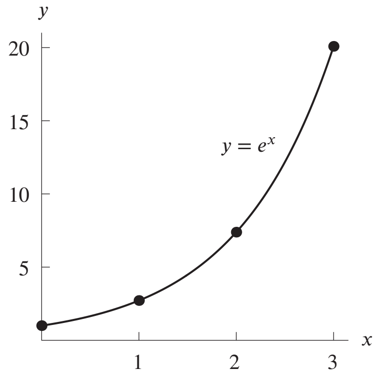


Figure 1.48

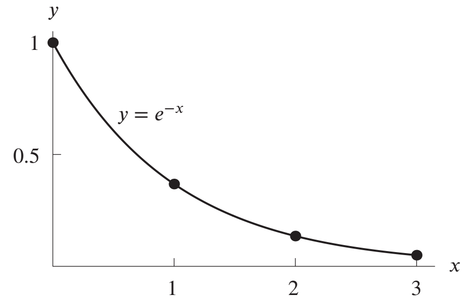


Figure 1.49

(c) See Table 1.5.

Table 1.5

x	0	1	2	3
e^{-x}	1	0.37	0.14	0.05

(d) See Figure 1.49. The function $y = e^{-x}$ is an exponential decay function.

20. The values of $f(x)$ given seem to increase by a factor of 1.4 for each increase of 1 in x, so we expect an exponential function with base 1.4. To assure that $f(0) = 4.30$, we multiply by the constant, obtaining

$$f(x) = 4.30(1.4)^x.$$

21. Each increase of 1 in t seems to cause $g(t)$ to decrease by a factor of 0.8, so we expect an exponential function with base 0.8. To make our solution agree with the data at $t = 0$, we need a coefficient of 5.50, so our completed equation is

$$g(t) = 5.50(0.8)^t.$$

22. Table A and Table B could represent linear functions of x. Table A could represent the constant linear function $y = 2.2$ because all y values are the same. Table B could represent a linear function of x with slope equal to $11/4$. This is because x values that differ by 4 have corresponding y values that differ by 11, and x values that differ by 8 have corresponding y values that differ by 22. In Table C, y decreases and then increases as x increases, so the table cannot represent a linear function. Table D does not show a constant rate of change, so it cannot represent a linear function.

23. Table D is the only table that could represent an exponential function of x. This is because, in Table D, the ratio of y values is the same for all equally spaced x values. Thus, the y values in the table have a constant percent rate of decrease:

$$\frac{9}{18} = \frac{4.5}{9} = \frac{2.25}{4.5} = 0.5.$$

Table A represents a constant function of x, so it cannot represent an exponential function. In Table B, the ratio between y values corresponding to equally spaced x values is not the same. In Table C, y decreases and then increases as x increases. So neither Table B nor Table C can represent exponential functions.

24. **(a)** See Figure 1.50.

(b) No. The points in the plot do not lie even approximately on a straight line, so a linear model is a poor choice.

(c) No. The plot shows that H is a decreasing function of Y that might be leveling off to an asymptote at $H = 0$. These are features of an exponential decay function, but their presence does not show that H is an exponential function of Y. We can do a more precise check by dividing each value of H by the previous year's H.

$$\frac{\text{Houses in 2011}}{\text{Houses in 2010}} = \frac{13 \text{ million}}{18.3 \text{ million}} = 0.710$$
$$\frac{\text{Houses in 2012}}{\text{Houses in 2011}} = \frac{7.8 \text{ million}}{13 \text{ million}} = 0.600$$
$$\frac{\text{Houses in 2013}}{\text{Houses in 2012}} = \frac{3.9 \text{ million}}{7.8 \text{ million}} = 0.500$$
$$\frac{\text{Houses in 2014}}{\text{Houses in 2013}} = \frac{1 \text{ million}}{3.9 \text{ million}} = 0.256$$
$$\frac{\text{Houses in 2015}}{\text{Houses in 2014}} = \frac{0.5 \text{ million}}{1 \text{ million}} = 0.500.$$

If H were an exponential function of Y, the ratios would be approximately constant. Since they are not approximately constant, we see that an exponential model is a poor choice.

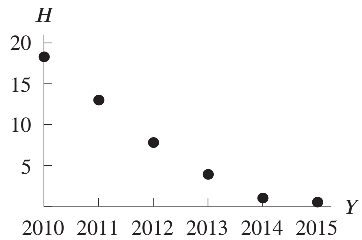


Figure 1.50

25. **(a)** We divide the two equations to solve for a:

$$\frac{P_0a^3}{P_0a^2} = \frac{75}{50}$$
$$a = 1.5$$

Now that we know the value of a, we can use either of the equations to solve for P_0. Using the first equation, we have:

$$P_0a^3 = 75$$
$$P_0(1.5^3) = 75$$
$$P_0 = \frac{75}{1.5^3} = 22.222$$

We see that $a = 1.5$ and $P_0 = 22.222$.

(b) The initial quantity is 22.222 and the quantity is growing at a rate of 50% per unit time.

26. **(a)** We divide the two equations to solve for a:

$$\frac{P_0a^4}{P_0a^3} = \frac{18}{20}$$
$$a = 0.9$$

Now that we know the value of a, we can use either of the equations to solve for P_0. Using the first equation, we have:

$$P_0a^4 = 18$$
$$P_0(0.9^4) = 18$$
$$P_0 = \frac{18}{0.9^4} = 27.435$$

We see that $a = 0.9$ and $P_0 = 27.435$.

(b) The initial quantity is 27.435 and the quantity is decaying at a rate of 10% per unit time.

27. From the statements given, we have the equations: $P_0a^5 = 320$ and $P_0a^3 = 500$.

(a) We divide the two equations to solve for a:

$$\begin{aligned}\frac{P_0a^5}{P_0a^3} &= \frac{320}{500}\\ a^2 &= 0.64\\ a &= 0.64^{1/2} = 0.8\end{aligned}$$

Now that we know the value of a, we can use either of the equations to solve for P_0. Using the first equation, we have:

$$\begin{aligned}P_0a^5 &= 320\\ P_0(0.8^5) &= 320\\ P_0 &= \frac{320}{0.8^5} = 976.563\end{aligned}$$

We see that $a = 0.8$ and $P_0 = 976.563$.

(b) The initial quantity is 976.563 and the quantity is decaying at a rate of 20% per unit time.

28. From the statements given, we have the equations: $P_0a^3 = 1600$ and $P_0a^1 = 1000$.

(a) We divide the two equations to solve for a:

$$\begin{aligned}\frac{P_0a^3}{P_0a^1} &= \frac{1600}{1000}\\ a^2 &= 1.6\\ a &= 1.6^{1/2} = 1.2649111 \approx 1.265.\end{aligned}$$

Now that we know the value of a, we can use either of the equations to solve for P_0. From the first equation, we have:

$$\begin{aligned}P_0a^3 &= 1600\\ P_0(1.2649111^3) &= 1600\\ P_0 &= \frac{1600}{1.2649111^3} = 790.569.\end{aligned}$$

We see that $a = 1.265$ and $P_0 = 790.569$.

(b) The initial quantity is 790.569 and the quantity is growing at a rate of 26.5% per unit time.

29. If the population increases exponentially, the formula for the population at time t can be written in the form

$$P(t) = P_0e^{kt}$$

where P_0 is the population at time $t = 0$ and k is the continuous growth rate; we measure time in years. If we let 1998 be the initial time $t = 0$, we have $P_0 = 5.937$, so

$$P(t) = 5.937e^{kt}.$$

In 2014, time $t = 16$, so we have $P(16) = 7.238$. Thus we get

$$\begin{aligned}7.238 &= 5.937e^{16k}\\ e^{16k} &= \frac{7.238}{5.937}\\ \ln e^{16k} &= \ln\left(\frac{7.238}{5.937}\right)\\ k &= \frac{\ln(7.238/5.937)}{16} = 0.0124.\end{aligned}$$

Thus, the formula for the population, assuming exponential growth, is

$$P(t) = 5.937e^{0.0124t} \text{ billion.}$$

Using our formula for the year 2015, time $t = 17$, we get

$$P(17) = 5.937e^{0.0124(17)} = 5.937e^{0.2108} = 7.33 \text{ billion.}$$

Thus, we see that we were not too far off the mark when we approximated the population by an exponential function.

30. Use an exponential function of the from $P = P_0 a^t$ for the number of passengers. At $t = 0$, the number of passengers is $P_0 = 190{,}205$, so

$$P = 190{,}205a^t.$$

After $t = 5$ years, the number of passengers is 174,989, so

$$\begin{aligned} 174{,}989 &= 190{,}205a^5 \\ a^5 &= \frac{174{,}989}{190{,}205} = 0.920002 \\ a &= (0.920002)^{1/5} = 0.983. \end{aligned}$$

Then

$$P = 190{,}205(0.932)^t,$$

which means the annual percentage decrease over this period is $1 - 0.983 = 0.017$ or 1.7%.

31. Since we are looking for an annual percent increase, we use an exponential function. If V represents the value of the company, and t represent the number of years since it was started, we have $V = Ca^t$. The initial value of the company was \$4000, so $V = 4000a^t$. To find a, we use the fact that $V = 14{,}000{,}000$ when $t = 40$:

$$\begin{aligned} V &= 4000a^t \\ 14{,}000{,}000 &= 4000a^{40} \\ 3500 &= a^{40} \\ a &= (3500)^{1/40} = 1.226. \end{aligned}$$

The value of the company increased at an average rate of 22.6% per year.

32. Let y represent the number of zebra mussels t years since 2014.

(a) The slope of the line is $(3186 - 2700)/1 = 486$ and the vertical intercept is 2700. We have $y = 2700 + 486t$. The slope of the line, 486 zebra mussels per year, represents the rate at which the zebra mussel population is growing.

(b) When $t = 0$, we have $y = 2700$, so the exponential function is in the form $y = 2700a^t$. When $t = 1$ we have $y = 3186$, and we substitute to get $3186 = 2700a^1$ so $a = 1.18$. The exponential function is $y = 2700(1.18)^t$ and the zebra mussel population is growing at 18% per year.

33. **(a)** The slope of the line is

$$\text{Slope} = \frac{\Delta W}{\Delta t} = \frac{371{,}374 - 236{,}733}{2014 - 2011} = 44{,}880.3.$$

The initial value (when $t = 0$) is 236,733, so the linear formula is

$$W = 236{,}733 + 44{,}880.3t.$$

The rate of growth of wind generating capacity is 44,880.3 megawatts per year.

(b) If the function is exponential, we have $W = W_0 a^t$. The initial value (when $t = 0$) is 236,733, so we have $W = 236{,}733a^t$. We use the fact that $W = 371{,}374$ when $t = 3$ to find a:

$$\begin{aligned} 371{,}374 &= 236{,}733a^3 \\ 1.56875 &= a^3 \\ a &= 1.56875^{1/3} = 1.16194. \end{aligned}$$

The exponential formula is

$$W = 236{,}733(1.16194)^t.$$

The percent rate of growth of wind generating capacity is 16.2% per year. (Alternately, if we used the form $W = W_0 e^{kt}$, the formula would be $W = 236{,}733e^{0.1501t}$, giving a continuous rate of 15.01% per year.)

(c) See Figure 1.51.

(d) The linear function, $236{,}733 + 44{,}880.3t$, gives a predicted generation in 2015, when $t = 4$, of approximately $236{,}733 + 44{,}880.3 \cdot 4 = 416{,}254$ megawatts, 18,690 less than the real power generation of 434,944 megawatts. The exponential model predicts the capacity in 2015 to be $W = 236{,}733(1.16194)^4 \approx 431{,}500$, which is about 3400 megawatts less than the real figure, so the exponential model is better.

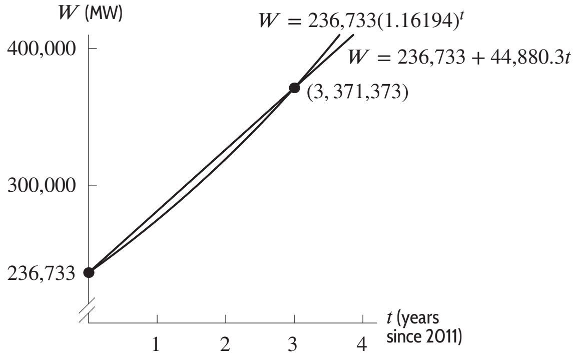


Figure 1.51

34. (a) We see that f cannot be a linear function, since $f(x)$ increases by different amounts $(24 - 16 = 8$ and $36 - 24 = 12)$ as x increases by one. Could f be an exponential function? We look at the ratios of successive $f(x)$ values:

$$\frac{24}{16} = 1.5 \qquad \frac{36}{24} = 1.5 \qquad \frac{54}{36} = 1.5 \qquad \frac{81}{54} = 1.5.$$

Since the ratios are all equal to 1.5, this table of values could correspond to an exponential function with a base of 1.5. Since $f(0) = 16$, a formula for $f(x)$ is

$$f(x) = 16(1.5)^x.$$

Check by substituting $x = 0, 1, 2, 3, 4$ into this formula; you get the values given for $f(x)$.

(b) As x increases by one, $g(x)$ increases by 6 (from 14 to 20), then 4 (from 20 to 24), so g is not linear. We check to see if g could be exponential:

$$\frac{20}{14} = 1.43 \quad \text{and} \quad \frac{24}{20} = 1.2.$$

Since these ratios (1.43 and 1.2) are different, g is not exponential.

(c) For h, notice that as x increases by one, the value of $h(x)$ increases by 1.2 each time. So h could be a linear function with a slope of 1.2. Since $h(0) = 5.3$, a formula for $h(x)$ is

$$h(x) = 5.3 + 1.2x.$$

35. (a) A linear function must change by exactly the same amount whenever x changes by some fixed quantity. While $h(x)$ decreases by 3 whenever x increases by 1, $f(x)$ and $g(x)$ fail this test, since both change by different amounts between $x = -2$ and $x = -1$ and between $x = -1$ and $x = 0$. So the only possible linear function is $h(x)$, so it will be given by a formula of the type: $h(x) = mx + b$. As noted, $m = -3$. Since the y-intercept of h is 31, the formula for $h(x)$ is $h(x) = 31 - 3x$.

(b) An exponential function must grow by exactly the same factor whenever x changes by some fixed quantity. Here, $g(x)$ increases by a factor of 1.5 whenever x increases by 1. Since the y-intercept of $g(x)$ is 36, $g(x)$ has the formula $g(x) = 36(1.5)^x$. The other two functions are not exponential; $h(x)$ is not because it is a linear function, and $f(x)$ is not because it both increases and decreases.

36. (a) In this case we know that

$$f(1) - f(0) = 12.7 - 10.5 = 2.2$$

while

$$f(2) - f(1) = 18.9 - 12.7 = 6.2.$$

Thus, the function described by these data is not a linear one. Next we check if this function is exponential.

$$\frac{f(1)}{f(0)} = \frac{12.7}{10.5} \approx 1.21$$

while

$$\frac{f(2)}{f(1)} = \frac{18.9}{12.7} \approx 1.49$$

thus $f(x)$ is not an exponential function either.

(b) In this case we know that

$$s(0) - s(-1) = 30.12 - 50.2 = -20.08$$

while

$$s(1) - s(0) = 18.072 - 30.12 = -12.048.$$

Thus, the function described by these data is not a linear one. Next we check if this function is exponential.

$$\frac{s(0)}{s(-1)} = \frac{30.12}{50.2} = 0.6,$$
$$\frac{s(1)}{s(0)} = \frac{18.072}{30.12} = 0.6,$$

and

$$\frac{s(2)}{s(1)} = \frac{10.8432}{18.072} = 0.6.$$

Thus, $s(t)$ is an exponential function. We know that $s(t)$ will be of the form

$$s(t) = P_0 a^t$$

where P_0 is the initial value and $a = 0.6$ is the base. We know that

$$P_0 = s(0) = 30.12.$$

Thus,

$$s(t) = 30.12a^t.$$

Since $a = 0.6$, we have

$$s(t) = 30.12(0.6)^t.$$

(c) In this case we know that

$$\frac{g(2) - g(0)}{2 - 0} = \frac{24 - 27}{2} = \frac{-3}{2} = -1.5,$$
$$\frac{g(4) - g(2)}{4 - 2} = \frac{21 - 24}{2} = \frac{-3}{2} = -1.5,$$

and

$$\frac{g(6) - g(4)}{6 - 4} = \frac{18 - 21}{2} = \frac{-3}{2} = -1.5.$$

Thus, $g(u)$ is a linear function. We know that

$$g(u) = m \cdot u + b$$

where m is the slope and b is the vertical intercept, or the value of the function at zero. So

$$b = g(0) = 27$$

and from the above calculations we know that

$$m = -1.5.$$

Thus,

$$g(u) = -1.5u + 27.$$

37. Assuming that, on average, the minimum wage has increased exponentially from its value of \$0.25 in 1938, we have

$$m = 0.25b^t,$$

where m is the minimum wage t years after 1938.

Since $t = 2004 - 1938 = 66$ in 2004, we have

$$5.15 = 0.25b^{66}.$$

Solving for b gives

$$b^{66} = \frac{5.15}{0.25}$$
$$b = \left(\frac{5.15}{0.25}\right)^{1/66} = 1.0469.$$

Thus, on average, the minimum wage has grown at 4.69% per year— higher than the inflation rate.

38. **(a)** Using $Q = Q_0(1-r)^t$ for loss, we have

$$Q = 10{,}000(1-0.1)^{10} = 10{,}000(0.9)^{10} = 3486.78.$$

The investment was worth $3486.78 after 10 years.

(b) Measuring time from the moment at which the stock begins to gain value and letting $Q_0 = 3486.78$, the value after t years is

$$Q = 3486.78(1+0.1)^t = 3486.78(1.1)^t.$$

We can estimate the value of t when $Q = 10{,}000$ by tracing along a graph of Q, giving $t \approx 11$. It will take about 11 years to get the investment back to $10,000.

39. **(a)** We know $w = 1248$ when $t = 1$ and $w = 25{,}827$ when $t = 24$. One algorithm for exponential regression gives

$$w = 1093.965(1.1408)^t.$$

(b) The annual growth rate is $0.1408 = 14.08\%$.

(c) The model predicts that in 2000 the number of cases would be

$$w = 1093.965(1.1408)^{20} \approx 15{,}247.$$

If the number of cases had doubled between 2000 and 2004, there would have been $15{,}247 \cdot 2 = 30{,}494$ cases in 2004. However, there were 25,827 cases in 2004. Thus the model does not confirm this report.

The reason is that the number of cases has been increasing at an average annual rate of 14.08% between 1980 and 2004, but at a higher rate between 2000 and 2004.

40. Direct calculation reveals that each 1000 foot increase in altitude results in a longer takeoff roll by a factor of about 1.096. Since the value of d when $h = 0$ (sea level) is $d = 670$, we are led to the formula

$$d = 670(1.096)^{h/1000},$$

where d is the takeoff roll, in feet, and h is the airport's elevation, in feet.

Alternatively, we can write

$$d = d_0 a^h,$$

where d_0 is the sea level value of d, $d_0 = 670$. In addition, when $h = 1000$, $d = 734$, so

$$734 = 670a^{1000}.$$

Solving for a gives

$$a = \left(\frac{734}{670}\right)^{1/1000} = 1.00009124,$$

so

$$d = 670(1.00009124)^h.$$

41. **(a)** Since the annual growth factor from 2009 to 2010 was $1 + (-0.193) = 0.807$ and $322(0.807) = 259.854$, the US consumed about 260 million gallons of biodiesel in 2010. Since the annual growth factor from 2010 to 2011 was $1 + 2.408 = 3.408$ and $260(3.408) = 886.08$, the US consumed approximately 886 million gallons of biodiesel in 2011.

(b) Completing the table of annual consumption of biodiesel and plotting the data gives Figure 1.52.

Year	2009	2010	2011	2012	2013	2014
Consumption of biodiesel (mn gal)	322	260	886	895	1404	1419

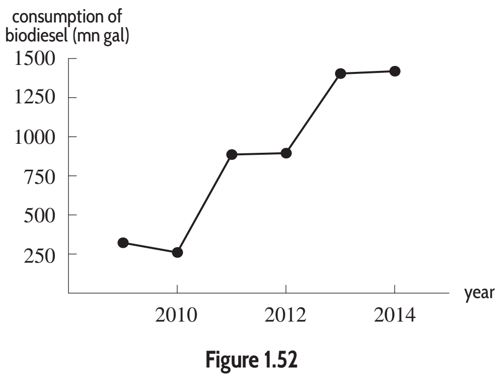


Figure 1.52

42. **(a)** False, because the annual percent growth is not constant over this interval.
(b) The US consumption of biodiesel more than tripled in 2011, since the annual percent growth in 2011 was over 200%.

43. **(a)** Since the annual growth factor from 2009 to 2010 was $1 - 0.049 = 0.951$ and $2.65(1 - 0.049) = 2.520$, the US consumed approximately 2.5 quadrillion BTUs of hydroelectric power in 2010. Since the annual growth factor from 2010 to 2011 was $1 + 0.224 = 1.224$ and $2.520(1 + 0.224) = 3.084$, the US consumed about 3.1 quadrillion BTUs of hydroelectric power in 2011.
(b) Completing the table of annual consumption of hydroelectric power and plotting the data gives Figure 1.53.

Year	2009	2010	2011	2012	2013	2014
Consumption of hydro. power (quadrillion BTU)	2.650	2.520	3.084	2.606	2.528	2.442

(c) The largest decrease in the US consumption of hydroelectric power occurred in 2011. In this year, the US consumption of hydroelectric power dropped by about 478 trillion BTUs to 2.606 quadrillion BTUs, down from 3.084 quadrillion BTUs in 2011.

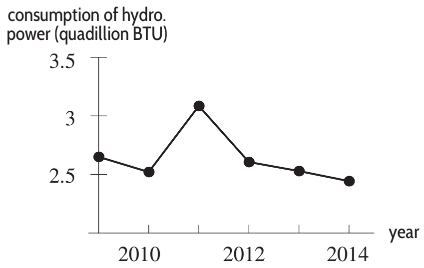


Figure 1.53

44. **(a)** From the figure we can read-off the approximate percent growth for each year over the previous year:

Year	2009	2010	2011	2012	2013	2014
% growth over previous yr	32	28	26	15	20	8

Since the annual growth factor from 2009 to 2010 was $1 + 0.28 = 1.28$ and $721(1 + 0.28) = 922.88$, the US consumed approximately 923 trillion BTUs of wind power energy in 2010. Since the annual growth factor from 2010 to 2011 was $1 + 0.26 = 1.26$ and $922.88(1 + 0.26) = 1162.829$, the US consumed about 1163 trillion BTUs of wind power energy in 2010.
(b) Completing the table of annual consumption of wind power and plotting the data gives Figure 1.54.

Year	2009	2010	2011	2012	2013	2014
Consumption of wind power (trillion BTU)	721	923	1163	1337	1605	1733

(c) The largest increase in the US consumption of wind power energy occurred in 2013. In this year the US consumption of wind power energy rose by about 268 trillion BTUs to 1605 trillion BTUs, up from 1337 trillion BTUs in 2012.

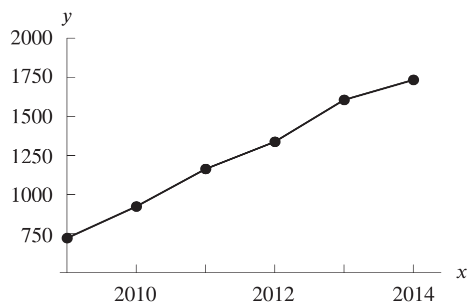


Figure 1.54

45. **(a)** The US consumption of wind power energy increased by at least 25% in 2009, 2010 and in 2011, relative to the previous year. Consumption did not decrease during the time period shown because all the annual percent growth values are positive, indicating a steady increase in the US consumption of wind power energy between 2009 and 2014.

(b) Yes. From 2008 to 2009 consumption increased by about 32%, which means $x(1+0.32)$ units of wind power energy were consumed in 2009 if x had been consumed in 2008. Similarly, $(x(1+0.32))(1+0.28)$ units of wind power energy were consumed in 2010 if x had been consumed in 2008 (because consumption increased by about 28% from 2009 to 2010). Repeating this process,

$$(x(1+0.32))(1+0.28)(1+0.26)$$

units of wind power energy were consumed in 2011 if x had been consumed in 2008 (because consumption increased by about 26% from 2010 to 2011). Since

$$(x(1+0.32))(1+0.28)(1+0.26) = x(2.129) = x(1+1.129),$$

the percent growth in wind power consumption was about 112.9%, or over 100%, in 2011 relative to consumption in 2008. Note that adding these three percents would be 86%, however there is a multiplier effect which makes the change well over 100%.

Solutions for Section 1.6

1. Taking natural logs of both sides we get

$$\ln 10 = \ln(2^t).$$

This gives

$$t \ln 2 = \ln 10$$

or in other words

$$t = \frac{\ln 10}{\ln 2} \approx 3.3219$$

2. Taking natural logs of both sides we get

$$\ln(5^t) = \ln 7.$$

This gives

$$t \ln 5 = \ln 7$$

or in other words

$$t = \frac{\ln 7}{\ln 5} \approx 1.209.$$

3. Taking natural logs of both sides we get

$$\ln 2 = \ln(1.02^t).$$

This gives

$$t \ln 1.02 = \ln 2$$

or in other words

$$t = \frac{\ln 2}{\ln 1.02} \approx 35.003.$$

4. Taking natural logs of both sides we get

$$\ln 130 = \ln(10^t).$$

This gives

$$t \ln 10 = \ln 130$$

or in other words

$$t = \frac{\ln 130}{\ln 10} \approx 2.1139.$$

5. Taking natural logs of both sides we get

$$\ln(e^t) = \ln 10$$

which gives

$$t = \ln 10 \approx 2.3026.$$

6. Dividing both sides by 25 we get

$$4 = 1.5^t.$$

Taking natural logs of both side gives

$$\ln(1.5^t) = \ln 4.$$

This gives

$$t \ln 1.5 = \ln 4$$

or in other words

$$t = \frac{\ln 4}{\ln 1.5} \approx 3.419.$$

7. Dividing both sides by 10 we get

$$5 = 3^t.$$

Taking natural logs of both sides gives

$$\ln(3^t) = \ln 5.$$

This gives

$$t \ln 3 = \ln 5$$

or in other words

$$t = \frac{\ln 5}{\ln 3} \approx 1.465.$$

8. Dividing both sides by 2 we get

$$2.5 = e^t.$$

Taking the natural log of both sides gives

$$\ln(e^t) = \ln 2.5.$$

This gives

$$t = \ln 2.5 \approx 0.9163.$$

9. Taking natural logs of both sides we get

$$\ln(e^{3t}) = \ln 100.$$

This gives

$$3t = \ln 100$$

or in other words

$$t = \frac{\ln 100}{3} \approx 1.535.$$

10. Dividing both sides by 6 gives

$$e^{0.5t} = \frac{10}{6} = \frac{5}{3}.$$

Taking natural logs of both sides we get

$$\ln(e^{0.5t}) = \ln\left(\frac{5}{3}\right).$$

This gives

$$0.5t = \ln\left(\frac{5}{3}\right) = \ln 5 - \ln 3$$

or in other words

$$t = 2(\ln 5 - \ln 3) \approx 1.0217.$$

11. We divide both sides by 100 and then take the natural logarithm of both sides and solve for t:

$$\begin{aligned} 40 &= 100e^{-0.03t} \\ 0.4 &= e^{-0.03t} \\ \ln(0.4) &= \ln(e^{-0.03t}) \\ \ln(0.4) &= -0.03t \\ t &= \frac{\ln(0.4)}{-0.03} = 30.54. \end{aligned}$$

12. Taking natural logs of both sides (and assuming $a > 0$, $b > 0$, and $b \neq 1$) gives

$$\ln(b^t) = \ln a.$$

This gives

$$t \ln b = \ln a$$

or in other words

$$t = \frac{\ln a}{\ln b}.$$

13. Dividing both sides by P we get

$$\frac{B}{P} = e^{rt}.$$

Taking the natural log of both sides gives

$$\ln(e^{rt}) = \ln\left(\frac{B}{P}\right).$$

This gives

$$rt = \ln\left(\frac{B}{P}\right) = \ln B - \ln P.$$

Dividing by r gives

$$t = \frac{\ln B - \ln P}{r}.$$

14. Dividing both sides by P we get

$$2 = e^{0.3t}.$$

Taking the natural log of both sides gives

$$\ln(e^{0.3t}) = \ln 2.$$

This gives

$$0.3t = \ln 2$$

or in other words

$$t = \frac{\ln 2}{0.3} \approx 2.3105.$$

15. Taking natural logs of both sides we get

$$\ln(7 \cdot 3^t) = \ln(5 \cdot 2^t)$$

which gives

$$\ln 7 + \ln(3^t) = \ln 5 + \ln(2^t)$$

or in other words

$$\ln 7 - \ln 5 = \ln(2^t) - \ln(3^t).$$

This gives

$$\ln 7 - \ln 5 = t \ln 2 - t \ln 3 = t(\ln 2 - \ln 3).$$

Thus, we get

$$t = \frac{\ln 7 - \ln 5}{\ln 2 - \ln 3} \approx -0.8298.$$

16. Taking natural logs of both sides we get

$$\ln(5e^{3t}) = \ln(8e^{2t}).$$

This gives

$$\ln 5 + \ln(e^{3t}) = \ln 8 + \ln(e^{2t})$$

or in other words

$$\ln(e^{3t}) - \ln(e^{2t}) = \ln 8 - \ln 5.$$

This gives

$$3t - 2t = \ln 8 - \ln 5$$

or in other words

$$t = \ln 8 - \ln 5 \approx 0.47.$$

17. Taking natural logs of both sides, we get

$$\ln(Ae^{2t}) = \ln(Be^{t})$$

so

$$\ln A + \ln(e^{2t}) = \ln B + \ln(e^{t})$$

or in other words

$$\ln A + 2t \ln e = \ln B + t \ln e.$$

Since $\ln e = 1$, this gives

$$\ln A + 2t = \ln B + t.$$

Thus, combining ts on the left side of the equation, we get

$$t = \ln B - \ln A = \ln(B/A).$$

18. We cannot take the log of 0. Moving the 5 to the other side of the equation, we have

$$2e^{t} = 5.$$

Then we take natural logs of both sides, giving

$$\ln(2e^{t}) = \ln 5$$

so

$$\ln 2 + \ln(e^{t}) = \ln 5$$

or in other words

$$\ln 2 + t \ln e = \ln 5.$$

Since $\ln e = 1$, this gives

$$\ln 2 + t = \ln 5.$$

Thus, we get

$$t = \ln 5 - \ln 2 = \ln(5/2) \approx 0.9163.$$

19. We cannot take the log of 0. Moving the $3e^{t}$ to the other side of the equation, we have

$$3e^{t} = 7.$$

Then we take natural logs of both sides, giving

$$\ln(3e^{t}) = \ln 7$$

so

$$\ln 3 + \ln(e^{t}) = \ln 7$$

or in other words

$$\ln 3 + t \ln e = \ln 7.$$

Since $\ln e = 1$, this gives

$$\ln 3 + t = \ln 7.$$

Thus, we get

$$t = \ln 7 - \ln 3 = \ln(7/3) \approx 0.8473.$$

20. Since we cannot take the log of 0, we move Qe^{-t} to the other side of the equation

$$Pe^{4t} = Qe^{-t}.$$

Then, taking natural logs of both sides, we get

$$\ln(Pe^{4t}) = \ln(Qe^{-t})$$

so

$$\ln P + \ln(e^{4t}) = \ln Q + \ln(e^{-t})$$

or in other words

$$\ln P + 4t \ln e = \ln Q - t \ln e.$$

Since $\ln e = 1$, this gives

$$\ln P + 4t = \ln Q - t.$$

Thus, combining ts on the left side of the equation, we get

$$\begin{aligned} 5t &= \ln Q - \ln P \\ t &= \frac{1}{5}(\ln Q - \ln P) = \frac{\ln(P/Q)}{5}. \end{aligned}$$

21. Initial quantity = 5; growth rate = 0.07 = 7%.

22. Initial quantity = 7.7; growth rate = −0.08 = −8% (decay).

23. Initial quantity = 15; growth rate = −0.06 = −6% (continuous decay).

24. Initial quantity = 3.2; growth rate = 0.03 = 3% (continuous).

25. Since $e^{0.25t} = \left(e^{0.25}\right)^t \approx (1.2840)^t$, we have $P = 15(1.2840)^t$. This is exponential growth since 0.25 is positive. We can also see that this is growth because $1.2840 > 1$.

26. Since $e^{-0.5t} = (e^{-0.5})^t \approx (0.6065)^t$, we have $P = 2(0.6065)^t$. This is exponential decay since −0.5 is negative. We can also see that this is decay because $0.6065 < 1$.

27. $P = P_0(e^{0.2})^t = P_0(1.2214)^t$. Exponential growth because $0.2 > 0$ or $1.2214 > 1$.

28. $P = 7(e^{-\pi})^t = 7(0.0432)^t$. Exponential decay because $-\pi < 0$ or $0.0432 < 1$.

29. Since we want $(1.5)^t = e^{kt} = (e^k)^t$, so $1.5 = e^k$, and $k = \ln 1.5 = 0.4055$. Thus, $P = 15e^{0.4055t}$. Since 0.4055 is positive, this is exponential growth.

30. We want $1.7^t = e^{kt}$ so $1.7 = e^k$ and $k = \ln 1.7 = 0.5306$. Thus $P = 10e^{0.5306t}$.

31. We want $0.9^t = e^{kt}$ so $0.9 = e^k$ and $k = \ln 0.9 = -0.1054$. Thus $P = 174e^{-0.1054t}$.

32. Since we want $(0.55)^t = e^{kt} = (e^k)^t$, so $0.55 = e^k$, and $k = \ln 0.55 = -0.5978$. Thus $P = 4e^{-0.5978t}$. Since −0.5978 is negative, this represents exponential decay.

33. We use the information to create two equations: $P_0e^{3k} = 140$ and $P_0e^{1k} = 100$.

(a) We divide the two equations to solve for k:

$$\begin{aligned} \frac{P_0e^{3k}}{P_0e^k} &= \frac{140}{100} \\ e^{2k} &= 1.4 \\ 2k &= \ln 1.4 \\ k &= \frac{\ln 1.4}{2} = 0.168 \end{aligned}$$

Now that we know the value of k, we can use either of the equations to solve for P_0. Using the first equation, we have:

$$\begin{aligned} P_0e^{3k} &= 140 \\ P_0e^{3(0.168)} &= 140 \\ P_0 &= \frac{140}{e^{0.504}} = 84.575 \end{aligned}$$

We see that $a = 0.168$ and $P_0 = 84.575$.

(b) The initial quantity is 84.575 and the quantity is growing at a continuous rate of 16.8% per unit time.

34. We use the information to create two equations: $P_0e^{4k} = 40$ and $P_0e^{3k} = 50$.

(a) We divide the two equations to solve for k:

$$\begin{aligned}\frac{P_0e^{4k}}{P_0e^{3k}} &= \frac{40}{50}\\ e^k &= 0.4\\ k &= \ln 0.4 = -0.916\end{aligned}$$

Now that we know the value of k, we can use either of the equations to solve for P_0. Using the first equation, we have:

$$\begin{aligned}P_0e^{4k} &= 40\\ P_0e^{4(-0.916)} &= 40\\ P_0 &= \frac{40}{e^{-3.664}} = 1560.684\end{aligned}$$

We see that $k = -0.916$ and $P_0 = 1560.684$.

(b) The initial quantity is 1560.684 and the quantity is decaying at a continuous rate of 91.6% per unit time.

35. **(a)** The continuous percent growth rate is 6%.

(b) We want $P_0a^t = 100e^{0.06t}$, so we have $P_0 = 100$ and $a = e^{0.06} = 1.0618$. The corresponding function is

$$P = 100(1.0618)^t.$$

The annual percent growth rate is 6.18%. An annual rate of 6.18% is equivalent to a continuous rate of 6%.

36. **(a)** Since $0.88 = 1 - 0.12$, the annual percent decay rate is 12%.

(b) We want $P_0e^{kt} = 25(0.88)^t$, so we have $P_0 = 25$ and $e^k = 0.88$. Since $e^k = 0.88$, we have $k = \ln(0.88) = -0.128$. The corresponding function is

$$P = 25e^{-0.128t}.$$

The continuous percent decay rate is 12.8%. A continuous rate of 12.8% is equivalent to an annual rate of 12%.

37. We find a with $a^t = e^{0.08t}$. Thus, $a = e^{0.08} = 1.0833$. The corresponding annual percent growth rate is 8.33%.

38. We find k with $e^{kt} = (1.10)^t$. We have $e^k = 1.10$ so $k = \ln(1.10) = 0.0953$. The corresponding continuous percent growth rate is 9.53%.

39. **(a)** Town D is growing fastest, since the rate of growth $k = 0.12$ is the largest.

(b) Town C is largest now, since $P_0 = 1200$ is the largest.

(c) Town B is decreasing in size, since k is negative ($k = -0.02$).

40. If production continues to decay exponentially at a continuous rate of 9.1% per year, the production $f(t)$ at time t years after 2008 is

$$f(t) = 84e^{-0.091t}.$$

In 2025, production is predicted to be

$$f(17) = 84e^{-0.091(17)} = 17.882 \text{ million barrels per day.}$$

41. **(a)** (i) $P = 1000(1.05)^t$; (ii) $P = 1000e^{0.05t}$

(b) (i) 1629; (ii) 1649

42. **(a)** $P = 5.97(1.0099)^t$

(b) Since $P = 5.97e^{kt} = 5.97(1.0099)^t$, we have

$$\begin{aligned}e^{kt} &= (1.0099)^t\\ kt &= t\ln(1.0099)\\ k &= 0.00985.\end{aligned}$$

Thus, $P = 5.97e^{0.00985t}$.

(c) The annual growth rate is 0.99%, while the continuous growth rate is 0.985%. Thus the growth rates are not equal, though for small growth rates (such as these), they are close. The annual growth rate is larger.

43. We find k with $e^{kt} = (1.029)^t$. We have $e^k = 1.029$ so $k = \ln(1.029) = 0.0286$. An equivalent formula for gross world product as a function of time is

$$W = 74.31e^{0.0286t}.$$

Notice that the annual growth rate of 2.9% is equivalent to a continuous growth rate of 2.86%.

44. We find k with $e^{kt} = (1.0109)^t$. We have $e^k = 1.0109$ so $k = \ln(1.0109) = 0.01084$. An equivalent formula for the population of the world as a function of time is

$$P = 7.32e^{0.01084t}.$$

Notice that the annual growth rate of 1.09% is equivalent to a continuous growth rate of 1.084%.

45. (a) Substituting $t = 0.5$ in $P(t)$ gives

$$\begin{aligned} P(0.5) &= 1000e^{-0.5(0.5)} \\ &= 1000e^{-0.25} \\ &\approx 1000(0.7788) \\ &= 778.8 \approx 779 \text{ trout.} \end{aligned}$$

Substituting $t = 1$ in $P(t)$ gives

$$\begin{aligned} P(1) &= 1000e^{-0.5} \\ &\approx 1000(0.6065) \\ &= 606.5 \approx 607 \text{ trout.} \end{aligned}$$

(b) Substituting $t = 3$ in $P(t)$ gives

$$\begin{aligned} P(3) &= 1000e^{-0.5(3)} \\ &= 1000e^{-1.5} \\ &\approx 1000(0.2231) \\ &= 223.1 \approx 223 \text{ trout.} \end{aligned}$$

This tells us that after 3 years the fish population has gone down to about 22% of the initial population.

(c) We are asked to find the value of t such that $P(t) = 100$. That is, we are asked to find the value of t such that

$$100 = 1000e^{-0.5t}.$$

Solving this gives

$$\begin{aligned} 100 &= 1000e^{-0.5t} \\ 0.1 &= e^{-0.5t} \\ \ln 0.1 &= \ln e^{-0.5t} = -0.5t \\ t &= \frac{\ln 0.1}{-0.5} \\ &\approx 4.6. \end{aligned}$$

Thus, after roughly 4.6 years, the trout population will have decreased to 100.

(d) A graph of the population is given in Figure 1.55.

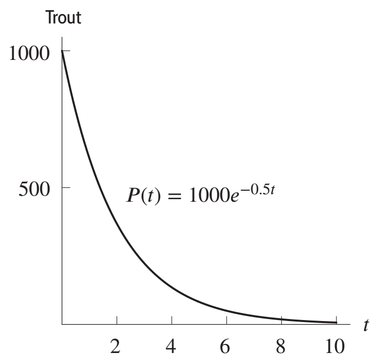


Figure 1.55

Looking at the graph, we see that as time goes on there are fewer and fewer trout in the pond. A possible cause for this may be the presence of predators, such as fishermen.The rate at which the fish die decreases due to the fact that there are fewer fish in the pond, which means that there are fewer fish that have to try to survive. Notice that the survival chances of any given fish remains the same, it is just the overall mortality that is decreasing.

46. **(a)** Since the percent rate of growth is constant and given as a continuous rate, we use the exponential function $S = 6.1e^{0.07t}$.

(b) In 2015, we have $t = 4$ and $S = 6.1e^{0.07\cdot 4} = 8.07$ billion dollars.

(c) Using the graph in Figure 1.56, we see that $S = 10$ at approximately $t = 7$. Solving $6.1e^{0.07t} = 10$, we have

$$\begin{aligned} e^{0.07t} &= \frac{10}{6.1} \\ 0.07t &= \ln\left(\frac{10}{6.1}\right) \\ t &= \frac{\ln(10/6.1)}{0.07} = 7.061 \text{ years.} \end{aligned}$$

In other words, assuming this growth rate continues, annual net sales are predicted to pass 10 billion dollars early in 2018.

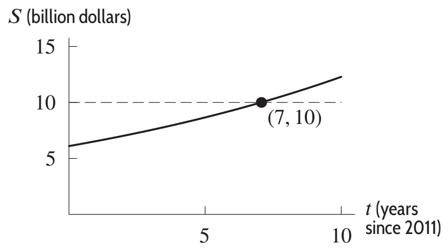


Figure 1.56

47. **(a)** The current revenue is $R(0) = 5e^{-0} = 5$ million dollars. In two years time the revenue will be $R(2) = 5e^{-0.15(2)} = 3.704$ million dollars.

(b) To find the time t at which the revenue has fallen to \$2.7 million we solve the equation $5e^{-0.15t} = 2.7$. Divide both sides by 5 giving

$$e^{-0.15t} = 0.54.$$

Now take natural logs of both sides:

$$-0.15t = -0.6162,$$

so $t = 4.108$. The revenue will have fallen to \$2.7 million early in the fifth year.

48. **(a)** We use the formula $P = P_0e^{kt}$, so we have

$$P = 50{,}000e^{0.045t},$$

where t is the number of years since 2014. See Figure 1.57.

(b) The year 2020 is when $t = 6$, so we have

$$P = 50{,}000e^{0.045(6)} = 65{,}498.2.$$

We can predict that the population will be about 65,500 in the year 2020.

(c) We see in Figure 1.57 that $P = 100{,}000$ approximately when $t = 15$, so the doubling time is about 15 years.

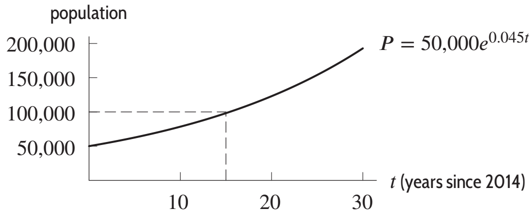


Figure 1.57: The doubling time is about 15 years

To find the doubling time more precisely, we use logarithms. Solving for the value of t for which $P = 100{,}000$:

$$\begin{aligned} 100{,}000 &= 50{,}000e^{0.045t} \\ 2 &= e^{0.045t} \\ \ln 2 &= 0.045t \\ t &= \frac{\ln 2}{0.045} = 15.403. \end{aligned}$$

The doubling time is 15.403 years.

49. **(a)** Since the percent increase in deaths during a year is constant for constant increase in pollution, the number of deaths per year is an exponential function of the quantity of pollution. If Q_0 is the number of deaths per year without pollution, then the number of deaths per year, Q, when the quantity of pollution is x micrograms per cubic meter of air is

$$Q = Q_0(1.0033)^x.$$

(b) We want to find the value of x making $Q = 2Q_0$, that is,

$$Q_0(1.0033)^x = 2Q_0.$$

Dividing by Q_0 and then taking natural logs yields

$$\ln\left((1.0033)^x\right) = x \ln 1.0033 = \ln 2,$$

so

$$x = \frac{\ln 2}{\ln 1.0033} = 210.391.$$

When there are 210.391 micrograms of pollutants per cubic meter of air, respiratory deaths per year are double what they would be in the absence of air pollution.

50. If C_0 is the concentration of NO_2 on the road, then the concentration x meters from the road is

$$C = C_0 e^{-0.0254x}.$$

We want to find the value of x making $C = C_0/2$, that is,

$$C_0 e^{-0.0254x} = \frac{C_0}{2}.$$

Dividing by C_0 and then taking natural logs yields

$$\ln\left(e^{-0.254x}\right) = -0.0254x = \ln\left(\frac{1}{2}\right) = -0.6931,$$

so

$$x = 27 \text{ meters}.$$

At 27 meters from the road the concentration of NO_2 in the air is half the concentration on the road.

51. **(a)** $B(t) = B_0 e^{0.067t}$
(b) $P(t) = P_0 e^{0.033t}$
(c) If the initial price is \$50, then

$$\begin{aligned} B(t) &= 50e^{0.067t} \\ P(t) &= 50e^{0.033t}. \end{aligned}$$

We want the value of t such that

$$\begin{aligned} B(t) &= 2P(t) \\ 50e^{0.067t} &= 2 \cdot 50e^{0.033t} \\ \frac{e^{0.067t}}{e^{0.033t}} &= e^{0.034t} = 2 \\ t &= \frac{\ln 2}{0.034} = 20.387 \text{ years}. \end{aligned}$$

Thus, when $t = 20.387$ the price of the textbook was predicted to be double what it would have been had the price risen by inflation only. This occurred in the year 2000.

52. The population of China, C, in billions, is given by

$$C = 1.355(1.0044)^t$$

where t is time measured from 2014, and the population of India, I, in billions, is given by

$$I = 1.255(1.0125)^t.$$

The two populations will be equal when $C = I$, thus, we must solve the equation:

$$1.355(1.004)^t = 1.255(1.0125)^t$$

for t, which leads to

$$\frac{1.355}{1.255} = \frac{(1.0125)^t}{(1.0044)^t} = \left(\frac{1.0125}{1.0044}\right)^t.$$

Taking logs on both sides, we get

$$t \ln \frac{1.0125}{1.0044} = \ln \frac{1.355}{1.255},$$

so

$$t = \frac{\ln(1.355/1.255)}{\ln(1.0125/1.0044)} = 9.54 \text{ years.}$$

This model predicts the population of India will exceed that of China in 2023.

53. If t is time in decades, then the number of vehicles, V, in millions, is given by

$$V = 246(1.155)^t.$$

For time t in decades, the number of people, P, in millions, is given by

$$P = 308.7(1.097)^t.$$

There is an average of one vehicle per person when $\frac{V}{P} = 1$, or $V = P$. Thus, we solve for t in the equation:

$$246(1.155)^t = 308.7(1.097)^t,$$

which leads to

$$\left(\frac{1.155}{1.097}\right)^t = \frac{(1.155)^t}{(1.097)^t} = \frac{308.7}{246}$$

Taking logs on both sides, we get

$$t \ln \frac{1.155}{1.097} = \ln \frac{308.7}{246},$$

so

$$t = \frac{\ln(308.7/246)}{\ln(1.155/1.097)} = 4.41 \text{ decades.}$$

This model predicts one vehicle per person in 2054

Solutions for Section 1.7

1. At the doubling time, $t = 10$, we have $p = 2p_0$. Thus

$$\begin{aligned} p_0 e^{10k} &= 2p_0 \\ e^{10k} &= 2 \\ 10k &= \ln 2 \\ k &= \frac{1}{10}\ln 2 = 0.0693. \end{aligned}$$

The function $p = p_0 e^{0.0693t}$ has doubling time equal to 10.

2. At the doubling time, $t = 0.4$, the we have $p = 2p_0$. Thus

$$\begin{aligned} p_0 e^{0.4k} &= 2p_0 \\ e^{0.4k} &= 2 \\ 0.4k &= \ln 2 \\ k &= \frac{1}{0.4}\ln 2 = 1.733. \end{aligned}$$

The function $p = p_0 e^{1.733t}$ has doubling time equal to 0.4.

3. In both cases the initial deposit was \$20. Compounding continuously earns more interest than compounding annually at the same interest rate. Therefore, curve A corresponds to the account which compounds interest continuously and curve B corresponds to the account which compounds interest annually. We know that this is the case because curve A is higher than curve B over the interval, implying that bank account A is growing faster, and thus is earning more money over the same time period.

4. Since $y(0) = Ce^0 = C$ we have that $C = 2$. Similarly, substituting $x = 1$ gives $y(1) = 2e^{\alpha}$ so

$$2e^{\alpha} = 1.$$

Rearranging gives $e^{\alpha} = 1/2$. Taking logarithms we get $\alpha = \ln(1/2) = -\ln 2 = -0.693$. Finally,

$$y(2) = 2e^{2(-\ln 2)} = 2e^{-2\ln 2} = \frac{1}{2}.$$

5. **(a)** If the interest is added only once a year (i.e. annually), then at the end of a year we have $1.015x$ where x is the amount we had at the beginning of the year. After two years, we'll have $1.015(1.015x)$ and after eight years, we'll have $(1.015)^8 x$. Since we started with \$1000, after eight years we'll have $(1.015)^8(1000) \approx \$1126.49$.

(b) If an initial deposit of $\$P_0$ is compounded continuously at interest rate r then the account will have $P = P_0 e^{rt}$ after t years. In this case, $P = P_0 e^{rt} = 1000e^{(0.015)(8)} \approx \$1127.50$.

6. We use the equation $B = Pe^{rt}$, where P is the initial principal, t is the time in years since the deposit is made to the account, r is the annual interest rate and B is the balance. After $t = 5$ years, we will have

$$B = (10{,}000)e^{(0.08)5} = (10{,}000)e^{0.4} \approx \$14{,}918.25.$$

7. We use the equation $B = Pe^{rt}$. We want to have a balance of $B = \$20{,}000$ in $t = 6$ years, with an annual interest rate of 10%.

$$\begin{aligned} 20{,}000 &= Pe^{(0.1)6} \\ P &= \frac{20{,}000}{e^{0.6}} \\ &\approx \$10{,}976.23. \end{aligned}$$

8. We know that the formula for the balance in an account after t years is

$$P(t) = P_0 e^{rt}$$

where P_0 is the initial deposit and r is the nominal rate. In our case the initial deposit is \$15,000 that is

$$P_0 = 15{,}000$$

and the nominal rate is

$$r = 0.015.$$

Thus we get

$$P(t) = 15{,}000e^{0.015t}.$$

We are asked to find t such that $P(t) = 20{,}000$, that is we are asked to solve

$$\begin{aligned} 20{,}000 &= 15{,}000e^{0.015t} \\ e^{0.015t} &= \frac{20{,}000}{15{,}000} = \frac{4}{3} \\ \ln e^{0.015t} &= \ln(4/3) \\ 0.015t &= \ln 4 - \ln 3 \\ t &= \frac{\ln 4 - \ln 3}{0.015} \approx 19.18. \end{aligned}$$

Thus after roughly 19.18 years there will be \$20,000 in the account.

9. The formula which models compounding interest continuously for t years is P_0e^{rt} where P_0 is the initial deposit and r is the interest rate. So if we want to double our money while getting 1.25% interest, we want to solve the following for t:

$$\begin{aligned} P_0e^{0.0125t} &= 2P_0 \\ e^{0.0125t} &= 2 \\ 0.0125t &= \ln(2) \\ t &= \frac{\ln 2}{0.0125} \approx 55.5 \text{ years} \end{aligned}$$

Alternatively, by the Rule of 70, we have

$$\text{Doubling time} \approx \frac{70}{1.25} = 56 \text{ years.}$$

10. We know that the formula for the total process is

$$P(t) = P_0a^t$$

where P_0 is the initial size of the process and $a = 1 + 0.07 = 1.07$. We are asked to find the doubling time. Thus, we solve

$$\begin{aligned} 2P_0 &= P_0(1.07)^t \\ 1.07^t &= 2 \\ \ln(1.07^t) &= \ln 2 \\ t\ln 1.07 &= \ln 2 \\ t &= \frac{\ln 2}{\ln 1.07} \approx 10.24. \end{aligned}$$

The doubling time is roughly 10.24 years.

Alternatively we could have used the “rule of 70” to estimate that the doubling time is approximately

$$\frac{70}{7} = 10 \text{ years.}$$

11. **(a)** The concentration decreases from 100 to 50 during the time from 1970 to 1975, a period of 5 years.

(b) The concentration decreases from 50 to 25 during the time from 1975 to 1980, a period of 5 years.

(c) The decay is exponential if the half-life of the concentration is the same beginning at any concentration level. We only checked at two levels, 100 and 50, but for those the half-lives were both 5 years. This is some evidence that the decay might be exponential.

Also, if we look at any other 5-year period, we can check that the concentration appears to decreases by half.

(d) Since the initial value of concentration is 100, an exponential model is of the form $C = 100e^{kt}$. We can evaluate k by using the value $C = 25$ at $t = 10$.

$$\begin{aligned} 25 &= 100e^{10k} \\ 0.25 &= e^{10k} \\ \ln(0.25) &= 10k \\ k &= -0.139. \end{aligned}$$

An exponential model is given by

$$C = 100e^{-0.139t}.$$

Alternative solution. Using the half-life $T = 5$ years, we have

$$C = 100 \cdot 2^{-t/T} = 100 \cdot 2^{-t/5}.$$

12. Every 2 hours, the amount of nicotine remaining is reduced by $1/2$. Thus, after 4 hours, the amount is $1/2$ of the amount present after 2 hours.

Table 1.6

t (hours)	0	2	4	6	8	10
Nicotine (mg)	0.4	0.2	0.1	0.05	0.025	0.0125

From the table it appears that it will take just over 6 hours for the amount of nicotine to reduce to 0.04 mg.

13. (a) We have $W = 371e^{0.168t}$.

(b) We substitute $W = 500$ and solve for t:

$$\begin{aligned} 500 &= 371e^{0.168t} \\ \frac{500}{371} &= e^{0.168t} \\ \ln\frac{500}{371} &= 0.168t \\ t &= \frac{\ln\frac{500}{371}}{0.168} = 1.78. \end{aligned}$$

Wind capacity is expected to pass 500 gigawatts in 2016.

14. Let $f(t)$ be oil consumption t years after 2010. Since consumption is exponential, and $f(0) = 8.938$ million barrels per day, we have

$$f(t) = 8.938e^{kt}.$$

In 2013, when $t = 3$, consumption was 10.480, so

$$\begin{aligned} f(3) &= 10.480 \\ 8.938e^{3t} &= 10.480 \\ 3t &= \ln\left(\frac{10.480}{8.938}\right) \\ k &= 0.05305. \end{aligned}$$

Our model predicts Chinese oil consumption in 2025, when $t = 15$, to be

$$f(15) = 8.938e^{0.05305(15)} = 19.808 \text{ million barrels per day.}$$

15. The doubling time for $N(t)$ is the time t such that $N(t) = 2N(0)$. Since $N(t) = N_0e^{t/500}$ and $N(0) = N_0$ we have

$$\begin{aligned} N_0e^{t/500} &= 2N_0 \\ e^{t/500} &= 2 \\ \frac{t}{500} &= \ln 2 \\ t &= 500\ln 2 = 347. \end{aligned}$$

During the 4 years from October 2002 to October 2006, the doubling time for the number of Wikipedia articles was about 347 days, a little less than a year.

16. (a) If the CD pays 9% interest over a 10-year period, then $r = 0.09$ and $t = 10$. We must find the initial amount P_0 if the balance after 10 years is $P = \$12{,}000$. Since the compounding is annual, we use

$$\begin{aligned} P &= P_0(1+r)^t \\ 12{,}000 &= P_0(1.09)^{10}, \end{aligned}$$

and solve for P_0:

$$P_0 = \frac{12{,}000}{(1.09)^{10}} \approx \frac{12{,}000}{2.36736} = 5068.93.$$

The initial deposit must be \$5068.93 if interest is compounded annually.

(b) On the other hand, if the CD interest is compounded continuously, we have

$$\begin{aligned} P &= P_0e^{rt} \\ 12{,}000 &= P_0e^{(0.09)(10)}. \end{aligned}$$

Solving for P_0 gives

$$P_0 = \frac{12{,}000}{e^{(0.09)(10)}} = \frac{12{,}000}{e^{0.9}} \approx \frac{12{,}000}{2.45960} = 4878.84.$$

The initial deposit must be \$4878.84 if the interest is compounded continuously. Notice that to achieve the same result, continuous compounding requires a smaller initial investment than annual compounding. This is to be expected, since continuous compounding yields more money.

17. **(a)** Since sales of electronic devices doubled in 12 years, sales in 1997 were $438/2 = 219$ million devices. Because part (b) asks for the annual percentage growth rate (not a continuous growth rate), we use an exponential function of the form $S(t) = S_0 b^t$, where b is the annual growth factor. We have

$$438 = 219b^{12},$$

so $2 = b^{12}$ and $b = 2^{1/12} = 1.05946$. Thus,

$$S(t) = 219(1.05946)^t.$$

(b) The annual growth rate was 5.946%.

18. **(a)** Let P represent the population of the world, and let t represent the number of years since 2014. Then we have $P = 7.17(1.011)^t$.

(b) According to this formula, the population of the world in the year 2020 (at $t = 6$) will be $P = 7.17(1.011)^6 = 7.66$ billion people.

(c) The graph is shown in Figure 1.58. The population of the world has doubled when $P = 14.34$; we see on the graph that this occurs at approximately $t = 63.4$. Under these assumptions, the doubling time of the world's population is about 63.4 years.

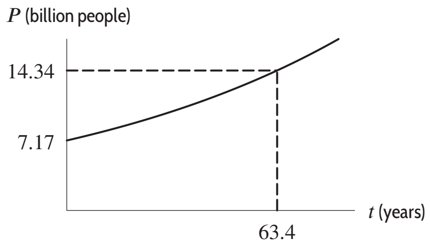


Figure 1.58

19. **(a)** Since the initial amount of caffeine is 100 mg and the exponential decay rate is −0.17, we have $A = 100e^{-0.17t}$.

(b) See Figure 1.59. We estimate the half-life by estimating t when the caffeine is reduced by half (so $A = 50$); this occurs at approximately $t = 4$ hours.

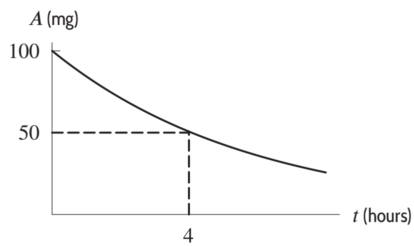


Figure 1.59

(c) We want to find the value of t when $A = 50$:

$$\begin{aligned} 50 &= 100e^{-0.17t} \\ 0.5 &= e^{-0.17t} \\ \ln 0.5 &= -0.17t \\ t &= 4.077. \end{aligned}$$

The half-life of caffeine is about 4.077 hours. This agrees with what we saw in Figure 1.59.

20. **(a)** Since $P(t)$ is an exponential function, it will be of the form $P(t) = P_0 a^t$, where P_0 is the initial population and a is the base. $P_0 = 200$, and a 5% growth rate means $a = 1.05$. Thus, we get

$$P(t) = 200(1.05)^t.$$

(b) The graph is shown in Figure 1.60.

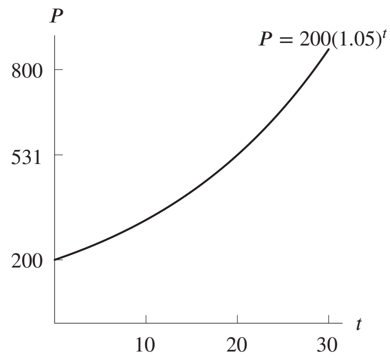


Figure 1.60

(c) Evaluating gives that $P(10) = 200(1.05)^{10} \approx 326$.
(d) From the graph we see that the population is 400 at about $t = 15$, so the doubling time appears to be about 15 years.

21. **(a)** The pressure P at 6194 meters is given in terms of the pressure P_0 at sea level to be

$$\begin{aligned} P &= P_0 e^{-0.00012h} \\ &= P_0 e^{(-0.00012)6194} \\ &= P_0 e^{-0.74328} \\ &\approx 0.4756 P_0 \quad \text{or about 47.6\% of sea level pressure.} \end{aligned}$$

(b) At $h = 12{,}000$ meters, we have

$$\begin{aligned} P &= P_0 e^{-0.00012h} \\ &= P_0 e^{(-0.00012)12{,}000} \\ &= P_0 e^{-1.44} \\ &\approx 0.2369 P_0 \quad \text{or about 23.7\% of sea level pressure.} \end{aligned}$$

22. We use $P = P_0 e^{kt}$. Since the half-life is 3 days, we can find k:

$$\begin{aligned} P &= P_0 e^{kt} \\ 0.50 P_0 &= P_0 e^{k(3)} \\ 0.50 &= e^{3k} \\ \ln(0.50) &= 3k \\ k &= \frac{\ln(0.50)}{3} \approx -0.231. \end{aligned}$$

After one day,

$$P = P_0 e^{-0.231(1)} = P_0(0.79).$$

About 79% of the original dose is in the body one day later. After one week (at $t = 7$), we have

$$P = P_0 e^{-0.231(7)} = P_0(0.20).$$

After one week, 20% of the dose is still in the body.

23. If the rate of increase is $r\%$ then each year the output will increase by $1 + r/100$ so that after five years the level of output is

$$20{,}000\left(1+\frac{r}{100}\right)^5.$$

To achieve a final output of 30,000, the value of r must satisfy the equation

$$20{,}000\left(1+\frac{r}{100}\right)^5 = 30{,}000.$$

Dividing both sides by 20,000 gives

$$\left(1+\frac{r}{100}\right)^5 = 1.5,$$

so $(1 + \frac{r}{100}) = 1.5^{1/5} = 1.0845$. Solving for r gives an annual growth rate of 8.45%.

24. **(a)** Using the formula for exponential decay, $A = A_0e^{-kt}$, with $A_0 = 10.32$. Since the half-life is 12 days, when $t = 12$ we have $A = \frac{10.32}{2} = 5.16$. Substituting this into the formula, we have

$$\begin{aligned} 5.16 &= 10.32e^{-12k} \\ 0.5 &= e^{-12k}, \quad \text{and, taking ln of both sides} \\ \ln 0.5 &= -12k \\ k &= -\frac{\ln(0.5)}{12} \approx 0.057762. \end{aligned}$$

The full equation is

$$A = 10.32e^{-0.057762t}.$$

(b) We want to solve for t when $A = 1$. Substituting into the equation from part (a) yields

$$\begin{aligned} 1 &= 10.32e^{-0.057762t} \\ \frac{1}{10.32} \approx 0.096899 &= e^{-0.057762t} \\ \ln 0.096899 &= -0.057762t \\ t &\approx \frac{-2.33408}{-0.057762} = 40.41 \text{ days.} \end{aligned}$$

25. **(a)** Since $P(t)$ is an exponential function, it will be of the form $P(t) = P_0a^t$. We have $P_0 = 1$, since 100% is present at time $t = 0$, and $a = 0.976$, because each year 97.6% of the contaminant remains. Thus,

$$P(t) = (0.976)^t.$$

(b) The graph is shown in Figure 1.61.

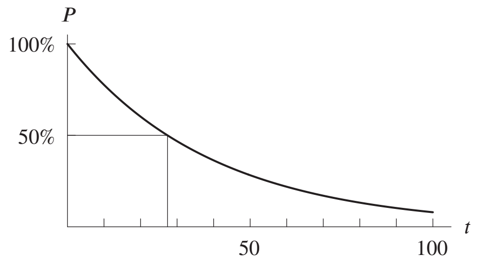


Figure 1.61

(c) The half-life is about 29 years, since $(0.976)^{29} \approx 0.5$.
(d) At time $t = 100$ there is about 9% remaining, since $(0.976)^{100} \approx 0.09$.

26. **(a)** We use an exponential function in the form $H = H_0 e^{kt}$. Since t is in years since 1985, the initial value of H is 2.5 and we have $H = 2.5e^{kt}$. We use the fact that $H = 36.7$ when $t = 30$ to find k:

$$\begin{aligned} H &= 2.5e^{kt} \\ 36.7 &= 2.5e^{k\cdot 30} \\ 14.68 &= e^{30k} \\ \ln(14.68) &= 30k \\ k &= \frac{\ln(14.68)}{30} = 0.0895. \end{aligned}$$

The formula is $H = 2.5e^{0.0895t}$. See Figure 1.62.

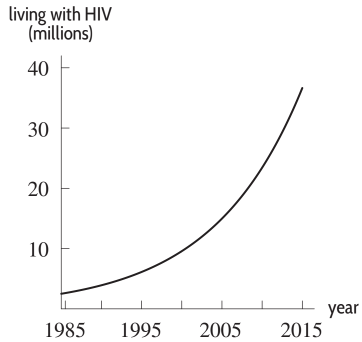


Figure 1.62: HIV infections worldwide

(b) The annual continuous percent change is an increase of approximately 8.95% per year.

27. **(a)** At $t = 0$, the population is 1000. The population doubles (reaches 2000) at about $t = 4$, so the population doubled in about 4 years.

(b) At $t = 3$, the population is about 1700. The population reaches 3400 at about $t = 7$. The population doubled in about 4 years.

(c) No matter when you start, the population doubles in 4 years.

28. We have $P_0 = 500$, so $P = 500e^{kt}$. We can use the fact that $P = 1{,}500$ when $t = 2$ to find k:

$$\begin{aligned} 1{,}500 &= 500e^{kt} \\ 3 &= e^{2k} \\ \ln 3 &= 2k \\ k &= \frac{\ln 3}{2} \approx 0.5493. \end{aligned}$$

The size of the population is given by

$$P = 500e^{0.5493t}.$$

At $t = 5$, we have

$$P = 500e^{0.5493(5)} \approx 7794.$$

29. We know that a function modeling the quantity of substance at time t will be an exponential decay function satisfying the fact that at $t = 10$ hours, 4% of the substance has decayed, or in other words 96% of the substance is left. Thus, a formula for the quantity of substance left at time t is

$$Q(t) = Q_0 e^{kt},$$

where Q_0 is the initial quantity. We solve for k:

$$\begin{aligned} 0.96Q_0 &= Q_0 e^{k\cdot 10} \\ 0.96 &= e^{k\cdot 10} \\ \ln 0.96 &= 10k \\ k &= \frac{\ln 0.96}{10} \approx -0.004. \end{aligned}$$

We are asked to find the time t at which the quantity of material will be half of the initial quantity. Thus, we are asked to solve

$$\begin{aligned} 0.5Q_0 &= Q_0 e^{-0.004t} \\ 0.5 &= e^{-0.004t} \\ \ln 0.5 &= -0.004t \\ t &= \frac{\ln 0.5}{-0.004} \approx 173. \end{aligned}$$

Thus, the half life of the material is roughly 173 hours.

30. We use $Q = Q_0 e^{kt}$ and let $t = 0$ represent 8 am. Then $Q_0 = 100$, so

$$Q = 100e^{kt}.$$

For the husband, the half-life is four hours. We solve for k:

$$\begin{aligned} 50 &= 100e^{k \cdot 4} \\ 0.5 &= e^{4k} \\ \ln 0.5 &= 4k \\ k &= \frac{\ln 0.5}{4} \approx -0.173. \end{aligned}$$

We have $Q = 100e^{-0.173t}$. At 10 pm, we have $t = 14$ so the amount of caffeine for the husband is

$$Q = 100e^{-0.173(14)} = 8.87 \text{ mg} \approx 9 \text{ mg}.$$

For the pregnant woman, the half-life is 10. Solving for k as above, we obtain

$$k = \frac{\ln 0.5}{10} = -0.069.$$

At 10 pm, the amount of caffeine for the pregnant woman is

$$Q = 100e^{-0.069(14)} \approx 38 \text{ mg}.$$

31. Since the amount of strontium-90 remaining halves every 29 years, we can solve for the decay constant;

$$\begin{aligned} 0.5P_0 &= P_0 e^{-29k} \\ k &= \frac{\ln(1/2)}{-29}. \end{aligned}$$

Knowing this, we can look for the time t in which $P = 0.10P_0$, or

$$\begin{aligned} 0.10P_0 &= P_0 e^{\ln(0.5)t/29} \\ t &= \frac{29\ln(0.10)}{\ln(0.5)} = 96.336 \text{ years}. \end{aligned}$$

32. Since the population of koalas is growing exponentially, we have $P = P_0 e^{kt}$. Since the initial population is 5000, we have $P_0 = 5000$, so

$$P = 5000e^{kt}.$$

We use the fact that $P = 27{,}000$ in 2005 (when $t = 9$) to find k:

$$27{,}000 = 5000e^{k(9)}$$

Divide both sides by 5000:

$$\frac{27{,}000}{5000} = e^{9k}$$

Take logs of both sides:

$$\ln\left(\frac{27{,}000}{5000}\right) = 9k$$

Divide both sides by 9:

$$k = \frac{\ln(27{,}000/5000)}{9} = 0.187.$$

The koala population increased at a rate of 18.7% per year. The population is given by

$$P = 5000e^{0.187t},$$

where t is in years since 1996. In the year 2020, we have $t = 24$, and using the exact value of $k = (\ln(27/5))/9$, we have

$$P = 5000e^{24\ln(27/5)/9} = 448{,}766 \text{ koalas}.$$

Thus, if growth continues at the same rate, there will be nearly half a million koalas in 2020. As a result, local authorities are trying to find new ways to slow down their rate of growth.

33. **(a)** Assuming the US population grows exponentially, we have population $P(t) = 281.4e^{kt}$ at time t years after 2000. Using the 2013 population, we have

$$\begin{aligned} 316.1 &= 281.4e^{13k} \\ k &= \frac{\ln(316.1/281.4)}{13} = 0.00894. \end{aligned}$$

We want to find the time t in which

$$\begin{aligned} 350 &= 281.4e^{0.00894t} \\ t &= \frac{\ln(350/281.4)}{0.00926} = 24.4 \text{ years}. \end{aligned}$$

This model predicts the population to go over 350 million 24.4 years after 2000, in the year 2024.

(b) Evaluate $P = 281.4e^{0.00894t}$ for $t = 20$ to find $P = 336.49$ million people.

34. We know that the world population is an exponential function over time. Thus the function for world population will be of the form

$$P(t) = P_0a^t$$

where P_0 is the initial population, t is measured in years after 2015 and a is the growth factor. We know that the initial population is the population in the year 2015 so

$$P_0 = 7.32 \text{ billion}.$$

We are also told that in the year 2023 the population will be 8 billion, that is

$$P(8) = 8 \text{ billion}.$$

Solving for a we get

$$\begin{aligned} 8 &= 7.32a^8 \\ \frac{8}{7.32} &= a^8 \\ \left(\frac{8}{7.32}\right)^{1/8} &= a \\ a &\approx 1.0112 \end{aligned}$$

Since the growth factor is 1.0112, we know the annual growth rate is projected to be 1.12%.

35. We assume exponential decay and solve for k using the half-life:

$$e^{-k(5730)} = 0.5 \quad \text{so} \quad k = 1.21 \cdot 10^{-4}.$$

Now find t, the age of the painting:

$$e^{-1.21\cdot 10^{-4}t} = 0.995, \quad \text{so} \quad t = \frac{\ln 0.995}{-1.21 \cdot 10^{-4}} = 41.43 \text{ years}.$$

Since Vermeer died in 1675, the painting is a fake.

36. **(a)** The 1.3 represents the number of food bank users, in thousands, when $t = 0$, that is in 2006.

(b) The continuous growth rate is 0.81, or 81%, per year.

(c) Since $e^{0.81t} = (e^{0.81})^t = 2.248^t$, we see that the annual growth factor is 2.248. Thus, the annual growth rate is $2.248-1 = 1.248 = 124.8\%$ per year.

(d) Since the annual growth rate is over 100% and the annual growth factor is 2.248, the number of users more than doubles each year. The doubling time is less than one year.

37. **(a)** We assume $f(t) = Ae^{-kt}$, where A is the initial population, so $A = 100{,}000$. When $t = 110$, there were 3200 tigers, so

$$3200 = 100{,}000e^{-k\cdot 110}$$

Solving for k gives

$$e^{-k\cdot 110} = \frac{3200}{100{,}000} = 0.0132$$
$$k = -\frac{1}{110}\ln(0.0132) = 0.0313 = 3.13\%$$

so

$$f(t) = 100{,}000e^{-0.0313t}.$$

(b) In 2000, the predicted number of tigers was

$$f(100) = 100{,}000e^{-0.0313(100)} = 4372.$$

In 2010, we know the number of tigers was 3200. The predicted percent reduction is

$$\frac{3200-4372}{4372} = -0.268 = -26.8\%.$$

Thus the actual decrease is larger than the predicted decrease.

38. For an exponentially growing population, $p = p_0e^{kt}$, that doubles in n days, we find the continuous growth rate k in terms of n. Since $p = 2p_0$ when $t = n$, we have

$$\begin{aligned}
2p_0 &= p_0e^{kn}\\
e^{kn} &= 2\\
kn &= \ln 2\\
k &= \frac{1}{n}\ln 2\\
p &= p_0e^{(\ln 2)t/n}.
\end{aligned}$$

Doubling time for the phytoplankton population, P, is 0.5 days, and for the foraminifera population, F, it is 5 days. If they both initially have the same population C, then after t days we have

$$\begin{aligned}
P &= Ce^{(\ln 2)t/0.5} = Ce^{1.3863t}\\
F &= Ce^{(\ln 2)t/5} = Ce^{0.13863t}.
\end{aligned}$$

(a) We have

$$\begin{aligned}
P &= 2F\\
Ce^{1.3863t} &= 2Ce^{0.13863t}\\
e^{1.3863t} &= 2e^{0.13863t}\\
1.3863t &= \ln 2 + 0.13863t\\
1.24767t &= \ln 2\\
t &= 0.556 \text{ days}.
\end{aligned}$$

The phytoplankton population is double the foraminifera population after 0.556 days, a little over 13 hours.

(b) We have

$$\begin{aligned}
P &= 1000F\\
Ce^{1.3863t} &= 1000Ce^{0.13863t}\\
e^{1.3863t} &= 1000e^{0.13863t}\\
1.3863t &= \ln 1000 + 0.13863t\\
1.24767t &= \ln 1000\\
t &= 5.537 \text{ days}.
\end{aligned}$$

The phytoplankton population is 1000 times the foraminifera population after 5.537 days, about five and a half days.

39. Let $P(t)$ be the world population in billions t years after 2010.

(a) Assuming exponential growth, we have

$$P(t) = 6.9e^{kt}.$$

In 2050, we have $t = 40$ and we expect the population then to be 9 billion, so

$$9 = 6.9e^{k\cdot 40}.$$

Solving for k, we have

$$e^{k\cdot 40} = \frac{9}{6.9}$$
$$k = \frac{1}{40}\ln\left(\frac{9}{6.9}\right) = 0.00664 = 0.664\% \text{ per year.}$$

(b) The "Day of 8 Billion" should occur when

$$8 = 6.9e^{0.00664t}.$$

Solving for t gives

$$e^{0.00664t} = \frac{8}{6.9}$$
$$t = \frac{\ln(8/6.9)}{0.00664} = 22.277 \text{ years.}$$

So the "Day of 8 Billion" should be 22.277 years after the end of 2010. This is 22 years and $0.277 \cdot 365 \approx 101$ days; so 101 days into 2032. That is, April 11, 2032. This solution does not take leap years into account.

40. **(a)** Since there are 5 years between 2005 and 2010 we let t be the number of years since 2005 and get:

$$618{,}505 = 246{,}363e^{5r}.$$

Solving for r, we get

$$\frac{618{,}505}{246{,}363} = e^{5r}$$
$$\ln\left(\frac{618{,}505}{246{,}363}\right) = 5r$$
$$r = 0.18410.$$

Substituting $t = 1, 2, 3, 4$ into

$$246{,}363\, e^{(0.18410)t},$$

we find the remaining table values:

Year	2005	2006	2007	2008	2009	2010
Number of E85 vehicles	246,363	296,132	355,956	427,864	514,300	618,505

(b) If N is the number of E85-powered vehicles in 2004, then

$$246{,}363 = Ne^{0.18410t}$$

or

$$N = \frac{246{,}363}{e^{0.18410}} = 204{,}958 \text{ vehicles.}$$

(c) From the table, we can see that the number of E85 vehicles slightly more than doubled from 2005 to 2009, so the percent growth between these years should be slightly over 100%:

$$\text{Percent growth from 2005 to 2009} = 100\left(\frac{514{,}300}{246{,}363} - 1\right) = 1.108575 = 108.575\%.$$

41. We have

$$\text{Future value} = 10{,}000e^{0.03\cdot 8} = \$12{,}712.49.$$

42. We have

$$\text{Future value} = 20{,}000e^{0.038\cdot 15} = \$35{,}365.34.$$

43. We have

$$\begin{aligned}\text{Future value} &= \text{Present value} \cdot e^{0.04\cdot 5}\\ 8000 &= \text{Present value} \cdot e^{0.04\cdot 5}\\ \text{Present value} &= \frac{8000}{e^{0.04\cdot 5}} = \$6549.85.\end{aligned}$$

44. We have

$$\begin{aligned}\text{Future value} &= \text{Present value} \cdot e^{0.032\cdot 10}\\ 20{,}000 &= \text{Present value} \cdot e^{0.032\cdot 10}\\ \text{Present value} &= \frac{20{,}000}{e^{0.032\cdot 10}} = \$14{,}522.98.\end{aligned}$$

45. **(a)** The total present value for each of the two choices are in the following table. Choice 1 is the preferred choice since it has the larger present value.

Choice 1			Choice 2	
Year	Payment	Present value	Payment	Present value
0	1500	1500	1900	1900
1	3000	3000/(1.05) = 2857.14	2500	2500/(1.05) = 2380.95
	Total	4357.14	Total	4280.95

(b) The difference between the choices is an extra \$400 now (\$1900 in Choice 2 instead of \$1500 in Choice 1) versus an extra \$500 one year from now (\$3000 in Choice 1 instead of \$2500 in Choice 2). Since $400 \times 1.25 = 500$, Choice 2 is better at interest rates above 25%.

46. **(a)** (i) We know that the formula for the account balance at time t is given by

$$P(t) = P_0e^{rt}$$

(since the account is being compounded continuously), where P_0 is the initial deposit and r is the annual rate. In our case the initial deposit is

$$P_0 = 24$$

and the annual rate is

$$r = 0.05.$$

Thus, the formula for the balance is

$$P(t) = 24e^{0.05t}.$$

We are asked for the amount of money in the account in the year 2012, that is we are asked for the amount of money in the account 386 years after the initial deposit. This gives

$$\begin{aligned}\text{amount of money in the account in the year 2012} &= P(386)\\ &= 24e^{0.05(386)}\\ &= 24e^{19.3}\\ &\approx 24(240{,}925{,}906)\\ &= 5{,}782{,}217{,}428.\end{aligned}$$

Thus, in the year 2012, there would be about 5.78 billion dollars in the account.

(ii) Here the annual rate is
$$r = 0.07.$$
Thus, the formula for the balance is
$$P(t) = 24e^{0.07t}.$$
Again, we want $P(386)$. This gives
$$\begin{aligned}\text{Amount of money in the account in the year 2012} &= P(386)\\ &= 24e^{0.07(386)}\\ &= 24e^{27.02}\\ &\approx 24(332{,}531{,}537{,}200)\\ &= 13{,}027{,}111{,}872{,}500.\end{aligned}$$
Thus, in the year 2012, there would be about 13.02 trillion dollars in the account.

(b) Here the annual rate is
$$r = 0.06.$$
Thus, the formula for the balance is
$$P(t) = 24e^{0.06t}.$$
We are asked to solve for t such that
$$P(t) = 1{,}000{,}000{,}000.$$
Solving we get
$$\begin{aligned}1{,}000{,}000{,}000 &= B(t) = 24e^{0.06t}\\ \frac{1{,}000{,}000{,}000}{24} &= e^{0.06t}\\ e^{0.06t} &\approx 41{,}666{,}666.7\\ \ln e^{0.06t} &\approx \ln 41{,}666{,}666.7\\ 0.06t &\approx \ln 41{,}666{,}666.7\\ t &\approx \frac{\ln 41{,}666{,}666.7}{0.06} \approx 292.4.\end{aligned}$$
Thus, there would be one million dollars in the account 292.4 years after the initial deposit, that is roughly in 1871.

47. (a) Using the Rule of 70, we get that the doubling time of the investment is
$$\frac{70}{8} = 8.75.$$
That is, it would take about 8.75 years for the investment to double.

(b) We know that the formula for the balance at the end of t years is
$$B(t) = Pa^t$$
where P is the initial investment and a is 1+(interest per year). We are asked to solve for the doubling time, which amounts to asking for the time t at which
$$B(t) = 2P.$$
Solving, we get
$$\begin{aligned}2P &= P(1 + 0.08)^t\\ 2 &= 1.08^t\\ \ln 2 &= \ln 1.08^t\\ t\ln 1.08 &= \ln 2\\ t &= \frac{\ln 2}{\ln 1.08} \approx 9.01.\end{aligned}$$
Thus, the actual doubling time is 9.01 years. And so our estimation by the "rule of 70" was off by a quarter of a year.

48. You should choose the payment which gives you the highest present value. The immediate lump-sum payment of $2800 obviously has a present value of exactly $2800, since you are getting it now. We can calculate the present value of the installment plan as:

$$\begin{aligned} \text{PV} &= 1000e^{-0.03(0)} + 1000e^{-0.03(1)} + 1000e^{-0.03(2)} \\ &\approx \$2912.21. \end{aligned}$$

Since the installment payments offer a higher present value, you should accept the installment option.

49. **(a)** Option 1 is the best option since money received now can earn interest.

(b) In Option 1, the entire $2000 earns 5% interest for 1 year, so we have:

$$\text{Future value of Option 1 } = 2000e^{0.05\cdot 1} = \$2102.54.$$

In Option 2, the payment in one year does not earn interest, but the payment made now earns 5% interest for one year. We have

$$\text{Future value of Option 2 } = 1000 + 1000e^{0.05\cdot 1} = 1000 + 1051.27 = \$2051.27.$$

Since Option 3 is paid all in the future, we have

$$\text{Future value of Option 3 } = \$2000.$$

As we expected, Option 1 has the highest future value.

(c) In Option 1, the entire $2000 is paid now, so we have:

$$\text{Present value of Option 1 } = \$2000.$$

In Option 2, the payment now has a present value of $1000 but the payment in one year has a lower present value. We have

$$\text{Present value of Option 2 } = 1000 + 1000e^{-0.05\cdot 1} = 1000 + 951.23 = \$1951.23.$$

Since Option 3 is paid all in the future, we have

$$\text{Present value of Option 3 } = 2000e^{-0.05\cdot 1} = \$1902.46.$$

Again, we see that Option 1 has the highest value.

Alternatively, we could have computed the present values directly from the future values found in part (b).

50. **(a)** The following table gives the present value of the cash flows. The total present value of the cash flows is $116,224.95.

Year	Payment	Present value
1	50,000	$50{,}000/(1.075) = 46,511.63$
2	40,000	$40{,}000/(1.075)^2 = 34,613.30$
3	25,000	$25{,}000/(1.075)^3 = 20,124.01$
4	20,000	$20{,}000/(1.075)^4 = 14,976.01$
	Total	116,224.95

(b) The present value of the cash flows, $116,224.95, is larger than the price of the machine, $97,000, so the recommendation is to buy the machine.

51. **(a)** We calculate the future values of the two options:

$$\begin{aligned} \text{FV}_1 &= 6e^{0.1(3)} + 2e^{0.1(2)} + 2e^{0.1(1)} + 2e^{0.1(0)} \\ &\approx 8.099 + 2.443 + 2.210 + 2 \\ &= \$14.752 \text{ million.} \end{aligned}$$

$$\begin{aligned} \text{FV}_2 &= e^{0.1(3)} + 2e^{0.1(2)} + 4e^{0.1(1)} + 6e^{0.1(0)} \\ &\approx 1.350 + 2.443 + 4.421 + 6 \\ &= \$14.214 \text{ million.} \end{aligned}$$

As we can see, the first option gives a higher future value, so he should choose Option 1.

(b) From the future value we can easily derive the present value using the formula $\text{PV} = \text{FV}e^{-rt}$. So the present value is

$$\text{Option 1: PV} = 14.752e^{0.1(-3)} \approx \$10.929 \text{ million.}$$

$$\text{Option 2: PV} = 14.214e^{0.1(-3)} \approx \$10.530 \text{ million.}$$

52. In effect, your friend is offering to give you \$17,000 now in return for the \$19,000 lottery payment one year from now. Since $19{,}000/17{,}000 = 1.11764\cdots$, your friend is charging you 11.7% interest per year, compounded annually. You can expect to get more by taking out a loan as long as the interest rate is less than 11.7%. In particular, if you take out a loan, you have the first lottery check of \$19,000 plus the amount you can borrow to be paid back by a single payment of \$19,000 at the end of the year. At 8.25% interest, compounded annually, the present value of 19,000 one year from now is $19{,}000/(1.0825) = 17{,}551.96$. Therefore the amount you can borrow is the total of the first lottery payment and the loan amount, that is, $19{,}000 + 17{,}551.96 = 36{,}551.96$. So you do better by taking out a one-year loan at 8.25% per year, compounded annually, than by accepting your friend's offer.

53. The following table contains the present value of each of the payments, though it does not take into account the resale value if you buy the machine.

Buy			Lease	
Year	Payment	Present Value	Payment	Present Value
0	12000	12000	2650	2650
1	580	$580/(1.0775) = 538.28$	2650	$2650/(1.0775) = 2459.40$
2	464	$464(1.0775)^2 = 399.65$	2650	$2650/(1.0775)^2 = 2282.50$
3	290	$290/(1.0775)^3 = 231.82$	2650	$2650/(1.0775)^3 = 2118.33$
	Total	13,169.75	Total	9510.23

Now we consider the \$5000 resale.

$$\text{Present value of resale} = \frac{5000}{(1.0775)^3} = 3996.85.$$

The net present value associated with buying the machine is the present value of the payments minus the present value of the resale price, which is

$$\text{Present value of buying} = 13{,}169.75 - 3996.85 = 9172.90$$

Since the present value of the expenses associated with buying (\$9172.90) is smaller than the present value of leasing (\$9510.23), you should buy the machine.

54. The following table contains the present value of each of the expenses. Since the total present value of the expenses, \$352.01, is less than the price of the extended warranty, it is not worth purchasing the extended warranty.

Present value of expenses		
Year	Expense	Present value
2	150	$150/(1.05)^2 = 136.05$
3	250	$250/(1.05)^3 = 215.96$
	Total	352.01

Solutions for Section 1.8

1. **(a)** We have $f(g(x)) = f(3x+2) = 5(3x+2) - 1 = 15x + 9$.
(b) We have $g(f(x)) = g(5x-1) = 3(5x-1) + 2 = 15x - 1$.
(c) We have $f(f(x)) = f(5x-1) = 5(5x-1) - 1 = 25x - 6$.

2. **(a)** We have $f(g(x)) = f(x^2+8) = (x^2+8) - 2 = x^2 + 6$.
(b) We have $g(f(x)) = g(x-2) = (x-2)^2 + 8 = x^2 - 4x + 12$.
(c) We have $f(f(x)) = f(x-2) = (x-2) - 2 = x - 4$.

3. **(a)** We have $f(g(x)) = f(e^{2x} = 3e^{2x}$.
(b) We have $g(f(x)) = g(3x) = e^{2(3x)} = e^{6x}$.
(c) We have $f(f(x)) = f(3x) = 3(3x) = 9x$.

4. Notice that $f(2) = 4$ and $g(2) = 5$.

(a) $f(2) + g(2) = 4 + 5 = 9$
(b) $f(2) \cdot g(2) = 4 \cdot 5 = 20$
(c) $f(g(2)) = f(5) = 5^2 = 25$
(d) $g(f(2)) = g(4) = 3(4) - 1 = 11$.

5. **(a)** $g(2 + h) = (2 + h)^2 + 2(2 + h) + 3 = 4 + 4h + h^2 + 4 + 2h + 3 = h^2 + 6h + 11$.
(b) $g(2) = 2^2 + 2(2) + 3 = 4 + 4 + 3 = 11$, which agrees with what we get by substituting $h = 0$ into (a).
(c) $g(2 + h) - g(2) = (h^2 + 6h + 11) - (11) = h^2 + 6h$.

6. **(a)** $f(t + 1) = (t + 1)^2 + 1 = t^2 + 2t + 1 + 1 = t^2 + 2t + 2$.
(b) $f(t^2 + 1) = (t^2 + 1)^2 + 1 = t^4 + 2t^2 + 1 + 1 = t^4 + 2t^2 + 2$.
(c) $f(2) = 2^2 + 1 = 5$.
(d) $2f(t) = 2(t^2 + 1) = 2t^2 + 2$.
(e) $(f(t))^2 + 1 = \left(t^2 + 1\right)^2 + 1 = t^4 + 2t^2 + 1 + 1 = t^4 + 2t^2 + 2$.

7. **(a)** $f(g(1)) = f(1 + 1) = f(2) = 2^2 = 4$
(b) $g(f(1)) = g(1^2) = g(1) = 1 + 1 = 2$
(c) $f(g(x)) = f(x + 1) = (x + 1)^2$
(d) $g(f(x)) = g(x^2) = x^2 + 1$
(e) $f(t)g(t) = t^2(t + 1)$

8. **(a)** $f(g(1)) = f(1^2) = f(1) = \sqrt{1 + 4} = \sqrt{5}$
(b) $g(f(1)) = g(\sqrt{1 + 4}) = g(\sqrt{5}) = (\sqrt{5})^2 = 5$
(c) $f(g(x)) = f(x^2) = \sqrt{x^2 + 4}$
(d) $g(f(x)) = g(\sqrt{x + 4}) = (\sqrt{x + 4})^2 = x + 4$
(e) $f(t)g(t) = (\sqrt{t + 4})t^2 = t^2\sqrt{t + 4}$

9. **(a)** $f(g(1)) = f(1^2) = f(1) = e^1 = e$
(b) $g(f(1)) = g(e^1) = g(e) = e^2$
(c) $f(g(x)) = f(x^2) = e^{x^2}$
(d) $g(f(x)) = g(e^x) = (e^x)^2 = e^{2x}$
(e) $f(t)g(t) = e^t t^2$

10. **(a)** $f(g(1)) = f(3 \cdot 1 + 4) = f(7) = \dfrac{1}{7}$
(b) $g(f(1)) = g(1/1) = g(1) = 7$
(c) $f(g(x)) = f(3x + 4) = \dfrac{1}{3x + 4}$
(d) $g(f(x)) = g\left(\dfrac{1}{x}\right) = 3\left(\dfrac{1}{x}\right) + 4 = \dfrac{3}{x} + 4$
(e) $f(t)g(t) = \dfrac{1}{t}(3t + 4) = 3 + \dfrac{4}{t}$

11. **(a)** We see in the tables that $g(1) = 6$ and $f(6) = 5$, so $f(g(1)) = f(6) = 5$.
(b) We see in the tables that $f(1) = 5$ and $g(5) = 2$, so $g(f(1)) = g(5) = 2$.
(c) We see in the tables that $g(4) = 3$ and $f(3) = 3$, so $f(g(4)) = f(3) = 3$.
(d) We see in the tables that $f(4) = 3$ and $g(3) = 4$, so $g(f(4)) = g(3) = 4$.
(e) We see in the tables that $g(6) = 1$ and $f(1) = 5$, so $f(g(6)) = f(1) = 5$.
(f) We see in the tables that $f(6) = 5$ and $g(5) = 2$, so $g(f(6)) = g(5) = 2$.

12. **(a)** We see in the tables that $g(0) = 2$ and $f(2) = 3$, so $f(g(0)) = f(2) = 3$.
(b) We see in the tables that $g(1) = 3$ and $f(3) = 4$, so $f(g(1)) = f(3) = 4$.
(c) We see in the tables that $g(2) = 5$ and $f(5) = 11$, so $f(g(2)) = f(5) = 11$.
(d) We see in the tables that $f(2) = 3$ and $g(3) = 8$, so $g(f(2)) = g(3) = 8$.
(e) We see in the tables that $f(3) = 4$ and $g(4) = 12$, so $g(f(3)) = g(4) = 12$.

13.

(a)

x	$f(x)+3$
0	13
1	9
2	6
3	7
4	10
5	14

(b)

x	$f(x-2)$
2	10
3	6
4	3
5	4
6	7
7	11

(c)

x	$5g(x)$
0	10
1	15
2	25
3	40
4	60
5	75

(d)

x	$-f(x)+2$
0	−8
1	−4
2	−1
3	−2
4	−5
5	−9

(e)

x	$g(x-3)$
3	2
4	3
5	5
6	8
7	12
8	15

(f)

x	$f(x)+g(x)$
0	12
1	9
2	8
3	12
4	19
5	26

14. **(a)** Let $u = 5t^2 - 2$. We now have $y = u^6$.
(b) Let $u = -0.6t$. We now have $P = 12e^u$.
(c) Let $u = q^3 + 1$. We now have $C = 12 \ln u$.

15. **(a)** Let $u = 3x - 1$. We now have $y = 2^u$.
(b) Let $u = 5t^2 + 10$. We now have $P = \sqrt{u}$.
(c) Let $u = 3r + 4$. We now have $w = 2 \ln u$.

16. $m(z+1) - m(z) = (z+1)^2 - z^2 = 2z + 1$.

17. $m(z+h) - m(z) = (z+h)^2 - z^2 = 2zh + h^2$.

18. $m(z) - m(z-h) = z^2 - (z-h)^2 = 2zh - h^2$.

19. $m(z+h) - m(z-h) = (z+h)^2 - (z-h)^2 = z^2 + 2hz + h^2 - (z^2 - 2hz + h^2) = 4hz$.

20. We see in the graph of $g(x)$ that $g(1) = 2$ so we have $f(g(1)) = f(2) = 4$.

21. We see in the graph of $f(x)$ that $f(1) = 5$ so we have $g(f(1)) = g(5) = 6$.

22. We see in the graph of $g(x)$ that $g(4) = 5$ so we have $f(g(4)) = f(5) = 1$.

23. We see in the graph of $f(x)$ that $f(4) = 2$ so we have $g(f(4)) = g(2) = 3$.

24. We see in the graph of $f(x)$ that $f(2) = 4$ so we have $f(f(2)) = f(4) = 2$.

25. We see in the graph of $g(x)$ that $g(2) = 3$ so we have $g(g(2)) = g(3) = 4$.

26. $f(g(1)) = f(2) \approx 0.4$.

27. $g(f(2)) \approx g(0.4) \approx 1.1$.

28. $f(f(1)) \approx f(-0.4) \approx -0.9$.

29. We see in the graph of $g(x)$ that $g(3) \approx -2.5$ so we have $f(g(3)) = f(-2.5) \approx 0$.

30. To find $f(g(-3))$, we notice that $g(-3) = 3$ and $f(3) = 0$ so $f(g(-3)) = f(3) = 0$. We find the other entries in the table similarly.

x	−3	−2	−1	0	1	2	3
$f(g(x))$	0	1	1	3	1	1	0
$g(f(x))$	0	−2	−2	−3	−2	−2	0

31. The tree has $B = y - 1$ branches on average and each branch has $n = 2B^2 - B = 2(y-1)^2 - (y-1)$ leaves on average. Therefore

$$\text{Average number of leaves } = Bn = (y-1)(2(y-1)^2 - (y-1)) = 2(y-1)^3 - (y-1)^2.$$

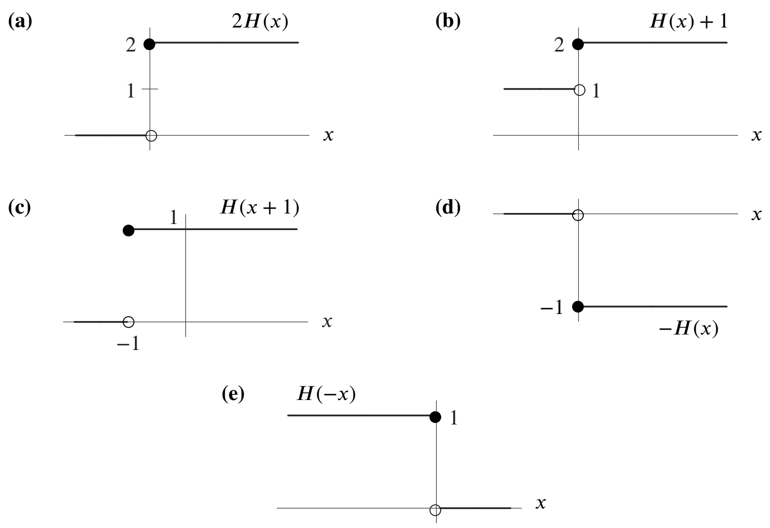
32. We have $v(10) = 65$ but the graph of u only enables us to evaluate $u(x)$ for $0 \le x \le 50$. There is not enough information to evaluate $u(v(10))$.

33. We have approximately $v(40) = 15$ and $u(15) = 18$ so $u(v(40)) = 18$.

34. We have approximately $u(10) = 13$ and $v(13) = 60$ so $v(u(10)) = 60$.

35. We have $u(40) = 60$ but the graph of v only enables us to evaluate $v(x)$ for $0 \le x \le 50$. There is not enough information to evaluate $v(u(40))$.

36.



37. The graph of $y = f(x)$ is shifted vertically upward by 1 unit. See Figure 1.63.

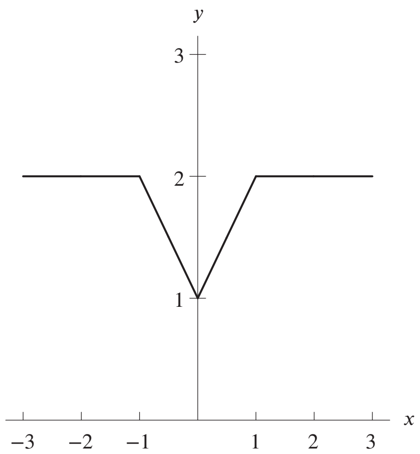


Figure 1.63: Graph of $y = f(x) + 1$

38. The graph of $y = f(x)$ is shifted horizontally to the right by 2 units. See Figure 1.64.

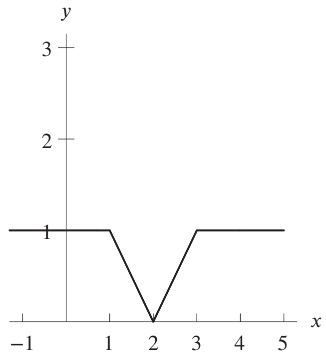


Figure 1.64: Graph of $y = f(x - 2)$

39. The graph of $y = f(x)$ is stretched vertically 3 units. See Figure 1.65.

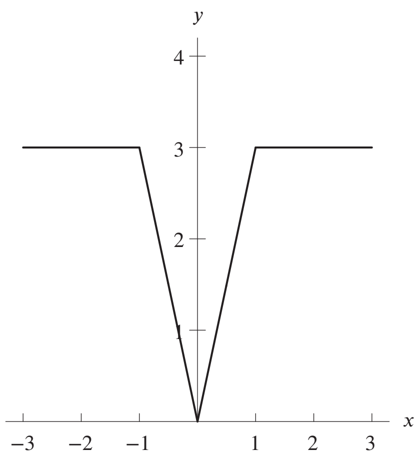


Figure 1.65: Graph of $y = 3f(x)$

40. The graph of $y = f(x)$ is shifted horizontally to the left by 1 unit and is shifted vertically down by 2 units. See Figure 1.66.

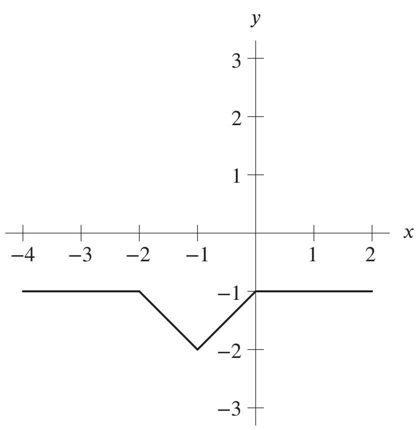


Figure 1.66: Graph of $y = f(x + 1) - 2$

41. The graph of $y = f(x)$ is reflected over the x-axis and shifted vertically up by 3 units. See Figure 1.67.

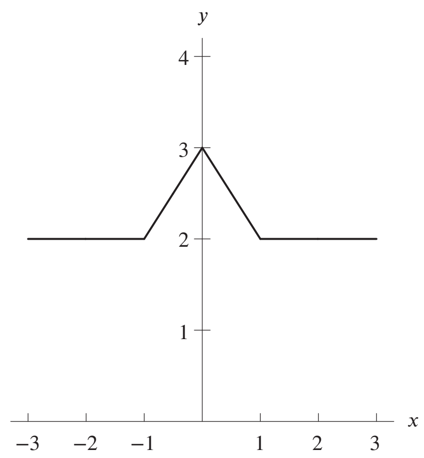


Figure 1.67: Graph of $y = -f(x) + 3$

42. The graph of $y = f(x)$ is reflected over the x-axis, stretched by 2 units, and shifted horizontally to the right by 1 units. See Figure 1.68.

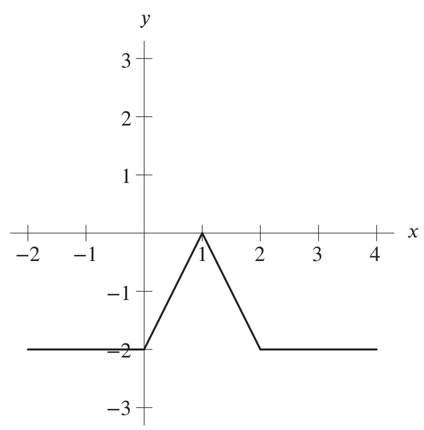
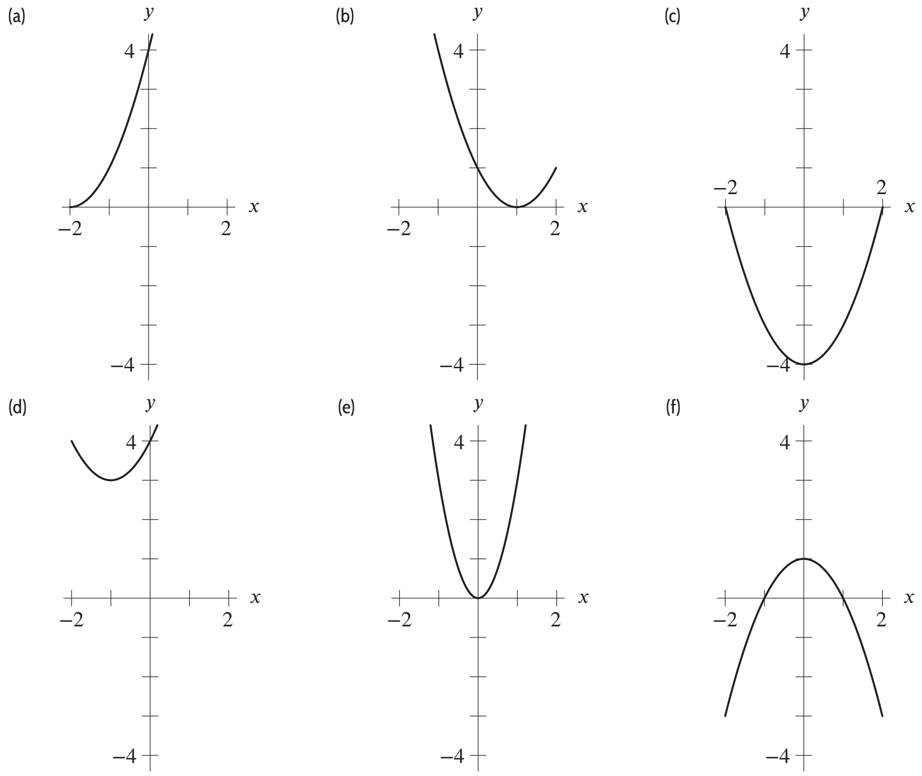
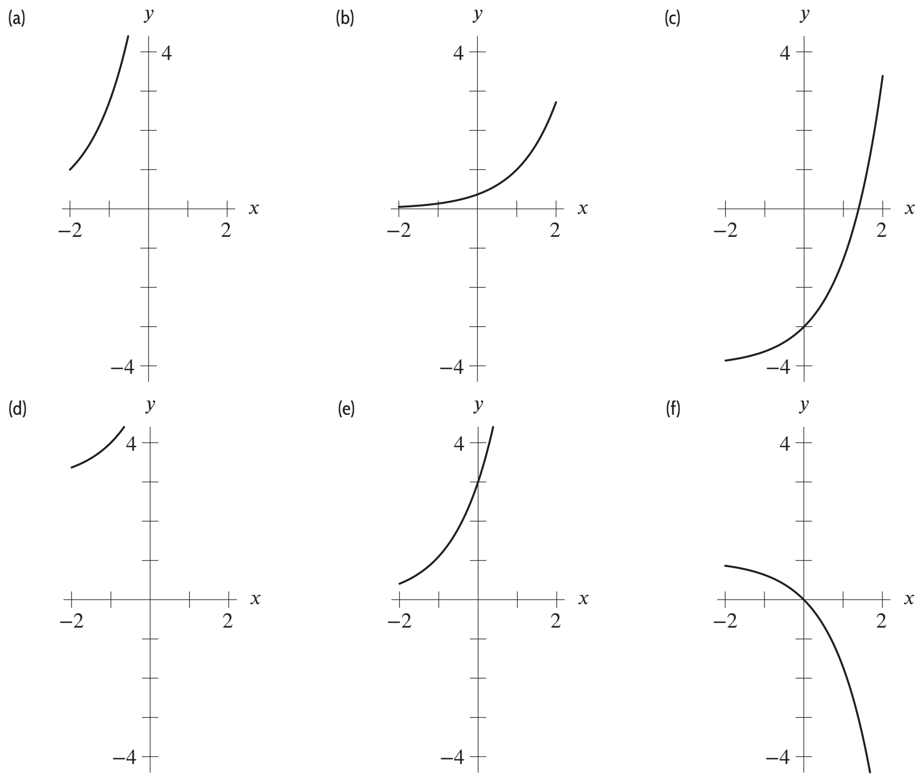


Figure 1.68: Graph of $y = -2f(x - 1)$

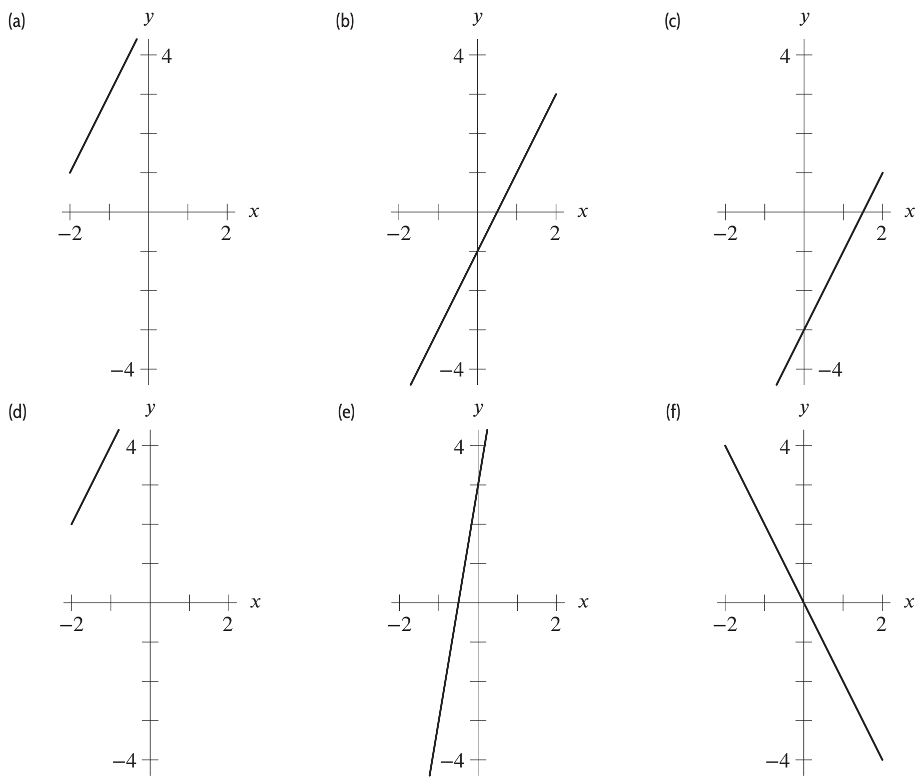
43.



44.



45.



46. See Figure 1.69.

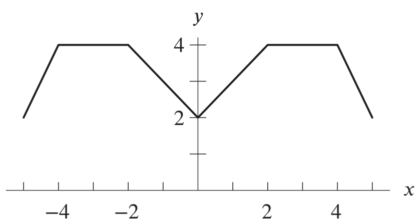


Figure 1.69: Graph of $y = f(x) + 2$

47. See Figure 1.70.

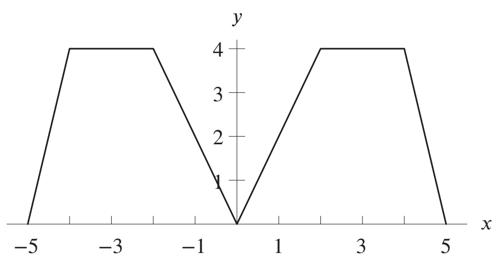


Figure 1.70: Graph of $y = 2f(x)$

48. See Figure 1.71.

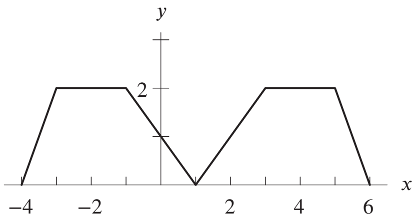


Figure 1.71: Graph of $y = f(x - 1)$

49. See Figure 1.72.

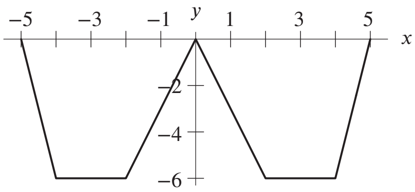


Figure 1.72: Graph of $y = -3f(x)$

50. See Figure 1.73.

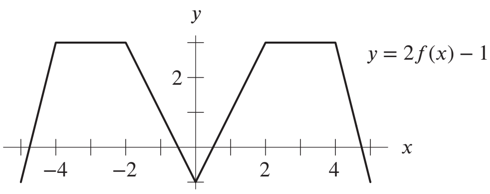


Figure 1.73

51. See Figure 1.74.

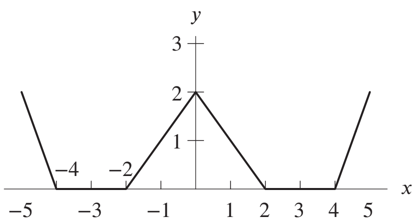


Figure 1.74: Graph of $2 - f(x)$

52. The graph shows the concentration leveling off to a.

(a) The value of a represents the value at which the concentration levels off.
(b) We see a is positive.
(c) At $t = 0$, we have $f(0) = a - be^{k \cdot 0} = a - b$. Since the graph goes through the origin, $a - b = 0$ so $a = b$.
(d) Since the concentration of f levels off to a and $f(t) = a - be^{kt}$, the term be^{kt} must decay to zero. The value of b is positive, so to make the term be^{kt} decay to zero, k must be negative.

53. **(a)** Since the IV line is started 4 hours later, the drug saturation curve is as shown in Figure 1.75.
(b) The function g is a horizontal shift of f.
(c) Since g is f shifted 4 hours to the right, we have $g(t) = f(t - 4)$ for $t \geq 4$.

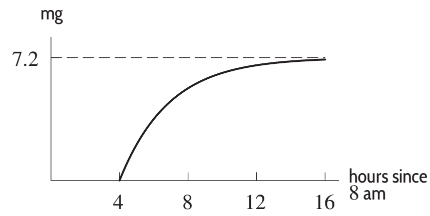


Figure 1.75

54. **(a)** The equation is $y = 2x^2 + 1$. Note that its graph is narrower than the graph of $y = x^2$ which appears in gray. See Figure 1.76.

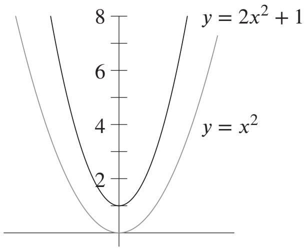


Figure 1.76

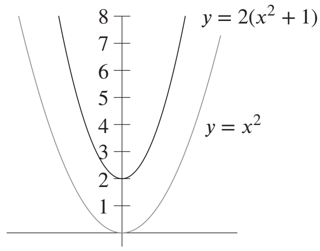


Figure 1.77

(b) $y = 2(x^2 + 1)$ moves the graph up one unit and *then* stretches it by a factor of two. See Figure 1.77.
(c) No, the graphs are not the same. Since $2(x^2 + 1) = (2x^2 + 1) + 1$, the second graph is always one unit higher than the first.

55. The volume of the balloon t minutes after inflation began is: $g(f(t))$ ft^3.

56. The volume of the balloon if its radius were twice as big is: $g(2r)$ ft^3.

57. The time elapsed is: $f^{-1}(30)$ min.

58. The time elapsed is: $f^{-1}(g^{-1}(10{,}000))$ min.

59. **(a)** We find an exponential model in the form $c(t) = c_0 a^t$. Using $c(0) = 25$ and $c(1) = 21.8$, we have

$$c(t) = 25a^t$$
$$21.8 = 25a^1 = 25a.$$

Thus

$$a = \frac{21.8}{25} = 0.872$$
$$c(t) = 25(0.872)^t.$$

Alternately, we could use $c(t) = c_0 e^{kt}$. Then we have $21.8 = 25e^{k\cdot 1} = 25e^k$, giving

$$k = \ln\left(\frac{21.8}{25}\right) = \ln(0.872) = -0.136966$$
$$c(t) = 25e^{-0.136966t}.$$

(b) We find t so that $25(0.872)^t = 10$, giving

$$(0.872)^t = \frac{10}{25}$$
$$t\ln(0.872) = \ln\left(\frac{10}{25}\right)$$
$$t = \frac{\ln(10/25)}{\ln(0.872)} = 6.6899 = 6.690 \text{ years}.$$

Alternatively, solving $25e^{-0.136966t} = 10$ gives

$$t = -\frac{\ln(10/25)}{0.136966} = 6.6899 = 6.690 \text{ years.}$$

(c) If $D(t) = c(2t)$, then

$$D(t) = 25(0.872)^{2t} = 25\left(0.872^2\right)^t = 25(0.7604)^t.$$

Alternatively,

$$D(t) = 25e^{-0.136966(2t)} = 25e^{-0.273932t}.$$

(d) Starting three years earlier, but with t measured from the original time, we have $E(t) = c(t+3)$, so

$$E(t) = 25(0.872)^{t+3} = 25(0.872)^3(0.872)^t = 16.576(0.872)^t.$$

Alternatively,

$$E(t) = 25e^{-0.136966(t+3)} = 25e^{-0.136966(3)}e^{-0.136966t} = 16.576e^{-0.136966t}.$$

Solutions for Section 1.9

1. $y = (1/5)x$; $k = 1/5$, $p = 1$.

2. $y = 5x^{1/2}$; $k = 5$, $p = 1/2$.

3. $y = 8x^{-1}$; $k = 8$, $p = -1$.

4. $y = 3x^{-2}$; $k = 3$, $p = -2$.

5. Not a power function.

6. $y = \frac{3}{8}x^{-1}$; $k = \frac{3}{8}$, $p = -1$.

7. $y = 9x^{10}$; $k = 9$, $p = 10$.

8. $y = \frac{5}{2}x^{-1/2}$; $k = \frac{5}{2}$, $p = -1/2$.

9. Not a power function.

10. $y = 0.2x^2$; $k = 0.2$, $p = 2$.

11. $y = 5^3 \cdot x^3 = 125x^3$; $k = 125$, $p = 3$.

12. Not a power function because of the +4.

13. For some constant k, we have $S = kh^2$.

14. We know that E is proportional to v^3, so $E = kv^3$, for some constant k.

15. If distance is d, then $v = \dfrac{d}{t}$.

16. For some constant k, we have $F = \dfrac{k}{d^2}$.

17. **(a)** This is a graph of $y = x^3$ shifted to the right 2 units and up 1 unit. A possible formula is $y = (x-2)^3 + 1$.
(b) This is a graph of $y = -x^2$ shifted to the left 3 units and down 2 units. A possible formula is $y = -(x+3)^2 - 2$.

18. Since

$$S = kM^{2/3}$$

we have

$$18{,}600 = k(70^{2/3})$$

and so

$$k = 1095.$$

We have $S = 1095M^{2/3}$. If $M = 60$, then

$$S = 1095(60^{2/3}) = 16{,}782 \text{ cm}^2.$$

19. If we let N represent the number of species of lizard on an island and A represent the area of the island, we have

$$N = kA^{1/4}.$$

A graph is shown in Figure 1.78. The function is increasing and concave down. Larger islands have more species of lizards. For small islands, the number of species increases rapidly as the size of the island increases. For larger islands, the increase is much less.

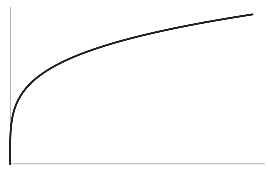


Figure 1.78

20. Let M = blood mass and B = body mass. Then $M = k \cdot B$. Using the fact that $M = 150$ when $B = 3000$, we have

$$\begin{aligned} M &= k \cdot B \\ 150 &= k \cdot 3000 \\ k &= 150/3000 = 0.05. \end{aligned}$$

We have $M = 0.05B$. For a human with $B = 70$, we have $M = 0.05(70) = 3.5$ kilograms of blood.

21. **(a)** Since daily calorie consumption, C, is proportional to the 0.75 power of weight, W, we have

$$C = kW^{0.75}.$$

(b) Since the exponent for this power function is positive and less than one, the graph is increasing and concave down. See Figure 1.79.

(c) We use the information about the human to find the constant of proportionality k. With W in pounds we have

$$\begin{aligned} 1800 &= k(150)^{0.75} \\ k &= \frac{1800}{150^{0.75}} = 42.0. \end{aligned}$$

The function is

$$C = 42.0W^{0.75}.$$

For a horse weighing 700 lbs, the daily calorie requirement is

$$C = 42.0 \cdot 700^{0.75} = 5{,}715.745 \text{ calories}.$$

For a rabbit weighing 9 lbs, the daily calorie requirement is

$$C = 42.0 \cdot 9^{0.75} = 218.238 \text{ calories}.$$

(d) Because the exponent is less than one, the graph is concave down. A mouse has a faster metabolism and needs to consume more calories per pound of weight.

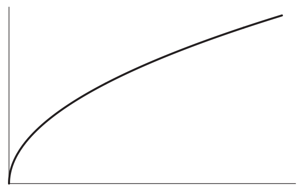


Figure 1.79

22. If $y = k \cdot x^3$, then we have $y/x^3 = k$ with k a constant. In other words, the ratio

$$\frac{y}{x^3} = \frac{\text{Weight}}{(\text{Length})^3}$$

should all be approximately equal (and the ratio will be the constant of proportionality k). For the first fish, we have

$$\frac{y}{x^3} = \frac{332}{(33.5)^3} = 0.0088.$$

If we check all 11 data points, we get the following values of the ratio y/x^3: 0.0088, 0.0088, 0.0087, 0.0086, 0.0086, 0.0088, 0.0087, 0.0086, 0.0087, 0.0088, 0.0088.These numbers are indeed approximately constant with an average of about 0.0087. Thus, $k \approx 0.0087$ and the allometric equation $y = 0.0087x^3$ fits this data well.

23. Since N is inversely proportional to the square of L, we have

$$N = \frac{k}{L^2}.$$

As L increases, N decreases, so there are more species at small lengths.

24. The specific heat is larger when the atomic weight is smaller, so we test to see if the two are inversely proportional to each other by calculating their product. This method works because if y is inversely proportional to x, then

$$y = \frac{k}{x} \quad \text{for some constant } k, \quad \text{so} \quad yx = k.$$

Table 1.7 shows that the product is approximately 6 in each case, so we conjecture that $sw \approx 6$, or $s \approx 6/w$.

Table 1.7

Element	Lithium	Magnesium	Aluminum	Iron	Silver	Lead	Mercury
w	6.9	24.3	27.0	55.8	107.9	207.2	200.6
s (cal/deg-gm)	0.92	0.25	0.21	0.11	0.056	0.031	.033
sw	6.3	6.1	5.7	6.1	6.0	6.4	6.6

25. **(a)** T is proportional to the fourth root of B, and so

$$T = k\sqrt[4]{B} = kB^{1/4}.$$

(b) $148 = k \cdot (5230)^{1/4}$ and so $k = 148/(5230)^{1/4} = 17.4$.

(c) Since $T = 17.4B^{1/4}$, for a human with $B = 70$ we have

$$T = 17.4(70)^{1/4} = 50.3 \text{ seconds}$$

It takes about 50 seconds for all the blood in the body to circulate and return to the heart.

26. **(a)** Since P is inversely proportional to R, we have

$$P = k\frac{1}{R}.$$

(b) If $k = 300{,}000$, we have $P = 300{,}000/R$. If $R = 1$, the population of the city is 300,000 people, since $P = 300{,}000/1$. The population of the second largest city is about 150,000 since $P = 300{,}000/2$. The population of the third largest city is about 100,000 since $P = 300{,}000/3$.

(c) If $k = 6$ million, the population of the largest city is about 6 million, the population of the second largest city is 6 million divided by 2, which is 3 million, and the population of the third largest city is 6 million divided by 3, which is 2 million.

(d) The constant k represents the population of the largest city.

27. **(a)** Since N is proportional to the 0.77 power of P , we have

$$N = kP^{0.77}.$$

(b) Since $P_A = 10 \cdot P_B$, we have

$$N_A = k(P_A)^{0.77} = k(10P_B)^{0.77} = k(10)^{0.77}(P_B)^{0.77} = (10)^{0.77}(k(P_B)^{0.77}) = (10)^{0.77} \cdot (N_B) = 5.888 \cdot N_B.$$

City A has about 5.888 times as many gas stations as city B.

(c) Since the exponent for this power function is positive and less than one, the graph is increasing and concave down. The town of 10,000 people has more gas stations per person.

28. **(a)** We know that the demand function is of the form

$$q = mp + b$$

where m is the slope and b is the vertical intercept. We know that the slope is

$$m = \frac{q(30) - q(25)}{30 - 25} = \frac{460 - 500}{5} = \frac{-40}{5} = -8.$$

Thus, we get

$$q = -8p + b.$$

Substituting in the point $(30, 460)$ we get

$$460 = -8(30) + b = -240 + b,$$

so that

$$b = 700.$$

Thus, the demand function is

$$q = -8p + 700.$$

(b) We know that the revenue is given by

$$R = pq$$

where p is the price and q is the demand. Thus, we get

$$R = p(-8p + 700) = -8p^2 + 700p.$$

(c) Figure 1.80 shows the revenue as a function of price.

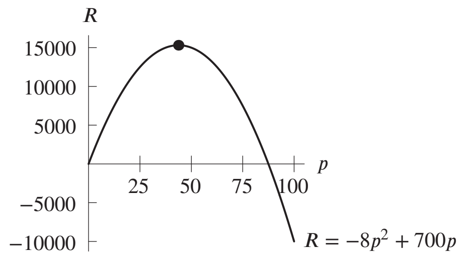


Figure 1.80

Looking at the graph, we see that maximal revenue is attained when the price charged for the product is roughly \$44. At this price the revenue is roughly \$15,300.

29. **(a)** We know that

$$q = 3000 - 20p,$$
$$C = 10{,}000 + 35q,$$

and

$$R = pq.$$

Thus, we get

$$\begin{aligned} C &= 10{,}000 + 35q \\ &= 10{,}000 + 35(3000 - 20p) \\ &= 10{,}000 + 105{,}000 - 700p \\ &= 115{,}000 - 700p \end{aligned}$$

and

$$\begin{aligned} R &= pq \\ &= p(3000 - 20p) \\ &= 3000p - 20p^2. \end{aligned}$$

(b) The graph of the cost and revenue functions are shown in Figure 1.81.

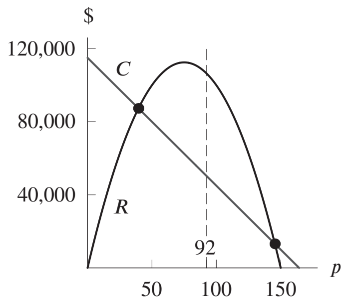


Figure 1.81

(c) It makes sense that the revenue function looks like a parabola, since if the price is too low, although a lot of people will buy the product, each unit will sell for a pittance and thus the total revenue will be low. Similarly, if the price is too high, although each sale will generate a lot of money, few units will be sold because consumers will be repelled by the high prices. In this case as well, total revenue will be low. Thus, it makes sense that the graph should rise to a maximum profit level and then drop.

(d) The club makes a profit whenever the revenue curve is above the cost curve in Figure 1.81. Thus, the club makes a profit when it charges roughly between \$40 and \$145.

(e) We know that the maximal profit occurs when the difference between the revenue and the cost is the greatest. Looking at Figure 1.81, we see that this occurs when the club charges roughly \$92.

Solutions for Section 1.10

1.

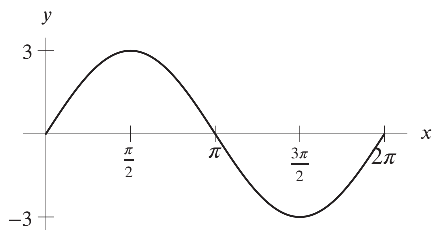


Figure 1.82

See Figure 1.82. The amplitude is 3; the period is 2π.

2.

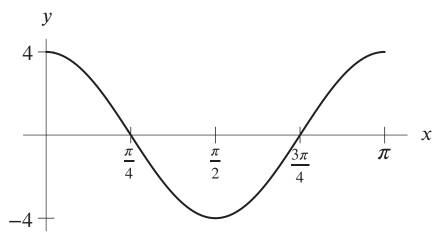


Figure 1.83

See Figure 1.83. The amplitude is 4; the period is π.

3.

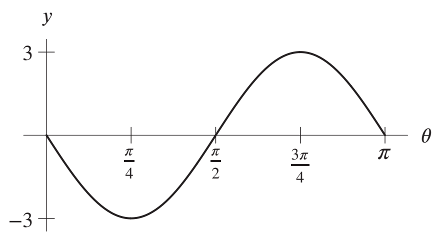


Figure 1.84

See Figure 1.84. The amplitude is 3; the period is π.

4.

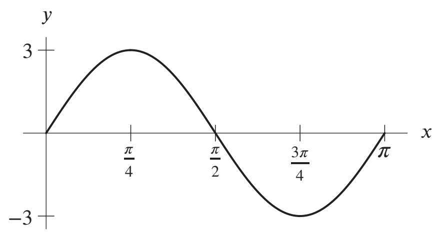


Figure 1.85

See Figure 1.85. The amplitude is 3; the period is π.

5.

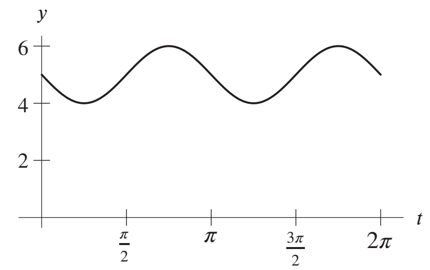


Figure 1.86

See Figure 1.86. The amplitude is 1; the period is π.

6.

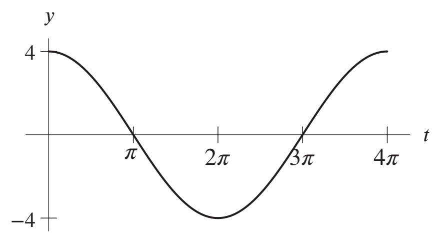


Figure 1.87

See Figure 1.87. The amplitude is 4; the period is 4π.

7. **(a)** The graph *looks* like a periodic function because it appears to repeat itself, reaching approximately the same minimum and maximum each year.
   (b) The maximum occurs in the 2nd quarter and the minimum occurs in the 4th quarter. It seems reasonable that people would drink more beer in the summer and less in the winter. Thus, production would be highest the quarter just before summer and lowest the quarter just before winter.
   (c) The period is 4 quarters or 1 year.

$$\text{Amplitude} = \frac{\max - \min}{2} \approx \frac{52 - 42}{2} = 5 \text{ million barrels}$$

8. It makes sense that sunscreen sales would be lowest in the winter (say from December through February) and highest in the summer (say from June to August). It also makes sense that sunscreen sales would depend almost completely on time of year, so the function should be periodic with a period of 1 year.

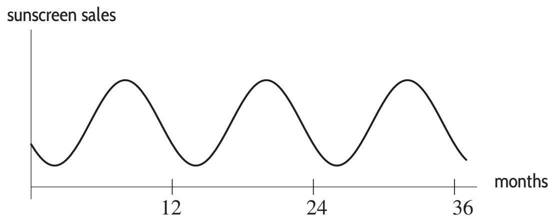


Figure 1.88

9. **(a)** The function appears to vary between 5 and -5, and so the amplitude is 5.
(b) The function begins to repeat itself at $x = 8$, and so the period is 8.
(c) The function is at its highest point at $x = 0$, so we use a cosine function. It is centered at 0 and has amplitude 5, so we have $f(x) = 5\cos(Bx)$. Since the period is 8, we have $8 = 2\pi/B$ and $B = \pi/4$. The formula is

$$f(x) = 5\cos\left(\frac{\pi}{4}x\right).$$

10. **(a)** Since C represent the vertical shift, $C = 27.5°$C.
(b) The amplitude, A, is the distance above and below the midline at C. Since the data rises $36.7 - 27.5 = 9.2°$C above $27.5°$C, and falls $27.5 - 18.4 = 9.1°$C. We take A to be the average

$$A = \frac{9.2 + 9.1}{2} = 9.15°\text{C}.$$

(c) Since the period is 12 months,

$$12 = \frac{2\pi}{B}$$

so $B = \pi/6$.
(d) The function is

$$H(n) = 9.15\cos\left(\frac{\pi}{6}n\right) + 27.5.$$

See Figure 1.89.

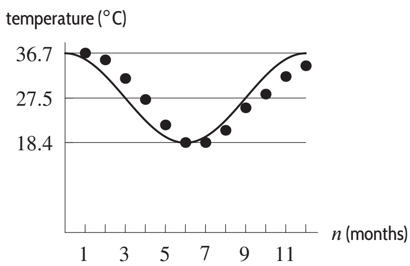


Figure 1.89

11. Since the volume of the function varies between 2 and 4 liters, the amplitude is 1 and the graph is centered at 3. Since the period is 3, we have

$$3 = \frac{2\pi}{B} \quad \text{so} \quad B = \frac{2\pi}{3}.$$

The correct formula is (b). This function is graphed in Figure 1.90 and we see that it has the right characteristics.

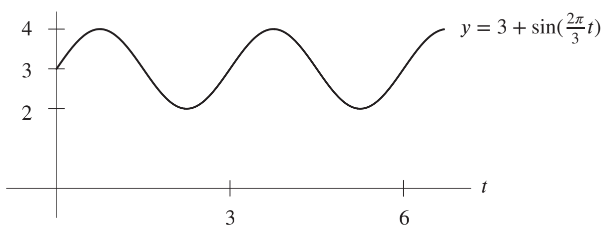


Figure 1.90

12. If we graph the data, it certainly looks like there's a pattern from $t = 20$ to $t = 45$ which begins to repeat itself after $t = 45$. And the fact that $f(20) = f(45)$ helps to reinforce this theory.

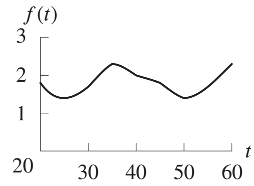


Figure 1.91

Since it looks like the graph repeats after 45, the period is $(45 - 20) = 25$. The amplitude $= \frac{1}{2}(\max - \min) = \frac{1}{2}(2.3 - 1.4) = 0.45$. Assuming periodicity,

$$f(15) = f(15 + 25) = f(40) = 2.0$$
$$f(75) = f(75 - 25) = f(50) = 1.4$$
$$f(135) = f(135 - 25 - 25 - 25) = f(60) = 2.3.$$

13. Since the temperature oscillates over time, we use a trigonometric function. The function is at its lowest point at $t = 0$, so we use a negative cosine curve. The amplitude is about $(26 - (-6))/2 = 16$ and the midline is halfway between -6 and 26, at 10° C. We have $H = -16\cos(Bt) + 10$. The period is 12 months so the coefficient $B = 2\pi/12 = \pi/6$. Thus, the formula is

$$H = -16\cos\left(\frac{\pi}{6}t\right) + 10.$$

14. The levels of both hormones certainly look periodic. In each case, the period seems to be about 28 days. Estrogen appears to peak around the 12th day. Progesterone appears to peak from the 17th through 21st day. (Note: the days given in the answer are approximate so your answer may differ slightly.)

15. Let b be the brightness of the star, and t be the time measured in days from a when the star is at its peak brightness. Because of the periodic behavior of the star,

$$b(t) = A\cos(Bt) + C.$$

The oscillations have amplitude $A = 0.35$, shift $C = 4.0$,and period 5.4, so $5.4B = 2\pi$ and $B = 2\pi/5.4$. This gives

$$b(t) = 0.35\cos\left(\frac{2\pi}{5.4}t\right) + 4.$$

16. **(a)** See Figure 1.92.
(b) The number of species oscillates between a high of 28 and a low of 10. The amplitude is $(28 - 10)/2 = 9$. The period is 12 months or one year.
(c) The graph is at its maximum at $t = 0$ so we use a cosine function. The amplitude is 9 and the graph is shifted up 19 units, so we have $N = 19 + 9\cos(Bt)$. The period is 12 so $B \cdot 12 = 2\pi$ and we have $B = \pi/6$. The formula is

$$N = 19 + 9\cos\left(\frac{\pi}{6}t\right).$$

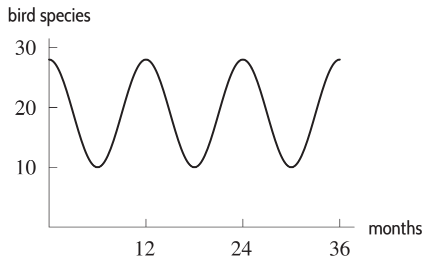


Figure 1.92

17. The graph looks like a sine function of amplitude 7 and period 10. If the equation is of the form

$$y = 7\sin(kt),$$

then $2\pi/k = 10$, so $k = \pi/5$. Therefore, the equation is

$$y = 7\sin\left(\frac{\pi t}{5}\right).$$

18. The graph looks like an upside-down sine function with amplitude $(90 - 10)/2 = 40$ and period π. Since the function is oscillating about the line $x = 50$, the equation is

$$x = 50 - 40\sin(2t).$$

19. This graph is a cosine curve with period 6π and amplitude 5, so it is given by $f(x) = 5\cos\left(\frac{x}{3}\right)$.

20. This graph is a sine curve with period 8π and amplitude 2, so it is given by $f(x) = 2\sin\left(\frac{x}{4}\right)$.

21. This graph is an inverted sine curve with amplitude 4 and period π, so it is given by $f(x) = -4\sin(2x)$.

22. The graph is a sine curve which has been shifted up by 2, so $f(x) = (\sin x) + 2$.

23. This graph is an inverted cosine curve with amplitude 8 and period 20π, so it is given by $f(x) = -8\cos\left(\frac{x}{10}\right)$.

24. This graph has period 5, amplitude 1 and no vertical shift or horizontal shift from $\sin x$, so it is given by

$$f(x) = \sin\left(\frac{2\pi}{5}x\right).$$

25. This graph has period 6, amplitude 5 and no vertical or horizontal shift, so it is given by

$$f(x) = 5\sin\left(\frac{2\pi}{6}x\right) = 5\sin\left(\frac{\pi}{3}x\right).$$

26. The graph is a cosine curve with period $2\pi/5$ and amplitude 2, so it is given by $f(x) = 2\cos(5x)$.

27. This graph has period 8, amplitude 3, and a vertical shift of 3 with no horizontal shift. It is given by

$$f(x) = 3 + 3\sin\left(\frac{2\pi}{8}x\right) = 3 + 3\sin\left(\frac{\pi}{4}x\right).$$

28. This can be represented by a sine function of amplitude 3 and period 18. Thus,

$$f(x) = 3\sin\left(\frac{\pi}{9}x\right).$$

29. The graph is a stretched sine function, so it starts at the midline, $y = 0$, when $x = 0$.
The amplitude is k, so the sinusoidal graph oscillates between a minimum of $-k$ and a maximum of k.
Since k is positive, the function begins by increasing. The period is 2π. See Figure 1.93.

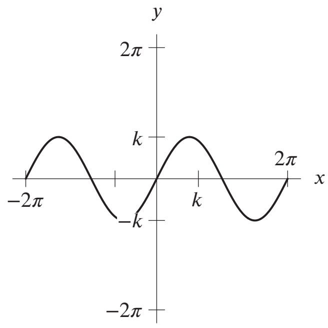


Figure 1.93

30. The graph is a stretched cosine function reflected about the x-axis, so it starts at its minimum when $x = 0$.
The amplitude is k, so the sinusoidal graph oscillates between a minimum of $-k$ and a maximum of k.
The period is 2π. See Figure 1.94.

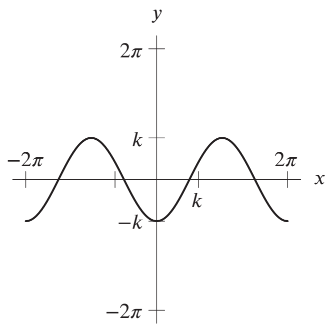


Figure 1.94

31. The graph is a stretched cosine function shifted vertically, so it starts at its maximum when $x = 0$.
We begin by finding the midline, which is k, since the sine function is shifted up by k. The amplitude is k, so the sinusoidal graph oscillates between a minimum of $k - k = 0$ and a maximum of $k + k = 2k$.
The period is 2π. See Figure 1.95.

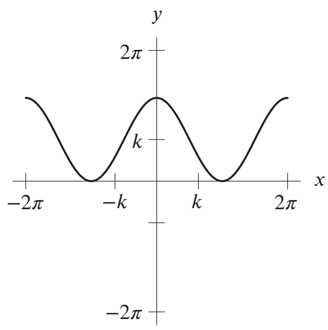


Figure 1.95

32. The graph is a stretched sine function shifted vertically, so it starts at its midline when $x = 0$.

We begin by finding the midline, which is $-k$, since the sine function is shifted up by $-k$ (down by k). The amplitude is k, so the sinusoidal graph oscillates between a minimum of $-k - k = -2k$ and a maximum of $-k + k = 0$.

Since k is positive, the function begins by increasing. The period is 2π. See Figure 1.96.

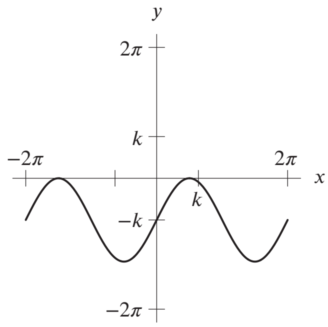


Figure 1.96

33. **(a)** D = the average depth of the water.
(b) A = the amplitude = $15/2 = 7.5$.
(c) Period = 12.4 hours. Thus $(B)(12.4) = 2\pi$ so $B = 2\pi/12.4 \approx 0.507$.
(d) C is the time of a high tide.

34. The depth function is a vertical shift of a function of the form $A\sin(Bt)$.

The vertical shift is $A = (5.5 + 8.5)/2 = 7$.
The amplitude is $A = 8.5 - 7 = 1.5$.
The period is $6 = 2\pi/B$ so $B = 2\pi/6 = \pi/3$. Thus, we have

$$\text{Depth } = 7 + 1.5\sin\left(\frac{\pi}{3}t\right).$$

35. We use a cosine of the form

$$H = A\cos(Bt) + C$$

and choose B so that the period is 24 hours, so $2\pi/B = 24$ giving $B = \pi/12$.

The temperature oscillates around an average value of 60° F, so $C = 60$. The amplitude of the oscillation is 20° F. To arrange that the temperature be at its lowest when $t = 0$, we take A negative, so $A = -20$. Thus

$$A = 60 - 20\cos\left(\frac{\pi}{12}t\right).$$

36. **(a)**

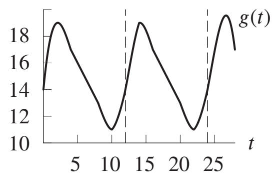


Figure 1.97

From the graph in Figure 1.97, the period appears to be about 12, and the table tells us that $g(0) = 14 = g(12) = g(24)$, which supports this guess.

$$\text{Amplitude} \approx \frac{\max - \min}{2} \approx \frac{19 - 11}{2} = 4$$

.

(b) To estimate $g(34)$, notice that $g(34) = g(24 + 10) = g(10) = 11$ (assuming the function is periodic with period 12). Likewise, $g(60) = g(24 + 12 + 12 + 12) = 14$.

37. **(a)** The monthly mean CO_2 increased about 10 ppm between December 2005 and December 2010. This is because the black curve shows that the December 2005 monthly mean was about 381 ppm, while the December 2010 monthly mean was about 391 ppm. The difference between these two values, $391 - 381 = 10$, gives the overall increase.

(b) The average rate of increase is given by

$$\text{Average monthly increase of monthly mean} = \frac{391 - 381}{60 - 0} = \frac{1}{6} \text{ ppm/month.}$$

This tells us that the slope of a linear equation approximating the black curve is $1/6$. Since the vertical intercept is about 381, a possible equation for the approximately linear black curve is

$$y = \frac{1}{6}t + 381,$$

where t is measured in months since December 2005.

(c) The period of the seasonal CO_2 variation is about 12 months since this is approximately the time it takes for the function given by the blue curve to complete a full cycle. The amplitude is about 3.5 since, looking at the blue curve, the average distance between consecutive maximum and minimum values is about 7 ppm. So a possible sinusoidal function for the seasonal CO_2 cycle is

$$y = 3.5 \sin\left(\frac{\pi}{6}t\right).$$

(d) Taking $f(t) = 3.5 \sin\left(\frac{\pi}{6}t\right)$ and $g(t) = \frac{1}{6}t + 381$, we have

$$h(t) = 3.5 \sin\left(\frac{\pi}{6}t\right) + \frac{1}{6}t + 381.$$

See Figure 1.98.

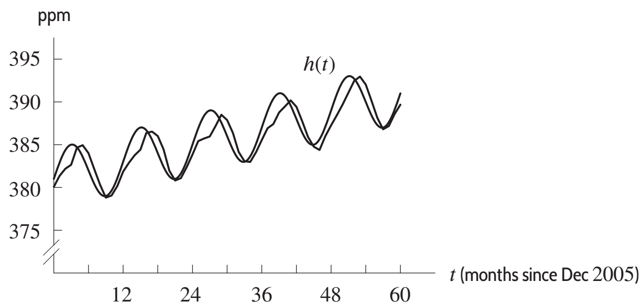


Figure 1.98

STRENGTHEN YOUR UNDERSTANDING: Digital only

1. False, the domain is the set of inputs of a function.
2. False, $f(10)$ is the value of the car when it is 10 years old.
3. True, the point $(2, 4)$ is on the graph since $f(2) = 2^2 = 4$.
4. False, since we include the points 3 and 4, the set of numbers is written $[3, 4]$.
5. True. Plugging in $r = 0$ to find the intercept on the D axis gives $D = 10$.
6. False, a function could be given by a graph or table, or it could be described in words.
7. True, plugging in $x = 3$, we have $f(3) = 3^2 + 2(3) + 1 = 9 + 6 + 1 = 16$.
8. True, consider the function $f(x) = x^2 - 1$. This has horizontal intercepts at $x = \pm 1$.
9. False, if a graph has two vertical intercepts, then it cannot correspond to a function, since the function would have two values at zero.
10. True. The vertical intercept is the value of C when $q = 0$, or the cost to produce no items.

11. True, the slope of a linear function is given by rise over run, where rise is the difference in the function values and run is the difference in the corresponding inputs.

12. False, the linear function $m(x)$ has slope 3.

13. True, any constant function is a linear function with slope zero.

14. True. We first check that the slope is -1, and then make sure that the point $(2, 5)$ is on the graph. Since $-(2) + 7 = 5$, the line $y = -x + 7$ passes through the point $(2, 5)$.

15. True, for every increase in s by 2, the function $h(s)$ decreases by 4, so $h(s)$ could correspond to a linear function with slope -2.

16. False, any two linear functions which have the same slope and different y-intercepts will not intersect, since they are parallel.

17. False, consider the linear function $y = x - 1$. The slope of this function is 1, and the y-intercept is -1.

18. False, the slope of a linear function $f(x)$ is given by $\frac{f(x_2)-f(x_1)}{x_2-x_1}$. Plugging in 4 for x_1 and 9 for x_2, we see that the slope is $\frac{3-2}{9-4} = \frac{1}{5}$.

19. False, the units of the slope are acres per meter.

20. True, since a slope of 3 for a linear function $y = f(x)$ means increasing the x coordinate by 1 causes the y coordinate to increase by 3.

21. False. Since the slope of this linear function is negative, the function is decreasing.

22. True, this is just the definition of the average rate of change.

23. False, the average rate of change is given by $\frac{C(b)-C(a)}{b-a}$. The units of the numerator are dollars, and the units of the denominator are students, so the units of the average rate of change is in dollars per student.

24. True, the average rate of change is given by the formula $\frac{s(b)-s(a)}{b-a}$. Substituting $b = 2$ and $a = -1$, we get $\frac{2^2-(-1)^2}{2-(-1)} = \frac{4-1}{2+1} = \frac{3}{3} = 1 > 0$.

25. True, consider $f(x) = \sqrt{x}$.

26. False, if $Q(r)$ was concave up, then the average rate of change would be increasing. However, the average rate of change from $r = 1$ to $r = 2$ is $\frac{15-10}{2-1} = 5$, and the average rate of change from $r = 2$ to $r = 3$ is $\frac{17-15}{3-2} = 2$.

27. False, the velocity includes the direction of the particle, whereas the speed does not.

28. True, the rate of change formula is the same as the formula for the slope of the secant line.

29. True, for any values $t_1 < t_2$, $\frac{s(t_2)-s(t_1)}{t_2-t_1} = \frac{3t_2+2-(3t_1+2)}{t_2-t_1} = \frac{3(t_2-t_1)}{t_2-t_1} = 3$.

30. False, consider the function $f(x) = 1/x$ for $x > 0$. $f(x)$ is concave up, but it is a decreasing function.

31. True. This is the definition of relative change.

32. False. Percent change does not have units.

33. True. When quantity increases by 100 from 500 to 600, the cost increases by 15%, which is $0.15 \cdot 1000 = 150$ dollars. Cost goes from \$1000 to \$1150.

34. False. Relative change can be positive (if the quantity is increasing) or negative (if the quantity is decreasing.)

35. False. Relative rate of change is given as a percent per hour.

36. False, profit is the difference between revenue and cost.

37. True, this is the definition of revenue.

38. False, although the cost function depends on the quantity produced q, there can also be fixed costs of production which are present even when nothing is being produced.

39. False, as q increases $C(q)$ should increase, since producing more units will cost more than producing less.

40. True, since the more units sold, the more revenue is produced.

41. False. It is possible that supply is greater than demand.

42. True.

43. True, both marginal cost and marginal revenue have units dollars per item.

44. True, since the additional profit in selling one more item is the additional revenue from that item minus the additional cost to produce the item.

45. False. Imposing a sales tax shifts the curves which can change where they intersect.

46. True, this is a function of the form $P_0 a^t$ which is the definition of an exponential function.

47. True.

48. False, an exponential function can be increasing or decreasing.

49. False, the percent growth rate is 3%.

50. True, substituting $x = 2$ and $x = 1$, we have $\frac{Q(2)}{Q(1)} = \frac{35(1/3)^2}{35(1/3)} = 1/3$.

51. True, substituting $s = 0$, we have $R(0) = 16$.

52. True, the first two decimal digits of e are 2.7.

53. True.

54. True.

55. False, $Q(r)$ has constant differences in successive values so it is linear. It does not have constant ratios of successive values so it is not exponential.

56. False, $\ln(x)$ is only defined for $x > 0$.

57. True, since $e^0 = 1$.

58. False, for example, $\ln(2) = \ln(1+1) \neq \ln(1) + \ln(1) = 0 + 0 = 0$.

59. True, since $\ln x = c$ means $e^c = x$.

60. True, since as x increases, the exponent e needs to be raised to in order to equal x also increases.

61. True. We take ln of both sides and bring the exponent t down.

62. True, an exponential growth function is one which has the form $P = P_0 e^{kt}$ for k a positive constant.

63. True, when $b > 0$, $e^{\ln b} = b$ and $\ln(e^b) = b$.

64. False, since if $B = 1$, then $\ln(2B) = \ln(2) \neq 2\ln(B) = 2\ln(1) = 0$.

65. True, by the properties of logarithms.

66. True, the doubling time depends only on the rate of growth, which in this case is the same.

67. False. It will take two half-lives, or 12 years, for the quantity to decay to 25 mg.

68. False, the half-life only depends on the rate of decay, which is the same in this case ($k = -0.6$). So, they have the same half-life.

69. False, if an initial investment Q is compounded annually with interest rate r for t years, then the value of the investment is $Q(1+r)^t$. So, we should have $1000(1.03)^t$ as the value of the account.

70. False, if the rate of growth is larger, the doubling time will be shorter.

71. True, the doubling time is the value of t which gives double the initial value for P. Since the initial value is 5, we find the value of t when $P = 10$.

72. True, since a deposit made today (present value) will always grow with interest over time (future value).

73. True, since we have a continuous rate of growth, we use the formula $P_0 e^{kt}$. In our case, $P_0 = 1000$, $k = .03$, and $t = 5$, giving $1000e^{.15}$.

74. True, since the longer the initial payment is invested, the more interest will be accumulated.

75. False, since less money needs to be invested today to grow to \$1000 ten years from now than to grow to \$1000 five years from now.

76. False, since $f(x+5)$ shifts the graph of f to the left by 5, while $f(x) + 5$ shifts the graph of f up by 5.

77. True, since $f(x+k)$ is just a shift of the graph of $f(x)$ k units to the left, this shift will not affect the increasing nature of the function f. So, $f(x+k)$ is increasing.

78. True, since $-2g(t)$ flips and stretches the concave up graph of $g(t)$.

79. True, crossing the x-axis at $x = 1$ means that $f(1) = 0$. If we multiply both sides by 5, we see that $5f(1) = 0$, which means that $5f(x)$ crosses the x-axis at $x = 1$.

80. True, since the graph of $f(x+1)$ is the graph of $f(x)$ shifted to the left by 1.

81. False. We have $g(3+h) = (3+h)^2 = 9 + 6h + h^2$ so $g(3+h) \neq 9 + h^2$ for many values of h.

82. False, $f(g(t)) = (t+1)^2 = t^2 + 2t + 1$ and $g(f(t)) = t^2 + 1$ which are clearly not equal.

83. True, we have $f(g(x)) = f(\ln x) = (\ln x)^3 - 5$.

84. True, we can check that $g(u(x)) = g(3x^2 + 2) = (3x^2 + 2)^3 = h(x)$.

85. False, since $f(x+h) = (x+h)^2 - 1 = x^2 + 2xh + h^2 - 1$, we see that $f(x+h) - f(x) = 2xh + h^2$ which is not equal to h^2.

86. True.

87. False, if A is inversely proportional to B, then $A = k/B$ for some non-zero constant k.

88. True, a power function has the form $g(x) = k \cdot x^p$.

89. False, this is an exponential function.

90. False. It can be written as the power function $h(x) = 3x^{-1/2}$.

91. False. It can be written as a power function in the form $g(x) = (3/2)x^{-2}$

92. True, since $f(x) = (3/2)x^{1/2} = 1.5x^{1/2}$.

93. True.

94. True.

95. True. If p is proportional to q, then $p = kq$ for nonzero constant k, so $p/q = k$, a constant.

96. False, the amplitude is the coefficient of the trig function, which in this case is 3.

97. True, for a trig function of the form $\cos(kx) + b$, the period is given by $2\pi/k$. Since $k = 1$ in this case, the period is 2π.

98. False, if we substitute $t = 0$, then the first expression is not defined, whereas the second expression is $3/5$. Thus, they cannot be equal.

99. True. It is shifted $\pi/2$ to the left.

100. False, for a trig function of the form $A\cos(kx) + b$, the period is given by $2\pi/k$. Since $k = 5$ in this case, the period is $2\pi/5$.

101. False, since the period of $y = \sin(2t)$ is π and the period of $\sin(t)$ is 2π, the period is half.

102. True, since both have amplitude of 5. The second function has a vertical shift of 8.

103. False, one way to see that the graphs are different is that $(\sin x)^2$ is never negative (as it is a square). On the other hand, $y = \sin(x^2)$ is both positive and negative.

104. False, the function $\sin(x)$ oscillates between -1 and 1.

105. True.

PROJECTS FOR CHAPTER ONE

1. **(a)** Compounding daily (continuously),

$$\begin{aligned} P &= P_0 e^{rt} \\ &= \$450{,}000 e^{(0.06)(213)} \\ &\approx \$1.5977 \cdot 10^{11}. \end{aligned}$$

This amounts to approximately \$160 billion.

(b) Compounding yearly,

$$\begin{aligned} A &= \$450{,}000\,(1+0.06)^{213} \\ &= \$450{,}000(1.06)^{213} \approx \$450{,}000(245{,}555.29) \\ &\approx \$1.10499882 \cdot 10^{11}. \end{aligned}$$

This is only about \$110.5 billion.

(c) We first wish to find the interest that will accrue during 1990. For 1990, the principal is $\$1.105 \cdot 10^{11}$. At 6% annual interest, during 1990 the money will earn

$$0.06 \cdot \$1.105 \cdot 10^{11} = \$6.63 \cdot 10^{9}.$$

The number of seconds in a year is

$$\left(365\frac{\text{days}}{\text{year}}\right)\left(24\frac{\text{hours}}{\text{day}}\right)\left(60\frac{\text{mins}}{\text{hour}}\right)\left(60\frac{\text{secs}}{\text{min}}\right) = 31{,}536{,}000 \text{ sec.}$$

Thus, over 1990, interest is accumulating at the rate of

$$\frac{\$6.63 \cdot 10^9}{31{,}536{,}000\ \text{sec}} \approx \$210.24\ /\text{sec}.$$

2. **(a)** Since the population center is moving west at 50 miles per 10 years, or 5 miles per year, if we start in 2000, when the center is in Edgar Springs, its distance d west of Edgar Springs t years after 2000 is given by

$$d = 5t.$$

(b) It moved over 1000 miles over the 210 years from 1790 to 2000, so its average speed was greater than $1000/210 = 4.8$ miles/year, somewhat slower than its present rate.

(c) According to the function in (a), after 400 years, the population center would be 2000 miles west of Edgar Springs, in the Pacific Ocean, which is impossible.

3. Assuming that T_r decays exponentially, we have

$$T_r = Ae^{-kt}.$$

We find the values of A and k from the data in the table in the problem. We know that:

$$\begin{aligned} Ae^{-k\cdot 4} &= 37 && \text{since } T_r = 37 \text{ at } t = 4 \\ Ae^{-k\cdot 19.5} &= 13 && \text{since } T_r = 13 \text{ at } t = 19.5 \\ \text{so} \quad \frac{Ae^{-k\cdot 4}}{Ae^{-k\cdot 19.5}} &= \frac{37}{13} && \text{dividing} \\ e^{15.5k} &= \frac{37}{13} \\ k &= \frac{1}{15.5}\ln\left(\frac{37}{13}\right) = 0.06748. \end{aligned}$$

Having found k, we can now find A:

$$\begin{aligned} Ae^{-0.06748(4)} &= 37 && \text{since } T_r = 37 \text{ at } t = 4 \\ A &= 48.5 \text{ ng/ml}. \end{aligned}$$

Thus, the concentration of tryptase t hours after surgery is modeled by the exponential decay function

$$T_r = 48.5e^{-0.06748t} \text{ ng/ml}.$$

The greatest concentration is 48.5 ng/ml, which occurs at $t = 0$ when the patient leaves surgery. We can diagnose anaphylaxis since this peak level is above 45 ng/ml.