Chapter 6 The Definite Integral

6.1 Antidifferentiation

1. $f(x) = x \Rightarrow F(x) = \frac{1}{2}x^2 + C$

2. $f(x) = 9x^8 \Rightarrow F(x) = x^9 + C$

3. $f(x) = e^{3x} \Rightarrow F(x) = \frac{1}{3}e^{3x} + C$

4. $f(x) = e^{-3x} \Rightarrow F(x) = -\frac{1}{3}e^{-3x} + C$

5. $f(x) = 3 \Rightarrow F(x) = 3x + C$

6. $f(x) = -4x \Rightarrow F(x) = -2x^2 + C$

7. $\int 4x^3 dx = x^4 + C$

8. $\int \frac{x}{3} dx = \int \frac{1}{3} x dx = \frac{1}{6}x^2 + C$

9. $\int 7 dx = 7x + C$

10. $\int k^2 dx = k^2 x + C$

11. $\int \frac{x}{c} dx = \int \frac{1}{c} x dx = \frac{1}{2c}x^2 + C$

12. $\int x \cdot x^2 dx = \int x^3 dx = \frac{1}{4}x^4 + C$

13. $$\begin{aligned}\int\left(\frac{2}{x}+\frac{x}{2}\right)dx &= \int\left(2\cdot\frac{1}{x}+\frac{1}{2}\cdot x\right)dx \\ &= 2\int\frac{1}{x}dx + \frac{1}{2}\int x dx \\ &= 2\ln|x| + \frac{1}{4}x^2 + C\end{aligned}$$

14. $\int \frac{1}{7x} dx = \int \frac{1}{7}\cdot\frac{1}{x} dx = \frac{1}{7}\ln|x| + C$

15. $\int x\sqrt{x} dx = \int x^{3/2} dx = \frac{2}{5}x^{5/2} + C$

16. $$\begin{aligned}\int\left(\frac{2}{\sqrt{x}}+2\sqrt{x}\right)dx &= \int\left(2x^{-1/2}+2x^{1/2}\right)dx \\ &= 2\int x^{-1/2}dx + 2\int x^{1/2}dx \\ &= 4x^{1/2} + \frac{4}{3}x^{3/2} + C \\ &= 4\sqrt{x} + \frac{4}{3}x^{3/2} + C\end{aligned}$$

17. $$\begin{aligned}\int\left(x-2x^2+\frac{1}{3x}\right)dx &= \int\left(x-2x^2+\frac{1}{3}\cdot\frac{1}{x}\right)dx \\ &= \int x dx - 2\int x^2 dx + \frac{1}{3}\int\frac{1}{x}dx \\ &= \frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{1}{3}\ln|x| + C\end{aligned}$$

18. $$\begin{aligned}\int\left(\frac{7}{2x^3}-\sqrt[3]{x}\right)dx &= \int\left(\frac{7}{2}x^{-3}-x^{1/3}\right)dx \\ &= \frac{7}{2}\int x^{-3}dx - \int x^{1/3}dx \\ &= \frac{7}{2}\left(-\frac{1}{2}\right)x^{-2} - \frac{1}{\frac{1}{3}+1}x^{4/3} + C \\ &= -\frac{7}{4}x^{-2} - \frac{3}{4}x^{4/3} + C\end{aligned}$$

19. $\int 3e^{-2x} dx = -\frac{3}{2}e^{-2x} + C$

20. $\int e^{-x} dx = -e^{-x} + C$

21. $\int e dx = ex + C$

22. $\int \frac{7}{2e^{2x}} dx = \frac{7}{2}\int e^{-2x} dx = -\frac{7}{4}e^{-2x} + C$

23. $$\begin{aligned}\int -2\left(e^{2x}+1\right)dx &= -2\int e^{2x}dx - 2\int 1 dx \\ &= -2\left(\frac{1}{2}e^{2x}\right) - 2x + C \\ &= -e^{2x} - 2x + C\end{aligned}$$

24. $$\begin{aligned}&\int\left(-3e^{-x}+2x-\frac{e^{0.5x}}{2}\right)dx \\ &= -3\int e^{-x}dx + 2\int x dx - \frac{1}{2}\int e^{0.5x}dx \\ &= 3e^{-x} + x^2 - \frac{1}{2}\left(\frac{1}{0.5}\right)e^{0.5x} + C \\ &= 3e^{-x} + x^2 - e^{0.5x} + C\end{aligned}$$

25. $\frac{d}{dt}\left[ke^{-2t}\right] = -2ke^{-2t} = 5e^{-2t} \Rightarrow k = -\frac{5}{2}$

26. $\frac{d}{dt}\left[ke^{t/10}\right] = \frac{1}{10}ke^{t/10} = 3e^{t/10} \Rightarrow k = 30$

27. $\frac{d}{dx}\left[ke^{4x-1}\right] = 4ke^{4x-1} = 2e^{4x-1} \Rightarrow k = \frac{1}{2}$

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28. $\frac{d}{dx}\left[\frac{k}{e^{3x+1}}\right] = \frac{d}{dx}\left[ke^{-(3x+1)}\right] = -3ke^{-(3x+1)}$
$-3ke^{-(3x+1)} = 4e^{-(3x+1)} \Rightarrow k = -\frac{4}{3}$

29. $\frac{d}{dx}\left[k(5x-7)^{-1}\right] = -k(5x-7)^{-2}(5)$
$= -5k(5x-7)^{-2} = (5x-7)^{-2}$
$k = -\frac{1}{5}$

30. $\frac{d}{dx}\left[k(x+1)^{3/2}\right] = \frac{3}{2}k(x+1)^{1/2} = (x+1)^{1/2} \Rightarrow$
$k = \frac{2}{3}$

31. $\frac{d}{dx}\left[k\ln|4-x|\right] = \frac{k}{4-x}(-1) = \frac{-k}{4-x} = \frac{1}{4-x} \Rightarrow$
$k = -1$

32. $\frac{d}{dx}\left[\frac{k}{(8-x)^3}\right] = \frac{d}{dx}\left[k(8-x)^{-3}\right]$
$= -3k(8-x)^{-4}(-1)$
$= 3k(8-x)^{-4} = 7(8-x)^{-4} \Rightarrow$
$k = \frac{7}{3}$

33. $\frac{d}{dx}\left[k(3x+2)^5\right] = 5k(3x+2)^4(3)$
$= 15k(3x+2)^4 = (3x+2)^4 \Rightarrow$
$k = \frac{1}{15}$

34. $\frac{d}{dx}\left[k(2x-1)^4\right] = 4k(2x-1)^3(2) = 8k(2x-1)^3$
$= (2x-1)^3 \Rightarrow k = \frac{1}{8}$

35. $\frac{d}{dx}[k\ln|2+x|] = \frac{k}{2+x} = \frac{3}{2+x} \Rightarrow k = 3$

36. $\frac{d}{dx}\left[k\ln|2-3x|\right] = \frac{k}{2-3x}(-3)$
$= \frac{-3k}{2-3x} = \frac{5}{2-3x} \Rightarrow k = -\frac{5}{3}$

37. $f'(t) = t^{3/2} \Rightarrow f(t) = \frac{2}{5}t^{5/2} + C$

38. $f'(t) = \frac{4}{6+t} \Rightarrow f(t) = 4\ln|6+t| + C$

39. $f'(t) = 0 \Rightarrow f(t) = C$

40. $f'(t) = t^2 - 5t - 7 \Rightarrow f(t) = \frac{1}{3}t^3 - \frac{5}{2}t^2 - 7t + C$

41. $f'(x) = 0.5e^{-0.2x} \Rightarrow f(x) = -2.5e^{-0.2x} + C$
$f(0) = 0 \Rightarrow -2.5e^{-0.2\cdot 0} + C = 0 \Rightarrow C = 2.5$
Thus, $f(x) = -2.5e^{-0.2x} + 2.5$.

42. $f'(x) = 2x - e^{-x} \Rightarrow f(x) = x^2 + e^{-x} + C$
$f(0) = 1 \Rightarrow 0^2 + e^{-0} + C = 1 \Rightarrow C = 0$
Thus, $f(x) = x^2 + e^{-x}$.

43. $f'(x) = x \Rightarrow f(x) = \frac{1}{2}x^2 + C$
$f(0) = 3 \Rightarrow \frac{1}{2}\cdot 0^2 + C = 3 \Rightarrow C = 3$
Thus, $f(x) = \frac{1}{2}x^2 + 3$.

44. $f'(x) = 8x^{1/3} \Rightarrow f(x) = 6x^{4/3} + C$
$f(1) = 4 \Rightarrow 6\cdot 1^{4/3} + C = 4 \Rightarrow C = -2$
Thus, $f(x) = 6x^{4/3} - 2$.

45. $f'(x) = x^{1/2} + 1 \Rightarrow f(x) = \frac{2}{3}x^{3/2} + x + C$
$f(4) = 0 \Rightarrow \frac{2}{3}4^{3/2} + 4 + C = 0 \Rightarrow$
$\frac{2}{3}\cdot 8 + 4 + C = 0 \Rightarrow C = -\frac{28}{3}$
Thus, $f(x) = \frac{2}{3}x^{3/2} + x - \frac{28}{3}$.

46. $f'(x) = x^2 + x^{1/2} \Rightarrow f(x) = \frac{1}{3}x^3 + \frac{2}{3}x^{3/2} + C$
$f(1) = 3 \Rightarrow \frac{1}{3}\cdot 1^3 + \frac{2}{3}\cdot 1^{3/2} + C = 3 \Rightarrow C = 2$
Thus, $f(x) = \frac{1}{3}x^3 + \frac{2}{3}x^{3/2} + 2$.

47. $f(x) = \int \frac{2}{x}\,dx = 2\ln|x| + C$
$f(1) = 2 \Rightarrow 2\ln|1| + C = 2 \Rightarrow C = 2$
Thus, $f(x) = 2\ln|x| + 2$.

48. $f(x) = \int \frac{1}{3}\,dx = \frac{1}{3}x + C$
$f(6) = 3 \Rightarrow \frac{1}{3}(6) + C = 3 \Rightarrow C = 1$
Thus, $f(x) = \frac{1}{3}x + 1$.

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49. $\frac{d}{dx}\left(\frac{1}{x}+C\right)=-\frac{1}{x^2}\neq \ln x$

$\frac{d}{dx}(x\ln x - x + C)=(\ln x+1)-1=\ln x$

$\frac{d}{dx}\left(\frac{1}{2}(\ln x)^2+C\right)=\frac{\ln x}{x}\neq \ln x$

The answer is (b).

50. $\frac{d}{dx}\left(\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}+C\right)$

$=(x+1)^{3/2}-(x+1)^{1/2}$

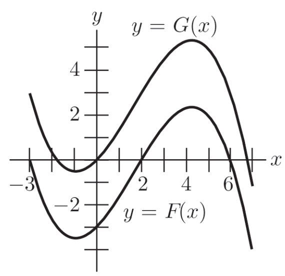
$=\sqrt{x+1}(x+1-1)=x\sqrt{x+1}$

$\frac{d}{dx}\left(\frac{1}{2}x^2\cdot\frac{2}{3}(x+1)^{3/2}+C\right)$

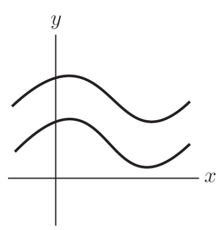
$=\frac{2}{3}x(x+1)^{3/2}+\frac{1}{2}x^2(x+1)^{1/2}\neq x\sqrt{x+1}$

The answer is (a).

51.



52.



53. $g(x) = f(x) + 3$

$g'(x)=f'(x)\Rightarrow g'(5)=f'(5)=\frac{1}{4}$

54. $h(x) = g(x) - f(x) = f(x) + 2 - f(x) = 2 \Rightarrow$
$h'(x)=0$

55. a. $\int(96-32t)dt=96t-16t^2+C$

The initial height is 256 feet, so $C = 256$.

Thus, $s(t)=-16t^2+96t+256$.

b. Setting $s(t) = 0$, $-16t^2+96t+256=0\Rightarrow$ $t^2-6t-16=0\Rightarrow$ $(t-8)(t+2) = 0$. The only solution that is sensible is $t = 8$ seconds.

c. Since $s'(t)=96-32t$, $s(t)$ has a maximum when $s'(t)=0\Rightarrow 96-32t=0\Rightarrow t=3$.

The ball will reach a maximum height of $s(3) = 400$ ft.

56. a. $\int -32t\ dt=-16t^2+C$

$s(0) = 400 = -16(0) + C \Rightarrow C = 400$

Thus, $s(t)=-16t^2+400$.

b. The rock hits the ground when

$s(t)=-16t^2+400=0\Rightarrow 16t^2=400\Rightarrow$

$t^2=25\Rightarrow t=5$ seconds.

c. $v(5) = -32(5) = -160$ ft/sec

57. $P(t)=\int\left(60+2t-\frac{1}{4}t^2\right)dt$

$=60t+t^2-\frac{1}{12}t^3+C$

$P(0)=0\Rightarrow 60\cdot 0+0^2-\frac{1}{12}\cdot 0^3+C=0\Rightarrow$

$C=0$

Thus, $P(t)=60t+t^2-\frac{1}{12}t^3$.

58. $P(t)=\int\left(40+2t-\frac{1}{5}t^2\right)dt$

$=40t+t^2-\frac{1}{15}t^3+C$

Assuming the output is 0 at time $t = 0$, we have

$P(0)=0\Rightarrow 40\cdot 0+0^2-\frac{1}{15}\cdot 0^3+C=0\Rightarrow$

$C=0$

Thus, $P(t)=40t+t^2-\frac{1}{15}t^3$.

59. $f(t)=\int 10e^{-0.4t}dt=-\frac{100}{4}e^{-0.4t}+C$

$=-25e^{-0.4t}+C$

$f(0)=-5\Rightarrow -25e^{-0.4\cdot 0}+C=-5\Rightarrow C=20$

Thus, $f(t)=-25e^{-0.4t}+20$ and the temperature at time t is $-25e^{-0.4t}+20$°C.

60. $P(t)=\int(120t-3t^2)\ dt=60t^2-t^3+C$

$P(0)=100\Rightarrow 60\cdot 0^2-0^3+C=100\Rightarrow C=100$

Thus, $P(t)=60t^2-t^3+100$.

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61. $P(x) = \int (1.30 + .06x - .0018x^2)\ dx$
$= 1.30x + .03x^2 - .0006x^3 + C$
$P(0) = -95 \Rightarrow$
$1.30 \cdot 0 + .03 \cdot 0^2 - .0006 \cdot 0^3 + C = -95 \Rightarrow$
$C = -95$
Thus, $P(x) = -.0006x^3 + .03x^2 + 1.30x - 95$.

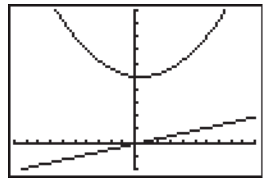
62. $C(x) = \int (.2x + 100)\ dx = .1x^2 + 100x + C$
$C(0) = 200 \Rightarrow .1 \cdot 0^2 + 100(0) + C = 200 \Rightarrow$
$C = 200$
Thus, $C(x) = .1x^2 + 100x + 200$.

63. $f(t) = \int 94e^{0.016t} dt = \dfrac{94}{.016} e^{0.016t} + C$
$= 5875e^{0.016t} + C$
Since consumption is reckoned from 1980, we have $f(0) = 0 = 5875(1) + C \Rightarrow C = -5875$.
Thus,
$f(t) = 5875e^{0.016t} - 5875 = 5875\left(e^{0.016t} - 1\right)$.

64. $T(t) = \int 17.04e^{0.016x} dt = \dfrac{17.04}{0.016} e^{0.016t}$
$= 1065e^{0.016t} + C$
Since consumption is reckoned from 1987, we have $f(0) = 0 = 1065e^{0.016t} + C \Rightarrow C = -1065$.
Thus,
$f(t) = 1065e^{0.016t} - 1065 = 1065\left(e^{0.016t} - 1\right)$

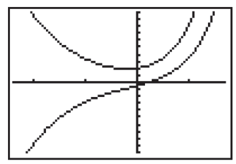
65. $\int C'(x)dx = C(x) = 1000x + 25x^2 + C_1$
$C(0) = C_1 =$ fixed cost $= 10,000 \Rightarrow$
$1000 \cdot 0 + 25 \cdot 0^2 + C_1 = 10,000 \Rightarrow C_1 = 10,000$
$C(x) = 25x^2 + 1000x + 10,000$

66. $F(x) = x^2 + 50e^{-0.02x}$



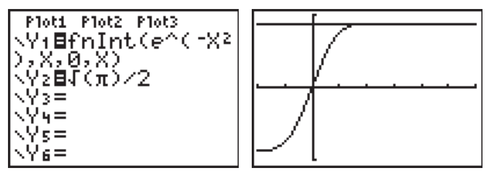
[−10, 10] by [−20, 100]

67. $F(x) = \dfrac{1}{2}e^{2x} - e^{-x} + \dfrac{1}{6}x^3$



[−2.4, 1.7] by [−10, 10]

68.



[−2, 6] by [−1, 1]

6.2 The Definite Integral and Net Change of a Function

1. $\int_0^1 \left(2x - \frac{3}{4}\right) dx = \left(x^2 - \frac{3}{4}x\right)\Big|_0^1 = \left(1^2 - \frac{3}{4}(1)\right) - \left(0^2 - \frac{3}{4}(0)\right) = \frac{1}{4}$

2. $\int_{-1}^2 \left(\frac{x^2}{3} - \frac{2}{9}x\right) dx = \left(\frac{x^3}{9} - \frac{1}{9}x^2\right)\Big|_{-1}^2 = \left[\left(\frac{2^3}{9} - \frac{1}{9}(2)^2\right) - \left(\frac{(-1)^3}{9} - \frac{1}{9}(-1)^2\right)\right] = \frac{4}{9} - \left(-\frac{2}{9}\right) = \frac{6}{9} = \frac{2}{3}$

3. $\int_1^4 \left(3\sqrt{t} + 4t\right) dt = \int_1^4 \left(3t^{1/2} + 4t\right) dt = \left(2t^{3/2} + 2t^2\right)\Big|_1^4 = \left(2(4)^{3/2} + 2(4)^2\right) - \left(2(1)^{3/2} + 2(1)^2\right) = 48 - 4 = 44$

4. $\int_1^9 \frac{1}{\sqrt{x}} dx = \int_1^9 x^{-1/2}\ dx = \left(2x^{1/2}\right)\Big|_1^9 = 2(9)^{1/2} - 2(1)^{1/2} = 6 - 2 = 4$

5. $\int_1^2 \left(-\frac{3}{x^2}\right) dx = \int_1^2 \left(-3x^{-2}\right) dx = \left(3x^{-1}\right)\Big|_1^2 = \left(\frac{3}{x}\right)\Big|_1^2 = \frac{3}{2} - 3 = -\frac{3}{2}$

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6. $\int_1^8 \left(-x+\sqrt[3]{x}\right)dx = \int_1^8 \left(-x+x^{1/3}\right)dx = \left(-\frac{x^2}{2}+\frac{3x^{4/3}}{4}\right)\Bigg|_1^8 = \left(-\frac{8^2}{2}+\frac{3(8)^{4/3}}{4}\right)-\left(-\frac{1^2}{2}+\frac{3(1)^{4/3}}{4}\right)$

$= -20-\frac{1}{4} = -\frac{81}{4}$

7. $\int_1^2 \left(\frac{5-2x^3}{x^6}\right)dx = \int_1^2 \left(\frac{5}{x^6}-\frac{2x^3}{x^6}\right)dx = \int_1^2 \left(5x^{-6}-2x^{-3}\right)dx = \left(-x^{-5}+x^{-2}\right)\Big|_1^2$

$= \left(-\frac{1}{2^5}+\frac{1}{2^2}\right)-\left(-\frac{1}{1^5}+\frac{1}{1^2}\right) = -\frac{1}{32}+\frac{1}{4} = \frac{7}{32}$

8. $\int_1^4 \left(\frac{x^2-\sqrt{x}}{x}\right)dx = \int_1^4 \left(\frac{x^2}{x}-\frac{x^{1/2}}{x}\right)dx = \int_1^4 \left(x-x^{-1/2}\right)dx = \left(\frac{x^2}{2}-2x^{1/2}\right)\Bigg|_1^4$

$= \left(\frac{4^2}{2}-2(4)^{1/2}\right)-\left(\frac{1^2}{2}-2(1)^{1/2}\right) = (8-4)-\left(\frac{1}{2}-2\right)$

$= 4-\left(-\frac{3}{2}\right) = \frac{11}{2}$

9. $\int_{-1}^0 \left(3e^{3t}+t\right)dt = \left(e^{3t}+\frac{t^2}{2}\right)\Bigg|_{-1}^0 = \left(e^{3(0)}+\frac{0^2}{2}\right)-\left(e^{3(-1)}+\frac{(-1)^2}{2}\right) = 1-\left(\frac{1}{e^3}+\frac{1}{2}\right) = \frac{1}{2}-\frac{1}{e^3}$

10. $\int_{-2}^2 \frac{2}{e^{2t}}\,dt = 2\int_{-2}^2 e^{-2t}dt = 2\left(-\frac{1}{2}e^{-2t}\right)\Bigg|_{-2}^2 = \left(-e^{-2t}\right)\Big|_{-2}^2 = -e^{-4}+e^4$

11. $\int_1^2 \frac{2}{x}\,dx = 2\ln|x|\Big|_1^2 = 2\ln 2 - 2\ln 1 = 2\ln 2 - 0 = \ln 2^2 = \ln 4$

12. $\int_{-2}^{-1} \left(\frac{1+x}{x}\right)dx = \int_{-2}^{-1}\left(\frac{1}{x}+1\right)dx = \left(\ln|x|+x\right)\Big|_{-2}^{-1} = \left(\ln|-1|-1\right)-\left(\ln|-2|-2\right) = -1-\ln 2+2 = 1-\ln 2$

13. $\int_0^1 \frac{e^x+e^{0.5x}}{e^{2x}}\,dx = \int_0^1 \left(\frac{e^x}{e^{2x}}+\frac{e^{0.5x}}{e^{2x}}\right)dx = \int_0^1 \left(e^{-x}+e^{-1.5x}\right)dx = \left(-e^{-x}-\frac{e^{-1.5x}}{1.5}\right)\Bigg|_0^1$

$= \left(-e^{-1}-\frac{e^{-1.5}}{1.5}\right)-\left(-e^0-\frac{e^0}{1.5}\right) = -\frac{1}{e}-\frac{e^{-1.5}}{1.5}-\left(-1-\frac{1}{1.5}\right)$

$= -\frac{1}{e}-\frac{2e^{-1.5}}{3}+\frac{5}{3}$

14. $\int_0^{\ln 2} \frac{e^x+e^{-x}}{2}\,dx = \int_0^{\ln 2}\left(\frac{e^x}{2}+\frac{e^{-x}}{2}\right)dx = \left(\frac{e^x}{2}-\frac{e^{-x}}{2}\right)\Bigg|_0^{\ln 2} = \left(\frac{e^{\ln 2}}{2}-\frac{e^{-\ln 2}}{2}\right)-\left(\frac{e^0}{2}-\frac{e^{-(0)}}{2}\right)$

$= \left(\frac{2}{2}-\frac{1}{2(2)}\right)-\left(\frac{1}{2}-\frac{1}{2}\right) = \frac{3}{4}$

15. $\int_0^4 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^4 f(x)\,dx = 3.5+5 = 8.5$

16. $\int_{-1}^{10} f(x)\,dx = \int_{-1}^{10} f(x)\,dx - \int_{-1}^{1} f(x)\,dx = 4-0 = 4$

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17. $\int_1^3 (2f(x)-3g(x))\,dx = \int_1^3 (2f(x))\,dx - \int_1^3 (3g(x))\,dx = 2\int_1^3 f(x)\,dx - 3\int_1^3 g(x)\,dx = 2(3)-3(-1)=9$

18. $\int_{-0.5}^3 (2g(x)+f(x))\,dx = \int_{-0.5}^3 (2g(x))\,dx + \int_{-0.5}^3 f(x)\,dx = 2\int_{-0.5}^3 g(x)\,dx + \int_{-0.5}^3 f(x)\,dx \Rightarrow$
$-4 = 2\int_{-0.5}^3 g(x)\,dx + 0 \Rightarrow \int_{-0.5}^3 g(x)\,dx = -2$

19.
$$\begin{aligned}
2\int_1^2\left(3x+\frac{1}{2}x^2-x^3\right)dx+3\int_1^2\left(x^2-2x+7\right)dx &= \int_1^2\left(6x+x^2-2x^3\right)dx+\int_1^2\left(3x^2-6x+21\right)dx\\
&= \int_1^2\left(6x+x^2-2x^3+3x^2-6x+21\right)dx\\
&= \int_1^2\left(-2x^3+4x^2+21\right)dx = \left(-\frac{1}{2}x^4+\frac{4}{3}x^3+21x\right)\Big|_1^2\\
&= \left(-\frac{1}{2}(2)^4+\frac{4}{3}(2)^3+21(2)\right)-\left(-\frac{1}{2}(1)^4+\frac{4}{3}(1)^3+21(1)\right)\\
&= \frac{134}{3}-\frac{131}{6}=\frac{137}{6}
\end{aligned}$$

20.
$$\begin{aligned}
\int_0^1(4x-2)\,dx+3\int_0^1(x-1)\,dx &= \int_0^1(4x-2)\,dx+\int_0^1(3x-3)\,dx\\
&= \int_0^1(4x-2+3x-3)\,dx = \int_0^1(7x-5)\,dx\\
&= \left(\frac{7x^2}{2}-5x\right)\Big|_0^1 = \left(\frac{7(1)^2}{2}-5(1)\right)-\left(\frac{7(0)^2}{2}-5(0)\right)\\
&= \frac{7}{2}-5=-\frac{3}{2}
\end{aligned}$$

21.
$$\begin{aligned}
\int_{-1}^0\left(x^3+x^2\right)dx+\int_0^1\left(x^3+x^2\right)dx &= \int_{-1}^1\left(x^3+x^2\right)dx = \left(\frac{x^4}{4}+\frac{x^3}{3}\right)\Big|_{-1}^1 = \left(\frac{1^4}{4}+\frac{1^3}{3}\right)-\left(\frac{(-1)^4}{4}+\frac{(-1)^3}{3}\right)\\
&= \frac{7}{12}+\frac{1}{12}=\frac{2}{3}
\end{aligned}$$

22.
$$\begin{aligned}
\int_0^1(7x+4)\,dx+\int_1^2(7x+5)\,dx &= \int_0^1 7x\,dx+\int_0^1 4\,dx+\int_1^2 7x\,dx+\int_1^2 5\,dx\\
&= \int_0^1 7x\,dx+\int_1^2 7x\,dx+\int_0^1 4\,dx+\int_1^2 5\,dx\\
&= \int_0^2 7x\,dx+\int_0^1 4\,dx+\int_1^2 5\,dx\\
&= \frac{7x^2}{2}\Big|_0^2+4x\Big|_0^1+5x\Big|_1^2 = \left(\frac{7(2)^2}{2}-\frac{7(0)^2}{2}\right)+\left(4(1)-4(0)\right)+\left(5(2)-5(1)\right)=23
\end{aligned}$$

23. $f(3)-f(1)=\int_1^3(-2x+3)\,dx=\left(-x^2+3x\right)\Big|_1^3=\left(-3^2+3(3)\right)-\left(-1^2+3(1)\right)=-2$

24. $f(4)-f(2)=\int_2^4 73\,dx = 73x\Big|_2^4 = 73(4)-73(2)=146$

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25. $f(1)-f(-1)=\int_{-1}^{1}\left(-.5t+e^{-2t}\right)dt=\left(-\frac{1}{4}t^2-\frac{1}{2}e^{-2t}\right)\Big|_{-1}^{1}=\left(-\frac{1}{4}(1)^2-\frac{1}{2}e^{-2(1)}\right)-\left(-\frac{1}{4}(-1)^2-\frac{1}{2}e^{-2(-1)}\right)$

$$=-\frac{1}{4}-\frac{e^{-2}}{2}-\left(-\frac{1}{4}-\frac{e^2}{2}\right)=\frac{e^2-e^{-2}}{2}$$

26. $f(3)-f(0)=\int_0^3\left(-12t-\frac{1}{e^t}\right)dt=\left(-6t^2+\frac{1}{e^t}\right)\Big|_0^3=\left(-6(3)^2+\frac{1}{e^3}\right)-\left(-6(0)^2+\frac{1}{e^0}\right)=-54+\frac{1}{e^3}-1=-55+\frac{1}{e^3}$

27. $\int_0^2 f(x)\,dx=\int_0^1 f(x)\,dx+\int_1^2 f(x)\,dx=\int_0^1 1\,dx+\int_1^2 x\,dx=x\Big|_0^1+\frac{x^2}{2}\Big|_1^2=1+2-\frac{1}{2}=\frac{5}{2}$

28. $\int_0^3 f(x)\,dx=\int_0^1 f(x)\,dx+\int_1^3 f(x)\,dx=\int_0^1\left(1-x^2\right)dx+\int_1^3\left((1-x)(x-3)\right)dx$

$$=\int_0^1\left(1-x^2\right)dx+\int_1^3\left(-x^2+4x-3\right)dx=\left(x-\frac{x^3}{3}\right)\Big|_0^1+\left(-\frac{x^3}{3}+2x^2-3x\right)\Big|_1^3$$

$$=\left[\left(1-\frac{1}{3}\right)-\left(0-\frac{0}{3}\right)\right]+\left[\left(-\frac{3^3}{3}+2(3)^2-3(3)\right)-\left(-\frac{1^3}{3}+2(1)^2-3(1)\right)\right]$$

$$=\frac{2}{3}+\left(0-\left(-\frac{4}{3}\right)\right)=2$$

29. $\int_{-1}^1 f(t)\,dt=\int_{-1}^0 f(t)\,dt+\int_0^1 f(t)\,dt=\int_{-1}^0(1+t)\,dt+\int_0^1(1-t)\,dt$

$$=\left(t+\frac{t^2}{2}\right)\Big|_{-1}^0+\left(t-\frac{t^2}{2}\right)\Big|_0^1=-\left(-1+\frac{1}{2}\right)+\left(1-\frac{1}{2}\right)=1$$

30. $\int_{-1}^2 f(t)\,dt=\int_{-1}^1 f(t)\,dt+\int_1^2 f(t)\,dt=\int_{-1}^1\left(t^2-1\right)dt+\int_1^2(t-1)\,dt$

$$=\left(\frac{t^3}{3}-t\right)\Big|_{-1}^1+\left(\frac{t^2}{2}-t\right)\Big|_1^2=\left[\left(\frac{1}{3}-1\right)-\left(-\frac{1}{3}+1\right)\right]+\left[\left(\frac{4}{2}-2\right)-\left(\frac{1}{2}-1\right)\right]=-\frac{4}{3}+\frac{1}{2}=-\frac{5}{6}$$

31. Let $s(t)$ represent the position function. We know that $s'(t)=v(t)=-32t,$ so the change in position is given by $s(4)-s(2)=\int_2^4(-32t)\,dt=\left(-16t^2\right)\Big|_2^4=-16(4)^2-\left(-16(2)^2\right)=-192.$
The rock fell 192 feet during the time interval $2\le t\le 4.$

32. a. Let $s(t)$ represent the position function. We know that $s'(t)=v(t)=-32t+75,$ so the change in position is given by

$$s(3)-s(0)=\int_0^3(-32t+75)\,dt=\left(-16t^2+75t\right)\Big|_0^3=\left(-16(3)^2+75(3)\right)-\left(-16(0)^2+75(0)\right)=81.$$

The ball rose 81 feet during the time interval $0\le t\le 3.$

b. $s(3)=s(0)+\left(s(3)-s(0)\right)=6+81=87$
The ball was 87 feet high at time $t=3.$

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33. a. Let $s(t)$ represent the position function. We know that $s'(t) = v(t) = -32t + 75,$ so the change in position is given by

$$s(3) - s(1) = \int_1^3 (-32t + 75)\,dt = \left(-16t^2 + 75t\right)\Big|_1^3 = \left(-16(3)^2 + 75(3)\right) - \left(-16(1)^2 + 75(1)\right) = 81 - 59 = 22.$$

b. During the time interval $1 \le t \le 3,$ the ball rose 22 feet. Therefore, at time $t = 3$, the ball is 22 feet higher than its position at time $t = 1$.

c. $s(5) - s(1) = \int_1^5 (-32t + 75)\,dt = \left(-16t^2 + 75t\right)\Big|_1^5 = \left(-16(5)^2 + 75(5)\right) - \left(-16(1)^2 + 75(1)\right) = -25 - 59 = -84$

During the time interval $1 \le t \le 5,$ the ball fell 84 feet. Therefore, at time $t = 5$, the ball is 84 feet lower than its position at time $t = 1$.

34. Let $s(t)$ represent the position function. We know that $s'(t) = v(t) = 45 - 45e^{0.2t},$ so the distance traveled during the first nine seconds is given by

$$\begin{aligned} s(9) - s(0) &= \int_0^9 \left(45 - 45e^{-0.2t}\right)dt = 45\int_0^9 \left(1 - e^{-t/5}\right)dt = 45\left[\left(t + 5e^{-t/5}\right)\Big|_0^9\right] = 45\left[\left(9 + 5e^{-9/5}\right) - \left(0 + 5e^{-0/5}\right)\right] \\ &= 45\left(9 + 5e^{-9/5} - 5\right) = 45\left(4 + 5e^{-9/5}\right) \approx 217.2 \end{aligned}$$

The skydiver fell about 217.2 feet during the first nine seconds.

35. a. Let $C(x)$ represent the cost function. The cost increase is given by

$$\begin{aligned} C(3) - C(1) &= \int_1^3 C'(x)\,dx = \int_1^3 \left(.1x^2 - x + 12\right)dx = \left(\frac{x^3}{30} - \frac{x^2}{2} + 12x\right)\Bigg|_1^3 \\ &= \left(\frac{3^3}{30} - \frac{3^2}{2} + 12(3)\right) - \left(\frac{1^3}{30} - \frac{1^2}{2} + 12(1)\right) = 32.4 - 11.53 = 20.87 \end{aligned}$$

The cost will increase \$20.87 if the company goes from a production level of 1 to 3 items per day.

b. $C(3) = C(1) + \left(C(3) - C(1)\right) = 15 + 20.87 = 35.87$

The cost of producing three items is \$35.87.

36. Let $C(x)$ represent the cost function. The cost increase is given by

$$\begin{aligned} C(20) - C(15) &= \int_{15}^{20} C'(x)\,dx = \int_{15}^{20} \left(32 + \frac{x}{20}\right)dx = \left(32x + \frac{x^2}{40}\right)\Bigg|_{15}^{20} = \left(32(20) + \frac{20^2}{40}\right) - \left(32(15) + \frac{15^2}{40}\right) \\ &= 650 - 485.625 = 164.375 \end{aligned}$$

The cost will increase \$164,375 if the company goes from a production level of 15 to 20 items per day.

37. Let $T(t)$ represent the value of the investment during a given time interval. Then $T'(t) = R(t),$ and the increase in value is given by

$$\begin{aligned} T(10) - T(0) &= \int_0^{10} T'(t)\,dt = \int_0^{10} R(t)\,dt = \int_0^{10} \left(700e^{0.07t} + 1000\right)dt \\ &= \left(\frac{700}{.07}e^{0.07t} + 1000t\right)\Bigg|_0^{10} = \left(10,000e^{0.07t} + 1000t\right)\Big|_0^{10} \\ &= \left(10,000e^{0.07(10)} + 1000(10)\right) - \left(10,000e^{0.07(0)} + 1000(0)\right) \\ &\approx 30137.50 - 10,000 = 20137.50 \end{aligned}$$

The investment increased by \$20,137.50.

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38. Let $T(t)$ represent the value of the property during a given time interval. Then $T'(t) = R(t)$, and the decrease in value from 2015 (t = 0) to 2021 (t = 6) is given by

$$T(6) - T(0) = \int_0^6 T'(t)\,dt = \int_0^6 R(t)\,dt = \int_0^6 \left(-8e^{-0.04t}\right)dt$$

$$= \left(\frac{8}{0.04}e^{-0.04t}\right)\Bigg|_0^6 = \left(200e^{-0.04t}\right)\Big|_0^6 = \left(200e^{-0.04(6)}\right) - \left(200e^{-0.04(0)}\right) = 157.326 - 200 = -42.674$$

The property decreased in value by $42,674.

39. a. $$P(10) - P(0) = \int_0^{10}\left(\frac{7}{300}e^{t/25} - \frac{1}{80}e^{t/16}\right) = \left(\frac{7}{12}e^{t/25} - \frac{1}{5}e^{t/16}\right)\Bigg|_0^{10}$$

$$= \left(\frac{7}{12}e^{10/25} - \frac{1}{5}e^{10/16}\right) - \left(\frac{7}{12}e^{0/25} - \frac{1}{5}e^{0/16}\right) = \left(\frac{7}{12}e^{10/25} - \frac{1}{5}e^{10/16}\right) - \left(\frac{7}{12} - \frac{1}{5}\right)$$

$$= \frac{1}{60}\left(35e^{2/5} - 12e^{5/8} - 23\right) \approx .11325$$

The population increased about .11325 million or 113,250 from 2000 to 2010.

b. $$P(40) - P(10) = \int_{10}^{40}\left(\frac{7}{300}e^{t/25} - \frac{1}{80}e^{t/16}\right) = \left(\frac{7}{12}e^{t/25} - \frac{1}{5}e^{t/16}\right)\Bigg|_{10}^{40}$$

$$= \left(\frac{7}{12}e^{40/25} - \frac{1}{5}e^{40/16}\right) - \left(\frac{7}{12}e^{10/25} - \frac{1}{5}e^{10/16}\right) = \left(\frac{7}{12}e^{8/5} - \frac{1}{5}e^{5/2}\right) - \left(\frac{7}{12}e^{2/5} - \frac{1}{5}e^{5/8}\right)$$

$$= \frac{7}{12}e^{2/5}\left(35e^{6/5} - 1\right) - \frac{1}{5}\left(e^{5/2} - e^{5/8}\right) \approx -.0438182$$

The population will decrease by about .043812 million or about 43,812 people due to emigration.

40. a. $$P(20) - P(0) = \int_0^{20} -4.1107e^{0.03t}\,dt = \left(-\frac{4.1107}{.03}e^{0.03t}\right)\Bigg|_0^{20} = -\frac{4.1107}{.03}e^{0.03(20)} + \frac{4.1107}{.03} \approx -112.649$$

In twenty years you will have paid $112,649 towards the loan.

b. $$P(20) = P(0) - \left(P(20) - P(0)\right) = 200{,}000 - 112{,}649 = 87{,}351$$

$87,351 is still remaining on the loan.

c. $$P(30) - P(0) = \int_0^{30} -4.1107e^{0.03t}\,dt = \left(-\frac{4.1107}{.03}e^{0.03t}\right)\Bigg|_0^{30} = -\frac{4.1107}{.03}e^{0.03(30)} + \frac{4.1107}{.03} \approx -200$$

Thus, the principal has been repaid.

41. $$P(t) = P(0) + \int_0^t -4.1107e^{0.03t}\,dt = 200 + \left(-\frac{4.1107}{.03}e^{0.03t}\right)\Bigg|_0^t = 200 - \frac{4.1107}{.03}e^{0.03t} + \frac{4.1107}{.03}$$

$$\approx 337.023 - 137.023e^{0.03t} \text{ thousand dollars}$$

42. Let $T(t)$ represent the amount of radioactive material in grams during a given time interval. Then $T'(t) = R(t)$, and the decrease in the amount of radioactive material in the first ten years is given by

$$T(10) - T(0) = \int_0^{10} T'(t)\,dt = \int_0^{10} R(t)\,dt = \int_0^{10}\left(-e^{-.1t}\right)dt$$

$$= \left(10e^{-.1t}\right)\Big|_0^{10} = \left(10e^{-.1(10)}\right) - \left(10e^{-.1(0)}\right) = \frac{10}{e} - 10 = -6.321$$

The radioactive material decayed by 6.321 grams during the first ten years.

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43. Let $T(t)$ represent the amount of salt in grams during a given time interval. Then $T'(t) = r(t)$, and the amount of salt that was eliminated during the first two minutes is given by

$$T(2) - T(0) = \int_0^2 T'(t)\,dt = \int_0^2 r(t)\,dt = \int_0^2 \left(-\left(t + \frac{1}{2}\right)\right)dt = \int_0^2 \left(-t - \frac{1}{2}\right)dt$$
$$= \left(-\frac{t^2}{2} - \frac{1}{2}t\right)\Bigg|_0^2 = \left(-\frac{2^2}{2} - \frac{1}{2}(2)\right) - \left(-\frac{0^2}{2} - \frac{1}{2}(0)\right) = -3$$

Three grams of salt were eliminated in the first two minutes.

44. Let $h(t)$ represent the depth of the water in the tank during a given time interval. The decrease in the depth of the water in the tank during the time interval $2 \le t \le 4$ is given by

$$h(4) - h(2) = \int_2^4 h'(t)\,dt = \int_2^4 \left(-\frac{t}{2}\right)dt = \left(-\frac{t^2}{4}\right)\Bigg|_2^4 = \left(-\frac{4^2}{4}\right) - \left(-\frac{2^2}{4}\right) = -3$$

The water level dropped by three inches.

6.3 The Definite Integral and Area Under a Graph

1. a. $A = lw = 3(2) = 6$

b. $A = \int_1^4 2\,dx = 2x\Big|_1^4 = 2(4) - 2(1) = 6$

2. a. $A = lw = 3(1) = 3$

b. $A = \int_{-1}^2 1\,dx = x\Big|_{-1}^2 = 2 - (-1) = 3$

3. a. $A = \frac{1}{2}bh = \frac{1}{2}(2)(2) = 2$

b. $A = \int_{-2}^0 (-x)\,dx = -\frac{x^2}{2}\Bigg|_{-2}^0 = 0 - (-2) = 2$

4. a. $A = \frac{1}{2}bh = \frac{1}{2}(4)(4) = 8$

b.
$$A = \int_{-2}^2 (x+2)\,dx = \left(\frac{x^2}{2} + 2x\right)\Bigg|_{-2}^2$$
$$= \left(\frac{2^2}{2} + 2(2)\right) - \left(\frac{(-2)^2}{2} + 2(-2)\right)$$
$$= 6 - (-2) = 8$$

5. a. $A = \frac{1}{2}b_1h_1 + \frac{1}{2}b_2h_2 = \frac{1}{2}(1)(1) + \frac{1}{2}(1)(1) = 1$

b.
$$A = \int_0^1 (1-x)\,dx + \int_1^2 (x-1)\,dx$$
$$= \left(x - \frac{x^2}{2}\right)\Bigg|_0^1 + \left(\frac{x^2}{2} - x\right)\Bigg|_1^2$$
$$= \frac{1}{2} + \left[\left(\frac{2^2}{2} - 2\right) - \left(\frac{1^2}{2} - 1\right)\right] = \frac{1}{2} + \frac{1}{2} = 1$$

6. a. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(2)(3+2) = 5$

b.
$$A = \int_0^2 2\,dx + \int_2^3 (6 - 2x)\,dx$$
$$= 2x\Big|_0^2 + \left(6x - x^2\right)\Big|_2^3$$
$$= 4 + \left[\left(6(3) - 3^2\right) - \left(6(2) - 2^2\right)\right]$$
$$= 4 + 1 = 5$$

7. $\int_1^2 \frac{1}{x}\,dx$

8. $\int_0^3 \left((-x)(x-3)\right)dx$

9. $\int_1^2 \ln x\,dx$

10. $\int_{-1}^2 e^{-x}\,dx$

11. $\int_1^3 \left(x + \frac{1}{x}\right)dx$

12. $\int_0^1 (x+1)\,dx + \int_1^2 (3-x)\,dx$

13. $\int_1^2 \frac{1}{x}\,dx = \ln|x|\Big|_1^2 = \ln 2 - \ln 1 = \ln 2$

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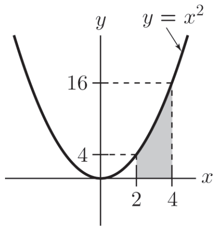
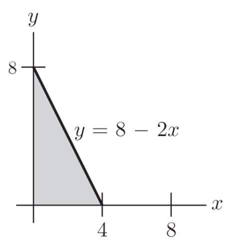
14. $\int_0^3 ((-x)(x-3))\,dx = \int_0^3 (-x^2+3x)\,dx$
$= \left(-\frac{x^3}{3}+\frac{3x^2}{2}\right)\Big|_0^3$
$= -\frac{3^3}{3}+\frac{3(3)^2}{2} = \frac{9}{2}$

15. $\int_1^3 \left(x+\frac{1}{x}\right)dx = \left(\frac{x^2}{2}+\ln|x|\right)\Big|_1^3$
$= \left(\frac{3^2}{2}+\ln|3|\right)-\left(\frac{1^2}{2}+\ln|1|\right)$
$= \left(\frac{9}{2}+\ln 3\right)-\frac{1}{2}- = 4+\ln 3$

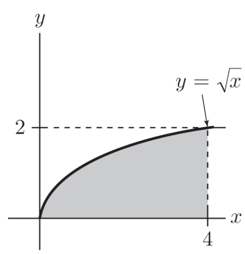
16.



17.



18.



19. $\int_2^3 4x\,dx = (2x^2)\Big|_2^3 = 2(9)-2(4)=10$

20. $\int_{-1}^1 3x^2\,dx = x^3\Big|_{-1}^1 = 1-(-1)=2$

21. $\int_0^1 (3x^2+x+2e^{x/2})\,dx$
$= \left(x^3+\frac{x^2}{2}+4e^{x/2}\right)\Big|_0^1$
$= \left(\frac{3}{2}+4e^{1/2}\right)-4 = -\frac{5}{2}+4e^{1/2}$

22. $\int_0^4 \sqrt{x}\,dx = \left(\frac{2x^{3/2}}{3}\right)\Big|_0^4 = \frac{2(4)^{3/2}}{3} = \frac{16}{3}$

23. $\int_1^4 (x-3)^4\,dx = \frac{(x-3)^5}{5}\Big|_1^4 = \frac{(4-3)^5}{5}-\frac{(1-3)^5}{5}$
$= \frac{1}{5}-\left(-\frac{32}{5}\right) = \frac{33}{5}$

24. $\int_{-1/3}^0 e^{3x}\,dx = \frac{e^{3x}}{3}\Big|_{-1/3}^0 = \frac{1}{3}-\frac{e^{-1}}{3} = \frac{1}{3}-\frac{1}{3e}$
$= \frac{e-1}{3e}$

25. $\int_0^b x^3\,dx = 4 \Rightarrow \frac{x^4}{4}\Big|_0^b = 4 \Rightarrow \frac{b^4}{4} = 4 \Rightarrow$
$b^4 = 16 \Rightarrow b = 2$

26. $\int_0^b x^2\,dx = \int_0^b x^3\,dx \Rightarrow \frac{x^3}{3}\Big|_0^b = \frac{x^4}{4}\Big|_0^b \Rightarrow$
$\frac{b^3}{3} = \frac{b^4}{4} \Rightarrow \frac{4}{3} = b$

27. $\Delta x = \frac{2-0}{4} = .5$
The first midpoint is that of [0, .5] which is .25, so the midpoints are .25, .75, 1.25, 1.75.

28. $\Delta x = \frac{3-0}{6} = .5$
The first midpoint is .25, so the midpoints are .25, .75, 1.25, 1.75, 2.25, 2.75.

29. $\Delta x = \frac{4-1}{5} = .6$
The first midpoint is that of [1, 1.6] which is 1.3, so the midpoints are 1.3, 1.9, 2.5, 3.1, 3.7.

30. $\Delta x = \frac{5-3}{5} = .4$
The first midpoint is that of [3, 3.4] which is 3.2, so the midpoints are 3.2, 3.6, 4, 4.4, 4.8.

31. $\Delta x = .5$
The midpoints are 1.25, 1.75, 2.25, 2.75.
Area
$= .5[f(1.25)+f(1.75)+f(2.25)+f(2.75)]$
$= .5\left[(1.25)^2+(1.75)^2+(2.25)^2+(2.75)^2\right]$
$= 8.625$

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32. $\Delta x = 1$; the midpoints are $-1.5, -.5, .5, 1.5$.
$\text{Area } = 1\left[(-1.5)^2 + (-.5)^2 + (.5)^2 + (1.5)^2\right] = 5$

33. $\Delta x = .4$; the left endpoints are 1, 1.4, 1.8, 2.2, 2.6.
$\text{Area} = .4\left[1^3 + (1.4)^3 + (1.8)^3 + (2.2)^3 + (2.6)^3\right] = 15.12$

34. $\Delta x = .2$; the right endpoints are .2, .4, .6, .8, 1.
$\text{Area} = .2\left[(.2)^3 + (.4)^3 + (.6)^3 + (.8)^3 + 1^3\right] = .36$

35. $\Delta x = .2$; the right endpoints are 2.2, 2.4, 2.6, 2.8, 3.
$\text{Area} = .2\left[e^{-2.2} + e^{-2.4} + e^{-2.6} + e^{-2.8} + e^{-3}\right] \approx .077278$

36. $\Delta x = .4$; the left endpoints are 2, 2.4, 2.8, 3.2, 3.6.
$\text{Area} = .4\left[\ln 2 + \ln 2.4 + \ln 2.8 + \ln 3.2 + \ln 3.6\right] \approx 2.0169$

37. midpoints: 1, 3, 5, 7; $\Delta x = 2$
$\left[f(1) + f(3) + f(5) + f(7)\right]\Delta x = \left[4 + 8 + 6 + 2\right]2 = 40$

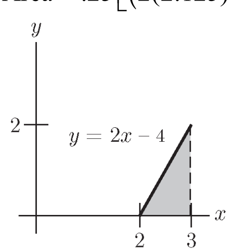
38. left endpoints: 3, 4, 5, 6; $\Delta x = 1$
$\left[f(3) + f(4) + f(5) + f(6)\right]\Delta x = \left[8 + 7 + 6 + 4\right]1 = 25$

39. right endpoints: 5, 6, 7, 8, 9; $\Delta x = 1$
$\left[f(5) + f(6) + f(7) + f(8) + f(9)\right]\Delta x = \left[6 + 4 + 2 + 1 + 2\right]1 = 15$

40. midpoints: 2, 4, 6; $\Delta x = 2$
$\left[f(2) + f(4) + f(6)\right]\Delta x = \left[7 + 7 + 4\right]2 = 36$

41. $\Delta x = .75$; the left endpoints are 1, 1.75, 2.5, 3.25.
$\text{Area} = .75\left[(4-1) + (4-1.75) + (4-2.5) + (4-3.25)\right] = 5.625$
The midpoints are 1.375, 2.125, 2.875, 3.625
$\text{Area} = .75\left[(4-1.375) + (4-2.125) + (4-2.875) + (4-3.625)\right] = 4.5$

42. $\Delta x = .25$; the right endpoints are 2.25, 2.5, 2.75, 3.
$\text{Area} = .25\left[\left(2(2.25)-4\right) + \left(2(2.5)-4\right) + \left(2(2.75)-4\right) + \left(2(3)-4\right)\right] = 1.25$
The midpoints are 2.125, 2.375, 2.625, 2.875.
$\text{Area} = .25\left[\left(2(2.125)-4\right) + \left(2(2.375)-4\right) + \left(2(2.625)-4\right) + \left(2(.875)-4\right)\right] = 1$



The base of the triangle is 1 and the height is 2, so $A = \frac{1}{2}bh = \frac{1}{2}\cdot 1\cdot 2 = 1$.

43. $\Delta x = .4$; the midpoints are $-.8, -.4, 0, .4, .8$.
$\text{Area} = 0.4\left[\left(1-(-.8)^2\right)^{1/2} + \left(1-(-.4)^2\right)^{1/2} + \left(1-(0)^2\right)^{1/2} + \left(1-(.4)^2\right)^{1/2} + \left(1-(.8)^2\right)^{1/2}\right] \approx 1.61321$
The error is $1.61321 - 1.57080 = .04241$.

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44. $\Delta x = .2$; the midpoints are .1, .3, .5, .7, .9.

Area = $.2\left[\sqrt{1-(.1)^2} + \sqrt{1-(.3)^2} + \sqrt{1-(.5)^2} + \sqrt{1-(.7)^2} + \sqrt{1-(.9)^2}\right] \approx .79300$

The error is .79300 – .78540 = .0076.

45. $A = 20(106) + 40(101) + 40(100) + 40(113) + 20(113) = 16{,}940 \text{ ft}^2$

46. First find the total area.

$A = 10(35+30+25+23+22+25+30+36+42) = 2680 \text{ ft}^2$

Therefore, the area of each lot must be 2680/2 = 1340 ft².

$10(35)+10(30)+10(25)+10(23)+10(22) = 1350,$ so build the fence 50 feet from the left side of the lot.

47. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

$n = 1: 1^2 = \dfrac{1(1+1)(2(1)+1)}{6} \Rightarrow 1 = \dfrac{6}{6}$

$n = 2: 1^2 + 2^2 = \dfrac{2(2+1)(2(2)+1)}{6} \Rightarrow 5 = \dfrac{30}{6}$

$n = 3: 1^2 + 2^2 + 3^2 = \dfrac{3(3+1)(2(3)+1)}{6} \Rightarrow 14 = \dfrac{84}{6}$

$n = 4: 1^2 + 2^2 + 3^2 + 4^2 = \dfrac{4(4+1)(2(4)+1)}{6} \Rightarrow 30 = \dfrac{180}{6}$

The formula can be proven for all values of n using mathematical induction.

48. $S_n = \left[f(x_1) + f(x_2) + \cdots + f(x_n)\right]\Delta x$

a. Since we are working with right endpoints, we have

$x_1 = 0 + \Delta x = 0 + \dfrac{1}{n} = \dfrac{1}{n},\ x_2 = x_1 + \dfrac{1}{n} = \dfrac{1}{n} + \dfrac{1}{n} = \dfrac{2}{n},\ x_3 = x_2 + \dfrac{1}{n} = \dfrac{2}{n} + \dfrac{1}{n} = \dfrac{3}{n},\ \ldots,$

$x_n = x_{n-1} + \dfrac{1}{n} = \dfrac{n-1}{n} + \dfrac{1}{n} = \dfrac{n}{n}.$

$f(x_1) = \left(\dfrac{1}{n}\right)^2,\ f(x_2) = \left(\dfrac{2}{n}\right)^2,\ f(x_3) = \left(\dfrac{3}{n}\right)^2,\ \ldots f(x_n) = \left(\dfrac{n}{n}\right)^2$

Substituting into the formula for S_n gives

$$S_n = \left(\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \left(\frac{3}{n}\right)^2 + \cdots + \left(\frac{n}{n}\right)^2\right)\frac{1}{n} = \left(\frac{1^2}{n^2} + \frac{2^2}{n^2} + \frac{3^2}{n^2} + \cdots + \frac{n^2}{n^2}\right)\frac{1}{n} = \frac{1}{n^3}\left(1^2 + 2^2 + 3^2 + \cdots + n^2\right)$$

b. Substituting the formula from exercise 47 gives

$$S_n = \frac{1}{n^3}\left(1^2 + 2^2 + 3^2 + \cdots + n^2\right) = \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) = \frac{n(n+1)(2n+1)}{6n^3}$$

c.
$$\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3} = \lim_{n\to\infty} \frac{2n^3 + n^2 + n}{6n^3} = \lim_{n\to\infty} \frac{2n^3}{6n^3} + \lim_{n\to\infty} \frac{n^2}{6n^3} + \lim_{n\to\infty} \frac{n}{6n^3}$$
$$= \lim_{n\to\infty} \frac{1}{3} + \lim_{n\to\infty} \frac{1}{6n} + \lim_{n\to\infty} \frac{1}{6n^2} = \frac{1}{3} + 0 + 0 = \frac{1}{3}$$

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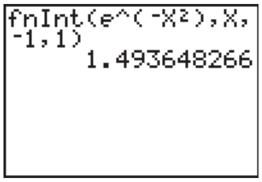
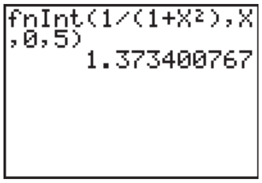
49. 

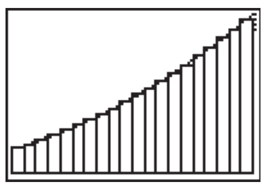
The area under the graph is about 1.494

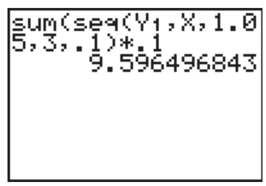
50. 

The area under the graph is about 1.373.

In exercises 51 and 52, the figures were created on a TI-84 Plus using the program RIEMANN.8xp downloaded from http://www.calcblog.com. Similar programs are available at www.ticalc.org.

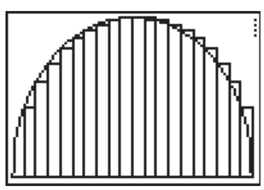
51. There are 20 intervals, so $\Delta x = \dfrac{2}{20} = .1.$ Since we are using the midpoints of the subintervals, $x_1 = 1.05.$ On the calculator, set $Y_1 = x\sqrt{1+x^2}.$ Use the **sum(** and **seq(** as shown to find the sum.

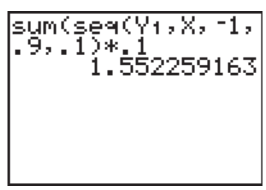




The area is approximately 9.60 square units.

52. There are 20 intervals, so $\Delta x = \dfrac{2}{20} = .1.$ Since we are using the left endpoints of the subintervals, $x_1 = -1$ and $x_{20} = .9.$ On the calculator, set $Y_1 = x\sqrt{1+x^2}.$ Use the **sum(** and **seq(** as shown to find the sum.



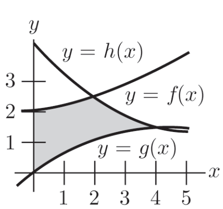


The area is approximately 1.55 square units.

6.4 Areas in the *xy*-Plane

1. $A = \int_1^2 f(x)\,dx + \int_3^4 [-f(x)]\,dx$

2. $A = \int_2^3 [f(x) - g(x)]\,dx$

3. 

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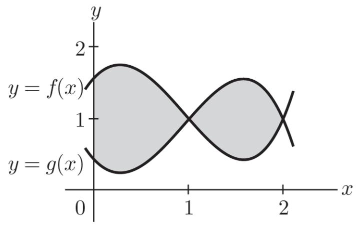
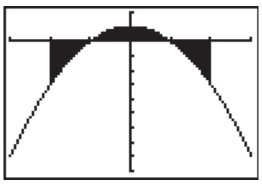
4.



5. $\int_0^7 f(x)\ dx$ is clearly positive since there is more area above the x-axis.

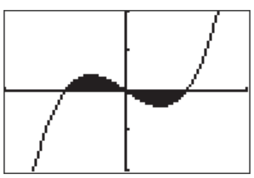
6. $\int_0^7 g(x)\ dx$ is clearly negative since there is more area below the x-axis.

7.



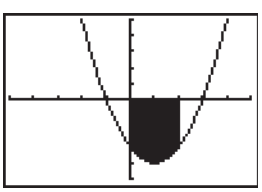
$$A = -\int_{-2}^{-1}\left(1-x^2\right)\ dx + \int_{-1}^{1}\left(1-x^2\right)\ dx - \int_{1}^{2}\left(1-x^2\right)\ dx = -\left(x-\frac{x^3}{3}\right)\Bigg|_{-2}^{-1} + \left(x-\frac{x^3}{3}\right)\Bigg|_{-1}^{1} - \left(x-\frac{x^3}{3}\right)\Bigg|_{1}^{2}$$
$$= -\left[\left(-1+\frac{1}{3}\right)-\left(-2+\frac{8}{3}\right)\right]+\left[\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)\right]-\left[\left(2-\frac{8}{3}\right)-\left(1-\frac{1}{3}\right)\right]$$
$$= -\left(-\frac{4}{3}\right)+\frac{4}{3}-\left(-\frac{4}{3}\right)=4$$

8.



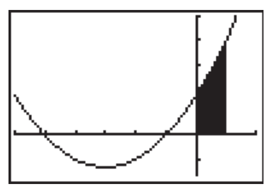
$$A = \int_{-1}^{0}\left(x\left(x^2-1\right)\right)\ dx - \int_{0}^{1}\left(x\left(x^2-1\right)\right)\ dx$$
$$= \int_{-1}^{0}\left(x^3-x\right)\ dx - \int_{0}^{1}\left(x^3-x\right)\ dx$$
$$= \left(\frac{x^4}{4}-\frac{x^2}{2}\right)\Bigg|_{-1}^{0} - \left(\frac{x^4}{4}-\frac{x^2}{2}\right)\Bigg|_{0}^{1}$$
$$= \left[0-\left(\frac{1}{4}-\frac{1}{2}\right)\right]-\left[\left(\frac{1}{4}-\frac{1}{2}\right)-0\right]=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

9.



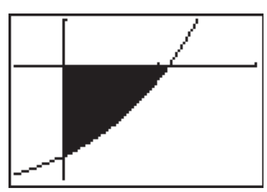
$$A = -\int_0^2\left(x^2-2x-3\right)\ dx = -\left(\frac{x^3}{3}-x^2-3x\right)\Bigg|_0^2$$
$$= -\left(\frac{8}{3}-4-6\right)-0=\frac{22}{3}$$

10.



$$A = \int_0^1\left(x^2+6x+5\right)\ dx = \left(\frac{x^3}{3}+3x^2+5x\right)\Bigg|_0^1$$
$$= \left(\frac{1}{3}+3+5\right)-0=\frac{25}{3}$$

11.



$$A = -\int_0^{\ln 3}\left(e^x-3\right)\ dx = -\left(\left(e^x-3x\right)\Big|_0^{\ln 3}\right)$$
$$= -\left[\left(e^{\ln 3}-3\ln 3\right)-1\right]$$
$$= -\left(3-3\ln 3-1\right)=3\ln 3-2$$

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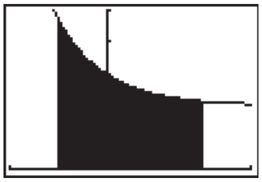
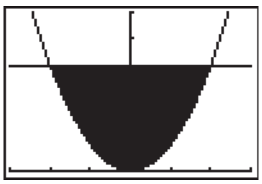
12.



$$A = \int_{-1}^{2}\left(e^{-x}+2\right)\,dx = \left(-e^{-x}+2x\right)\Big|_{-1}^{2}$$
$$= \left(-e^{-2}+4\right)-\left(-e^{1}-2\right) = 6+e-\frac{1}{e^2} \approx 8.6$$

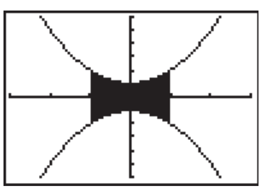
13.



Since $y = 8$ lies above $y = 2x^2$ on [–2, 2], we calculate

$$\int_{-2}^{2}(8-2x^2)\,dx = \left(8x-\frac{2}{3}x^3\right)\Big|_{-2}^{2}$$
$$= 16-\frac{16}{3}-\left(-16+\frac{16}{3}\right)$$
$$= 32-\frac{32}{3} = \frac{96}{3}-\frac{32}{3} = \frac{64}{3}$$

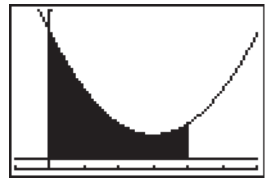
14.



Since $y = x^2+1$ lies above $y = -x^2-1$ on [–1, 1], we calculate

$$\int_{-1}^{1}\left(\left(x^2+1\right)-\left(-x^2-1\right)\right)dx$$
$$= \int_{-1}^{1}\left(2x^2+2\right)dx = \left(\frac{2}{3}x^3+2x\right)\Big|_{-1}^{1}$$
$$= \left(\frac{2}{3}+2\right)-\left(-\frac{2}{3}-2\right) = \frac{16}{3}$$

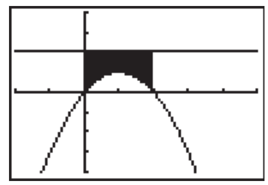
15.



Since $y = x^2-6x+12$ lies above $y = 1$ on [0, 4], we calculate

$$\int_{0}^{4}\left((x^2-6x+12)-1\right)dx = \left(\frac{1}{3}x^3-3x^2+11x\right)\Big|_{0}^{4}$$
$$= \frac{64}{3}-48+44-0 = \frac{52}{3}$$

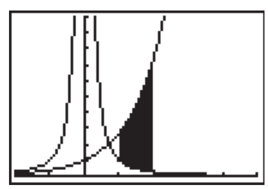
16.



Since $y = 2$ lies above $y = x(2-x)$ on [0, 2], we calculate

$$\int_{0}^{2}[2-x(2-x)]\,dx = \int_{0}^{2}(x^2-2x+2)\,dx$$
$$= \left(\frac{1}{3}x^3-x^2+2x\right)\Big|_{0}^{2}$$
$$= \left(\frac{8}{3}-4+4\right)-0 = \frac{8}{3}$$

17.



Since $y = e^x$ lies above $y = \frac{1}{x^2}$ on [1, 2], we calculate

$$\int_{1}^{2}\left(e^x-\frac{1}{x^2}\right)dx = \int_{1}^{2}\left(e^x-x^{-2}\right)dx$$
$$= \left(e^x+\frac{1}{x}\right)\Big|_{1}^{2}$$
$$= \left(e^2+\frac{1}{2}\right)-(e+1) = e^2-e-\frac{1}{2}$$

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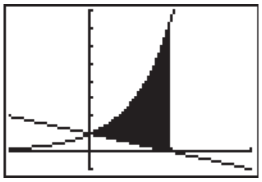
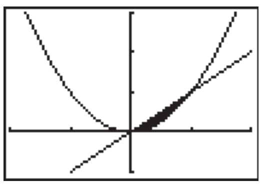
18.



Since $y = e^{2x}$ lies above $y = 1 - x$ on [0, 1], we calculate

$$\int_0^1 [e^{2x} - (1-x)]\,dx = \int_0^1 \left(e^{2x} - 1 + x\right)dx$$
$$= \left(\frac{1}{2}e^{2x} - x + \frac{1}{2}x^2\right)\Bigg|_0^1$$
$$= \left(\frac{1}{2}e^2 - 1 + \frac{1}{2}\right) - \frac{1}{2}$$
$$= \frac{1}{2}e^2 - 1$$

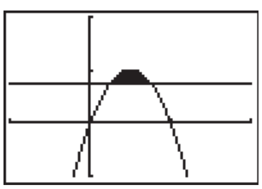
19.



To find the points of intersection, solve
$x^2 = x \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow$.
$x = 0$ or $x = 1$
Thus, we want to integrate from $x = 0$ to $x = 1$ with $y = x$ above $y = x^2$, so $\int_0^1 \left(x - x^2\right)dx$.

$$\int_0^1 \left(x - x^2\right)dx = \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Bigg|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

20.



Solve $4x(1-x) = \frac{3}{4} \Rightarrow 4x^2 - 4x + \frac{3}{4} = 0 \Rightarrow$

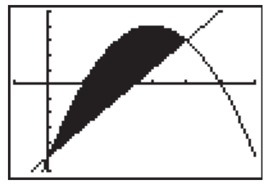
$$x = \frac{4 \pm \sqrt{16 - 4(4)\left(\frac{3}{4}\right)}}{8} = \frac{4 \pm 8}{8} \Rightarrow$$

$x = \frac{1}{4}$ or $x = \frac{3}{4}$

Thus, the interval is $\left[\frac{1}{4}, \frac{3}{4}\right]$ and since $y = 4x(1-x)$ lies above $y = \frac{3}{4}$, we integrate $\int_{1/4}^{3/4}\left(4x(1-x) - \frac{3}{4}\right)dx$.

$$\int_{1/4}^{3/4}\left(4x(1-x) - \frac{3}{4}\right)dx$$
$$= \int_{1/4}^{3/4}\left(4x - 4x^2 - \frac{3}{4}\right)dx$$
$$= \left(2x^2 - \frac{4}{3}x^3 - \frac{3}{4}x\right)\Bigg|_{1/4}^{3/4}$$
$$= \left(\frac{18}{16} - \frac{9}{16} - \frac{9}{16}\right) - \left(\frac{2}{16} - \frac{1}{48} - \frac{3}{16}\right) = \frac{1}{12}$$

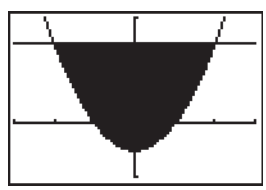
21.



Solve:
$2x - 5 = -x^2 + 6x - 5 \Rightarrow$
$2x - 5 - (-x^2 + 6x - 5) = 0 \Rightarrow x^2 - 4x = 0 \Rightarrow$ Th
$x(x-4) = 0 \Rightarrow x = 0$ or $x = 4$
us, we should integrate over [0, 4] and since $y = -x^2 + 6x - 5$ lies above $y = 2x - 5$ on [0, 4], we have $\int_0^4 \left[-x^2 + 6x - 5 - (2x-5)\right]dx$.

$$\int_0^4 \left[-x^2 + 6x - 5 - (2x-5)\right]dx$$
$$= \int_0^4 (-x^2 + 4x)\,dx = \left(-\frac{1}{3}x^3 + 2x^2\right)\Bigg|_0^4$$
$$= -\frac{64}{3} + 32 = \frac{32}{3}$$

22.



Solve $x^2 - 1 = 3 \Rightarrow x^2 - 4 = 0 \Rightarrow$
$(x+2)(x-2) = 0 \Rightarrow x = -2$ or $x = 2$
Thus, we should integrate over [–2, 2] and since $y = 3$ lies above $y = x^2 - 1$ on [–2, 2], we have

$$\int_{-2}^2 [3 - (x^2 - 1)]\,dx = \int_{-2}^2 [3 - x^2 + 1]\,dx$$
$$= \int_{-2}^2 \left(4 - x^2\right)dx$$
$$= \left(4x - \frac{1}{3}x^3\right)\Bigg|_{-2}^2$$
$$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) = \frac{32}{3}$$

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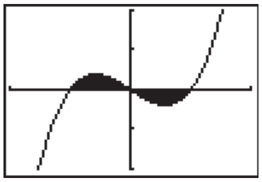
23.



Solve $x(x^2-1)=0 \Rightarrow x(x-1)(x+1)=0 \Rightarrow$
$x=0$ or $x=1$ or $x=-1$
We should integrate over [–1, 0] and [0, 1].
Since $y=x(x^2-1)$ lies above the x-axis on [–1, 0], we have

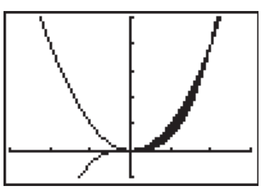
$$\int_{-1}^{0}\left[x(x^2-1)-0\right]dx=\int_{-1}^{0}\left(x^3-x\right)dx$$
$$=\left(\frac{1}{4}x^4-\frac{1}{2}x^2\right)\Bigg|_{-1}^{0}$$
$$=0-\left(\frac{1}{4}-\frac{1}{2}\right)=\frac{1}{4}$$

The x-axis lies above $y=x(x^2-1)$ on [0, 1], so we have

$$\int_{0}^{1}\left[0-x(x^2-1)\right]dx=\int_{0}^{1}\left(x-x^3\right)dx$$
$$=\left(\frac{1}{2}x^2-\frac{1}{4}x^4\right)\Bigg|_{0}^{1}$$
$$=\left(\frac{1}{2}-\frac{1}{4}\right)=\frac{1}{4}$$

Thus, the total area bounded by these curves is $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$.

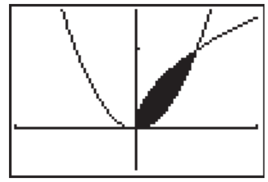
24.



$x^3=2x^2 \Rightarrow x^3-2x^2=0 \Rightarrow x^2(x-2)=0 \Rightarrow$
$x=0$ or $x=2$. Thus, we should integrate over [0, 2] and since $y=2x^2$ lies above $y=x^3$ on [0, 2], we have

$$\int_{0}^{2}(2x^2-x^3)\,dx=\left(\frac{2}{3}x^3-\frac{1}{4}x^4\right)\Bigg|_{0}^{2}$$
$$=\left(\frac{16}{3}\right)-\left(\frac{16}{4}\right)=\frac{4}{3}$$

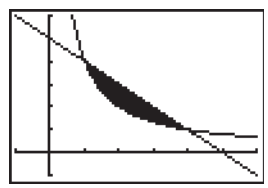
25.



$8x^2=\sqrt{x} \Rightarrow 64x^4-x=0 \Rightarrow$
$x(4x-1)(16x^2+4x+1)=0 \Rightarrow x=0$ or $x=\frac{1}{4}$
(Note that there is no real solution for $16x^2+4x+1=0$.) Thus, we should integrate over $\left[0,\frac{1}{4}\right]$ and since $y=\sqrt{x}$ lies above $y=8x^2$ on $\left[0,\frac{1}{4}\right]$, we have

$$\int_{0}^{1/4}\left(\sqrt{x}-8x^2\right)dx=\int_{0}^{1/4}\left(x^{1/2}-8x^2\right)dx$$
$$=\left(\frac{2}{3}x^{3/2}-\frac{8}{3}x^3\right)\Bigg|_{0}^{1/4}$$
$$=\frac{2}{3}\left(\frac{1}{4}\right)^{3/2}-\frac{8}{3}\left(\frac{1}{4}\right)^3$$
$$=\frac{1}{12}-\frac{1}{24}=\frac{1}{24}$$

26.



Solve
$\frac{4}{x}=5-x \Rightarrow 4=5x-x^2 \Rightarrow x^2-5x+4=0 \Rightarrow$
$(x-1)(x-4)=0 \Rightarrow x=1$ or $x=4$. Thus, we should integrate over $[1,4]$ and since $y=5-x$ lies above $y=\frac{4}{x}$ on $[1,4]$, we have

$$\int_{1}^{4}\left(5-x-\frac{4}{x}\right)dx=\left(5x-\frac{1}{2}x^2-4\ln x\right)\Bigg|_{1}^{4}$$
$$=(20-8-4\ln 4)-\left(5-\frac{1}{2}\right)$$
$$=\frac{15}{2}-4\ln 4$$

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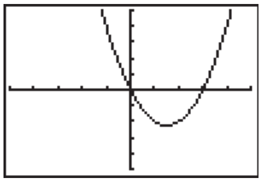
27.



First solve $x^2 - 3x = 0$ to find any x-intercepts. $x(x-3) = 0$ so the x-intercepts are at $x = 0, 3$.

a. $y = x^2 - 3x$ lies under the x-axis on the interval, so calculate $\int_0^3 [0-(x^2-3x)]\,dx$

$$= \int_0^3 (-x^2+3x)\,dx = \left(-\frac{1}{3}x^3 + \frac{3}{2}x^2\right)\Big|_0^3$$
$$= -\frac{27}{3} + \frac{27}{2} = \frac{27}{6} = \frac{9}{2}$$

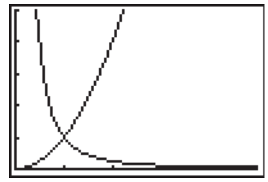
b. On [3, 4] $y = x^2 - 3x$ lies above the x-axis so, using results from part (a), we wish to calculate

$$\int_0^3 \left[-(x^2-3x)\right]dx + \int_3^4 (x^2-3x)\,dx$$
$$= \frac{9}{2} + \left(\frac{1}{3}x^3 - \frac{3}{2}x^2\right)\Big|_3^4$$
$$= \frac{9}{2} + \left(\frac{64}{3} - 24 - \left(\frac{27}{3} - \frac{27}{2}\right)\right)$$
$$= \frac{9}{2} + \left(\frac{64}{3} - 24 + \frac{9}{2}\right) = \frac{19}{3}$$

c. On [–2, 0], $y = x^2 - 3x$ lies above the x-axis so, using results from (a) and (b)

$$\int_{-2}^0 (x^2-3x)\,dx + \int_0^3 -(x^2-3x)\,dx$$
$$= \left(\frac{1}{3}x^3 - \frac{3}{2}x^2\right)\Big|_{-2}^0 + \frac{9}{2}$$
$$= -\left(-\frac{8}{3} - \frac{12}{2}\right) + \frac{9}{2}$$
$$= \frac{26}{3} + \frac{9}{2} = \frac{79}{6}$$

28.



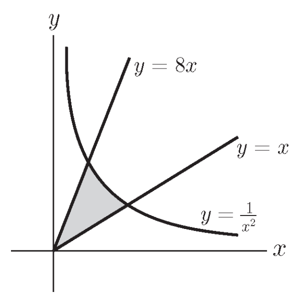
a. On [1, 4], $y = \frac{1}{x^2}$ lies below $y = x^2$ so

$$\int_1^4 \left(x^2 - \frac{1}{x^2}\right)dx = \left(\frac{1}{3}x^3 + \frac{1}{x}\right)\Big|_1^4$$
$$= \frac{64}{3} + \frac{1}{4} - \left(\frac{1}{3} + 1\right)$$
$$= 21 - \frac{3}{4} = \frac{81}{4}$$

b. On $\left[\frac{1}{2}, 1\right]$, $y = \frac{1}{x^2}$ lies above $y = x^2$, so

$$\int_{1/2}^1 \left(\frac{1}{x^2} - x^2\right)dx + \int_1^4 \left(x^2 - \frac{1}{x^2}\right)dx$$
$$= \left(-\frac{1}{x} - \frac{1}{3}x^3\right)\Big|_{1/2}^1 + \frac{81}{4}$$
$$= -1 - \frac{1}{3} - \left(-2 - \frac{1}{24}\right) + \frac{81}{4} = \frac{503}{24}$$

29.



First solve: $\frac{1}{x^2} = 8x$. This has a solution at $x = \frac{1}{2}$. Next solve: $\frac{1}{x^2} = x$. This has a solution at $x = 1$. Thus, the area should be calculated by the following sum of integrals:

$$\int_0^{1/2} (8x - x)\,dx + \int_{1/2}^1 \left(\frac{1}{x^2} - x\right)dx$$
$$= \left(4x^2 - \frac{1}{2}x^2\right)\Big|_0^{1/2} + \left(-\frac{1}{x} - \frac{1}{2}x^2\right)\Big|_{1/2}^1$$
$$= 1 - \frac{1}{8} + \left(-1 - \frac{1}{2} - \left(-2 - \frac{1}{8}\right)\right) = \frac{3}{2}$$

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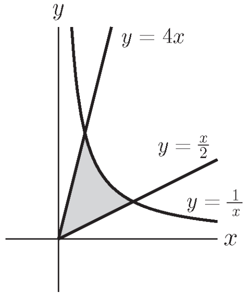
30.

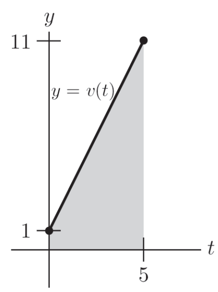


First solve: $4x = \frac{1}{x}$. This has a solution at $x = \frac{1}{2}$. Next solve: $\frac{x}{2} = \frac{1}{x}$. This has a solution at $x = \sqrt{2}$. Thus, the area should be calculated by $\int_0^{1/2}\left(4x - \frac{x}{2}\right) dx + \int_{1/2}^{\sqrt{2}}\left(\frac{1}{x} - \frac{x}{2}\right) dx$

$$= \left(2x^2 - \frac{1}{4}x^2\right)\Bigg|_0^{1/2} + \left(\ln x - \frac{1}{4}x^2\right)\Bigg|_{1/2}^{\sqrt{2}}$$
$$= \frac{1}{2} - \frac{1}{16} + \ln 2^{1/2} - \frac{1}{2} - \left(\ln\frac{1}{2} - \frac{1}{16}\right)$$
$$= \frac{1}{2}\ln 2 - (\ln 1 - \ln 2) = \frac{3}{2}\ln 2$$

31. a. $\int_0^5 (2t+1)\ dt = \left(t^2 + t\right)\Big|_0^5 = 30$ ft

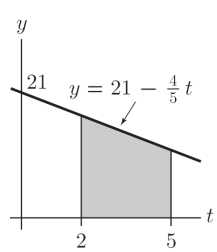
b.



32. a. $\int_2^5\left(21 - \frac{4}{5}t\right)dt = \left(21t - \frac{2}{5}t^2\right)\Bigg|_2^5$

$$= \left((105-10) - \left(42 - \frac{8}{5}\right)\right)$$
$$= \frac{273}{5} \approx 54 \text{ mowers}$$

b.



33. a. $\int_2^8\left(\frac{3}{32}x^2 - x + 200\right)dx$

$$= \left(\frac{1}{32}x^3 - \frac{1}{2}x^2 + 200x\right)\Bigg|_2^8$$
$$= 16 - 32 + 1600 - \left(\frac{1}{4} - 2 + 400\right)$$
$$= \$1185.75$$

b. It is the area under the marginal cost curve from $x = 2$ to $x = 8$.

34. a. $\int_5^8 (100 + 50x - 3x^2)\ dx$

$$= (100x + 25x^2 - x^3)\Big|_5^8$$
$$= (800 + 1600 - 512) - (500 + 625 - 125)$$
$$= \$888$$

b. It is the area under the marginal profit function from $x = 5$ to $x = 8$.

35. $\int_{44}^{48} M(x)dx$ represents the increase in profits resulting from increasing the production level from 44 to 48 units.

36. $\int_0^{100} M(x)dx$ represents the total variable costs of producing 100 units of goods.

37. a. $\int_0^2\left(12 + \frac{4}{(t+3)^2}\right) dt = \left(12t - \frac{4}{t+3}\right)\Bigg|_0^2$

$$= 24 - \frac{4}{5} - \left(-\frac{4}{3}\right)$$
$$= \frac{360}{15} - \frac{12}{15} + \frac{20}{15}$$
$$= \frac{368}{15} \approx 24.5$$

b. The area represents the amount the temperature falls during the first 2 hours.

38. a. $\int_1^9\left(40 + \frac{8}{(t+1)^2}\right) dt = \left(40t - \frac{8}{t+1}\right)\Bigg|_1^9$

$$= 360 - \frac{8}{10} - (40 - 4)$$
$$= \frac{3232}{10} = 323.2$$

b. The area represents the distance traveled during the time from $t = 1$ to $t = 9$.

39. $\int_0^{20} 76.2e^{0.03t}dt = 2540e^{0.03t}\Big|_0^{20}$

$$\approx 2088 \text{ million cubic meters}$$

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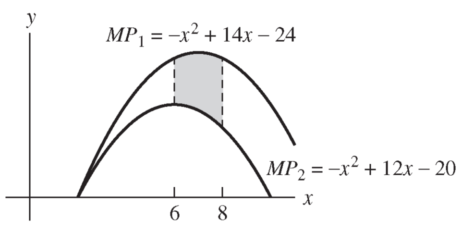
40. From exercise 39, we have $c(t) = 76.2e^{0.03t}$. We are given $g(t) = 50 - 6.03e^{0.09t}$. Thus,

$$\int_0^{20}\left[76.2e^{0.03t} - \left(50 - 6.03e^{0.09t}\right)\right]dt$$
$$= \int_0^{20}\left(76.2e^{0.03t} - 50 + 6.03e^{0.09t}\right)dt.$$

41. $$\int_5^{10} M_2(x)\,dx - \int_5^{10} M_1(x)\,dx$$
$$= \int_5^{10}\left(M_2(x) - M_1(x)\right)dx$$
$$= \int_5^{10}\left(\left(2x^2 - 2.4x + 8\right) - \left(2x^2 - 3x + 11\right)\right)dx$$
$$= \int_5^{10}(.6x - 3)\,dx = \left.\left(\frac{.6x^2}{2} - 3x\right)\right|_5^{10}$$
$$= \left.\left(.3x^2 - 3x\right)\right|_5^{10}$$
$$= \left(.3(10)^2 - 3(10)\right) - \left(.3(5)^2 - 3(5)\right)$$
$$= 0 - (-7.5) = 7.5$$

The net change in profit is \$7.5 thousand or \$7500.

42.

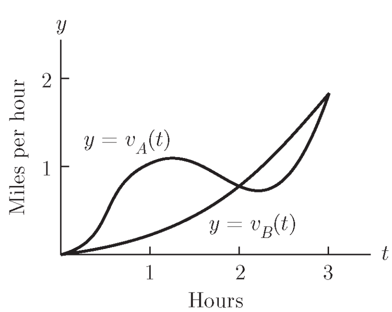


$$\int_6^8\left[(-x^2 + 14x - 24) - (-x^2 + 12x - 20)\right]dx$$
$$= \int_6^8 (2x - 4)\,dx = \left.(x^2 - 4x)\right|_6^8 = 20$$

The company should *not* adopt the new plan. The area is 20, and it represents the additional profit from using the original plan.

43. *A* is the difference between the two heights after 10 seconds.

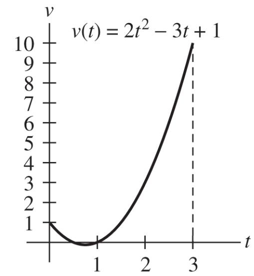
44.



a. The area between the two curves from $t = 0$ to $t = 1$ represents the distance between the two cars after 1 hour.

b. The distance between the cars will be the greatest after 2 hours.

45. a.



Let $s(t)$ represent the position of the object at time *t*, measured from its initial position. The required displacement is $s(3) - s(0)$, the net change in position over the interval $0 \le t \le 3$.

$$s(3) - s(0) = \int_0^3\left(2t^2 - 3t + 1\right)dt$$
$$= \left.\left(\frac{2t^3}{3} - \frac{3t^2}{2} + t\right)\right|_0^3$$
$$= \frac{2(3)^3}{3} - \frac{3(3)^2}{2} + 3 = \frac{15}{2}$$

The object is displaced 7.5 feet higher after three seconds than it was at the start.

b. Note that $v(t) \le 0$ for $\frac{1}{2} \le t \le 1$. So, the object changes direction at time $t = \frac{1}{2}$, so we must compute the net displacements on the three intervals $\left[0, \frac{1}{2}\right]$, $\left[\frac{1}{2}, 1\right]$, and $[1, 3]$. On the interval $\left[0, \frac{1}{2}\right]$ the net displacement is

$$\int_0^{1/2}\left(2t^2 - 3t + 1\right)dt = \left.\left(\frac{2t^3}{3} - \frac{3t^2}{2} + t\right)\right|_0^{1/2}$$
$$= \frac{2\left(\frac{1}{2}\right)^3}{3} - \frac{3\left(\frac{1}{2}\right)^2}{2} + \frac{1}{2}$$
$$= \frac{5}{24}$$

The object moved $\frac{5}{24}$ ft upward during this time interval.

(*continued on next page*)

(*continued*)

On the interval $\left[\frac{1}{2}, 1\right]$, we have

$$\int_{1/2}^{1}\left(2t^2-3t+1\right)dt$$
$$=\left(\frac{2t^3}{3}-\frac{3t^2}{2}+t\right)\Bigg|_{1/2}^{1}$$
$$=\left(\frac{2(1)^3}{3}-\frac{3(1)^2}{2}+1\right)-\frac{5}{24}=-\frac{1}{24}$$

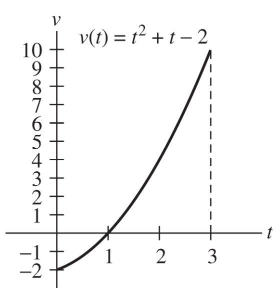
The object moved $\frac{1}{24}$ ft downward during this time interval.

On the interval $[1, 3]$, we have

$$\int_{1}^{3}\left(2t^2-3t+1\right)dt$$
$$=\left(\frac{2t^3}{3}-\frac{3t^2}{3}+t\right)\Bigg|_{1}^{3}$$
$$=\left(\frac{2(3)^3}{3}-\frac{3(3)^2}{2}+3\right)-\frac{1}{6}=\frac{22}{3}$$

The object moved $\frac{22}{3}$ ft upward during this time interval. Thus, the total distance traveled was $\frac{5}{24}+\frac{1}{24}+\frac{22}{3}=\frac{91}{12}\approx 7.583$ ft.

46. a.



Let $s(t)$ represent the position of the object at time t, measured from its initial position. The required displacement is $s(3)-s(0)$, the net change in position over the interval $0 \le t \le 3$.

$$s(3)-s(0)=\int_0^3\left(t^2+t-2\right)dt$$
$$=\left(\frac{t^3}{3}+\frac{t^2}{2}-2t\right)\Bigg|_0^3$$
$$=\frac{3^3}{3}+\frac{3^2}{2}-2(3)=\frac{15}{2}$$

The object is 7.5 feet higher after three seconds than it was at the start. The value of the definite integral over the interval $[0, 3]$ is equal to the area above the t-axis bounded by the graph of $v(t)$ minus the area below the t-axis bounded by the graph of $v(t)$.

b. Note that $v(t) \le 0$ for $[0, 1]$, so we must compute the net displacements on the two intervals $[0, 1]$, and $[1, 3]$. On the interval $[0, 1]$ the net displacement is

$$\int_0^1\left(t^2+t-2\right)dt=\left(\frac{t^3}{3}+\frac{t^2}{2}-2t\right)\Bigg|_0^1$$
$$=\left(\frac{1}{3}+\frac{1}{2}-2\right)-0=-\frac{7}{6}$$

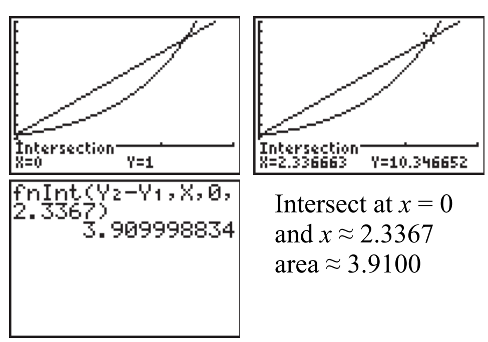
The object moved $\frac{7}{6}$ ft downward during this time interval.

On the interval $[1, 3]$, we have

$$\int_1^3\left(t^2+t-2\right)dt$$
$$=\left(\frac{t^3}{3}+\frac{t^2}{2}-2t\right)\Bigg|_1^3$$
$$=\left(\frac{3^3}{3}+\frac{3^2}{2}-2(3)\right)-\left(-\frac{7}{6}\right)=\frac{26}{3}$$

The object moved $\frac{26}{3}$ ft during this interval. Thus, the total distance traveled was $\frac{7}{6}+\frac{26}{3}=\frac{59}{6}\approx 9.83$ ft. In terms of area, the total distance traveled is equal to the area below the t-axis bounded by the graph of $v(t)$ from 0 to 1 plus the area above the t-axis bounded by the graph of $v(t)$ from 1 to 3.

47.



Intersect at $x = 0$ and $x \approx 2.3367$
area ≈ 3.9100

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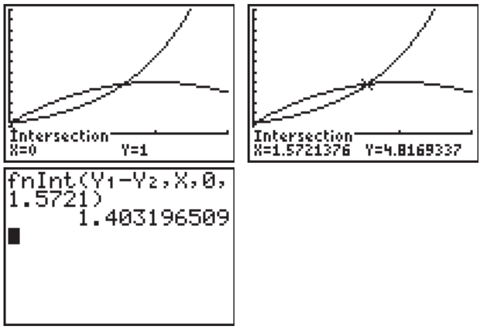
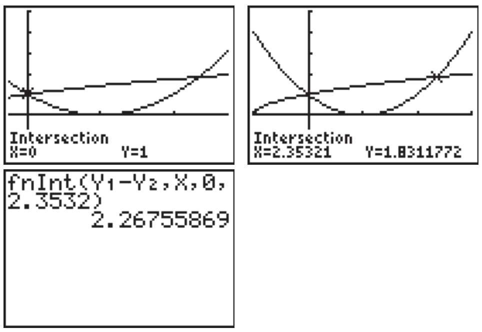
48.



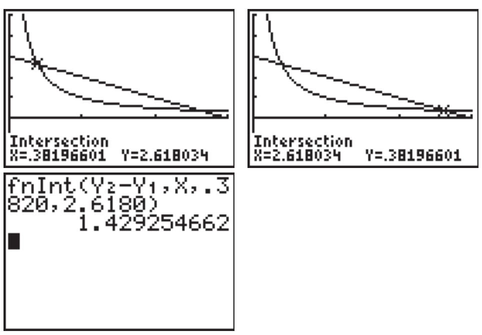
Intersect at $x = 0$ and $x \approx 1.5721$
area ≈ 1.4032

49.



Intersect at $x = 0$ and $x \approx 2.3532$
area ≈ 2.2676

50.



Intersect at $x \approx .3820$ and $x \approx 2.6180$
area ≈ 1.4293

6.5 Applications of the Definite Integral

1. Average $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\ dx = \dfrac{1}{3}\int_0^3 x^2 dx$

$$= \left.\frac{x^3}{9}\right|_0^3 = 3$$

2. $\dfrac{1}{1-(-1)}\displaystyle\int_{-1}^{1}(1-x)\,dx = \dfrac{1}{2}\cdot\left.\left(x - \dfrac{1}{2}x^2\right)\right|_{-1}^{1}$

$$= \frac{1}{2}\left(\left(1-\frac{1}{2}\right)-\left(-1-\frac{1}{2}\right)\right)$$
$$= 1$$

3. $\dfrac{1}{4-0}\displaystyle\int_0^4 100e^{-.5x}dx = \dfrac{1}{4}\left.\left(-200e^{-0.5x}\right)\right|_0^4$

$$= \frac{1}{4}\left(-200e^{-2} + 200\right)$$
$$= 50 - 50e^{-2} = 50\left(1 - e^{-2}\right)$$

4. $\dfrac{1}{1-0}\displaystyle\int_0^1 2\ dx = 2x\Big|_0^1 = 2$

5. $\dfrac{1}{3-\frac{1}{3}}\displaystyle\int_{1/3}^{3}\dfrac{1}{x}\ dx = \dfrac{3}{8}\left.(\ln x)\right|_{1/3}^{3} = \dfrac{3}{8}\left(\ln 3 - \ln\dfrac{1}{3}\right)$

$$= \frac{3}{8}(\ln 3 + \ln 3) = \frac{3}{4}\ln 3$$

6. $\dfrac{1}{9-1}\displaystyle\int_1^9 \dfrac{1}{\sqrt{x}}\ dx = \dfrac{1}{8}\int_1^9 x^{-1/2}\ dx$

$$= \left.\frac{1}{8}\left(2x^{1/2}\right)\right|_1^9 = \frac{1}{8}(6-2) = \frac{1}{2}$$

7. $\dfrac{1}{12}\displaystyle\int_0^{12}\left(47 + 4t - \dfrac{1}{3}t^2\right)\ dt$

$$= \frac{1}{12}\left.\left(47t + 2t^2 - \frac{1}{9}t^3\right)\right|_0^{12} = 55°$$

8. $\dfrac{1}{50}\displaystyle\int_0^{50} 3e^{0.02t}dt = \dfrac{1}{50}\cdot\dfrac{3}{(.02)}\cdot\left. e^{0.02t}\right|_0^{50}$

$$\approx 5.1548 \text{ million}$$

9. $P(t) = P(0)e^{kt}$

Find k: $1 = 2e^{k(1690)} \Rightarrow \dfrac{1}{2} = e^{1690k} \Rightarrow$

$1690k = \ln\dfrac{1}{2} \Rightarrow k = -.00041.$

The average value

$$= \frac{1}{1000}\int_0^{1000} 100e^{-0.00041t}dt$$

$$= \frac{1}{10}\cdot\left.\frac{e^{-0.00041t}}{-.00041}\right|_0^{1000} \approx 82 \text{ grams}$$

10. Recall that $A = Pe^{rt}$ for when interest is compounded continually.

$$\frac{1}{20}\int_0^{20} 100e^{.05t}dt = \left.\frac{5e^{.05t}}{.05}\right|_0^{20} \approx \$171.83$$

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11. $p(20) = 3 - \frac{20}{10} = 1$

$$\int_0^{20}\left(3 - \frac{x}{10} - 1\right)dx = \int_0^{20}\left(2 - \frac{x}{10}\right)dx$$
$$= \left(2x - \frac{x^2}{20}\right)\Bigg|_0^{20}$$
$$= 40 - 20 = \$20$$

12. $p(20) = 2 - 20 + 50 = 32$

$$\int_0^{20}\left(\frac{x^2}{200} - x + 50 - 32\right)dx$$
$$= \int_0^{20}\left(\frac{x^2}{200} - x + 18\right)dx$$
$$= \left(\frac{x^3}{600} - \frac{1}{2}x^2 + 18x\right)\Bigg|_0^{20}$$
$$= \frac{8000}{600} - 200 + 360 \approx \$173.33$$

13. $p(40) = \frac{500}{40+10} - 3 = 7$

$$\int_0^{40}\left(\frac{500}{x+10} - 3 - 7\right)dx$$
$$= \int_0^{40}\left(\frac{500}{x+10} - 10\right)dx$$
$$= \left[500\ln(x+10) - 10x\right]\Big|_0^{40}$$
$$= 500\ln 50 - 400 - 500\ln 10 \approx \$404.72$$

14. $p(350) = \sqrt{16 - (.02)350} = 3$

$$\int_0^{350}\left[(16 - .02x)^{1/2} - 3\right]dx$$
$$= \left(\frac{2}{3}\left(-\frac{1}{.02}\right)(16 - .02x)^{3/2} - 3x\right)\Bigg|_0^{350}$$
$$= -\frac{2}{3}\cdot\frac{9^{3/2}}{.02} - 1050 + \frac{2}{3}\cdot\frac{16^{3/2}}{.02} \approx \$183.33$$

15. $p(200) = 0.01(200) + 3 = 5$

$$\int_0^{200}[5 - (.01x + 3)]\,dx$$
$$= \int_0^{200}(2 - .01x)\,dx$$
$$= \left(2x - \frac{.01}{2}x^2\right)\Bigg|_0^{200}$$
$$= 400 - .005(40,000) = \$200$$

16. $p(3) = \frac{9}{9} + 1 = 2$

$$\int_0^{3}\left(2 - \left(\frac{x^2}{9} + 1\right)\right)dx = \int_0^{3}\left(1 - \frac{x^2}{9}\right)dx$$
$$= \left(x - \frac{x^3}{27}\right)\Bigg|_0^{3} = 3 - 1 = \$2$$

17. $p(10) = \frac{10}{2} + 7 = 12$

$$\int_0^{10}\left(12 - \left(\frac{x}{2} + 7\right)\right)dx = \int_0^{10}\left(5 - \frac{x}{2}\right)dx$$
$$= \left(5x - \frac{x^2}{4}\right)\Bigg|_0^{10}$$
$$= 50 - 25 = \$25$$

18. $p(36) = 1 + \frac{1}{2}\sqrt{36} = 4$

$$\int_0^{36}\left(4 - \left(1 + \frac{1}{2}x^{1/2}\right)\right)dx = \int_0^{36}\left(3 - \frac{1}{2}x^{1/2}\right)dx$$
$$= \left(3x - \frac{1}{3}x^{3/2}\right)\Bigg|_0^{36}$$
$$= 108 - \frac{216}{3} = \$36$$

19. First find the point of intersection of the functions:

$$12 - \frac{x}{50} = \frac{x}{20} + 5 \Rightarrow 7 = \frac{x}{50} + \frac{x}{20} \Rightarrow$$
$$7 = \frac{2x + 5x}{100} \Rightarrow 700 = 7x \Rightarrow x = 100$$
$$p(100) = 12 - \frac{100}{50} = 10$$

Thus, the functions intersect at (100, 10).

$$\text{C.S.} = \int_0^{100}\left(12 - \frac{x}{50} - 10\right)dx$$
$$= \int_0^{100}\left(2 - \frac{x}{50}\right)dx = \left(2x - \frac{x^2}{100}\right)\Bigg|_0^{100}$$
$$= 200 - 100 = \$100$$
$$\text{P.S.} = \int_0^{100}\left(10 - \left(\frac{x}{20} + 5\right)\right)dx$$
$$= \int_0^{100}\left(5 - \frac{x}{20}\right)dx = \left(5x - \frac{x^2}{40}\right)\Bigg|_0^{100}$$
$$= 500 - 250 = \$250$$

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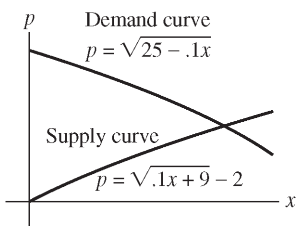
20. First find the point of intersection of the functions:

$$\sqrt{25-.1x} = \sqrt{.1x+9}-2$$
$$25-.1x = .1x+9+4-4\sqrt{.1x+9}$$
$$4\sqrt{.1x+9} = .2x-12$$
$$16(.1x+9) = .04x^2+144-4.8x$$
$$.04x^2-6.4x=0 \Rightarrow x^2-160x=0 \Rightarrow$$
$$x(x-160)=0 \Rightarrow x=0 \text{ or } x=160$$

$x = 0$ is an extraneous solution.

$$p(160) = \sqrt{.1\cdot 160+9}-2 = 3$$

Thus, the functions intersect at (160, 3). Verify graphically:



$$\text{C.S.} = \int_0^{160}\left[(25-.1x)^{1/2}-3\right]dx$$
$$= \left(\frac{-2}{.3}(25-.1x)^{3/2}-3x\right)\Bigg|_0^{160}$$
$$= \frac{-2}{.3}(27)-480+\frac{2}{.3}(125) = \$173.33$$
$$\text{P.S.} = \int_0^{160}\left[3-((.1x+9)^{1/2}-2)\right]dx$$
$$= \int_0^{160}(5-(.1x+9)^{1/2})\ dx$$
$$= \left(5x-\frac{2}{.3}(.1x+9)^{3/2}\right)\Bigg|_0^{160}$$
$$= 800-\frac{2}{.3}(125)+\frac{2}{.3}(27) = \$146.67$$

For exercises 21−26, the future value of a continuous income stream of k dollars per year for N years at interest rate r compounded continuously is $\int_0^N Ke^{r(N-t)}dt.$

21. $\int_0^3 1000e^{0.05(3-t)}dt = -20,000e^{0.05(3-t)}\Big|_0^3 = \$3236.68$

22. $\int_0^2 2000e^{0.06(2-t)}dt = -33,333e^{0.06(2-t)}\Big|_0^2 = \$4249.90$

23. $\int_0^4 16,000e^{0.08(4-t)}dt = -200,000e^{0.08(4-t)}\Big|_0^4 = \$75,426$

24. $\int_0^6 14,000e^{0.045(6-t)}dt = -\frac{14,000}{0.045}e^{0.045(6-t)}\Big|_0^6 = \$96,433$

25. Solve $140,000 = \int_0^x 5000e^{.1(x-t)}dt$ for x.

$$140,000 = -50,000e^{0.1(x-t)}\Big|_0^x \Rightarrow$$
$$140,000 = -50,000\left(1-e^{0.1x}\right)$$
$$2.8 = e^{0.1x}-1 \Rightarrow e^{0.1x} = 3.8 \Rightarrow$$
$$0.1x = \ln 3.8 \Rightarrow x = 10\ln 3.8 \approx 13.35$$

It will take about 13.35 years until the value of the investment reaches $140,000.

26. Solve $100,000 = \int_0^{10} xe^{0.0425(10-t)}dt$ for x.

$$100,000 = \int_0^{10} xe^{0.0425(10-t)}dt$$
$$100,000 = -\frac{x}{.0425}e^{0.0425(10-t)}\Bigg|_0^{10} \Rightarrow$$
$$100,000 = -\frac{x}{0.0425}\left(1-e^{0.425}\right)$$
$$\frac{4250}{x} = e^{0.425}-1$$
$$x = \frac{4250}{e^{0.425}-1} \approx \$8025.07$$

About $8025.07 should be invested per year.

For exercises 27−36, recall that the volume of the solid of revolution obtained form revolving the region below the graph of $y = g(x)$ from $x = a$ to $x = b$ about the x-axis is $\int_a^b \pi\left[g(x)\right]^2 dx.$

27. $\int_0^2 \pi(x+1)^2\,dx = \pi\int_0^2\left(x^2+2x+1\right)dx$

$$= \pi\left(\frac{x^3}{3}+x^2+x\right)\Bigg|_0^2$$
$$= \pi\left(\frac{8}{3}+4+2\right)-0 = \frac{26\pi}{3}$$

28. $\int_0^1 \pi\left(-x^2+1\right)^2 dx = \pi\int_0^1\left(x^4-2x^2+1\right)dx$

$$= \pi\left(\frac{x^5}{5}-\frac{2x^3}{3}+x\right)\Bigg|_0^1$$
$$= \pi\left(\frac{1}{5}-\frac{2}{3}+1\right)-0 = \frac{8\pi}{15}$$

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29. $\int_{-2}^{2} \pi\left[\sqrt{4-x^2}\right]^2 dx$

$= \int_{-2}^{2} \pi\left(4-x^2\right) dx = \left(4\pi x - \frac{\pi x^3}{3}\right)\Big|_{-2}^{2}$

$= \left(8\pi - \frac{8\pi}{3}\right) - \left(-8\pi + \frac{8\pi}{3}\right) = \frac{32\pi}{3}$

30. $\int_{-r}^{r} \pi\left(\sqrt{r^2-x^2}\right)^2 dx$

$= \int_{-r}^{r} \pi\left(r^2-x^2\right) dx = \left(\pi r^2 x - \frac{\pi x^3}{3}\right)\Big|_{-r}^{r}$

$= \pi r^3 - \frac{\pi r^3}{3} - \left(-\pi r^3 + \frac{\pi r^3}{3}\right)$

$= 2\pi r^3 - \frac{2\pi r^3}{3} = \frac{4}{3}\pi r^3$

31. $\int_{1}^{2} \pi\left(x^2\right)^2 dx = \int_{1}^{2} \pi x^4 dx = \frac{\pi}{5}x^5\Big|_{1}^{2}$

$= \frac{32\pi}{5} - \frac{\pi}{5} = \frac{31\pi}{5}$

32. $\int_{0}^{h} \pi(kx)^2\ dx = \frac{1}{3}\pi k^2 x^3\Big|_{0}^{h} = \frac{1}{3}\pi k^2 h^3$

33. $\int_{0}^{4} \pi\left(\sqrt{x}\right)^2\ dx = \int_{0}^{4} \pi x\ dx = \frac{\pi}{2}x^2\Big|_{0}^{4} = 8\pi$

34. $\int_{0}^{2} \pi\left(2x-x^2\right)^2 dx = \int_{0}^{2} \pi\left(x^4-4x^3+4x^2\right) dx$

$= \pi\left(\frac{x^5}{5} - x^4 + \frac{4}{3}x^3\right)\Big|_{0}^{2}$

$= \pi\left(\frac{32}{5} - 16 + \frac{32}{3}\right)$

$= \pi\left(\frac{16}{15}\right) = \frac{16\pi}{15}$

35. $\int_{0}^{1} \pi(2x+1)^2 dx = \int_{0}^{1} \pi(4x^2+4x+1)\ dx$

$= \pi\left(\frac{4}{3}x^3 + 2x^2 + x\right)\Big|_{0}^{1}$

$= \pi\left(\frac{4}{3} + 2 + 1\right) = \frac{13\pi}{3}$

36. $\int_{0}^{1} \pi\left(e^{-x}\right)^2 dx = \int_{0}^{1} \pi e^{-2x} dx = -\frac{\pi}{2}e^{-2x}\Big|_{0}^{1}$

$= -\frac{\pi}{2}e^{-2} + \frac{\pi}{2} = \frac{\pi}{2}\left(1 - \frac{1}{e^2}\right)$

37. $\left[8.25^3 + 8.75^3 + 9.25^3 + 9.75^3\right](.5)$

$n = 4;\ a = 8 \Rightarrow \frac{b-8}{4} = .5 \Rightarrow b = 10;\ f(x) = x^3$

38. $\left[\frac{3}{1} + \frac{3}{1.5} + \frac{3}{2} + \frac{3}{2.5} + \frac{3}{3} + \frac{3}{3.5}\right](.5)$

$n = 6; a = 1 \Rightarrow \frac{b-1}{6} = 0.5 \Rightarrow b = 4;\ f(x) = \frac{3}{x}$

39. $\left[\left(5+e^5\right) + \left(6+e^6\right) + \left(7+e^7\right)\right](1)$

$n = 3;\ a = 4 \Rightarrow \frac{b-4}{3} = 1 \Rightarrow b = 7$

$f(x) = x + e^x$

40. $\left[3(.3)^2 + 3(.9)^2 + 3(1.5)^2 + 3(2.1)^2 + 3(2.7)^2\right](.6)$

$n = 5; a = 0 \Rightarrow \frac{b-0}{5} = .6 \Rightarrow b = 3;\ f(x) = 3x^2$

41. The sum is approximately

$\int_{0}^{3} (3-x)^2 dx = \int_{0}^{3} (x^2 - 6x + 9)\ dx$

$= \left(\frac{1}{3}x^3 - 3x^2 + 9x\right)\Big|_{0}^{3}$

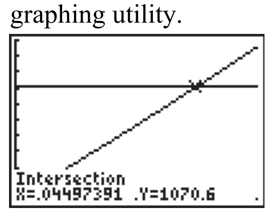
$= \frac{27}{3} - 27 + 27 = 9$

42. The sum is approximately

$\int_{0}^{1} \left(2x + x^3\right)\ dx = \left(x^2 + \frac{1}{4}x^4\right)\Big|_{0}^{1} = 1 + \frac{1}{4} = \frac{5}{4}.$

43. a. $\frac{1}{3}\int_{0}^{3} 1000e^{rt} dt = \frac{1000}{3r}e^{rt}\Big|_{0}^{3} = \frac{1000}{3r}\left(e^{3r} - 1\right)$

b. Solve $\frac{1000}{3r}\left(e^{3r} - 1\right) = 1070.60$ using a graphing utility.



[0, .06] by [1000, 1100]

$r \approx .045 = 4.5\%$

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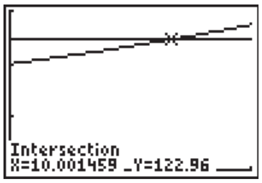
44. Avg. amt. $= \frac{1}{x}\int_0^x 100e^{0.04t}\,dt = \frac{2500}{x}e^{0.04t}\Big|_0^x$

$= \frac{2500}{x}\left(e^{0.04x} - 1\right)$

Solve $\frac{2500}{x}(e^{0.04x} - 1) = 122.96$ using a graphing utility.



[0, 15] by [0, 150]

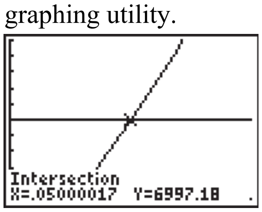
$x \approx 10$ years

45. a. $\int_0^6 1000e^{r(6-t)}\,dt = \frac{1000}{-r}e^{r(6-t)}\Big|_0^6$

$= \frac{1000}{-r}\left(1 - e^{6r}\right)$

$= \frac{1000}{r}\left(e^{6r} - 1\right)$

b. Solve $\frac{1000}{r}\left(e^{6r} - 1\right) = 6997.18$ using a graphing utility.



[0, .1] by [6500, 7500]

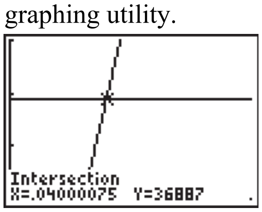
$r \approx 0.05 = 5\%$

46. $\int_0^{10} 3000e^{r(10-t)}\,dt = \frac{3000}{-r}e^{r(10-t)}\Big|_0^{10}$

$= \frac{3000}{-r}\left(1 - e^{10r}\right)$

$= \frac{3000}{r}\left(e^{10r} - 1\right)$

Solve $\frac{3000}{r}\left(e^{10r} - 1\right) = 36{,}887$ using a graphing utility.



[0, .1] by [35,000, 38,000]

$r \approx 0.04$

Chapter 6 Fundamental Concept Check Exercises

1. To antidifferentiate a function $f(x)$ means to find a function $F(x)$ such that $F'(x) = f(x)$.

2. a. $\int h(x)\,dx = \frac{x^{r+1}}{r+1} + C$

b. $\int h(x)\,dx = \frac{e^{kx}}{k} + C$

c. $\int h(x)\,dx = \ln|x| + C$

d. $\int h(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

e. $\int h(x)\,dx = k\int f(x)\,dx$

3. *a* denotes the left endpoint of the interval, *b* denotes the right endpoint of the interval, *n* denotes the number of intervals, and Δx denotes the length of one subinterval.

4. Suppose $f(x) \ge 0$. To approximate the integral $\int_a^b f(x)\,dx$, which represents the area under the graph of *f*, above the *x*-axis, from *a* to *b*, we can use rectangles of equal width Δx and height $f(x_i)$. Each rectangle has area $f(x_i)\Delta x$.

The sum of the areas of the rectangles is the Riemann sum $f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$, which approximates the area of the region under the graph of *f*.

5. The area under the rate of change function $f(x)$ is equal to the net change in the function $F(x)$. For example, the area under the velocity function $v(t)$ from *a* to *b* is equal to the net change in position or $s(b) - s(a)$.

6. The definite integral has the form $\int_a^b f(x)\,dx = F(b) - F(a)$, where *F* is any antidervative of *f*.

7. An indefinite integral has the form $\int f(x)\,dx = F(x) + C$. It is the family of antiderivatives of *f*. A definite integral has the form $\int_a^b f(x)\,dx = F(b) - F(a)$. It is a number.

8. Suppose *f* is a continuous function on $[a, b]$. The Reimann sums of *f* on $[a, b]$ approach the value of the definite integral $\int_a^b f(x)\,dx = F(b) - F(a)$ as the number of partitions of the interval $[a, b]$ increase indefinitely.

9. $F(x)\Big|_a^b = F(b) - F(a)$ is the value of the definite integral of *f* on $[a, b]$.

10. The area of the region bounded by the graph of $y = f(x)$ on top, the graph of $y = g(x)$ at the bottom, from $x = a$ to $x = b$ is given by $\int_a^b \left[f(x) - g(x)\right]dx.$ If the limits *a* and *b* are not given, we determine them by finding the first coordinates of the points of intersection of the graphs of *f* and *g*.

11. **a.** The average value of a function is given by $\dfrac{1}{b-a}\int_a^n f(x)\,dx.$

b. The consumer's surplus for a commodity having demand curve $p = f(x)$ is given by $\int_0^A \left[f(x) - B\right]dx,$ where the quantity demanded is *A* and the price is $B = f(A)$.

c. The future value of a continuous income stream of *K* dollars per year for *N* years at interest rate *r* compounded continuously is $\int_0^N Ke^{r(N-t)}\,dt.$

d. The volume of a solid of revolution is given by $\pi\int_a^b \left[f(x)\right]^2 dx.$

Chapter 6 Review Exercises

1. $\int 3^2\,dx = 9x + C$

2. $\int (x^2 - 3x + 2)dx = \dfrac{1}{3}x^3 - \dfrac{3}{2}x^2 + 2x + C$

3. $\int \sqrt{x+1}\,dx = \dfrac{2}{3}(x+1)^{3/2} + C$

4. $\int \dfrac{2}{x+4}\,dx = 2\ln|x+4| + C$

5. $2\int (x^3 + 3x^2 - 1)dx = \dfrac{1}{2}x^4 + 2x^3 - 2x + C$

6. $\int \sqrt[5]{x+3}\,dx = \dfrac{5}{6}(x+3)^{6/5} + C$

7. $\int e^{-x/2}\,dx = -2e^{-x/2} + C$

8. $\int \dfrac{5}{\sqrt{x-7}}\,dx = 10\sqrt{x-7} + C$

9. $\int (3x^4 - 4x^3)\,dx = \dfrac{3}{5}x^5 - x^4 + C$

10. $\int (2x+3)^7\,dx = \dfrac{1}{16}(2x+3)^8 + C$

11. $\int \sqrt{4-x}\,dx = -\dfrac{2}{3}(4-x)^{3/2} + C$

12. $\int \left(\dfrac{5}{x} - \dfrac{x}{5}\right)dx = 5\ln|x| - \dfrac{x^2}{10} + C$

13. $\int_{-1}^{1} (x+1)^2\,dx = \dfrac{1}{3}(x+1)^3\Big|_{-1}^{1} = \dfrac{8}{3} - 0 = \dfrac{8}{3}$

14. $\int_0^{1/8} \sqrt[3]{x}\,dx = \dfrac{3}{4}x^{4/3}\Big|_0^{1/8} = \dfrac{3}{64} - 0 = \dfrac{3}{64}$

15.
$$\int_{-1}^{2} \sqrt{2x+4}\,dx = \sqrt{2}\int_{-1}^{2}\sqrt{x+2}\,dx = \frac{2}{3}\sqrt{2}(x+2)^{3/2}\Big|_{-1}^{2} = \frac{2}{3}\sqrt{2}\left(4^{3/2} - 1\right) = \frac{14}{3}\sqrt{2}$$

16.
$$2\int_0^1 \left(\frac{2}{x+1} - \frac{1}{x+4}\right)dx = \left[4\ln(x+1) - 2\ln(x+4)\right]\Big|_0^1 = 4\ln 2 - 2\ln 5 - (4\ln 1 - 2\ln 4) = 2\ln\frac{16}{5}$$

17. $\int_1^2 \dfrac{4}{x^5}\,dx = -\dfrac{1}{x^4}\Big|_1^2 = -\dfrac{1}{16} - (-1) = \dfrac{15}{16}$

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18. $\frac{2}{3}\int_0^8 \sqrt{x+1}dx = \frac{4}{9}(x+1)^{3/2}\Big|_0^8 = 12 - \frac{4}{9} = \frac{104}{9}$

19. $\int_1^4 \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_1^4 = -\frac{1}{4} + 1 = \frac{3}{4}$

20. $\int_3^6 e^{2-(x/3)}dx = -3e^{2-(x/3)}\Big|_3^6 = -3 + 3e = 3(e-1)$

21. $\int_0^5 (5+3x)^{-1}\,dx = \frac{1}{3}\ln(5+3x)\Big|_0^5 = \frac{1}{3}\ln 20 - \frac{1}{3}\ln 5 = \frac{1}{3}\ln 4$

22. $\int_{-2}^2 \frac{3}{2e^{3x}}dx = -\frac{1}{2e^{3x}}\Big|_{-2}^2 = -\frac{1}{2e^6} - \left(-\frac{1}{2e^{-6}}\right) = \frac{1}{2}\left(e^6 - e^{-6}\right)$

23. $\int_0^{\ln 2} (e^x - e^{-x})dx = (e^x + e^{-x})\Big|_0^{\ln 2} = 2 + \frac{1}{2} - (1+1) = \frac{1}{2}$

24. $\int_{\ln 2}^{\ln 3} \left(e^x + e^{-x}\right)dx = \left(e^x - e^{-x}\right)\Big|_{\ln 2}^{\ln 3} = 3 - \frac{1}{3} - \left(2 - \frac{1}{2}\right) = \frac{7}{6}$

25. $\int_0^{\ln 3} \frac{e^x + e^{-x}}{e^{2x}}dx = \left(-e^{-x} - \frac{1}{3}e^{-3x}\right)\Big|_0^{\ln 3} = -\frac{1}{3} - \frac{1}{81} - \left(-1 - \frac{1}{3}\right) = \frac{80}{81}$

26. $\int_0^1 \frac{3+e^{2x}}{e^x}\,dx = \left(-3e^{-x} + e^x\right)\Big|_0^1 = -3e^{-1} + e - (-3+1) = 2 + e - \frac{3}{e}$

27. $\int_1^2 (3x-2)^{-3}\,dx = \left(-\frac{1}{6}(3x-2)^{-2}\right)\Big|_1^2 = -\frac{1}{6}\cdot\frac{1}{16} + \frac{1}{6} = \frac{15}{96} = \frac{5}{32}$

28. $\int_1^9 \left(1+\sqrt{x}\,\right)dx = \left(x + \frac{2}{3}x^{3/2}\right)\Big|_1^9 = 9 + 18 - \left(1 + \frac{2}{3}\right) = 26 - \frac{2}{3} = \frac{76}{3}$

29. $\int_0^1 \left(\sqrt{x} - x^2\right)dx = \left(\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right)\Big|_0^1 = \frac{2}{3} - \frac{1}{3} - (0-0) = \frac{1}{3}$

30. $y = x^3$ lies above $y = \frac{1}{2}x^3 + 2x$ on [–2, 0] and below on [0, 2]. Thus, we calculate

$$\int_{-2}^0 \left[x^3 - \left(\frac{1}{2}x^3 + 2x\right)\right]dx + \int_0^2 \left[\frac{1}{2}x^3 + 2x - x^3\right]dx = \int_{-2}^0 \left(\frac{1}{2}x^3 - 2x\right)dx + \int_0^2 \left(-\frac{1}{2}x^3 + 2x\right)dx$$

$$= \left(\frac{1}{8}x^4 - x^2\right)\Big|_{-2}^0 + \left(-\frac{1}{8}x^4 + x^2\right)\Big|_0^2$$

$$= 0 - 0 - (2-4) + (-2) + 4 - (0+0) = 4$$

31. $\int_0^{\ln 2} (e^x - e^{-x})dx = (e^x + e^{-x})\Big|_0^{\ln 2} = 2 + \frac{1}{2} - (1+1) = \frac{1}{2}$

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32. Set $\sqrt{x} = x^2 \Rightarrow x = x^4 \Rightarrow x(x^3 - 1) = 0 \Rightarrow x = 0$ or $x = 1$. Thus, the graphs intersect at $x = 0, 1$. On (0, 1), $y = \sqrt{x}$ lies above $y = x^2$, and below on (1, 1.21]. Thus, we calculate

$$\int_0^1 \left[\sqrt{x} - x^2\right] dx + \int_1^{1.21} \left(x^2 - \sqrt{x}\right) dx = \left(\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right)\Bigg|_0^1 + \left(\frac{1}{3}x^3 - \frac{2}{3}x^{3/2}\right)\Bigg|_1^{1.21}$$

$$\approx \frac{2}{3} - \frac{1}{3} - (0-0) + .5905 - .8873 - \left(\frac{1}{3} - \frac{2}{3}\right) = \frac{1.109561}{3} \approx 0.370$$

33. $4 - x^2$ and $1 - x^2$ are even, so the area is given by

$$2\int_0^2 \left(4 - x^2\right) dx - 2\int_0^1 \left(1 - x^2\right) dx = 2\left[\left(4x - \frac{1}{3}x^3\right)\Bigg|_0^2 - \left(x - \frac{1}{3}x^3\right)\Bigg|_0^1\right] = 2\left[8 - \frac{8}{3} - \left(1 - \frac{1}{3}\right)\right] = \frac{28}{3}$$

34. $$\int_{1/2}^2 \left[\left(1 - \frac{1}{x}\right) - \left(x^2 - \frac{3}{2}x - \frac{1}{2}\right)\right] dx = \int_{1/2}^2 \left(-x^2 + \frac{3}{2}x + \frac{3}{2} - \frac{1}{x}\right) dx = \left(-\frac{1}{3}x^3 + \frac{3}{4}x^2 + \frac{3}{2}x - \ln x\right)\Bigg|_{1/2}^2$$

$$= -\frac{8}{3} + 3 + 3 - \ln 2 - \left(-\frac{1}{24} + \frac{3}{16} + \frac{3}{4} - \ln\frac{1}{2}\right) = \frac{39}{16} - \ln 4$$

35. $$\int_0^1 (e^x - ex)dx = \left(e^x - \frac{e}{2}x^2\right)\Bigg|_0^1 = e - \frac{e}{2} - (1-0) = \frac{e}{2} - 1$$

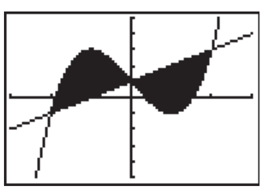
36. $y = 2x^3 - x^2 - 6x$ lies above $y = x^3$ on [–2, 0] and below on [0, 3]. Thus, we calculate

$$\int_{-2}^0 \left[2x^3 - x^2 - 6x - (x^3)\right] dx + \int_0^3 \left[x^3 - (2x^3 - x^2 - 6x)\right] dx$$

$$= \int_{-2}^0 \left[x^3 - x^2 - 6x\right] dx + \int_0^3 \left[-x^3 + x^2 + 6x)\right] dx = \left(\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2\right)\Bigg|_{-2}^0 + \left(-\frac{1}{4}x^4 + \frac{1}{3}x^3 + 3x^2\right)\Bigg|_0^3$$

$$= 0 - \left(4 + \frac{8}{3} - 12\right) + \left(-\frac{81}{4}\right) + 9 + 27 - 0 = \frac{253}{12}$$

37.



$x^3 - 3x + 1 = x + 1 \Rightarrow x^3 - 4x = 0 \Rightarrow x(x^2 - 4) = 0 \Rightarrow x = -2$ or $x = 0$ or $x = 2$

Thus, the graphs intersect at $x = 0, \pm 2$. On [–2, 0], $y = x^3 - 3x + 1$ lies above $y = x + 1$, and below on [0, 2].

$$\int_{-2}^0 \left[(x^3 - 3x + 1) - (x + 1)\right] dx + \int_0^2 \left[(x + 1) - (x^3 - 3x + 1)\right] dx$$

$$= \int_{-2}^0 (x^3 - 4x)\ dx + \int_0^2 (-x^3 + 4x)\ dx = \left(\frac{1}{4}x^4 - 2x^2\right)\Bigg|_{-2}^0 + \left(-\frac{1}{4}x^4 + 2x^2\right)\Bigg|_0^2 = 0 - (4 - 8) + (-4 + 8) = 8$$

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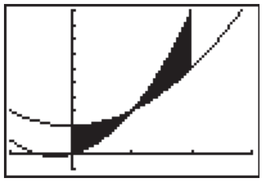
38. 

$2x^2 + x = x^2 + 2 \Rightarrow x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0 \Rightarrow x = -2$ or $x = 1$

Thus, on the interval [0, 2], the graphs intersect at $x = 1$. On (0, 1), $y = x^2 + 2$ lies above $y = 2x^2 + x$ and below on (1, 2).

$$\int_0^1 \left[x^2 + 2 - (2x^2 + x)\right] dx + \int_1^2 \left[2x^2 + x - (x^2 + 2)\right] dx$$

$$= \int_0^1 \left(-x^2 - x + 2\right) dx + \int_1^2 \left(x^2 + x - 2\right) dx = \left(-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right)\Bigg|_0^1 + \left(\frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x\right)\Bigg|_1^2$$

$$= -\frac{1}{3} - \frac{1}{2} + 2 + \frac{8}{3} + 2 - 4 - \left(\frac{1}{3} + \frac{1}{2} - 2\right) = \frac{6}{3} - \frac{2}{2} + 2 = 3$$

39. $\int (x-5)^2\, dx = \frac{1}{3}(x-5)^3 + C$

$f(8) = 2 = \frac{1}{3}(3)^3 + C = 9 + C \Rightarrow C = -7$

$f(x) = \frac{1}{3}(x-5)^3 - 7$

40. $\int e^{-5x} dx = -\frac{1}{5}e^{-5x} + C$

$f(0) = 1 = -\frac{1}{5} + C \Rightarrow C = \frac{6}{5}$

$f(x) = \frac{6}{5} - \frac{1}{5}e^{-5x}$

41. **a.** $y' = 4t \Rightarrow y = 2t^2 + C$

b. $y' = 4y \Rightarrow y = Ce^{4t}$

c. $y' = e^{4t} \Rightarrow y = \frac{1}{4}e^{4t} + C$

42. Theorem II of section 6.1 states if $F'(x) = 0$ for all x in an interval I, then there is a constant C such that $F(x) = C$ for all x in I.

$y' = f'(t) = kt(f(t))$.

Using the hint, we have

$$\frac{d}{dt}\left[f(t)e^{-kt^2/2}\right]$$
$$= f(t)(-kt)e^{-kt^2/2} + f'(x)e^{-kt^2/2}$$
$$= e^{-kt^2/2}\left[-f(t)kt + f'(t)\right]$$
$$= e^{-kt^2/2}\left[-f(t)kt + kt(f(t))\right]$$
$$= e^{-kt^2/2}(0) = 0$$

Since only constant functions have a zero derivative, $f(t)e^{-kt^2/2} = C$ for some C. Thus, $f(t) = Ce^{kt^2/2}$.

43. $C(x) = \int (.04x + 150)\ dx = .02x^2 + 150x + C$

$C(0) = 500 = 0 + 0 + C = C$

Thus, $C(x) = .02x^2 + 150x + 500$ dollars.

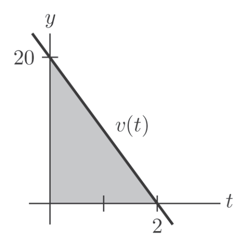
44. $\int_{10}^{20} (400 - 3x^2)\ dx = (400x - x^3)\Big|_{10}^{20} = -3000$

Thus, a loss of $3000 would result.

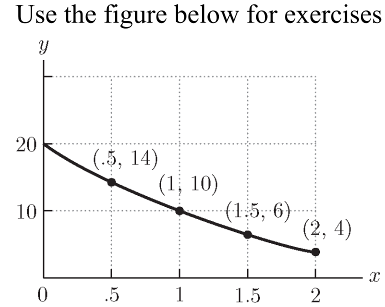
45. It represents the total quantity of drug (in cubic centimeters) injected during the first 4 minutes.

46. $v(t) = -9.8t + 20$

a. $\int_0^2 (-9.8t + 20)\ dt = (-4.9t^2 + 20t)\Big|_0^2$
$= -19.6 + 40 = 20.4$ m

b. 

Use the figure below for exercises 47 and 48.



47. $\left[f(0) + f(.5) + f(1) + f(1.5)\right]\Delta x$
$= (20 + 14 + 10 + 6)(.5) = 25$

48. $\left[f(.5) + f(1) + f(1.5) + f(2)\right]\Delta x$
$= (14 + 10 + 6 + 4)(.5) = 17$

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49. $\Delta x = 1$; the midpoints are .5, 1.5

Area $= \left[\frac{1}{0.5+2} + \frac{1}{1.5+2}\right] \approx 0.68571$

$$\int_0^2 \frac{1}{x+2}\,dx = \ln(x+2)\Big|_0^2 = \ln 4 - \ln 2 = \ln\frac{4}{2} = \ln 2 \approx .69315$$

50. $\Delta x = 0.2$; the midpoints are 0.1, 0.3, 0.5, 0.7, 0.9

Area =

$$\left[e^{2(0.1)} + e^{2(0.3)} + e^{2(0.5)} + e^{2(0.7)} + e^{2(0.9)}\right]\cdot(.2) \approx 3.17333$$

$$\int_0^1 e^{2x}dx = \frac{1}{2}e^{2x}\Big|_0^1 = \frac{1}{2}e^2 - \frac{1}{2} \approx 3.1945$$

51. $p(400) = \sqrt{25 - 0.04(400)} = 3$

$$\text{C.S.} = \int_0^{400} (\sqrt{25-0.04x} - 3)\ dx = \left(\frac{2}{-0.12}(25-0.04x)^{3/2} - 3x\right)\Bigg|_0^{400} = \frac{2}{-0.12}(27) - 1200 - \frac{2}{-0.12}(125) \approx \$433.33$$

52. $\displaystyle \frac{1}{10}\int_0^{10} 3000e^{.04t}dt = \int_0^{10} 300e^{.04t}dt = 7500e^{.04t}\Big|_0^{10} = 7500(e^{.4} - e^0) \approx \$3688.69$

53. $\displaystyle \frac{1}{\frac{1}{2}-\frac{1}{3}}\int_{1/3}^{1/2} \frac{1}{x^3}\ dx = 6\int_{1/3}^{1/2}\frac{1}{x^3}\ dx = 6\left[-\frac{1}{2x^2}\Big|_{1/3}^{1/2}\right] = 6\left(-2+\frac{9}{2}\right) = 15$

54. The sum is approximated by

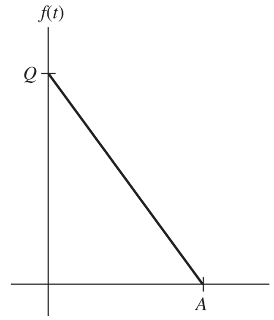
$$\int_0^1 3e^{-x}dx = -3e^{-x}\Big|_0^1 = -3e^{-1} + 3 = 3\left(1 - e^{-1}\right).$$

55. $\displaystyle \int_a^c f(x)\ dx = .68 - .42 = .26$

$$\int_a^d f(x)\ dx = .68 - .42 + 1.7 = 1.96$$

56. $\displaystyle \int_0^1 \pi(1-x^2)^2\,dx = \int_0^1 \pi(x^4 - 2x^2 + 1)\ dx = \pi\left(\frac{x^5}{5} - \frac{2}{3}x^3 + x\right)\Bigg|_0^1 = \pi\left(\frac{1}{5} - \frac{2}{3} + 1\right) = \frac{8\pi}{15}$

57. a. Inventory is decreasing, so the slope is $-\frac{Q}{A}$. From the graph we can see that $f(t) = Q - \frac{Q}{A}t$.



b. $\displaystyle \frac{1}{A}\int_0^A\left(Q - \frac{Q}{A}t\right)\ dt = \frac{1}{A}\left(Qt - \frac{Q}{2A}t^2\right)\Bigg|_0^A = \frac{Q}{A}A - \frac{QA^2}{2A^2} = \frac{Q}{2}$

58. a. $\displaystyle f(t) = Q - \int_0^t rt\ dt = Q - \frac{rt^2}{2}$

b. $\displaystyle 0 = Q - \frac{rA^2}{2} \Rightarrow r = \frac{2Q}{A^2}$

c. $\displaystyle f(t) = Q - \frac{\frac{2Q}{A^2}t^2}{2} = Q - \frac{Qt^2}{A^2}$

$$\frac{1}{A}\int_0^A\left(Q - \frac{Qt^2}{A^2}\right)\ dt = \frac{1}{A}\left(Qt - \frac{Qt^3}{3A^2}\right)\Bigg|_0^A = Q - \frac{Q}{3} = \frac{2}{3}Q$$

59. a. $g(3)$ is the area under the curve $y = \frac{1}{1+t^2}$ from $t = 0$ to $t = 3$.

b. $\displaystyle g'(x) = \frac{1}{1+x^2}$

60. a. $h(0)$ is the area under one-quarter of the unit circle. $h(1)$ is the area under one-half of the unit circle.

b. $h'(x) = \sqrt{1-x^2}$

61. The sum is approximated by

$$\int_0^3 5000e^{-0.1t}dt = -50{,}000e^{-0.1t}\Big|_0^3 \approx 12{,}959 \approx 13{,}000$$

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62. $\Delta x = \frac{1}{n}$, with left endpoints $t_i = i\Delta x$

$$\text{Sum} = \Delta x e^0 + \Delta x e^{\Delta x} + \Delta x e^{2\Delta x} + \cdots + \Delta x e^{(n-1)\Delta x}$$
$$= \Delta x\left[e^{t_0} + e^{t_1} + e^{t_2} + \cdots + e^{t_{n-1}}\right]$$
$$\approx \int_0^1 e^x dx = e - 1$$

63. $\Delta x = \frac{1}{n}$, with left endpoints $t_i = i\Delta x$

$$\text{Sum} = \Delta x\left[1 + (1+\Delta x)^3 + \cdots + (1+(n-1)\Delta x)^3\right]$$
$$= \Delta x\left[1 + (1+t_1)^3 + \cdots + (1+t_{n-1})^3\right]$$
$$\approx \int_0^1 (1+x)^3\, dx = \left.\frac{(1+x)^4}{4}\right|_0^1 = \frac{15}{4}$$

64. Using the figure on the left, the average value of $f(x) = 4$ on $2 \le x \le 6$.

65. True; $3 \le f(x) \le 4$

$$\int_0^5 3\ dx \le \int_0^5 f(x)\ dx \le \int_0^5 4\ dx$$
$$15 \le \int_0^5 f(x)\ dx \le 20\text{, so } 3 \le \frac{1}{5}\int_0^5 f(x)\ dx \le 4$$

66. a.
$$\int_{t_1}^{t_2} (20 - 4t_1)\ dt = (20-4t_1)t\Big|_{t_1}^{t_2}$$
$$= (20-4t_1)t_2 - (20-4t_1)t_1$$
$$= (20-4t_1)\Delta t$$

b. Let $R(t)$ = the amount of water added up to time t. Then $R'(t) = r(t)$ and $\int_0^5 r(t)\ dt = R(5) - R(0)$ = the total amount of water added to the tank from $t = 0$ to $t = 5$.

67.
$$\int_0^{35} 860e^{0.04t} dt = 21{,}500e^{0.04t}\Big|_0^{35}$$
$$= 21{,}500(e^{1.4} - 1)$$
$$= 65{,}687 \text{ cubic kilometers}$$

68.
$$\int_0^1 4500e^{0.09(1-t)} dt = -50{,}000e^{0.09(1-t)}\Big|_0^1$$
$$= \$4708.71$$

69. $\int (3x^2 - 2x + 1)dx = x^3 - x^2 + x + C$

Now $f(1) = 1$, so

$$(x^3 - x^2 + x + C)\Big|_{x=1} = 1 \Rightarrow$$
$$1 - 1 + 1 + C = 1 \Rightarrow C = 0$$

So, $f(x) = x^3 - x^2 + x.$

70. The slope of the line connecting (0, 0) and (a, a^2) is $m = \frac{a^2 - 0}{a - 0} = a$. The equation of the line is $y - 0 = a(x - 0) \Rightarrow y = ax.$ The shaded area is equal to 1, so we have

$$\int_0^a \left(ax - x^2\right) dx = 1$$
$$\left.\left(\frac{a}{2}x^2 - \frac{1}{3}x^3\right)\right|_0^a = 1$$
$$\frac{a^3}{2} - \frac{a^3}{3} - 0 = 1 \Rightarrow \frac{a^3}{6} = 1 \Rightarrow a = \sqrt[3]{6}$$

71.
$$\int_0^{b^2} \sqrt{x}dx + \int_0^b x^2 dx = \frac{2}{3}x^{3/2}\Big|_0^{b^2} + \frac{1}{3}x^3\Big|_0^b$$
$$= \frac{2}{3}(b^2)^{3/2} - 0 + \frac{1}{3}b^3 - 0$$
$$= \frac{2}{3}|b|^3 + \frac{1}{3}b^3$$
$$\left(\text{since } (b^2)^{1/2} = |b|\right)$$
$$= \frac{2}{3}b^3 + \frac{1}{3}b^3$$
$$\left(\begin{array}{l}|b| = b \text{ since } b\\ \text{is positive.}\end{array}\right)$$
$$= b^3$$

72.
$$\int_0^{b^n} \sqrt[n]{x}dx + \int_0^b x^n dx$$
$$= \left.\left(\frac{n}{n+1}x^{(n+1)/n}\right)\right|_0^{b^n} + \left.\left(\frac{1}{n+1}x^{n+1}\right)\right|_0^b$$
$$= \frac{n}{n+1}\left(b^n\right)^{(n+1)/n} - 0 + \frac{1}{n+1}b^{n+1} - 0$$
$$= \frac{n}{n+1}b^{n+1} + \frac{1}{n+1}b^{n+1} = b^{n+1}$$

[Note: $\left(b^n\right)^{1/n} = \begin{cases} b, & n \text{ is odd} \\ |b|, & n \text{ is even}\end{cases}$, but $|b| = b$ since b is positive. So, $\left(b^n\right)^{1/n} = b.$]

73.
$$\int_0^1\left(\sqrt{x} - x^2\right)dx = \int_0^1 \sqrt{x}dx - \int_0^1 x^2 dx$$
$$= \left.\left(\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right)\right|_0^1$$
$$= \left(\frac{2}{3} - \frac{1}{3}\right) - 0 = \frac{1}{3}$$

74.
$$\int_0^1\left(\sqrt[n]{x} - x^n\right)dx = \left.\left(\frac{n}{n+1}x^{\frac{n+1}{n}} - \frac{1}{n+1}x^{n+1}\right)\right|_0^1$$
$$= \frac{n}{n+1} - \frac{1}{n+1} - 0 = \frac{n-1}{n+1}$$

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