Chapter 8 The Trigonometric Functions

8.1 Radian Measure of Angles

1. $30° = 30 \cdot \frac{\pi}{180}$ radians $= \frac{\pi}{6}$ radian

$120° = 120 \cdot \frac{\pi}{180}$ radians $= \frac{2\pi}{3}$ radians

$315° = 315 \cdot \frac{\pi}{180}$ radians $= \frac{7\pi}{4}$ radians

2. $18° = 18 \cdot \frac{\pi}{180}$ radians $= \frac{\pi}{10}$ radians

$72° = 72 \cdot \frac{\pi}{180}$ radians $= \frac{2\pi}{5}$ radians

$150° = 150 \cdot \frac{\pi}{180}$ radians $= \frac{5\pi}{6}$ radians

3. $450° = 450 \cdot \frac{\pi}{180}$ radians $= \frac{5\pi}{2}$ radians

$-210° = -210 \cdot \frac{\pi}{180}$ radians $= -\frac{7\pi}{6}$ radians

$-90° = -90 \cdot \frac{\pi}{180}$ radians $= -\frac{\pi}{2}$ radians

4. $990° = 990 \cdot \frac{\pi}{180}$ radians $= \frac{11\pi}{2}$ radians

$-270° = -270 \cdot \frac{\pi}{180}$ radians $= -\frac{3\pi}{2}$ radians

$-540° = -540 \cdot \frac{\pi}{180}$ radians $= -3\pi$ radians

5. $t = 8 \cdot \frac{\pi}{2} = 4\pi$ radians

6. $t = -3 \cdot \frac{\pi}{2} = -\frac{3\pi}{2}$ radians

7. $t = 7 \cdot \frac{\pi}{2} = \frac{7\pi}{2}$ radians

8. $t = 9 \cdot \frac{\pi}{2} = \frac{9\pi}{2}$ radians

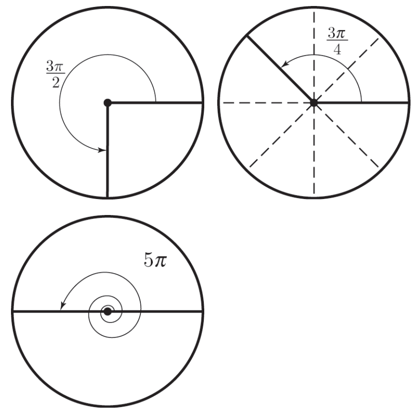
9. $t = -6 \cdot \frac{\pi}{2} = -3\pi$ radians

10. $t = -5 \cdot \frac{\pi}{2} = -\frac{5\pi}{2}$ radians

11. $t = 2 \cdot \frac{\pi}{3} = \frac{2\pi}{3}$ radians

12. $t = 5 \cdot \frac{\pi}{4} = \frac{5\pi}{4}$ radians

13.



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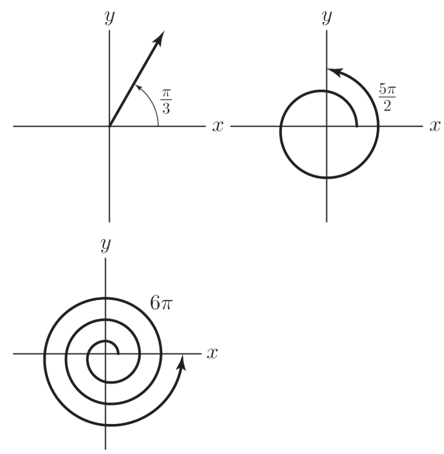

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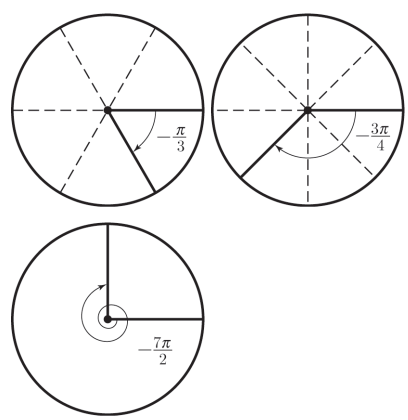

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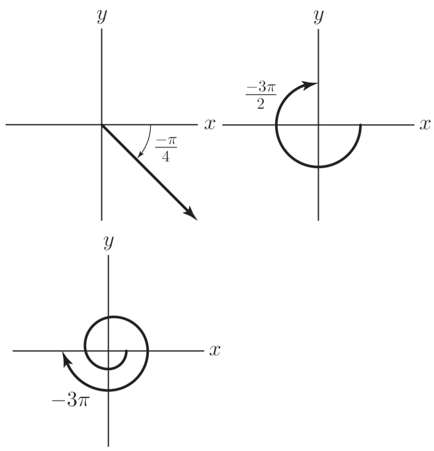

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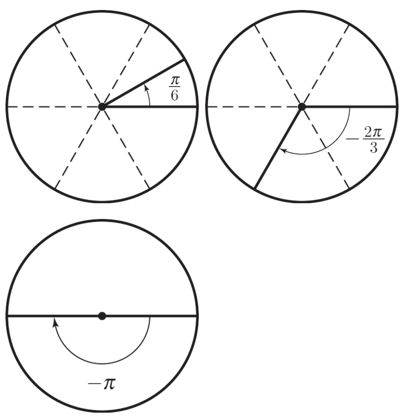

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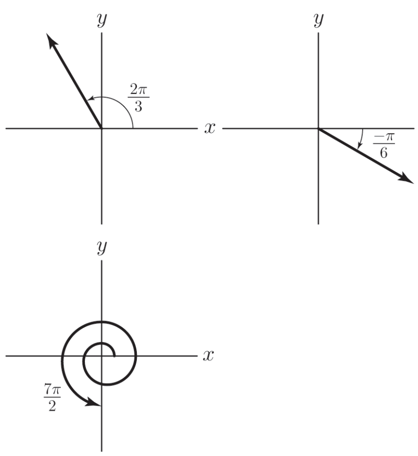


8.2 The Sine and the Cosine

1. $\sin t = \frac{1}{2}; \cos t = \frac{\sqrt{3}}{2}$

2. $\sin t = \frac{2}{3}; \cos t = \frac{\sqrt{5}}{3}$

3. $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$

$\sin t = \frac{2}{\sqrt{13}}; \cos t = \frac{3}{\sqrt{13}}$

4. $r = \sqrt{x^2 + y^2}$, so

$x = \sqrt{r^2 - y^2} = \sqrt{4^2 - 1^2} = \sqrt{15}$

$\sin t = \frac{1}{4}; \cos t = \frac{\sqrt{15}}{4}$

5. $r = \sqrt{x^2 + y^2}$, so

$x = \sqrt{r^2 - y^2} = \sqrt{13^2 - 12^2} = 5$

$\sin t = \frac{12}{13}; \cos t = \frac{5}{13}$

6. $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 5^2} = \sqrt{34}$

$\sin t = \frac{5}{\sqrt{34}}; \cos t = \frac{3}{\sqrt{34}}$

7. $r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$

$\sin t = \frac{1}{\sqrt{5}}; \cos t = -\frac{2}{\sqrt{5}}$

8. $r = \sqrt{x^2 + y^2} = \sqrt{2^2 + (-3)^2} = \sqrt{13}$

$\sin t = -\frac{3}{\sqrt{13}}; \cos t = \frac{2}{\sqrt{13}}$

9. $r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (2)^2} = \sqrt{8}$

$\sin t = \frac{2}{\sqrt{8}} = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos t = \frac{-2}{\sqrt{8}} = -\frac{2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2}$

10. $r = \sqrt{x^2 + y^2} = \sqrt{(.6)^2 + (.8)^2} = 1$

$\sin t = \frac{.8}{1} = .8; \cos t = \frac{.6}{1} = .6$

11. $r = \sqrt{x^2 + y^2} = \sqrt{(-0.6)^2 + (-0.8)^2} = 1$

$\sin t = \frac{-.8}{1} = -.8; \cos t = \frac{-.6}{1} = -.6$

12. $r = \sqrt{x^2 + y^2} = \sqrt{(.8)^2 + (-.6)^2} = 1$

$\sin t = \frac{-.6}{1} = -.6; \cos t = \frac{.8}{1} = .8$

13. $a = 12$, $b = 5$, and $c = 13$

$\sin t = \frac{b}{c} = \frac{5}{13} \approx .3846$

$t = .4$ radians

14. $t = 1.1$ and $c = 10.0$;
$\sin t = \frac{b}{c} \Rightarrow \sin 1.1 = \frac{b}{10.0} \Rightarrow$
$b \approx .89121(10.0) = 8.9$

15. $t = 1.1$ and $b = 3.2$
$\sin t = \frac{b}{c} \Rightarrow \sin 1.1 = \frac{3.2}{c} \Rightarrow c \approx \frac{3.2}{.89121} = 3.6$

16. $t = .4$ and $c = 5.0$
$\cos t = \frac{a}{c} \Rightarrow \cos 0.4 = \frac{a}{5.0} \Rightarrow$
$a \approx (.92106)(5.0) = 4.6$

17. $t = .4$, $a = 10.0$
$\cos .4 = \frac{10.0}{c} \Rightarrow c \approx \frac{10.0}{.92106} = 10.9$

18. $t = .9$, $c = 20.0$
$\sin t = \frac{b}{c} \Rightarrow \sin 0.9 = \frac{b}{20.0} \Rightarrow$
$b \approx (.78333)(20.0) = 15.7$
$\cos t = \frac{a}{c} \Rightarrow \cos 0.9 = \frac{a}{20.0} \Rightarrow$
$a \approx (.62161)(20) = 12.4$

19. $t = .5$, $a = 2.4$
$\cos t = \frac{a}{c} \Rightarrow \cos .5 = \frac{2.4}{c} \Rightarrow$
$c \approx \frac{2.4}{.87758} = 2.7$
$\sin t = \frac{b}{c} \Rightarrow \sin .5 = \frac{b}{2.73} \Rightarrow$
$b \approx (.47943)(2.73) = 1.3$

20. $t = 1.1$, $b = 3.5$
$\sin t = \frac{b}{c} \Rightarrow \sin 1.1 = \frac{3.5}{c} \Rightarrow$
$c \approx \frac{3.5}{.89121} = 3.9$
$\cos t = \frac{a}{c} \Rightarrow \cos 1.1 = \frac{a}{3.93} \Rightarrow$
$a \approx (0.45360)(3.93) = 1.8$

21. $\cos t = \cos\left(-\frac{\pi}{6}\right) \Rightarrow t = \frac{\pi}{6}$

22. $\cos t = \cos\left(\frac{3\pi}{2}\right) \Rightarrow t = \frac{\pi}{2}$

23. $\cos t = \cos\left(\frac{5\pi}{4}\right) \Rightarrow t = \frac{3\pi}{4}$

24. $\cos t = \cos\left(-\frac{4\pi}{6}\right) \Rightarrow t = \frac{2\pi}{3}$

25. $\cos t = \cos\left(-\frac{5\pi}{8}\right) \Rightarrow t = \frac{5\pi}{8}$

26. $\cos t = \cos\left(-\frac{3\pi}{4}\right) \Rightarrow t = \frac{3\pi}{4}$

27. $\sin t = \sin\left(\frac{3\pi}{4}\right) \Rightarrow t = \frac{\pi}{4}$

28. $\sin t = \sin\left(\frac{7\pi}{6}\right) \Rightarrow t = -\frac{\pi}{6}$

29. $\sin t = \sin\left(-\frac{4\pi}{3}\right) \Rightarrow t = \frac{\pi}{3}$

30. $\sin t = -\sin\left(\frac{3\pi}{8}\right) \Rightarrow t = -\frac{3\pi}{8}$

31. $\sin t = -\sin\left(\frac{\pi}{6}\right) \Rightarrow t = -\frac{\pi}{6}$

32. $\sin t = -\sin\left(-\frac{\pi}{3}\right) \Rightarrow t = \frac{\pi}{3}$

33. $\sin t = \cos t \Rightarrow t = \frac{\pi}{4}$

34. $\sin t = -\cos t \Rightarrow t = -\frac{\pi}{4}$

35. $\cos t$ decreases from 1 to –1 as t increases from 0 to π .

36. As t increases from π to $\frac{3\pi}{2}$, $\sin t$ decreases from 0 to –1. From $\frac{3\pi}{2}$ to 2π, $\sin t$ increases from –1 to 0.

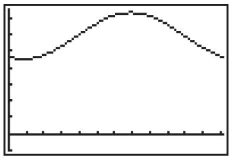
37. $\sin 5\pi = 0$; $\sin(-2\pi) = 0$
$\sin\left(\frac{17\pi}{2}\right) = 1; \sin\left(\frac{-13\pi}{2}\right) = -1$

38. $\cos 5\pi = -1$; $\cos(-2\pi) = 1$;
$\cos\left(\frac{17\pi}{2}\right) = 0; \cos\left(\frac{-13\pi}{2}\right) = 0$

39. $\sin(.19) = \sqrt{1 - \cos^2 .19} = .2$
$\cos(.19 - 4\pi) = \cos(.19) = .98$
$\cos(-.19) = \cos(.19) = .98$
$\sin(-.19) = -\sin(.19) = -.2$

40. $\sin(-.42) = -\sin(.42) = -.41$
$\sin(6\pi - .42) = \sin(-.42) = -.41$
$\cos(.42) = \sqrt{1 - \sin^2 .42} = .91$

41. a.

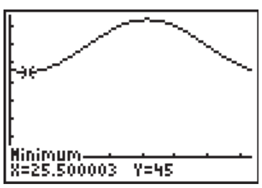


$[0, 365]$ *by* $[-10, 75]$

b. $59 + 14\cos\left[\dfrac{2\pi}{365}(45 - 208)\right]$
$\approx 59 + 14\cos(-2.81) \approx 45.76$
The temperature on February 14 is approximately 46°.

c. The coldest temperature will be when cosine has the value –1, and the warmest temperature will be when cosine has the value 1.
59 + 14(–1) = 45° is the coldest tap water temperature
59 + 14(1) = 73° is the warmest tap water temperature

d.



Estimate: $t = 25.5$

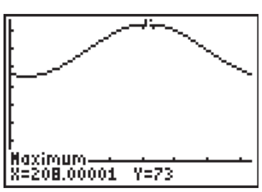
Since $\cos(-\pi) = -1$, $\dfrac{2\pi}{365}(t - 208) = -\pi$

$t - 208 = -\dfrac{365}{2}$

$t = -\dfrac{365}{2} + 208 = 25.5$

Tap water is the coldest on January 26.

e.



Estimate: $t = 208$

Since $\cos(0) = 1$, $\dfrac{2\pi}{365}(t - 208) = 0$

$t - 208 = 0$; $t = 208$

Tap water is the warmest on July 27.

f. $59 + 14\cos\left[\dfrac{2\pi}{365}(t - 208)\right] = 59$

$$\cos\left[\frac{2\pi}{365}(t - 208)\right] = 0$$

$$\frac{2\pi}{365}(t - 208) = \frac{\pi}{2}$$

$$t - 208 = \frac{365}{4}$$

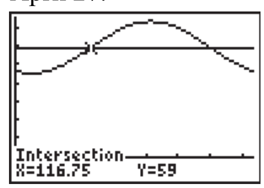
$$t = 299.25$$

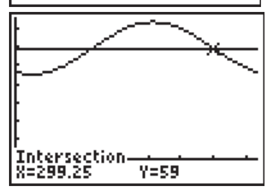
$$\frac{2\pi}{365}(t - 208) = \frac{3\pi}{2}$$

$$t - 208 = \frac{1095}{4}$$

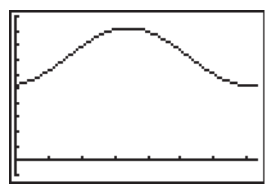
$t = 481.75$, which is equivalent to $481.75 - 365 = 116.75$.

Tap water is 59° on October 27 and April 27.





42. a.

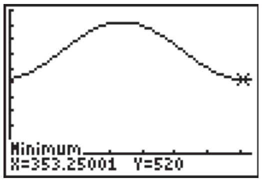


$[0, 365]$ by $[-100, 1000]$

b. $720 + 200\sin\left(\dfrac{2\pi}{365}(45 - 79.5)\right) \approx 608.08$

c. The shortest amount of daylight will be when sine has the value –1, and the longest amount of daylight will be when sine has the value 1.
720 + 200(–1) = 520 minutes of daylight
720 + 200(1) = 920 minutes of daylight

d.



Estimate: $t = 353.25$

Since $\sin\left(\frac{3\pi}{2}\right) = -1$,

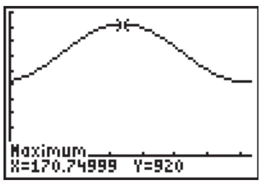
$$\frac{2\pi}{365}(t - 79.5) = \frac{3\pi}{2} \Rightarrow$$

$$t - 79.5 = \frac{1095}{4} \Rightarrow t = 353.25$$

$t = 353.25$

The shortest amount of daylight is on December 20.

e.



Estimate: $t = 170.75$

Since $\sin\left(\frac{\pi}{2}\right) = 1, \frac{2\pi}{365}(t - 79.5) = \frac{\pi}{2} \Rightarrow$

$$t - 79.5 = \frac{365}{4} \Rightarrow t = 170.75$$

The longest amount of daylight is on June 20.

f. $720 + 200\sin\left[\frac{2\pi}{365}(t - 79.5)\right] = 720$

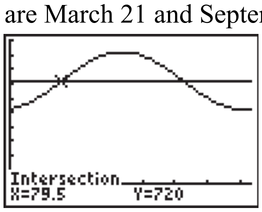
$$\sin\left[\frac{2\pi}{365}(t - 79.5)\right] = 0$$

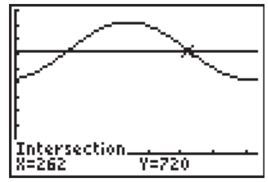
$$\frac{2\pi}{365}(t - 79.5) = 0 \qquad \frac{2\pi}{365}(t - 79.5) = \pi$$

$$t = 79.5 \qquad t - 79.5 = \frac{365}{2}$$

$$t = 262$$

The two days during which the amount of daylight equals the amount of darkness are March 21 and September 19.





8.3 Differentiation and Integration of sin t and cos t

1. $\frac{d}{dt}\sin 4t = (\cos 4t)(4) = 4\cos 4t$

2. $\frac{d}{dt}(2\cos 2t) = 2(-\sin 2t)(2) = -4\sin 2t$

3. $\frac{d}{dt}4\sin t = 4(\cos t) = 4\cos t$

4. $\frac{d}{dt}\cos(-4t) = -\sin(-4t)(-4) = 4\sin(-4t)$

5. $\frac{d}{dt}2\cos 3t = 2(-\sin 3t)(3) = -6\sin 3t$

6. $\frac{d}{dt}\left(-\frac{\sin 3t}{3}\right) = \frac{d}{dt}\left(-\frac{1}{3}\sin 3t\right) = -\frac{1}{3}(\cos 3t)(3)$
$= -\cos 3t$

7. $\frac{d}{dt}(t + \cos \pi t) = 1 + (-\sin \pi t)(\pi) = 1 - \pi\sin \pi t$

8. $\frac{d}{dt}(t\cos t) = t(-\sin t) + (1)\cos t = \cos t - t\sin t$

9. $\frac{d}{dt}\sin(\pi - t) = \cos(\pi - t)(-1) = -\cos(\pi - t)$

10. $\frac{d}{dt}\left(\frac{\cos(2x+2)}{2}\right) = \frac{d}{dt}\left(\frac{1}{2}\cos(2x+2)\right)$
$= \frac{1}{2}\left(-\sin(2x+2)\right)(2)$
$= -\sin(2x+2)$

11. $\frac{d}{dt}\cos^3 t = (3\cos^2 t)(-\sin t) = -3\cos^2 t\sin t$

12. $\frac{d}{dt}\sin^3 t^2 = \frac{d}{dt}\left(\sin t^2\right)^3$
$= 3\left(\sin t^2\right)^2\left(\cos t^2\right)(2t)$
$= 6t\cos t^2\sin^2 t^2$

13. $\frac{d}{dx}\sin\sqrt{x-1} = \cos(x-1)^{1/2}\cdot\frac{1}{2}(x-1)^{-1/2}(1)$
$= \frac{\cos\sqrt{x-1}}{2\sqrt{x-1}}$

14. $\frac{d}{dx}\cos e^x = -\sin e^x(e^x) = -e^x\sin(e^x)$

15. $\frac{d}{dx}\sqrt{\sin(x-1)} = \frac{1}{2}\left(\sin(x-1)\right)^{-1/2}\left(\cos(x-1)\right)(1)$
$= \frac{\cos(x-1)}{2\sqrt{\sin(x-1)}}$

16. $\frac{d}{dx}e^{\cos x} = e^{\cos x}(-\sin x) = -(\sin x)e^{\cos x}$

17. $\frac{d}{dx}(1+\cos t)^8 = 8(1+\cos t)^7(-\sin t)$
$= -8\sin t(1+\cos t)^7$

18. $\frac{d}{dt}\sqrt[3]{\sin \pi t} = \frac{1}{3}(\sin \pi t)^{-2/3}(\cos \pi t)(\pi)$
$= \frac{1}{3}\pi\cos \pi t(\sin \pi t)^{-2/3} = \frac{\pi\cos \pi t}{3\sqrt[3]{(\sin \pi t)^2}}$

19. $\frac{d}{dx}\cos^2 x^3 = 2\left(\cos x^3\right)\left(-\sin x^3\right)\left(3x^2\right)$
$= -6x^2\sin x^3\cos x^3$

20. $\frac{d}{dx}\left(\cos^2 x+\sin^2 x\right) = \frac{d}{dx}(1) = 0$

21. $\frac{d}{dx}(e^x\sin x) = e^x\cos x + e^x\sin x$
$= e^x(\cos x+\sin x)$

22. $\frac{d}{dx}(\cos x+\sin x)^2$
$= 2(\cos x+\sin x)(-\sin x+\cos x)$
$= 2(\cos^2 x-\sin^2 x)$

23. $\frac{d}{dx}\left[\sin(2x)\cos(3x)\right]$
$= \sin(2x)\cdot\left[-\sin(3x)\right](3)$
$+\left[2\cos(2x)\right]\cos(3x)$
$= 2\cos(2x)\cos(3x) - 3\sin(2x)\sin(3x)$

24. $\frac{d}{dx}\left(\frac{1+x}{\cos x}\right) = \frac{\cos x(1)-(1+x)(-\sin x)}{\cos^2 x}$
$= \frac{\cos x+\sin x+x\sin x}{\cos^2 x}$

25. $\frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) = \frac{\cos t(\cos t)-\sin t(-\sin t)}{\cos^2 t}$
$= \frac{\cos^2 t+\sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$

26. $\frac{d}{dx}\cos\left[e^{2x+3}\right] = -\sin\left[e^{2x+3}\right]\left(e^{2x+3}\right)(2)$
$= -2e^{2x+3}\sin\left(e^{2x+3}\right)$

27. $\frac{d}{dt}\ln(\cos t) = \frac{1}{\cos t}(-\sin t) = -\frac{\sin t}{\cos t} = -\cot t$

28. $\frac{d}{dt}\ln(\sin 2t) = \frac{1}{\sin 2t}(\cos 2t)(2) = \frac{2\cos 2t}{\sin 2t}$
$= 2\cot 2t$

29. $\frac{d}{dt}\sin(\ln t) = \cos(\ln t)\cdot\left(\frac{1}{t}\right) = \frac{\cos(\ln t)}{t}$

30. $\frac{d}{dt}\left[(\cos t)\ln t\right] = \cos t\cdot\frac{1}{t} - \sin t(\ln t)$
$= \frac{\cos t}{t} - (\sin t)\ln t$

31. $y = \cos 3x$

slope $= \frac{dy}{dx} = -(\sin 3x)(3) = -3\sin 3x$

When $x = \frac{13\pi}{6}$,

slope $= -3\sin\left(3\cdot\frac{13\pi}{6}\right) = -3(1) = -3.$

32. $y = \sin 2x$

slope $= \frac{dy}{dx} = (\cos 2x)(2) = 2\cos 2x$

When $x = \frac{5\pi}{4}$,

slope $= 2\cos\left(2\cdot\frac{5\pi}{4}\right) = 2(0) = 0.$

33. $y = 3\sin x + \cos 2x$

When $x = \frac{\pi}{2}$,

$y = 3\sin\frac{\pi}{2} + \cos 2\left(\frac{\pi}{2}\right) = 3 + (-1) = 2.$

slope $= \frac{dy}{dx} = 3\cos x + (-\sin 2x)2$
$= 3\cos x - 2\sin 2x$

When $x = \frac{\pi}{2}$,

slope $= 3\cos\frac{\pi}{2} - 2\sin 2\cdot\frac{\pi}{2} = 0 - 0 = 0.$

The equation of the tangent line is

$y - 2 = 0\left(x - \frac{\pi}{2}\right)$ or $y = 2.$

34. $y = 3\sin 2x - \cos 2x$

When $x = \frac{3\pi}{4}$,

$$y = 3\sin 2\left(\frac{3\pi}{4}\right) - \cos 2\left(\frac{3\pi}{4}\right) = -3 - 0 = -3.$$

$$\text{slope} = \frac{dy}{dx} = 3(\cos 2x)2 - (-\sin 2x)2 = 6\cos 2x + 2\sin 2x$$

When $x = \frac{3\pi}{4}$,

$$\text{slope} = 6\cos 2\left(\frac{3\pi}{4}\right) + 2\sin 2\left(\frac{3\pi}{4}\right) = 0 - 2 = -2$$

The equation of the tangent line is

$y - (-3) = -2\left(x - \frac{3\pi}{4}\right)$ or $y = -2x + \frac{3\pi}{2} - 3$.

35. $\int \cos 2x \; dx = \frac{1}{2}\sin 2x + C$

36. $$\int 3\sin 3x \; dx = 3\left(-\frac{1}{3}\cos 3x\right) + C = -\cos 3x + C$$

37. $$\int -\frac{1}{2}\cos\frac{x}{7} \; dx = -\frac{1}{2}\left(7\sin\frac{x}{7}\right) + C = -\frac{7}{2}\sin\frac{x}{7} + C$$

38. $\int 2\sin\frac{x}{2}dx = 2\left(-2\cos\frac{x}{2}\right) + C = -4\cos\frac{x}{2} + C$

39. $\int (\cos x - \sin x)dx = \sin x + \cos x + C$

40. $$\begin{aligned}\int\left(2\sin 3x + \frac{\cos 2x}{2}\right)dx &= \int\left(2\sin 3x + \frac{1}{2}\cos 2x\right)dx \\ &= 2\left(-\frac{1}{3}\cos 3x\right) + \frac{1}{2}\left(\frac{1}{2}\sin 2x\right) + C \\ &= -\frac{2}{3}\cos 3x + \frac{1}{4}\sin 2x + C\end{aligned}$$

41. $$\begin{aligned}\int\big(-\sin x + 3\cos(-3x)\big)\,dx &= \cos x - \sin(-3x) + C \\ &= \cos x + \sin 3x + c\end{aligned}$$

42. $$\int \sin(-2x)dx = \frac{1}{2}\cos(-2x) + C = \frac{1}{2}\cos(2x) + C$$

43. $\int \sin(4x+1)\; dx = -\frac{1}{4}\cos(4x+1) + C$

44. $$\begin{aligned}\int \cos\left(\frac{x-2}{2}\right)dx = \int \cos\left(\frac{1}{2}x - 1\right)dx &= 2\sin\left(\frac{1}{2}x - 1\right) + C \\ &= 2\sin\left(\frac{x-2}{2}\right) + C\end{aligned}$$

45. $\int_0^{\pi/2} \cos t\, dt = \sin t\Big|_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = 1$

46. $$\begin{aligned}\int_0^{\pi/4} \sin 2t\, dt &= -\frac{1}{2}\cos 2t\Big|_0^{\pi/4} \\ &= -\frac{1}{2}\left[\cos\left(2\cdot\frac{\pi}{4}\right) - \cos(2\cdot 0)\right] \\ &= -\frac{1}{2}\left(\cos\frac{\pi}{2} - \cos 0\right) \\ &= -\frac{1}{2}(-1) = \frac{1}{2}\end{aligned}$$

47. a. $P = 100 + 20\cos 6t$

$$\frac{dP}{dt} = (-20\sin 6t)6 = -120\sin 6t$$

$$\frac{d^2P}{dt^2} = (-120\cos 6t)6 = -720\cos 6t$$

Setting $\frac{dP}{dt} = 0$ gives $-120\sin 6t = 0 \Rightarrow$

$\sin 6t = 0 \Rightarrow 6t = 0, \pi, 2\pi, 3\pi, \ldots \Rightarrow$

$t = 0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \ldots$

When $t = 0$,

$$\frac{d^2P}{dt^2} = -720\cos 6(0) = -720.$$

When $t = \frac{\pi}{6}$,

$$\frac{d^2P}{dt^2} = -720\cos 6\left(\frac{\pi}{6}\right) = -720(-1) = 720.$$

When $t = \frac{\pi}{3}$,

$$\frac{d^2P}{dt^2} = -720\cos 6\left(\frac{\pi}{3}\right) = -720(1) = -720.$$

When $t = \frac{\pi}{2}$,

$$\frac{d^2P}{dt^2} = -720\cos 6\left(\frac{\pi}{2}\right) = -720(-1) = 720.$$

(*continued on next page*)

(*continued*)

Thus $t = 0$ and $t = \frac{\pi}{3}$ give relative maximum values for P. The maximum value is

$$P = 100 + 20\cos 6\left(\frac{\pi}{3}\right)$$
$$= 100 + 20(1) = 120$$

$t = \frac{\pi}{6}$ and $t = \frac{\pi}{2}$ give minimum values for P. The minimum value is

$$P = 100 + 20\cos 6\left(\frac{\pi}{6}\right) = 100 - 20 = 80.$$

b. The length of time between two maximum values of P is $\frac{\pi}{3}$ seconds. The heart beats every $\frac{\pi}{3}$ seconds. Therefore the heart rate is $\frac{180}{\pi} \approx 57$ beats per minute.

48. $BMR(t) = .4 + .2\cos\left(\frac{\pi t}{12}\right)$;

$$BM = \int BMR(t)\ dt$$

$$\int \left[.4 + .2\cos\left(\frac{\pi t}{12}\right)\right] dt = .4t + \frac{2.4}{\pi}\sin\left(\frac{\pi t}{12}\right) + C$$

$$\int_0^{24}\left[.4 + .2\cos\left(\frac{\pi t}{12}\right)\right] dt$$
$$= \left[.4t + \frac{2.4}{\pi}\sin\left(\frac{\pi t}{12}\right)\right]\Bigg|_0^{24}$$
$$= \left[.4(24) + \frac{2.4}{\pi}(0)\right] - \left[.4(0) + \frac{2.4}{\pi}(0)\right]$$
$$= 9.6 \text{ kcal}$$

49. Since $\sin\frac{\pi}{2} = 1$,

$$\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}+h\right)-1}{h} = \lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}+h\right)-\sin\left(\frac{\pi}{2}\right)}{h}.$$

$f'(a) \approx \frac{f(a+\Delta x) - f(a)}{\Delta x}$ so

$f(x) = \sin x$ and $\Delta x = h,\ a = \frac{\pi}{2}.$

Therefore,

$$\left(\sin(a)\right)' = \frac{\sin(a+h)-\sin(a)}{h}$$
$$= \frac{\sin\left(\frac{\pi}{2}+h\right)-\sin\left(\frac{\pi}{2}\right)}{h}$$

but $(\sin(a))' = \left(\sin\left(\frac{\pi}{2}\right)\right)' = \cos\left(\frac{\pi}{2}\right) = 0$.

50. $f(x) = \cos(x)$, $\Delta x = h$, $a = \pi$
So,

$$\left(\cos(a)\right)' = \lim_{h\to 0}\frac{\cos(\pi+h)-\cos\pi}{h}$$
$$= \lim_{h\to 0}\frac{\cos(\pi+h)+1}{h} \Rightarrow$$
$$\left(\cos(\pi)\right)' = -\sin\pi = 0$$

51. a. $f(18) = 54 + 23\sin\left[\frac{2\pi}{52}(18-12)\right]$
$\approx 54 + 23\sin(.72) \approx 69.17 \approx 69°$

b. $f'(t) = 23\cos\left[\frac{2\pi}{52}(t-12)\right]\left(\frac{2\pi}{52}\right)$

$$f'(20) = 23\cos\left[\frac{2\pi}{52}(20-12)\right]\left(\frac{2\pi}{52}\right)$$
$$\approx 23\cos(.97)\left(\frac{2\pi}{52}\right) \approx 1.6$$

The temperature is increasing 1.6 degrees per week.

c. $54 + 23\sin\left[\frac{2\pi}{52}(t-12)\right] = 39$

$$\sin\left(\frac{2\pi}{52}(t-12)\right) = -0.65 \Rightarrow$$

$\frac{2\pi}{52}(t-12) = -0.71$ or $\frac{2\pi}{52}(t-12) = 0.71 + \pi$
$t - 12 = -5.88$ $\quad$ $t - 12 = 31.88$
$t = 6.12$ $\quad$ $t = 43.88$

Average weekly temperature is 39° during weeks 6 and 44.

d. $23\cos\left[\frac{2\pi}{52}(t-12)\right]\left(\frac{2\pi}{52}\right)=-1$

$$\cos\left[\frac{2\pi}{52}(t-12)\right]=-0.36$$

$\frac{2\pi}{52}(t-12)=1.94 \Rightarrow t-12=16.06 \Rightarrow$
$t=28.06$ or
$\frac{2\pi}{52}(t-12)=2\pi-1.94 \Rightarrow$
$t-12=35.94 \Rightarrow t=47.94$
Average weekly temperature is falling 1° per week during weeks 28 and 48.

e. The average weekly temperature is greatest when sine is 1 and least when sine is –1. Since

$\sin\left(\frac{\pi}{2}\right)=1$ and $\sin\left(\frac{3\pi}{2}\right)=-1,$

$\frac{2\pi}{52}(t-12)=\frac{\pi}{2}$ or $\frac{2\pi}{52}(t-12)=\frac{3\pi}{2}$
$t-12=13$ $t-12=39$
$t=25$ $t=51$
The average weekly temperature is greatest at week 25 and least at week 51.

f. The average weekly temperature is increasing fastest when cosine is 1 and decreasing fastest when cosine is –1. Since cos (0) = 1 and cos (π) = –1,

$\frac{2\pi}{52}(t-12)=0$ or $\frac{2\pi}{52}(t-12)=\pi$
$t=12$ $t-12=26$
$t=38$
The average weekly temperature is increasing fastest at week 12 and decreasing fastest at week 38.

52. a. $f(42)=12.18+2.725\sin\left[\frac{2\pi}{52}(42-12)\right]$
$\approx 12.18+2.725\sin(3.62)\approx 10.9$ hr

b. $f'(t)=2.725\cos\left[\frac{2\pi}{52}(t-12)\right]\left(\frac{2\pi}{52}\right)$

$f'(32)=2.725\cos\left[\frac{2\pi}{52}(32-12)\right]\left(\frac{2\pi}{52}\right)$
$\approx 2.725\cos(2.42)\left(\frac{2\pi}{52}\right)$
$\approx -.25$ hours per week

c. $12.18+2.725\sin\left[\frac{2\pi}{52}(t-12)\right]=14$

$$\sin\left[\frac{2\pi}{52}(t-12)\right]=.67 \Rightarrow$$

$\frac{2\pi}{52}(t-12)=.73 \Rightarrow t-12=6.04$
$t=18.04$ or
$\frac{2\pi}{52}(t-12)=\pi-.73 \Rightarrow$
$t-12=19.96 \Rightarrow t=31.96$
There are 14 hours of daylight per day during weeks 18 and 32.

d. $2.725\cos\left[\frac{2\pi}{52}(t-12)\right]\left(\frac{2\pi}{52}\right)=.25$

$$\cos\left[\frac{2\pi}{52}(t-12)\right]=.76 \Rightarrow$$

$\frac{2\pi}{52}(t-12)=0.71 \Rightarrow t-12=5.88 \Rightarrow$
$t=17.88$ or
$\frac{2\pi}{52}(t-12)=2\pi-0.71 \Rightarrow$
$t-12=46.12 \Rightarrow t=58.12$, which is equivalent to $t=6.12$
The number of hours of daylight is increasing 15 minutes per week during weeks 18 and 6.

e. The days are longest when sine is 1 and shortest when sine is –1. Since

$\sin\left(\frac{\pi}{2}\right)=1$ and $\sin\left(\frac{3\pi}{2}\right)=-1,$

$\frac{2\pi}{52}(t-12)=\frac{\pi}{2}$ or $\frac{2\pi}{52}(t-12)=\frac{3\pi}{2}$
$t=25$ $t=51$
Days are longest at week 25 and shortest at week 51.

f. The number of hours of daylight is increasing fastest when cosine is 1 and decreasing fastest when cosine is –1. Since cos (0) = 1 and cos (π) = –1,

$\frac{2\pi}{52}(t-12)=0$ or $\frac{2\pi}{52}(t-12)=\pi$
$t=12$ $t=38$
The number of hours of daylight is increasing fastest at week 12 and decreasing fastest at week 38.

8.4 The Tangent and Other Trigonometric Functions

1. $\sec t=\frac{1}{\cos t}=\frac{\text{hyp.}}{\text{adj.}}$

2. $\cot t = \dfrac{\cos t}{\sin t} = \dfrac{\text{adj.}}{\text{opp.}}$

3. $(\text{adj.})^2 + 5^2 = 13^2 \Rightarrow \text{adj.} = 12$

$\tan t = \dfrac{\sin t}{\cos t} = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{5}{12}$

$\sec t = \dfrac{1}{\cos t} = \dfrac{\text{hyp.}}{\text{adj.}} = \dfrac{13}{12}$

4. $3^2 + (\text{opp.})^2 = 4^2 \Rightarrow \text{opp.} = \sqrt{7}$

$\tan t = \dfrac{\sin t}{\cos t} = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{\sqrt{7}}{3}$

$\sec t = \dfrac{1}{\cos t} = \dfrac{\text{hyp.}}{\text{adj.}} = \dfrac{4}{3}$

5. $(-2)^2 + (1)^2 = (\text{hyp.})^2 \Rightarrow \text{hyp.} = \sqrt{5}$

$\tan t = \dfrac{\sin t}{\cos t} = \dfrac{\text{opp.}}{\text{adj.}} = -\dfrac{1}{2}$

$\sec t = \dfrac{1}{\cos t} = \dfrac{\text{hyp.}}{\text{adj.}} = -\dfrac{\sqrt{5}}{2}$

6. $2^2 + (-3)^2 = (\text{hyp.})^2 \Rightarrow \text{hyp.} = \sqrt{13}$

$\tan t = \dfrac{\sin t}{\cos t} = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{-3}{2}$

$\sec t = \dfrac{1}{\cos t} = \dfrac{\text{hyp.}}{\text{adj.}} = \dfrac{\sqrt{13}}{2}$

7. $(-2)^2 + 2^2 = (\text{hyp.})^2 \Rightarrow \text{hyp.} = \sqrt{8} = 2\sqrt{2}$

$\tan t = \dfrac{\sin t}{\cos t} = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{2}{-2} = -1$

$\sec t = \dfrac{1}{\cos t} = \dfrac{\text{hyp.}}{\text{adj.}} = \dfrac{2\sqrt{2}}{-2} = -\sqrt{2}$

8. $(0.6)^2 + (0.8)^2 = (\text{hyp.})^2 \Rightarrow \text{hyp.} = 1$

$\tan t = \dfrac{\sin t}{\cos t} = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{.8}{.6} = \dfrac{4}{3}$

$\sec t = \dfrac{1}{\cos t} = \dfrac{\text{hyp.}}{\text{adj.}} = \dfrac{1}{.6} = \dfrac{5}{3}$

9. $(-.6)^2 + (-.8)^2 = (\text{hyp.})^2 \Rightarrow \text{hyp.} = 1$

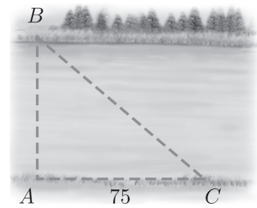
$\tan t = \dfrac{\sin t}{\cos t} = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{-.8}{-.6} = \dfrac{4}{3}$

$\sec t = \dfrac{1}{\cos t} = \dfrac{\text{hyp.}}{\text{adj.}} = \dfrac{1}{-.6} = -\dfrac{5}{3}$

10. $(0.8)^2 + (-0.6)^2 = (\text{hyp.})^2 \Rightarrow \text{hyp.} = 1$

$\tan t = \dfrac{\sin t}{\cos t} = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{-0.6}{0.8} = -\dfrac{3}{4}$

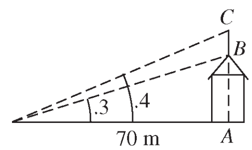
$\sec t = \dfrac{1}{\cos t} = \dfrac{\text{hyp.}}{\text{adj.}} = \dfrac{1}{0.8} = \dfrac{5}{4}$

11. 

$40° = 40° \times \dfrac{\pi}{180°} = \dfrac{2\pi}{9} \approx 0.7$

$\tan(0.7) = \dfrac{\sin t}{\cos t} = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{AB}{75}$

$AB \approx 75 \tan(0.7) \approx 63$ feet

12. 

Let AC be the distance between the ground and the top of the spire, and AB be the distance between the ground and the top of the church.

$\tan(.4) = \dfrac{AC}{70} \Rightarrow AC = 70\tan(.4)$

$\tan(.3) = \dfrac{AB}{70} \Rightarrow AB = 70\tan(.3)$

The height of the spire is
70 tan (.4) – 70 tan (.3) ≈ 7.94 meters.

13. $\dfrac{d}{dt}\sec t = \dfrac{d}{dt}\left(\dfrac{1}{\cos t}\right) = \dfrac{0-(-\sin t)(1)}{\cos^2 t}$
$= \tan t \sec t$

14. $\dfrac{d}{dt}\csc t = \dfrac{d}{dt}\left(\dfrac{1}{\sin t}\right) = \dfrac{0-(\cos t)(1)}{\sin^2 t}$
$= -\cot t \csc t$

15. $\dfrac{d}{dt}\cot t = \dfrac{d}{dt}\left(\dfrac{\cos t}{\sin t}\right)$
$= \dfrac{-\sin t(\sin t) - \cos t(\cos t)}{\sin^2 t}$
$= \dfrac{-\sin^2 t - \cos^2 t}{\sin^2 t} = \dfrac{-1}{\sin^2 t}$
$= -\csc^2 t$

16. $\dfrac{d}{dt}\cot 3t = \dfrac{d}{dt}\left(\dfrac{\cos 3t}{\sin 3t}\right) = \dfrac{(-\sin 3t)(3)\sin 3t - \cos 3t(\cos 3t)(3)}{\sin^2 3t} = \dfrac{-3(\sin^2 3t + \cos^2 3t)}{\sin^2 3t}$
$= \dfrac{-(3)(1)}{\sin^2 3t} = -3\csc^2 3t$

17. $\dfrac{d}{dt}\tan 4t = \dfrac{d}{dt}\left(\dfrac{\sin 4t}{\cos 4t}\right) = \dfrac{(\cos 4t)(4)\cos 4t - \sin 4t(-\sin 4t)(4)}{\cos^2 4t} = \dfrac{4(\cos^2 4t + \sin^2 4t)}{\cos^2 4t}$
$= \dfrac{4(1)}{\cos^2 4t} = 4\sec^2 4t$

18. $\dfrac{d}{dt}\tan \pi t = \sec^2(\pi t)\cdot \pi = \pi\sec^2(\pi t)$

19. $\dfrac{d}{dx}\left[3\tan(\pi - x)\right] = 3\sec^2(\pi - x)(-1) = -3\sec^2(\pi - x)$

20. $\dfrac{d}{dx}\left[5\tan(2x+1)\right] = 5\sec^2(2x+1)(2) = 10\sec^2(2x+1)$

21. $\dfrac{d}{dx}\left[4\tan(x^2+x+3)\right] = 4\sec^2(x^2+x+3)(2x+1) = (8x+4)\sec^2(x^2+x+3)$

22. $\dfrac{d}{dx}\left[3\tan(1-x^2)\right] = 3\sec^2(1-x^2)(-2x) = -6x\sec^2(1-x^2)$

23. $\dfrac{d}{dx}\tan\sqrt{x} = \dfrac{d}{dx}\tan x^{1/2} = \sec^2 x^{1/2}\left(\dfrac{1}{2}x^{-1/2}\right) = \dfrac{\sec^2\sqrt{x}}{2\sqrt{x}}$

24. $\dfrac{d}{dx}\left[2\tan\sqrt{x^2-4}\right] = \dfrac{d}{dx}2\tan(x^2-4)^{1/2} = 2\sec^2(x^2-4)^{1/2}\left(\dfrac{1}{2}\left(x^2-4\right)^{-1/2}(2x)\right) = \dfrac{2x\sec^2\sqrt{x^2-4}}{\sqrt{x^2-4}}$

25. $\dfrac{d}{dx}\left[x\tan x\right] = x\sec^2 x + (1)\tan x = x\sec^2 x + \tan x$

26. $\dfrac{d}{dx}\left[e^{3x}\tan 2x\right] = e^{3x}\sec^2 2x(2) + e^{3x}(3)\tan 2x = e^{3x}\left(2\sec^2 2x + 3\tan 2x\right)$

27. $\dfrac{d}{dx}\tan^2 x = \dfrac{d}{dx}(\tan x)^2 = 2\left(\tan x\right)\left(\sec^2 x\right) = 2\tan x\sec^2 x$

28. $\dfrac{d}{dx}\sqrt{\tan x} = \dfrac{d}{dx}\left(\tan x\right)^{1/2} = \dfrac{1}{2}\left(\tan x\right)^{-1/2}\left(\sec^2 x\right) = \dfrac{\sec^2 x}{2\sqrt{\tan x}}$

29. $\dfrac{d}{dt}(1+\tan 2t)^3 = 3\left(1+\tan 2t\right)^2\left(\sec^2 2t\right)(2) = 6\sec^2\left(2t\right)\left[1+\tan\left(2t\right)\right]^2$

30. $\dfrac{d}{dt}\tan^4 3t = \dfrac{d}{dt}(\tan 3t)^4 = 4\left(\tan 3t\right)^3\left(\sec^2 3t\right)(3) = 12(\tan^3 3t)\sec^2 3t$

31. $\frac{d}{dx}\ln(\tan t+\sec t)=\frac{d}{dx}\ln\left(\tan t+\frac{1}{\cos t}\right)=\frac{1}{\tan t+\sec t}\left(\sec^2 t+\frac{\sin t}{\cos^2 t}\right)$

$=\frac{1}{\tan t+\sec t}\left(\sec^2 t+\frac{\sin t}{\cos t}\cdot\frac{1}{\cos t}\right)=\frac{1}{\tan t+\sec t}\left(\sec^2 t+\tan t\sec t\right)$

$=\frac{1}{\tan t+\sec t}\left(\sec t+\tan t\right)\sec t=\sec t$

32. $\frac{d}{dx}\left[\ln(\tan t)\right]=\frac{1}{\tan t}\sec^2 t=\frac{\cos t}{\sin t}\cdot\frac{1}{\cos^2 t}$

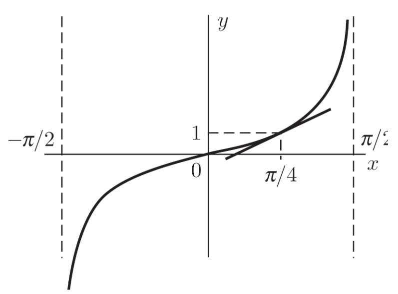
$=\frac{1}{\sin t\cos t}=\csc t\sec t$

33. a. $y=\tan x\Rightarrow\frac{dy}{dx}=\sec^2 x$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}}=\sec^2\frac{\pi}{4}=2$

$y-1=2\left(x-\frac{\pi}{4}\right)$

b.

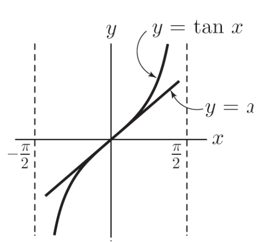


34. a. $y=\tan x\Rightarrow\frac{dy}{dx}=\sec^2 x\Rightarrow$

$\left.\frac{dy}{dx}\right|_{x=0}=\sec^2 0=1$

$y-0=1(x-0)\Rightarrow y=x$

b.



35. $\int\sec^2 3x\ dx=\frac{1}{3}\tan 3x+C$

36. $\int\sec^2(2x+1)dx=\frac{1}{2}\tan(2x+1)+C$

37. $\int_{-\pi/4}^{\pi/4}\sec^2 x\ dx=(\tan x)\Big|_{-\pi/4}^{\pi/4}=1-(-1)=2$

38. $\int_{-\pi/8}^{\pi/8}\sec^2\left(x+\frac{\pi}{8}\right)dx=\tan\left(x+\frac{\pi}{8}\right)\Big|_{-\pi/8}^{\pi/8}$

$=1-0=1$

39. $\int\frac{1}{\cos^2 x}dx=\int\sec^2 x\,dx=\tan x+C$

40. $\int\frac{3}{\cos^2 2x}dx=3\int\sec^2 2x\,dx=\frac{3}{2}\tan 2x+C$

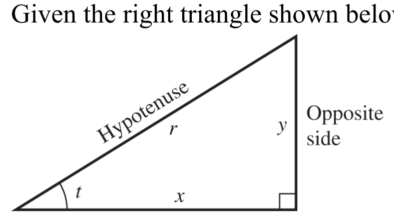
Chapter 8 Fundamental Concept Check Exercises

1. On a circle of radius 1, a central angle of one radian is determined by an arc of length 1 along the circumference.

2. To convert degree measure to radian measure, multiply the angle measure by $\frac{\pi}{180°}$.

$d=d°\cdot\frac{\pi}{180°}$ radians.

3. Given the right triangle shown below.

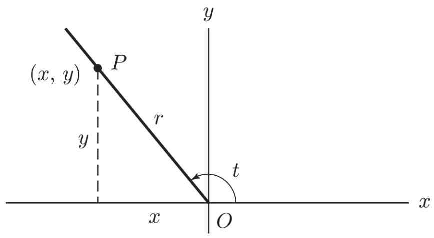


$\sin\theta=\frac{\text{Opposite}}{\text{Hypotenuse}}=\frac{y}{r}$

$\cos\theta=\frac{\text{Adjacent}}{\text{Hypotenuse}}=\frac{x}{r}$

$\tan\theta=\frac{\text{Opposite}}{\text{Adjacent}}=\frac{y}{x}$

4.



Given a number t, consider an angle of t radians placed in standard position as shown above. Let $P(x, y)$ be a point on the terminal side of the angle. Let r be the length of the segment OP; that is $r = \sqrt{x^2 + y^2}$. Then, sint t, cos t, and tan t are defined as the following ratios: $\sin t = \frac{y}{r}$, $\cos t = \frac{x}{r}$, $\tan t = \frac{y}{x}$.

5. We say that the sine and cosine curves are periodic with period 2π because their graphs repeat every 2π units. Therefore $\sin(t + 2\pi) = \sin t$ and $\cos(t + 2\pi) = \cos t$.

6. Sin t: As t increases from $t = 0$ to $t = \frac{\pi}{2}$, sin t increases from 0 to 1. As t increases from $t = \frac{\pi}{2}$ to $t = \pi$, sin t decreases from 1 to 0. As t increases from $t = \pi$ to $t = \frac{3\pi}{2}$, sin t decreases from 0 to −1. As t increases from $t = \frac{3\pi}{2}$ to $t = 2\pi$ sin t increases from −1 to 0. The graph is periodic with period 2π, so it repeats.

Cos t: As t increases from $t = 0$ to $t = \frac{\pi}{2}$, cos t decreases from 1 to 0. As t increases from $t = \frac{\pi}{2}$ to $t = \pi$, cos t decreases from 0 to −1. As t increases from $t = \pi$ to $t = \frac{3\pi}{2}$, cos t increases from −1 to 0. As t increases from $t = \frac{3\pi}{2}$ to $t = 2\pi$ cos t increases from 0 to 1. The graph is periodic with period 2π, so it repeats. The graph of cos t is the graph of sin t shifted $\frac{\pi}{2}$ units left.

7. $\sin(-t) = -\sin t$; $\cos(-t) = \cos t$

$\sin\left(\frac{\pi}{2} - t\right) = \cos t$; $\cos\left(\frac{\pi}{2} - t\right) = \sin t$

$\sin(t \pm 2\pi) = \sin t$; $\cos\left(t \tan^{-1} 2\pi\right) = \cos t$

$\frac{\sin t}{\cos t} = \tan t$

$\sin^2 t + \cos^2 t = 1$

$\sin 2t = 2\sin t \cos t$

$\cos 2t = \cos^2 t - \sin^2 t$

$\cos 2t = 2\cos^2 t - 1$

$\cos 2t = 1 - 2\sin^2 t$

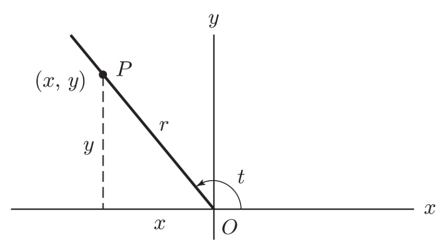
$\sin(t + u) = \sin t \cos u + \cos t \sin u$

$\sin(t - u) = \sin t \cos u - \cos t \sin u$

$\cos(t + u) = \cos t \cos u - \sin t \sin u$

$\cos(t - u) = \cos t \cos u + \sin t \sin u$

8.



$\cot t = \frac{x}{y} = \frac{\cos t}{\sin t}$, $\sec t = \frac{r}{x} = \frac{1}{\cos t}$,

$\csc t = \frac{r}{y} = \frac{1}{\sin t}$

9. $\tan^2 t + 1 = \sec^2 t$

10. $\frac{d}{dt}\left[\sin(g(t))\right] = \left[\cos(g(t))\right]g'(t)$

$\frac{d}{dt}\left[\cos(g(t))\right] = \left[-\sin(g(t))\right]g'(t)$

$\frac{d}{dt}\left[\tan(g(t))\right] = \left[\sec^2(g(t))\right]g'(t)$

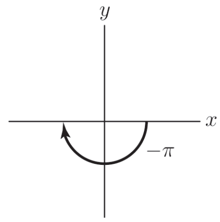
Chapter 8 Review Exercises

1. $t = \frac{3\pi}{2}$

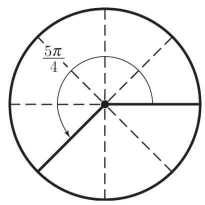
2. $t = -\frac{7\pi}{2}$

3. $t = -\frac{3\pi}{4}$

4.



5.



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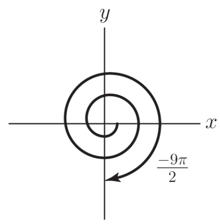


7. $3^2 + 4^2 = (\text{hyp.})^2 \Rightarrow \text{hyp.} = 5$

$\sin t = \dfrac{\text{opp.}}{\text{hyp.}} = \dfrac{4}{5}$; $\cos t = \dfrac{\text{adj.}}{\text{hyp.}} = \dfrac{3}{5}$

$\tan t = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{4}{3}$

8. $(-.6)^2 + (.8)^2 = (\text{hyp.})^2 \Rightarrow \text{hyp.} = 1$

$\sin t = \dfrac{\text{opp.}}{\text{hyp.}} = .8$; $\cos t = \dfrac{\text{adj.}}{\text{hyp.}} = -.6$

$\tan t = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{.8}{-.6} = -\dfrac{4}{3}$

9. $(-.6)^2 + (-.8)^2 = (\text{hyp.})^2 \Rightarrow \text{hyp.} = 1$

$\sin t = \dfrac{\text{opp.}}{\text{hyp.}} = -.8; \cos t = \dfrac{\text{adj.}}{\text{hyp.}} = -.6$

$\tan t = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{-.8}{-.6} = \dfrac{4}{3}$

10. $3^2 + (-4)^2 = (\text{hyp.})^2; \text{hyp.} = 5$

$\sin t = \dfrac{\text{opp.}}{\text{hyp.}} = -\dfrac{4}{5}; \cos t = \dfrac{\text{adj.}}{\text{hyp.}} = \dfrac{3}{5}$

$\tan t = \dfrac{\text{opp.}}{\text{adj.}} = -\dfrac{4}{3}$

11. $\sin t = \dfrac{1}{5}; (\text{opp.})^2 + (\text{adj.})^2 = (\text{hyp.})^2$

$1 + (\text{adj.})^2 = 25; \text{adj.} = \pm\sqrt{24} = \pm 2\sqrt{6}$

$\cos t = \dfrac{\text{adj.}}{\text{hyp.}} = \pm\dfrac{2\sqrt{6}}{5}$

12. $\cos t = -\dfrac{2}{3}; (\text{opp.})^2 + (\text{adj.})^2 = (\text{hyp.})^2$

$(\text{opp.})^2 + 4 = 9; \text{opp.} = \pm\sqrt{5}; \sin t = \dfrac{\pm\sqrt{5}}{3}$

13. $\dfrac{\pi}{4}, \dfrac{5\pi}{4}, -\dfrac{3\pi}{4}, -\dfrac{7\pi}{4}$

14. $\dfrac{3\pi}{4}, \dfrac{7\pi}{4}, -\dfrac{\pi}{4}, -\dfrac{5\pi}{4}$

15. negative

16. positive

17. Let r be the length of the rafter needed to support the roof.

$r^2 = (15)^2 + [15(\tan 23°)]^2 \Rightarrow r \approx 16.3$ ft

18. Let t be the height of the tree.
$t = 60(\tan 53°) \approx 79.62$ feet

19. $f(t) = 3 \sin t$; $\dfrac{d}{dt} 3\sin t = 3\cos t$

20. $f(t) = \sin 3t$; $\dfrac{d}{dt}\sin 3t = (\cos 3t)(3) = 3\cos 3t$

21. $f(t) = \sin\sqrt{t} = \sin t^{1/2}$

$\dfrac{d}{dt}\sin t^{1/2} = \left(\cos\left(t^{1/2}\right)\right)\left(\dfrac{1}{2}\right)t^{-1/2} = \dfrac{\cos\sqrt{t}}{2\sqrt{t}}$

22. $f(t) = \cos t^3$

$\dfrac{d}{dt}\cos t^3 = (-\sin t^3)(3t^2) = -3t^2 \sin t^3$

23. $g(x) = x^3 \sin x$

$\dfrac{d}{dx}(x^3 \sin x) = x^3 \cos x + 3x^2 \sin x$

24. $g(x) = \sin(-2x)\cos 5x$

$\dfrac{d}{dx}[\sin(-2x)\cos 5x]$
$= \sin(-2x)(-\sin 5x)(5) + \cos(-2x)(-2)\cos 5x$
$= -5 \sin 5x \sin(-2x) - 2\cos(-2x)\cos 5x$
$= 5 \sin 5x \sin 2x - 2\cos 5x \cos 2x$

25. $f(x) = \dfrac{\cos 2x}{\sin 3x}$

$\dfrac{d}{dx}\left(\dfrac{\cos 2x}{\sin 3x}\right)$
$= \dfrac{(-\sin 2x)(2)\sin 3x - (\cos 2x)(\cos 3x)(3)}{\sin^2 3x}$
$= -\dfrac{2\sin 2x \sin 3x + 3\cos 2x \cos 3x}{\sin^2 3x}$

26. $f(x)=\dfrac{\cos x-1}{x^3}$

$$\frac{d}{dx}\left(\frac{\cos x-1}{x^3}\right)=\frac{(-\sin x)(x^3)-3x^2(\cos x-1)}{x^6}$$
$$=\frac{-x^3\sin x-3x^2(\cos x-1)}{x^6}$$

27. $f(x)=\cos^3 4x$

$$\frac{d}{dx}\cos^3 4x=(3\cos^2 4x)(-\sin 4x)(4)$$
$$=-12\cos^2 4x\sin 4x$$

28. $f(x)=\tan^3 2x$

$$\frac{d}{dx}\tan^3 2x=3(\tan^2 2x)(\sec^2 2x)(2)$$
$$=6\tan^2(2x)\sec^2(2x)$$

29. $y=\tan(x^4+x^2)$

$$\frac{d}{dx}\tan(x^4+x^2)=\left(\sec^2\left(x^4+x^2\right)\right)\left(4x^3+2x\right)$$
$$=(4x^3+2x)\sec^2(x^4+x^2)$$

30. $y=\tan e^{-2x}$

$$\frac{d}{dx}\tan e^{-2x}=(\sec^2 e^{-2x})e^{-2x}(-2)$$
$$=-2e^{-2x}\sec^2\left(e^{-2x}\right)$$

31. $y=\sin(\tan x)$

$$\frac{d}{dx}\sin(\tan x)=\cos(\tan x)\sec^2 x$$

32. $y=\tan(\sin x)$

$$\frac{d}{dx}\tan(\sin x)=\sec^2(\sin x)\cos x$$

33. $y=\sin x\tan x$

$$\frac{d}{dx}[\sin x\tan x]=\sin x\sec^2 x+\cos x\tan x$$
$$=\sin x\sec^2 x+\sin x$$

34. $y=(\ln x)\cos x$

$$\frac{d}{dx}[(\ln x)\cos x]=\ln x(-\sin x)+\frac{1}{x}\cos x$$
$$=\frac{\cos x}{x}-(\ln x)\sin x$$

35. $y=\ln(\sin x)$

$$\frac{d}{dx}\ln(\sin x)=\frac{1}{\sin x}(\cos x)=\cot x$$

36. $y=\ln(\cos x)$

$$\frac{d}{dx}\ln(\cos x)=\frac{1}{\cos x}(-\sin x)=-\tan x$$

37. $y=e^{3x}\sin^4 x$

$$\frac{d}{dx}[e^{3x}\sin^4 x]$$
$$=e^{3x}(4\sin^3 x)(\cos x)+e^{3x}(3)\sin^4 x$$
$$=4e^{3x}\sin^3 x\cos x+3e^{3x}\sin^4 x$$

38. $y=\sin^4 e^{3x}$

$$\frac{d}{dx}\sin^4 e^{3x}=\left(4\sin^3 e^{3x}\right)\left(\cos e^{3x}\right)\left(e^{3x}\right)(3)$$
$$=12e^{3x}\left(\cos e^{3x}\right)\left(\sin^3 e^{3x}\right)$$

39. $f(t)=\dfrac{\sin t}{\tan 3t}$

$$\frac{d}{dt}\left(\frac{\sin t}{\tan 3t}\right)=\frac{\cos t\tan 3t-(\sec^2 3t)(3)\sin t}{\tan^2 3t}$$
$$=\frac{\cos t\tan 3t-3\sin t\sec^2 3t}{\tan^2 3t}$$

40. $f(t)=\dfrac{\tan 2t}{\cos t}$

$$\frac{d}{dt}\left(\frac{\tan 2t}{\cos t}\right)=\frac{(\sec^2 2t)(2)\cos t-(-\sin t)\tan 2t}{\cos^2 t}$$
$$=\frac{2\cos t\sec^2 2t+\sin t\tan 2t}{\cos^2 t}$$

41. $f(t)=e^{\tan t}$

$$\frac{d}{dt}e^{\tan t}=e^{\tan t}(\sec^2 t)$$

42. $f(t)=e^t\tan t$

$$\frac{d}{dt}[e^t\tan t]=e^t(\sec^2 t)+e^t\tan t$$
$$=e^t(\sec^2 t+\tan t)$$

43. $f(t)=\sin^2 t$

$f'(t)=2\sin t\cos t$

$f''(t)=2[(\sin t)(-\sin t)+\cos t\cos t]$

$=2(\cos^2 t-\sin^2 t)$

44. $y = 3 \sin 2t + \cos 2t$
$y' = 3(\cos 2t)(2) + (-\sin 2t)(2)$
$= 6\cos 2t - 2\sin 2t$
$y'' = 6[(-\sin 2t)(2)] - 2(\cos 2t)(2)$
$= -12\sin 2t - 4\cos 2t$
$= -4(3\sin 2t + \cos 2t)$
$-4y = -12\sin 2t - 4\cos 2t$
Therefore y'' and $-4y$ are equal.

45. $f(s, t) = \sin s \cos 2t$; $\dfrac{\partial f}{\partial s} = \cos s \cos 2t$

$\dfrac{\partial f}{\partial t} = \sin s\,(-\sin 2t)(2) = -2 \sin s \sin 2t$

46. $z = \sin wt$; $\dfrac{\partial z}{\partial w} = t \cos wt$; $\dfrac{\partial z}{\partial t} = w \cos wt$

47. $f(s, t) = t \sin st$; $\dfrac{\partial f}{\partial s} = t(\cos st)(t) = t^2 \cos st$

$\dfrac{\partial f}{\partial t} = t(\cos st)(s) + (1)\sin st = st\cos st + \sin st$

48. $\sin(s + t) = \sin s \cos t + \cos s \sin t$
$\dfrac{\partial}{\partial t}\sin(s+t) = \cos(s+t)$
$\dfrac{\partial}{\partial t}[\sin s\cos t + \cos s \sin t]$
$= \sin s(-\sin t) + \cos s \cos t$
$= \cos s\cos t - \sin s \sin t$
Thus, $\cos(s + t) = \cos s \cos t - \sin s \sin t$.

49. $y = \tan t = \dfrac{\sin t}{\cos t}$

When $t = \dfrac{\pi}{4}$, $y = \dfrac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = 1$

slope $= y' = \sec^2 t = \sec^2\left(\dfrac{\pi}{4}\right) = 2$

The tangent line is $y - 1 = 2\left(t - \dfrac{\pi}{4}\right)$.

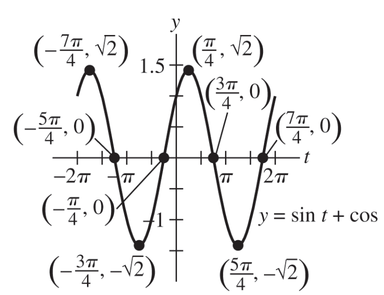
50. **a.** $f(t) = \sin t + \cos t$
$f'(t) = \cos t - \sin t$
$\cos t - \sin t = 0 \Rightarrow \cos t = \sin t \Rightarrow$
$t = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, -\dfrac{3\pi}{4}, -\dfrac{7\pi}{4}$

b. $f''(t) = -\cos t - \sin t$

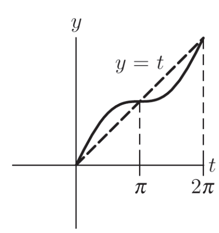
$f''\left(\dfrac{\pi}{4}\right) = -\sqrt{2}$, so the curve is concave down at $t = \dfrac{\pi}{4}$. $f''\left(\dfrac{5\pi}{4}\right) = \sqrt{2}$, so the curve is concave up at $t = \dfrac{5\pi}{4}$. Similarly, $f''\left(-\dfrac{7\pi}{4}\right) = -\sqrt{2}$, so the curve is concave down at $t = -\dfrac{7\pi}{4}$, and $f''\left(-\dfrac{3\pi}{4}\right) = \sqrt{2}$, so the curve is concave up at $t = \dfrac{3\pi}{4}$.

c. $f''(t) = -\cos t - \sin t = 0 \Rightarrow$
$-\cos t = \sin t \Rightarrow t = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}, -\dfrac{\pi}{4}, -\dfrac{5\pi}{4}$

The inflection points are at $\left(\dfrac{3\pi}{4}, 0\right)$, $\left(\dfrac{7\pi}{4}, 0\right)$, $\left(-\dfrac{\pi}{4}, 0\right)$, and $\left(-\dfrac{5\pi}{4}, 0\right)$.



51.



52. $y = 2 + \sin 3t$

Area under the curve is $\int_0^{\pi/2} (2+\sin 3t)\,dt$.

$$\int_0^{\pi/2} (2+\sin 3t)\,dt = \left[2t - (\cos 3t)\left(\frac{1}{3}\right)\right]\Bigg|_0^{\pi/2} = \left[2t - \frac{1}{3}\cos 3t\right]\Bigg|_0^{\pi/2} = \pi - \frac{1}{3}(0) - \left(0 - \frac{1}{3}\right) = \frac{1}{3} + \pi$$

53. The desired area is

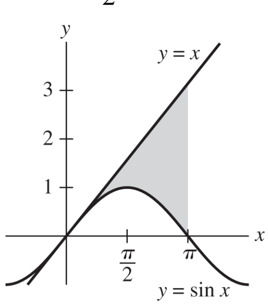
$$\int_0^{\pi} \sin t\,dt + \int_{\pi}^{2\pi} (-\sin t)\,dt = (-\cos t)\Big|_0^{\pi} + \cos t\Big|_{\pi}^{2\pi} = 2 + 2 = 4$$

54. The desired area is

$$\int_0^{\pi/2} \cos t\,dt + \int_{\pi/2}^{3\pi/2} (-\cos t)\,dt = \sin t\Big|_0^{\pi/2} + (-\sin t)\Big|_{\pi/2}^{3\pi/2} = 1 + 2 = 3$$

55. It is easy to check that the line $y = x$ is tangent to the graph of $y = \sin x$ at $x = 0$. From the graph of $y = \sin x$, it is clear that $y = \sin x$ lies below $y = x$ for $x \geq 0$. So the area between these two curves from $x = 0$ to $x = \pi$ is given by

$$\text{Area} = \int_0^{\pi} (x - \sin x)\,dx = \left[\frac{x^2}{2} + \cos x\right]\Bigg|_0^{\pi} = \left[\frac{\pi^2}{2} + \cos\pi\right] - [0 + \cos 0] = \frac{\pi^2}{2} + (-1) - [0+1] = \frac{\pi^2}{2} - 2$$



Exercises 56–58 refer to the function $V(t) = 3 + 0.05\sin\left(160\pi t - \frac{\pi}{2}\right)$, where $V(t)$ is the lung volume in liters and t is measured in minutes.

56. a. $V(0) = 2.95$, $V\left(\frac{1}{320}\right) = 3$, $V\left(\frac{1}{160}\right) = 3.05$, $V\left(\frac{1}{80}\right) = 2.95$

b. $V'(t) = 0.05\cos\left(160\pi t - \frac{\pi}{2}\right)(160\pi)$

Setting $V'(t) = 0$ gives

$$\cos\left(160\pi t - \frac{\pi}{2}\right) = 0 \Rightarrow$$
$$160\pi t - \frac{\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \Rightarrow$$
$$160\pi t = \pi, 2\pi, 3\pi, \ldots \Rightarrow$$
$$t = \frac{1}{160}, \frac{2}{160}, \frac{3}{160}, \ldots$$

$$V''(t) = -8\pi\sin\left(160\pi t - \frac{\pi}{2}\right)(160\pi) = -1280\pi^2 \sin\left(160\pi t - \frac{\pi}{2}\right)$$

At $t = \frac{1}{160}$, $V''(t) < 0$, so this value of t gives a relative maximum, and the maximum lung volume is $V\left(\frac{1}{160}\right) = 3.05$ liters.

57. a. $V'(t) = 8\pi\cos\left(160\pi t - \frac{\pi}{2}\right)$

b. Inspiration (in the first cycle) occurs from $t = 0$ to $t = \frac{1}{160}$. To find the maximum rate of inspiration, we need to find the maximum of $V'(t)$ on $\left[0, \frac{1}{160}\right]$.

(*continued on next page*)

(*continued*)

$$V''(t) = -1280\pi^2 \sin\left(160\pi t - \frac{\pi}{2}\right)$$

Setting $V''(t) = 0$ gives $160\pi t - \frac{\pi}{2} = 0, \pi, 2\pi, \ldots \Rightarrow 160\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \Rightarrow t = \frac{1}{320}, \frac{3}{320}, \frac{5}{320}, \ldots$ Among these values, only $\frac{1}{320}$ is within $\left[0, \frac{1}{160}\right]$. Since $V'(t) = 0$ at the end points of the interval, $t = \frac{1}{320}$ must give the maximum value of $V'(t)$. Thus, the maximum rate of air flow is

$V'\left(\frac{1}{320}\right) = 8\pi$ liters/min.

c. The average value of $V'(t)$ on $\left[0, \frac{1}{160}\right]$ is

$$\frac{1}{\frac{1}{160}}\int_0^{1/160} V'(t)\, dt$$
$$= 160\int_0^{1/160} 8\pi \cos\left(160\pi t - \frac{\pi}{2}\right) dt$$
$$= -8\sin\left(160\pi t - \frac{\pi}{2}\right)\Big|_0^{1/160}$$
$$= 8(1+1) = 16 \text{ liters/min}$$

58. During one minute, there will be 80 inspirations. Each inspiration represents $V\left(\frac{1}{160}\right) - V(0) = 3 + 0.05 - (3 - 0.05) = .1$ liter. Therefore, the minute volume is 0.1(80) = 8 liters. In Exercise 57(b), we found that the peak respiratory flow was 8π liters/min. Thus the first statement is verified. In Exercise 57(c), we found the mean inspiratory flow to be $16 = 8 \cdot 2$ liters/min, verifying the second statement.

59. $\int \sin(\pi - x)dx = \cos(\pi - x) + C$

60. $\int (3\cos 3x - 2\sin 2x)dx = \sin 3x + \cos 2x + C$

61. $\int_0^{\pi/2} \cos 6x\, dx = \left(\frac{1}{6}\sin 6x\right)\Big|_0^{\pi/2} = 0 - 0 = 0$

62. $\int \cos(6 - 2x)dx = -\frac{1}{2}\sin(6 - 2x) + C$

63. $\int_0^{\pi} [x - 2\cos(\pi - 2x)]dx$
$$= \left[\frac{1}{2}x^2 + \sin(\pi - 2x)\right]\Big|_0^{\pi}$$
$$= \frac{\pi^2}{2} + 0 - (0) = \frac{\pi^2}{2}$$

64. $\int_{-\pi}^{\pi} (\cos 3x + 2\sin 7x)dx$
$$= \left(\frac{1}{3}\sin 3x - \frac{2}{7}\cos 7x\right)\Big|_{-\pi}^{\pi}$$
$$= 0 - \left(-\frac{2}{7}\right) - \left[0 - \left(-\frac{2}{7}\right)\right] = 0$$

65. $\int \sec^2 \frac{x}{2} dx = 2\tan\frac{x}{2} + C$

66. $\int 2\sec^2 2x\, dx = \tan 2x + C$

67. $\int_0^{\pi/4} (\cos x - \sin x)dx = (\sin x + \cos x)\Big|_0^{\pi/4}$
$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0 + 1)$$
$$= \sqrt{2} - 1$$

68. $\int_0^{\pi/4} \sin x\; dx + \int_{\pi/4}^{\pi/2} \cos x\; dx$
$$= (-\cos x)\Big|_0^{\pi/4} + (\sin x)\Big|_{\pi/4}^{\pi/2}$$
$$= -\frac{\sqrt{2}}{2} - (-1) + 1 - \left(\frac{\sqrt{2}}{2}\right) = 2 - \sqrt{2}$$

69. $\int_{\pi/4}^{\pi/2} (\sin x - \cos x)dx + \int_{\pi/2}^{\pi} \sin x\, dx$
$$= (-\cos x - \sin x)\Big|_{\pi/4}^{\pi/2} + (-\cos x)\Big|_{\pi/2}^{\pi}$$
$$= 0 - 1 - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) + [-(-1)] - (0) = \sqrt{2}$$

70. $\int_{\pi/2}^{\pi} (0 - \cos x)dx = -\int_{\pi/2}^{\pi} \cos x\, dx = -(\sin x)\Big|_{\pi/2}^{\pi}$
$$= -[0 - (1)] = 1$$

71. Average $= \dfrac{1}{b-a}\displaystyle\int_a^b f(t)\,dt$

$$= \frac{1}{2\pi - 0}\int_0^{2\pi}\left(1 + \sin 2t - \frac{1}{3}\cos 2t\right)dt$$
$$= \frac{1}{2\pi}\left[t - \frac{1}{2}\cos 2t - \frac{1}{6}\sin 2t\right]\Bigg|_0^{2\pi}$$
$$= \frac{1}{2\pi}\left[2\pi - \frac{1}{2} - 0 - \left(0 - \frac{1}{2} - 0\right)\right] = 1$$

72. $$\frac{1}{\pi - 0}\int_0^{\pi}(t - \cos 2t)\,dt = \frac{1}{\pi}\left[\frac{1}{2}t^2 - \frac{1}{2}\sin 2t\right]\Bigg|_0^{\pi}$$
$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} - 0 - (0 - 0)\right]$$
$$= \frac{\pi}{2}$$

73. $$\frac{1}{\frac{3\pi}{4} - 0}\int_0^{3\pi/4}\left[1000 + 200\sin 2\left(t - \frac{\pi}{4}\right)\right]dt$$
$$= \frac{4}{3\pi}\left[1000t - 100\cos 2\left(t - \frac{\pi}{4}\right)\right]\Bigg|_0^{3\pi/4}$$
$$= \frac{4}{3\pi}\left[750\pi + 100 - (0 - 0)\right] = 1000 + \frac{400}{3\pi}$$

74. $$\frac{1}{0 - (-\pi)}\int_{-\pi}^{0}(\cos t + \sin t)\,dt$$
$$= \frac{1}{\pi}\left[\sin t - \cos t\right]\Big|_{-\pi}^{0}$$
$$= \frac{1}{\pi}\left[0 - 1 - (0 - (-1))\right] = -\frac{2}{\pi}$$

75. Substitute $\tan^2 x = \sec^2 x - 1$.
$$\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = \tan x - x + C$$

76. Substitute $\tan^2 3x = \sec^2 3x - 1$.
$$\int \tan^2 3x\,dx = \int (\sec^2 3x - 1)\,dx$$
$$= \frac{1}{3}\tan 3x - x + C$$

77. Substitute $1 + \tan^2 x = \sec^2 x$.
$$\int (1 + \tan^2 x)\,dx = \int \sec^2 x\,dx = \tan x + C$$

78. Substitute $1 + \tan^2 x = \sec^2 x$.
$$\int (2 + \tan^2 x)\,dx = \int (1 + 1 + \tan^2 x)\,dx$$
$$= \int (1 + \sec^2 x)\,dx$$
$$= x + \tan x + C$$

79. Substitute $\tan^2 x = \sec^2 x - 1$.
$$\int_0^{\pi/4} \tan^2 x\,dx = \int_0^{\pi/4}(\sec^2 x - 1)\,dx$$
$$= (\tan x - x)\Big|_0^{\pi/4}$$
$$= 1 - \frac{\pi}{4} - (0 - 0) = 1 - \frac{\pi}{4}$$

80. Substitute $\tan^2 x = \sec^2 x - 1$.
$$\int_0^{\pi/4}(2 + 2\tan^2 x)\,dx$$
$$= \int_0^{\pi/4}\left[2 + 2(\sec^2 x - 1)\right]dx$$
$$= \int_0^{\pi/4} 2\sec^2 x\,dx = 2\tan x\Big|_0^{\pi/4} = 2 - 0 = 2$$