Chapter 1 Nature of Light

1-1. a) $\lambda = \dfrac{h}{p} = \dfrac{h}{m\,v} = \dfrac{6.63\times10^{-34}\,\mathrm{J\cdot s}}{(0.05\,\mathrm{kg})\,(20\,\mathrm{m/s})} = 6.63\times10^{-34}\ \mathrm{m}$

b) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2\,m\,E}} = \dfrac{6.63\times10^{-34}\,\mathrm{J\cdot s}}{[(2\cdot 9.11\times10^{-31}\,\mathrm{kg})\,(10\cdot 1.602\times10^{-19}\,\mathrm{J})]^{1/2}} = 3.88\times10^{-10}\,\mathrm{m}$

1-2. $P = \dfrac{\text{Energy}}{\text{time}} = \dfrac{n\,h\,\nu}{t} = \dfrac{n\,h\,c}{t\lambda} = \dfrac{100\left(6.63\times10^{-34}\,\mathrm{J\cdot s}\right)\left(3\times10^{8}\mathrm{m/s}\right)}{(1\,\mathrm{s})\,(550\times10^{-9}\mathrm{m})} = 3.62\times10^{-17}\,\mathrm{W}$

1-3. The energy of a photon is given by $E = h\,\nu = h\,c/\lambda$

$$\text{At } \lambda = 380\,\mathrm{nm}:\quad E = \frac{\left(6.63\times10^{-34}\,\mathrm{J\cdot s}\right)\left(3\times10^{8}\mathrm{m/s}\right)}{380\times10^{-9}\,\mathrm{m}} = \left(5.23\times10^{-19}\mathrm{J}\right)\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\mathrm{J}} = 3.27\,\mathrm{eV}$$

$$\text{At } \lambda = 770\,\mathrm{nm}:\quad E = \frac{\left(6.63\times10^{-34}\,\mathrm{J\cdot s}\right)\left(3\times10^{8}\mathrm{m/s}\right)}{770\times10^{-9}\,\mathrm{m}} = \left(2.58\times10^{-19}\mathrm{J}\right)\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\mathrm{J}} = 1.61\,\mathrm{eV}$$

1-4. $p = E/c = m\,c^2/c = m\,c = 2.73\times10^{-22}\,\mathrm{kg\cdot m/s}, \quad \lambda = \dfrac{h}{p} = \dfrac{h\,c}{E} = \dfrac{h\,c}{m\,c^2} = \dfrac{h}{m\,c} = 2.43\times10^{-12}\,\mathrm{m}$

1-5. $E_{v=0} = m\,c^2 = \left(9.109\times10^{-31}\,\mathrm{kg}\right)\left(2.998\times10^{8}\,\mathrm{m/s}\right)^2 = \left(8.187\times10^{-14}\,\mathrm{J}\right)\dfrac{1\,\mathrm{MeV}}{1.602\times10^{-19}\,\mathrm{J}} = .511\,\mathrm{MeV}$

1-6. $c\,p = \sqrt{E^2 - m^2\,c^4}$, where $E = E_K + m\,c^2 = (1+0.511))\mathrm{MeV}$. So $c\,p = \sqrt{1.511^2 - 0.511^2}\,\mathrm{MeV}$

That is, $c\,p = 1.422\,\mathrm{MeV}$ and $p = 1.422\,\mathrm{MeV}/c$.

1-7. $\lambda = \dfrac{h\,c}{E} = \dfrac{\left(6.626\times10^{-34}\,\mathrm{J\cdot s}\right)\left(2.998\times10^{8}\,\mathrm{m/s}\right)}{E}\left(\dfrac{1\,\mathrm{eV}}{1.602\times10^{-19}\,\mathrm{J}}\right)\left(\dfrac{1\ \text{Å}}{10^{-10}\,\mathrm{m}}\right) = \dfrac{12,400}{E}\left(\text{Å}\cdot\mathrm{eV}\right)$

1-8. $E_K = m\,c^2\left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right) = m\,c^2\left[\left(1-v^2/c^2\right)^{-1/2} - 1\right] \simeq m\,c^2\left[\left(1-(-1/2)\,v^2/c^2\right) - 1\right] = \dfrac{1}{2}m\,v^2$

1-9. The total energy of the proton is,

$$E = E_K + m_p c^2 = 2 \times 10^9 \left(\frac{1.60 \times 10^{-19}\,\mathrm{J}}{1\,\mathrm{eV}} \right) + \left(1.67 \times 10^{-27}\,\mathrm{kg}\right)\left(3.00 \times 10^8\,\mathrm{m/s}\right)^2 = 4.71 \times 10^{-10}\,\mathrm{J}$$

a) $$p = \frac{\sqrt{E^2 - m_p^2 c^4}}{c} = \frac{\left[\left(4.71 \times 10^{-10}\,\mathrm{J}\right)^2 - \left(1.67 \times 10^{-27}\,\mathrm{kg}\right)^2 \left(3.00 \times 10^8\,\mathrm{m}\right)^2\right]^{-1/2}}{3.00 \times 10^8\,\mathrm{m/s}}$$

$p = 1.49 \times 10^{-18}\,\mathrm{kg \cdot m/s}$

b) $\lambda = h/p = \left(6.63 \times 10^{-34}\,\mathrm{J \cdot s}\right)/\left(1.49 \times 10^{-18}\,\mathrm{kg \cdot m/s}\right) = 4.45 \times 10^{-16}\,\mathrm{m}$

c) $\lambda_{\text{photon}} = h\,c/E = \left(6.63 \times 10^{-34}\,\mathrm{J \cdot s}\right)\left(3.00 \times 10^8\,\mathrm{m/s}\right)/\left(4.71 \times 10^{-10}\right) = 4.22 \times 10^{-16}\,\mathrm{m}$

1-10. $$n_{\text{photons}} = \frac{\text{Energy}}{h\,\nu} = \frac{\text{Energy}}{h\,c/\lambda} = \frac{\left(1000\,\mathrm{W/m^2}\right)\left(10^{-4}\,\mathrm{m^2}\right)}{(6.63 \times 10^{-34}\,\mathrm{J})\,(3.00 \times 10^8\,\mathrm{m/s})/(550 \times 10^{-9}\,\mathrm{m})} = 2.77 \times 10^{17}$$

1-11. $$\frac{n_1}{n_2} = \frac{E_e/h\,\nu_1}{E_e/h\,\nu_2} = \frac{E_e\,\lambda_1/h\,c}{E_e\,\lambda_2/h\,c} = \frac{\lambda_1}{\lambda_2}$$

1-12. The wavelength range is 380 nm to 770 nm. The corresponding frequencies are

$$\nu_{770} = \frac{c}{\lambda} = \frac{3.00 \times 10^8\,\mathrm{m/s}}{770 \times 10^{-9}\,\mathrm{m}} = 3.89 \times 10^{14}\,\mathrm{Hz} \quad \nu_{380} = \frac{c}{\lambda} = \frac{3.00 \times 10^8\,\mathrm{m/s}}{380 \times 10^{-9}\,\mathrm{m}} = 7.89 \times 10^{14}\mathrm{Hz}$$

1-13. The wavelength of the radio waves is $\lambda = c/\nu = \left(3.00 \times 10^8\,\mathrm{m/s}\right)/\left(100 \times 10^6\,\mathrm{Hz}\right) = 3\,\mathrm{m}$. The length of the half-wave antenna is then $\lambda/2 = 1.5\,\mathrm{m}$.

1-14. The wavelength is $\lambda = c/\nu = \left(3.0 \times 10^8\,\mathrm{m/s}\right)/\left(90 \times 10^6\,\mathrm{Hz}\right) = 3.33\,\mathrm{m}$. The length of each of the rods is then $\lambda/4 = 0.83\,\mathrm{m}$.

1-15. a) $t = D_l/c = (90 \times 10^3/3.0 \times 10^8)\,\mathrm{s} = 3.0 \times 10^{-4}\,\mathrm{s}$. b) $D_s = v_s\,t = (340)\left(3.0 \times 10^{-4}\right)\mathrm{m} = 0.10\,\mathrm{m}$

1-16. a) $I_e = \frac{\Phi_e}{\Delta\omega} = \frac{500\,\mathrm{W}}{4\,\pi\,\mathrm{sr}} = 39.8\,\mathrm{W/sr}$ b) $M_e = \frac{\Phi_e}{A} = \frac{500\,\mathrm{W}}{5 \times 10^{-4}\,\mathrm{m^2}} = 10^6\,\mathrm{W/m^2}$

c) $E_e = \frac{\Phi_e}{A} = \frac{\Phi_e}{4\,\pi\,r^2} = \frac{500\,\mathrm{W}}{4\,\pi(2\,\mathrm{m})^2} = 9.95\,\mathrm{W/m^2}$ e) $\Phi_e = E_e\,A = \left(9.95\,\mathrm{W/m^2}\right)\pi\,(0.025\,\mathrm{m})^2 = .0195\,\mathrm{W}$

1-17. a) The half-angle divergence $\theta_{1/2}$ can be found from the relation

$$\tan(\theta_{1/2}) \approx \theta_{1/2} = \frac{r_{\text{spot}}}{L_{\text{room}}} = \frac{0.0025\,\mathrm{m}}{15\,\mathrm{m}} = 1.67 \times 10^{-4}\,\mathrm{rad} = .0096^\circ$$

b) The solid angle is $\Delta\omega = \frac{A_{\text{spot}}}{L_{\text{room}}^2} = \frac{\pi\,r_{\text{spot}}^2}{L_{\text{room}}^2} = \frac{\pi\,(0.0025\,\mathrm{m})^2}{(15\,\mathrm{m})^2} = 8.73 \times 10^{-8}\,\mathrm{sr}$.

c) The irradiance on the wall is $E_e = \frac{\Phi_e}{A_{\text{spot}}} = \frac{\Phi_e}{\pi\,r_{\text{spot}}^2} = \frac{0.0015\,\mathrm{W}}{\pi\,(0.0025\,\mathrm{m})^2} = 76.4\,\mathrm{W/m^2}$.

d) The radiance is (approximating differentials as increments)

$$L_e \approx \frac{\Phi_e}{\Delta\omega\,\Delta A_{\text{laser}}\cos\theta} = \frac{0.0015\,\mathrm{W}}{(8.73 \times 10^{-8}\,\mathrm{sr})\left(\pi\,(0.00025\,\mathrm{m})^2\right)\cos(0)} = 8.75 \times 10^{10}\,\frac{\mathrm{W}}{\mathrm{m^2 \cdot sr}}$$