109. Using the fnInt feature on a calculator, we get:
$$\int_{-1}^{1}\left(3+\sqrt{1-x^2}\right)dx \approx 7.571.$$

110. Using the fnInt feature on a calculator, we get:
$$\int_{0}^{8} x\left(x-5\right)^4\,dx \approx 885.333.$$

111. Using the fnInt feature on a calculator, we get:
$$\int_{-2}^{2} x^{2/3}\left(\frac{5}{2}-x\right)dx \approx 9.524.$$

112. Using the fnInt feature on a calculator, we get:
$$\int_{2}^{4}\frac{x^2-4}{x^2-3}\,dx \approx 1.507.$$

113. Using the fnInt feature on a calculator, we get:
$$\int_{-10}^{10}\frac{8}{x^2+4}\,dx \approx 10.987.$$

114. $\boxed{tw}$ Let $F(x)$ be an anti-derivative of $f(x)$ on the interval $a \le x \le b$. Then, by the Fundamental Theorem, we have:
$$\begin{aligned}{}_a^b\, f(x)\,dx &= F(b)-F(a)\\ &= -F(a)+F(b)\\ &= -\ \ F(a)\quad F(b)\\ &= -\ {}_b^a\, f(x)\,dx.\end{aligned}$$

Exercise Set 4.4

1.
$$\begin{aligned}\int_1^5 f(x)\,dx &= \int_1^3 f(x)\,dx+\int_3^5 f(x)\,dx\\ &= \int_1^3 (2x+1)\,dx+\int_3^5 (10-x)\,dx\\ &= \left[x^2+x\right]_1^3+\left[10x-\frac{1}{2}x^2\right]_3^5\\ &= \left[\left((3)^2+(3)\right)-\left((1)^2+(1)\right)\right]+\\ &\quad \left[\left(10(5)-\frac{1}{2}(5)^2\right)-\left(10(3)-\frac{1}{2}(3)^2\right)\right]\\ &= (12-2)+\left(\frac{75}{2}-\frac{51}{2}\right)\\ &= 10+12\\ &= 22\end{aligned}$$

2.
$$\begin{aligned}\int_1^5 f(x)\,dx &= \int_1^4 f(x)\,dx+\int_3^4 f(x)\,dx\\ &= \int_1^4 (x+5)\,dx+\int_4^5\left(11-\frac{1}{2}x\right)dx\\ &= \left[\frac{1}{2}x^2+5x\right]_1^4+\left[11x-\frac{1}{4}x^2\right]_4^5\\ &= \frac{45}{2}+\frac{35}{4}\\ &= \frac{125}{4}=31.25\end{aligned}$$

3.
$$\begin{aligned}\int_{-2}^3 g(x)\,dx &= \int_{-2}^0 g(x)\,dx+\int_0^3 g(x)\,dx\\ &= \int_{-2}^0 \left(x^2+4\right)dx+\int_0^3 (4-x)\,dx\\ &= \left[\frac{x^3}{3}+4x\right]_{-2}^0+\left[4x-\frac{1}{2}x^2\right]_0^3\\ &= \left[\left(\frac{(0)^3}{3}+4(0)\right)-\left(\frac{(-2)^3}{3}+4(-2)\right)\right]+\\ &\quad \left[\left(4(3)-\frac{1}{2}(3)^2\right)-\left(4(0)-\frac{1}{2}(0)^2\right)\right]\\ &= \left(0-\left(-\frac{32}{3}\right)\right)+\left(\frac{15}{2}-0\right)\\ &= \frac{109}{6}\\ &= 18\tfrac{1}{6}\end{aligned}$$

4.
$$\begin{aligned}\int_{-2}^3 g(x)\,dx &= \int_{-2}^0 g(x)\,dx+\int_0^3 g(x)\,dx\\ &= \int_{-2}^0 \left(-x^2+5\right)dx+\int_0^3 (x+5)\,dx\\ &= \left[-\frac{x^3}{3}+5x\right]_{-2}^0+\left[\frac{1}{2}x^2+5x\right]_0^3\\ &= \left(0-\left(-\frac{22}{3}\right)\right)+\left(\frac{39}{2}-0\right)\\ &= \frac{22}{3}+\frac{39}{2}\\ &= \frac{161}{6}\\ &= 26\tfrac{5}{6}\end{aligned}$$

5. $$\int_{-6}^{4} f(x)\,dx = \int_{-6}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx$$
$$= \int_{-6}^{1}\left(-x^2-6x+7\right)dx + \int_{1}^{4}\left(\frac{3}{2}x-1\right)dx$$
$$= \left[-\frac{x^3}{3}-3x^2+7x\right]_{-6}^{1} + \left[\frac{3}{4}x^2-x\right]_{1}^{4}$$
$$= \left[\left(-\frac{(1)^3}{3}-3(1)^2+7(1)\right)-\left(-\frac{(-6)^3}{3}-3(-6)^2+7(-6)\right)\right]+\left[\left(\frac{3}{4}(4)^2-(4)\right)-\left(\frac{3}{4}(1)^2-(1)\right)\right]$$
$$= \left(\frac{11}{3}-(-78)\right)+\left(8-\left(-\frac{1}{4}\right)\right)$$
$$= \frac{245}{3}+\frac{33}{4}$$
$$= \frac{1079}{12}$$
$$= 89\tfrac{11}{12}$$

6. $$\int_{-6}^{4} f(x)\,dx = \int_{-6}^{-1} f(x)\,dx + \int_{-1}^{4} f(x)\,dx$$
$$= \int_{-6}^{-1}(-x-1)\,dx + \int_{-1}^{4}\left(-x^2+4x+5\right)dx$$
$$= \left[-\frac{1}{2}x^2-x\right]_{-6}^{-1} + \left[-\frac{x^3}{3}+2x^2+5x\right]_{-1}^{4}$$
$$= \left(\frac{1}{2}-(-12)\right)+\left(\frac{92}{3}-\left(-\frac{8}{3}\right)\right)$$
$$= \frac{275}{6} = 45\tfrac{5}{6}$$

7. To calculate the points of intersection, we set $f(x)$ equal to $g(x)$ and solve.
$$f(x) = g(x)$$
$$9 = x^2$$
$$\sqrt{9} = \sqrt{x^2}$$
$$\pm 3 = x$$
The graphs intersect at $x = -3$ and $x = 3$.

8. $$f(x) = g(x)$$
$$8 = \frac{1}{2}x^2$$
$$16 = x^2$$
$$\pm 4 = x$$
The graphs intersect at $x = -4$ and $x = 4$.

9. To calculate the points of intersection, we set $f(x)$ equal to $g(x)$ and solve.
$$f(x) = g(x)$$
$$7 = x^2-3x+2$$
$$0 = x^2-3x-5$$
Applying the quadratic formula we have:
$$x = \frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-5)}}{2(1)}$$
$$= \frac{3\pm\sqrt{29}}{2}$$
The graphs intersect at $x = \dfrac{3+\sqrt{29}}{2}$ and $x = \dfrac{3-\sqrt{29}}{2}$.

10. $$f(x) = g(x)$$
$$-6 = x^2+3x+13$$
$$0 = x^2+3x+19$$
This equation has no real solution; therefore, the graphs do not intersect.

11. To calculate the points of intersection, we set $f(x)$ equal to $g(x)$ and solve.
$$f(x) = g(x)$$
$$x^2-x-5 = x+10$$
$$x^2-2x-15 = 0$$
$$(x-5)(x+3) = 0$$
$$x-5=0 \quad \text{or} \quad x+3=0$$
$$x=5 \quad \text{or} \quad x=-3$$
The graphs intersect at $x = -3$ and $x = 5$.

12. $$f(x) = g(x)$$
$$x^2-7x+20 = 2x+6$$
$$x^2-9x+14 = 0$$
$$(x-7)(x-2) = 0$$
$$x-2=0 \quad \text{or} \quad x-7=0$$
$$x=2 \quad \text{or} \quad x=7$$
The graphs intersect at $x = 2$ and $x = 7$.

13. $g(x) \ge f(x)$ on $[-1,0]$ and $f(x) \ge g(x)$ on $[0,1]$. We use two integrals to find the total area.

$$\int_{-1}^{0}\left[0-\left(2x+x^2-x^3\right)\right]dx + \int_{0}^{1}\left[\left(2x+x^2-x^3\right)-0\right]dx$$

$$\int_{-1}^{0}\left[0-\left(2x+x^2-x^3\right)\right]dx + \int_{0}^{1}\left[\left(2x+x^2-x^3\right)-0\right]dx$$

$$= \int_{-1}^{0}\left(-2x-x^2+x^3\right)dx + \int_{0}^{1}\left(2x+x^2-x^3\right)dx$$

$$= \left[-x^2-\frac{x^3}{3}+\frac{x^4}{4}\right]_{-1}^{0} + \left[x^2+\frac{x^3}{3}-\frac{x^4}{4}\right]_{0}^{1}$$

$$= \left[-(0)^2-\frac{(0)^3}{3}+\frac{(0)^4}{4}\right] - \left[-(-1)^2-\frac{(-1)^3}{3}+\frac{(-1)^4}{4}\right] + \left[(1)^2+\frac{(1)^3}{3}-\frac{(1)^4}{4}\right] - \left[(0)^2+\frac{(0)^3}{3}-\frac{(0)^4}{4}\right]$$

$$= \left[0-\left(-\frac{5}{12}\right)\right]+\left[\frac{13}{12}-0\right]$$

$$= \frac{5}{12}+\frac{13}{12}$$

$$= \frac{18}{12}$$

$$= \frac{3}{2}$$

14. $$\int_{-5}^{-1}\left[\left(x^3+3x^2-9x-12\right)-\left(4x+3\right)\right]dx + \int_{-1}^{3}\left[\left(4x+3\right)-\left(x^3+3x^2-9x-12\right)\right]dx$$

$$= \int_{-5}^{-1}\left(x^3+3x^2-13x-15\right)dx + \int_{-1}^{3}\left(-x^3-3x^2+13x+15\right)dx$$

$$= \left[\frac{x^4}{4}+x^3-\frac{13}{2}x^2-15x\right]_{-5}^{-1} + \left[-\frac{x^4}{4}-x^3+\frac{13}{2}x^2+15x\right]_{-1}^{3}$$

Applying the Fundamental Theorem of Calculus we have:

$$= \left[\frac{(-1)^4}{4}+(-1)^3-\frac{13}{2}(-1)^2-15(-1)\right] - \left[\frac{(-5)^4}{4}+(-5)^3-\frac{13}{2}(-5)^2-15(-5)\right] + \left[-\frac{(3)^4}{4}-(3)^3+\frac{13}{2}(3)^2+15(3)\right] - \left[-\frac{(-1)^4}{4}-(-1)^3+\frac{13}{2}(-1)^2+15(-1)\right]$$

$$= \left[\frac{31}{4}\right]-\left[\frac{-225}{4}\right]+\left[\frac{225}{4}\right]-\left[-\frac{31}{4}\right]$$

$$= 128$$

15. $g(x) \ge f(x)$ over the entire region. We find the area.

$$\int_{-1}^{4}\left[\left(x+28\right)-\left(x^4-8x^3+18x^2\right)\right]dx$$

$$= \int_{-1}^{4}\left(-x^4+8x^3-18x^2+x+28\right)dx$$

$$= \left[-\frac{x^5}{5}+2x^4-6x^3+\frac{x^2}{2}+28x\right]_{-1}^{4}$$

$$= \left[-\frac{(4)^5}{5}+2(4)^4-6(4)^3+\frac{(4)^2}{2}+28(4)\right] - \left[-\frac{(-1)^5}{5}+2(-1)^4-6(-1)^3+\frac{(-1)^2}{2}+28(-1)\right]$$

$$= \frac{216}{5}+\frac{193}{10}$$

$$= \frac{625}{10}$$

$$= 62\tfrac{1}{2}$$

16. $$\int_{1}^{4}\left[\left(4x-x^2\right)-\left(x^2-6x+8\right)\right]dx$$

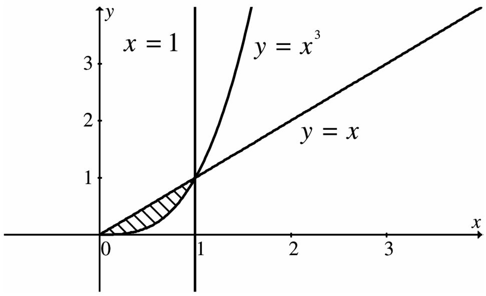
$$= \int_{1}^{4}\left(-2x^2+10x-8\right)dx$$

$$= \left[-\frac{2}{3}x^3+5x^2-8x\right]_{1}^{4}$$

$$= \left[-\frac{2}{3}(4)^3+5(4)^2-8(4)\right] - \left[-\frac{2}{3}(1)^3+5(1)^2-8(1)\right]$$

$$= \frac{27}{3} = 9$$

17. First graph the system of equations and shade the region bounded by the graphs.



Here the boundaries are easily determined by looking at the graph, or by solving the following:

$x = x^3$

$0 = x^3 - x$

$0 = x\left(x^2 - 1\right)$

$x = 0$ or $x^2 - 1 = 0$

$x = 0$ or $x = \pm 1$

$x = 0$ or $x = 1$

Note $x \geq x^3$ over the interval $[0,1]$. We compute the area as follows:

$$\int_0^1 \left(x - x^3\right) dx$$

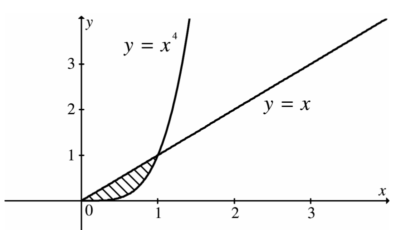
$$= \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$= \left(\frac{(1)^2}{2} - \frac{(1)^4}{4}\right) - \left(\frac{(0)^2}{2} - \frac{(0)^4}{4}\right)$$

$$= \frac{1}{2} - \frac{1}{4}$$

$$= \frac{1}{4}.$$

18.



The boundaries are easily determined from the graph.

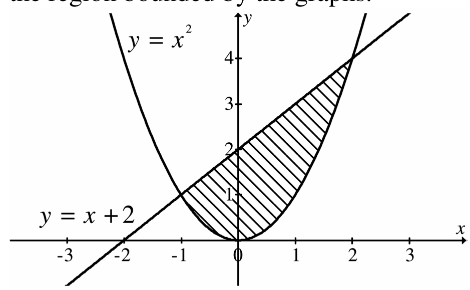
$$\int_0^1 \left(x - x^4\right) dx$$

$$= \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$

$$= \left(\frac{(1)^2}{2} - \frac{(1)^5}{5}\right) - \left(\frac{(0)^2}{2} - \frac{(0)^5}{5}\right)$$

$$= \frac{1}{2} - \frac{1}{5} = \frac{3}{10}$$

19. First graph the system of equations and shade the region bounded by the graphs.



Here the boundaries are easily determined by looking at the graph, or solving the following equation:

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1$ or $x = 2$

Note $(x+2) \geq x^2$ over the interval $[-1,2]$. We compute the area as follows:

$$\int_{-1}^2 \left((x+2) - x^2\right) dx$$

$$= \int_{-1}^2 \left(-x^2 + x + 2\right) dx$$

$$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^2$$

$$= \left(-\frac{(2)^3}{3} + \frac{(2)^2}{2} + 2(2)\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2(-1)\right)$$

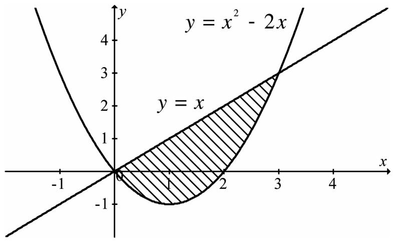
$$= \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$= \frac{10}{3} - \left(-\frac{7}{6}\right)$$

$$= \frac{27}{6}$$

$$= 4\tfrac{1}{2}.$$

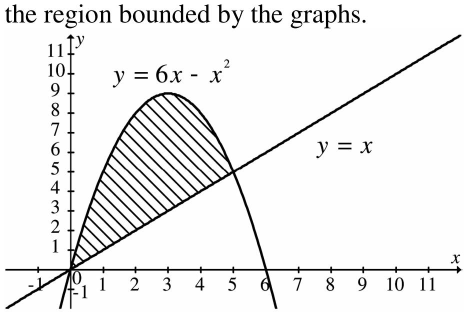
20.



The boundaries are easily determined from the graph.

$$\int_0^3 \left[x-\left(x^2-2x\right)\right]dx$$
$$=\int_0^3 \left(-x^2+3x\right)dx$$
$$=\left[-\frac{x^3}{3}+\frac{3}{2}x^2\right]_0^3$$
$$=\left(-\frac{(3)^3}{3}+\frac{3}{2}(3)^2\right)-\left(-\frac{(0)^3}{3}+\frac{3}{2}(0)^2\right)$$
$$=\frac{9}{2}=4\tfrac{1}{2}$$

21. First graph the system of equations and shade the region bounded by the graphs.



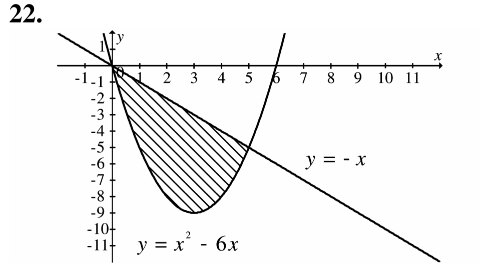
Here the boundaries are easily determined by looking at the graph, or by solving the following equation:

$$x=6x-x^2$$
$$x^2-5x=0$$
$$x(x-5)=0$$
$$x=0 \text{ or } x=5$$

Note $\left(6x-x^2\right)\geq x$ over the interval $[0,5]$. We compute the area as follows:

$$\int_0^5 \left(\left(6x-x^2\right)-x\right)dx$$
$$=\int_0^5 \left(-x^2+5x\right)dx$$
$$=\left[-\frac{x^3}{3}+\frac{5}{2}x^2\right]_0^5$$
$$=\left(-\frac{(5)^3}{3}+\frac{5}{2}(5)^2\right)-\left(-\frac{(0)^3}{3}+\frac{5}{2}(0)^2\right)$$
$$=\left(-\frac{125}{3}+\frac{125}{2}\right)-0$$
$$=\frac{125}{6}$$
$$=20\tfrac{5}{6}.$$

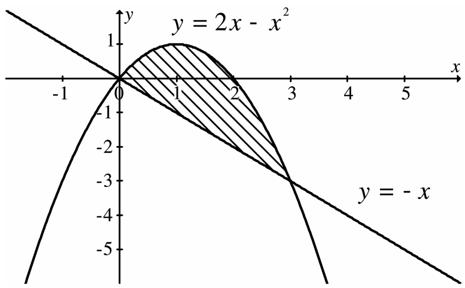
22.



Here the boundaries are easily determined by looking at the graph. Note $-x\geq\left(x^2-6x\right)$ over the interval $[0,5]$. We compute the area as follows:

$$\int_0^5 \left(-x-\left(x^2-6x\right)\right)dx$$
$$=\int_0^5 \left(-x^2+5x\right)dx$$
$$=\left[-\frac{x^3}{3}+\frac{5}{2}x^2\right]_0^5$$
$$=\left(-\frac{(5)^3}{3}+\frac{5}{2}(5)^2\right)-\left(-\frac{(0)^3}{3}+\frac{5}{2}(0)^2\right)$$
$$=\frac{125}{6}=20\tfrac{5}{6}.$$

23. First graph the system of equations and shade the region bounded by the graphs.



Here the boundaries are easily determined by looking at the graph, or by solving the following equation:

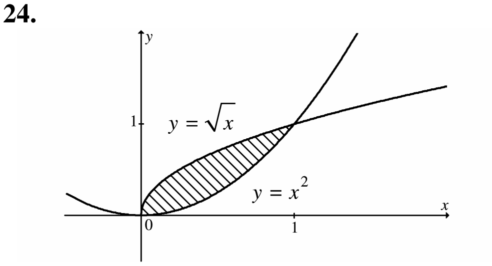
$$-x = 2x - x^2$$
$$x^2 - 3x = 0$$
$$x(x-3) = 0$$
$$x = 0 \text{ or } x = 3$$

Note $(2x - x^2) \geq -x$ over the interval $[0,3]$.

We compute the area as follows:

$$\int_0^3 \left(\left(2x - x^2\right) - (-x)\right) dx$$
$$= \int_0^3 \left(-x^2 + 3x\right) dx$$
$$= \left[-\frac{x^3}{3} + \frac{3}{2}x^2\right]_0^3$$
$$= \left(-\frac{(3)^3}{3} + \frac{3}{2}(3)^2\right) - \left(-\frac{(0)^3}{3} + \frac{3}{2}(0)^2\right)$$
$$= \left(-\frac{27}{3} + \frac{27}{2}\right) - 0$$
$$= \frac{27}{6}$$
$$= 4\tfrac{1}{2}.$$

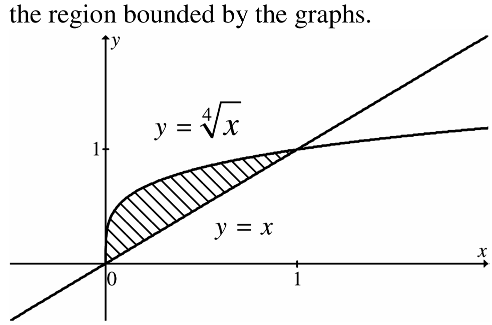
24.



Here the boundaries are easily determined by looking at the graph. Note $\sqrt{x} \geq x^2$ over the interval $[0,1]$. We compute the area as follows:

$$\int_0^1 \left(\sqrt{x} - x^2\right) dx$$
$$= \int_0^1 \left(x^{1/2} - x^2\right) dx$$
$$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1$$
$$= \left(\frac{2}{3}(1)^{3/2} - \frac{(1)^3}{3}\right) - \left(\frac{2}{3}(0)^{3/2} - \frac{(0)^3}{3}\right)$$
$$= \frac{1}{3}.$$

25. First graph the system of equations and shade the region bounded by the graphs.



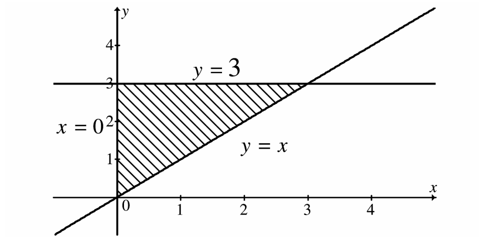
Here the boundaries are easily determined by looking at the graph, or by solving the following equation:

$$x = \sqrt[4]{x}$$
$$x^4 = x$$
$$x^4 - x = 0$$
$$x(x^3 - 1) = 0$$
$$x = 0 \text{ or } x = 1$$

Note $\sqrt[4]{x} \geq x$ over the interval $[0,1]$. We compute the area as follows:

$$\int_0^1 \left(\sqrt[4]{x} - x\right) dx$$
$$= \int_0^1 \left(x^{1/4} - x\right) dx$$
$$= \left[\frac{4}{5}x^{5/4} - \frac{x^2}{2}\right]_0^1$$
$$= \left(\frac{4}{5}(1)^{5/4} - \frac{(1)^2}{2}\right) - \left(\frac{4}{5}(0)^{5/4} - \frac{(0)^2}{2}\right)$$
$$= \left(\frac{4}{5} - \frac{1}{2}\right) - 0$$
$$= \frac{3}{10}.$$

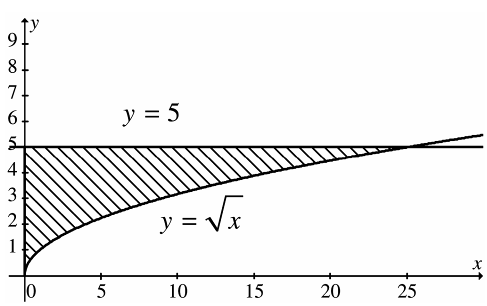
26.



Here the boundaries are easily determined by looking at the graph. Note $3 \geq x$ over the interval $[0,3]$. We compute the area as follows:

$$\int_0^3 (3-x)\,dx$$
$$=\left[3x-\frac{x^2}{2}\right]_0^3$$
$$=\left(3(3)-\frac{(3)^2}{2}\right)-\left(3(0)-\frac{(0)^2}{2}\right)$$
$$=9-\frac{9}{2}$$
$$=\frac{9}{2}=4\tfrac{1}{2}.$$

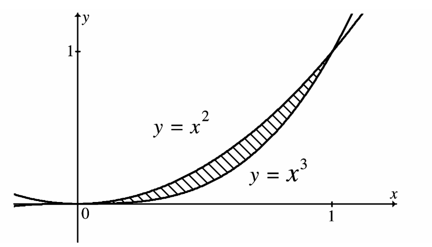
27. First graph the system of equations and shade the region bounded by the graphs.



Here the boundaries are easily determined by looking at the graph. Note $5 \geq \sqrt{x}$ over the interval $[0,25]$. We compute the area as follows:

$$\int_0^{25}\left(5-\sqrt{x}\right)dx$$
$$=\int_0^{25}\left(5-x^{1/2}\right)dx$$
$$=\left[5x-\frac{2}{3}x^{3/2}\right]_0^{25}$$
$$=\left(5(25)-\frac{2}{3}(25)^{3/2}\right)-\left(5(0)-\frac{2}{3}(0)^{3/2}\right)$$
$$=\left(125-\frac{250}{3}\right)-0$$
$$=\frac{125}{3}$$
$$=41\tfrac{2}{3}.$$

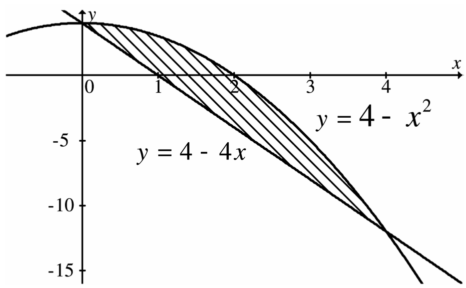
28.



Here the boundaries are easily determined by looking at the graph. Note $x^2 \geq x^3$ over the interval $[0,1]$. We compute the area as follows:

$$\int_0^1\left(x^2-x^3\right)dx$$
$$=\left[\frac{x^3}{3}-\frac{x^4}{4}\right]_0^1$$
$$=\left(\frac{(1)^3}{3}-\frac{(1)^4}{4}\right)-\left(\frac{(0)^3}{3}-\frac{(0)^4}{4}\right)$$
$$=\frac{1}{3}-\frac{1}{4}$$
$$=\frac{1}{12}.$$

29. First graph the system of equations and shade the region bounded by the graphs.



Here the boundaries are easily determined by looking at the graph, or by solving the following:

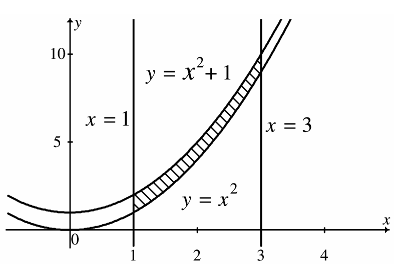
$$4-4x=4-x^2$$
$$x^2-4x=0$$
$$x(x-4)=0$$
$$x=0 \quad \text{or} \quad x=4.$$

Note $(4-x^2)\geq(4-4x)$ over the interval $[0,4]$.

We compute the area as follows:

$$\int_0^4\left[(4-x^2)-(4-4x)\right]dx$$
$$=\int_0^4(-x^2+4x)dx$$
$$=\left[-\frac{x^3}{3}+2x^2\right]_0^4$$
$$=\left(-\frac{(4)^3}{3}+2(4)^2\right)-\left(-\frac{(0)^3}{3}+2(0)^2\right)$$
$$=\left(-\frac{64}{3}+32\right)-0$$
$$=\frac{32}{3}$$
$$=10\tfrac{2}{3}.$$

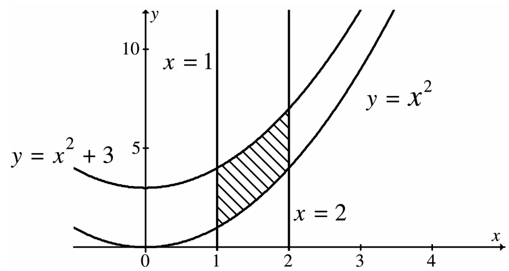
30.



Here the boundaries are easily determined by looking at the graph. Note $x^2+1\geq x^2$ over the interval $[1,3]$. We compute the area as follows:

$$\int_1^3\left[(x^2+1)-(x^2)\right]dx$$
$$=\int_1^3 1dx$$
$$=[x]_1^3$$
$$=(3-1)$$
$$=2.$$

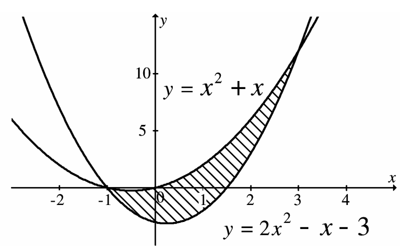
31. First graph the system of equations and shade the region bounded by the graphs.



Here the boundaries are easily determined by looking at the graph. Note $(x^2+3)\geq(x^2)$ over the interval $[1,2]$. We compute the area as follows:

$$\int_1^2\left[(x^2+3)-(x^2)\right]dx$$
$$=\int_1^2 3dx$$
$$=[3x]_1^2$$
$$=(3(2)-3(1))$$
$$=3.$$

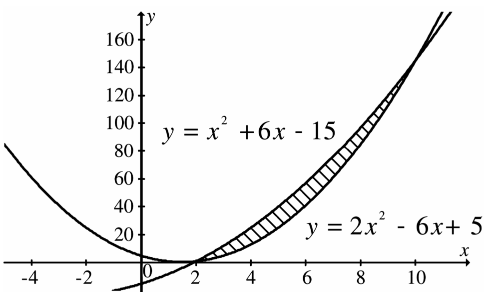
32.



Here the boundaries are easily determined by looking at the graph. Note $x^2+x\geq 2x^2-x-3$ over the interval $[-1,3]$. We compute the area as follows:

$$\int_{-1}^{3}\left[\left(x^2+x\right)-\left(2x^2-x-3\right)\right]dx$$
$$=\int_{-1}^{3}\left(-x^2+2x+3\right)dx$$
$$=\left[-\frac{x^3}{3}+x^2+3x\right]_{-1}^{3}$$
$$=\left(-\frac{(3)^3}{3}+(3)^2+3(3)\right)-\left(-\frac{(-1)^3}{3}+(-1)^2+3(-1)\right)$$
$$=(-9+9+9)-\left(\frac{1}{3}+1-3\right)$$
$$=\frac{32}{3}=10\tfrac{2}{3}.$$

33. First graph the system of equations and shade the region bounded by the graphs.



Here the boundaries are determined by solving the following equation:

$$2x^2-6x+5=x^2+6x-15$$
$$x^2-12x+20=0$$
$$(x-2)(x-10)=0$$
$$x=2 \text{ or } x=10$$

Note $\left(x^2+6x-15\right)\geq\left(2x^2-6x+5\right)$ over the interval $[2,10]$. We compute the area as follows:

$$\int_{2}^{10}\left[\left(x^2+6x-15\right)-\left(2x^2-6x+5\right)\right]dx$$
$$=\int_{2}^{10}\left(-x^2+12x-20\right)dx$$
$$=\left[-\frac{x^3}{3}+6x^2-20x\right]_{2}^{10}$$
$$=\left(-\frac{(10)^3}{3}+6(10)^2-20(10)\right)-\left(-\frac{(2)^3}{3}+6(2)^2-20(2)\right)$$
$$=\frac{200}{3}-\left(-\frac{56}{3}\right)$$
$$=\frac{256}{3}$$
$$=85\tfrac{1}{3}.$$

34. The average value is:

$$y_{\text{av}}=\frac{1}{b-a}\int_a^b f(x)\,dx$$
$$=\frac{1}{1-(-1)}\int_{-1}^{1}2x^3dx$$
$$=\frac{1}{2}\left[\frac{1}{2}x^4\right]_{-1}^{1}$$
$$=\frac{1}{2}\left[\frac{1}{2}(1)^4-\frac{1}{2}(-1)^4\right]$$
$$=\frac{1}{2}[0]$$
$$=0.$$

35. The average value is:

$$y_{\text{av}}=\frac{1}{b-a}\int_a^b f(x)\,dx$$
$$=\frac{1}{2-(-2)}\int_{-2}^{2}\left(4-x^2\right)dx$$
$$=\frac{1}{4}\left[4x-\frac{x^3}{3}\right]_{-2}^{2}$$
$$=\frac{1}{4}\left[\left(4(2)-\frac{(2)^3}{3}\right)-\left(4(-2)-\frac{(-2)^3}{3}\right)\right]$$
$$=\frac{1}{4}\left[\frac{16}{3}-\left(-\frac{16}{3}\right)\right]$$
$$=\frac{8}{3}.$$

36. The average value is:

$$\begin{aligned} y_{\text{av}} &= \frac{1}{b-a}\int_a^b f(x)\,dx \\ &= \frac{1}{1-(0)}\int_0^1 e^x\,dx \\ &= \frac{1}{1}\left[e^x\right]_0^1 \\ &= \left[e^1 - e^0\right] \\ &= e-1, \text{ or approximately } 1.718. \end{aligned}$$

37. The average value is:

$$\begin{aligned} y_{\text{av}} &= \frac{1}{b-a}\int_a^b f(x)\,dx \\ &= \frac{1}{1-(0)}\int_0^1 e^{-x}\,dx \\ &= \frac{1}{1}\left[-e^{-x}\right]_0^1 \\ &= \left[-e^{-1} - \left(-e^0\right)\right] \\ &= -e^{-1}+1, \text{ or approximately } 0.632. \end{aligned}$$

38. The average value is:

$$\begin{aligned} y_{\text{av}} &= \frac{1}{b-a}\int_a^b f(x)\,dx \\ &= \frac{1}{2-(0)}\int_0^2 \left(x^2 - x + 1\right)dx \\ &= \frac{1}{2}\left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^2 \\ &= \frac{1}{2}\left[\left(\frac{(2)^3}{3} - \frac{(2)^2}{2} + (2)\right) - \left(\frac{(0)^3}{3} - \frac{(0)^2}{2} + (0)\right)\right] \\ &= \frac{1}{2}\left[\frac{8}{3} - 0\right] \\ &= \frac{4}{3}. \end{aligned}$$

39. The average value is:

$$\begin{aligned} y_{\text{av}} &= \frac{1}{b-a}\int_a^b f(x)\,dx \\ &= \frac{1}{4-(0)}\int_0^4 \left(x^2 + x - 2\right)dx \\ &= \frac{1}{4}\left[\frac{x^3}{3} + \frac{x^2}{2} - 2x\right]_0^4 \\ &= \frac{1}{4}\left[\left(\frac{(4)^3}{3} + \frac{(4)^2}{2} - 2(4)\right) - \left(\frac{(0)^3}{3} + \frac{(0)^2}{2} - 2(0)\right)\right] \\ &= \frac{1}{4}\left[\frac{64}{3} - 0\right] \\ &= \frac{16}{3}. \end{aligned}$$

40. The average value is:

$$\begin{aligned} y_{\text{av}} &= \frac{1}{b-a}\int_a^b f(x)\,dx \\ &= \frac{1}{2-(0)}\int_0^2 (mx+1)\,dx \\ &= \frac{1}{2}\left[\frac{m}{2}x^2 + x\right]_0^2 \\ &= \frac{1}{2}\left[\left(\frac{m}{2}(2)^2 + (2)\right) - \left(\frac{m}{2}(0)^2 + (0)\right)\right] \\ &= \frac{1}{2}\left[(2m+2) - 0\right] \\ &= m+1. \end{aligned}$$

41. The average value is:

$$\begin{aligned} y_{\text{av}} &= \frac{1}{b-a}\int_a^b f(x)\,dx \\ &= \frac{1}{a-(0)}\int_0^a (4x+5)\,dx \\ &= \frac{1}{a}\left[2x^2 + 5x\right]_0^a \\ &= \frac{1}{a}\left[\left(2(a)^2 + 5(a)\right) - \left(2(0)^2 + 5(0)\right)\right] \\ &= \frac{1}{a}\left[\left(2a^2 + 5a\right) - 0\right] \\ &= 2a+5. \end{aligned}$$

42. The average value is:

$$\begin{aligned} y_{\text{av}} &= \frac{1}{b-a}\int_a^b f(x)\,dx \\ &= \frac{1}{1-(0)}\int_0^1 x^n dx, \qquad n \neq 0 \\ &= \frac{1}{1}\left[\frac{1}{n+1}x^{n+1}\right]_0^1 \\ &= \left[\frac{1}{n+1}(1)^{n+1} - \frac{1}{n+1}(0)^{n+1}\right] \\ &= \frac{1}{n+1} - 0 \\ &= \frac{1}{n+1}. \end{aligned}$$

43. The average value is:

$$\begin{aligned} y_{\text{av}} &= \frac{1}{b-a}\int_a^b f(x)\,dx \\ &= \frac{1}{2-(1)}\int_1^2 x^n dx, \qquad n \neq 0 \\ &= \frac{1}{1}\left[\frac{1}{n+1}x^{n+1}\right]_1^2 \\ &= \left[\frac{1}{n+1}(2)^{n+1} - \frac{1}{n+1}(1)^{n+1}\right] \\ &= \frac{2^{n+1}}{n+1} - \frac{1}{n+1} \\ &= \frac{2^{n+1}-1}{n+1}. \end{aligned}$$

44. The average value is:

$$\begin{aligned} y_{\text{av}} &= \frac{1}{b-a}\int_a^b f(x)\,dx \\ &= \frac{1}{5-(1)}\int_1^5 \frac{n}{x}\,dx \\ &= \frac{1}{4}\left[n\ln x\right]_1^5 \\ &= \frac{1}{4}\left[n\ln 5 - n\ln 1\right] \\ &= \frac{1}{4}\left[n\ln 5 - 0\right] \\ &= \frac{n}{4}(\ln 5). \end{aligned}$$

45. a) We find total profit by integrating:

$$\begin{aligned} P(10) &= R(10) - C(10) \\ &= \int_0^{10}\left[R'(t) - C'(t)\right]dt \\ &= \int_0^{10}\left[\left(100e^t\right) - \left(100 - 0.2t\right)\right]dt \\ &= \int_0^{10}\left[100e^t - 100 + 0.2t\right]dt \\ &= \left[100e^t - 100t + 0.1t^2\right]_0^{10} \\ &= \left(100e^{10} - 100(10) + 0.1(10)^2\right) - \\ &\qquad \left(100e^0 - 100(0) + 0.1(0)^2\right) \\ &= \left(100e^{10} - 990\right) - (100) \\ &= 100e^{10} - 1090 \\ &\approx 2{,}201{,}556.58 \end{aligned}$$

The total profit for the first 10 days is approximately \$2,201,556.58.

b) The average daily profit for the first ten days is given by

$$\begin{aligned} P_{\text{av}} &= \frac{1}{10-0}\int_0^{10}\left[R'(t) - C'(t)\right]dt \\ &\approx \frac{1}{10}\left[2{,}201{,}556.58\right] \qquad \text{From part (a).} \\ &\approx 220{,}155.66 \end{aligned}$$

The average daily profit over the first 10 days is approximately \$220,155.66.

46. a) We find total profit by integrating:

$$\begin{aligned} P(10) &= \int_0^{10}\left[R'(t) - C'(t)\right]dt \\ &= \int_0^{10}\left[\left(75e^t - 2t\right) - \left(75 - 3t\right)\right]dt \\ &= \int_0^{10}\left[75e^t - 75 + t\right]dt \\ &= \left[75e^t - 75t + \frac{1}{2}t^2\right]_0^{10} \\ &= \left(75e^{10} - 75(10) + \frac{1}{2}(10)^2\right) - \\ &\qquad \left(75e^0 - 75(0) + \frac{1}{2}(0)^2\right) \\ &= \left(75e^{10} - 700\right) - (75) \\ &= 75e^{10} - 775 \\ &\approx 1{,}651{,}209.94 \end{aligned}$$

The total profit for the first 10 days is approximately \$1,651,209.94.

b) The average daily profit for the first ten days is given by

$$P_{\text{av}} = \frac{1}{10-0}\int_0^{10}\left[R'(t)-C'(t)\right]dt$$
$$\approx \frac{1}{10}\left[1{,}651{,}209.94\right] \qquad \text{From part (a).}$$
$$\approx 165{,}120.99$$

The average daily profit over the first 10 days is approximately $165,120.99.

47. We find the average weekly sales for the first 5 weeks as follows:

$$S_{\text{av}} = \frac{1}{5-0}\int_0^5 S(t)\,dt$$
$$= \frac{1}{5}\int_0^5 9e^t\,dt$$
$$= \frac{1}{5}\left[9e^t\right]_0^5$$
$$= \frac{1}{5}\left[9e^5-9e^0\right]$$
$$= \frac{1}{5}\left[9e^5-9\right]$$
$$= \frac{9}{5}\left[e^5-1\right]$$
$$\approx 265.3437.$$

Therefore, average weekly sales for the first 5 weeks are $265.3437 hundred, or $26,534.37.

48.
$$R_{\text{av}} = \frac{1}{4-0}\int_0^4 R(t)\,dt$$
$$= \frac{1}{4}\int_0^4 0.5e^t\,dt$$
$$= \frac{1}{4}\left[0.5e^t\right]_0^4$$
$$= \frac{1}{8}\left[e^4-e^0\right]$$
$$= \frac{1}{8}\left[e^4-1\right]$$
$$\approx 6.69977$$

Therefore, average monthly revenue for the first 4 months is $6.69977 thousand, or $6699.77.

49. We find the average weekly sales for weeks 2 through 5 by integrating over the interval $[1,5]$ as follows:

$$S_{\text{av}} = \frac{1}{5-1}\int_1^5 S(t)\,dt$$
$$= \frac{1}{4}\int_1^5 9e^t\,dt$$
$$= \frac{1}{4}\left[9e^t\right]_1^5$$
$$= \frac{1}{4}\left[9e^5-9e^1\right]$$
$$= \frac{9}{4}\left[e^5-e\right]$$
$$\approx 327.8135.$$

Therefore, average weekly sales for weeks 2 though 5 week are $327.8135 hundred, or $32,781.35.

50.
$$R_{\text{av}} = \frac{1}{5-2}\int_2^5 R(t)\,dt$$
$$= \frac{1}{3}\int_2^5 0.5e^t\,dt$$
$$= \frac{1}{3}\left[0.5e^t\right]_2^5$$
$$= \frac{1}{3}\left[0.5e^5-0.5e^2\right]$$
$$= \frac{1}{6}\left[e^5-e^2\right]$$
$$\approx 23.50402$$

Therefore, average monthly revenue for months 3 through 5 is $23.50402 thousand, or $23,504.02.

51. a) We notice $-0.003t^2 \ge -0.009t^2$, which means $M'(t) \ge m'(t)$ so Ben has the higher rate of memorization.

b) $M(10)-m(10) = \int_0^{10}\left[M'(t)-m'(t)\right]dt$

$$\int_0^{10}\left[\left(-0.003t^2+0.2t\right)-\left(-0.009t^2+0.2t\right)\right]dt$$
$$= \int_0^{10} 0.006t^2\,dt$$
$$= \left[0.002t^3\right]_0^{10}$$
$$= 0.002(10)^3-0.002(0)^3$$
$$= 2-0$$
$$= 2$$

c) $m_{\text{av}} = \dfrac{1}{10-0}\int_0^{10} m'(t)\,dt$

$m_{\text{av}} = \dfrac{1}{10-0}\int_0^{10}\left(-0.009t^2+0.2t\right)dt$

$= \dfrac{1}{10}\left[-0.003t^3+0.1t^2\right]_0^{10}$

$= \dfrac{1}{10}\left[\left(-0.003(10)^3+0.1(10)^2\right)-\left(-0.003(0)^3+0.1(0)^2\right)\right]$

$= \dfrac{1}{10}\left[(-3+10)-(0+0)\right]$

$= \dfrac{1}{10}(7)$

$= \dfrac{7}{10} = 0.7$

Alice averaged memorizing about 0.7 words per minute during the first 10 minutes.

d) $m_{\text{av}} = \dfrac{1}{10-0}\int_0^{10} M'(t)\,dt$

$m_{\text{av}} = \dfrac{1}{10-0}\int_0^{10}\left(-0.003t^2+0.2t\right)dt$

$= \dfrac{1}{10}\left[-0.001t^3+0.1t^2\right]_0^{10}$

$= \dfrac{1}{10}\left[\left(-0.001(10)^3+0.1(10)^2\right)-\left(-0.001(0)^3+0.1(0)^2\right)\right]$

$= \dfrac{1}{10}\left[(-1+10)-(0+0)\right]$

$= \dfrac{1}{10}(9)$

$= \dfrac{9}{10} = 0.9$

Ben averaged memorizing about 0.9 words per minute during the first 10 minutes.

52. a) Notice $10t \ge t^2$ for $0 \le t \le 10$, therefore, $S(t) \ge s(t)$. Bonnie will have the higher test score.

b) $s_{\text{av}} = \dfrac{1}{10-7}\int_7^{10} s(t)\,dt$

$s_{\text{av}} = \dfrac{1}{10-7}\int_7^{10} t^2\,dt$

$= \dfrac{1}{3}\left[\dfrac{t^3}{3}\right]_7^{10}$

$= \dfrac{1}{9}\left[t^3\right]_7^{10}$

$= \dfrac{1}{9}\left[10^3-7^3\right]$

$= 73.$

Antonio scores 73, on average, when he studies for 7 – 10 hours.

c) $S_{\text{av}} = \dfrac{1}{10-6}\int_6^{10} S(t)\,dt$

$s_{\text{av}} = \dfrac{1}{10-6}\int_6^{10} 10t\,dt$

$= \dfrac{1}{4}\left[5t^2\right]_6^{10}$

$= \dfrac{1}{4}\left[5(10)^2-5(6)^2\right]$

$= \dfrac{1}{4}[320]$

$= 80.$

Bonnie scores an 80, on average, when she studies for 6 – 10 hours.

d) We integrate as follows:

$\dfrac{1}{10-0}\int_0^{10}\left[10t-t^2\right]dt$

$= \dfrac{1}{10}\left[5t^2-\dfrac{t^3}{3}\right]_0^{10}$

$= \dfrac{1}{10}\left[\left(5(10)^2-\dfrac{(10)^3}{3}\right)-\left(5(0)^2-\dfrac{(0)^3}{3}\right)\right]$

$= \dfrac{1}{10}\left(\dfrac{500}{3}\right)$

≈ 16.67

Their test scores will be approximately 16.7 points apart.

53. a) $W(0) = -6(0)^2+12(0)+90 = 90.$

At the beginning of the interval, the keyboarder's speed is 90 words per minute.

b) First, we find the derivative.

$W'(t) = -12t+12$

Next, we set the derivative equal to zero to find the critical value.

$$W'(t) = 0$$
$$-12t + 12 = 0$$
$$t = 1$$

The only critical value occurs when $t = 1$.

We also know that $W''(t) = -12 < 0$.

Therefore, by the Max-Min Principle 2, we know that an absolute maximum occurs at $t = 1$.

$$W(1) = -6(1)^2 + 12(1) + 90 = 96$$

Thus, the maximum speed is 96 words per minute, occurring 1 minute into the interval.

c) $W_{\text{av}} = \dfrac{1}{5-0}\displaystyle\int_0^5 W(t)\,dt$

$$W_{\text{av}} = \frac{1}{5-0}\int_0^5 \left(-6t^2 + 12t + 90\right)dt$$
$$= \frac{1}{5}\left[-2t^3 + 6t^2 + 90t\right]_0^5$$
$$= \frac{1}{5}\left[\left(-2(5)^3 + 6(5)^2 + 90(5)\right) - \left(-2(0)^3 + 6(0)^2 + 90(0)\right)\right]$$
$$= \frac{1}{5}[350 - 0]$$
$$= 70$$

The keyboarder's average speed over the 5 minute interval is 70 words per minute.

54. Letting $t = 0$ represent the year 2000, we are looking for the average over the interval $1 \le t \le 5$.

$$P_{\text{av}} = \frac{1}{5-1}\int_1^5 P(t)\,dt$$
$$= \frac{1}{4}\int_1^5 282.3e^{0.01t}\,dt$$
$$= \frac{1}{4}\left[\frac{282.3}{0.01}e^{0.01t}\right]_1^5$$
$$= \frac{1}{4}\left[28{,}230e^{0.01t}\right]_1^5$$
$$= \frac{28{,}230}{4}\left[e^{0.01(5)} - e^{0.01(1)}\right]$$
$$\approx 7057.5[0.04122093]$$
$$\approx 290.92$$

The average population from 2001 to 2005 was approximately 290.92 million people.

55. a) $C(0) = 42.03e^{-0.01050(0)} = 42.03$

The initial dosage is 42.03 micrograms per milliliter.

b) $C_{\text{av}} = \dfrac{1}{120-10}\displaystyle\int_{10}^{120} C(t)\,dt$

$$C_{\text{av}} = \frac{1}{110}\int_{10}^{120}\left(42.03e^{-0.01050t}\right)dt$$
$$= \frac{1}{110}\left[\frac{42.03}{-0.01050}e^{-0.01050t}\right]_{10}^{120}$$
$$= \frac{1}{110}\left[\left(\frac{42.03}{-0.01050}e^{-0.01050(120)}\right) - \frac{42.03}{-0.01050}e^{-0.01050(10)}\right]$$
$$\approx \frac{1}{110}[2468.4439]$$
$$\approx 22.44$$

The average amount of phenylbutazone in the calf's body for the time between 10 and 120 hours is about 22.44 micrograms per milliliter.

56. $T_{\text{av}} = \dfrac{1}{12-0}\displaystyle\int_0^{12} T(x)\,dx$

$$T_{\text{av}} = \frac{1}{12}\int_0^{12}\left(43.5 - 18.4x + 8.57x^2 - 0.996x^3 + 0.0338x^4\right)dx$$
$$= \frac{1}{12}\left[43.5x - 9.2x^2 + \frac{8.57}{3}x^3 - 0.249x^4 + 0.00676x^5\right]_0^{12}$$
$$\approx \frac{1}{12}[652.36032 - 0]$$
$$\approx 54.36$$

The average temperature in New York over the whole year is approximately 54.4 degrees.

57. a) We determine the average temperature as follows:

$$\frac{1}{10-0}\int_0^{10}\left(-t^2+5t+40\right)dt$$

$$=\frac{1}{10}\left[-\frac{t^3}{3}+\frac{5}{2}t^2+40t\right]_0^{10}$$

$$=\frac{1}{10}\left[\left(-\frac{(10)^3}{3}+\frac{5}{2}(10)^2+40(10)\right)-\left(-\frac{(0)^3}{3}+\frac{5}{2}(0)^2+40(0)\right)\right]$$

$$=\frac{1}{10}\left[\frac{950}{3}-0\right]$$

$$=\frac{95}{3}\approx 31.67$$

The average temperature over the 10 hour period is 31.7 degrees.

b) First, we find the critical values.

$$f'(t)=-2t+5$$

$$f'(t)=0$$

$$-2t+5=0$$

$$t=\frac{5}{2}=2.5$$

Since there is only one critical value and $f''(t)=-2<0$, we know that it represents a maximum over the closed interval. Therefore, the minimum temperature must be at an endpoint. We have:

$$f(0)=-(0)^2+5(0)+40=40$$

$$f(10)=-(10)^2+5(10)+40=-10$$

The minimum temperature over the 10 hour period is minus 10 degrees.

c) From part (b), we know that the maximum temperature occurs when $t=2.5$. Substituting we have:

$$f(2.5)=-(2.5)^2+5(2.5)+40=46.25$$

Therefore, the maximum temperature over the 10 hour period is 46.25 degrees.

58. $E_{\text{av}}=\dfrac{1}{5-1}\displaystyle\int_1^5 E(t)\,dt$

$$E_{\text{av}}=\frac{1}{4}\int_1^5 2t^2dt$$

$$=\frac{1}{4}\left[\frac{2}{3}t^3\right]_1^5$$

$$=\frac{1}{6}\left[t^3\right]_1^5$$

$$=\frac{1}{6}\left[5^3-1^3\right]$$

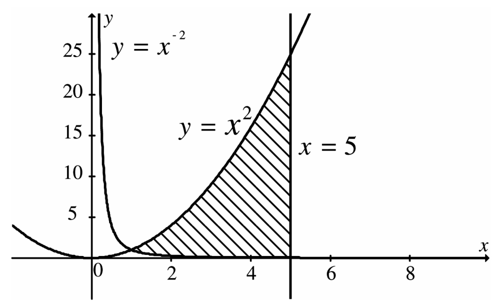
$$=\frac{1}{6}[125-1]$$

$$=\frac{124}{6}$$

$$\approx 20.667$$

The average emission over the period is approximately 20.667 billions pollution particulates per year.

59. First graph the system of equations and shade the region bounded by the graphs.



Here the boundaries are determined by looking at the graph. Note $x^2\ge x^{-2}$ over the interval $[1,5]$. We compute the area as follows:

$$\int_1^5\left(x^2-x^{-2}\right)dx$$

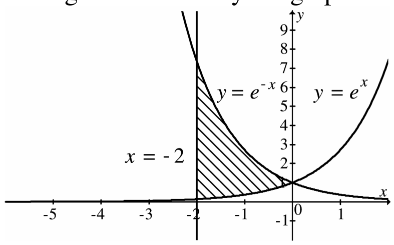
$$=\left[\frac{x^3}{3}+x^{-1}\right]_1^5$$

$$=\left(\frac{(5)^3}{3}+(5)^{-1}\right)-\left(\frac{(1)^3}{3}+(1)^{-1}\right)$$

$$=\frac{608}{15}$$

$$=40\tfrac{8}{15}.$$

60. First graph the system of equations and shade the region bounded by the graphs.



Here the boundaries are determined by looking at the graph. Note $e^{-x} \ge e^{x}$ over the interval $\left[-2, 0\right]$. We compute the area as follows:

$$\int_{-2}^{0}\left(e^{-x} - e^{x}\right)dx$$

$$= \left[-e^{-x} - e^{x}\right]_{-2}^{0}$$

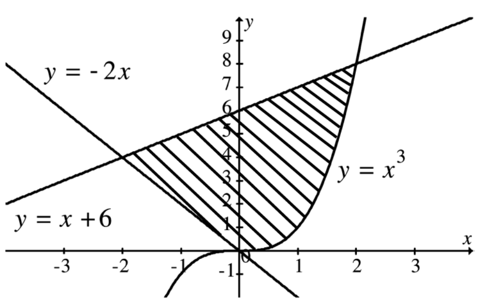
$$= \left(-e^{-0} - e^{0}\right) - \left(-e^{-(-2)} - e^{(-2)}\right)$$

$$= \left(-1 - 1\right) - \left(-e^{2} - e^{-2}\right)$$

$$= e^{2} + e^{-2} - 2$$

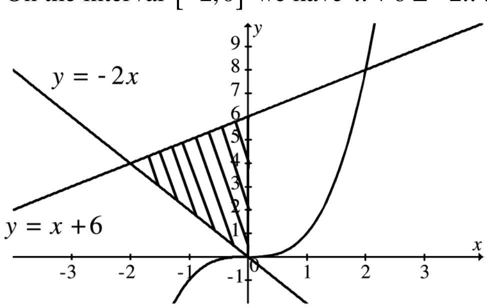
$$\approx 5.524.$$

61. First graph the system of equations and shade the region bounded by the graphs.

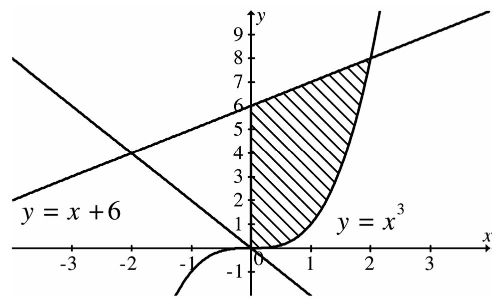


Here the boundaries are determined by looking at the graph. We will split the interval $\left[-2, 2\right]$ up into two parts.

On the interval $\left[-2, 0\right]$ we have $x + 6 \ge -2x$.



On the interval $\left[0, 2\right]$ we have $x + 6 \ge x^3$.



We compute the area as follows:

$$\int_{-2}^{0}\left[\left(x+6\right) - \left(-2x\right)\right]dx + \int_{0}^{2}\left[\left(x+6\right) - x^{3}\right]dx$$

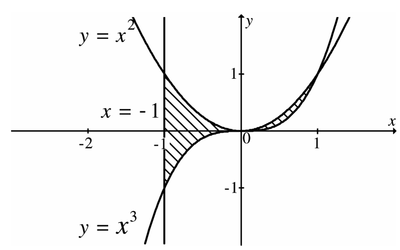
$$= \int_{-2}^{0}\left(3x + 6\right)dx + \int_{0}^{2}\left[-x^{3} + x + 6\right]dx$$

$$= \left[\frac{3}{2}x^{2} + 6x\right]_{-2}^{0} + \left[-\frac{x^{4}}{4} + \frac{x^{2}}{2} + 6x\right]_{0}^{2}$$

$$= \left(0 - \left(-6\right)\right) + \left(10 - 0\right)$$

$$= 16$$

62.



Here the boundaries are determined by looking at the graph. Note $x^{2} \ge x^{3}$ over the interval $\left[-1, 1\right]$. We compute the area as follows:

$$\int_{-1}^{1}\left(x^{2} - x^{3}\right)dx$$

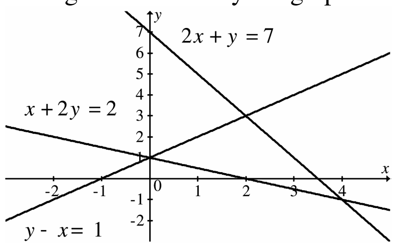
$$= \left[\frac{x^{3}}{3} - \frac{x^{4}}{4}\right]_{-1}^{1}$$

$$= \left(\frac{(1)^{3}}{3} - \frac{(1)^{4}}{4}\right) - \left(\frac{(-1)^{3}}{3} - \frac{(-1)^{4}}{4}\right)$$

$$= \frac{1}{12} - \left(-\frac{7}{12}\right)$$

$$= \frac{2}{3}.$$

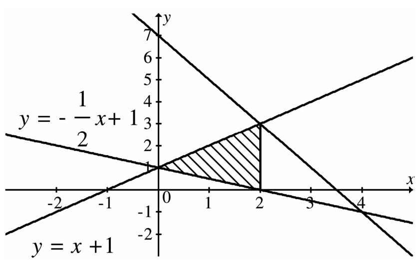
63. First graph the system of equations and shade the region bounded by the graphs.



Here the boundaries are determined by looking at the graph. We will split the interval $[0,4]$ up into two parts. Solving each equation for *y*, we have

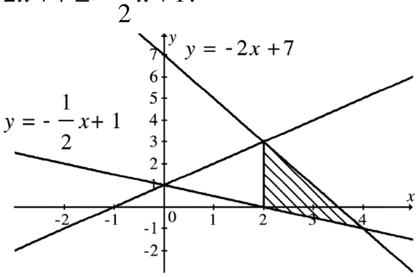
$$x+2y=2 \to y=-\frac{x}{2}+1$$
$$y-x=1 \to y=x+1$$
$$2x+y=7 \to y=-2x+7$$

On the interval $[0,2]$ we have $x+1 \ge -\frac{x}{2}+1$.



On the interval $[2,4]$ we have

$$-2x+7 \ge -\frac{1}{2}x+1.$$



We compute the area as follows:

$$\int_0^2 \left[(x+1)-\left(-\frac{1}{2}x+1\right)\right]dx + \int_2^4 \left[(-2x+7)-\left(-\frac{1}{2}x+1\right)\right]dx$$
$$=\int_0^2 \frac{3}{2}x\,dx+\int_2^4\left[-\frac{3}{2}x+6\right]dx$$
$$=\left[\frac{3}{4}x^2\right]_0^2+\left[-\frac{3}{4}x^2+6x\right]_2^4$$
$$=(3-0)+(12-9)$$
$$=6$$

64. First we find the coordinates of the relative extrema.

$$y=3x^5-20x^3$$
$$y'=15x^4-60x^2$$

The derivative exists for all real numbers. We solve $y'=0$.

$$15x^4-60x^2=0$$
$$15x^2\left(x^2-4\right)=0$$
$$15x^2(x+2)(x-2)=0$$
$$x=0 \quad \text{or} \quad x=-2 \quad \text{or} \quad x=2$$

We use the Second Derivative Test.

$$y''=60x^3-120x$$

When $x=-2$,

$$y''=60(-2)^3-120(-2)=-240<0$$

Therefore there is a relative maximum at $x=-2$.

When $x=2$,

$$y''=60(2)^3-120(2)=240>0$$

Therefore there is a relative minimum at $x=2$.

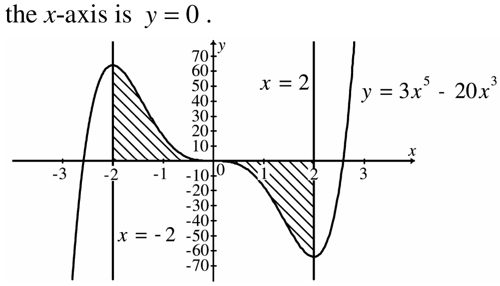
When $x=0, y''=0$ so the test fails. We use the First Derivative Test and test a value in $(-2,0)$ and $(0,2)$.

Test $-1: 15(-1)^4-60(-1)^2=-45<0$

Test $1: 15(1)^4-60(1)^2=-45<0$

Thus the function is decreasing on both intervals, so there is no relative extremum at $x=0$.

We graph the region noting that the equation of the *x*-axis is $y=0$.



On the interval $[-2,0]$, $3x^5-20x^3 \ge 0$, and on the interval $[0,2]$, $0 \ge 3x^5-20x^3$.

We compute the area as follows:

$$\int_{-2}^{0}\left[\left(3x^5-20x^3\right)-(0)\right]dx+$$
$$\int_{0}^{2}\left[(0)-\left(3x^5-20x^3\right)\right]dx$$
$$=\int_{-2}^{0}\left(3x^5-20x^3\right)dx+\int_{0}^{2}\left[-3x^5+20x^3\right]dx$$
$$=\left[\frac{1}{2}x^6-5x^4\right]_{-2}^{0}+\left[-\frac{1}{2}x^6+5x^4\right]_{0}^{2}$$
$$=(0-0)-(32-80)+(-32+80)-(-0+0)$$
$$=0+48+48+0$$
$$=96.$$

65. First we find the coordinates of the relative extrema.

$y=x^3-3x+2$

$y'=3x^2-3$

The derivative exists for all real numbers. We solve $y'=0$.

$3x^2-3=0$

$x^2-1=0$

$x=\pm 1$

$x=-1$ or $x=1$

We use the Second Derivative Test.

$y''=6x$

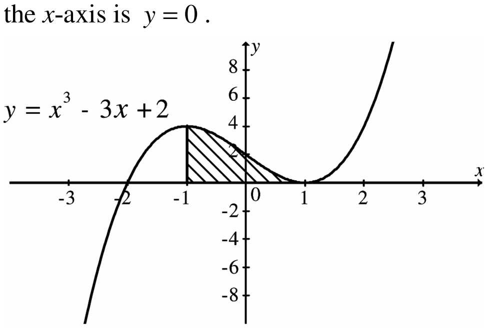
When $x=-1, y''=6(-1)=-6<0$

Therefore there is a relative maximum at $x=-1$.

When $x=1, y''=6(1)=6>0$

Therefore there is a relative minimum at $x=1$.

We graph the region noting that the equation of the x-axis is $y=0$.



On the interval $[-1,1]$, $x^3-3x+2\ge 0$. We compute the area as follows:

$$\int_{-1}^{1}\left(x^3-3x+2\right)dx$$
$$=\left[\frac{x^4}{4}-\frac{3}{2}x^2+2x\right]_{-1}^{1}$$
$$=\left(\frac{(1)^4}{4}-\frac{3}{2}(1)^2+2(1)\right)-$$
$$\left(\frac{(-1)^4}{4}-\frac{3}{2}(-1)^2+2(-1)\right)$$
$$=\left(\frac{3}{4}\right)-\left(-\frac{13}{4}\right)$$
$$=4.$$

66. $Q=\int_0^R 2\pi\cdot\frac{p}{4Lv}\left(R^2-r^2\right)\cdot r\,dr$

$$Q=\frac{2\pi p}{4Lv}\int_0^R\cdot\left(R^2r-r^3\right)dr$$
$$=\frac{\pi p}{2Lv}\left[\frac{R^2r^2}{2}-\frac{r^4}{4}\right]_0^R$$
$$=\frac{\pi p}{2Lv}\left[\left(\frac{R^2\cdot R^2}{2}-\frac{R^4}{4}\right)-\left(\frac{R^2\cdot 0^2}{2}-\frac{0^4}{4}\right)\right]$$
$$=\frac{\pi p}{2Lv}\left[\frac{R^4}{4}-0\right]$$
$$=\frac{\pi p R^4}{8Lv}$$

67. $\int_1^2\left[\left(3x^2+5x\right)-\left(3x+K\right)\right]dx$

$$=\int_1^2\left(3x^2+2x-K\right)dx$$
$$=\left[x^3+x^2-Kx\right]_1^2$$
$$=\left[2^3+2^2-K\cdot 2\right]-\left[1^3+1^2-K\right]$$
$$=[12-2K]-[2-K]$$
$$=10-K$$

Since

$\int_1^2\left[\left(3x^2+5x\right)-\left(3x+K\right)\right]dx=6$

We solve for K by substituting.

$10-K=6$

$-K=-4$

$K=4$

68. From the graph we find that the interval we are concerned with is approximately $[-4, 0.8]$.

Over this interval $\sqrt{16-x^2} \ge x^2+4x$. We use the fnInt function on the calculator to find

$\int_{-4}^{0.8}\left[\sqrt{16-x^2}-\left(x^2+4x\right)\right]dx \approx 24.961.$

69. From the graph we find that the interval we are concerned with is $[0,2]$. Over this interval

$x\sqrt{4-x^2} \ge \dfrac{-4x}{x^2+1}$. We use the fnInt function on the calculator to find

$\int_{0}^{2}\left[x\sqrt{4-x^2}-\dfrac{-4x}{x^2+1}\right]dx \approx 5.8855.$

70. From the graph we find that the interval we are concerned with is approximately $[-1.54, 1.35]$. Over this interval

$1-x+8x^2-4x^4 \ge 2x^2+x-4$. We use the fnInt function on the calculator to find

$\int_{-1.54}^{1.35}\left[\left(1-x+8x^2-4x^4\right)-\left(2x^2+x-4\right)\right]dx$

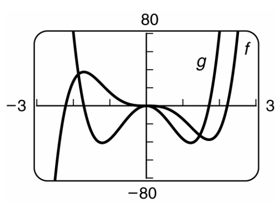
$\approx 16.708.$

71. From the graph we find that the interval we are concerned with is $[-1,1]$. Over this interval

$\sqrt{1-x^2} \ge 1-x^2$. We use the fnInt function on the calculator to find

$\int_{-1}^{1}\left[\sqrt{1-x^2}-\left(1-x^2\right)\right]dx \approx 0.2375.$

72. a)



b) From the graph we estimate that the first coordinates of the three points of intersection of the graphs are $a \approx -1.8623, b \approx 0, c \approx 1.4594$.

c) Using the fnInt feature on a calculator, we find that

$\int_{-1.8623}^{0}\left[3.8x^5-18.6x^3-\left(19x^4-55.8x^2\right)\right]dx$

≈ 64.524

d) Using the fnInt feature on a calculator, we find that

$\int_{0}^{1.4594}\left[3.8x^5-18.6x^3-\left(19x^4-55.8x^2\right)\right]dx$

≈ 17.683

Exercise Set 4.5

1. $\int\left(8+x^3\right)^5 3x^2dx$

Let $u=8+x^3$, then $du=3x^2dx$.

$=\int u^5du$ Substitution: $\begin{matrix}u=8+x^3\\ du=3x^2dx\end{matrix}$

$=\dfrac{1}{6}u^6+C$ Formula A

$=\dfrac{1}{6}\left(8+x^3\right)^6+C$ Reverse substitution

2. $\int\left(x^2-7\right)^6 2xdx$

$=\int u^6du$ Substitution: $\begin{matrix}u=x^2-7\\ du=2xdx\end{matrix}$

$=\dfrac{1}{7}u^7+C$

$=\dfrac{1}{7}\left(x^2-7\right)^7+C$

3. $\int\left(x^2-6\right)^7 xdx$

Let $u=x^2-6$, then $du=2xdx$. We do not have $2xdx$. We only have xdx and need to supply a 2. We do this by multiplying by $\frac{1}{2}\cdot 2$ as follows.

$=\dfrac{1}{2}\cdot 2\int\left(x^2-6\right)^7 xdx$ Multiplying by 1

$=\dfrac{1}{2}\int\left(x^2-6\right)^7 2xdx$ $\left[a\int f(x)\,dx=\int af(x)\,dx\right]$

$=\dfrac{1}{2}\int u^7du$ Substitution: $\begin{matrix}u=x^2-6\\ du=2xdx\end{matrix}$

$=\dfrac{1}{16}u^8+C$ Formula A

$=\dfrac{1}{16}\left(x^2-6\right)^8+C$ Reverse substitution

4. $\int\left(x^3+1\right)^4 x^2dx$

$=\frac{1}{3}\cdot 3\int\left(x^3+1\right)^4 x^2dx$

$=\frac{1}{3}\int\left(x^3+1\right)^4 3x^2dx$

$=\frac{1}{3}\int u^4du$ Substitution: $\substack{u=x^3+1\\du=3x^2dx}$

$=\frac{1}{15}u^5+C$

$=\frac{1}{15}\left(x^3+1\right)^5+C$

5. $\int\left(3t^4+2\right)t^3dt$

Let $u=3t^4+2,$ then $du=12t^3dt.$ We do not have $12t^3dt.$ We only have t^3dt and need to supply a 12. We do this by multiplying by $\frac{1}{12}\cdot 12$ as follows.

$=\frac{1}{12}\cdot 12\int\left(3t^4+2\right)^7 t^3dt$ Multiplying by 1

$=\frac{1}{12}\int\left(3t^4+2\right)12t^3dt$

$=\frac{1}{12}\int udu$ Substitution: $\substack{u=3t^4+2\\du=12t^3dt}$

$=\frac{1}{24}u^2+C$ Formula A

$=\frac{1}{24}\left(3t^4+2\right)^2+C$ Reverse substitution

6. $\int\left(2t^5-3\right)t^4dt$

$=\frac{1}{10}\cdot 10\int\left(2t^5-3\right)t^4dt$

$=\frac{1}{10}\int\left(2t^5-3\right)\cdot 10t^4dt$

$=\frac{1}{10}\int udu$ Substitution: $\substack{u=2t^5-3\\du=10t^4dt}$

$=\frac{1}{20}u^2+C$

$=\frac{1}{20}\left(2t^5-3\right)^2+C$

7. $\int\frac{2}{1+2x}dx$

Let $u=1+2x,$ then $du=2dx.$

$=\int\frac{du}{u}$ Substitution: $\substack{u=1+2x\\du=2dx}$

$=\ln u+C$ Formula C, $u>0.$

$=\ln\left(1+2x\right)+C$ Reverse substitution

8. $\int\frac{5}{5x+7}dx$

$=\int\frac{du}{u}$ Substitution: $\substack{u=5x+7\\du=5dx}$

$=\ln u+C$

$=\ln\left(5x+7\right)+C$

9. $\int\left(\ln x\right)^3\frac{1}{x}dx$

Let $u=\ln x,$ then $du=\frac{1}{x}dx.$

$=\int u^3du$ Substitution: $\substack{u=\ln x\\du=\frac{1}{x}dx}$

$=\frac{1}{4}u^4+C$ Formula A

$=\frac{1}{4}\left(\ln x\right)^4+C$ Reverse substitution

10. $\int\left(\ln x\right)^7\frac{1}{x}dx$

$=\int u^7du$ Substitution: $\substack{u=\ln x\\du=\frac{1}{x}dx}$

$=\frac{1}{8}u^8+C$

$=\frac{1}{8}\left(\ln x\right)^8+C$

11. $\int e^{3x}dx$

Let $u=3x,$ then $du=3dx.$ We do not have $3dx.$ We only have dx and need to supply a 3. We do this by multiplying by $\frac{1}{3}\cdot 3$ as follows.

$= \frac{1}{3} \cdot 3\int e^{3x} dx$ Multiplying by 1

$= \frac{1}{3}\int e^{3x} \cdot 3dx$

$= \frac{1}{3}\int e^{u} du$ Substitution: $\begin{smallmatrix} u=3x \\ du=3dx \end{smallmatrix}$

$= \frac{1}{3}e^{u} + C$ Formula B

$= \frac{1}{3}e^{3x} + C$ Reverse substitution

12. $\int e^{7x} dx$

$= \frac{1}{7} \cdot 7\int e^{7x} dx$

$= \frac{1}{7}\int e^{7x} \cdot 7dx$

$= \frac{1}{7}\int e^{u} du$ Substitution: $\begin{smallmatrix} u=7x \\ du=7dx \end{smallmatrix}$

$= \frac{1}{7}e^{u} + C$

$= \frac{1}{7}e^{7x} + C$

13. $\int e^{x/3} dx$

Let $u = \frac{x}{3}$, then $du = \frac{1}{3}dx$. We do not have $\frac{1}{3}dx$. We only have dx and need to supply a $\frac{1}{3}$. We do this by multiplying by $\frac{1}{3} \cdot 3$ as follows.

$= \frac{1}{3} \cdot 3\int e^{x/3} dx$ Multiplying by 1

$= 3\int e^{x/3} \cdot \frac{1}{3}dx$

$= 3\int e^{u} du$ Substitution: $\begin{smallmatrix} u=x/3 \\ du=1/3dx \end{smallmatrix}$

$= 3e^{u} + C$ Formula B

$= 3e^{x/3} + C$ Reverse substitution

14. $\int e^{x/2} dx$

$= \frac{1}{2} \cdot 2\int e^{x/2} dx$

$= 2\int e^{x/2} \cdot \frac{1}{2}dx$

$= 2\int e^{u} du$ Substitution: $\begin{smallmatrix} u=x/2 \\ du=1/2dx \end{smallmatrix}$

$= 2e^{u} + C$

$= 2e^{x/2} + C$

15. $\int x^4 e^{x^5} dx$

Let $u = x^5$, then $du = 5x^4 dx$. We do not have $5x^4 dx$. We only have $x^4 dx$ and need to supply a 5. We do this by multiplying by $\frac{1}{5} \cdot 5$ as follows.

$= \frac{1}{5} \cdot 5\int x^4 e^{x^5} dx$ Multiplying by 1

$= \frac{1}{5}\int 5x^4 e^{x^5} dx$

$= \frac{1}{5}\int e^{u} du$ Substitution: $\begin{smallmatrix} u=x^5 \\ du=5x^4 dx \end{smallmatrix}$

$= \frac{1}{5}e^{u} + C$ Formula B

$= \frac{1}{5}e^{x^5} + C$ Reverse substitution

16. $\int x^3 e^{x^4} dx$

$= \frac{1}{4} \cdot 4\int x^3 e^{x^4} dx$

$= \frac{1}{4}\int 4x^3 e^{x^4} dx$

$= \frac{1}{4}\int e^{u} du$ Substitution: $\begin{smallmatrix} u=x^4 \\ du=4x^3 dx \end{smallmatrix}$

$= \frac{1}{4}e^{u} + C$

$= \frac{1}{4}e^{x^4} + C$

17. $\int te^{-t^2} dt$

Let $u = -t^2$, then $du = -2tdt$. We do not have $-2tdt$. We only have tdt and need to supply a -2. We do this by multiplying by $\left(-\frac{1}{2}\right) \cdot (-2)$ as follows.

$=\left(-\frac{1}{2}\right)\cdot(-2)\int te^{-t^2}\,dt$ Multiplying by 1

$=-\frac{1}{2}\int(-2t)e^{-t^2}\,dt$

$=-\frac{1}{2}\int e^u\,du$ Substitution: $\begin{smallmatrix}u=-t^2\\ du=-2t\,dt\end{smallmatrix}$

$=-\frac{1}{2}e^u+C$ Formula B

$=-\frac{1}{2}e^{-t^2}+C$ Reverse substitution

18. $\int t^2e^{-t^3}\,dt$

$=\left(-\frac{1}{3}\right)\cdot(-3)\int t^2e^{-t^3}\,dt$

$=-\frac{1}{3}\int(-3t^2)e^{-t^3}\,dt$

$=-\frac{1}{3}\int e^u\,du$ Substitution: $\begin{smallmatrix}u=-t^3\\ du=-3t^2dt\end{smallmatrix}$

$=-\frac{1}{3}e^u+C$

$=-\frac{1}{3}e^{-t^3}+C$

19. $\int\frac{1}{5+2x}\,dx$

Let $u=5+2x$, then $du=2dx$. We do not have $2dx$. We only have dx and need to supply a 2. We do this by multiplying by $\frac{1}{2}\cdot 2$ as follows.

$=\frac{1}{2}\cdot 2\int\frac{1}{5+2x}\,dx$ Multiplying by 1

$=\frac{1}{2}\int\frac{2dx}{5+2x}$

$=\frac{1}{2}\int\frac{du}{u}$ Substitution: $\begin{smallmatrix}u=5+2x\\ du=2dx\end{smallmatrix}$

$=\frac{1}{2}\ln u+C$ Formula C, $u>0$.

$=\frac{1}{2}\ln(5+2x)+C$ Reverse substitution

20. $\int\frac{1}{2+8x}\,dx$

$=\frac{1}{8}\cdot 8\int\frac{1}{2+8x}\,dx$

$=\frac{1}{8}\int\frac{8dx}{2+8x}$

$=\frac{1}{8}\int\frac{du}{u}$ Substitution: $\begin{smallmatrix}u=2+8x\\ du=8dx\end{smallmatrix}$

$=\frac{1}{8}\ln u+C$

$=\frac{1}{8}\ln(2+8x)+C$

21. $\int\frac{dx}{12+3x}$

Let $u=12+3x$, then $du=3dx$. We do not have $3dx$. We only have dx and need to supply a 3. We do this by multiplying by $\frac{1}{3}\cdot 3$ as follows.

$=\frac{1}{3}\cdot 3\int\frac{dx}{12+3x}$ Multiplying by 1

$=\frac{1}{3}\int\frac{3dx}{12+3x}$

$=\frac{1}{3}\int\frac{du}{u}$ Substitution: $\begin{smallmatrix}u=12+3x\\ du=3dx\end{smallmatrix}$

$=\frac{1}{3}\ln u+C$ Formula C, $u>0$.

$=\frac{1}{3}\ln(12+3x)+C$ Reverse substitution

22. $\int\frac{dx}{1+7x}$

$=\frac{1}{7}\cdot 7\int\frac{dx}{1+7x}$

$=\frac{1}{7}\int\frac{7dx}{1+7x}$

$=\frac{1}{7}\int\frac{du}{u}$ Substitution: $\begin{smallmatrix}u=1+7x\\ du=7dx\end{smallmatrix}$

$=\frac{1}{7}\ln u+C$

$=\frac{1}{7}\ln(1+7x)+C$

23. $\int\frac{dx}{1-x}$

Let $u=1-x$, then $du=-dx$. We do not have $-dx$. We only have dx and need to supply a -1. We do this by multiplying by $(-1)(-1)$ as follows.

$= (-1)\cdot(-1)\int \frac{dx}{1-x}$ Multiplying by 1

$= -1\cdot\int \frac{-1dx}{1-x}$

$= -\int \frac{du}{u}$ Substitution: $\substack{u=1-x \\ du=-dx}$

$= -\ln u + C$ Formula C, $u > 0$.

$= -\ln(1-x) + C$ Reverse substitution

24. $\int \frac{dx}{4-x}$

$= (-1)\cdot(-1)\int \frac{dx}{4-x}$

$= -\int \frac{-dx}{4-x}$

$= -\int \frac{du}{u}$ Substitution: $\substack{u=4-x \\ du=-dx}$

$= -\ln u + C$

$= -\ln(4-x) + C$

25. $\int t\left(t^2-1\right)^5 dt$

Let $u = t^2 - 1$, then $du = 2tdt$. We do not have $2tdt$. We only have tdt and need to supply a 2. We do this by multiplying by $\frac{1}{2}\cdot 2$ as follows.

$= \frac{1}{2}\cdot 2\int t\left(t^2-1\right)^5 dt$ Multiplying by 1

$= \frac{1}{2}\int 2t\left(t^2-1\right)^5 dt$

$= \frac{1}{2}\int u^5 du$ Substitution: $\substack{u=t^2-1 \\ du=2tdt}$

$= \frac{1}{12}u^6 + C$ Formula A

$= \frac{1}{12}\left(t^2-1\right)^6 + C$ Reverse substitution

26. $\int t^2\left(t^3-1\right)^7 dt$

$= \frac{1}{3}\cdot 3\int t^2\left(t^3-1\right)^7 dt$

$= \frac{1}{3}\int 3t^2\left(t^3-1\right)^7 dt$

$= \frac{1}{3}\int u^7 du$ Substitution: $\substack{u=t^3-1 \\ du=3t^2dt}$

$= \frac{1}{24}u^8 + C$

$= \frac{1}{24}\left(t^3-1\right)^8 + C$

27. $\int \left(x^4+x^3+x^2\right)^7\left(4x^3+3x^2+2x\right)dx$

Let $u = x^4+x^3+x^2$, then $du = \left(4x^3+3x^2+2x\right)dx$.

$= \int u^7 du$ Substitution: $\substack{u=x^4+x^3+x^2 \\ du=\left(4x^3+3x^2+2x\right)dx}$

$= \frac{1}{8}u^8 + C$ Formula A

$= \frac{1}{8}\left(x^4+x^3+x^2\right)^8 + C$ Reverse substitution

28. $\int \left(x^3-x^2-x\right)^9\left(3x^2-2x-1\right)dx$

$= \int u^9 du$ Substitution: $\substack{u=x^3-x^2-x \\ du=\left(3x^2-2x^2-1\right)dx}$

$= \frac{1}{10}u^{10} + C$ Formula A

$= \frac{1}{10}\left(x^3-x^2-x\right)^{10} + C$ Reverse substitution

29. $\int \frac{e^x dx}{4+e^x}$

Let $u = 4+e^x$, then $du = e^x dx$.

$= \int \frac{du}{u}$ Substitution: $\substack{u=4+e^x \\ du=e^x dx}$

$= \ln u + C$ Formula C, $u > 0$.

$= \ln\left(4+e^x\right) + C$ Reverse substitution

30. $\int \frac{e^t dt}{3+e^t}$

$= \int \frac{du}{u}$ Substitution: $\substack{u=3+e^t \\ du=e^t dt}$

$= \ln u + C$ Formula C, $u > 0$.

$= \ln\left(3+e^t\right) + C$ Reverse substitution

31. $\int \frac{\ln x^2}{x}dx$

Using properties of logarithms, we have

$\int \frac{2\cdot\ln x}{x}dx = 2\int \frac{\ln x}{x}dx$

Let $u = \ln x$, then $du = \frac{1}{x}dx$.

$= 2\int u du$ Substitution: $\substack{u=\ln x \\ du=\frac{1}{x}dx}$

$= u^2 + C$ Formula A

$= (\ln x)^2 + C$ Reverse substitution

32. $\int \frac{(\ln x)^2}{x}dx$

$= \int u^2 du$ Substitution: $\begin{smallmatrix} u=\ln x \\ du=\frac{1}{x}dx \end{smallmatrix}$

$= \frac{1}{3}u^3 + C$

$= \frac{1}{3}(\ln x)^3 + C$

33. $\int \frac{dx}{x \ln x}$

Let $u = \ln x,$ then $du = \frac{1}{x}dx.$

$= \int \frac{1}{\ln x} \cdot \left(\frac{1}{x}dx\right)$

$= \int \frac{1}{u}du$ Substitution: $\begin{smallmatrix} u=\ln x \\ du=\frac{1}{x}dx \end{smallmatrix}$

$= \ln u + C$ Formula C, $u > 0$.

$= \ln(\ln x) + C$ Reverse substitution

34. $\int \frac{dx}{x \ln x^2}$

$= \int \frac{dx}{2x \ln x}$

$= \frac{1}{2}\int \frac{dx}{x \ln x}$

$= \frac{1}{2}\int \frac{1}{u}du$ Substitution: $\begin{smallmatrix} u=\ln x \\ du=\frac{1}{x}dx \end{smallmatrix}$

$= \frac{1}{2}\ln u + C$

$= \frac{1}{2}\ln(\ln x) + C$

35. $\int x\sqrt{ax^2 + b}\, dx$

Let $u = ax^2 + b,$ then $du = 2axdx.$ We do not have $2axdx.$ We only have xdx and need to supply a $2a$. We do this by multiplying by $\frac{1}{2a} \cdot 2a$ as follows.

$= \frac{1}{2a} \cdot 2a\int x\sqrt{ax^2 + b}\, dx$ Multiplying by 1

$= \frac{1}{2a}\int 2ax\sqrt{ax^2 + b}\, dx$

$= \frac{1}{2a}\int \sqrt{u}\, du$ Substitution: $\begin{smallmatrix} u=ax^2+b \\ du=2axdx \end{smallmatrix}$

$= \frac{1}{2a}\int u^{1/2}du$

$= \frac{1}{2a} \cdot \frac{2}{3}u^{3/2} + C$ Formula A

$= \frac{1}{3a}(ax^2 + b)^{3/2} + C$ Reverse substitution

36. $\int \sqrt{ax + b}\, dx$

$= \frac{1}{a} \cdot a\int \sqrt{ax + b}\, dx$

$= \frac{1}{a}\int a \cdot \sqrt{ax + b}\, dx$

$= \frac{1}{a}\int \sqrt{u}\, du$ Substitution: $\begin{smallmatrix} u=ax+b \\ du=adx \end{smallmatrix}$

$= \frac{1}{a}\int u^{1/2}du$

$= \frac{1}{a} \cdot \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3a}(ax + b)^{3/2} + C$

37. $\int P_0 e^{kt}dt = P_0 \cdot \int e^{kt}dt$

Let $u = kt,$ then $du = kdt.$ We do not have $kdt.$ We only have dt and need to supply a k. We do this by multiplying by $\frac{1}{k} \cdot k$ as follows.

$= \frac{1}{k} \cdot k \cdot P_0\int e^{kt}dt$ Multiplying by 1

$= \frac{1}{k} \cdot P_0\int e^{kt} \cdot kdt$

$= \frac{P_0}{k}\int e^u du$ Substitution: $\begin{smallmatrix} u=kt \\ du=kdt \end{smallmatrix}$

$= \frac{P_0}{k}e^u + C$ Formula B

$= \frac{P_0}{k}e^{kt} + C$ Reverse substitution

38. $\int be^{ax}dx$

$= b\int e^{ax}dx$

$= \frac{1}{a}\cdot a\cdot b\int e^{ax}dx$

$= \frac{b}{a}\int e^{ax}\cdot a\,dx$

$= \frac{b}{a}\int e^{u}du$ Substitution: $\begin{smallmatrix} u=ax \\ du=a\,dx \end{smallmatrix}$

$= \frac{b}{a}e^{u}+C$

$= \frac{b}{a}e^{ax}+C$

39. $\int \frac{x^3dx}{(2-x^4)^7}$

Let $u=2-x^4$, then $du=-4x^3dx.$ We do not have $-4x^3dx.$ We only have x^3dx and need to supply a -4. We do this by multiplying by $\frac{1}{-4}\cdot -4$ as follows.

$= \left(-\frac{1}{4}\right)\cdot(-4)\int \frac{x^3dx}{(2-x^4)^7}$ Multiplying by 1

$= -\frac{1}{4}\int \frac{-4x^3dx}{(2-x^4)^7}$

$= -\frac{1}{4}\int \frac{du}{u^7}$ Substitution: $\begin{smallmatrix} u=2-x^4 \\ du=-4x^3dx \end{smallmatrix}$

$= -\frac{1}{4}\int u^{-7}du$

$= -\frac{1}{4}\cdot\frac{1}{-6}u^{-6}+C$ Formula A

$= \frac{1}{24}(2-x^4)^{-6}+C$ Reverse substitution

$= \frac{1}{24(2-x^4)^6}+C$

40. $\int \frac{3x^2dx}{(1+x^3)^5}$

$= \int \frac{du}{u^5}$ Substitution: $\begin{smallmatrix} u=1+x^3 \\ du=3x^2dx \end{smallmatrix}$

$= \int u^{-5}du$

$= \cdot\frac{1}{-4}u^{-4}+C$

$= -\frac{1}{4}(1+x^3)^{-4}+C$

$= -\frac{1}{4(1+x^3)^4}+C$

41. $\int 12x\sqrt[5]{1+6x^2}\,dx$

Let $u=1+6x^2$, then $du=12xdx.$

$= \int \sqrt[5]{u}\,du$ Substitution: $\begin{smallmatrix} u=1+6x^2 \\ du=12xdx \end{smallmatrix}$

$= \int u^{1/5}du$

$= \frac{5}{6}u^{6/5}+C$ Formula A

$= \frac{5}{6}(1+6x^2)^{6/5}+C$ Reverse substitution

42. $\int 5x\cdot\sqrt[4]{1-x^2}\,dx$

$= \left(-\frac{1}{2}\right)(-2)\int 5x\cdot\sqrt[4]{1-x^2}\,dx$

$= -\frac{5}{2}\int -2x\cdot\sqrt[4]{1-x^2}\,dx$

$= -\frac{5}{2}\int \sqrt[4]{u}\,du$ Substitution: $\begin{smallmatrix} u=1-x^2 \\ du=-2xdx \end{smallmatrix}$

$= -\frac{5}{2}\int u^{1/4}du$

$= -2u^{5/4}+C$

$= -2(1-x^2)^{5/4}+C$

43. $\int_0^1 2xe^{x^2}dx$

We first find the indefinite integral

$\int 2xe^{x^2}dx$

$= \int e^{u}du$ Substitution: $\begin{smallmatrix} u=x^2 \\ du=2xdx \end{smallmatrix}$

$= e^{u}+C$ Formula B

$= e^{x^2}+C$ Reverse Substitution

Next, we evaluate the definite integral on $[0,1]$.

$$\int_0^1 2xe^{x^2}\,dx = \left[e^{x^2}\right]_0^1 \qquad \text{Let } C=0.$$
$$= e^{(1)^2} - e^{(0)^2}$$
$$= e^1 - e^0$$
$$= e-1$$

44. $\int_0^1 3x^2 e^{x^3}\,dx$

We first find the indefinite integral

$$\int 3x^2 e^{x^3}\,dx$$
$$= \int e^u\,du \qquad \text{Substitution: } \begin{smallmatrix} u=x^3 \\ du=3x^2dx \end{smallmatrix}$$
$$= e^u + C$$
$$= e^{x^3} + C$$

Next, we evaluate the definite integral on $[0,1]$.

$$\int_0^1 3x^2 e^{x^3}\,dx = \left[e^{x^3}\right]_0^1 \qquad \text{Let } C=0.$$
$$= e^{(1)^3} - e^{(0)^3}$$
$$= e^1 - e^0$$
$$= e-1$$

45. $\int_0^1 x\left(x^2+1\right)^5 dx$

We first find the indefinite integral

$$\int x\left(x^2+1\right)^5 dx$$
$$= \frac{1}{2}\cdot 2\int x\left(x^2+1\right)^5 dx \qquad \text{Multiplying by 1}$$
$$= \frac{1}{2}\int 2x\left(x^2+1\right)^5 dx$$
$$= \frac{1}{2}\int u^5\,du \qquad \text{Substitution: } \begin{smallmatrix} u=x^2+1 \\ du=2xdx \end{smallmatrix}$$
$$= \frac{1}{12}u^6 + C \qquad \text{Formula A}$$
$$= \frac{1}{12}\left(x^2+1\right)^6 + C \qquad \text{Reverse Substitution}$$

Next, we evaluate the definite integral on $[0,1]$.

$$\int_0^1 x\left(x^2+1\right)^5 dx$$
$$= \left[\frac{1}{12}\left(x^2+1\right)^6\right]_0^1 \qquad \text{Let } C=0.$$
$$= \left[\frac{1}{12}\left(1^2+1\right)^6 - \frac{1}{12}\left(0^2+1\right)^6\right]$$
$$= \frac{1}{12}(2)^6 - \frac{1}{12}(1)^6$$
$$= \frac{16}{3} - \frac{1}{12}$$
$$= \frac{21}{4}$$

46. $\int_1^2 x\left(x^2-1\right)^7 dx$

We first find the indefinite integral

$$\int x\left(x^2-1\right)^7 dx$$
$$= \frac{1}{2}\cdot 2\int x\left(x^2-1\right)^7 dx$$
$$= \frac{1}{2}\int 2x\left(x^2-1\right)^7 dx$$
$$= \frac{1}{2}\int u^7\,du \qquad \text{Substitution: } \begin{smallmatrix} u=x^2-1 \\ du=2xdx \end{smallmatrix}$$
$$= \frac{1}{16}u^8 + C$$
$$= \frac{1}{16}\left(x^2-1\right)^8 + C$$

Next, we evaluate the definite integral on $[1,2]$.

$$\int_1^2 x\left(x^2-1\right)^7 dx$$
$$= \left[\frac{1}{16}\left(x^2-1\right)^8\right]_1^2$$
$$= \left[\frac{1}{16}\left(2^2-1\right)^8 - \frac{1}{16}\left(1^2-1\right)^8\right]$$
$$= \frac{1}{16}(3)^8 - \frac{1}{16}(0)^8$$
$$= \frac{6561}{16} - 0$$
$$= \frac{6561}{16}$$

47. $\int_0^4 \frac{dt}{1+t}$

We first find the indefinite integral

$$\int \frac{dt}{1+t}$$

$= \int \frac{du}{u}$ Substitution: $\begin{smallmatrix} u=1+t \\ du=dt \end{smallmatrix}$

$= \ln u + C$ Formula C, $u > 0$

$= \ln(1+t) + C$ Reverse Substitution

Next, we evaluate the definite integral on $[0,4]$.

$$\int_0^4 \frac{dt}{1+t}$$

$= \left[\ln(1+t)\right]_0^4$ Let $C = 0$.

$= \left[\ln(1+4) - \ln(1+0)\right]$

$= \ln(5) - \ln(1)$

$= \ln 5 - 0$

$= \ln 5$

48. $\int_0^2 e^{4x} dx$

We first find the indefinite integral

$$\int e^{4x} dx$$

$= \frac{1}{4} \cdot 4 \int e^{4x} dx$

$= \frac{1}{4} \int 4e^{4x} dx$

$= \int e^u du$ Substitution: $\begin{smallmatrix} u=4x \\ du=4dx \end{smallmatrix}$

$= e^u + C$

$= e^{4x} + C$

Next, we evaluate the definite integral on $[0,2]$.

$\int_0^1 e^{4x} dx = \left[\frac{1}{4} e^{4x}\right]_0^2$ Let $C = 0$.

$= \frac{1}{4} e^{4(2)} - \frac{1}{4} e^{4(0)}$

$= \frac{1}{4} e^8 - \frac{1}{4} e^0$

$= \frac{1}{4}\left(e^8 - 1\right)$

49. $\int_1^4 \frac{2x+1}{x^2+x-1} dx$

We first find the indefinite integral

$$\int \frac{2x+1}{x^2+x-1} dx$$

$= \int \frac{du}{u}$ Substitution: $\begin{smallmatrix} u=x^2+x-1 \\ du=2x+1dx \end{smallmatrix}$

$= \ln u + C$ Formula C, $u > 0$

$= \ln\left(x^2+x-1\right) + C$ Reverse Substitution

Next, we evaluate the definite integral on $[1,4]$.

$$\int_1^4 \frac{2x+1}{x^2+x-1} dx$$

$= \left[\ln\left(x^2+x-1\right)\right]_1^4$ Let $C = 0$.

$= \left[\ln\left((4)^2+(4)-1\right) - \ln\left((1)^2+(1)-1\right)\right]$

$= \ln(19) - \ln(1)$

$= \ln 19 - 0$

$= \ln 19$

50. $\int_1^3 \frac{2x+3}{x^2+3x} dx$

We first find the indefinite integral

$$\int \frac{2x+3}{x^2+3x} dx$$

$= \int \frac{du}{u}$ Substitution: $\begin{smallmatrix} u=x^2+3x \\ du=2x+3dx \end{smallmatrix}$

$= \ln u + C$ Formula C, $u > 0$

$= \ln\left(x^2+3x\right) + C$ Reverse Substitution

Next, we evaluate the definite integral on $[1,3]$.

$$\int_1^3 \frac{2x+3}{x^2+3x} dx$$

$= \left[\ln\left(x^2+3x\right)\right]_1^3$ Let $C = 0$.

$= \left[\ln\left((3)^2+3(3)\right) - \ln\left((1)^2+3(1)\right)\right]$

$= \ln(18) - \ln(4)$

$= \ln\left(\frac{18}{4}\right)$

$= \ln \frac{9}{2}$

51. $\int_0^b e^{-x}dx$

We first find the indefinite integral

$\int e^{-x}dx$

$= -\int -e^{-x}dx$ Multipling by 1

$= -\int e^u du$ Substitution: $\substack{u=-x \\ du=-dx}$

$= -e^u + C$ Formula B

$= -e^{-x} + C$ Reverse Substitution

Next, we evaluate the definite integral on $[0,b]$.

$\int_0^b e^{-x}dx$

$= \left[-e^{-x}\right]_0^b$ Let $C = 0$.

$= -e^{-(b)} - \left(-e^{-(0)}\right)$

$= -e^{-b} + e^0$

$= 1 - e^{-b}$

52. $\int_0^b 2e^{-2x}dx$

We first find the indefinite integral

$\int 2e^{-2x}dx$

$= -\int -2e^{-2x}dx$

$= -\int e^u du$ Substitution: $\substack{u=-2x \\ du=-2dx}$

$= -e^u + C$ Formula B

$= -e^{-2x} + C$ Reverse Substitution

Next, we evaluate the definite integral on $[0,b]$.

$\int_0^b 2e^{-2x}dx$

$= \left[-e^{-2x}\right]_0^b$ Let $C = 0$.

$= -e^{-2(b)} - \left(-e^{-2(0)}\right)$

$= -e^{-2b} + e^0$

$= 1 - e^{-2b}$

53. $\int_0^b me^{-mx}dx$

We first find the indefinite integral

$\int me^{-mx}dx$

$= -\int -me^{-mx}dx$ Multipling by 1

$= -\int e^u du$ Substitution: $\substack{u=-mx \\ du=-mdx}$

$= -e^u + C$ Formula B

$= -e^{-mx} + C$ Reverse Substitution

Next, we evaluate the definite integral on $[0,b]$.

$\int_0^b me^{-mx}dx$

$= \left[-e^{-mx}\right]_0^b$ Let $C = 0$.

$= -e^{-m(b)} - \left(-e^{-m(0)}\right)$

$= -e^{-mb} + e^0$

$= 1 - e^{-mb}$

54. $\int_0^b ke^{-kx}dx$

We first find the indefinite integral

$\int ke^{-kx}dx$

$= -\int -ke^{-kx}dx$ Multipling by 1

$= -\int e^u du$ Substitution: $\substack{u=-kx \\ du=-kdx}$

$= -e^u + C$ Formula B

$= -e^{-kx} + C$ Reverse Substitution

Next, we evaluate the definite integral on $[0,b]$.

$\int_0^b ke^{-kx}dx$

$= \left[-e^{-kx}\right]_0^b$ Let $C = 0$.

$= -e^{-k(b)} - \left(-e^{-k(0)}\right)$

$= -e^{-kb} + e^0$

$= 1 - e^{-kb}$

55. $\int_0^4 (x-6)^2\, dx$

We first find the indefinite integral

$\int (x-6)^2\, dx$

$= \int u^2 du$ Substitution: $\substack{u=x-6 \\ du=dx}$

$= \frac{1}{3}u^3 + C$ Formula A

$= \frac{1}{3}(x-6)^3 + C$ Reverse Substitution

Next, we evaluate the definite integral on $[0,4]$.

$$\int_0^4 (x-6)^2\,dx$$
$$=\left[\frac{1}{3}(x-6)^3\right]_0^4 \quad \text{Let } C=0.$$
$$=\left[\frac{1}{3}((4)-6)^3-\frac{1}{3}((0)-6)^3\right]$$
$$=\frac{1}{3}(-2)^3-\frac{1}{3}(-6)^3$$
$$=-\frac{8}{3}-\left(-\frac{216}{3}\right)$$
$$=\frac{208}{3}$$

56. $\int_0^3 (x-5)^2\,dx$

We first find the indefinite integral

$$\int (x-5)^2\,dx$$
$$=\int u^2\,du \quad \text{Substitution: } {}^{u=x-5}_{du=dx}$$
$$=\frac{1}{3}u^3+C$$
$$=\frac{1}{3}(x-5)^3+C \quad \text{Reverse Substitution}$$

Next, we evaluate the definite integral on $[0,3]$.

$$\int_0^3 (x-5)^2\,dx$$
$$=\left[\frac{1}{3}(x-5)^3\right]_0^3 \quad \text{Let } C=0.$$
$$=\left[\frac{1}{3}((3)-5)^3-\frac{1}{3}((0)-5)^3\right]$$
$$=\frac{1}{3}(-2)^3-\frac{1}{3}(-5)^3$$
$$=-\frac{8}{3}-\left(-\frac{125}{3}\right)$$
$$=\frac{117}{3}$$
$$=39$$

57. $\int_0^2 \frac{3x^2\,dx}{\left(1+x^3\right)^5}$

We first find the indefinite integral

$$\int \frac{3x^2\,dx}{\left(1+x^3\right)^5}$$
$$=\int \frac{du}{u^5} \quad \text{Substitution: } {}^{u=1+x^3}_{du=3x^2dx}$$
$$=\int u^{-5}\,du$$
$$=-\frac{1}{4}u^{-4}+C \quad \text{Formula A}$$
$$=-\frac{1}{4}\left(1+x^3\right)^{-4}+C \quad \text{Reverse Substitution}$$
$$=-\frac{1}{4\left(1+x^3\right)^4}+C$$

Next, we evaluate the definite integral on $[0,2]$.

$$\int_0^2 \frac{3x^2\,dx}{\left(1+x^3\right)^5}$$
$$=\left[-\frac{1}{4\left(1+x^3\right)^4}\right]_0^2 \quad \text{Let } C=0.$$
$$=\left[\left(-\frac{1}{4\left(1+(2)^3\right)^4}\right)-\left(-\frac{1}{4\left(1+(0)^3\right)^4}\right)\right]$$
$$=\left(-\frac{1}{4(9)^4}\right)-\left(-\frac{1}{4(1)^4}\right)$$
$$=\left(-\frac{1}{26{,}244}\right)-\left(-\frac{1}{4}\right)$$
$$=-\frac{1}{26{,}244}+\frac{6561}{26{,}244}$$
$$=\frac{6560}{26{,}244}$$
$$=\frac{1640}{6561}$$

58. $\int_{-1}^{0} \frac{x^3\,dx}{\left(2-x^4\right)^7}$

We first find the indefinite integral

$$\int \frac{x^3\,dx}{\left(2-x^4\right)^7}$$

$$= -\frac{1}{4}\int \frac{-4x^3\,dx}{\left(2-x^4\right)^7}$$

$$= -\frac{1}{4}\int \frac{du}{u^7} \qquad \text{Substitution: } \begin{matrix} u=2-x^4 \\ du=-4x^3\,dx \end{matrix}$$

$$= -\frac{1}{4}\int u^{-7}\,du$$

$$= \frac{1}{24}u^{-6} + C$$

$$= \frac{1}{24\left(2-x^4\right)^6} + C$$

We evaluate the definite integral on $[-1,0]$.

$$\int_{-1}^{0} \frac{x^3\,dx}{\left(2-x^4\right)^7}$$

$$= \left[\frac{1}{24\left(2-x^4\right)^6}\right]_{-1}^{0} \qquad \text{Let } C = 0.$$

$$= \left[\left(\frac{1}{24\left(2-(0)^4\right)^6}\right) - \left(\frac{1}{24\left(2-(-1)^4\right)^6}\right)\right]$$

$$= \left(\frac{1}{24(2)^6}\right) - \left(\frac{1}{24(1)^6}\right)$$

$$= \left(\frac{1}{1536}\right) - \left(\frac{1}{24}\right)$$

$$= -\frac{21}{512}$$

59. $\int_{0}^{\sqrt{7}} 7x\cdot\sqrt[3]{1+x^2}\,dx$

We first find the indefinite integral

$$\int 7x\cdot\sqrt[3]{1+x^2}\,dx$$

$$= 7\int x\cdot\sqrt[3]{1+x^2}\,dx$$

$$= \frac{7}{2}\int 2x\cdot\sqrt[3]{1+x^2}\,dx$$

$$= \frac{7}{2}\int \sqrt[3]{1+x^2}\,(2x\,dx)$$

$$= \frac{7}{2}\int \sqrt[3]{u}\,du \qquad \text{Substitution: } \begin{matrix} u=1+x^2 \\ du=2x\,dx \end{matrix}$$

$$= \frac{7}{2}\int u^{1/3}\,du$$

$$= \frac{21}{8}u^{4/3} + C \qquad \text{Formula A}$$

$$= \frac{21}{8}\left(1+x^2\right)^{4/3} + C \qquad \text{Reverse Substitution}$$

We evaluate the definite integral on $\left[0,\sqrt{7}\right]$.

$$\int_{0}^{\sqrt{7}} 7x\sqrt[3]{1+x^2}\,dx$$

$$= \left[\frac{21}{8}\left(1+x^2\right)^{4/3}\right]_{0}^{\sqrt{7}} \qquad \text{Let } C = 0.$$

$$= \left[\left(\frac{21}{8}\left(1+\left(\sqrt{7}\right)^2\right)^{4/3}\right) - \left(\frac{21}{8}\left(1+(0)^2\right)^{4/3}\right)\right]$$

$$= \left(\frac{21}{8}(8)^{4/3}\right) - \left(\frac{21}{8}(1)^{4/3}\right)$$

$$= (42) - \left(\frac{21}{8}\right)$$

$$= \frac{315}{8}$$

60. $\int_{0}^{1} 12x\cdot\sqrt[5]{1-x^2}\,dx$

We first find the indefinite integral

$$\int 12x\cdot\sqrt[5]{1-x^2}\,dx$$

$$= 6\int 2x\cdot\sqrt[5]{1-x^2}\,dx$$

$$= -6\int \sqrt[5]{1-x^2}\,(-2x\,dx)$$

$$= -6\int \sqrt[5]{u}\,du \qquad \text{Substitution: } \begin{matrix} u=1-x^2 \\ du=-2x\,dx \end{matrix}$$

$$= -6\int u^{1/5}\,du$$

$$= -5u^{6/5} + C \qquad \text{Formula A}$$

$$= -5\left(1-x^2\right)^{6/5} + C \qquad \text{Reverse Substitution}$$

Next, we evaluate the definite integral on $[0,1]$.

$$\int_0^1 12x\cdot\sqrt[5]{1-x^2}\,dx$$
$$=\left[-5\left(1-x^2\right)^{6/5}\right]_0^1 \quad \text{Let } C=0.$$
$$=\left[\left(-5\left(1-(1)^2\right)^{6/5}\right)-\left(-5\left(1-(0)^2\right)^{6/5}\right)\right]$$
$$=(0)-(-5)$$
$$=5$$

61. Left to the student.

62. $D(x)=\int\frac{-2000x}{\sqrt{25-x^2}}dx$

Let $u=25-x^2$, then $du=-2xdx$.
Note that $-2000=-2\cdot 1000$.

$$D(x)=\int\frac{1000}{\sqrt{25-x^2}}(-2xdx)$$
$$=\int\frac{1000}{\sqrt{u}}du \qquad \text{Substituting: } \begin{smallmatrix}u=25-x^2\\ du=-2xdx\end{smallmatrix}$$
$$=1000\int u^{-1/2}du$$
$$=1000\frac{u^{1/2}}{1/2}+C$$
$$=2000u^{1/2}+C$$
$$=2000\sqrt{25-x^2}+C$$

We use the condition $D(3)=13,000$ to find C.

$$D(3)=13,000$$
$$2000\sqrt{25-(3)^2}+C=13,000$$
$$2000\sqrt{16}+C=13,000$$
$$8000+C=13,000$$
$$C=5000$$

Thus, $D(x)=2000\sqrt{25-x^2}+5000.$

63. a) We will evaluate each integral separately and then do the operations to find $P(t)$.

First, we will find revenue from the sale of products.

$$R(t)=\int R'(t)dt$$
$$=\int 4000t\,dt$$
$$=2000t^2+C$$

We use the initial condition to find C.

$$R(0)=0$$
$$2000(0)^2+C=0$$
$$C=0$$

Thus, the revenue from the sale of product is $R(t)=2000t^2$.

The revenue from the sale of the machine is the salvage value of the machine, which is given by $V(t)=200,000-25,000e^{0.1t}$.

The cost of the machine was $250,000.
Putting these three functions together, the total profit from the machine, in dollars, is

$$P(t)=R(t)+V(t)-C(t)$$
$$=\left(2000t^2\right)+\left(200,000-25,000e^{0.1t}\right)-(250,000)$$
$$=2000t^2-50,000-25,000e^{0.1t}.$$

b) Substituting 10 for t, we have:

$$P(10)=2000(10)^2-50,000-25,000e^{0.1(10)}$$
$$=150,000-25,000e$$
$$\approx 82,042.95$$

After 10 years the total profit from the machine is $82,042.95.

64. $\frac{dP}{dx}=\frac{9000-3000x}{\left(x^2-6x+10\right)^2}$

We integrate to find total profit.

$$P(x)=\int\frac{9000-3000x}{\left(x^2-6x+10\right)^2}dx$$

Let $u=x^2-6x+10$, then $du=(2x-6)dx$

Note that $9000-3000x=-1500(2x-6)$.

$$P(x)=\int\frac{-1500}{\left(x^2-6x+10\right)^2}\cdot(2x-6)dx$$
$$=\int\frac{-1500}{u^2}\cdot du \qquad \text{Substitution } \begin{smallmatrix}u=x^2-6x+10\\ du=(2x-6)dx\end{smallmatrix}$$
$$=-1500\int u^{-2}du$$
$$=1500u^{-1}+C$$
$$=1500\left(x^2-6x+10\right)^{-1}+C$$
$$=\frac{1500}{x^2-6x+10}+C$$

We use the condition $P(3)=1500$ to find C.

$$P(3) = 1500$$
$$\frac{1500}{(3)^2 - 6(3) + 10} + C = 1500$$
$$1500 + C = 1500$$
$$C = 0$$

Thus, the total profit function is

$$P(x) = \frac{1500}{x^2 - 6x + 10}.$$

65. a) $\int_0^{105} D(t)\,dt = \int_0^{105} 100{,}000e^{0.025t}\,dt$

Find the indefinite integral first.

$$\int 100{,}000e^{0.025t}\,dt$$
$$= \frac{1}{0.025}\cdot(0.025)\int 100{,}000e^{0.025t}\,dt$$
$$= 4{,}000{,}000\int e^{0.025t}(0.025)\,dt$$
$$= 4{,}000{,}000\int e^{u}\,du \qquad \text{Substitution: } \begin{matrix} u = 0.025t \\ du = 0.025dt \end{matrix}$$
$$= 4{,}000{,}000e^{u} + C$$
$$= 4{,}000{,}000e^{0.025t} + C$$

Next, we evaluate the definite integral on $[0,105]$.

$$\int_0^{105} 100{,}000e^{0.025t}\,dt$$
$$= \left[4{,}000{,}000e^{0.025t}\right]_0^{105} \qquad \text{Let } C = 0.$$
$$= 4{,}000{,}000\left[e^{0.025(105)} - e^{0.025(0)}\right]$$
$$= 4{,}000{,}000\left[e^{2.625} - e^{0}\right]$$
$$= 4{,}000{,}000\left[e^{2.625} - 1\right]$$
$$\approx 51{,}218{,}297$$

There were approximately 51,218,297 divorces from 1900 to 2005.

b) We found the indefinite integral in part (a), so we simply need to evaluate the definite integral over the interval $[80,106]$

$$\int_{80}^{106} 100{,}000e^{0.025t}\,dt$$
$$= \left[4{,}000{,}000e^{0.025t}\right]_{80}^{106} \qquad \text{From part (a).}$$
$$= 4{,}000{,}000\left[e^{0.025(106)} - e^{0.025(80)}\right]$$
$$= 4{,}000{,}000\left[e^{2.65} - e^{2}\right]$$
$$\approx 4{,}000{,}000[6.764982546]$$
$$\approx 27{,}059{,}930$$

There were approximately 27,059,930 divorces from 1980 to 2006.

66. $\int_{-2}^{2} -x\sqrt{4-x^2}\,dx$

We notice that $-x\sqrt{4-x^2} \geq 0$, on $[-2,0]$ and $0 \geq -x\sqrt{4-x^2}$, on $[0,2]$. We will divide the interval into two parts.

First on the interval $[-2,0]$, we find the indefinite integral.

$$\int\left(-x\sqrt{4-x^2} - 0\right)dx$$
$$= \int\left(-x\sqrt{4-x^2}\right)dx$$
$$= \frac{1}{2}\int\sqrt{4-x^2}\,(-2x)\,dx$$
$$= \frac{1}{2}\int\sqrt{u}\,du \qquad \text{Substitution: } \begin{matrix} u = 4 - x^2 \\ du = -2xdx \end{matrix}$$
$$= \frac{1}{2}\int u^{1/2}\,du$$
$$= \frac{1}{3}u^{3/2} + C$$
$$= \frac{1}{3}\left(4-x^2\right)^{3/2} + C$$

Next, we will evaluate the integral on $[-2,0]$.

$$\int_{-2}^{0} -x\sqrt{4-x^2}\,dx$$
$$= \left[\frac{1}{3}\left(4-x^2\right)^{3/2}\right]_{-2}^{0}$$
$$= \left[\left(\frac{1}{3}\left(4-(0)^2\right)^{3/2}\right) - \left(\frac{1}{3}\left(4-(-2)^2\right)^{3/2}\right)\right]$$
$$= \frac{1}{3}(4)^{3/2} - \frac{1}{3}(0)^{3/2}$$
$$= \frac{8}{3}$$

Next, on the interval $[0,2]$, we find the indefinite integral.

$$\int\left(0-\left(-x\sqrt{4-x^2}\right)\right)dx$$
$$=\int\left(x\sqrt{4-x^2}\right)dx$$
$$=-\frac{1}{2}\int\sqrt{4-x^2}\,(-2x)\,dx$$
$$=-\frac{1}{2}\int\sqrt{u}\,du \qquad \text{Substitution: } \begin{smallmatrix}u=4-x^2\\ du=-2x\,dx\end{smallmatrix}$$
$$=-\frac{1}{2}\int u^{1/2}du$$
$$=-\frac{1}{3}u^{3/2}+C$$
$$=-\frac{1}{3}\left(4-x^2\right)^{3/2}+C$$

Next, we will evaluate the integral on $[0,2]$.

$$\int_0^2 x\sqrt{4-x^2}\,dx$$
$$=\left[-\frac{1}{3}\left(4-x^2\right)^{3/2}\right]_0^2$$
$$=\left[\left(-\frac{1}{3}\left(4-(2)^2\right)^{3/2}\right)-\left(-\frac{1}{3}\left(4-(0)^2\right)^{3/2}\right)\right]$$
$$=-\frac{1}{3}(0)^{3/2}+\frac{1}{3}(4)^{3/2}$$
$$=\frac{8}{3}$$

Therefore, the total area is:

$$\int_{-2}^0 -x\sqrt{4-x^2}\,dx+\int_0^2 x\sqrt{4-x^2}\,dx$$
$$=\frac{8}{3}+\frac{8}{3}$$
$$=\frac{16}{3}=5\tfrac{1}{3}.$$

67. $\int_{-4}^4 x\sqrt{16-x^2}\,dx$

We notice that $0\ge x\sqrt{16-x^2}$, on $[-4,0]$ and $x\sqrt{16-x^2}\ge 0$, on $[0,4]$. We will divide the interval into two parts.
First on the interval $[-4,0]$, we find the indefinite integral.

$$\int\left(0-x\sqrt{16-x^2}\right)dx$$
$$=\int\left(-x\sqrt{16-x^2}\right)dx$$
$$=\frac{1}{2}\int\sqrt{16-x^2}\,(-2x)\,dx$$
$$=\frac{1}{2}\int\sqrt{u}\,du \qquad \text{Substitution: } \begin{smallmatrix}u=16-x^2\\ du=-2x\,dx\end{smallmatrix}$$
$$=\frac{1}{2}\int u^{1/2}du$$
$$=\frac{1}{3}u^{3/2}+C$$
$$=\frac{1}{3}\left(16-x^2\right)^{3/2}+C$$

Next, we will evaluate the integral on $[-4,0]$.

$$\int_{-4}^0 -x\sqrt{16-x^2}\,dx$$
$$=\left[\frac{1}{3}\left(16-x^2\right)^{3/2}\right]_{-4}^0$$
$$=\left[\left(\frac{1}{3}\left(16-(0)^2\right)^{3/2}\right)-\left(\frac{1}{3}\left(16-(-4)^2\right)^{3/2}\right)\right]$$
$$=\frac{1}{3}(16)^{3/2}-\frac{1}{3}(0)^{3/2}$$
$$=\frac{64}{3}$$

Next, on the interval $[0,4]$, we find the indefinite integral.

$$\int\left(x\sqrt{16-x^2}-0\right)dx$$
$$=\int\left(x\sqrt{16-x^2}\right)dx$$
$$=-\frac{1}{2}\int\sqrt{16-x^2}\,(-2x)\,dx$$
$$=-\frac{1}{2}\int\sqrt{u}\,du \qquad \text{Substitution: } \begin{smallmatrix}u=16-x^2\\ du=-2x\,dx\end{smallmatrix}$$
$$=-\frac{1}{2}\int u^{1/2}du$$
$$=-\frac{1}{3}u^{3/2}+C$$
$$=-\frac{1}{3}\left(16-x^2\right)^{3/2}+C$$

Next, we will evaluate the integral on $[0,4]$.

$$\int_0^4 x\sqrt{16-x^2}\,dx$$
$$=\left[-\frac{1}{3}\left(16-x^2\right)^{3/2}\right]_0^4$$
$$=\left[\left(-\frac{1}{3}\left(16-(4)^2\right)^{3/2}\right)-\left(-\frac{1}{3}\left(16-(0)^2\right)^{3/2}\right)\right]$$
$$=-\frac{1}{3}(0)^{3/2}+\frac{1}{3}(16)^{3/2}$$
$$=\frac{64}{3}$$

Therefore, the total area is:

$$\int_{-4}^0 -x\sqrt{16-x^2}\,dx+\int_0^4 x\sqrt{16-x^2}\,dx$$
$$=\frac{64}{3}+\frac{64}{3}=\frac{128}{3}.$$

68. $\int \frac{1}{ax+b}dx$

$=\frac{1}{a}\int\frac{adx}{ax+b}$

$=\frac{1}{a}\int\frac{du}{u}$ Substitution: $u=ax+b$, $du=adx$

$=\frac{1}{a}\ln u+C$

$=\frac{1}{a}\ln(ax+b)+C$

69. $\int 5x\sqrt{1-4x^2}\,dx=5\int x\sqrt{1-4x^2}\,dx$

Let $u=1-4x^2$, then $du=-8xdx$. We do not have $-8xdx$. We only have xdx and need to supply a -8. We do this by multiplying by $\frac{1}{-8}\cdot -8$ as follows.

$=\left(-\frac{1}{8}\right)(-8)5\int x\sqrt{1-4x^2}\,dx$

$=-\frac{5}{8}\int\sqrt{1-4x^2}\,(-8x)\,dx$

$=-\frac{5}{8}\int\sqrt{u}\,du$ Substitution: $u=1-4x^2$, $du=-8xdx$

$=-\frac{5}{8}\int u^{1/2}du$

$=-\frac{5}{12}u^{3/2}+C$ Formula A

$=-\frac{5}{12}\left(1-4x^2\right)^{3/2}+C$ Reverse substitution

70. $\int\frac{e^{\sqrt{t}}}{\sqrt{t}}dt$

$=2\int\frac{e^{\sqrt{t}}}{2\sqrt{t}}dt$

$=2\int e^u du$ Substitution: $u=\sqrt{t}$, $du=\frac{1}{2\sqrt{t}}dt$

$=2e^u+C$

$=2e^{\sqrt{t}}+C$

71. $\int\frac{x^2}{e^{x^3}}dx=\int x^2e^{-x^3}dx$

Let $u=-x^3$, then $du=-3x^2dx$. We do not have $-3x^2dx$. We only have x^2dx and need to supply a -3. We do this by multiplying by $\frac{1}{-3}\cdot -3$ as follows.

$=-\frac{1}{3}\cdot(-3)\int x^2e^{-x^3}dx$

$=-\frac{1}{3}\int e^{-x^3}\left(-3x^2\right)dx$

$=-\frac{1}{3}\int e^u du$ Substitution: $u=-x^3$, $du=-3x^2dx$

$=-\frac{1}{3}e^u+C$ Formula B

$=-\frac{1}{3}e^{-x^3}+C$ Reverse substitution

72. $\int\frac{(\ln x)^{99}}{x}dx$

$=\int u^{99}du$ Substitution: $u=\ln x$, $du=\frac{1}{x}dx$

$=\frac{1}{100}u^{100}+C$

$=\frac{1}{100}(\ln x)^{100}+C$

73. $\int\frac{e^{1/t}}{t^2}dt$

Let $u=\frac{1}{t}=t^{-1}$, then $du=-t^{-2}dt=-\frac{1}{t^2}dt$. We do not have $-\frac{1}{t^2}dt$. We only have $\frac{1}{t^2}dt$ and need to supply a -1. We do this by multiplying by $(-1)\cdot(-1)$ as follows.

$= (-1)\int(-1)\frac{e^{1/t}}{t^2}\,dt$

$= -\int e^u\,du$ Substitution: $\begin{smallmatrix} u=\frac{1}{t} \\ du=-\frac{1}{t^2}dt \end{smallmatrix}$

$= -e^u + C$ Formula B

$= -e^{1/t} + C$ Reverse substitution

74. $\int (e^t+2)e^t\,dt$

$= \int u\,du$ Substitution: $\begin{smallmatrix} u=e^t+2 \\ du=e^t dt \end{smallmatrix}$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}(e^t+2)^2 + C$

75. $\int \frac{dx}{x(\ln x)^4}$

Let $u = \ln x$, then $du = \frac{1}{x}dx$.

$= \int \frac{du}{u^4}$ Substitution: $\begin{smallmatrix} u=\ln x \\ du=\frac{1}{x}dx \end{smallmatrix}$

$= \int u^{-4}\,du$

$= \frac{1}{-3}u^{-3} + C$

$= -\frac{1}{3}(\ln x)^{-3} + C$

76. $\int \frac{t^2\,dt}{\sqrt[4]{2+t^3}}\,dt$

$= \frac{1}{3}\int \frac{3t^2\,dt}{\sqrt[4]{2+t^3}}\,dt$

$= \frac{1}{3}\int \frac{du}{\sqrt[4]{u}}$ Substitution: $\begin{smallmatrix} u=2+t^3 \\ du=3t^2 dt \end{smallmatrix}$

$= \frac{1}{3}\int u^{-1/4}\,du$

$= \frac{4}{9}u^{3/4} + C$

$= \frac{4}{9}(2+t^3)^{3/4} + C$

77. $\int x^2\sqrt{x^3+1}\,dx$

Let $u = x^3+1$, then $du = 3x^2dx$. We do not have $3x^2dx$ We only have x^2dx and need to supply a 3. We do this by multiplying by $\frac{1}{3}\cdot 3$ as follows.

$= \frac{1}{3}\cdot 3\int x^2\sqrt{x^3+1}\,dx$

$= \frac{1}{3}\int \sqrt{x^3+1}(3x^2)\,dx$

$= \frac{1}{3}\int \sqrt{u}\,du$ Substitution: $\begin{smallmatrix} u=x^3+1 \\ du=3x^2dx \end{smallmatrix}$

$= \frac{1}{3}\int u^{1/2}\,du$

$= \frac{2}{9}u^{3/2} + C$ Formula A

$= \frac{2}{9}(x^3+1)^{3/2} + C$

78. $\int \frac{\left[(\ln x)^2 + 3(\ln x) + 4\right]}{x}dx$

Let $u = \ln x$, then $du = \frac{1}{x}dx$.

$= \int\left[(u)^2 + 3(u) + 4\right]du$ Substitution: $\begin{smallmatrix} u=\ln x \\ du=\frac{1}{x}dx \end{smallmatrix}$

$= \frac{1}{3}u^3 + \frac{3}{2}u^2 + 4u + C$

$= \frac{1}{3}(\ln x)^3 + \frac{3}{2}(\ln x)^2 + 4\ln x + C$

79. $\int \frac{x-3}{(x^2-6x)^{1/3}}dx$

Let $u = x^2-6x$, then $du = (2x-6)dx$. We do not have $(2x-6)dx$. We only have $(x-3)dx$ and need to supply a 2. We do this by multiplying by $\frac{1}{2}\cdot 2$ as follows.

$$= \frac{1}{2} \cdot 2 \int \frac{x-3}{\left(x^2-6x\right)^{1/3}} dx$$

$$= \frac{1}{2} \int \frac{2x-6}{\left(x^2-6x\right)^{1/3}} dx$$

$$= \frac{1}{2} \int \frac{1}{u^{1/3}} du \qquad \text{Substitution: } \begin{array}{l} u = x^2-6x \\ du = (2x-6)dx \end{array}$$

$$= \frac{1}{2} \int u^{-1/3} du$$

$$= \frac{1}{2} \cdot \frac{u^{2/3}}{2/3} + C$$

$$= \frac{3}{4} \left(x^2-6x\right)^{2/3} + C$$

80. $\displaystyle\int \frac{t^3 \ln\left(t^4+8\right)}{t^4+8} dt$

Let $u = \ln\left(t^4+8\right)$, then $du = \dfrac{4t^3}{t^4+8} dt.$ We need to supply a 4, by multiplying by $\frac{1}{4} \cdot 4$

$$= \frac{1}{4} \int \frac{4t^3 \ln\left(t^4+8\right)}{t^4+8} dt$$

$$= \frac{1}{4} \int u\, du \qquad \text{Substitution: } \begin{array}{l} u = \ln\left(t^4+8\right) \\ du = \frac{4t^3}{t^4+8} dt \end{array}$$

$$= \frac{1}{8} u^2 + C$$

$$= \frac{1}{8} \left(\ln\left(t^4+8\right)\right)^2 + C$$

81. $\displaystyle\int \frac{t^2+2t}{(t+1)^2} dt = \int \left[1 - \frac{1}{(t+1)^2}\right] dt$ See the hint.

$$= \int dt - \int \frac{1}{(t+1)^2} dt$$

Let $u = t+1$, then $du = dt$.

$$= \int du - \int \frac{1}{u^2} du \qquad \text{Substitution: } \begin{array}{l} u = t+1 \\ du = dt \end{array}$$

$$= \int du - \int u^{-2} du$$

$$= u - \left[-u^{-1}\right] + K$$

$$= t + 1 + (t+1)^{-1} + K$$

$$= t + \frac{1}{t+1} + C \qquad [C = K+1]$$

82. $\displaystyle\int \frac{x^2+6x}{(x+3)^2} dx$

$$\int \frac{x^2+6x+9-9}{(x+3)^2} dx \qquad \text{Adding } 9-9 \text{ in the numerator.}$$

$$\int \left[\frac{x^2+6x+9}{(x+3)^2} - \frac{9}{(x+3)^2}\right] dx$$

$$= \int \left[\frac{(x+3)^2}{(x+3)^2} - \frac{9}{(x+3)^2}\right] dx$$

$$= \int \left[1 - \frac{9}{(x+3)^2}\right] dx$$

$$= \int dx - \int \frac{9}{(x+3)^2} dx$$

Let $u = x+3$, then $du = dx$.

$$= \int du - \int \frac{9}{u^2} du \qquad \text{Substitution: } \begin{array}{l} u = x+3 \\ du = dx \end{array}$$

$$= \int du - 9 \int u^{-2} du + K$$

$$= u + 9u^{-1} + K$$

$$= x + 9(x+3)^{-1} + K + 3$$

$$= x + \frac{9}{x+3} + C \qquad [C = K+3]$$

83. $\displaystyle\int \frac{x+3}{x+1} dx = \int \left[1 + \frac{2}{x+1}\right] dx$ See the hint.

$$= \int dx + \int \frac{2}{x+1} dx$$

Let $u = x+1$, then $du = dx$.

$$= \int du + 2 \int \frac{du}{u} \qquad \text{Substitution: } \begin{array}{l} u = x+1 \\ du = dx \end{array}$$

$$= u + 2 \ln u + K$$

$$= x + 1 + 2 \ln(x+1) + K$$

$$= x + 2 \ln(x+1) + C \qquad [C = K+1]$$

84. $\int \frac{t-5}{t-4}dt$

First, we divide algebraically to see

$$t-4\overline{)t-5}^{\;1}$$
$$\underline{-(t-4)}$$
$$-1$$

Thus, $\frac{t-5}{t-4} = 1 - \frac{1}{t-4}$, and

$$\int \frac{t-5}{t-4}dt$$
$$= \int \left[1 - \frac{1}{t-4}\right]dt$$
$$= \int dt - \int \frac{dt}{t-4}$$

Let $u = t-4$, then $du = dt$.

$$= \int du - \int \frac{du}{u} \quad \text{Substitution: } \substack{u=t-4 \\ du=dt}$$
$$= u - (\ln u + C_2)$$
$$= t - 4 - \ln(t-4) + K$$
$$= t - \ln(t-4) + C \qquad [C = K-4]$$

85. $\int \frac{dx}{x(\ln x)^n}, \quad n \neq -1$

Let $u = \ln x$, then $du = \frac{1}{x}dx$.

$$= \int \frac{du}{u^n} \quad \text{Substitution: } \substack{u=\ln x \\ du=\frac{1}{x}dx}$$
$$= \int u^{-n}du$$
$$= \frac{u^{-n+1}}{-n+1} + C$$
$$= \frac{(\ln x)^{-n+1}}{-n+1} + C$$

86. $\int \frac{dx}{e^x+1} = \int \frac{e^{-x}}{1+e^{-x}}dx$

Let $u = 1+e^{-x}$, then $du = -e^{-x}dx$.

$$= -\int \frac{-e^{-x}}{1+e^{-x}}dx$$
$$= -\int \frac{du}{u} \quad \text{Substitution: } \substack{u=1+e^{-x} \\ du=-e^{-x}dx}$$
$$= -\ln u + C$$
$$= -\ln(1+e^{-x}) + C$$

87. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$

Let $u = e^x + e^{-x}$, then $du = e^x - e^{-x}dx$.

$$= \int \frac{du}{u} \quad \text{Substitution: } \substack{u=e^x+e^{-x} \\ du=(e^x-e^{-x})dx}$$
$$= \ln u + C$$
$$= \ln(e^x + e^{-x}) + C$$

88. $\int \frac{(\ln x)^n}{x}dx, \quad n \neq -1$

Let $u = \ln x$, then $du = \frac{1}{x}dx$.

$$= \int u^n du \quad \text{Substitution: } \substack{u=\ln x \\ du=\frac{1}{x}dx}$$
$$= \frac{1}{n+1}u^{n+1} + C$$
$$= \frac{1}{n+1}(\ln x)^{n+1} + C$$
$$= \frac{(\ln x)^{n+1}}{n+1} + C$$

89. $\int \frac{dx}{x \ln x[\ln(\ln x)]}$

Let $u = \ln(\ln x)$, then $du = \frac{1}{x \ln x}dx$.

$$= \int \frac{du}{u} \quad \text{Substitution: } \substack{u=\ln(\ln x) \\ du=\frac{1}{x\ln x}dx}$$
$$= \ln u + C$$
$$= \ln[\ln(\ln x)] + C$$

90. $\int \frac{e^{-mx}}{1-ae^{-mx}}dx$

Let $u = 1-ae^{-mx}$, then $du = ame^{-mx}dx$.

$$= \frac{1}{am}\int \frac{ame^{-mx}}{1-ae^{-mx}}dx$$
$$= \frac{1}{am}\int \frac{du}{u} \quad \text{Substitution: } \substack{u=1-ae^{-mx} \\ du=ame^{-mx}dx}$$
$$= \frac{1}{am}\ln u + C$$
$$= \frac{1}{am}\left[\ln(1-ae^{-mx})\right] + C$$

91. $\int 9x\left(7x^2+9\right)^n dx, \qquad n \neq -1$

$= 9\int x\left(7x^2+9\right)^n dx$

Let $u = 7x^2+9$, then $du = 14xdx$. We do not have $14xdx$. We only have xdx and need to supply a 14. We do this by multiplying by $\frac{1}{14}\cdot 14$ as follows.

$= \frac{1}{14}\cdot 14\cdot 9\int x\left(7x^2+9\right)^n dx$

$= \frac{9}{14}\int 14x\left(7x^2+9\right)^n dx$

$= \frac{9}{14}\int u^n\, du \qquad$ Substitution: $\substack{u=7x^2+9 \\ du=14xdx}$

$= \frac{9}{14}\cdot\frac{u^{n+1}}{n+1}+C$

$= \frac{9}{14(n+1)}\left(7x^2+9\right)^{n+1}+C$

92. $\int 5x^2\left(2x^3-7\right)^n dx, \qquad n \neq -1$

$= 5\int x^2\left(2x^3-7\right)^n dx$

Let $u = 2x^3-7$, then $du = 6x^2dx$.

$= \frac{1}{6}\cdot 6\cdot 5\int x^2\left(2x^3-7\right)^n dx$

$= \frac{5}{6}\int 6x^2\left(2x^3-7\right)^n dx$

$= \frac{5}{6}\int u^n\, du \qquad$ Substitution: $\substack{u=2x^3-7 \\ du=6x^2dx}$

$= \frac{5}{6}\cdot\frac{u^{n+1}}{n+1}+C$

$= \frac{5}{6(n+1)}\left(2x^3-7\right)^{n+1}+C$

93. $\boxed{tw}$ No, this is not a theorem. Let $f(x) = x^2$.

Then $\int 2\left[f(x)\right]dx = \int 2x^2dx = \frac{2x^3}{3}+C;$

however, $\left[f(x)\right]^2+C = \left[x^2\right]^2+C = x^4+C$

Since $\frac{2x^3}{3}$ and x^4 do not differ by only a constant, the given statement is not true.

Exercise Set 4.6

1. $\int 4xe^{4x}dx = \int x\left(4e^{4x}dx\right)$

Let

$u = x \qquad$ and $\qquad dv = 4e^{4x}dx$

Then

$du = dx \qquad$ and $\qquad v = e^{4x}$

Using the Integration-by-Parts Formula gives

$$\int \underbrace{x}_{u}\underbrace{\left(4e^{4x}dx\right)}_{dv} = \underbrace{x\cdot e^{4x}}_{u\cdot v} - \int \underbrace{e^{4x}}_{v}\underbrace{dx}_{du}$$

$$= xe^{4x} - \frac{1}{4}e^{4x} + C.$$

2. $\int 3xe^{3x}dx = \int x\left(3e^{3x}dx\right)$

Let

$u = x \qquad$ and $\qquad dv = 3e^{3x}dx$

Then

$du = dx \qquad$ and $\qquad v = e^{3x}$

Using the Integration-by-Parts Formula gives

$$\int \underbrace{x}_{u}\underbrace{\left(3e^{3x}dx\right)}_{dv} = \underbrace{x\cdot e^{3x}}_{u\cdot v} - \int \underbrace{e^{3x}}_{v}\underbrace{dx}_{du}$$

$$= xe^{3x} - \frac{1}{3}e^{3x} + C.$$

3. $\int x^3\left(3x^2\right)dx$

Let

$u = x^3 \qquad$ and $\qquad dv = 3x^2dx$

Then

$du = 3x^2dx \qquad$ and $\qquad v = x^3$

Using the Integration-by-Parts Formula gives

$$\int \underbrace{x^3}_{u}\underbrace{\left(3x^2dx\right)}_{dv} = \underbrace{x^3}_{u}\cdot\underbrace{x^3}_{v} - \int \underbrace{x^3}_{v}\underbrace{3x^2dx}_{du}$$

$$= x^6 - \int 3x^5dx$$

$$= x^6 - 3\cdot\frac{x^6}{6} + C$$

$$= x^6 - \frac{x^6}{2} + C$$

$$= \frac{x^6}{2} + C.$$

It should be noted that this problem can also be worked with substitution or formula A.

Integrate using Substitution

$$\int x^3\left(3x^2\right)dx$$

$$= \int u\,du \qquad \text{Substitution: } {\scriptstyle u=x^3 \atop du=3x^2dx}$$

$$= \frac{1}{2}u^2 + C$$

$$= \frac{1}{2}\left(x^3\right)^2 + C$$

$$= \frac{x^6}{2} + C$$

Integrate using formula A.

$$\int x^3\left(3x^2\right)dx$$

$$= \int 3x^5\,dx$$

$$= 3\cdot\frac{x^6}{6} + C \qquad \text{Using Formula A}$$

$$= \frac{x^6}{2} + C$$

4. $\int x^2\left(2x\right)dx$

Let

$u = x^2$ and $dv = 2xdx$

Then

$du = 2xdx$ and $v = x^2$

Using the Integration-by-Parts Formula gives

$$\int \underbrace{x^2}_{u}\left(\underbrace{2xdx}_{dv}\right) = \underbrace{x^2}_{u}\cdot\underbrace{x^2}_{v} - \int \underbrace{x^2}_{v}\,\underbrace{2xdx}_{du}$$

$$= x^4 - \int 2x^3dx$$

$$= x^4 - 2\cdot\frac{x^4}{4} + C$$

$$= x^4 - \frac{1}{2}x^4 + C$$

$$= \frac{1}{2}x^4 + C.$$

It should be noted that this problem can also be worked with substitution or formula A.

5. $\int xe^{5x}dx$

Let

$u = x$ and $dv = e^{5x}dx$

Then

$du = dx$ and $v = \frac{1}{5}e^{5x}$

We integrated dv using the formula

$$\int be^{ax}dx = \frac{b}{a}e^{ax} + C\,.$$

Using the Integration-by-Parts Formula gives

$$\int \underbrace{x}_{u}\left(\underbrace{e^{5x}dx}_{dv}\right) = \underbrace{x}_{u}\cdot\underbrace{\frac{1}{5}e^{5x}}_{v} - \int \underbrace{\frac{1}{5}e^{5x}}_{v}\underbrace{dx}_{du}$$

$$= x\cdot\frac{1}{5}e^{5x} - \frac{1}{5}\cdot\frac{1}{5}e^{5x} + C$$

$$\left(\int be^{ax}dx = \frac{b}{a}e^{ax} + C\right)$$

$$= \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + C.$$

6. $\int 2xe^{4x}dx$

Let

$u = 2x$ and $dv = e^{4x}dx$

Then

$du = 2dx$ and $v = \frac{1}{4}e^{4x}$

Using the Integration-by-Parts Formula gives

$$\int \underbrace{2x}_{u}\left(\underbrace{e^{4x}dx}_{dv}\right) = \underbrace{2x}_{u}\cdot\underbrace{\frac{1}{4}e^{4x}}_{v} - \int \underbrace{\frac{1}{4}e^{4x}}_{v}\underbrace{\left(2dx\right)}_{du}$$

$$= \frac{1}{2}xe^{4x} - \frac{1}{2}\cdot\frac{1}{4}e^{4x} + C$$

$$= \frac{1}{2}xe^{4x} - \frac{1}{8}e^{4x} + C.$$

7. $\int xe^{-2x}dx$

Let

$u = x$ and $dv = e^{-2x}dx$

Then

$du = dx$ and $v = -\frac{1}{2}e^{-2x}$

We integrated dv using the formula

$$\int be^{ax}dx = \frac{b}{a}e^{ax} + C\,.$$

Using the Integration-by-Parts Formula gives

$$\int \underbrace{x}_{u}\left(\underbrace{e^{-2x}dx}_{dv}\right) = \underbrace{x}_{u}\cdot\underbrace{\left(-\frac{1}{2}e^{-2x}\right)}_{v} - \int \underbrace{\left(-\frac{1}{2}e^{-2x}\right)}_{v}\underbrace{dx}_{du}$$

$$= -\frac{1}{2}xe^{-2x} - \left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}e^{-2x}\right) + C$$

$$\left(\int be^{ax}dx = \frac{b}{a}e^{ax} + C\right)$$

$$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C.$$

8. $\int xe^{-x}dx$

Let

$u = x$ and $dv = e^{-x}dx$

Then

$du = dx$ and $v = -e^{-x}$

Using the Integration-by-Parts Formula gives

$$\int \underset{u}{x}\left(\underset{dv}{e^{-x}dx}\right) = \underset{u}{x} \cdot \underset{v}{\left(-e^{-x}\right)} - \int \underset{v}{\left(-e^{-x}\right)}\underset{du}{dx}$$
$$= -xe^{-x} - (-1)\cdot\left(-e^{-x}\right) + C$$
$$= -xe^{-x} - e^{-x} + C.$$

9. $\int x^2 \ln x\,dx$

Let

$u = \ln x$ and $dv = x^2dx$

Then

$du = \frac{1}{x}dx$ and $v = \frac{1}{3}x^3$

Using the Integration-by-Parts Formula gives

$$\int \underset{u}{\ln x}\left(\underset{dv}{x^2\,dx}\right) = \underset{u}{\ln x} \cdot \underset{v}{\left(\frac{1}{3}x^3\right)} - \int \underset{v}{\left(\frac{1}{3}x^3\right)}\underset{du}{\left(\frac{1}{x}dx\right)}$$
$$= \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^2dx$$
$$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$$
$$= \frac{x^3\ln x}{3} - \frac{x^3}{9} + C.$$

10. $\int x^3 \ln x\,dx$

Let

$u = \ln x$ and $dv = x^3dx$

Then

$du = \frac{1}{x}dx$ and $v = \frac{1}{4}x^4$

Using the Integration-by-Parts Formula gives

$$\int \underset{u}{\ln x}\left(\underset{dv}{x^3\,dx}\right) = \underset{u}{\ln x} \cdot \underset{v}{\left(\frac{1}{4}x^4\right)} - \int \underset{v}{\left(\frac{1}{4}x^4\right)}\underset{du}{\left(\frac{1}{x}dx\right)}$$
$$= \frac{1}{4}x^4\ln x - \int \frac{1}{4}x^3dx$$
$$= \frac{x^4\ln x}{4} - \frac{x^4}{16} + C.$$

11. $\int x\ln\sqrt{x}\,dx = \int x\ln x^{1/2}\,dx$

Let

$u = \ln x^{1/2}$ and $dv = xdx$

Then

$du = \frac{1}{x^{1/2}}\cdot\frac{1}{2}x^{-1/2}dx = \frac{1}{2x}dx$ and $v = \frac{1}{2}x^2$

Using the Integration-by-Parts Formula gives

$$\int \underset{u}{\ln x^{1/2}}\,\underset{dv}{(xdx)} = \underset{u}{\ln x^{1/2}} \cdot \underset{v}{\left(\frac{x^2}{2}\right)} - \int \underset{v}{\left(\frac{x^2}{2}\right)}\underset{du}{\left(\frac{1}{2x}dx\right)}$$
$$= \frac{x^2\ln x^{1/2}}{2} - \int \frac{1}{4}xdx$$
$$= \frac{x^2\ln x^{1/2}}{2} - \frac{1}{8}x^2 + C$$
$$= \frac{x^2\ln x}{4} - \frac{1}{8}x^2 + C \qquad \left(\ln x^{1/2} = \frac{1}{2}\ln x\right)$$

12. $\int x^2\ln x^3\,dx$

Let

$u = \ln x^3$ and $dv = x^2dx$

Then

$du = \frac{3}{x}dx$ and $v = \frac{1}{3}x^3$

Using the Integration-by-Parts Formula gives

$$\int \underset{u}{\ln x^3}\,\underset{dv}{\left(x^2dx\right)} = \underset{u}{\ln x^2} \cdot \underset{v}{\left(\frac{x^3}{3}\right)} - \int \underset{v}{\left(\frac{x^3}{3}\right)}\underset{du}{\left(\frac{3}{x}dx\right)}$$
$$= \frac{x^3\ln x^3}{3} - \int x^2dx$$
$$= \frac{x^3\ln x^3}{3} - \frac{1}{3}x^3 + C$$
$$= x^3\ln x - \frac{x^3}{3} + C \qquad \left(\ln x^3 = 3\ln x\right)$$

13. $\int \ln(x+5)\,dx$

Let

$u = \ln(x+5)$ and $dv = dx$

Then

$du = \frac{1}{x+5}dx$ and $v = x+5$

Choosing $x+5$ as an antiderivative of dv

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{\ln(x+5)}\,\overset{dv}{dx} =$$

$$\overset{u}{\ln(x+5)}\cdot\overset{v}{(x+5)} - \int \overset{v}{(x+5)}\overset{du}{\left(\frac{1}{x+5}dx\right)}$$

$$= (x+5)\ln(x+5) - \int dx$$

$$= (x+5)\ln(x+5) - x + C.$$

14. $\int \ln(x+4)\,dx$

Let

$$u = \ln(x+4) \quad \text{and} \quad dv = dx$$

Then

$$du = \frac{1}{x+4}dx \quad \text{and} \quad v = x+4$$

Choosing $x+4$ as an antiderivative of dv

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{\ln(x+4)}\,\overset{dv}{dx} =$$

$$\overset{u}{\ln(x+4)}\cdot\overset{v}{(x+4)} - \int \overset{v}{(x+4)}\overset{du}{\left(\frac{1}{x+4}dx\right)}$$

$$= (x+4)\ln(x+4) - \int dx$$

$$= (x+4)\ln(x+4) - x + C.$$

15. $\int (x+2)\ln x\,dx$

Let

$$u = \ln x \quad \text{and} \quad dv = (x+2)dx$$

Then

$$du = \frac{1}{x}dx \quad \text{and} \quad v = \frac{x^2}{2} + 2x$$

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{\ln x}\cdot\overset{dv}{(x+2)dx} =$$

$$\overset{u}{\ln(x)}\cdot\overset{v}{\left(\frac{x^2}{2}+2x\right)} - \int \overset{v}{\left(\frac{x^2}{2}+2x\right)}\overset{du}{\left(\frac{1}{x}dx\right)}$$

$$= \left(\frac{x^2}{2}+2x\right)\ln x - \int\left(\frac{x}{2}+2\right)dx$$

$$= \left(\frac{x^2}{2}+2x\right)\ln x - \int \frac{x}{2}dx - \int 2dx$$

$$= \left(\frac{x^2}{2}+2x\right)\ln x - \frac{x^2}{4} - 2x + C.$$

16. $\int (x+1)\ln x\,dx$

Let

$$u = \ln x \quad \text{and} \quad dv = (x+1)dx$$

Then

$$du = \frac{1}{x}dx \quad \text{and} \quad v = \frac{x^2}{2} + x$$

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{\ln x}\cdot\overset{dv}{(x+1)dx} =$$

$$\overset{u}{\ln(x)}\cdot\overset{v}{\left(\frac{x^2}{2}+x\right)} - \int \overset{v}{\left(\frac{x^2}{2}+x\right)}\overset{du}{\left(\frac{1}{x}dx\right)}$$

$$= \left(\frac{x^2}{2}+x\right)\ln x - \int\left(\frac{x}{2}+1\right)dx$$

$$= \left(\frac{x^2}{2}+x\right)\ln x - \int \frac{x}{2}dx - \int dx$$

$$= \left(\frac{x^2}{2}+x\right)\ln x - \frac{x^2}{4} - x + C.$$

17. $\int (x-1)\ln x\,dx$

Let

$$u = \ln x \quad \text{and} \quad dv = (x-1)dx$$

Then

$$du = \frac{1}{x}dx \quad \text{and} \quad v = \frac{x^2}{2} - x$$

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{\ln x}\cdot\overset{dv}{(x-1)dx} =$$

$$\overset{u}{\ln(x)}\cdot\overset{v}{\left(\frac{x^2}{2}-x\right)} - \int \overset{v}{\left(\frac{x^2}{2}-x\right)}\overset{du}{\left(\frac{1}{x}dx\right)}$$

$$= \left(\frac{x^2}{2}-x\right)\ln x - \int\left(\frac{x}{2}-1\right)dx$$

$$= \left(\frac{x^2}{2}-x\right)\ln x - \int \frac{x}{2}dx + \int 1dx$$

$$= \left(\frac{x^2}{2}-x\right)\ln x - \frac{x^2}{4} + x + C.$$

18. $\int (x-2)\ln x\,dx$

Let

$$u = \ln x \qquad \text{and} \qquad dv = (x-2)\,dx$$

Then

$$du = \frac{1}{x}dx \qquad \text{and} \qquad v = \frac{x^2}{2} - 2x$$

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{\ln x} \cdot \overset{dv}{(x-2)\,dx} =$$

$$\overset{u}{\ln(x)} \cdot \overset{v}{\left(\frac{x^2}{2} - 2x\right)} - \int \overset{v}{\left(\frac{x^2}{2} - 2x\right)} \overset{du}{\left(\frac{1}{x}dx\right)}$$

$$= \left(\frac{x^2}{2} - 2x\right)\ln x - \int \left(\frac{x}{2} - 2\right)dx$$

$$= \left(\frac{x^2}{2} - 2x\right)\ln x - \int \frac{x}{2}dx + \int 2dx$$

$$= \left(\frac{x^2}{2} - 2x\right)\ln x - \frac{x^2}{4} + 2x + C.$$

19. $\int x\sqrt{x+2}\,dx$

Let

$$u = x \quad \text{and} \quad dv = \sqrt{x+2}dx = (x+2)^{1/2}\,dx$$

Then

$$du = dx \quad \text{and} \quad v = \frac{(x+2)^{3/2}}{3/2} = \frac{2}{3}(x+2)^{3/2}$$

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{x}\overset{dv}{\left(\sqrt{x+2}dx\right)} =$$

$$\overset{u}{x} \cdot \overset{v}{\frac{2}{3}(x+2)^{3/2}} - \int \overset{v}{\frac{2}{3}(x+2)^{3/2}}\,\overset{du}{dx}$$

$$= \frac{2}{3}x(x+2)^{3/2} - \frac{2}{3}\int (x+2)^{3/2}\,dx$$

$$= \frac{2}{3}x(x+2)^{3/2} - \frac{2}{3}\frac{(x+2)^{5/2}}{5/2} + C$$

$$= \frac{2}{3}x(x+2)^{3/2} - \frac{4}{15}(x+2)^{5/2} + C.$$

20. $\int x\sqrt{x+5}\,dx$

Let

$$u = x \quad \text{and} \quad dv = \sqrt{x+5}dx = (x+5)^{1/2}\,dx$$

Then

$$du = dx \quad \text{and} \quad v = \frac{(x+5)^{3/2}}{3/2} = \frac{2}{3}(x+5)^{3/2}$$

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{x}\overset{dv}{\left(\sqrt{x+5}dx\right)} =$$

$$\overset{u}{x} \cdot \overset{v}{\frac{2}{3}(x+5)^{3/2}} - \int \overset{v}{\frac{2}{3}(x+5)^{3/2}}\,\overset{du}{dx}$$

$$= \frac{2}{3}x(x+5)^{3/2} - \frac{2}{3}\int (x+5)^{3/2}\,dx$$

$$= \frac{2}{3}x(x+5)^{3/2} - \frac{2}{3}\frac{(x+5)^{5/2}}{5/2} + C$$

$$= \frac{2}{3}x(x+5)^{3/2} - \frac{4}{15}(x+5)^{5/2} + C.$$

21. $\int x^3 \ln(2x)\,dx$

Let

$$u = \ln(2x) \qquad \text{and} \qquad dv = x^3 dx$$

Then

$$du = \frac{1}{2x} \cdot 2dx = \frac{1}{x}dx \quad \text{and} \quad v = \frac{1}{4}x^4$$

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{\ln(2x)}\overset{dv}{\left(x^3 dx\right)} = \overset{u}{\ln(2x)} \cdot \overset{v}{\left(\frac{x^4}{4}\right)} - \int \overset{v}{\left(\frac{x^4}{4}\right)}\overset{du}{\left(\frac{1}{x}dx\right)}$$

$$= \frac{x^4}{4}\ln(2x) - \int \frac{x^3}{4}dx$$

$$= \frac{x^4}{4}\ln(2x) - \frac{x^4}{16} + C.$$

22. $\int x^2 \ln(5x)\,dx$

Let

$u = \ln(5x)$ and $dv = x^2 dx$

Then

$du = \frac{1}{5x}\cdot 5dx = \frac{1}{x}dx$ and $v = \frac{1}{3}x^3$

Using the Integration-by-Parts Formula gives

$$\underbrace{\int \ln(5x)}_{u}\underbrace{\left(x^2dx\right)}_{dv} = \underbrace{\ln(5x)}_{u}\cdot\underbrace{\left(\frac{x^3}{3}\right)}_{v} - \int\underbrace{\left(\frac{x^3}{3}\right)}_{v}\underbrace{\left(\frac{1}{x}dx\right)}_{du}$$

$$= \frac{x^3}{3}\ln(5x) - \int\frac{x^2}{3}dx$$

$$= \frac{x^3}{3}\ln(5x) - \frac{x^3}{9} + C.$$

23. $\int x^2 e^x dx$

Let

$u = x^2$ and $dv = e^x dx$

Then

$du = 2xdx$ and $v = e^x$

Using the Integration-by-Parts Formula gives

$$\int \underbrace{x^2}_{u}\underbrace{\left(e^x dx\right)}_{dv} = \underbrace{x^2}_{u}\underbrace{e^x}_{v} - \int \underbrace{2x}_{du}\underbrace{e^x}_{v}dx$$

We will use integration by parts again to evaluate $\int 2xe^x dx$

Let

$u = 2x$ and $dv = e^x dx$

Then

$du = 2dx$ and $v = e^x$

Using the Integration-by-Parts Formula gives

$$\int \underbrace{2x}_{u}\underbrace{\left(e^x dx\right)}_{dv} = \underbrace{2x}_{u}\underbrace{e^x}_{v} - \int \underbrace{2}_{du}\underbrace{e^x}_{v}dx$$

$$= 2xe^x - 2e^x + C_1$$

Thus,

$$\int x^2e^x dx = x^2e^x - \int 2xe^x dx$$

$$= x^2e^x - \left(2xe^x - 2e^x + C_1\right)$$

$$= x^2e^x - 2xe^x + 2e^x - C_1$$

$$= x^2e^x - 2xe^x + 2e^x + C \quad \left[C = -C_1\right]$$

Note, Since we have an integral $\int f(x)g(x)dx$, where $f(x)$ can be differentiated repeatedly to a derivative that is eventually 0 and $g(x)$ can be integrated repeatedly easily, we can use tabular integration as shown below:

$f(x)$ and Repeated Derivatives	Sign of Product	$g(x)$ and Repeated Integrals
x^2	+	e^x
$2x$	−	e^x
2	+	e^x
0		e^x

We add the products along the arrows, making the alternate sign changes.

$$\int x^2e^x dx = x^2e^x - 2xe^x + 2e^x + C$$

24. $\int (\ln x)^2\,dx$

Let

$u = (\ln x)^2$ and $dv = dx$

Then

$du = \frac{2\ln x}{x}dx$ and $v = x$

Using the Integration-by-Parts Formula gives

$$\int \underbrace{(\ln x)^2}_{u}\underbrace{dx}_{dv} = \underbrace{(\ln x)^2}_{u}\cdot\underbrace{x}_{v} - \int\underbrace{(x)}_{v}\underbrace{\left(\frac{2\ln x}{x}dx\right)}_{du}$$

$$= (\ln x)^2\cdot x - \int 2\ln x dx$$

$$= x(\ln x)^2 - 2\int \ln x dx$$

$$= x(\ln x)^2 - 2\left(x\ln x - x + C_1\right)$$

See Example 1.

$$= x(\ln x)^2 - 2x\ln x + 2x + C. \quad \left(C = -2C_1\right)$$

25. $\int x^2 e^{2x} dx$

Let

$u = x^2 \quad$ and $\quad dv = e^{2x} dx$

Then

$du = 2x dx$ and $v = \frac{1}{2}e^{2x} \quad \left(\int be^{ax} = \frac{b}{a}e^{ax} + C \right)$

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{x^2} \left(\overset{dv}{e^{2x} dx}\right) = \overset{u}{x^2} \cdot \overset{v}{\frac{1}{2}e^x} - \int \overset{v}{\left(\frac{1}{2}e^{2x}\right)} \cdot \overset{du}{2x dx}$$

$$\frac{1}{2}x^2 e^x - \int xe^{2x} dx$$

We will use integration by parts again to evaluate $\int xe^{2x} dx$

Let

$u = x \quad$ and $\quad dv = e^{2x} dx$

Then

$du = dx$ and $v = \frac{1}{2}e^{2x} \quad \left(\int be^{ax} = \frac{b}{a}e^{ax} + C \right)$

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{x} \left(\overset{dv}{e^{2x} dx}\right) = \overset{u}{x} \overset{v}{\left(\frac{1}{2}e^{2x} \right)} - \int \overset{v}{\left(\frac{1}{2}e^{2x} \right)} \overset{du}{dx}$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{2} \cdot \frac{1}{2}e^{2x} + C_1$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C_1$$

Thus,

$$\int x^2 \left(e^{2x} dx\right) = x^2 \cdot \frac{1}{2}e^x - \int \left(\frac{1}{2}e^{2x}\right) \cdot 2x dx$$

$$= x^2 \cdot \frac{1}{2}e^x - \int \left(xe^{2x}\right) dx$$

$$= \frac{1}{2}x^2 e^x - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C_1 \right)$$

$$= \frac{1}{2}x^2 e^x - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} - C_1$$

$$= \frac{1}{2}x^2 e^x - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

$$[C = -C_1]$$

Alternatively, we can also use tabular integration as shown below:

$f(x)$ and Repeated Derivatives	Sign of Product	$g(x)$ and Repeated Integrals
x^2		e^{2x}
	$+$	
$2x$		$\frac{1}{2}e^{2x}$
	$-$	
2		$\frac{1}{4}e^{2x}$
	$+$	
0		$\frac{1}{8}e^{2x}$

We add the products along the arrows, making the alternate sign changes.

$$\int x^2 e^{2x} dx$$

$$= x^2 \left(\frac{1}{2}e^{2x} \right) - 2x \left(\frac{1}{4}e^{2x} \right) + 2 \left(\frac{1}{8}e^{2x} \right) + C$$

$$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

26. $\int x^{-5} \ln x \, dx$

Let

$u = \ln x \quad$ and $\quad dv = x^{-5} dx$

Then

$du = \frac{1}{x} dx \quad$ and $\quad v = \frac{1}{-4}x^{-4}$

Using the Integration-by-Parts Formula gives

$$\int \overset{u}{\ln x} \left(\overset{dv}{x^{-5} dx}\right) = \overset{u}{\ln x} \cdot \overset{v}{\left(-\frac{x^{-4}}{4} \right)} - \int \overset{v}{\left(-\frac{x^{-4}}{4} \right)} \overset{du}{\left(\frac{1}{x} dx \right)}$$

$$= -\frac{x^{-4}}{4}(\ln x) - \int -\frac{x^{-5}}{4} dx$$

$$= -\frac{x^{-4}}{4}(\ln x) + \frac{1}{4}\int x^{-5} dx$$

$$= -\frac{x^{-4}}{4}(\ln x) + \frac{1}{4} \cdot \left(-\frac{1}{4}x^{-4} \right) + C$$

$$= -\frac{x^{-4}}{4}(\ln x) - \frac{x^{-4}}{16} + C.$$

27. $\int x^3 e^{-2x}\,dx$

This integral requires multiple applications of the Integration-by-Parts formula. However, since $f(x) = x^3$ is easily differentiable and to a derivative that is eventually 0 and $g(x) = e^{-2x}$ can be integrated easily using $\int be^{ax} = \frac{b}{a}e^{ax} + C$ we will use tabular integration as shown to simplify our work.

$f(x)$ and Repeated Derivatives	Sign of Product	$g(x)$ and Repeated Integrals
x^3	$+$	e^{-2x}
$3x^2$	$-$	$-\frac{1}{2}e^{-2x}$
$6x$	$+$	$\frac{1}{4}e^{-2x}$
6	$-$	$-\frac{1}{8}e^{-2x}$
0		$\frac{1}{16}e^{-2x}$

We add the products along the arrows, making the alternate sign changes to obtain

$$\int x^3 e^{-2x}\,dx$$

$$= x^3\left(-\frac{1}{2}e^{-2x}\right) - 3x^2\left(\frac{1}{4}e^{-2x}\right) + 6x\left(-\frac{1}{8}e^{-2x}\right) - 6\left(\frac{1}{16}e^{-2x}\right) + C$$

$$= -\frac{1}{2}x^3e^{-2x} - \frac{3}{4}x^2e^{-2x} - \frac{3}{4}xe^{-2x} - \frac{3}{8}e^{-2x} + C.$$

28. $\int x^5 e^{4x}\,dx$

This integral requires multiple applications of the Integration-by-Parts formula. However, we will use tabular integration as shown to simplify our work.

$f(x)$ and Repeated Derivatives	Sign of Product	$g(x)$ and Repeated Integrals
x^5	$+$	e^{4x}
$5x^4$	$-$	$\frac{1}{4}e^{4x}$
$20x^3$	$+$	$\frac{1}{16}e^{4x}$
$60x^2$	$-$	$\frac{1}{64}e^{4x}$
$120x$	$+$	$\frac{1}{256}e^{4x}$
120	$-$	$\frac{1}{1024}e^{4x}$
0		$\frac{1}{4096}e^{4x}$

We add the products along the arrows, making the alternate sign changes to obtain

$$\int x^5 e^{4x}\,dx$$

$$= x^5\left(\frac{1}{4}e^{4x}\right) - 5x^4\left(\frac{1}{16}e^{4x}\right) + 20x^3\left(\frac{1}{64}e^{4x}\right) - 60x^2\left(\frac{1}{256}e^{4x}\right) + 120x\left(\frac{1}{1024}e^{4x}\right) - 120\left(\frac{1}{4096}e^{4x}\right) + C$$

$$= \frac{1}{4}x^5e^{4x} - \frac{5}{16}x^4e^{4x} + \frac{5}{16}x^3e^{4x} - \frac{15}{64}x^2e^{4x} + \frac{15}{128}xe^{4x} - \frac{15}{512}e^{4x} + C.$$

29. $\int (x^4+4)e^{3x}\,dx$

This integral requires multiple applications of the Integration-by-Parts formula. However, since $f(x)=x^4+4$ is easily differentiable and to a derivative that is eventually 0 and $g(x)=e^{3x}$ can be integrated easily using $\int be^{ax} = \frac{b}{a}e^{ax}+C$ we will use tabular integration as shown to simplify our work.

$f(x)$ and Repeated Derivatives	Sign of Product	$g(x)$ and Repeated Integrals
x^4+4		e^{3x}
	+	
$4x^3$		$\frac{1}{3}e^{3x}$
	−	
$12x^2$		$\frac{1}{9}e^{3x}$
	+	
$24x$		$\frac{1}{27}e^{3x}$
	−	
24		$\frac{1}{81}e^{3x}$
	+	
0		$\frac{1}{243}e^{3x}$

We add the products along the arrows, making the alternate sign changes to obtain

$$\int (x^4+4)e^{3x}dx$$
$$=(x^4+4)\left(\frac{1}{3}e^{3x}\right)-4x^3\left(\frac{1}{9}e^{3x}\right)+$$
$$12x^2\left(\frac{1}{27}e^{3x}\right)-24x\left(\frac{1}{81}e^{3x}\right)+$$
$$24\left(\frac{1}{243}e^{3x}\right)+C$$
$$=\frac{1}{3}(x^4+4)e^{3x}-\frac{4}{9}x^3e^{3x}+\frac{4}{9}x^2e^{3x}-$$
$$\frac{8}{27}xe^{3x}+\frac{8}{81}e^{3x}+C.$$

30. $\int (x^3-x+1)e^{-x}\,dx$

This integral requires multiple applications of the Integration-by-Parts formula. However, we will use tabular integration as shown to simplify our work.

$f(x)$ and Repeated Derivatives	Sign of Product	$g(x)$ and Repeated Integrals
x^3-x+1		e^{-x}
	+	
$3x^2-1$		$-e^{-x}$
	−	
$6x$		e^{-x}
	+	
6		$-e^{-x}$
	−	
0		e^{-x}

We add the products along the arrows, making the alternate sign changes to obtain

$$\int (x^3-x+1)e^{-x}dx$$
$$=(x^3-x+1)(-e^{-x})-(3x^2-1)(e^{-x})+$$
$$6x(-e^{-x})-6(e^{-x})+C$$
$$=-x^3e^{-x}+xe^{-x}-e^{-x}-3x^2e^{-x}+e^{-x}-$$
$$6xe^{-x}-6e^{-x}+C$$
$$=-x^3e^{-x}-3x^2e^{-x}-5xe^{-x}-6e^{-x}+C.$$

31. $\int_1^2 x^2\ln x\,dx$

In Exercise 9 we found the indefinite integral

$$\int x^2\ln x\,dx=\frac{x^3\ln x}{3}-\frac{x^3}{9}+C.$$

We use the indefinite integral to evaluate the definite integral as follows:

$$\int_1^2 x^2\ln x\,dx$$
$$=\left[\frac{x^3\ln x}{3}-\frac{x^3}{9}\right]_1^2 \qquad \text{Use } C=0.$$
$$=\left[\frac{(2)^3\ln(2)}{3}-\frac{(2)^3}{9}\right]-\left[\frac{(1)^3\ln(1)}{3}-\frac{(1)^3}{9}\right]$$
$$=\left[\frac{8}{3}(\ln 2)-\frac{8}{9}\right]-\left[0-\frac{1}{9}\right]$$
$$=\frac{8}{3}\ln 2-\frac{8}{9}+\frac{1}{9}$$
$$=\frac{8}{3}\ln 2-\frac{7}{9}.$$

32. $\int_1^2 x^3 \ln x dx$

In Exercise 10, we found the definite integral

$\int x^3 \ln x dx = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C.$

We evaluate the indefinite integral as follows

$$\int_1^2 x^3 \ln x dx$$
$$= \left[\frac{x^4 \ln x}{4} - \frac{x^4}{16}\right]_1^2$$
$$= \left[\frac{(2)^4 \ln(2)}{4} - \frac{(2)^4}{16}\right] - \left[\frac{(1)^4 \ln(1)}{4} - \frac{(1)^4}{16}\right]$$
$$= \left[\frac{16}{4}\ln 2 - \frac{16}{16}\right] - \left[0 - \frac{1}{16}\right]$$
$$= 4\ln 2 - 1 + \frac{1}{16}$$
$$= 4\ln 2 - \frac{15}{16}.$$

33. $\int_2^6 \ln(x+8) dx$

First, we find the indefinite integral.

$\int \ln(x+8) dx$

Let

$u = \ln(x+8)$ and $dv = dx$

Then

$du = \frac{1}{x+8} dx$ and $v = x+8$

Choosing $x+8$ as an antiderivative of dv

Using the Integration-by-Parts Formula gives

$$\int \underbrace{\ln(x+8)}_{u} \underbrace{dx}_{dv} =$$
$$\underbrace{\ln(x+8)}_{u} \cdot \underbrace{(x+8)}_{v} - \int \underbrace{(x+8)}_{v} \underbrace{\left(\frac{1}{x+8} dx\right)}_{du}$$
$$= (x+8)\ln(x+8) - \int dx$$
$$= (x+8)\ln(x+8) - x + C.$$

We now use the indefinite integral to evaluate the definite integral. To simplify our work, we choose the constant of integration C to be 0.

$$\int_2^6 \ln(x+8) dx$$
$$= \left[(x+8)\ln(x+8) - x\right]_2^6$$
$$= \left[((6)+8)\ln((6)+8) - (6)\right] - \left[((2)+8)\ln((2)+8) - (2)\right]$$
$$= [14\ln 14 - 6] - [10\ln 10 - 2]$$
$$= 14\ln 14 - 10\ln 10 - 4.$$

34. $\int_0^5 \ln(x+7) dx$

First, we find the indefinite integral.

$\int \ln(x+7) dx$

Let

$u = \ln(x+7)$ and $dv = dx$

Then

$du = \frac{1}{x+7} dx$ and $v = x+7$

Choosing $x+7$ as an antiderivative of dv

Using the Integration-by-Parts Formula gives

$$\int \underbrace{\ln(x+7)}_{u} \underbrace{dx}_{dv} =$$
$$\underbrace{\ln(x+7)}_{u} \cdot \underbrace{(x+7)}_{v} - \int \underbrace{(x+7)}_{v} \underbrace{\left(\frac{1}{x+7} dx\right)}_{du}$$
$$= (x+7)\ln(x+7) - \int dx$$
$$= (x+7)\ln(x+7) - x + C.$$

Evaluate the definite integral.

$$\int_0^5 \ln(x+7) dx$$
$$= \left[(x+7)\ln(x+7) - x\right]_0^5$$
$$= \left[((5)+7)\ln((5)+7) - (5)\right] - \left[((0)+7)\ln((0)+7) - (0)\right]$$
$$= [12\ln 12 - 5] - [7\ln 7 - 0]$$
$$= 12\ln 12 - 7\ln 7 - 5.$$

35. $\int_0^1 xe^x dx$

First, we find the indefinite integral $\int xe^x dx$.

Let

$u = x$ and $dv = e^x dx$

Then

$du = dx$ and $v = e^x$

Using the Integration-by-Parts Formula gives

$$\overset{u}{\int x}\left(\overset{dv}{e^x dx}\right) = \overset{u}{x}\cdot \overset{v}{e^x} - \int \overset{v}{e^x}\, \overset{du}{dx}$$
$$= xe^x - e^x + C.$$

We now use the indefinite integral to evaluate the definite integral. To simplify our work, we choose the constant of integration C to be 0.

$$\int_0^1 xe^x dx$$
$$= \left[xe^x - e^x\right]_0^1$$
$$= \left[(1)e^{(1)} - e^{(1)}\right] - \left[(0)e^{(0)} - e^{(0)}\right]$$
$$= [e-e] - [0-1]$$
$$= 1.$$

36. $\int_0^1 \left(x^3 + 2x^2 + 3\right)e^{-2x} dx$

First we find the indefinite integral using tabular integration as shown.

$f(x)$ and Repeated Derivatives	Sign of Product	$g(x)$ and Repeated Integrals
$x^3 + 2x^2 + 3$	+	e^{-2x}
$3x^2 + 4x$	−	$-\frac{1}{2}e^{-2x}$
$6x + 4$	+	$\frac{1}{4}e^{-2x}$
6	−	$-\frac{1}{8}e^{-2x}$
0		$\frac{1}{16}e^{-2x}$

We add the products along the arrows, making the alternate sign changes to obtain

$$\int \left(x^3 + 2x^2 + 3\right)e^{-2x} dx$$
$$= \left(x^3 + 2x^2 + 3\right)\left(-\frac{1}{2}e^{-2x}\right) -$$
$$\left(3x^2 + 4x\right)\left(\frac{1}{4}e^{-2x}\right) + (6x+4)\left(-\frac{1}{8}e^{-2x}\right)$$
$$-6\left(\frac{1}{16}e^{-2x}\right) + C$$
$$= e^{-2x}\left(-\frac{1}{2}x^3 - \frac{7}{4}x^2 - \frac{7}{4}x - \frac{19}{8}\right) + C.$$

Evaluate the definite integral.

$$\int_0^1 \left(x^3 + 2x^2 + 3\right)e^{-2x} dx$$
$$= \left[e^{-2x}\left(-\frac{1}{2}x^3 - \frac{7}{4}x^2 - \frac{7}{4}x - \frac{19}{8}\right)\right]_0^1$$
$$= \left[e^{-2(1)}\left(-\frac{1}{2}(1)^3 - \frac{7}{4}(1)^2 - \frac{7}{4}(1) - \frac{19}{8}\right)\right] -$$
$$\left[e^{-2(0)}\left(-\frac{1}{2}(0)^3 - \frac{7}{4}(0)^2 - \frac{7}{4}(0) - \frac{19}{8}\right)\right]$$
$$= \left[e^{-2}\left(-\frac{1}{2} - \frac{7}{4} - \frac{7}{4} - \frac{19}{8}\right)\right] -$$
$$\left[e^{0}\left(-0 - 0 - 0 - \frac{19}{8}\right)\right]$$
$$= \left[e^{-2}\left(-\frac{51}{8}\right)\right] - \left[-\frac{19}{8}\right]$$
$$= \frac{-51e^{-2} + 19}{8}.$$

37. $C(x) = \int C'(x)\,dx = \int 4x\sqrt{x+3}\,dx$

Let

$$u = 4x \quad \text{and} \quad dv = \sqrt{x+3}\,dx = (x+3)^{1/2}\,dx$$

Then

$$du = 4dx \quad \text{and} \quad v = \frac{(x+3)^{3/2}}{3/2} = \frac{2}{3}(x+3)^{3/2}$$

Using the Integration-by-Parts Formula gives

$$\int 4x\left(\sqrt{x+3}\,dx\right)$$
$$= 4x\cdot\frac{2}{3}(x+3)^{3/2} - \int \frac{2}{3}(x+3)^{3/2}\cdot 4dx$$
$$= \frac{8}{3}x(x+3)^{3/2} - \frac{8}{3}\int (x+3)^{3/2}\,dx$$
$$= \frac{8}{3}x(x+3)^{3/2} - \frac{8}{3}\frac{(x+3)^{5/2}}{5/2} + K \quad \text{Use } K \text{ to avoid confusion with cost.}$$
$$= \frac{8}{3}x(x+3)^{3/2} - \frac{16}{15}(x+3)^{5/2} + K.$$

Next, we use the condition $C(13) = 1126.40$ to find the constant of integration K.

$$C(13) = 1126.40$$

$$\frac{8}{3}(13)\left((13)+3\right)^{3/2} - \frac{16}{15}\left((13)+3\right)^{5/2} + K = 1126.40$$

$$\frac{104}{3}\cdot(16)^{3/2} - \frac{16}{15}(16)^{5/2} + K = 1126.40$$

$$\frac{104}{3}(64) - \frac{16}{15}(1024) + K = 1126.40$$

$$\frac{6656}{3} - \frac{16,384}{15} + K = 1126.40$$

$$\frac{16,896}{15} + K = 1126.40$$

$$1126.40 + K = 1126.40$$

$$K = 0$$

Therefore, the total cost function is

$$C(x) = \frac{8}{3}x(x+3)^{3/2} - \frac{16}{15}(x+3)^{5/2}$$

38. $P(x) = \int P'(x)\,dx = \int 1000x^2 e^{-0.2x}\,dx$

We will use tabular integration as shown below:

$f(x)$ and Repeated Derivatives	Sign of Product	$g(x)$ and Repeated Integrals
$1000x^2$	+	$e^{-0.2x}$
$2000x$	−	$-5e^{-0.2x}$
2000	+	$25e^{-0.2x}$
0		$-125e^{-0.2x}$

We add the products along the arrows, making the alternate sign changes.

$$\int 1000x^2 e^{-0.2x}\,dx$$
$$= 1000x^2\left(-5e^{-0.2x}\right) - 2000x\left(25e^{-0.2x}\right) +$$
$$2000\left(-125e^{-0.2x}\right) + C$$
$$= e^{-0.2x}\left(-5000x^2 - 50,000x - 250,000\right) + C$$

We use the condition $P(0) = -2000$ to determine C.

$$P(0) = -2000$$
$$e^{-0.2(0)}\left(-5000(0)^2 - 50,000(0) - 250,000\right)$$
$$+ C = -2000$$
$$-250,000 + C = -2000$$
$$C = 248,000$$

Thus, the total profit function is:

$$P(x) = -5000x^2 e^{-0.2x} - 50,000xe^{-0.2x} -$$
$$250,000e^{-0.2x} + 248,000.$$

39. a) $K(t) = 10te^{-t}$

The number of kilowatt hours the family uses in the first T hours of the day is given by $\int_0^T K(t)\,dt$. We find the indefinite integral first.

$$\int K(t)\,dt = \int 10te^{-t}\,dt$$

Let

$u = 10t$ and $dv = e^{-t}dt$

Then

$du = 10dt$ and $v = -e^{-t}$

Using the Integration-by-Parts Formula gives

$$\int 10te^{-t}\,dt = 10t\cdot\left(-e^{-t}\right) - \int -e^{-t}\cdot 10dt$$
$$= -10te^{-t} + 10\int e^{-t}\,dt$$
$$= -10te^{-t} - 10e^{-t} + C$$

Next, we evaluate the definite integral.

$$\int_0^T K(t)\,dt$$
$$= \left[-10te^{-t} - 10e^{-t}\right]_0^T$$
$$= \left[-10Te^{-T} - 10e^{-T}\right] - \left[-10(0)e^{-0} - 10e^{-0}\right]$$
$$= -10Te^{-T} - 10e^{-T} + 10.$$

b) Substitute 4 for T.

$$\int_0^4 K(t)\,dt = -10(4)e^{-(4)} - 10e^{-(4)} + 10$$
$$= -40e^{-4} - 10e^{-4} + 10$$
$$\approx 9.084$$

The family uses approximately 9.084 kW-h during the first 4 hours of the day.

40. a) $\int_0^T E(t)\,dt = \int_0^T te^{-kt}\,dt$. We find the indefinite integral first.

$$\int te^{-kt}\,dt$$

Let

$u = t$ and $dv = e^{-kt}dt$

Then

$du = dt$ and $v = -\frac{1}{k}e^{-kt}$

Using the Integration-by-Parts Formula gives

$$\int te^{-kt}dt = t\cdot\left(-\frac{1}{k}e^{-kt}\right)-\int -\frac{1}{k}e^{-kt}dt$$
$$= -\frac{1}{k}te^{-kt}+\frac{1}{k}\int e^{-kt}dt$$
$$= -\frac{1}{k}te^{-kt}+\frac{1}{k}\left(-\frac{1}{k}e^{-kt}\right)+C$$
$$= -\frac{1}{k}te^{-kt}-\frac{1}{k^2}e^{-kt}+C$$
$$= -e^{-kt}\left(\frac{t}{k}+\frac{1}{k^2}\right)+C.$$

Next, we evaluate the definite integral.

$$\int_0^T E(t)\,dt$$
$$=\left[-e^{-kt}\left(\frac{t}{k}+\frac{1}{k^2}\right)\right]_0^T$$
$$=\left[-e^{-kT}\left(\frac{T}{k}+\frac{1}{k^2}\right)\right]-\left[-e^{-k\cdot 0}\left(\frac{(0)}{k}+\frac{1}{k^2}\right)\right]$$
$$=-e^{-kT}\left(\frac{T}{k}+\frac{1}{k^2}\right)-\left[-\frac{1}{k^2}\right]$$
$$=-e^{-kT}\left(\frac{T}{k}+\frac{1}{k^2}\right)+\frac{1}{k^2}.$$

b) Substitute 10 for T and 0.2 for k.

$$\int_0^{10} E(t)\,dt=\int_0^{10} te^{-0.2t}dt$$
$$=-e^{-(0.2)(10)}\left(\frac{10}{0.2}+\frac{1}{(0.2)^2}\right)+\frac{1}{(0.2)^2}$$
$$=-e^{-2}(50+25)+25$$
$$=-75e^{-2}+25$$
$$\approx 14.850.$$

After 10 hours the total excretion of the drug is approximately 14.850 mg.

41. $\int \sqrt{x}\ln x\,dx=\int x^{1/2}\ln x\,dx$

Let

$u=\ln x$ and $dv=x^{1/2}dx$

Then

$du=\frac{1}{x}dx$ and $v=\frac{2}{3}x^{3/2}$

Using the Integration-by-Parts Formula gives

$$\int \ln x\left(x^{1/2}\,dx\right)$$
$$=\ln x\cdot\left(\frac{2}{3}x^{3/2}\right)-\int\left(\frac{2}{3}x^{3/2}\right)\left(\frac{1}{x}dx\right)$$
$$=\frac{2}{3}x^{3/2}\ln x-\frac{2}{3}\int x^{1/2}dx$$
$$=\frac{2}{3}x^{3/2}\ln x-\frac{2}{3}\left(\frac{2}{3}x^{3/2}\right)+C$$
$$=\frac{2}{3}x^{3/2}\ln x-\frac{4}{9}x^{3/2}+C.$$

42. $\int \frac{te^t}{(t+1)^2}dt=\int te^t(t+1)^{-2}\,dt$

Let

$u=te^t$ and $dv=(t+1)^{-2}\,dt$

Then

$du=\left(te^t+e^t\right)dt$ and $v=\frac{(t+1)^{-1}}{-1}$

or

$du=e^t(t+1)\,dt$ and $v=-\frac{1}{t+1}$

Using the Integration-by-Parts Formula gives

$$\int \frac{te^t}{(t+1)^2}dt$$
$$=te^t\left(-\frac{1}{t+1}\right)-\int\left(-\frac{1}{t+1}\right)e^t(t+1)\,dt$$
$$=-\frac{te^t}{t+1}+\int e^t\,dt$$
$$=-\frac{te^t}{t+1}+e^t+C$$
$$=\frac{e^t}{t+1}+C.$$

43. $\int \frac{\ln x}{\sqrt{x}}dx=\int x^{-1/2}\ln x\,dx$

Let

$u=\ln x$ and $dv=x^{-1/2}dx$

Then

$du=\frac{1}{x}dx$ and $v=2x^{1/2}$

Using the Integration-by-Parts Formula gives

$$\int \ln x\left(x^{-1/2}\,dx\right)$$
$$= \ln x\cdot\left(2x^{1/2}\right)-\int\left(2x^{1/2}\right)\left(\frac{1}{x}\,dx\right)$$
$$= 2x^{1/2}\ln x-2\int x^{-1/2}dx$$
$$= 2x^{1/2}\ln x-2\left(2x^{1/2}\right)+C$$
$$= 2x^{1/2}\ln x-4x^{1/2}+C$$
$$= 2\sqrt{x}\left(\ln x\right)-4\sqrt{x}+C.$$

44. $\displaystyle\int\frac{13t^2-48}{\sqrt[5]{4t+7}}\,dt=\int\left(13t^2-48\right)\left(4t+7\right)^{-1/5}\,dt$

We will use tabular integration as shown to simplify our work.

$f(t)$ and Repeated Derivatives	Sign of Product	$g(t)$ and Repeated Integrals
$13t^2-48$	+	$(4t+7)^{-1/5}$
$26t$	−	$\frac{5}{16}(4t+7)^{4/5}$
26	+	$\frac{25}{576}(4t+7)^{9/5}$
0		$\frac{125}{32,256}(4t+7)^{14/5}$

$$\int\frac{13t^2-48}{\sqrt[5]{4t+7}}\,dt$$
$$=\left(13t^2-48\right)\left[\frac{5}{16}(4t+7)^{4/5}\right]-26t\left[\frac{25}{576}(4t+7)^{9/5}\right]$$
$$26\left[\frac{125}{32,256}(4t+7)^{14/5}\right]+C$$
$$=\frac{5}{16}\left(13t^2-48\right)(4t+7)^{4/5}-\frac{325}{288}t(4t+7)^{9/5}+$$
$$\frac{1625}{16,128}(4t+7)^{14/5}+C.$$

45. $\displaystyle\int\left(27x^3+83x-2\right)\sqrt[6]{3x+8}\,dx$

$=\left(27x^3+83x-2\right)\left(3x+8\right)^{1/6}\,dx$

We will use tabular integration as shown to simplify our work.

$f(x)$ and Repeated Derivatives	Sign of Product	$g(x)$ and Repeated Integrals
$27x^3+83x-2$	+	$(3x+8)^{1/6}$
$81x^2+83$	−	$\frac{2}{7}(3x+8)^{7/6}$
$162x$	+	$\frac{4}{91}(3x+8)^{13/6}$
162	−	$\frac{8}{1729}(3x+8)^{19/6}$
0		$\frac{16}{43,225}(3x+8)^{25/6}$

$$\int\left(27x^3+83x-2\right)\sqrt[6]{3x+8}\,dx$$
$$=\frac{2}{7}\left(27x^3+83x-2\right)(3x+8)^{7/6}-$$
$$\frac{4}{91}\left(81x^2+83\right)(3x+8)^{13/6}+$$
$$\frac{1296}{1729}x(3x+8)^{19/6}-\frac{2592}{43,225}(3x+8)^{25/6}+C$$

46. $\displaystyle\int x^2\left(\ln x\right)^2\,dx$

Let

$u=\left(\ln x\right)^2$ and $dv=x^2dx$

Then

$du=\dfrac{2\ln x}{x}dx$ and $v=\dfrac{x^3}{3}$

$$\int x^2\left(\ln x\right)^2dx=\frac{x^3\left(\ln x\right)^2}{3}-\int\frac{2}{3}x^2\ln x\,dx$$

Use integration by parts again to evaluate $\displaystyle\int\frac{2}{3}x^2\ln x\,dx$.

Let

$u=\ln x$ and $dv=\dfrac{2}{3}x^2dx$

Then

$du=\dfrac{1}{x}dx$ and $v=\dfrac{2}{9}x^3$

$$\int \frac{2}{3}x^2 \ln x dx = \frac{2}{9}x^3 \ln x - \int \frac{2}{9}x^2 dx$$
$$= \frac{2}{9}x^3 \ln x - \frac{2}{27}x^3 + K$$

Therefore,

$$\int x^2 (\ln x)^2 \, dx =$$
$$= \frac{x^3 (\ln x)^2}{3} - \left(\frac{2}{9}x^3 \ln x - \frac{2}{27}x^3 + K\right)$$
$$= \frac{x^3 (\ln x)^2}{3} - \frac{2}{9}x^3 \ln x + \frac{2}{27}x^3 - K$$
$$= \frac{x^3}{3}(\ln x)^2 - \frac{2x^3}{9}\ln x + \frac{2x^3}{27} + C \quad (C = -K)$$

47. $\int x^n (\ln x)^2 \, dx, \quad n \neq -1$

Let

$u = (\ln x)^2$ and $dv = x^n dx$

Then

$du = \frac{2 \ln x}{x} dx$ and $v = \frac{x^{n+1}}{n+1}$

$$\int x^n (\ln x)^2 \, dx = \frac{x^{n+1}(\ln x)^2}{n+1} - \int \frac{2x^n \ln x}{n+1} dx$$
$$= \frac{x^{n+1}(\ln x)^2}{n+1} - \frac{2}{n+1}\int x^n \ln x dx$$

Use integration by parts again to evaluate $\int x^n \ln x dx$.

Let

$u = \ln x$ and $dv = x^n dx$

Then

$du = \frac{1}{x} dx$ and $v = \frac{x^{n+1}}{n+1}$

$$\int x^n \ln x dx = \frac{x^{n+1} \ln x}{n+1} - \int \frac{x^n}{n+1} dx$$
$$= \frac{x^{n+1} \ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + K$$

Therefore,

$$\int x^n (\ln x)^2 \, dx =$$
$$= \frac{x^{n+1}(\ln x)^2}{n+1} - \frac{2}{n+1}\left(\frac{x^{n+1}\ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + K\right)$$
$$= \frac{x^{n+1}}{n+1}(\ln x)^2 - \frac{2x^{n+1}}{(n+1)^2}(\ln x) + \frac{2x^{n+1}}{(n+1)^3} + C$$
$$\left(C = \frac{-2K}{n+1}\right)$$

48. $\int x^n \ln x dx$.

Let

$u = \ln x$ and $dv = x^n dx$

Then

$du = \frac{1}{x} dx$ and $v = \frac{x^{n+1}}{n+1}$

$$\int x^n \ln x dx = \frac{x^{n+1} \ln x}{n+1} - \int \frac{x^n}{n+1} dx$$
$$= \frac{x^{n+1} \ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C$$

49. $\int x^n e^x dx$

Let

$u = x^n$ and $dv = e^x dx$

Then

$du = nx^{n-1} dx$ and $v = e^x$

$$\int x^n e^x dx = x^n e^x - \int e^x \left(nx^{n-1}\right) dx$$
$$= x^n e^x - n\int x^{n-1} e^x dx.$$

50. $\int (\ln x)^n \, dx$

Let

$u = (\ln x)^n$ and $dv = dx$

Then

$du = \frac{n}{x}(\ln x)^{n-1} \, dx$ and $v = x$

$$\int (\ln x)^n \, dx = x(\ln x)^n - \int x \cdot \frac{n}{x}(\ln x)^{n-1} \, dx$$
$$= x(\ln x)^n - n\int (\ln x)^{n-1} \, dx.$$

51. $\boxed{tw}$ No; let $f(x) = 2$ and $g(x) = x$. Then $\int f(x)g(x)dx = \int 2xdx = x^2 + C_1$, but $\int f(x)dx \cdot \int g(x)dx = \int 2dx \cdot \int xdx =$ $(2x + C_2)\left(\frac{x^2}{2} + C_3\right)$.

Since $x^2 + C_1$ and $(2x + C_2)\left(\frac{x^2}{2} + C_3\right)$ do not differ by only a constant, the given statement is not true.

52. $\boxed{tw}$ Answers will vary.

53. Using the fnInt feature on a calculator, we find that $\int_1^{10} x^5 \ln x dx \approx 355{,}986.$

Exercise Set 4.7

1. $\int xe^{-3x}dx$

This integral fits Formula 6 in Table 1.

$$\int xe^{ax}dx = \frac{1}{a^2}\cdot e^{ax}\left(ax-1\right)+C$$

In the given integral, $a=-3$, so we have, by the formula,

$$\begin{aligned}\int xe^{-3x}dx &= \frac{1}{(-3)^2}\cdot e^{(-3)x}\left((-3)x-1\right)+C\\ &= \frac{1}{9}e^{-3x}\left(-3x-1\right)+C\\ &= -\frac{1}{9}e^{-3x}\left(3x+1\right)+C.\end{aligned}$$

2. $\int 2xe^{3x}dx = 2\int xe^{3x}dx$

This integral fits Formula 6 in Table 1.

$$\int xe^{ax}dx = \frac{1}{a^2}\cdot e^{ax}\left(ax-1\right)+C$$

In the given integral, $a=3$, so we have, by the formula,

$$\begin{aligned}2\int xe^{3x}dx &= 2\cdot\frac{1}{(3)^2}\cdot e^{(3)x}\left(3x-1\right)+C\\ &= \frac{2}{9}e^{3x}\left(3x-1\right)+C.\end{aligned}$$

3. $\int 6^x dx$

This integral fits Formula 11 in Table 1.

$$\int a^x dx = \frac{a^x}{\ln a}+C,\quad a>0, a\neq 1$$

In the given integral, $a=6$, so we have, by the formula,

$$\int 6^x dx = \frac{6^x}{\ln 6}+C.$$

4. $\int \frac{1}{\sqrt{x^2-9}}dx$

This integral fits Formula 13 in Table 1.

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left|x+\sqrt{x^2-a^2}\right|+C$$

In the given integral, $a^2=9$, so we have, by the formula,

$$\int \frac{1}{\sqrt{x^2-9}}dx = \ln\left|x+\sqrt{x^2-9}\right|+C.$$

5. $\int \frac{1}{25-x^2}dx$

This integral fits Formula 15 in Table 1.

$$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+C$$

In the given integral, $a^2=25$, or $a=5$, so we have, by the formula,

$$\begin{aligned}\int \frac{1}{25-x^2}dx &= \int \frac{1}{5^2-x^2}dx\\ &= \frac{1}{2\cdot 5}\ln\left|\frac{5+x}{5-x}\right|+C\\ &= \frac{1}{10}\ln\left|\frac{5+x}{5-x}\right|+C.\end{aligned}$$

6. $\int \frac{1}{x\sqrt{4+x^2}}dx$

This integral fits Formula 16 in Table 1.

$$\int \frac{1}{x\sqrt{a^2+x^2}}dx = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right|+C$$

In the given integral, $a^2=4$, or $a=2$, so we have, by the formula,

$$\begin{aligned}\int \frac{1}{x\sqrt{4+x^2}}dx &= \int \frac{1}{x\sqrt{2^2+x^2}}dx\\ &= -\frac{1}{2}\ln\left|\frac{2+\sqrt{4+x^2}}{x}\right|+C\end{aligned}$$

7. $\int \frac{x}{3-x}dx$

This integral fits Formula 18 in Table 1.

$$\int \frac{x}{a+bx}dx = \frac{a}{b^2}+\frac{x}{b}-\frac{a}{b^2}\ln|a+bx|+C$$

In the given integral, $a=3$ and $b=-1$, so we have, by the formula,

$$\int \frac{x}{3+(-1)x}dx$$
$$=\frac{3}{(-1)^2}+\frac{x}{(-1)}-\frac{3}{(-1)^2}\ln|3-x|+C$$
$$=3-x-3\ln|3-x|+C.$$

8. $\int \frac{x}{(1-x)^2}dx$

This integral fits Formula 19 in Table 1.

$$\int \frac{x}{(a+bx)^2}dx = \frac{a}{b^2(a+bx)}+\frac{1}{b^2}\ln|a+bx|+C$$

In the given integral, $a=1$ and $b=-1$, so we have, by the formula,

$$\int \frac{x}{(1-x)^2}dx$$
$$=\frac{1}{(-1)^2(1-x)}+\frac{1}{(-1)^2}\ln|1-x|+C$$
$$=\frac{1}{(1-x)}+\ln|1-x|+C.$$

9. $\int \frac{1}{x(8-x)^2}dx$

This integral fits Formula 21 in Table 1.

$$\int \frac{1}{x(a+bx)^2}dx = \frac{1}{a(a+bx)}+\frac{1}{a^2}\ln\left|\frac{x}{a+bx}\right|+C$$

In the given integral, $a=8$ and $b=-1$, so we have, by the formula,

$$\int \frac{1}{x(8-x)^2}dx$$
$$=\frac{1}{8(8-x)}+\frac{1}{(8)^2}\ln\left|\frac{x}{8-x}\right|+C$$
$$=\frac{1}{8(8-x)}+\frac{1}{64}\ln\left|\frac{x}{8-x}\right|+C$$

10. $\int \sqrt{x^2+9}\,dx$

This integral fits Formula 22 in Table 1.

$$\int \sqrt{x^2+a^2}\,dx$$
$$=\frac{1}{2}\left[x\sqrt{x^2+a^2}+a^2\ln\left|x+\sqrt{x^2+a^2}\right|\right]+C$$

In the given integral, $a^2=9$ and $a=3$, so we have, by the formula,

$$\int \sqrt{x^2+9}\,dx$$
$$=\frac{1}{2}\left[x\sqrt{x^2+9}+9\ln\left|x+\sqrt{x^2+9}\right|\right]+C$$

11. $\int \ln(3x)\,dx$

$$=\int(\ln 3+\ln x)\,dx$$
$$=\int \ln 3\,dx+\int \ln x\,dx$$
$$=\ln 3\int dx+\int \ln x\,dx$$

The integral in the first term is the integral of a constant and we will integrate accordingly. The integral in the second term fits Formula 8 in Table 1.

$$\int \ln x\,dx = x\ln x - x + C$$

so we have, by the formula,

$$\int \ln(3x)\,dx$$
$$=\ln 3\int dx+\int \ln x\,dx$$
$$=\ln 3\cdot x + C_1 + x\ln x - x + C_2$$
$$=(\ln 3)x + x\ln x - x + C \qquad (C=C_1+C_2)$$

12. $\int \ln\left(\frac{4}{5}x\right)dx$

$$=\int\left(\ln\left(\frac{4}{5}\right)+\ln x\right)dx$$
$$=\ln\left(\frac{4}{5}\right)\int dx+\int \ln x\,dx$$

The integral in the second term fits Formula 8 in Table 1.

$$\int \ln x\,dx = x\ln x - x + C$$

so we have, by the formula,

$$\int \ln\left(\frac{4}{5}x\right)dx$$
$$= \ln\frac{4}{5}\int dx + \int \ln x\,dx$$
$$= \ln\frac{4}{5}\cdot x + C_1 + x\ln x - x + C_2$$
$$= \left(\ln\frac{4}{5}\right)x + x\ln x - x + C. \qquad (C = C_1 + C_2)$$

13. $\int x^4 \ln x\,dx$

This integral fits Formula 10 in Table 1.

$$\int x^n \ln x\,dx = x^{n+1}\left[\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right] + C$$

In the given integral, $n = 4$, so we have, by the formula,

$$\int x^4 \ln x\,dx = x^{4+1}\left[\frac{\ln x}{4+1} - \frac{1}{(4+1)^2}\right] + C$$
$$= x^5\left[\frac{\ln x}{5} - \frac{1}{(5)^2}\right] + C$$
$$= \frac{x^5}{5}(\ln x) - \frac{x^5}{25} + C.$$

14. $\int x^3 e^{-2x}dx$

This integral fits Formula 7 in Table 1.

$$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}dx + C$$

In the given integral, $n = 3$ and $a = -2$, so we have, by the formula,

$$\int x^3 e^{-2x}\,dx = \frac{x^3 e^{-2x}}{-2} - \frac{3}{-2}\int x^{3-1}e^{-2x}dx + C_1$$
$$= -\frac{1}{2}x^3e^{-2x} + \frac{3}{2}\int x^2 e^{-2x}dx + C_1$$

We will apply Formula 7 again, this time with $n = 2$ and $a = -2$

$$= -\frac{1}{2}x^3e^{-2x} +$$
$$\frac{3}{2}\left[\frac{x^2e^{-2x}}{-2} - \frac{2}{-2}\int x^{2-1}e^{-2x}dx + C_2\right] + C_1$$
$$= -\frac{1}{2}x^3e^{-2x} - \frac{3}{4}x^2e^{-2x} + \frac{3}{2}\int xe^{-2x}dx$$
$$+ C_2 + C_1$$

Now we apply Formula 6

$$\int xe^{ax}dx = \frac{1}{a^2}\cdot e^{ax}(ax-1) + C$$

with $a = -2$

$$= -\frac{1}{2}x^3e^{-2x} - \frac{3}{4}x^2e^{-2x} +$$
$$\frac{3}{2}\left[\frac{1}{(-2)^2}\cdot e^{-2x}(-2x-1) + C_3\right] + C_2 + C_1$$
$$= -\frac{1}{2}x^3e^{-2x} - \frac{3}{4}x^2e^{-2x} - \frac{3}{4}xe^{-2x} - \frac{3}{8}e^{-2x} + C.$$
$$\left(C = C_1 + C_2 + \tfrac{3}{2}C_3\right)$$

15. $\int x^3 \ln x\,dx$

This integral fits Formula 10 in Table 1.

$$\int x^n \ln x\,dx = x^{n+1}\left[\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right] + C$$

In the given integral, $n = 3$, so we have, by the formula,

$$\int x^3 \ln x\,dx = x^{3+1}\left[\frac{\ln x}{3+1} - \frac{1}{(3+1)^2}\right] + C$$
$$= x^4\left[\frac{\ln x}{4} - \frac{1}{(4)^2}\right] + C$$
$$= \frac{x^4}{4}(\ln x) - \frac{x^4}{16} + C.$$

16. $\int 5x^4 \ln x\,dx = 5\int x^4 \ln x\,dx$

This integral fits Formula 10 in Table 1.

$$\int x^n \ln x\,dx = x^{n+1}\left[\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right] + C$$

In the given integral, $n = 4$, so we have, by the formula,

$$5\int x^4 \ln x\,dx = 5\cdot x^{4+1}\left[\frac{\ln x}{4+1} - \frac{1}{(4+1)^2}\right] + C$$
$$= x^5(\ln x) - \frac{x^5}{5} + C.$$

17. $\int \frac{dx}{\sqrt{x^2+7}}$

This integral fits Formula 12 in Table 1.

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left|x+\sqrt{x^2+a^2}\right|+C$$

In the given integral, $a^2=7$, so we have, by the formula,

$$\int \frac{1}{\sqrt{x^2+7}}\,dx = \ln\left|x+\sqrt{x^2+7}\right|+C.$$

18. $\int \frac{3dx}{x\sqrt{1-x^2}} = 3\int \frac{dx}{x\sqrt{1-x^2}}$

This integral fits Formula 17 in Table 1.

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|+C$$

In the given integral, $a^2=1$ and $a=1$, so we have, by the formula,

$$3\int \frac{1}{x\sqrt{1-x^2}}\,dx = 3\cdot\left(-\frac{1}{1}\right)\ln\left|\frac{1+\sqrt{1-x^2}}{x}\right|+C$$
$$= -3\ln\left|\frac{1+\sqrt{1-x^2}}{x}\right|+C$$

19. $\int \frac{10dx}{x(5-7x)^2} = 10\int \frac{dx}{x(5-7x)^2}$

This integral fits Formula 21 in Table 1.

$$\int \frac{1}{x(a+bx)^2}\,dx = \frac{1}{a(a+bx)}+\frac{1}{a^2}\ln\left|\frac{x}{a+bx}\right|+C$$

In the given integral, $a=5$ and $b=-7$, so we have, by the formula,

$$10\int \frac{1}{x(5-7x)^2}\,dx$$
$$=10\left[\frac{1}{5(5-7x)}+\frac{1}{(5)^2}\ln\left|\frac{x}{5-7x}\right|\right]+C$$
$$=\frac{2}{5-7x}+\frac{2}{5}\ln\left|\frac{x}{5-7x}\right|+C.$$

20. $\int \frac{2}{5x(7x+2)}\,dx = \frac{2}{5}\int \frac{1}{x(2+7x)}\,dx$

This integral fits Formula 20 in Table 1.

$$\int \frac{1}{x(a+bx)}\,dx = \frac{1}{a}\ln\left|\frac{x}{a+bx}\right|+C$$

In the given integral, $a=2$ and $b=7$, so we have, by the formula,

$$\frac{2}{5}\int \frac{1}{x(2+7x)}\,dx = \frac{2}{5}\left[\frac{1}{2}\ln\left|\frac{x}{2+7x}\right|\right]+C$$
$$=\frac{1}{5}\ln\left|\frac{x}{2+7x}\right|+C$$
$$=\frac{1}{5}\ln\left|\frac{x}{7x+2}\right|+C.$$

21. $\int \frac{-5}{4x^2-1}\,dx = -5\int \frac{1}{4x^2-1}\,dx$

This integral almost fits Formula 14 in table 1.

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C$$

However, the x^2 coefficient needs to be 1. We factor out 4 as follows and then we apply formula 14.

$$-5\int \frac{1}{4x^2-1}\,dx = -5\int \frac{1}{4\left(x^2-\frac{1}{4}\right)}\,dx$$
$$=-\frac{5}{4}\int \frac{1}{x^2-\frac{1}{4}}\,dx$$

In the given integral, $a^2=\frac{1}{4}$ and $a=\frac{1}{2}$, so we have, by the formula,

$$-\frac{5}{4}\int \frac{1}{x^2-\frac{1}{4}}\,dx = -\frac{5}{4}\left[\frac{1}{2\left(\frac{1}{2}\right)}\ln\left|\frac{x-\frac{1}{2}}{x+\frac{1}{2}}\right|\right]+C$$
$$=-\frac{5}{4}\ln\left|\frac{x-\frac{1}{2}}{x+\frac{1}{2}}\right|+C$$

22. $\int \sqrt{9t^2-1}\,dt$

This integral almost fits Formula 22 in table 1.

$$\int \sqrt{x^2-a^2}\,dx$$
$$=\frac{1}{2}\left[x\sqrt{x^2-a^2}-a^2\ln\left|x+\sqrt{x^2-a^2}\right|\right]+C$$

However, the x^2 coefficient needs to be 1. We factor out 9 as follows and then we apply formula 22.

$$\int \sqrt{9\left(t^2-\tfrac{1}{9}\right)}\,dt = \int 3\sqrt{t^2-\tfrac{1}{9}}\,dt = 3\int \sqrt{t^2-\tfrac{1}{9}}\,dt$$

In the given integral, $a^2=\frac{1}{9}$ and $a=\frac{1}{3}$, so we have, by the formula,

$$3\int \sqrt{t^2-\tfrac{1}{9}}\,dt$$
$$=3\cdot\frac{1}{2}\left[t\sqrt{t^2-\tfrac{1}{9}}-\frac{1}{9}\ln\left|t+\sqrt{t^2-\tfrac{1}{9}}\right|\right]+C$$
$$=\frac{3}{2}\left[t\sqrt{t^2-\tfrac{1}{9}}-\frac{1}{9}\ln\left|t+\sqrt{t^2-\tfrac{1}{9}}\right|\right]+C$$

23. $\int \sqrt{4m^2+16}dm$

This integral almost fits Formula 22 in table 1.

$$\int \sqrt{x^2+a^2}\,dx$$
$$=\frac{1}{2}\left[x\sqrt{x^2+a^2}+a^2\ln\left|x+\sqrt{x^2+a^2}\right|\right]+C$$

However, the m^2 coefficient needs to be 1. We factor out 4 as follows and then we apply formula 22.

$$\int \sqrt{4\left(m^2+4\right)}dm=\int 2\sqrt{m^2+4}dm$$
$$=2\int \sqrt{m^2+4}dm$$

In the given integral, $a^2=4$ and $a=2$, so we have, by the formula,

$$2\int \sqrt{m^2+4}\,dm$$
$$=2\cdot\frac{1}{2}\left[m\sqrt{m^2+4}+4\ln\left|m+\sqrt{m^2+4}\right|\right]+C$$
$$=m\sqrt{m^2+4}+4\ln\left|m+\sqrt{m^2+4}\right|+C.$$

24. $\int \frac{3\ln x}{x^2}dx=3\int x^{-2}\ln x dx$

This integral fits Formula 10 in Table 1.

$$\int x^n \ln x dx=x^{n+1}\left[\frac{\ln x}{n+1}-\frac{1}{(n+1)^2}\right]+C$$

In the given integral, $n=-2$, so we have, by the formula,

$$3\int x^{-2}\ln x dx$$
$$=3\cdot x^{-2+1}\left[\frac{\ln x}{-2+1}-\frac{1}{(-2+1)^2}\right]+C$$
$$=3\cdot x^{-1}\left[\frac{\ln x}{-1}-\frac{1}{(-1)^2}\right]+C$$
$$=\frac{3}{x}(-\ln x-1)+C$$
$$=-\frac{3}{x}(\ln x+1)+C.$$

25. $\int \frac{-5\ln x}{x^3}dx=-5\int x^{-3}\ln x dx$

This integral fits Formula 10 in Table 1.

$$\int x^n \ln x dx=x^{n+1}\left[\frac{\ln x}{n+1}-\frac{1}{(n+1)^2}\right]+C$$

In the given integral, $n=-3$, so we have, by the formula,

$$-5\int x^{-3}\ln x dx$$
$$=-5\cdot x^{-3+1}\left[\frac{\ln x}{-3+1}-\frac{1}{(-3+1)^2}\right]+C$$
$$=-5\cdot x^{-2}\left[\frac{\ln x}{-2}-\frac{1}{(-2)^2}\right]+C$$
$$=\frac{5}{2x^2}(\ln x)+\frac{5}{4x^2}+C$$

26. $\int (\ln x)^4\,dx$

This integral fits Formula 9 in Table 1.

$$\int (\ln x)^n=x(\ln x)^n-n\int (\ln x)^{n-1}\,dx+C,$$
$$n\neq -1$$

In the given integral, $n=4$, so we have, by the formula,

$$\int (\ln x)^4=x(\ln x)^4-4\int (\ln x)^{4-1}\,dx+C_1$$
$$=x(\ln x)^4-4\int (\ln x)^3\,dx+C_1$$

We continue to apply Formula 9.

$$x(\ln x)^4-4\int (\ln x)^{4-1}\,dx+C$$
$$=x(\ln x)^4-4\left[x(\ln x)^3-3\int (\ln x)^{3-1}\,dx\right]+C$$
$$=x(\ln x)^4-4x(\ln x)^3+12\int (\ln x)^2\,dx+C$$

We apply Formula 9 again.

$$=x(\ln x)^4-4x(\ln x)^3+$$
$$12\left[x(\ln x)^2-2\int \ln x dx\right]+C$$
$$=x(\ln x)^4-4x(\ln x)^3+12x(\ln x)^2-$$
$$24\int \ln x dx+C$$

Now we apply formula 8,

$$\int \ln x dx=x\ln x-x+C.$$
$$=x(\ln x)^4-4x(\ln x)^3+12x(\ln x)^2-$$
$$24\left[x\ln x-x\right]+C$$
$$=x(\ln x)^4-4x(\ln x)^3+12x(\ln x)^2-$$
$$24x\ln x+24x+C.$$

27. $\int \frac{e^x}{x^{-3}}dx=\int x^3e^x dx$

This integral fits Formula 7 in Table 1.

$$\int x^n e^{ax}\,dx=\frac{x^n e^{ax}}{a}-\frac{n}{a}\int x^{n-1}e^{ax}dx+C$$

In the given integral, $n=3$ and $a=1$, so we have, by the formula,

$$\int x^3 e^x\,dx = \frac{x^3 e^x}{1} - \frac{3}{1}\int x^{3-1} e^x\,dx + C$$
$$= x^3 e^x - 3\int x^2 e^x\,dx + C$$

We will apply Formula 7 again, this time with $n = 2$ and $a = 1$

$$x^3 e^x - 3\left[\frac{x^2 e^x}{1} - \frac{2}{1}\int x^{2-1} e^x\,dx\right] + C$$
$$= x^3 e^x - 3x^2 e^x + 6\int x e^x\,dx + C$$

Now we apply Formula 6

$$\int x e^{ax}\,dx = \frac{1}{a^2}\cdot e^{ax}\,(ax-1) + C$$

with $a = 1$

$$= x^3 e^x - 3x^2 e^x + 6\left[\frac{1}{(1)^2}\cdot e^x\,(x-1)\right] + C$$
$$= x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + C.$$

28. $\displaystyle\int \frac{3}{\sqrt{4x^2+100}}\,dx$

$$= 3\int \frac{1}{\sqrt{4\left(x^2+25\right)}}\,dx$$
$$= 3\int \frac{1}{2\sqrt{x^2+25}}\,dx$$
$$= \frac{3}{2}\int \frac{1}{\sqrt{x^2+25}}\,dx$$

This integral fits Formula 12 in Table 1.

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left|x+\sqrt{x^2+a^2}\right| + C$$

In the given integral, $a^2 = 25$, so we have, by the formula,

$$\frac{3}{2}\int \frac{1}{\sqrt{x^2+25}}\,dx = \frac{3}{2}\ln\left|x+\sqrt{x^2+25}\right| + C.$$

29. $\displaystyle\int x\sqrt{1+2x}\,dx$

This integral fits Formula 23 in Table 1.

$$\int x\sqrt{a+bx}\,dx = \frac{2}{15b^2}(3bx-2a)(a+bx)^{3/2} + C$$

In the given integral, $a = 1$ and $b = 2$, so we have, by the formula,

$$\int x\sqrt{1+2x}\,dx$$
$$= \frac{2}{15(2)^2}\left(3(2)x - 2(1)\right)(1+2x)^{3/2} + C$$
$$= \frac{2}{60}(6x-2)(1+2x)^{3/2} + C$$
$$= \frac{1}{30}\cdot 2(3x-1)(1+2x)^{3/2} + C$$
$$= \frac{1}{15}(3x-1)(1+2x)^{3/2} + C.$$

30. $\displaystyle\int x\sqrt{2+3x}\,dx$

This integral fits Formula 23 in Table 1.

$$\int x\sqrt{a+bx}\,dx = \frac{2}{15b^2}(3bx-2a)(a+bx)^{3/2} + C$$

In the given integral, $a = 2$ and $b = 3$, so we have, by the formula,

$$\int x\sqrt{2+3x}\,dx$$
$$= \frac{2}{15(3)^2}\left(3(3)x - 2(2)\right)(2+3x)^{3/2} + C$$
$$= \frac{2}{135}(9x-4)(2+3x)^{3/2} + C.$$

31. $S(x) = \int S'(x)\,dx$

$$= \int \frac{100x}{(20-x)^2}\,dx = 100\int \frac{x}{(20-x)^2}\,dx$$

This integral fits Formula 19 in Table 1.

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{a}{b^2\,(a+bx)} + \frac{1}{b^2}\ln|a+bx| + C$$

In the given integral, $a = 20$ and $b = -1$, so we have, by the formula,

$$100\int \frac{x}{(20-x)^2}\,dx$$
$$= 100\left[\frac{20}{(-1)^2\,(20-x)} + \frac{1}{(-1)^2}\ln|20-x|\right] + C$$
$$= \frac{2000}{(20-x)} + 100\ln|20-x| + C.$$

Next, we use the condition $S(19) = 2000$ to determine C.

$$S(19) = 2000$$
$$\frac{2000}{(20-(19))} + 100\ln|20-(19)| + C = 2000$$
$$\frac{2000}{1} + 100\ln|1| + C = 2000$$
$$2000 + C = 2000$$
$$C = 0$$

Thus, the supply function is

$$S(x) = \frac{2000}{(20-x)} + 100\ln|20-x| = 100\left[\frac{20}{20-x} + \ln|20-x|\right].$$

32. $p(t) = \int p'(t)\,dt = \int \frac{1}{t(2+t)^2}\,dt$

This integral fits Formula 21 in Table 1.

$$\int \frac{1}{x(a+bx)^2}\,dx = \frac{1}{a(a+bx)} + \frac{1}{a^2}\ln\left|\frac{x}{a+bx}\right| + C$$

In the given integral, $a = 2$ and $b = 1$, so we have, by the formula,

$$p(t) = \int \frac{1}{t(2+t)^2}\,dt$$
$$= \frac{1}{2(2+t)} + \frac{1}{2^2}\ln\left|\frac{t}{2+t}\right| + C$$
$$= \frac{1}{2(2+t)} + \frac{1}{4}\ln\left(\frac{t}{2+t}\right) + C$$

Note, we can drop the absolute value sign because $t > 0$.

We use the condition $p(2) = 0.8267$ to find C.

$$p(2) = 0.8267$$
$$\frac{1}{2(2+2)} + \frac{1}{4}\ln\left(\frac{2}{2+2}\right) + C = 0.8267$$
$$\frac{1}{8} + \frac{1}{4}\ln\left(\frac{1}{2}\right) + C = 0.8267$$
$$-0.0482867951 + C \approx 0.8267$$
$$C \approx 0.8750$$

Thus, $p(t) = \frac{1}{2(2+t)} + \frac{1}{4}\ln\left(\frac{t}{2+t}\right) + 0.8750$.

33. $\int \frac{8}{3x^2 - 2x}\,dx = 8\int \frac{1}{x(-2+3x)}\,dx$

This integral fits Formula 20 in Table 1.

$$\int \frac{1}{x(a+bx)}\,dx = \frac{1}{a}\ln\left|\frac{x}{a+bx}\right| + C$$

In the given integral, $a = -2$ and $b = 3$, so we have, by the formula,

$$8\int \frac{1}{x(-2+3x)}\,dx = 8\cdot\frac{1}{(-2)}\ln\left|\frac{x}{-2+3x}\right| + C$$
$$= -4\ln\left|\frac{x}{-2+3x}\right| + C.$$

34. $\int \frac{x\,dx}{4x^2 - 12x + 9} = \int \frac{x\,dx}{(2x-3)^2} = \int \frac{x\,dx}{(-3+2x)^2}$

This integral fits Formula 19 in Table 1.

$$\int \frac{x}{(a+bx)^2}\,dx = \frac{a}{b^2(a+bx)} + \frac{1}{b^2}\ln|a+bx| + C$$

In the given integral, $a = -3$ and $b = 2$, so we have, by the formula,

$$\int \frac{x}{(-3+2x)^2}\,dx$$
$$= \frac{-3}{(2)^2(-3+2x)} + \frac{1}{2^2}\ln|-3+2x| + C$$
$$= -\frac{3}{4(2x-3)} + \frac{1}{4}\ln|2x-3| + C.$$

35. $\int \frac{dx}{x^3 - 4x^2 + 4x}$

$$= \int \frac{dx}{x(x^2 - 4x + 4)}$$
$$= \int \frac{1}{x(x-2)^2}\,dx$$
$$= \int \frac{1}{x(-2+x)^2}\,dx$$

This integral fits Formula 21 in Table 1.

$$\int \frac{1}{x(a+bx)^2}\,dx = \frac{1}{a(a+bx)} + \frac{1}{a^2}\ln\left|\frac{x}{a+bx}\right| + C$$

In the given integral, $a = -2$ and $b = 1$, so we have, by the formula,

$$\int \frac{1}{x(-2+x)^2}dx$$
$$= \frac{1}{-2(-2+x)} + \frac{1}{(-2)^2}\ln\left|\frac{x}{-2+x}\right| + C$$
$$= \frac{1}{-2(x-2)} + \frac{1}{4}\ln\left|\frac{x}{x-2}\right| + C$$
$$= \frac{-1}{2(x-2)} + \frac{1}{4}\ln\left|\frac{x}{x-2}\right| + C.$$

36. $\int e^x\sqrt{e^{2x}+1}dx$

We substitute $u = e^x$ and $du = e^x dx$, and get $\sqrt{u^2+1}du$

This integral fits Formula 22 in Table 1.

$$\int \sqrt{x^2+a^2}dx$$
$$= \frac{1}{2}\left[x\sqrt{x^2+a^2} + a^2 \ln\left|x+\sqrt{x^2+a^2}\right|\right] + C$$

In the given integral, we have $u = x$ and $a^2 = 1$, so we have, by the formula,

$$\int \sqrt{u+1}du$$
$$= \frac{1}{2}\left[u\sqrt{u^2+1^2} + \ln\left(u+\sqrt{u^2+1^2}\right)\right] + C.$$
$$= \frac{1}{2}\left[e^x\sqrt{e^{2x}+1} + \ln\left(e^x+\sqrt{e^{2x}+1}\right)\right] + C.$$

37. $\int \frac{-e^{-2x}dx}{9-6e^{-x}+e^{-2x}}dx = \int \frac{e^{-x}\left(-e^{-x}\right)dx}{\left(-3+e^{-x}\right)^2}$

We substitute $u = e^{-x}$ and $du = -e^{-x}dx$, and get

$$\frac{u}{(-3+u)^2}du$$

Which fits Formula 19 in Table 1.

$$\int \frac{x}{(a+bx)^2}dx = \frac{a}{b^2(a+bx)} + \frac{1}{b^2}\ln|a+bx| + C$$

In the given integral, we have $u = x$, $a = -3$ and $b = 1$, so we have, by the formula,

$$\int \frac{e^{-x}\left(-e^{-x}\right)dx}{\left(-3+e^{-x}\right)^2} = \int \frac{u}{(-3+u)^2}du$$
$$= \frac{-3}{1^2(-3+u)} + \frac{1}{1^2}\ln|-3+u| + C$$
$$= \frac{-3}{1^2\left(-3+e^{-x}\right)} + \frac{1}{1^2}\ln\left|-3+e^{-x}\right| + C \quad \text{Substituting for } u$$
$$= \frac{-3}{e^{-x}-3} + \ln\left|e^{-x}-3\right| + C.$$

38. $\int \frac{\sqrt{(\ln x)^2+49}}{2x}dx$

We substitute $u = \ln x$ and $du = \frac{1}{x}dx$, and get

$$\frac{1}{2}\ \sqrt{u^2+49}du$$

This integral fits Formula 22 in Table 1.

$$\int \sqrt{x^2+a^2}dx$$
$$= \frac{1}{2}\left[x\sqrt{x^2+a^2} + a^2 \ln\left|x+\sqrt{x^2+a^2}\right|\right] + C$$

In the given integral, we have $u = x$ and $a^2 = 49$, so we have, by the formula,

$$\frac{1}{2}\int \sqrt{u^2+49}du$$
$$= \frac{1}{2}\cdot\frac{1}{2}\left[u\sqrt{(u)^2+49} + 49\ln\left(u+\sqrt{(u)^2+49}\right)\right] + C$$
$$= \frac{1}{2}\cdot\frac{1}{2}\left[\ln x\sqrt{(\ln x)^2+49} + 49\ln\left(\ln x+\sqrt{(\ln x)^2+49}\right)\right] + C \quad \text{Substituting for } u$$
$$= \frac{1}{4}\left[\ln x\sqrt{(\ln x)^2+49} + 49\ln\left(\ln x+\sqrt{(\ln x)^2+49}\right)\right] + C.$$