Exercise Set 1.3

1. a) $f(x) = 4x^2$

so,

$f(x+h) = 4(x+h)^2$ substituing $x+h$ for x

$= 4(x^2 + 2xh + h^2)$

$= 4x^2 + 8xh + 4h^2$

Then

$\dfrac{f(x+h) - f(x)}{h}$ Difference quotient

$= \dfrac{(4x^2 + 8xh + 4h^2) - 4x^2}{h}$ Substituting

$= \dfrac{8xh + 4h^2}{h}$

$= \dfrac{h(8x + 4h)}{h}$ Factoring the numerator

$= \dfrac{h}{h} \cdot (8x + 4h)$ Removing a factor = 1.

$= 8x + 4h,$ Simplified difference quotient

or $4(2x+h)$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$8x + 4h$ Simplified difference quotient

$= 8(5) + 4(2) = 48$

substitiuting 5 for x and 2 for h;

$= 8(5) + 4(1) = 44$

substitiuting 5 for x and 1 for h;

$= 8(5) + 4(0.1) = 40.4$

substitiuting 5 for x and 0.1 for h;

$= 8(5) + 4(0.01) = 40.04$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\dfrac{f(x+h) - f(x)}{h}$
5	2	48
5	1	44
5	0.1	40.4
5	0.01	40.04

2. a) $f(x) = 5x^2$

so,

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{5(x+h)^2 - 5x^2}{h}$

$= \dfrac{(5x^2 + 10xh + 5h^2) - 5x^2}{h}$

$= \dfrac{10xh + 5h^2}{h}$

$= \dfrac{h(10x + 5h)}{h}$

$= \dfrac{h}{h} \cdot (10x + 5h)$

$= 10x + 5h = 5(2x + h)$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$10x + 5h$ Simplified difference quotient

$= 10(5) + 5(2) = 60$

substitiuting 5 for x and 2 for h;

$= 10(5) + 5(1) = 55$

substitiuting 5 for x and 1 for h;

$= 10(5) + 5(0.1) = 50.5$

substitiuting 5 for x and 0.1 for h;

$= 10(5) + 5(0.01) = 50.05$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\dfrac{f(x+h) - f(x)}{h}$
5	2	60
5	1	55
5	0.1	50.5
5	0.01	50.05

3. a) $f(x) = -4x^2$

so,

$$f(x+h) = -4(x+h)^2 \quad \text{substituing } x+h \text{ for } x$$
$$= -4(x^2 + 2xh + h^2)$$
$$= -4x^2 - 8xh - 4h^2$$

Then

$$\frac{f(x+h) - f(x)}{h} \quad \text{Difference quotient}$$

$$= \frac{(-4x^2 - 8xh - 4h^2) - (-4x^2)}{h} \quad \text{Substituting}$$

$$= \frac{-8xh - 4h^2}{h}$$

$$= \frac{h(-8x - 4h)}{h} \quad \text{Factoring the numerator}$$

$$= \frac{h}{h} \cdot (-8x - 4h) \quad \text{Removing a factor = 1.}$$

$$= -8x - 4h, \quad \text{Simplified difference quotient}$$

or $-4(2x+h)$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-8x - 4h$ Simplified difference quotient

$= -8(5) - 4(2) = -48$

substitiuting 5 for x and 2 for h;

$= -8(5) - 4(1) = -44$

substitiuting 5 for x and 1 for h;

$= -8(5) - 4(0.1) = -40.4$

substitiuting 5 for x and 0.1 for h;

$= -8(5) - 4(0.01) = -40.04$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	−48
5	1	−44
5	0.1	−40.4
5	0.01	−40.04

4. a) $f(x) = -5x^2$

so,

$$\frac{f(x+h) - f(x)}{h} = \frac{-5(x+h)^2 - (-5x^2)}{h}$$
$$= \frac{(-5x^2 - 10xh - 5h^2) + 5x^2}{h}$$
$$= \frac{-10xh - 5h^2}{h}$$
$$= \frac{h(-10x - 5h)}{h}$$
$$= \frac{h}{h} \cdot (10x + 5h)$$
$$= -10x - 5h = -5(2x+h)$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-10x - 5h$ Simplified difference quotient

$= -10(5) - 5(2) = -60$

substitiuting 5 for x and 2 for h;

$= -10(5) - 5(1) = -55$

substitiuting 5 for x and 1 for h;

$= -10(5) - 5(0.1) = -50.5$

substitiuting 5 for x and 0.1 for h;

$= -10(5) - 5(0.01) = -50.05$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	−60
5	1	−55
5	0.1	−50.5
5	0.01	−50.05

5. a) $f(x)=x^2+x$

We substitute $x+h$ for x

$$f(x+h)=(x+h)^2+(x+h)$$
$$=(x^2+2xh+h^2)+x+h$$
$$=x^2+2xh+h^2+x+h$$

Then

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$
$$=\frac{(x^2+2xh+h^2+x+h)-(x^2+x)}{h}$$
$$=\frac{2xh+h^2+h}{h}$$
$$=\frac{h(2x+h+1)}{h} \quad \text{Factoring the numerator}$$
$$=\frac{h}{h}\cdot(2x+h+1) \quad \text{Removing a factor = 1.}$$
$$=2x+h+1 \quad \text{Simplified difference quotient}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x+h+1$ Simplified difference quotient

$=2(5)+(2)+1=13$

substitiuting 5 for x and 2 for h;

$=2(5)+(1)+1=12$

substitiuting 5 for x and 1 for h;

$=2(5)+(0.1)+1=11.1$

substitiuting 5 for x and 0.1 for h;

$=2(5)+(0.01)+1=11.01$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	13
5	1	12
5	0.1	11.1
5	0.01	11.01

6. a) $f(x)=x^2-x$

Then

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$
$$=\frac{((x+h)^2-(x+h))-(x^2-x)}{h}$$
$$=\frac{(x^2+2xh+h^2-x-h)-(x^2-x)}{h}$$
$$=\frac{2xh+h^2-h}{h}$$
$$=\frac{h(2x+h-1)}{h}$$
$$=\frac{h}{h}\cdot(2x+h-1)$$
$$=2x+h-1 \quad \text{Simplified difference quotient}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x+h-1$ Simplified difference quotient

$=2(5)+(2)-1=11$

substitiuting 5 for x and 2 for h;

$=2(5)+(1)-1=10$

substitiuting 5 for x and 1 for h;

$=2(5)+(0.1)-1=9.1$

substitiuting 5 for x and 0.1 for h;

$=2(5)+(0.01)-1=9.01$

substitiuting 5 for x and 0.01 for h.

The completed table is shown at the top of the next column.

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	11
5	1	10
5	0.1	9.1
5	0.01	9.01

7. a) $f(x)=\dfrac{2}{x}$

We substitute $x+h$ for x

$f(x+h)=\dfrac{2}{x+h}$

Then

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$=\dfrac{\left(\dfrac{2}{x+h}\right)-\left(\dfrac{2}{x}\right)}{h}$

$=\dfrac{\left(\dfrac{2}{x+h}\cdot\dfrac{x}{x}\right)-\left(\dfrac{2}{x}\cdot\dfrac{(x+h)}{(x+h)}\right)}{h}$ multiplying by 1

$=\dfrac{\left(\dfrac{2x}{x(x+h)}\right)-\left(\dfrac{2(x+h)}{x(x+h)}\right)}{h}$

$=\dfrac{\dfrac{-2h}{x(x+h)}}{h}$ adding fractions

$=\dfrac{\dfrac{-2h}{x(x+h)}}{\dfrac{h}{1}}$ $h=\dfrac{h}{1}$

$=\dfrac{-2h}{x(x+h)}\cdot\dfrac{1}{h}$ multiplying by the reciprocal

$=\dfrac{h}{h}\cdot\left(\dfrac{-2}{x(x+h)}\right)$ Removing a factor = 1.

$=\dfrac{-2}{x(x+h)}$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-\dfrac{2}{x(x+h)}$ Simplified difference quotient

$=-\dfrac{2}{5(5+2)}=-\dfrac{2}{35}$

substitiuting 5 for x and 2 for h;

$=-\dfrac{2}{5(5+1)}=-\dfrac{2}{30}=-\dfrac{1}{15}$

substitiuting 5 for x and 1 for h;

$=-\dfrac{2}{5(5+0.1)}=-\dfrac{2}{25.5}=-\dfrac{4}{51}$

substitiuting 5 for x and 0.1 for h;

$=-\dfrac{2}{5(5+0.01)}=-\dfrac{2}{25.05}=-\dfrac{40}{501}$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\dfrac{f(x+h)-f(x)}{h}$
5	2	$-\dfrac{2}{35}$
5	1	$-\dfrac{1}{15}$
5	0.1	$-\dfrac{4}{51}$
5	0.01	$-\dfrac{40}{501}$

8. a) $f(x)=\dfrac{9}{x}$

Then

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$=\dfrac{\left(\dfrac{9}{x+h}\right)-\left(\dfrac{9}{x}\right)}{h}$

$=\dfrac{\left(\dfrac{9}{x+h}\cdot\dfrac{x}{x}\right)-\left(\dfrac{9}{x}\cdot\dfrac{(x+h)}{(x+h)}\right)}{h}$

$=\dfrac{\left(\dfrac{9x}{x(x+h)}\right)-\left(\dfrac{9(x+h)}{x(x+h)}\right)}{h}$

$=\dfrac{\dfrac{-9h}{x(x+h)}}{h}$

$=\dfrac{-9h}{x(x+h)}\cdot\dfrac{1}{h}$

$=\dfrac{h}{h}\cdot\left(\dfrac{-9}{x(x+h)}\right)$

$=\dfrac{-9}{x(x+h)}$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-\dfrac{9}{x(x+h)}$ Simplified difference quotient

$= -\dfrac{9}{5(5+2)} = -\dfrac{9}{35}$

substitiuting 5 for x and 2 for h;

$= -\dfrac{9}{5(5+1)} = -\dfrac{9}{30} = -\dfrac{3}{10}$

substitiuting 5 for x and 1 for h;

$= -\dfrac{9}{5(5+0.1)} = -\dfrac{9}{25.5} = -\dfrac{6}{17}$

substitiuting 5 for x and 0.1 for h;

$= -\dfrac{9}{5(5+0.01)} = -\dfrac{9}{25.05} = -\dfrac{60}{167}$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\dfrac{f(x+h)-f(x)}{h}$
5	2	$-\dfrac{9}{35}$
5	1	$-\dfrac{3}{10}$
5	0.1	$-\dfrac{6}{17}$
5	0.01	$-\dfrac{60}{167}$

9. a) $f(x) = -2x+5$

We substitute $x+h$ for x

$f(x+h) = -2(x+h)+5$

$-2x-2h+5$

Then

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$= \dfrac{(-2x-2h+5)-(-2x+5)}{h}$

$= \dfrac{-2h}{h}$

$= -2$ Simplified difference quotient

b) The difference quotient is -2 for all values of x and h. Therefore, the completed table is shown at the top of the next column.

x	h	$\dfrac{f(x+h)-f(x)}{h}$
5	2	–2
5	1	–2
5	0.1	–2
5	0.01	–2

10. a) $f(x) = 2x+3$

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$= \dfrac{(2(x+h)+3)-(2x+3)}{h}$

$= \dfrac{(2x+2h+3)-(2x+3)}{h}$

$= \dfrac{2h}{h}$

$= 2$ Simplified difference quotient

b) The difference quotient is 2 for all values of x and h. Therefore, the completed table is:

x	h	$\dfrac{f(x+h)-f(x)}{h}$
5	2	2
5	1	2
5	0.1	2
5	0.01	2

11. a) $f(x) = 1-x^3$

so,

$f(x+h) = 1-(x+h)^3$ substituing $x+h$ for x

$= 1-(x^3+3x^2h+3xh^2+h^3)$

$= 1-x^3-3x^2h-3xh^2-h^3$

Then

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$= \dfrac{(1-x^3-3x^2h-3xh^2-h^3)-(1-x^3)}{h}$

$= \dfrac{-3x^2h-3xh^2-h^3}{h}$

$= \dfrac{h(-3x^2-3xh-h^2)}{h}$ Factoring the numerator

$= \dfrac{h}{h}\cdot(-3x^2-3xh-h^2)$ Removing a factor = 1.

$= -3x^2-3xh-h^2$

Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-3x^2 - 3xh - h^2$

$= -3(5)^2 - 3(5)(2) - (2)^2 = -109$

substitiuting 5 for x and 2 for h;

$= -3(5)^2 - 3(5)(1) - (1)^2 = -91$

substitiuting 5 for x and 1 for h;

$= -3(5)^2 - 3(5)(0.1) - (0.1)^2 = -76.51$

substitiuting 5 for x and 0.1 for h;

$= -3(5)^2 - 3(5)(0.01) - (0.01)^2 = -75.1501$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	–109
5	1	–91
5	0.1	–76.51
5	0.01	–75.1501

12. a) $f(x) = 12x^3$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$= \frac{\left(12(x+h)^3\right) - \left(12x^3\right)}{h}$

$= \frac{12\left(x^3 + 3x^2h + 3xh^2 + h^3\right) - \left(12x^3\right)}{h}$

$= \frac{36x^2h + 36xh^2 + 12h^3}{h}$

$= \frac{h\left(36x^2 + 36xh + 12h^2\right)}{h}$

$= \frac{h}{h} \cdot \left(36x^2 + 36xh + 12h^2\right)$

$= 36x^2 + 36xh + 12h^2$

Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$36x^2 + 36xh + 12h^2$

$= 36(5)^2 + 36(5)(2) + 12(2)^2 = 1308$

substitiuting 5 for x and 2 for h;

$= 36(5)^2 + 36(5)(1) + 12(1)^2 = 1092$

substitiuting 5 for x and 1 for h;

$= 36(5)^2 + 36(5)(0.1) + 12(0.1)^2 = 918.12$

substitiuting 5 for x and 0.1 for h;

$= 36(5)^2 + 36(5)(0.01) + 12(0.01)^2 = 901.8012$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	1308
5	1	1092
5	0.1	918.12
5	0.01	901.8012

13. a) $f(x) = x^2 - 3x$

We substitute $x + h$ for x

$f(x+h) = (x+h)^2 - 3(x+h)$

$= \left(x^2 + 2xh + h^2\right) - 3x - 3h$

$= x^2 + 2xh + h^2 - 3x - 3h$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$= \frac{\left(x^2 + 2xh + h^2 - 3x - 3h\right) - \left(x^2 - 3x\right)}{h}$

$= \frac{2xh + h^2 - 3h}{h}$

$= \frac{h(2x + h - 3)}{h}$ Factoring the numerator

$= \frac{h}{h} \cdot (2x + h - 3)$ Removing a factor = 1.

$= 2x + h - 3$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x + h - 3$ Simplified difference quotient

$= 2(5) + (2) - 3 = 9$

substitiuting 5 for x and 2 for h;

$= 2(5) + (1) - 3 = 8$

substitiuting 5 for x and 1 for h;

$= 2(5) + (0.1) - 3 = 7.1$

substitiuting 5 for x and 0.1 for h;

$= 2(5) + (0.01) - 3 = 7.01$

substitiuting 5 for x and 0.01 for h.

Using the values from the previous page, the completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	9
5	1	8
5	0.1	7.1
5	0.01	7.01

14. a) $f(x)=x^2-4x$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{\left((x+h)^2-4(x+h)\right)-\left(x^2-4x\right)}{h}$

$=\frac{\left(x^2+2xh+h^2-4x-4h\right)-\left(x^2-4x\right)}{h}$

$=\frac{2xh+h^2-4h}{h}$

$=\frac{h(2x+h-4)}{h}$

$=\frac{h}{h}\cdot(2x+h-4)$

$=2x+h-4$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x+h-4$ Simplified difference quotient

$=2(5)+(2)-4=8$

substitiuting 5 for x and 2 for h;

$=2(5)+(1)-4=7$

substitiuting 5 for x and 1 for h;

$=2(5)+(0.1)-4=6.1$

substitiuting 5 for x and 0.1 for h;

$=2(5)+(0.01)-4=6.01$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	8
5	1	7
5	0.1	6.1
5	0.01	6.01

15. a) $f(x)=x^2+4x-3$

We substitute $x+h$ for x

$f(x+h)=(x+h)^2+4(x+h)-3$

$=\left(x^2+2xh+h^2\right)+4x+4h-3$

$=x^2+2xh+h^2+4x+4h-3$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{\left(x^2+2xh+h^2+4x+4h-3\right)-\left(x^2+4x-3\right)}{h}$

$=\frac{2xh+h^2+4h}{h}$

$=\frac{h(2x+h+4)}{h}$ Factoring the numerator

$=\frac{h}{h}\cdot(2x+h+4)$ Removing a factor = 1

$=2x+h+4$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x+h+4$ Simplified difference quotient

$=2(5)+(2)+4=16$

substitiuting 5 for x and 2 for h;

$=2(5)+(1)+4=15$

substitiuting 5 for x and 1 for h;

$=2(5)+(0.1)+4=14.1$

substitiuting 5 for x and 0.1 for h;

$=2(5)+(0.01)+4=14.01$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	16
5	1	15
5	0.1	14.1
5	0.01	14.01

16. a) $f(x) = x^2 - 3x + 5$

Then

$\dfrac{f(x+h) - f(x)}{h}$ Difference quotient

$$= \frac{\left((x+h)^2 - 3(x+h) + 5\right) - \left(x^2 - 3x + 5\right)}{h}$$

$$= \frac{\left(x^2 + 2xh + h^2 - 3x - 3h + 5\right) - \left(x^2 - 3x + 5\right)}{h}$$

$$= \frac{2xh + h^2 - 3h}{h}$$

$$= \frac{h(2x + h - 3)}{h}$$

$$= \frac{h}{h} \cdot (2x + h - 3)$$

$= 2x + h - 3$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x + h - 3$ Simplified difference quotient

$= 2(5) + (2) - 3 = 9$

substitiuting 5 for x and 2 for h;

$= 2(5) + (1) - 3 = 8$

substitiuting 5 for x and 1 for h;

$= 2(5) + (0.1) - 3 = 7.1$

substitiuting 5 for x and 0.1 for h;

$= 2(5) + (0.01) - 3 = 7.01$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\dfrac{f(x+h) - f(x)}{h}$
5	2	9
5	1	8
5	0.1	7.1
5	0.01	7.01

17. a) To find the average rate of change from 1994 to 2000, we locate the corresponding points $(1994, 0)$ and $(2000, 20)$. Using these two points we calculate the average rate of change

$$\frac{20 - 0}{2000 - 1994} = \frac{20}{6} = \frac{10}{3} = 3\tfrac{1}{3}$$

The average rate of change of total employment from 1994 to 2000 increased approximately $3\tfrac{1}{3}\%$ per year.

b) To find the average rate of change from 2000 to 2005, we locate the corresponding points $(2000, 20)$ and $(2005, 22)$. Using these two points we calculate the average rate of change

$$\frac{22 - 20}{2005 - 2000} = \frac{2}{5} = 0.4$$

The average rate of change of total employment from 2000 to 2005 increased approximately 0.4% per year.

c) To find the average rate of change from 1994 to 2005, we locate the corresponding points $(1994, 0)$ and $(2005, 22)$. Using these two points we calculate the average rate of change

$$\frac{22 - 0}{2005 - 1994} = \frac{22}{11} = 2$$

The average rate of change of total employment from 1994 to 2005 increased approximately 2% per year.

18. a) Locate the points $(1994, 5)$ and $(2000, 45)$.

The average rate of change is

$$\frac{45 - 5}{2000 - 1994} = \frac{40}{6} = \frac{20}{3} = 6\tfrac{2}{3}$$

The average rate of change of construction employment from 1994 to 2000 was approximately $6\tfrac{2}{3}\%$ per year.

b) Locate the points $(2000, 45)$ and $(2005, 55)$.

The average rate of change is

$$\frac{55 - 45}{2005 - 2000} = \frac{10}{5} = 2$$

The average rate of change of construction employment from 2000 to 2005 was approximately 2% per year.

c) Locate the points $(1994, 5)$ and $(2005, 55)$.

The average rate of change is

$$\frac{55 - 5}{2005 - 1994} = \frac{50}{11} = 4\tfrac{6}{11} \approx 4.545$$

The average rate of change of construction employment from 1994 to 2005 was approximately $4\tfrac{6}{11}\%$, or about 4.5% per year.

19. a) To find the average rate of change from 1994 to 2000, we locate the corresponding points $(1994,1)$ and $(2000,40)$. Using these two points we calculate the average rate of change

$$\frac{40-1}{2000-1994}=\frac{39}{6}=\frac{13}{2}=6.5$$

The average rate of change of employment for professional services from 1994 to 2000 increased approximately 6.5% per year.

b) To find the average rate of change from 2000 to 2005, we locate the corresponding points $(2000,40)$ and $(2005,50)$.

Using these two points we calculate the average rate of change

$$\frac{50-40}{2005-2000}=\frac{10}{5}=2.0$$

The average rate of change of employment for professional services from 2000 to 2005 increased approximately 2.0% per year.

c) To find the average rate of change from 1994 to 2005, we locate the corresponding points $(1994,1)$ and $(2005,50)$. Using these two points we calculate the average rate of change

$$\frac{50-1}{2005-1994}=\frac{49}{11}\approx 4.4545$$

The average rate of change of employment for professional services from 1994 to 2005 increased approximately 4.5% per year.

20. a) Locate the points $(1994,1)$ and $(2000,20)$.

The average rate of change is

$$\frac{20-1}{2000-1994}=\frac{19}{6}\approx 3.167$$

The average rate of change of health care employment from 1994 to 2000 was approximately 3.2% per year.

b) Locate the points $(2000,20)$ and $(2005,40)$.

The average rate of change is

$$\frac{40-20}{2005-2000}=\frac{20}{5}=4$$

The average rate of change of health care employment from 2000 to 2005 was approximately 4% per year.

c) Locate the points $(1994,1)$ and $(2005,40)$.

The average rate of change is

$$\frac{40-1}{2005-1994}=\frac{39}{11}\approx 3.5455$$

The average rate of change of health care employment from 1994 to 2005 was approximately 3.5% per year.

21. a) To find the average rate of change from 1994 to 2000, we locate the corresponding points $(1994,0)$ and $(2000,17)$. Using these two points we calculate the average rate of change

$$\frac{17-0}{2000-1994}=\frac{17}{6}=2\tfrac{5}{6}$$

The average rate of change of education employment from 1994 to 2000 increased approximately $2\tfrac{5}{6}\%$ per year.

b) To find the average rate of change from 2000 to 2005, we locate the corresponding points $(2000,17)$ and $(2005,28)$. Using these two points we calculate the average rate of change

$$\frac{28-17}{2005-2000}=\frac{11}{5}=2\tfrac{1}{5}=2.2$$

The average rate of change of education employment from 2000 to 2005 increased approximately $2\tfrac{1}{5}$, or about 2.2% per year.

c) To find the average rate of change from 1994 to 2005, we locate the corresponding points $(1994,0)$ and $(2005,28)$. Using these two points we calculate the average rate of change

$$\frac{28-0}{2005-1994}=\frac{28}{11}=2\tfrac{6}{11}\approx 2.545455$$

The average rate of change of education employment from 1994 to 2005 increased approximately $2\tfrac{6}{11}\%$, or about 2.5% per year.

22. a) Locate the points $(1994,0)$ and $(2000,9)$.

The average rate of change is

$$\frac{9-0}{2000-1994}=\frac{9}{6}=\frac{3}{2}=1\tfrac{1}{2}$$

The average rate of change of government employment from 1994 to 2000 was approximately $1\tfrac{1}{2}\%$ per year.

b) Locate the points $(2000,9)$ and $(2005,8)$.
The average rate of change is
$$\frac{8-9}{2005-2000}=\frac{-1}{5}=-0.2$$
The average rate of change of government employment from 2000 to 2005 was approximately $-\frac{1}{5}\%$, or -0.2% per year.

c) Locate the points $(1994,0)$ and $(2005,8)$.
The average rate of change is
$$\frac{8-0}{2005-1994}=\frac{8}{11}\approx 0.73$$
The average rate of change of government employment from 1994 to 2005 was approximately $\frac{8}{11}\%$, or about 0.7% per year.

23. a) To find the average rate of change from 1994 to 2000, we locate the corresponding points $(1994,0)$ and $(2000,-10)$.
Using these two points we calculate the average rate of change.
$$\frac{-10-0}{2000-1994}=\frac{-10}{6}=-1\tfrac{2}{3}$$
The average rate of change of natural resources employment from 1994 to 2000 increased approximately $-1\frac{2}{3}\%$ per year.

b) To find the average rate of change from 2000 to 2005, we locate the corresponding points $(2000,-10)$ and $(2005,-4)$. Using these two points we calculate the average rate of change.
$$\frac{-4-(-10)}{2005-2000}=\frac{6}{5}=1\tfrac{1}{5}=1.2$$
The average rate of change of natural resources employment from 2000 to 2005 increased approximately $1\frac{1}{5}$, or about 1.2% per year.

c) To find the average rate of change from 1994 to 2005, we locate the corresponding points $(1994,0)$ and $(2005,-4)$. Using these two points we calculate the average rate of change.
$$\frac{-4-0}{2005-1994}=\frac{-4}{11}\approx -0.363$$
The average rate of change of natural resources employment from 1994 to 2005 increased approximately $-\frac{4}{11}\%$, or about -0.4% per year.

24. a) Locate the points $(1994,1)$ and $(2000,3)$.
The average rate of change is
$$\frac{3-1}{2000-1994}=\frac{2}{6}=\frac{1}{3}$$
The average rate of change of manufacturing employment from 1994 to 2000 was approximately $\frac{1}{3}\%$ per year.

b) Locate the points $(2000,3)$ and $(2005,-15)$.
The average rate of change is
$$\frac{-15-(3)}{2005-2000}=\frac{-18}{5}=-3\tfrac{3}{5}=-3.6$$
The average rate of change of manufacturing employment from 2000 to 2005 was approximately $-3\frac{3}{5}\%$, or -3.6% per year.

c) Locate the points $(1994,1)$ and $(2005,-15)$.
The average rate of change is
$$\frac{-15-1}{2005-1994}=\frac{-16}{11}=-1\tfrac{5}{11}\approx -1.455$$
The average rate of change of manufacturing employment from 1994 to 2005 was approximately $-1\frac{5}{11}\%$, or about -1.5% per year.

25. In order to find the average rate of change from 1960 to 1970, we use the data points $(1960,45.09)$ and $(1970,67.84)$. The average rate of change is
$$\frac{67.84-45.09}{1970-1960}=\frac{22.75}{10}=2.275\approx 2.3.$$
Between the years 1960 to 1970 the average rate of change in U.S. energy consumption increased about 2.3 quadrillion BTUs per year.

In order to find the average rate of change from 1980 to 1990, we use the data points $(1980,78.29)$ and $(1990,84.67)$. The average rate of change is
$$\frac{84.67-78.29}{1990-1980}=\frac{6.38}{10}=0.638\approx 0.6.$$
Between the years 1980 to 1990 the average rate of change in U.S. energy consumption increased about 0.6 quadrillion BTUs per year.

In order to find the average rate of change from 1990 to 2005, we use the data points $(1990, 84.67)$ and $(2005, 101.85)$. The average rate of change is
$\frac{101.85-84.67}{2005-1990}=\frac{17.18}{15}=1.1453333\approx 1.1$.
Between the years 1990 to 205 the average rate of change in U.S. energy consumption increased about 1.1 quadrillion BTUs per year.

26. Using the points $(1990, 41.1)$ and $(1995, 59.1)$ the average rate of change is
$\frac{59.1-41.1}{1995-1990}=\frac{18}{5}=3.6$
Between 1990 and 1995, the average rate of change of the U.S. trade deficit with Japan was 3.6 billion dollars per year.

Using the points $(1995, 59.1)$ and $(2000, 81.6)$ the average rate of change is
$\frac{81.6-59.1}{2000-1995}=\frac{22.5}{5}=4.5$
Between 1995 and 2000, the average rate of change of the U.S. trade deficit with Japan was 4.5 billion dollars per year.

Using the points $(2000, 81.6)$ and $(2004, 75.2)$ the average rate of change is
$\frac{75.2-81.6}{2004-2000}=\frac{-6.4}{4}=-1.6$
Between 2000 and 2004, the average rate of change of the U.S. trade deficit with Japan was -1.6 billion dollars per year.

27. a) From 0 units to 1 unit the average rate of change is
$\frac{70-0}{1-0}=70$ pleasure units per unit.
From 1 unit to 2 units the average rate of change is
$\frac{109-70}{2-1}=\frac{39}{1}=39$ pleasure units per unit.
From 2 units to 3 units the average rate of change is
$\frac{138-109}{3-2}=\frac{29}{1}=29$ pleasure units per unit.
From 3 units to 4 units the average rate of change is
$\frac{161-138}{4-3}=\frac{23}{1}=23$ pleasure units per unit.

b) $\boxed{tw}$ Answers will vary. As you consume more and more of a good, the additional utility, or amount of pleasure, associated with that good will start to fall. The additional satisfaction from the 1st to the 2nd slice of pizza is greater than the additional satisfaction from the 9th to the 10th slice of pizza.

28. a) $N(0)=0,\ N(1)=300$
$\frac{300-0}{1-0}=300$ units per thousand dollars.
$N(1)=300,\ N(2)=480$
$\frac{480-300}{2-1}=180$ units per thousand dollars.
$N(2)=480,\ N(3)=600$
$\frac{600-480}{3-2}=120$ units per thousand dollars.
$N(3)=600,\ N(4)=700$
$\frac{700-600}{4-3}=100$ units per thousand dollars.

b) $\boxed{tw}$ As spending on advertising increases, there are few increases in sales from each additional amount of spending.

29. $p(x)=0.03x^2+0.56x+8.63$

a) $p(4)=0.03(4)^2+0.56(4)+8.63=11.35$

b) $p(14)=0.03(14)^2+0.56(14)+8.63=22.35$

c) $P(14)-p(4)=22.35-11.35=11$

d) $\frac{p(14)-p(4)}{14-4}=\frac{11}{10}=1.1$
This result implies that the average price of a ticket between 1995 (x =4) and 2005 (x =14) grew at an average rate of \$1.10 per year.

30. $A(t)=2000(1.015)^{4t}$

a) $A(3)=2000(1.015)^{4(3)}=2391.236343$

b) $A(5)=2000(1.015)^{4(5)}=2693.710013$

c) $A(5)-A(3)=2693.710013-2391.236343$
$=302.4736702$

d) $\frac{A(5)-A(3)}{5-3}=\frac{302.4736702}{2}=151.236835$
The value of the account grew at a rate of \$151.24 per year between the 3rd and 5th year of the investment.

31. $A(t)=5000(1.14)^t$

$A(3)=5000(1.14)^3=7407.72$

$A(2)=5000(1.14)^2=6498.00$

$$\frac{A(3)-A(2)}{3-2}=\frac{7407.72-6498.00}{3-2}=909.72$$

If unpaid, the debt on the credit card will be growing at an average rate of \$909.72 per year between the 2nd and 3rd year.

32. $A(t)=3000(1.17)^t$

$A(4)=3000(1.17)^4=5621.66163$

$A(3)=3000(1.17)^3=4804.839$

$$\frac{A(4)-A(3)}{4-3}=\frac{5621.66-4804.84}{4-3}\approx 816.82$$

If unpaid, the debt on the credit card will be growing at an average rate of \$816.82 per year between the 3rd and 4th year.

33. $C(x)=-0.05x^2+50x$

First substitute 301 for *x*.

$$\begin{aligned}C(301)&=-0.05(301)^2+50(301)\\&=-4530.05+15,050\\&=10,519.95\end{aligned}$$

The total cost of producing 301 units is \$10,519.95.

Next substitute 300 for *x*.

$$\begin{aligned}C(300)&=-0.05(300)^2+50(300)\\&=-4500+15,000\\&=10,500.00\end{aligned}$$

The total cost of producing 300 units is \$10,500.00.

Now we can substitute to find the average rate of change.

$$\begin{aligned}\frac{C(301)-C(300)}{301-300}&=\frac{10,519.95-10,500}{301-300}\\&=\frac{19.95}{1}\\&=19.95\end{aligned}$$

Total cost will increase \$19.95 if the company produces the 301st unit.

34. $R(x)=-0.01x^2+1000x$

$$\begin{aligned}R(301)&=-0.01(301)^2+1000(301)\\&=300,093.99\end{aligned}$$

$$\begin{aligned}R(300)&=-0.01(300)^2+1000(300)\\&=299,100.00\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{R(301)-R(300)}{301-300}&=\frac{300,093.99-299,100}{1}\\&=993.99\end{aligned}$$

The average increase in revenue from selling the 301st unit is \$993.99.

35. Note: Answers will vary according to the values estimated from the graph.

a) Locate the points $(0,8)$ and $(12,20)$ on the girls growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{20-8}{12-0}=\frac{12}{12}=1.$$

The average growth rate of a girl during her first 12 months is 1 pound per month.

b) Locate the points $(12,20)$ and $(24,26.5)$ on the girls growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{26.5-20}{24-12}=\frac{6.5}{12}=0.541\overline{6}\approx 0.54.$$

The average growth rate of a girl during her second 12 months is approximately 0.54 pounds per month.

c) Locate the points $(0,8)$ and $(24,26.5)$ on the girls growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{26.5-8}{24-0}=\frac{18.5}{24}=0.7708\overline{3}\approx 0.77.$$

The average growth rate of a girl during her first 24 months is approximately 0.77 pounds per month.

d) We estimate the growth rate of a 12 month old girl to be approximately 0.67 pounds per month. This answer will vary depending upon your tangent line.

e) The graph indicates that the growth rate is fastest during the first 5 months.

36. Note: Answers will vary according to the values estimated from the graph.

a) Locate the points $(0,8)$ and $(15,25)$ on the boys growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{25-8}{15-0}=\frac{17}{15}=1.13\overline{3}\approx 1.13 .$$

The average growth rate of a boy during his first 15 months is 1.1 pound per month.

b) Locate the points $(15,25)$ and $(30,30)$ on the boys growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{30-25}{30-15}=\frac{5}{15}=0.33\overline{3}\approx 0.3 .$$

The average growth rate of a boy during his second 15 months is approximately 0.3 pounds per month.

c) Locate the points $(0,8)$ and $(30,30)$ on the boys growth median weight chart. Using these points we calculate the average growth rate.

$$\frac{30-8}{30-0}=\frac{22}{30}=0.733\overline{3}\approx 0.7 .$$

The average growth rate of a boy during his first 30 months is approximately 0.7 pounds per month.

d) $\boxed{tw}$ We estimate the growth rate of a 15 month old boy to be approximately 0.5 pounds per month. We used a straight edge to draw a line tangent to the curve at the point $(15,25)$ and found the slope of that line. This answer will vary depending upon your tangent line.

37. $H(w)=0.11w^{1.36}$

a) First we substitute 500 and 700 in for w to find the home range at the respective weights.

$$\begin{aligned} H(500) &= 0.11(500)^{1.36} \\ &= 0.11(4683.809314) \\ &= 515.2190246 \\ &\approx 515.22 \end{aligned}$$

$$\begin{aligned} H(700) &= 0.11(700)^{1.36} \\ &= 0.11(7401.731628) \\ &= 814.1904791 \\ &\approx 814.19 \end{aligned}$$

Next we use the function values to find the average rate at which the mammal's home range will increase.

$$\begin{aligned} \frac{H(700)-H(500)}{700-500} &= \frac{814.19-515.22}{700-500} \\ &= \frac{298.97}{200} \\ &\approx 1.49485 \end{aligned}$$

The average rate at which a carnivorous mammal's home range increases as the animal's weight grows from 500 g to 700 g is approximately 1.49 hectares per gram.

b) First we substitute 200 and 300 in for w to find the home range at the respective weights.

$$\begin{aligned} H(200) &= 0.11(200)^{1.36} \\ &= 0.11(1347.102971) \\ &= 148.1813269 \\ &\approx 148.18 \end{aligned}$$

$$\begin{aligned} H(300) &= 0.11(300)^{1.36} \\ &= 0.11(2338.217499) \\ &= 257.2039249 \\ &\approx 257.20 \end{aligned}$$

Next we use the function values to find the average rate at which the mammal's home range will increase

$$\begin{aligned} \frac{H(300)-H(200)}{300-200} &= \frac{257.20-148.18}{300-200} \\ &= \frac{109.02}{100} \\ &\approx 1.0902 \end{aligned}$$

The average rate at which a carnivorous mammal's home range increases as the animal's weight grows from 200 g to 300 g is approximately 1.09 hectares per gram.

38. $R(x)=11.74x^{1/4}$

a) First, find the function values.

$$R(40{,}000)=11.74(40{,}000)^{1/4}\approx 166.03$$

$$R(60{,}000)=11.74(60{,}000)^{1/4}\approx 183.74$$

Next find the average rate of change:

$$\begin{aligned} \frac{R(60{,}000)-R(40{,}000)}{60{,}000-40{,}000} &\approx \frac{183.74-166.03}{60{,}000-40{,}000} \\ &\approx 0.00089 \end{aligned}$$

The rate at which the maximum radar range changes as peak power increases from 40,000 W to 60,000 W is approximately 0.00089 miles per watt.

b) First, find the function values.
$R(50{,}000) = 11.74(50{,}000)^{1/4} \approx 175.55$
$R(60{,}000) = 11.74(60{,}000)^{1/4} \approx 183.74$
Next find the average rate of change:

$$\frac{R(60{,}000) - R(50{,}000)}{60{,}000 - 50{,}000} \approx \frac{183.74 - 175.55}{60{,}000 - 50{,}000} \approx \frac{8.19}{10{,}000} \approx 0.00082$$

The rate at which the maximum radar range changes as peak power increases from 50,000 W to 60,000 W is approximately 0.00082 miles per watt.

39. a) We locate the points $(0,0)$ and $(8,10)$ on the graph and use them to calculate the average rate of change.

$$\frac{10-0}{8-0} = \frac{10}{8} = \frac{5}{4} = 1.25$$

The average rate of change is 1.25 words per minute.

We locate the points $(8,10)$ and $(16,20)$ on the graph and use them to calculate the average rate of change.

$$\frac{20-10}{16-8} = \frac{10}{8} = \frac{5}{4} = 1.25$$

The average rate of change is 1.25 words per minute.

We locate the points $(16,20)$ and $(24,25)$ on the graph and use them to calculate the average rate of change.

$$\frac{25-20}{24-16} = \frac{5}{8} = 0.625$$

The average rate of change is 0.625 words per minute.

We locate the points $(24,25)$ and $(32,25)$ on the graph and use them to calculate the average rate of change.

$$\frac{25-25}{32-24} = \frac{0}{8} = 0$$

The average rate of change is 0 words per minute.

We locate the points $(32,25)$ and $(36,25)$ on the graph and use them to calculate the average rate of change.

$$\frac{25-25}{36-32} = \frac{0}{4} = 0$$

The average rate of change is 0 words per minute.

b) $\boxed{tw}$ Answers will vary. The person has reached a saturation point after 24 minutes. They cannot memorize any more words.

40. $s(t) = 10t^2$

a) First, find the function values.
$s(2) = 10(2)^2 = 40$
$s(5) = 10(5)^5 = 250$
Therefore,
$s(5) - s(2) = 250 - 40 = 210$.
This represents the distance the truck traveled, 210 miles, in the three hour period from $t = 2$ and $t = 5$.

b) We have:

$$\frac{s(5) - s(2)}{5-2} = \frac{250-40}{5-2} = \frac{210}{3} = 70$$

The average velocity of the truck for the three hour period between $t = 2$ and $t = 5$ was 70 miles per hour.

41. $s(t) = 16t^2$

a) First, we find the function values by substituting 3 and 5 in for *t* respectively.
$s(3) = 16(3)^2 = 16(9) = 144$
$s(5) = 16(5)^2 = 16(25) = 400$
Next we subtract the function values.
$s(5) - s(3) = 400 - 144 = 256$
The object will fall 256 feet in the two second time period between $t = 3$ and $t = 5$.

b) The average rate of change is calculated as

$$\frac{s(5) - s(3)}{5-3} = \frac{400-144}{5-3} = \frac{256}{2} = 128$$

The average velocity of the object during the two second time period from $t = 3$ to $t = 5$ is 128 feet per second.

42. a) We use the two points $(0, 30{,}680)$ and $(13.5, 31{,}077)$.

$$\frac{31{,}077 - 30{,}680}{13.5 - 0} = \frac{397}{13.5} \approx 29.4$$

The car averaged 29.4 miles per gallon on the trip.

b) To find the average rate of gas consumption in gallons per mile, we take the reciprocal of part (a).

$$\frac{13.5-0}{31{,}077-30{,}680}=\frac{13.5}{397}\approx 0.034$$

The car consumed approximately 0.034 gallons per mile on the trip.

43. a) For each curve, as t changes from 0 to 4, $P(t)$ changes from 0 to 500. Thus, the average growth rate for each country is

$$\frac{500-0}{4-0}=\frac{500}{4}=125.$$

The average growth rate for each country is approximately 125 million people per year.

b) $\boxed{tw}$ No; and average rate of change does not give an indication of the growth patterns of the two populations during the 4 years for which it was calculated.

c) For Country A:

As t changes from 0 to 1, $P(t)$ changes from 0 to 290. Thus the average growth rate is

$$\frac{290-0}{1-0}=290 \text{ million people per year.}$$

As t changes from 1 to 2, $P(t)$ changes from 290 to 250. Thus the average growth rate is

$$\frac{250-290}{2-1}=-40 \text{ million people per year.}$$

As t changes from 2 to 3, $P(t)$ changes from 250 to 200. Thus the average growth rate is

$$\frac{200-250}{3-2}=-50 \text{ million people per year.}$$

As t changes from 3 to 4, $P(t)$ changes from 200 to 500. Thus the average growth rate is

$$\frac{500-200}{4-3}=300 \text{ million people per year.}$$

For Country B:

As t changes from 0 to 1, $P(t)$ changes from 0 to 125. Thus the average growth rate is

$$\frac{125-0}{1-0}=125 \text{ million people per year.}$$

As t changes from 1 to 2, $P(t)$ changes from 125 to 250. Thus the average growth rate is

$$\frac{250-125}{2-1}=125 \text{ million people per year.}$$

As t changes from 2 to 3, $P(t)$ changes from 250 to 375. Thus the average growth rate is

$$\frac{375-250}{3-2}=125 \text{ million people per year.}$$

As t changes from 3 to 4, $P(t)$ changes from 375 to 500. Thus the average growth rate is

$$\frac{500-375}{4-3}=125 \text{ million people per year.}$$

d) $\boxed{tw}$ The statement most accurately reflects the information found in Country B. Country B shows linear growth, which implies a constant rate of change.

44. $\boxed{tw}$ Answers will vary. The first graph shows a constant average rate of change while the second graph has average rate of change that varies on an increasing trend. The first graph shows a decreasing rate of change, while the second graph shows a varying rate of change.

45. a) Tracing along the 4-year private school graph, we see that the largest increase in costs occurred in the 1985-86 year.

b) Tracing along the 4-year public school graph, we see that the largest increases in costs occurred during the 1975-76 year, the 2003-04 year and the 2004-05 year.

c) For the 4-year public school, the cost in 2005 dollars is approximately \$6000. To find out what the cost was in 1975 dollars assuming a 3% inflation rate over the 30 years we create the following equation using the simple compound interest formula from section R1 $C(1+r)^t=A$.

$$C(1+.03)^{30}=6000$$
$$C(1.03)^{30}=6000$$
$$C=\frac{6000}{(1.03)^{30}}$$
$$C=2471.92$$
$$C\approx 2472$$

The cost of attending a 4 year public school in 1975 was \$2472 in 1975 dollars. Likewise, for the 4-year private school, the cost in 2005 dollars is approximately \$13,000.

To find out what the cost was in 1975 dollars assuming a 3% inflation rate over the 30 years we create the following equation using the simple compound interest formula from section R1 $C(1+r)^t = A$.

$$C(1+.03)^{30} = 13{,}000$$
$$C(1.03)^{30} = 13{,}000$$
$$C = \frac{13{,}000}{(1.03)^{30}}$$
$$C = 5355.83$$
$$C \approx 5356$$

The cost of attending a 4 year private school in 1975 was \$5356 in 1975 dollars.

46. $f(x) = mx + b$

$$\frac{f(x+h)-f(x)}{h} = \frac{(m(x+h)+b)-(mx+b)}{h}$$
$$= \frac{mx+mh+b-mx-b}{h}$$
$$= \frac{mh}{h}$$
$$= m$$

47. $f(x) = ax^2 + bx + c$

Substituting $x+h$ for x we have,

$$f(x+h) = a(x+h)^2 + b(x+h) + c$$
$$= a(x^2 + 2xh + h^2) + bx + bh + c$$
$$= ax^2 + 2axh + ah^2 + bx + bh + c$$

Thus,

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$
$$= \frac{(ax^2 + 2axh + ah^2 + bx + bh + c)-(ax^2 + bx + c)}{h}$$
$$= \frac{2axh + ah^2 + bh}{h}$$
$$= \frac{h(2ax + ah + b)}{h} \quad \text{Factoring the numerator}$$
$$= 2ax + ah + b \quad \text{Simplified difference quotient}$$

48. $f(x) = ax^3 + bx^2$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{a(x+h)^3 + b(x+h)^2 - (ax^3 + bx^2)}{h}$$
$$= \frac{a(x^3 + 3x^2h + 3xh^2 + h^3) + b(x^2 + 2hx + h^2) - ax^3 - bx^2}{h}$$
$$= \frac{ax^3 + 3ax^2h + 3axh^2 + ah^3 + bx^2 + 2bhx + bh^2 - ax^3 - bx^2}{h}$$
$$= \frac{3ax^2h + 3axh^2 + ah^3 + 2bhx + bh^2}{h}$$
$$= \frac{h(3ax^2 + 3axh + ah^2 + 2bx + bh)}{h}$$
$$= 3ax^2 + 3axh + ah^2 + 2bx + bh$$

49. $f(x) = \sqrt{x}$

Substituting $x+h$ for x we have,

$$f(x+h) = \sqrt{x+h}$$

Thus,

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$
$$= \frac{\sqrt{x+h}-\sqrt{x}}{h}$$
$$= \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \quad \text{multiplying by 1}$$
$$= \frac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$
$$= \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$
$$= \frac{1}{\sqrt{x+h}+\sqrt{x}} \quad \text{Simplified difference quotient}$$

50. $f(x)=x^3+x$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{(x+h)^3+(x+h)-(x^3+x)}{h}$$

$$=\frac{x^3+3x^2h+3xh^2+h^3+x+h-x^3-x}{h}$$

$$=\frac{3x^2h+3xh^2+h^3+h}{h}$$

$$=\frac{h(3x^2+3xh+h^2+1)}{h}$$

$$=3x^2+3xh+h^2+1$$

51. $f(x)=\frac{1}{x^2}$

Substituting $x+h$ for x we have,

$$f(x+h)=\frac{1}{(x+h)^2}$$

Thus,

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$

$$=\frac{\left(\frac{1}{(x+h)^2}\right)-\left(\frac{1}{x^2}\right)}{h}$$

Next, we find a common denominator in the numerator.

$$=\frac{\left(\frac{1}{(x+h)^2}\cdot\frac{x^2}{x^2}\right)-\left(\frac{1}{x^2}\cdot\frac{(x+h)^2}{(x+h)^2}\right)}{h}$$

$$=\frac{\frac{x^2}{x^2(x+h)^2}-\frac{x^2+2xh+h^2}{x^2(x+h)^2}}{h}$$

$$=\frac{\frac{-2xh-h^2}{x^2(x+h)^2}}{h}$$

$$=\frac{-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}$$

$$=\frac{h(-2x-h)}{x^2(x+h)^2}\cdot\frac{1}{h}$$

$$=\frac{-2x-h}{x^2(x+h)^2} \quad \text{Simplified difference quotient}$$

52. $f(x)=\frac{1}{1-x}$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{\frac{1}{1-(x+h)}-\left(\frac{1}{1-x}\right)}{h}$$

$$=\frac{\frac{1}{1-x-h}\cdot\frac{1-x}{1-x}-\left(\frac{1}{1-x}\cdot\frac{1-x-h}{1-x-h}\right)}{h}$$

$$=\frac{\frac{1-x}{(1-x-h)(1-x)}-\frac{1-x-h}{(1-x-h)(1-x)}}{h}$$

$$=\frac{\frac{h}{(1-x-h)(1-x)}}{h}$$

$$=\frac{h}{(1-x-h)(1-x)}\cdot\frac{1}{h}$$

$$=\frac{1}{(1-x-h)(1-x)}$$

53. $f(x)=\frac{x}{1+x}$

Substituting $x+h$ for x, we have

$$f(x+h)=\frac{x+h}{1+(x+h)}.$$

Thus,

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$

$$=\frac{\frac{x+h}{1+(x+h)}-\left(\frac{x}{1+x}\right)}{h}$$

Next, we find a common denominator for the numerator.

$$=\frac{\frac{x+h}{1+x+h}\cdot\frac{1+x}{1+x}-\left(\frac{x}{1+x}\cdot\frac{1+x+h}{1+x+h}\right)}{h}$$

$$=\frac{\frac{(x+h)(1+x)}{(1+x+h)(1+x)}-\frac{x(1+x+h)}{(1+x+h)(1+x)}}{h}$$

$$=\frac{\frac{x^2+x+h+hx}{(1+x+h)(1+x)}-\frac{x+x^2+hx}{(1+x+h)(1+x)}}{h}$$

Next, we simplify the difference quotient at the top of the next page.

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{h}{(1+x+h)(1+x)}}{h}$$
$$=\frac{h}{(1+x+h)(1+x)}\cdot\frac{1}{h}$$
$$=\frac{1}{(1+x+h)(1+x)}$$

54. $f(x)=\sqrt{3-2x}$

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$
$$=\frac{\sqrt{3-2(x+h)}-\sqrt{3-2x}}{h}$$
$$=\frac{\sqrt{3-2(x+h)}-\sqrt{3-2x}}{h}\cdot\frac{\sqrt{3-2(x+h)}+\sqrt{3-2x}}{\sqrt{3-2(x+h)}+\sqrt{3-2x}}$$
$$=\frac{3-2(x+h)-(3-2x)}{h\left(\sqrt{3-2(x+h)}+\sqrt{3-2x}\right)}$$
$$=\frac{3-2x-2h-3+2x}{h\left(\sqrt{3-2(x+h)}+\sqrt{3-2x}\right)}$$
$$=\frac{-2h}{h\left(\sqrt{3-2(x+h)}+\sqrt{3-2x}\right)}$$
$$=\frac{-2}{\sqrt{3-2(x+h)}+\sqrt{3-2x}}$$

55. $f(x)=\dfrac{1}{\sqrt{x}}$

Substituting $x+h$ for x we have,

$$f(x+h)=\frac{1}{\sqrt{x+h}}$$

Thus,

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$
$$=\frac{\left(\frac{1}{\sqrt{x+h}}\right)-\left(\frac{1}{\sqrt{x}}\right)}{h}$$

Next, we find a common denominator in the numerator.

$$=\frac{\left(\frac{1}{\sqrt{x+h}}\cdot\frac{\sqrt{x}}{\sqrt{x}}\right)-\left(\frac{1}{\sqrt{x}}\cdot\frac{\sqrt{x+h}}{\sqrt{x+h}}\right)}{h}$$
$$=\frac{\frac{\sqrt{x}}{\sqrt{x}\sqrt{x+h}}-\frac{\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$$
$$=\frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$$
$$=\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\cdot\frac{1}{h} \quad \text{Simplifying the complex fraction}$$
$$=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

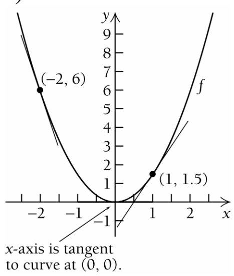
Next, we rationalize the numerator.

$$=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$$
$$=\frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$
$$=\frac{-h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$
$$=\frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

Exercise Set 1.4

1. $f(x)=\frac{3}{2}x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{3}{2}(x+h)^2-\frac{3}{2}x^2}{h}$$
$$=\frac{\frac{3}{2}\left(x^2+2xh+h^2\right)-\frac{3}{2}x^2}{h}$$
$$=\frac{\frac{3}{2}x^2+3xh+\frac{3}{2}h^2-\frac{3}{2}x^2}{h}$$
$$=\frac{3xh+\frac{3}{2}h^2}{h}$$
$$=\frac{h\left(3x+\frac{3}{2}h\right)}{h}$$
$$=3x+\frac{3}{2}h \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(3x+\frac{3}{2}h\right)$$
$$=3x$$

Thus, $f'(x)=3x$.

d) Find the values of the derivative by making the appropriate substitutions.

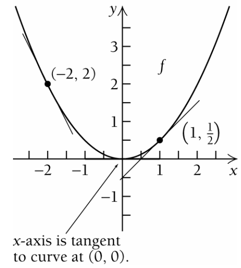
$f'(-2)=3(-2)=-6$ Substituting -2 for x

$f'(0)=3(0)=0$ Substituting 0 for x

$f'(1)=3(1)=3$ Substituting 1 for x

2. $f(x)=\frac{1}{2}x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{1}{2}(x+h)^2-\frac{1}{2}x^2}{h}$$
$$=\frac{\frac{1}{2}\left(x^2+2xh+h^2\right)-\frac{1}{2}x^2}{h}$$
$$=\frac{\frac{1}{2}x^2+xh+\frac{1}{2}h^2-\frac{1}{2}x^2}{h}$$
$$=\frac{xh+\frac{1}{2}h^2}{h}$$
$$=\frac{h\left(x+\frac{1}{2}h\right)}{h}$$
$$=x+\frac{1}{2}h \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(x+\frac{1}{2}h\right)$$
$$=x$$

Thus, $f'(x)=x$.

d) Find the values of the derivative by making the appropriate substitutions.

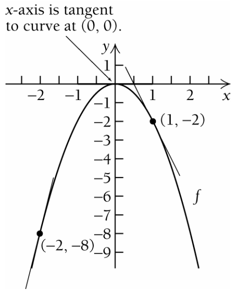
$f'(-2)=(-2)=-2$ Substituting -2 for x

$f'(0)=(0)=0$ Substituting 0 for x

$f'(1)=(1)=1$ Substituting 1 for x

3. $f(x) = -2x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{-2(x+h)^2 - (-2x^2)}{h}$$
$$= \frac{-2(x^2+2xh+h^2) - (-2x^2)}{h}$$
$$= \frac{-2x^2 - 4xh - 2h^2 + 2x^2}{h}$$
$$= \frac{-4xh - 2h^2}{h}$$
$$= \frac{h(-4x-2h)}{h}$$
$= -4x - 2h$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(-4x-2h) = -4x$$

Thus, $f'(x) = -4x$.

d) Find the values of the derivative by making the appropriate substitutions.

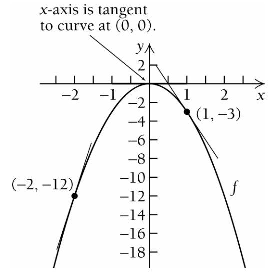
$f'(-2) = -4(-2) = 8$ Substituting -2 for x

$f'(0) = -4(0) = 0$ Substituting 0 for x

$f'(1) = -4(1) = -4$ Substituting 1 for x

4. $f(x) = -3x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{-3(x+h)^2 - (-3x^2)}{h}$$
$$= \frac{-3(x^2+2xh+h^2) - (-3x^2)}{h}$$
$$= \frac{-3x^2 - 6xh - 3h^2 + 3x^2}{h}$$
$$= \frac{-6xh - 3h^2}{h}$$
$$= \frac{h(-6x-3h)}{h}$$
$= -6x - 3h$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(-6x-3h) = -6x$$

Thus, $f'(x) = -6x$.

d) Find the values of the derivative by making the appropriate substitutions.

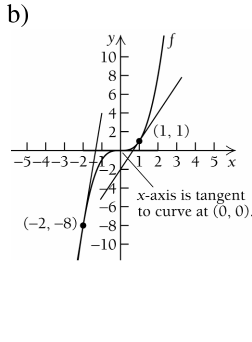
$f'(-2) = -6(-2) = 12$ Substituting -2 for x

$f'(0) = -6(0) = 0$ Substituting 0 for x

$f'(1) = -6(1) = -6$ Substituting 1 for x

5. $f(x) = x^3$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{(x+h)^3-(x^3)}{h}$$
$$=\frac{(x^3+3x^2h+3xh^2+h^3)-(x^3)}{h}$$
$$=\frac{3x^2h+3xh^2+h^3}{h}$$
$$=\frac{h(3x^2+3xh+h^2)}{h}$$
$$=3x^2+3xh+h^2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(3x^2+3xh+h^2)$$
$$=3x^2$$

Thus, $f'(x)=3x^2$.

d) Find the values of the derivative by making the appropriate substitutions.

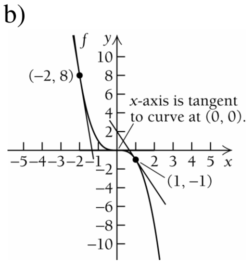
$f'(-2)=3(-2)^2=12$ Substituting -2 for x

$f'(0)=3(0)^2=0$ Substituting 0 for x

$f'(1)=3(1)^2=3$ Substituting 1 for x

6. $f(x)=-x^3$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{-(x+h)^3-(-x^3)}{h}$$
$$=\frac{-(x^3+3x^2h+3xh^2+h^3)-(-x^3)}{h}$$
$$=\frac{-x^3-3x^2h-3xh^2-h^3+x^3}{h}$$
$$=\frac{-3x^2h-3xh^2-h^3}{h}$$
$$=\frac{h(-3x^2-3xh-h^2)}{h}$$
$$=-3x^2-3xh-h^2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(-3x^2-3xh-h^2)$$
$$=-3x^2$$

Thus, $f'(x)=-3x^2$.

d) Find the values of the derivative by making the appropriate substitutions.

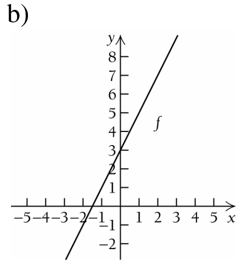
$f'(-2)=-3(-2)^2=-12$ Substituting -2 for x

$f'(0)=-3(0)^2=0$ Substituting 0 for x

$f'(1)=-3(1)^2=-3$ Substituting 1 for x

7. $f(x)=2x+3$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{2(x+h)+3-(2x+3)}{h}$$
$$=\frac{2x+2h+3-2x-3}{h}$$
$$=\frac{2h}{h}$$
$$=2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(2)$$
$$=2$$

Thus, $f'(x)=2$.

d) Since the derivative is a constant, the value of the derivative will be 2 regardless of the value of *x*.

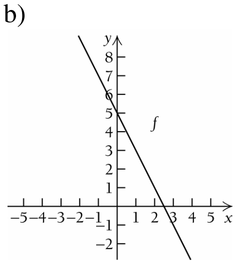
$f'(-2)=2$ Substituting -2 for *x*

$f'(0)=2$ Substituting 0 for *x*

$f'(1)=2$ Substituting 1 for *x*

8. $f(x)=-2x+5$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{-2(x+h)+5-(-2x+5)}{h}$$
$$=\frac{-2x-2h+5+2x-5}{h}$$
$$=\frac{-2h}{h}$$
$$=-2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(-2)$$
$$=-2$$

Thus, $f'(x)=-2$.

d) Since the derivative is a constant, the value of the derivative will be -2 regardless of the value of *x*.

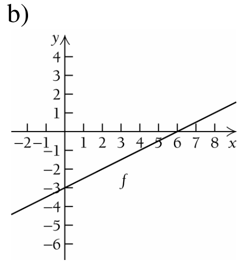
$f'(-2)=-2$ Substituting -2 for *x*

$f'(0)=-2$ Substituting 0 for *x*

$f'(1)=-2$ Substituting 1 for *x*

9. $f(x)=\frac{1}{2}x-3$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{1}{2}(x+h)-3-\left(\frac{1}{2}x-3\right)}{h}$$
$$=\frac{\frac{1}{2}x+\frac{1}{2}h-3-\frac{1}{2}x+3}{h}$$
$$=\frac{\frac{1}{2}h}{h}$$
$$=\frac{1}{2} \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(\frac{1}{2}\right)$$
$$=\frac{1}{2}$$

Thus, $f'(x)=\frac{1}{2}$.

d) Since the derivative is a constant, the value of the derivative will be $\frac{1}{2}$ regardless of the value of x.

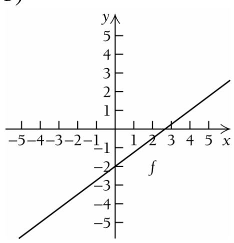
$f'(-2)=\frac{1}{2}$ Substituting -2 for x

$f'(0)=\frac{1}{2}$ Substituting 0 for x

$f'(1)=\frac{1}{2}$ Substituting 1 for x

10. $f(x)=\frac{3}{4}x-2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{\frac{3}{4}(x+h)-2-\left(\frac{3}{4}x-2\right)}{h}$$

$$=\frac{\frac{3}{4}x+\frac{3}{4}h-2-\frac{3}{4}x+2}{h}$$

$$=\frac{\frac{3}{4}h}{h}$$

$$=\frac{3}{4} \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(\frac{3}{4}\right)$$

$$=\frac{3}{4}$$

Thus, $f'(x)=\frac{3}{4}$.

d) Since the derivative is a constant, the value of the derivative will be $\frac{3}{4}$ regardless of the value of x.

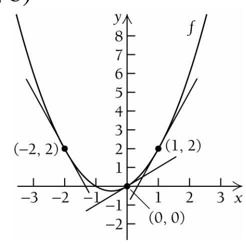
$f'(-2)=\frac{3}{4}$ Substituting -2 for x

$f'(0)=\frac{3}{4}$ Substituting 0 for x

$f'(1)=\frac{3}{4}$ Substituting 1 for x

11. $f(x)=x^2+x$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{(x+h)^2+(x+h)-(x^2+x)}{h}$$

$$=\frac{x^2+2xh+h^2+x+h-x^2-x}{h}$$

$$=\frac{2xh+h^2+h}{h}$$

$$=\frac{h(2x+h+1)}{h}$$

$$=2x+h+1 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(2x+h+1)$$

$$=2x+1$$

Thus, $f'(x)=2x+1$.

d) Find the values of the derivative by making the appropriate substitutions.

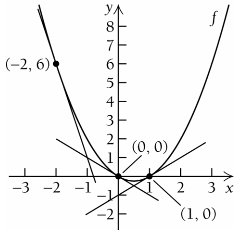
$f'(-2)=2(-2)+1=-3$

$f'(0)=2(0)+1=1$

$f'(1)=2(1)+1=3$

12. $f(x) = x^2 - x$

a), b)



c) Find the simplified difference quotient first. Referring to Exercise Set 1.3, Exercise 6 we know the simplified difference quotient is:

$$\frac{f(x+h)-f(x)}{h} = 2x+h-1$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(2x+h-1)$$
$$= 2x-1$$

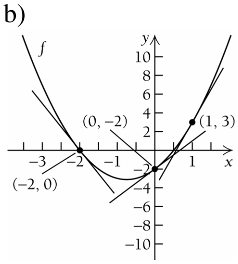
Thus, $f'(x) = 2x-1$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2) = 2(-2)-1 = -5$$
$$f'(0) = 2(0)-1 = -1$$
$$f'(1) = 2(1)-1 = 1$$

13. $f(x) = 2x^2 + 3x - 2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{\left(2(x+h)^2+3(x+h)-2\right)-\left(2x^2+3x-2\right)}{h}$$
$$= \frac{2\left(x^2+2xh+h^2\right)+3x+3h-2-2x^2-3x+2}{h}$$
$$= \frac{2x^2+4xh+2h^2+3x+3h-2-2x^2-3x+2}{h}$$
$$= \frac{4xh+2h^2+3h}{h}$$
$$= \frac{h(4x+2h+3)}{h}$$
$$= 4x+2h+3 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(4x+2h+3)$$
$$= 4x+3$$

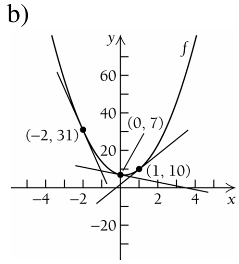
Thus, $f'(x) = 4x+3$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2) = 4(-2)+3 = -5$$
$$f'(0) = 4(0)+3 = 3$$
$$f'(1) = 4(1)+3 = 7$$

14. $f(x) = 5x^2 - 2x + 7$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(5(x+h)^2-2(x+h)+7\right)-\left(5x^2-2x+7\right)}{h}$$
$$=\frac{5\left(x^2+2xh+h^2\right)-2x-2h+7-5x^2+2x-7}{h}$$
$$=\frac{5x^2+10xh+5h^2-2x-2h+7-5x^2+2x-7}{h}$$
$$=\frac{10xh+5h^2-2h}{h}$$
$$=\frac{h(10x+5h-2)}{h}$$
$$=10x+5h-2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(10x+5h-2)$$
$$=10x-2$$

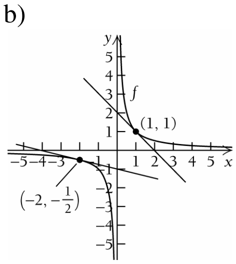
Thus, $f'(x)=10x-2$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2)=10(-2)-2=-22$$
$$f'(0)=10(0)-2=-2$$
$$f'(1)=10(1)-2=8$$

15. $f(x)=\dfrac{1}{x}$

a), b)



There is no tangent line for $x=0$

c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(\frac{1}{x+h}\right)-\left(\frac{1}{x}\right)}{h}$$
$$=\frac{\left(\frac{1}{x+h}\cdot\frac{x}{x}\right)-\left(\frac{1}{x}\cdot\frac{(x+h)}{(x+h)}\right)}{h}$$
$$=\frac{\left(\frac{x}{x(x+h)}\right)-\left(\frac{(x+h)}{x(x+h)}\right)}{h}$$
$$=\frac{\frac{-h}{x(x+h)}}{h}$$
$$=\frac{-h}{x(x+h)}\cdot\frac{1}{h}$$
$$=\frac{-1}{x(x+h)} \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(\frac{-1}{x(x+h)}\right)$$
$$=\frac{-1}{x(x+0)}$$
$$=\frac{-1}{x^2}$$

Thus, $f'(x)=\dfrac{-1}{x^2}$.

d) Find the values of the derivative by making the appropriate substitutions.

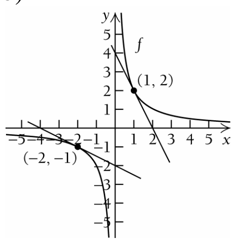
$$f'(-2)=\frac{-1}{(-2)^2}=-\frac{1}{4}$$

$$f'(0)=\frac{-1}{(0)^2}; \text{ Thus, } f'(0) \text{ does not exist.}$$

$$f'(1)=\frac{-1}{(1)^2}=-1$$

16. $f(x)=\frac{2}{x}$

a), b)



There is no tangent line for $x=0$

c) Find the simplified difference quotient first. Referring to Exercise Set 1.3, Exercise 7, we know that the simplified difference quotient is:

$$\frac{f(x+h)-f(x)}{h}=\frac{-2}{x(x+h)}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(\frac{-2}{x(x+h)}\right)$$
$$=\frac{-2}{x^2}$$

Thus, $f'(x)=\frac{-2}{x^2}$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2)=\frac{-2}{(-2)^2}=-\frac{1}{2}$$

$$f'(0)=\frac{-2}{(0)^2};\text{ Thus, } f'(0) \text{ does not exist.}$$

$$f'(1)=\frac{-2}{(1)^2}=-2$$

17. a) From Example 1 we know that $f'(x)=2x$. $f'(3)=2(3)=6$, so the slope of the line tangent to the curve at $(3,9)$ is 6. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-9=6(x-3)$$
$$y-9=6x-18$$
$$y=6x-9$$

b) $f'(-1)=2(-1)=-2$, so the slope of the line tangent to the curve at $(-1,1)$ is -2. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-1=-2(x-(-1))$$
$$y-1=-2x-2$$
$$y=-2x-1$$

c) $f'(10)=2(10)=20$, so the slope of the line tangent to the curve at $(10,100)$ is 20. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-100=20(x-10)$$
$$y-100=20x-200$$
$$y=20x-100$$

18. From Example 2 we know that $f'(x)=3x^2$.

a) At $(-2,-8)$: $f'(-2)=3(-2)^2=12$

$$y-y_1=m(x-x_1)$$
$$y-(-8)=12(x-(-2))$$
$$y+8=12x+24$$
$$y=12x+16$$

b) At $(0,0)$: $f'(0)=3(0)^2=0$

$$y-y_1=m(x-x_1)$$
$$y-(0)=0(x-(0))$$
$$y=0$$

c) At $(4,64)$: $f'(4)=3(4)^2=48$

$$y-y_1=m(x-x_1)$$
$$y-64=48(x-4)$$
$$y-64=48x-192$$
$$y=48x-128$$

19. From Exercise 16 we know that $f'(x)=\frac{-2}{x^2}$.

a) $f'(1)=\frac{-2}{(1)^2}=-2$, so the slope of the line tangent to the curve at $(1,2)$ is -2. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-2&=-2(x-1)\\ y-2&=-2x+2\\ y&=-2x+4 \end{aligned}$$

b) $f'(-1)=\frac{-2}{(-1)^2}=-2$, so the slope of the line tangent to the curve at $(-1,2)$ is -2. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-(-2)&=-2(x-(-1))\\ y+2&=-2x-2\\ y&=-2x-4 \end{aligned}$$

c) $f'(100)=\frac{-2}{(100)^2}=-0.0002$, so the slope of the line tangent to the curve at $(100,0.02)$ is -0.002. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-0.02&=-0.0002(x-100)\\ y-0.02&=-0.0002x+0.02\\ y&=-0.0002x+0.04 \end{aligned}$$

20. First, we find $f'(x)$.

$$\begin{aligned} \frac{f(x+h)-f(x)}{h}&=\frac{\left(\frac{-1}{x+h}\right)-\left(\frac{-1}{x}\right)}{h}\\ &=\frac{\left(\frac{-1}{x+h}\cdot\frac{x}{x}\right)-\left(\frac{-1}{x}\cdot\frac{(x+h)}{(x+h)}\right)}{h}\\ &=\frac{\left(\frac{-x}{x(x+h)}\right)-\left(\frac{-x-h}{x(x+h)}\right)}{h}\\ &=\frac{\frac{h}{x(x+h)}}{h}\\ &=\frac{h}{x(x+h)}\cdot\frac{1}{h}\\ &=\frac{1}{x(x+h)} \end{aligned}$$

Taking the limit as $h\to 0$ we have:

$$\begin{aligned} \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}&=\lim_{h\to 0}\frac{1}{x(x+h)}\\ &=\frac{1}{x^2} \end{aligned}$$

Thus, $f'(x)=\frac{1}{x^2}$.

a) At $(-1,1)$: $f'(-1)=\frac{1}{(-1)^2}=1$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-1&=1(x-(-1))\\ y-1&=x+1\\ y&=x+2 \end{aligned}$$

b) At $\left(2,-\frac{1}{2}\right)$: $f'(2)=\frac{1}{(2)^2}=\frac{1}{4}$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-\left(-\frac{1}{2}\right)&=\frac{1}{4}(x-2)\\ y+\frac{1}{2}&=\frac{1}{4}x-\frac{1}{2}\\ y&=\frac{1}{4}x-1 \end{aligned}$$

c) At $\left(5,-\frac{1}{5}\right)$: $f'(5)=\frac{1}{(5)^2}=\frac{1}{25}$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-\left(-\frac{1}{5}\right)&=\frac{1}{25}(x-5)\\ y+\frac{1}{5}&=\frac{1}{25}x-\frac{1}{5}\\ y&=\frac{1}{25}x-\frac{2}{5} \end{aligned}$$

21. First, we find $f'(x)$:

$$\begin{aligned} &\frac{f(x+h)-f(x)}{h}\\ &=\frac{\left(4-(x+h)^2\right)-\left(4-x^2\right)}{h}\\ &=\frac{4-\left(x^2+2xh+h^2\right)-4+x^2}{h}\\ &=\frac{4-x^2-2xh-h^2-4+x^2}{h}\\ &=\frac{-2xh-h^2}{h}\\ &=\frac{h(-2x-h)}{h}\\ &=-2x-h \quad \text{Simplified difference quotient} \end{aligned}$$

$$\begin{aligned} f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &=\lim_{h\to 0}(-2x-h)\\ &=-2x \end{aligned}$$

a) $f'(-1)=-2(-1)=2$, so the slope of the line tangent to the curve at $(-1,3)$ is 2. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-3&=2(x-(-1))\\ y-3&=2x+2\\ y&=2x+5 \end{aligned}$$

b) $f'(0)=-2(0)=0$, so the slope of the line tangent to the curve at $(0,4)$ is 0. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-4&=0(x-0)\\ y-4&=0\\ y&=4 \end{aligned}$$

c) $f'(5)=-2(5)=-10$, so the slope of the line tangent to the curve at $(5,-21)$ is -10. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-(-21)&=-10(x-5)\\ y+21&=-10x+50\\ y&=-10x+29 \end{aligned}$$

22. First, we will find $f'(x)$

$$\begin{aligned} &\frac{f(x+h)-f(x)}{h}\\ &=\frac{\left((x+h)^2-2(x+h)\right)-\left(x^2-2x\right)}{h}\\ &=\frac{x^2+2xh+h^2-2x-2h-x^2+2x}{h}\\ &=\frac{2xh+h^2-2h}{h}\\ &=\frac{h(2x+h-2)}{h}\\ &=2x+h-2 \quad \text{Simplified difference quotient} \end{aligned}$$

$$\begin{aligned} f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &=\lim_{h\to 0}(2x+h-2)\\ &=2x-2 \end{aligned}$$

Thus, $f'(x)=2x-2$.

a) At $(-2,8)$: $f'(-2)=2(-2)-2=-6$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-8&=-6(x-(-2))\\ y-8&=-6x-12\\ y&=-6x-4 \end{aligned}$$

b) At $(1,-1)$: $f'(1)=2(1)-2=0$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-(-1)&=0(x-1)\\ y+1&=0\\ y&=-1 \end{aligned}$$

c) At $(4,8)$: $f'(4)=2(4)-2=6$

$$y-y_1=m(x-x_1)$$
$$y-8=6(x-4)$$
$$y-8=6x-24$$
$$y=6x-16$$

23. Find the simplified difference quotient for $f(x)=mx+b$ first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{m(x+h)+b-(mx+b)}{h}$$
$$=\frac{mx+mh+b-mx-b}{h}$$
$$=\frac{mh}{h}$$
$$=m \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(m)$$
$$=m$$

Thus, $f'(x)=m$.

24. $f(x)=ax^2+bx$

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(a(x+h)^2+b(x+h)\right)-\left(ax^2+bx\right)}{h}$$
$$=\frac{a\left(x^2+2hx+h^2\right)+bx+bh-ax^2-bx}{h}$$
$$=\frac{ax^2+2ahx+ah^2+bx+bh-ax^2-bx}{h}$$
$$=\frac{2ahx+ah^2+bh}{h}$$
$$=\frac{h(2ax+ah+b)}{h}$$
$$=2ax+ah+b$$

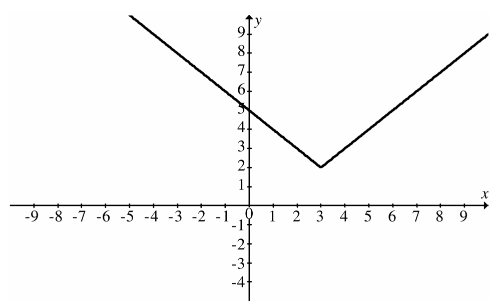
Next we take the limit as $h\to 0$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}(2ax+ah+b)$$
$$=2ax+b$$

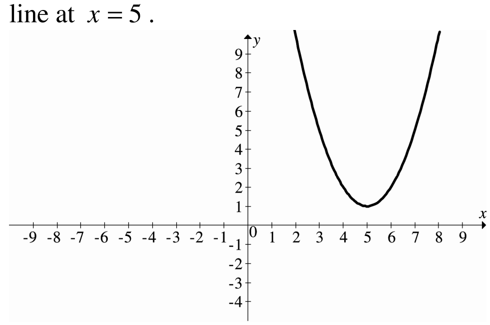
25. If a function has a "sharp point" or a "corner," it will not be differentiable at that point. Thus, the function is not differentiable at x_3, x_4, x_6. The function has a vertical tangent at x_{12}. Vertical lines have undefined slope, hence the function is not differentiable at x_{12}. Also, if a function is discontinuous at some point *a*, then it is not differentiable at *a*. The function is discontinuous at the point x_0, thus it is not differentiable at x_0.
Therefore, the graph is not differentiable at the points $x_0, x_3, x_4, x_6, x_{12}$.

26. The function is not differentiable at x_2, x_4, x_5 because there is a "sharp point" or a "corner" at each of those points. The function is not differentiable at x_7, x_8 because the function is discontinuous at those points.

27. The following graph is continuous but not differentiable, at $x=3$.



28. The following graph has a horizontal tangent line at $x=5$.



29. The postage function does not have any "sharp points" nor does it have any vertical tangents. However it is discontinuous at all natural numbers. Therefore the postage function is not differentiable for 1, 2, 3, 4, and so on.

30. The taxi cab function is not differentiable at 0, 0.2, 0.4, 0.6, and so on.

31. The ticket price function is continuous everywhere on its domain. It also does not have any "sharp points" or "corners" nor does it have any vertical tangents. Thus, the function is differentiable everywhere on it domain.

32. $\boxed{tw}$ The lines L_2, L_3, L_4, L_6 appear to be tangent lines. The slopes appear to be the same as the instantaneous rate of change of the function at the indicated points.

33. $\boxed{tw}$ The graph is left to the student. As the points Q approach P , the slopes of the secant lines approach the slope of the tangent line at P.

34. $f(x) = x^4$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{(x+h)^4 - x^4}{h}\\ &= \frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h}\\ &= \frac{4x^3h+6x^2h^2+4xh^3+h^4}{h}\\ &= \frac{h(4x^3+6x^2h+4xh^2+h^3)}{h}\\ &= 4x^3+6x^2h+4xh^2+h^3\end{aligned}$$

Thus,

$$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0} 4x^3+6x^2h+4xh^2+h^3\\ &= 4x^3\end{aligned}$$

35. $f(x) = \dfrac{1}{x^2}$

Find the difference quotient first.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}\\ &= \frac{\frac{1}{(x+h)^2}\cdot\frac{x^2}{x^2}-\frac{1}{x^2}\cdot\frac{(x+h)^2}{(x+h)^2}}{h}\\ &= \frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}\\ &= \frac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}\\ &= \frac{-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}\\ &= \frac{-2x-h}{x^2(x+h)^2}\end{aligned}$$

Next, we will find the limit of the difference quotient as $h \to 0$.

$$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{-2x-h}{x^2(x+h)^2}\\ &= \frac{-2x}{x^2(x+0)^2}\\ &= \frac{-2x}{x^4}\\ &= \frac{-2}{x^3}\end{aligned}$$

Thus, $f'(x) = \dfrac{-2}{x^3}$.

36. $f(x) = \dfrac{1}{1-x}$

We found the simplified difference quotient in Exercise 52 of Exercise Set 1.3. We now find the limit of the difference quotient as $h \to 0$.

$$\begin{aligned}\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{1}{(1-x)(1-x-h)}\\ &= \frac{1}{(1-x)(1-x-0)}\\ &= \frac{1}{(1-x)^2}\end{aligned}$$

Thus, $f'(x) = \dfrac{1}{(1-x)^2}$.

37. $f(x)=\dfrac{x}{1+x}$

We found the simplified difference quotient in Exercise 53 of Exercise Set 1.3. We now find the limit of the difference quotient as $h \to 0$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{1}{(x+1)(x+1+h)}$$
$$=\frac{1}{(x+1)(x+1+0)}$$
$$=\frac{1}{(x+1)^2}$$

Thus, $f'(x)=\dfrac{1}{(x+1)^2}$.

38. $f(x)=\dfrac{2x}{x+1}$

Find the simplified difference quotient.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(\dfrac{2(x+h)}{(x+h)+1}\right)-\left(\dfrac{2x}{x+1}\right)}{h}$$
$$=\frac{\left(\dfrac{2x+2h}{x+h+1}\cdot\dfrac{x+1}{x+1}\right)-\left(\dfrac{2x}{x+1}\cdot\dfrac{(x+h+1)}{(x+h+1)}\right)}{h}$$
$$=\frac{\left(\dfrac{2x^2+2hx+2x+2h}{(x+1)(x+h+1)}\right)-\left(\dfrac{2x^2+2hx+2x}{(x+1)(x+h+1)}\right)}{h}$$
$$=\frac{\dfrac{2h}{(x+1)(x+h+1)}}{h}$$
$$=\frac{2h}{(x+1)(x+h+1)}\cdot\frac{1}{h}$$
$$=\frac{2}{(x+1)(x+h+1)}$$

We now find the limit of the difference quotient as $h \to 0$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{2}{(x+1)(x+h+1)}$$
$$=\frac{2}{(x+1)^2}$$

Thus, $f'(x)=\dfrac{2}{(x+1)^2}$.

39. $f(x)=\sqrt{x}$

We found the simplified difference quotient in Exercise 49 of Exercise Set 1.3. We now find the limit of the difference quotient as $h \to 0$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$$
$$=\frac{1}{\sqrt{x+0}+\sqrt{x}}$$
$$=\frac{1}{2\sqrt{x}}$$

Thus, $f'(x)=\dfrac{1}{2\sqrt{x}}$

40. $f(x)=\dfrac{1}{\sqrt{x}}$

We found the simplified difference quotient in Exercise 55 of Exercise Set 1.3. We now find the limit of the difference quotient as $h \to 0$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$
$$=\frac{-1}{\sqrt{x}\sqrt{x+0}\left(\sqrt{x}+\sqrt{x+0}\right)}$$
$$=\frac{-1}{x\left(2\sqrt{x}\right)}$$
$$=\frac{-1}{2x\sqrt{x}}$$

Thus, $f'(x)=\dfrac{-1}{2x\sqrt{x}}$.

41. a) The domain of the rational function is restricted to those input values that do not result in division by 0. The domain for $f(x)=\dfrac{x^2-9}{x+3}$ consists of all real numbers except -3. Since $f(-3)$ does not exist, the function is not continuous at -3. Thus, the function is not differentiable at $x=-3$.

b) $\boxed{tw}$ Answers will vary. The simplest way of finding $f'(4)$ is to use the nDeriv function on your calculator. However, without using advanced technology the easiest way is to approximate the difference quotient using a very small value of h, and allowing the calculator to perform the basic computations. We illustrate using $h = 0.0001$. The difference quotient will be

$$\frac{f(x+h)-f(x)}{h} = \frac{f(4+0.0001)-f(4)}{0.0001}$$

Using the calculator to evaluate the function we have:

$f(4.0001) = 1.0001$

$f(4) = 1$

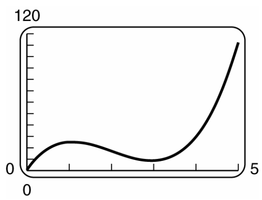
Plugging in these values we have:

$$\frac{f(4+0.0001)-f(4)}{0.0001} = \frac{1.0001-1}{0.0001} = 1$$

Therefore, $f'(4) = 1$.

42-47. Left to the student.

48. a) $V(t) = 5t^3 - 30t^2 + 45t + 5\sqrt{t}$



b) First we find the function values

$$V(1) = 5(1)^3 - 30(1)^2 + 45(1) + 5\sqrt{1}$$
$$= 5 - 30 + 45 + 5$$
$$= 25$$

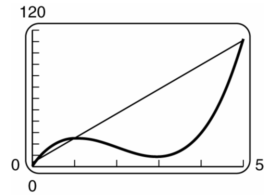
$$V(5) = 5(5)^3 - 30(5)^2 + 45(5) + 5\sqrt{5}$$
$$\approx 111.180$$

Now we can find the slope of the secant line passing through the two points.

$$m = \frac{V(5)-V(1)}{5-1}$$
$$= \frac{111.180-25}{5-1}$$
$$= \frac{86.180}{4}$$
$$= 21.545$$

Using the point-slope equation we have:

$$V - V_1 = m(t - t_1)$$
$$V - 25 = 21.545(t-1)$$
$$V - 25 = 21.545t - 21.545$$
$$V = 21.545t + 3.455$$



c) The average rate of change of the investment between year 1 and year 5 is the slope of the secant line between the two points. That is, the average rate of change is \$21.545 million per year.

d) For $(1, V(1))$ and $(4, V(4))$:

From part (b) we know $V(1) = 25$.

$$V(4) = 5(4)^3 - 30(4)^2 + 45(4) + 5\sqrt{4}$$
$$\approx 30$$

$$m = \frac{V(4)-V(1)}{4-1}$$
$$= \frac{30-25}{4-1}$$
$$= \frac{5}{3}$$
$$= 1.666667$$

$$V - V_1 = m(t - t_1)$$
$$V - 25 = 1.666667(t-1)$$
$$V - 25 = 1.666667t - 1.666667$$
$$V = 1.666667t + 23.33333$$

For $(1, V(1))$ and $(3, V(3))$:

From part (b) we know $V(1) = 25$.

$$V(3) = 5(3)^3 - 30(3)^2 + 45(3) + 5\sqrt{3}$$
$$\approx 8.660254$$

$$m = \frac{V(3)-V(1)}{3-1}$$
$$= \frac{8.660254-25}{3-1}$$
$$= \frac{-16.339746}{2}$$
$$= -8.169873$$