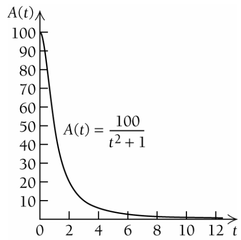


d) $\boxed{tw}$ No, the medication does not completely leave the bloodstream. We notice that $\lim_{t\to\infty} A(t) = 0$. This means that as t approaches infinity, A approaches 0, but does not actually reach the value 0. We also notice that the equation $A(t) = 0$ has no solution, so according to this model, the medication will never completely leave the bloodstream.

63. $E(n) = 9\cdot\dfrac{4}{n}$

a) Calculate each value for the given n.

$$E(9) = 9\cdot\frac{4}{9} = 4.00$$

$$E(8) = 9\cdot\frac{4}{8} = 4.50$$

$$E(7) = 9\cdot\frac{4}{7} \approx 5.14$$

$$E(6) = 9\cdot\frac{4}{6} = 6.00$$

$$E(5) = 9\cdot\frac{4}{5} = 7.20$$

$$E(4) = 9\cdot\frac{4}{4} = 9.00$$

$$E(3) = 9\cdot\frac{4}{3} = 12.00$$

$$E(2) = 9\cdot\frac{4}{2} = 18.00$$

$$E(1) = 9\cdot\frac{4}{1} = 36.00$$

$$E\left(\frac{2}{3}\right) = 9\cdot\frac{4}{\frac{2}{3}} = 9\left(4\cdot\frac{3}{2}\right) = 54.00$$

$$E\left(\frac{1}{3}\right) = 9\cdot\frac{4}{\frac{1}{3}} = 9\left(4\cdot\frac{3}{1}\right) = 108.00$$

We complete the table.

Innings Pitched (n)	Earned-Run average (E)
9	4.00
8	4.50
7	5.14
6	6.00
5	7.20
4	9.00
3	12.00
2	18.00
1	36.00
$\frac{2}{3}$	54.00
$\frac{1}{3}$	108.00

b) $\lim_{n\to 0} E(n) = \lim_{n\to 0} 9\cdot\dfrac{4}{n} = \lim_{n\to 0}\dfrac{36}{n} = \infty$

64. $\boxed{tw}$ Vertical asymptotes occur at values of the variable for which the function is undefined. Thus, they cannot be part of the graph.

65. $\boxed{tw}$ Asymptotes can be thought of as “limiting lines” for the graph of a function. The graphs and limits in Examples 1, 3 and 6 in section 2.3 of the text book illustrate vertical, horizontal and slant asymptotes.

66. $\displaystyle \lim_{x\to-\infty}\frac{-3x^2+5}{2-x} = \lim_{x\to-\infty}\frac{-3x+\frac{5}{x}}{\frac{2}{x}-1} =$

$$\frac{\lim_{x\to-\infty}(-3x)+0}{-1} = -\lim_{x\to-\infty}(-3x) = -\infty$$

67. $\displaystyle \lim_{x\to 0}\frac{|x|}{x}$

Using $|x| = \begin{cases}-x, & \text{for } x<0\\ x, & \text{for } x\ge 0\end{cases}$, we have

$\displaystyle \lim_{x\to 0^-}\frac{|x|}{x} = -1$ and $\displaystyle \lim_{x\to 0^+}\frac{|x|}{x} = 1$.

Therefore, $\displaystyle \lim_{x\to 0}\frac{|x|}{x}$ does not exist.

68. $\lim_{x \to -2} \frac{x^3+8}{x^2-4} = \lim_{x \to -2} \frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)}$

$= \lim_{x \to -2} \frac{(x^2-2x+4)}{(x-2)}$

$= \frac{(-2)^2 - 2(-2)+4}{(-2)-2}$.

$= -3$

69. We divide the numerator and denominator by x^2 the highest power of x in the denominator.

$$\lim_{x \to \infty} \frac{-6x^3+7x}{2x^2-3x-10} = \lim_{x \to \infty} \frac{-6x + \frac{7}{x}}{2 - \frac{3}{x} - \frac{10}{x^2}}$$

$$= \frac{\lim_{x \to \infty}(-6x)+0}{2-0-0}$$

$$= -\infty$$

70. $\lim_{x \to -\infty} \frac{-6x^3+7x}{2x^2-3x-10} = \lim_{x \to -\infty} \frac{-6x + \frac{7}{x}}{2 - \frac{3}{x} - \frac{10}{x^2}}$

$$= \frac{\lim_{x \to -\infty}(-6x)+0}{2-0-0}$$

$$= \infty$$

71. $\lim_{x \to 1} \frac{x^3-1}{x^2-1} = \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}$

$= \lim_{x \to 1} \frac{(x^2+x+1)}{(x+1)}$

$= \frac{(1)^2+(1)+1}{(1)+1}$

$= \frac{3}{2}$

72. $\lim_{x \to -\infty} \frac{7x^5+x-9}{6x+x^3} = \lim_{x \to -\infty} \frac{7x^2 + \frac{1}{x^2} - \frac{9}{x^3}}{\frac{6}{x^2}+1}$

$$= \frac{\lim_{x \to -\infty} 7x^2 + 0 - 0}{0+1}$$

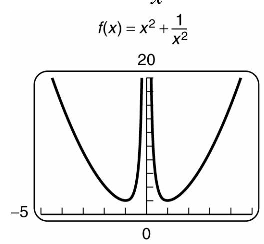
$$= \infty$$

73. $\lim_{x \to -\infty} \frac{2x^4+x}{x+1} = \lim_{x \to -\infty} \frac{2x^3+1}{1+\frac{1}{x}}$

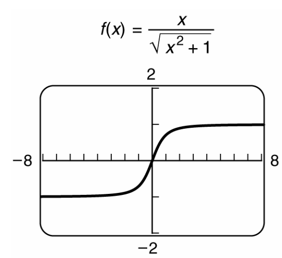
$$= \frac{\lim_{x \to -\infty} 2x^3+1}{1+0}$$

$$= -\infty$$

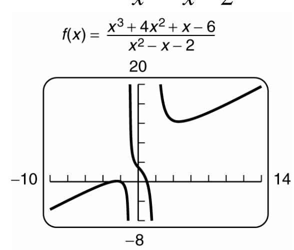
74. $f(x) = x^2 + \frac{1}{x^2}$



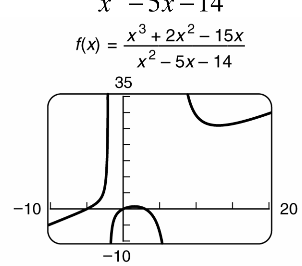
75. $f(x) = \frac{x}{\sqrt{x^2+1}}$



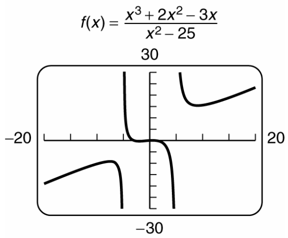
76. $f(x) = \frac{x^3+4x^2+x-6}{x^2-x-2}$



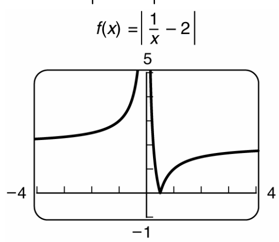
77. $f(x) = \frac{x^3+2x^2-15x}{x^2-5x-14}$



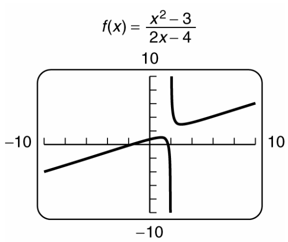
78. $f(x)=\dfrac{x^3+2x^2-3x}{x^2-25}$



79. $f(x)=\left|\dfrac{1}{x}-2\right|$



80. $f(x)=\dfrac{x^2-3}{2x-4}$



a) $(-1.732,0)$ and $(1.732,0)$

b) $(0,0.75)$

c) *Vertical.* $x=2$
Horizontal. None
Slant. $y=0.5x+1$

81. $\boxed{tw}$ $f(x)=\dfrac{\sqrt{x^2+3x+2}}{x-3}$

a) $\lim\limits_{x\to\infty} f(x)=1$; $\lim\limits_{x\to-\infty} f(x)=-1$

As x increases without bound,

$f(x)=\dfrac{\sqrt{x^2+3x+2}}{x-3}$ approaches

$\dfrac{\sqrt{x^2}}{x}=\dfrac{|x|}{x}=1.$

As x decreases without bound,

$f(x)=\dfrac{\sqrt{x^2+3x+2}}{x-3}$ approaches

$\dfrac{\sqrt{x^2}}{x}=\dfrac{|x|}{x}=-1.$

b) For values of x in $(-2,-1)$, $x^2+3x+2<0$
Therefore, the function values
$f(x)=\dfrac{\sqrt{x^2+3x+2}}{x-3}$ are not defined on $(-2,-1)$.

c) The domain of the function appears to be $(-\infty,-2]\cup[-1,3)\cup(3,\infty)$. We must throw out values that make the radicand negative, or the denominator 0.

d) From the graph it appears that
$\lim\limits_{x\to-2^-} f(x)=0$ and $\lim\limits_{x\to-1^+} f(x)=0$.
We verify this algebraically.

$$\lim_{x\to-2^-}\frac{\sqrt{x^2+3x+2}}{x-3}=\frac{\sqrt{(-2)^2+3(-2)+2}}{(-2)-3}=0$$

$$\lim_{x\to-1^+}\frac{\sqrt{x^2+3x+2}}{x-3}=\frac{\sqrt{(-1)^2+3(-1)+2}}{(-1)-3}=1$$

82. $f(x)=\dfrac{x^5+x-9}{x^3+6x}$

Using long division we have:

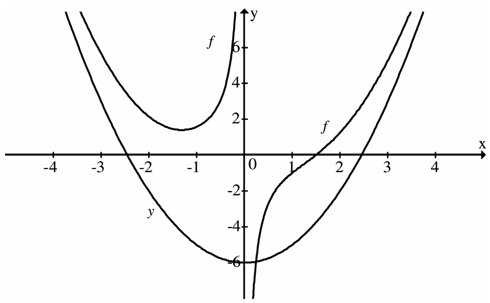
$$\begin{array}{r l}
 & x^2 \qquad -6 \\
x^3+6x \,\big) & x^5 \qquad\qquad + x - 9 \\
 & \underline{x^5+6x^3} \\
 & -6x^3+0x^2+x \\
 & \underline{-6x^3 \qquad +36x} \\
 & \qquad\qquad 37x-9
\end{array}$$

Therefore,

$$f(x)=\frac{x^5+x-9}{x^3+6x}=x^2-6+\frac{37}{x^3+6x}.$$

As $|x|$ gets large, $f(x)$ approaches x^2-6x. Therefore, the nonlinear asymptote is $y=x^2-6$.

Using a calculator, we graph the function, *f*, and the asymptote, *y*, below.



Exercise Set 2.4

1. a) The absolute maximum gasoline mileage is obtained at a speed of 55 mph.
b) The absolute minimum gasoline mileage is obtained at a speed of 5 mph.
c) At 70 mph, the fuel economy is 25 mpg.

2. Over the interval $[30,70]$, the sedan's absolute maximum fuel economy is 30 mpg at 55 mph. The sedan's absolute minimum fuel economy is 25 mpg at 70 mph.

3. $f(x)=5+x-x^2$; $[0,2]$

a) Find $f'(x)$

$f'(x)=1-2x$

b) Find the Critical Values. The derivative exists for all real numbers. Thus, we solve

$$f'(x)=0$$
$$1-2x=0$$
$$1=2x$$
$$\frac{1}{2}=x$$

c) List the critical values and endpoints. These values are $0,\ \frac{1}{2},$ and 2.

d) Evaluate $f(x)$ at each value in step (c).

$$f(0)=5+(0)-(0)^2=5$$
$$f\left(\frac{1}{2}\right)=5+\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)^2=\frac{21}{4}=5.25$$
$$f(2)=5+(2)-(2)^2=3$$

The largest of these values, $\frac{21}{4}$, is the absolute maximum, it occurs at $x=\frac{1}{2}$. The smallest of these values, 3, is the absolute minimum, it occurs at $x=2$.

4. $f(x)=4+x-x^2$; $[0,2]$

a) $f'(x)=1-2x$

b) $f'(x)$ exists for all real numbers. Solve:

$$1-2x=0$$
$$x=\frac{1}{2}$$

c) The critical value and the endpoints are: $0,\ \frac{1}{2},$ and 2.

d) Evaluate $f(x)$ for each value in (c).

$$f(0)=4+(0)-(0)^2=4$$
$$f\left(\frac{1}{2}\right)=4+\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)^2=\frac{17}{4}=4.25$$
$$f(2)=4+(2)-(2)^2=2$$

On the interval $[0,2]$, the absolute maximum is $\frac{17}{4}$, which occurs at $x=\frac{1}{2}$. The absolute minimum is 2, which occurs at $x=2$.

5. $f(x)=x^3-x^2-x+2$; $[-1,2]$

a) Find $f'(x)$

$f'(x)=3x^2-2x-1$

b) Find the Critical Values. The derivative exists for all real numbers. Thus, we solve

$$f'(x)=0$$
$$3x^2-2x-1=0$$
$$(3x+1)(x-1)=0$$
$$3x+1=0 \quad \text{or} \quad x-1=0$$
$$3x=-1 \quad \text{or} \quad x=1$$
$$x=-\frac{1}{3} \quad \text{or} \quad x=1$$

c) List the critical values and endpoints. These values are $-1,\ -\frac{1}{3},\ 1, \text{ and } 2$.

d) Evaluate $f(x)$ at each value in step (c).

$$f(-1)=(-1)^3-(-1)^2-(-1)+2=1$$
$$f\left(\frac{-1}{3}\right)=\left(\frac{-1}{3}\right)^3-\left(\frac{-1}{3}\right)^2-\left(\frac{-1}{3}\right)+2=\frac{59}{27}\approx 2.2$$
$$f(1)=(1)^3-(1)^2-(1)+2=1$$
$$f(2)=(2)^3-(2)^2-(2)+2=4$$

The largest of these values, 4,is the absolute maximum, it occurs at $x=2$. The smallest of these values, 1, is the absolute minimum, it occurs at $x=-1$ and $x=1$.

6. $f(x)=x^3+\frac{1}{2}x^2-2x+5; \quad [-2,1]$

a) $f'(x)=3x^2+x-2$

b) $f'(x)$ exists for all real numbers. Solve:

$$3x^2+x-2=0$$
$$(3x-2)(x+1)=0$$
$$x=\frac{2}{3} \quad \text{or} \quad x=-1$$

c) The critical value and the endpoints are: $-2,\ -1,\ \frac{2}{3}, \text{ and } 1$.

d) Evaluate $f(x)$ for each value in (c)

$$f(-2)=(-2)^3+\frac{1}{2}(-2)^2-2(-2)+5=3$$
$$f(-1)=(-1)^3+\frac{1}{2}(-1)^2-2(-1)+5=\frac{13}{2}\approx 6.5$$
$$f\left(\frac{2}{3}\right)=\left(\frac{2}{3}\right)^3+\frac{1}{2}\left(\frac{2}{3}\right)^2-2\left(\frac{2}{3}\right)+5=\frac{113}{27}\approx 4.2$$
$$f(1)=(1)^3+\frac{1}{2}(1)^2-2(1)+5=\frac{9}{2}=4.5$$

On the interval $[-2,1]$, the absolute maximum is $\frac{13}{2}$, which occurs at $x=-1$. The absolute minimum is 3, which occurs at $x=-2$.

7. $f(x)=x^3-x^2-x+3; \quad [-1,0]$

a) Find $f'(x)$

$$f'(x)=3x^2-2x-1$$

b) Find the Critical Values. The derivative exists for all real numbers. Thus, we solve

$$f'(x)=0$$
$$3x^2-2x-1=0$$
$$(3x+1)(x-1)=0$$
$$3x+1=0 \quad \text{or} \quad x-1=0$$
$$3x=-1 \quad \text{or} \quad x=1$$
$$x=-\frac{1}{3} \quad \text{or} \quad x=1$$

c) List the critical values and endpoints. The critical value $x=1$ is not in the interval, so we exclude it. We will test the values $-1,\ -\frac{1}{3}, \text{ and } 0$.

d) Evaluate $f(x)$ at each value in step (c).

$$f(-1)=(-1)^3-(-1)^2-(-1)+3=2$$
$$f\left(\frac{-1}{3}\right)=\left(\frac{-1}{3}\right)^3-\left(\frac{-1}{3}\right)^2-\left(\frac{-1}{3}\right)+3=\frac{86}{27}\approx 3.2$$
$$f(0)=(0)^3-(0)^2-(0)+3=3$$

On the interval $[-1,0]$, the absolute maximum is $\frac{86}{27}$, which occurs at $x=-\frac{1}{3}$. The absolute minimum is 2, which occurs at $x=-1$.

8. $f(x)=x^3+\frac{1}{2}x^2-2x+4; \quad [-2,0]$

a) $f'(x)=3x^2+x-2$

b) $f'(x)$ exists for all real numbers. Solve:

$$3x^2+x-2=0$$
$$(3x-2)(x+1)=0$$
$$x=\frac{2}{3} \quad \text{or} \quad x=-1$$

c) The critical value $x=\frac{2}{3}$ is not in the interval, so we exclude it. We will test the values: $-2,\ -1,$ and 0.

d) Evaluate $f(x)$ for each value in (c)

$$f(-2)=(-2)^3+\frac{1}{2}(-2)^2-2(-2)+4=2$$

$$f(-1)=(-1)^3+\frac{1}{2}(-1)^2-2(-1)+4=\frac{11}{2}\approx 5.5$$

$$f(0)=(0)^3+\frac{1}{2}(0)^2-2(0)+4=4$$

On the interval $[-2,0]$, the absolute maximum is $\frac{11}{2}$, which occurs at $x=-1$. The absolute minimum is 2, which occurs at $x=-2$.

9. $f(x)=5x-7;\qquad [-2,3]$

a) Find $f'(x)$

$f'(x)=5$

b) and c)
The derivative exists and is 5 for all real numbers. Note that the derivative is never 0. Thus, there are no critical values for $f(x)$, and the absolute maximum and absolute minimum will occur at the endpoints of the interval.

d) Evaluate $f(x)$ at the endpoints.

$f(-2)=5(-2)-7=-17$

$f(3)=5(3)-7=8$

On the interval $[-2,3]$, the absolute maximum is 8, which occurs at $x=3$. The absolute minimum is -17, which occurs at $x=-2$.

10. $f(x)=2x+4;\qquad [-1,1]$

a) $f'(x)=2$

b) and c)
$f'(x)=2$ for all real numbers; therefore, there are no critical points. The absolute maximum and minimum values occur at the endpoints.

d) Evaluate $f(x)$ at the endpoints.

$f(-1)=2(-1)+4=2$

$f(1)=2(1)+4=6$

On the interval $[-1,1]$, the absolute maximum is 6, which occurs at $x=1$. The absolute minimum is 2, which occurs at $x=-1$.

11. $f(x)=7-4x;\qquad [-2,5]$

a) Find $f'(x)$

$f'(x)=-4$

b) and c)
The derivative exists and is -4 for all real numbers. Note that the derivative is never 0. Thus, there are no critical values for $f(x)$, and the absolute maximum and absolute minimum will occur at the endpoints of the interval.

d) Evaluate $f(x)$ at the endpoints.

$f(-2)=7-4(-2)=15$

$f(5)=7-4(5)=-13$

On the interval $[-2,5]$, the absolute maximum is 15, which occurs at $x=-2$. The absolute minimum is -13, which occurs at $x=5$.

12. $f(x)=-2-3x;\qquad [-10,10]$

a) $f'(x)=-3$

b) and c)
$f'(x)=-3$ for all real numbers; therefore, there are no critical points. The absolute maximum and minimum values occur at the endpoints.

d) Evaluate $f(x)$ at the endpoints.

$f(-10)=-2-3(-10)=28$

$f(10)=-2-3(10)=-32$

On the interval $[-10,10]$, the absolute maximum is 28, which occurs at $x=-10$. The absolute minimum is -32, which occurs at $x=10$.

13. $f(x)=-5;\qquad [-1,1]$

Note for all values of x, $f(x)=-5$. Thus, the absolute maximum is -5 for $-1\le x\le 1$ and the absolute minimum is -5 for $-1\le x\le 1$.

14. $g(x)=24;\qquad [4,13]$

Note for all values of x, $g(x)=24$. Thus, the absolute maximum is 24 for $4\le x\le 13$ and the absolute minimum is 24 for $4\le x\le 13$.

15. $f(x)=x^2-6x-3;\qquad [-1,5]$

a) $f'(x)=2x-6$

b) $f'(x)$ exists for all real numbers. Solve:

$$2x-6=0$$
$$2x=6$$
$$x=3$$

c) The critical value and the endpoints are $-1,\ 3,$ and 5.

d) Evaluate $f(x)$ for each value in (c)

$$f(-1)=(-1)^2-6(-1)-3=4$$
$$f(3)=(3)^2-6(3)-3=-12$$
$$f(5)=(5)^2-6(5)-3=-8$$

On the interval $[-1,5]$, the absolute maximum is 4, which occurs at $x=-1$. The absolute minimum is -12, which occurs at $x=3$.

16. $f(x)=x^2-4x+5;\qquad [-1,3]$

a) $f'(x)=2x-4$

b) $f'(x)$ exists for all real numbers. Solve:

$$2x-4=0$$
$$x=2$$

c) The critical value and the endpoints are $-1,\ 2,$ and 3.

d) Evaluate $f(x)$ for each value in (c)

$$f(-1)=(-1)^2-4(-1)+5=10$$
$$f(2)=(2)^2-4(2)+5=1$$
$$f(3)=(3)^2-4(3)+5=2$$

On the interval $[-1,3]$, the absolute maximum is 10, which occurs at $x=-1$. The absolute minimum is 1, which occurs at $x=2$.

17. $f(x)=3-2x-5x^2;\qquad [-3,3]$

a) $f'(x)=-2-10x$

b) $f'(x)$ exists for all real numbers. Solve:

$$-2-10x=0$$
$$-10x=2$$
$$x=-\frac{1}{5}$$

c) The critical value and the endpoints are $-3,\ -\frac{1}{5},$ and 3.

d) Evaluate $f(x)$ for each value in (c)

$$f(-3)=3-2(-3)-5(-3)^2=-36$$
$$f\left(-\frac{1}{5}\right)=3-2\left(-\frac{1}{5}\right)-5\left(-\frac{1}{5}\right)^2=\frac{16}{5}=3.2$$
$$f(3)=3-2(3)-5(3)^2=-48$$

On the interval $[-3,3]$, the absolute maximum is $\frac{16}{5}$, which occurs at $x=-\frac{1}{5}$. The absolute minimum is -48, which occurs at $x=3$.

18. $f(x)=1+6x-3x^2;\qquad [0,4]$

a) $f'(x)=6-6x$

b) $f'(x)$ exists for all real numbers. Solve:

$$6-6x=0$$
$$x=1$$

c) The critical value and the endpoints are $0,\ 1,$ and 4.

d) Evaluate $f(x)$ for each value in (c)

$$f(0)=1+6(0)-3(0)^2=1$$
$$f(1)=1+6(1)-3(1)^2=4$$
$$f(4)=1+6(4)-3(4)^2=-23$$

On the interval $[0,4]$, the absolute maximum is 4, which occurs at $x=1$. The absolute minimum is -23, which occurs at $x=4$.

19. $f(x)=x^3-3x^2;\qquad [0,5]$

a) $f'(x)=3x^2-6x$

b) $f'(x)$ exists for all real numbers. Solve:

$$3x^2-6x=0$$
$$3x(x-2)=0$$
$$3x=0 \quad\text{or}\quad x-2=0$$
$$x=0 \quad\text{or}\quad x=2$$

c) The critical value and the endpoints are $0,\ 2,$ and 5. Note, since 0 is an endpoint of the interval, $x=0$ is included in this list as an endpoint, as well as a critical value.

d) Evaluate $f(x)$ for each value in (c)

$f(0)=(0)^3-3(0)^2=0$

$f(2)=(2)^3-3(2)^2=-4$

$f(5)=(5)^3-3(5)^2=50$

On the interval $[0,5]$, the absolute maximum is 50, which occurs at $x=5$. The absolute minimum is -4, which occurs at $x=2$.

20. $f(x)=x^3-3x+6;\quad [-1,3]$

a) $f'(x)=3x^2-3$

b) $f'(x)$ exists for all real numbers. Solve:

$3x^2-3=0$

$x^2-1=0$

$x=\pm 1$

c) The critical value and the endpoints are $-1,\ 1,$ and 3. Note, since -1 is an endpoint of the interval, $x=-1$ is included in this list as an endpoint, not a critical value.

d) Evaluate $f(x)$ for each value in (c)

$f(-1)=(-1)^3-3(-1)+6=8$

$f(1)=(1)^3-3(1)+6=4$

$f(3)=(3)^3-3(3)+6=24$

On the interval $[-1,3]$, the absolute maximum is 24, which occurs at $x=3$. The absolute minimum is 4, which occurs at $x=1$.

21. $f(x)=x^3-3x;\quad [-5,1]$

a) $f'(x)=3x^2-3$

b) $f'(x)$ exists for all real numbers. Solve:

$3x^2-3=0$

$x^2-1=0$

$x=\pm 1$

c) The critical value and the endpoints are $-5,\ -1,$ and 1. Note, since 1 is an endpoint of the interval, $x=1$ is included in this list as an endpoint, not a critical value.

d) Evaluate $f(x)$ for each value in (c)

$f(-5)=(-5)^3-3(-5)=-110$

$f(-1)=(-1)^3-3(-1)=2$

$f(1)=(1)^3-3(1)=-2$

On the interval $[-5,1]$, the absolute maximum is 2, which occurs at $x=-1$. The absolute minimum is -110, which occurs at $x=-5$.

22. $f(x)=3x^2-2x^3;\quad [-5,1]$

a) $f'(x)=6x-6x^2$

b) $f'(x)$ exists for all real numbers. Solve:

$6x-6x^2=0$

$6x(1-x)=0$

$x=0 \quad \text{or} \quad x=1$

c) The critical value and the endpoints are $-5,\ 0,$ and 1. Note, since 1 is an endpoint of the interval, $x=1$ is included in this list as an endpoint, not a critical value.

d) Evaluate $f(x)$ for each value in (c)

$f(-5)=3(-5)^2-2(-5)^3=325$

$f(0)=3(0)^2-2(0)^3=0$

$f(1)=3(1)^2-2(1)^3=1$

On the interval $[-5,1]$, the absolute maximum is 325, which occurs at $x=-5$. The absolute minimum is 0, which occurs at $x=0$.

23. $f(x)=1-x^3;\quad [-8,8]$

a) $f'(x)=-3x^2$

b) $f'(x)$ exists for all real numbers. Solve:

$-3x^2=0$

$x=0$

c) The critical value and the endpoints are $-8,\ 0,$ and 8.

d) Evaluate $f(x)$ for each value in (c)

$f(-8)=1-(-8)^3=513$

$f(0)=1-(0)^3=1$

$f(8)=1-(8)^3=-511$

On the interval $[-8,8]$, the absolute maximum is 513, which occurs at $x=-8$. The absolute minimum is -511, which occurs at $x=8$.

24. $f(x) = 2x^3; \quad [-10,10]$

a) $f'(x) = 6x^2$

b) $f'(x)$ exists for all real numbers. Solve:

$$6x^2 = 0$$
$$x = 0$$

c) The critical value and the endpoints are $-10,\ 0,\ \text{and } 10$.

d) Evaluate $f(x)$ for each value in (c)

$$f(-10) = 2(-10)^3 = -2000$$
$$f(0) = 2(0)^3 = 0$$
$$f(10) = 2(10)^3 = 2000$$

On the interval $[-10,10]$, the absolute maximum is 2000, which occurs at $x = 10$. The absolute minimum is -2000, which occurs at $x = -10$.

25. $f(x) = 12 + 9x - 3x^2 - x^3; \quad [-3,1]$

a) $f'(x) = 9 - 6x - 3x^2$

b) $f'(x)$ exists for all real numbers. Solve:

$$9 - 6x - 3x^2 = 0$$
$$x^2 + 2x - 3 = 0 \qquad \text{Divide by } -3.$$
$$(x+3)(x-1) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -3 \quad \text{or} \quad x = 1$$

c) The critical values and the endpoints are -3 and 1. Note, since the possible critical values are the endpoints of the interval, they are included in this list as endpoints, not as critical values.

d) Evaluate $f(x)$ for each value in (c)

$$f(-3) = 12 + 9(-3) - 3(-3)^2 - (-3)^3 = -15$$
$$f(1) = 12 + 9(1) - 3(1)^2 - (1)^3 = 17$$

On the interval $[-3,1]$, the absolute maximum is 17, which occurs at $x = 1$. The absolute minimum is -15, which occurs at $x = -3$.

26. $f(x) = x^3 - 6x^2 + 10; \quad [0,4]$

a) $f'(x) = 3x^2 - 12x$

b) $f'(x)$ exists for all real numbers. Solve:

$$3x^2 - 12x = 0$$
$$3x(x-4) = 0$$
$$x = 0 \quad \text{or} \quad x = 4$$

c) The critical values and the endpoints are 0 and 4. Note, since the possible critical values are the endpoints of the interval, they are included in this list as endpoints, not as critical values.

d) Evaluate $f(x)$ for each value in (c)

$$f(0) = (0)^3 - 6(0)^2 + 10 = 10$$
$$f(4) = (4)^3 - 6(4)^2 + 10 = -22$$

On the interval $[0,4]$, the absolute maximum is 10, which occurs at $x = 0$. The absolute minimum is -2, which occurs at $x = 4$.

27. $f(x) = x^4 - 2x^3; \quad [-2,2]$

a) $f'(x) = 4x^3 - 6x^2$

b) $f'(x)$ exists for all real numbers. Solve:

$$4x^3 - 6x^2 = 0$$
$$2x^2(2x-3) = 0$$
$$2x^2 = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$x = 0 \quad \text{or} \quad x = \frac{3}{2}$$

c) The critical values and the endpoints are $-2,\ 0,\ \frac{3}{2},\ \text{and } 2$.

d) Evaluate $f(x)$ for each value in (c)

$$f(-2) = (-2)^4 - 2(-2)^3 = 32$$
$$f(0) = (0)^4 - 2(0)^3 = 0$$
$$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^4 - 2\left(\frac{3}{2}\right)^3 = -\frac{27}{16} = -1.6875$$
$$f(2) = (2)^4 - 2(2)^3 = 0$$

On the interval $[-2,2]$, the absolute maximum is 32, which occurs at $x = -2$. The absolute minimum is $-\frac{27}{16}$, which occurs at $x = \frac{3}{2}$.

28. $f(x) = x^3 - x^4; \quad [-1,1]$

a) $f'(x) = 3x^2 - 4x^3$

b) $f'(x)$ exists for all real numbers. Solve:

$$3x^2 - 4x^3 = 0$$
$$x^2(3-4x) = 0$$
$$x^2 = 0 \quad \text{or} \quad 3-4x = 0$$
$$x = 0 \quad \text{or} \quad x = \frac{3}{4}$$

c) The critical values and the endpoints are $-1,\ 0,\ \frac{3}{4}$, and 1.

d) Evaluate $f(x)$ for each value in (c)

$$f(-1) = (-1)^3 - (-1)^4 = -2$$
$$f(0) = (0)^3 - (0)^4 = 0$$
$$f\left(\frac{3}{4}\right) = \left(\frac{3}{4}\right)^3 - \left(\frac{3}{4}\right)^4 = \frac{27}{256} \approx 0.105$$
$$f(1) = (1)^3 - (1)^4 = 0$$

e) On the interval $[-1,1]$, the absolute maximum is $\frac{27}{256}$, which occurs at $x = \frac{3}{4}$. The absolute minimum is -2, which occurs at $x = -1$.

29. $f(x) = x^4 - 2x^2 + 5; \qquad [-2,2]$

a) $f'(x) = 4x^3 - 4x$

b) $f'(x)$ exists for all real numbers. Solve:

$$4x^3 - 4x = 0$$
$$4x(x^2 - 1) = 0$$
$$4x = 0 \quad \text{or} \quad x^2 - 1 = 0$$
$$x = 0 \quad \text{or} \quad x = \pm 1$$

c) The critical values and the endpoints are $-2,\ -1,\ 0,\ 1$, and 2.

d) Evaluate $f(x)$ for each value in (c)

$$f(-2) = (-2)^4 - 2(-2)^2 + 5 = 13$$
$$f(-1) = (-1)^4 - 2(-1)^2 + 5 = 4$$
$$f(0) = (0)^4 - 2(0)^2 + 5 = 5$$
$$f(1) = (1)^4 - 2(1)^2 + 5 = 4$$
$$f(2) = (2)^4 - 2(2)^2 + 5 = 13$$

On the interval $[-2,2]$, the absolute maximum is 13, which occurs at $x = -2$ and $x = 2$. The absolute minimum is 4, which occurs at $x = -1$ and $x = 1$.

30. $f(x) = x^4 - 8x^2 + 3; \qquad [-3,3]$

a) $f'(x) = 4x^3 - 16x$

b) $f'(x)$ exists for all real numbers. Solve:

$$4x^3 - 16x = 0$$
$$4x(x^2 - 4) = 0$$
$$4x = 0 \quad \text{or} \quad x^2 - 4 = 0$$
$$x = 0 \quad \text{or} \quad x = \pm 2$$

c) The critical values and the endpoints are $-3,\ -2,\ 0,\ 2$, and 3.

d) Evaluate $f(x)$ for each value in (c)

$$f(-3) = (-3)^4 - 8(-3)^2 + 3 = 12$$
$$f(-2) = (-2)^4 - 8(-2)^2 + 3 = -13$$
$$f(0) = (0)^4 - 8(0)^2 + 3 = 3$$
$$f(2) = (2)^4 - 8(2)^2 + 3 = -13$$
$$f(3) = (3)^4 - 8(3)^2 + 3 = 12$$

On the interval $[-3,3]$, the absolute maximum is 12, which occurs at $x = -3$ and $x = 3$. The absolute minimum is -13, which occurs at $x = -2$ and $x = 2$.

31. $f(x) = (x+3)^{2/3} - 5; \qquad [-4,5]$

a) $f'(x) = \frac{2}{3}(x+3)^{-1/3} = \frac{2}{3(x+3)^{1/3}}$

b) $f'(x)$ does not exist for $x = -3$. The equation $f'(x) = 0$ has no solution, so $x = -3$ is the only critical value.

c) The critical values and the endpoints are $-4,\ -3$, and 5.

d) Evaluate $f(x)$ for each value in (c)

$$f(-4) = ((-4)+3)^{2/3} - 5 = -4$$
$$f(-3) = ((-3)+3)^{2/3} - 5 = -5$$
$$f(5) = ((5)+3)^{2/3} - 5 = -1$$

On the interval $[-4,5]$, the absolute maximum is -1, which occurs at $x = 5$. The absolute minimum is -5, which occurs at $x = -3$.

32. $f(x) = 1 - x^{2/3}; \qquad [-8,8]$

a) $f'(x) = -\frac{2}{3}x^{-1/3} = -\frac{2}{3x^{1/3}}$

b) $f'(x)$ does not exist for $x=0$. The equation $f'(x)=0$ has no solution, so $x=0$ is the only critical value.

c) The critical values and the endpoints are $-8, 0,$ and 8.

d) Evaluate $f(x)$ for each value in (c)

$$f(-8)=1-(-8)^{2/3}=-3$$

$$f(0)=1-(0)^{2/3}=1$$

$$f(8)=1-(8)^{2/3}=-3$$

On the interval $[-8,8]$, the absolute maximum is 1, which occurs at $x=0$. The absolute minimum is -3, which occurs at $x=-8$ and $x=8$.

33. $f(x)=x+\dfrac{1}{x}$; $\quad [1,20]$

a) $f'(x)=1-x^{-2}=1-\dfrac{1}{x^2}$

b) $f'(x)$ does not exist for $x=0$. However, $x=0$ is not in the interval. Solve $f'(x)=0$.

$$1-\frac{1}{x^2}=0$$

$$1=\frac{1}{x^2}$$

$$x^2=1$$

$$x=\pm 1$$

The critical value $x=-1$ is not in the interval, and the other critical value is an endpoint.

c) The critical values and the endpoints are 1 and 20.

d) Evaluate $f(x)$ for each value in (c)

$$f(1)=1+\frac{1}{1}=2$$

$$f(20)=20+\frac{1}{20}=\frac{401}{20}=20.05$$

On the interval $[1,20]$, the absolute maximum is 20.05, which occurs at $x=20$. The absolute minimum is 2, which occurs at $x=1$.

34. $f(x)=x+\dfrac{4}{x}$; $\quad [-8,-1]$

a) $f'(x)=1-4x^{-2}=1-\dfrac{4}{x^2}$

b) $f'(x)$ does not exist for $x=0$. However, $x=0$ is not in the interval. Solve $f'(x)=0$.

$$1-\frac{4}{x^2}=0$$

$$1=\frac{4}{x^2}$$

$$x^2=4$$

$$x=\pm 2$$

The only critical value in the interval is at $x=-2$

c) The critical values and the endpoints are $-8,\ -2$ and -1.

d) Evaluate $f(x)$ for each value in (c)

$$f(-8)=(-8)+\frac{4}{(-8)}=-\frac{17}{2}=8.5$$

$$f(-2)=(-2)+\frac{4}{(-2)}=-4$$

$$f(-1)=(-1)+\frac{4}{(-1)}=-5$$

On the interval $[-8,-1]$, the absolute maximum is -4, which occurs at $x=-2$. The absolute minimum is $-\dfrac{17}{2}$, which occurs at $x=-8$.

35. $f(x)=\dfrac{x^2}{x^2+1}$; $\quad [-2,2]$

a) $f'(x)=\dfrac{(x^2+1)(2x)-x^2(2x)}{(x^2+1)^2}$ Quotient Rule

$$=\frac{2x^3+2x-2x^3}{(x^2+1)^2}$$

$$=\frac{2x}{(x^2+1)^2}$$

b) $f'(x)$ exists for all real numbers. Solve:

$$f'(x)=0$$

$$\frac{2x}{(x^2+1)^2}=0$$

$$2x=0$$

$$x=0$$

c) The critical values and the endpoints are $-2,\ 0,$ and 2.

d) Evaluate $f(x)$ for each value in (c)

$$f(-2)=\frac{(-2)^2}{(-2)^2+1}=\frac{4}{5}$$

$$f(0)=\frac{(0)^2}{(0)^2+1}=0$$

$$f(2)=\frac{(2)^2}{(2)^2+1}=\frac{4}{5}$$

On the interval $[-2,2]$, the absolute maximum is $\frac{4}{5}$, which occurs at $x=-2$ and $x=2$. The absolute minimum is 0, which occurs at $x=0$.

36. $f(x)=\frac{4x}{x^2+1}$; $[-3,3]$

a) $$f'(x)=\frac{(x^2+1)(4)-4x(2x)}{(x^2+1)^2}$$
$$=\frac{4-4x^2}{(x^2+1)^2}$$

b) $f'(x)$ exists for all real numbers. Solve:

$$\frac{4-4x^2}{(x^2+1)^2}=0$$
$$4-4x^2=0$$
$$x^2-1=0$$
$$x=\pm1$$

c) The critical values and the endpoints are $-3,\ -1,\ 1$, and 3.

d) Evaluate $f(x)$ for each value in (c)

$$f(-3)=\frac{4(-3)}{(-3)^2+1}=-\frac{6}{5}$$

$$f(-1)=\frac{4(-1)}{(-1)^2+1}=-2$$

$$f(1)=\frac{4(1)}{(1)^2+1}=2$$

$$f(3)=\frac{4(3)}{(3)^2+1}=\frac{6}{5}$$

On the interval $[-3,3]$, the absolute maximum is 2, which occurs at $x=1$. The absolute minimum is -2, which occurs at $x=-1$.

37. $f(x)=(x+1)^{1/3}$; $[-2,26]$

a) $$f'(x)=\frac{1}{3}(x+1)^{-2/3}=\frac{1}{3(x+1)^{2/3}}$$

b) $f'(x)$ does not exist for $x=-1$. The equation $f'(x)=0$ has no solution, so $x=-1$ is the only critical value.

c) The critical values and the endpoints are $-2,\ -1$, and 26.

d) Evaluate $f(x)$ for each value in (c)

$$f(-2)=((-2)+1)^{1/3}=-1$$

$$f(-1)=((-1)+1)^{1/3}=0$$

$$f(26)=((26)+1)^{1/3}=3$$

On the interval $[-2,26]$, the absolute minimum is -1, which occurs at $x=-2$. The absolute maximum is 3, which occurs at $x=26$.

38. $f(x)=\sqrt[3]{x}=x^{1/3}$; $[8,64]$

a) $$f'(x)=\frac{1}{3}x^{-2/3}=\frac{1}{3x^{2/3}}$$

b) There are no critical values in the interval.

c) The endpoints are 8 and 64.

d) Evaluate $f(x)$ for each value in (c)

$$f(8)=(8)^{1/3}=2$$

$$f(64)=(64)^{1/3}=4$$

On the interval $[8,64]$, the absolute maximum is 4, which occurs at $x=64$. The absolute minimum is 2, which occurs at $x=8$.

39. – 48. Left to the student.

49. $f(x)=12x-x^2$

When no interval is specified , we use the real line $(-\infty,\infty)$.

a) Find $f'(x)$

$$f'(x)=12-2x$$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve

$f'(x)=0$.

$12-2x=0$

$12=2x$

$6=x$

The only critical value is $x=6$.

c) Since there is only one critical value, we can apply Max-Min Principle 2. First we find $f''(x)$.

$f''(x)=-2$.

The second derivative is constant, so $f''(6)=-2$. Since the second derivative is negative at 6, we have a maximum at $x=6$. Next, we find the function value at $x=6$.

$f(6)=12(6)-(6)^2=36$

Therefore, the absolute maximum is 36, which occurs at $x=6$. The function has no minimum value.

50. $f(x)=30x-x^2;\qquad(-\infty,\infty)$

a) $f'(x)=30-2x$

b) $f'(x)$ exists for all real numbers. Solve

$f'(x)=0$

$30-2x=0$

$x=15$

c) $f''(x)=-2$

$f''(15)=-2<0$

Thus, we have a maximum at $x=15$.

$f(15)=30(15)-(15)^2=225$

Therefore, the absolute maximum is 225, which occurs at $x=15$. There is no minimum value.

51. $f(x)=2x^2-40x+270$

When no interval is specified , we use the real line $(-\infty,\infty)$.

a) Find $f'(x)$

$f'(x)=4x-40$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve

$f'(x)=0$.

$4x-40=0$

$4x=40$

$x=10$

The only critical value is $x=10$. .

c) Since there is only one critical value, we can apply Max-Min Principle 2. First we find $f''(x)$.

$f''(x)=4$.

The second derivative is constant, so $f''(10)=4$. Since the second derivative is positive at 10, we have a minimum at $x=10$. Next, we find the function value at $x=10$.

$f(10)=2(10)^2-40(10)+270=70$

Therefore, the absolute minimum is 70, which occurs at $x=10$. The function has no maximum value.

52. $f(x)=2x^2-20x+340;\qquad(-\infty,\infty)$

a) $f'(x)=4x-20$

b) $f'(x)$ exists for all real numbers. Solve

$f'(x)=0$

$4x-20=0$

$x=5$

c) $f''(x)=4$

$f''(5)=4>0$

Thus, we have a minimum at $x=5$.

$f(5)=2(5)^2-20(5)+340=290$

Therefore, the absolute minimum is 290, which occurs at $x=5$. There is no maximum value.

53. $f(x)=x-\frac{4}{3}x^3;\qquad(0,\infty)$

a) Find $f'(x)$

$f'(x)=1-4x^2$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve

$f'(x)=0$.

$1-4x^2=0$

$-4x^2=-1$

$x^2=\frac{1}{4}$

$x=\pm\sqrt{\frac{1}{4}}$

$x=\pm\frac{1}{2}$

There are two critical values; however, the only critical value in $(0,\infty)$ is $x=\frac{1}{2}$.

c) Since there is only one critical value in the interval, we can apply Max-Min Principle 2. First we find $f''(x)$.

$f''(x)=-8x$.

The second derivative is constant, so $f''\left(\frac{1}{2}\right)=-4$. Since the second derivative is negative at $\frac{1}{2}$, we have a maximum at $x=\frac{1}{2}$. Next, we find the function value at $x=\frac{1}{2}$.

$$f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)-\frac{4}{3}\left(\frac{1}{2}\right)^3=\frac{1}{3}$$

Therefore, the absolute maximum is $\frac{1}{3}$, which occurs at $x=\frac{1}{2}$. The function has no minimum value.

54. $f(x)=16x-\frac{4}{3}x^3;\qquad (0,\infty)$

a) $f'(x)=16-4x^2$

b) $f'(x)$ exists for all real numbers. Solve

$$\begin{aligned} f'(x)&=0\\ 16-4x^2&=0\\ 4x^2&=16\\ x^2&=4\\ x&=\pm 2 \end{aligned}$$

$x=2$ is the only critical value on the interval $(0,\infty)$.

c) $f''(x)=-8x$

$f''(2)=-16<0$

Thus, we have a maximum at $x=2$.

$$f(2)=16(2)-\frac{4}{3}(2)^3=\frac{64}{3}$$

Therefore, the absolute maximum is $\frac{64}{3}$, which occurs at $x=2$. There is no minimum value.

55. $f(x)=x(60-x)=60x-x^2$

When no interval is specified , we use the real line $(-\infty,\infty)$.

a) Find $f'(x)$

$f'(x)=60-2x$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve $f'(x)=0$.

$$\begin{aligned} 60-2x&=0\\ 60&=2x\\ 30&=x \end{aligned}$$

The only critical value is $x=30$.

c) Since there is only one critical value, we can apply Max-Min Principle 2. First we find $f''(x)$.

$f''(x)=-2$.

The second derivative is constant, so $f''(30)=-2$. Since the second derivative is negative at 30, we have a maximum at $x=30$. Next, we find the function value at $x=30$.

$f(30)=30(60-30)=900$

Therefore, the absolute maximum is 900, which occurs at $x=30$. The function has no minimum value.

56. $f(x)=x(25-x)=25x-x^2;\qquad (-\infty,\infty)$

a) $f'(x)=25-2x$

b) $f'(x)$ exists for all real numbers. Solve

$$\begin{aligned} f'(x)&=0\\ 25-2x&=0\\ x&=\frac{25}{2} \end{aligned}$$

c) $f''(x)=-2$

$f''\left(\frac{25}{2}\right)=-2<0$

Thus, we have a maximum at $x=\frac{25}{2}$.

$$f\left(\frac{25}{2}\right)=\left(\frac{25}{2}\right)\left(25-\frac{25}{2}\right)=\frac{625}{4}=156.25$$

Therefore, the absolute maximum is $\frac{625}{4}$, which occurs at $x=\frac{25}{2}$. There is no minimum value.

57. $f(x)=\frac{1}{3}x^3-3x;\qquad [-2,2]$

a) Find $f'(x)$

$f'(x)=x^2-3$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve

$f'(x)=0$.

$x^2-3=0$

$x^2=3$

$x=\pm\sqrt{3}\approx\pm1.732$

Both critical values are in the interval $[-2,2]$.

c) The interval is closed and there is more than one critical value, so we use Max-Min Principle 1.
The critical points and the endpoints are $-2,\ -\sqrt{3},\ \sqrt{3}$, and 2.
Next, we find the function values at these points.

$$f(-2)=\frac{1}{3}(-2)^3-3(-2)=\frac{10}{3}=3.3\overline{3}$$

$$f(-\sqrt{3})=\frac{1}{3}(-\sqrt{3})^3-3(-\sqrt{3})$$
$$=-\sqrt{3}+3\sqrt{3}=2\sqrt{3}\approx3.464$$

$$f(\sqrt{3})=\frac{1}{3}(\sqrt{3})^3-3(\sqrt{3})$$
$$=\sqrt{3}-3\sqrt{3}=-2\sqrt{3}\approx-3.464$$

$$f(2)=\frac{1}{3}(2)^3-3(2)=-\frac{10}{3}=-3.3\overline{3}$$

The largest of these values, $2\sqrt{3}$, is the maximum. It occurs at $x=-\sqrt{3}$. The smallest of these values, $-2\sqrt{3}$, is the minimum. It occurs at $x=\sqrt{3}$.
Thus, the absolute maximum over the interval $[-2,2]$, is $2\sqrt{3}$, which occurs at $x=-\sqrt{3}$, and the absolute minimum over $[-2,2]$ is $-2\sqrt{3}$, which occurs at $x=\sqrt{3}$.

58. $f(x)=\frac{1}{3}x^3-5x;\qquad [-3,3]$

a) $f'(x)=x^2-5$

b) $f'(x)$ exists for all real numbers. Solve

$f'(x)=0$

$x^2-5=0$

$x^2=5$

$x=\pm\sqrt{5}$

Both critical values are in the interval $[-3,3]$.

c) The critical points and the endpoints are $-3,\ -\sqrt{5},\ \sqrt{5}$, and 3.
Next, we find the function values at these points.

$$f(-3)=\frac{1}{3}(-3)^3-5(-3)=6$$

$$f(-\sqrt{5})=\frac{1}{3}(-\sqrt{5})^3-5(-\sqrt{5})=\frac{10\sqrt{5}}{3}\approx7.454$$

$$f(\sqrt{5})=\frac{1}{3}(\sqrt{5})^3-5(\sqrt{5})=-\frac{10\sqrt{5}}{3}\approx-7.454$$

$$f(3)=\frac{1}{3}(3)^3-5(3)=-6$$

Thus, the absolute maximum over the interval $[-3,3]$, is $\frac{10\sqrt{5}}{3}$, which occurs at $x=-\sqrt{5}$, and the absolute minimum over $[-3,3]$ is $-\frac{10\sqrt{5}}{3}$, which occurs at $x=\sqrt{5}$.

59. $f(x)=-0.001x^2+4.8x-60$

When no interval is specified , we use the real line $(-\infty,\infty)$.

a) Find $f'(x)$

$f'(x)=-0.002x+4.8$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve

$f'(x)=0$.

$-0.002x+4.8=0$

$-0.002x=-4.8$

$x=2400$

The only critical value is $x=2400$.

c) Since there is only one critical value, we can apply Max-Min Principle 2. First we find $f''(x)$.

$f''(x)=-0.002$.

The second derivative is constant, so $f''(2400)=-0.002$.

Since the second derivative is negative at 2400, we have a maximum at $x = 2400$. Next, we find the function value at $x = 2400$.

$$f(2400) = -0.001(2400)^2 + 4.8(2400) - 60$$
$$= 5700$$

Therefore, the absolute maximum is 5700, which occurs at $x = 2400$. The function has no minimum value.

60. $f(x) = -0.01x^2 + 1.4x - 30$

a) $f'(x) = -0.02x + 1.4$

b) $f'(x)$ exists for all real numbers. Solve

$$f'(x) = 0$$
$$-0.02x + 1.4 = 0$$
$$x = 70$$

c) $f''(x) = -0.02$

$f''(70) = -0.02 < 0$

Thus, we have a maximum at $x = 70$.

$f(70) = -0.01(70)^2 + 1.4(70) - 30 = 19$

Therefore, the absolute maximum is 19, which occurs at $x = 70$. There is no minimum value.

61. $f(x) = -\frac{1}{3}x^3 + 6x^2 - 11x - 50; \quad (0,3)$

a) Find $f'(x)$

$f'(x) = -x^2 + 12x - 11$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve $f'(x) = 0$.

$$-x^2 + 12x - 11 = 0$$
$$x^2 - 12x + 11 = 0$$
$$(x-11)(x-1) = 0$$
$$x - 11 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = 11 \quad \text{or} \quad x = 1$$

c) The interval $(0,3)$ is not closed. The only critical value in the interval is $x = 1$. Therefore, we can apply Max-Min Principle 2. First, we find the second derivative.

$f''(x) = -2x + 12$

Evaluating the second derivative at $x = 1$, we have:

$f''(x) = -2(1) + 12 = 10 > 0$.

Since the second derivative is positive when $x = 1$, there is a minimum at $x = 1$.

Next, find the function value at $x = 1$.

$$f(1) = -\frac{1}{3}(1)^3 + 6(1)^2 - 11(1) - 50$$
$$= -\frac{166}{3} = -55.33\overline{3}$$

Thus, the absolute minimum over the interval $(0,3)$ is $-\frac{166}{3}$, which occurs at $x = 1$.

62. $f(x) = -x^3 + x^2 + 5x - 1; \quad (0,\infty)$

a) $f'(x) = -3x^2 + 2x + 5$

b) $f'(x)$ exists for all real numbers. Solve

$$f'(x) = 0$$
$$-3x^2 + 2x + 5 = 0$$
$$-(3x-5)(x+1) = 0$$
$$3x - 5 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = \frac{5}{3} \quad \text{or} \quad x = -1$$

$x = \frac{5}{3}$ is the only critical value on the interval $(0,\infty)$.

c) $f''(x) = -6x + 2$

$$f''\left(\frac{5}{3}\right) = -6\left(\frac{5}{3}\right) + 2 = -8 < 0$$

Therefore, a maximum occurs at $x = \frac{5}{3}$.

$$f\left(\frac{5}{3}\right) = -\left(\frac{5}{3}\right)^3 + \left(\frac{5}{3}\right)^2 + 5\left(\frac{5}{3}\right) - 1 = \frac{148}{27}$$

Thus, the absolute maximum over the interval $(0,\infty)$ is $\frac{148}{27}$, which occurs at $x = \frac{5}{3}$. The function has no minimum value.

63. $f(x) = 15x^2 - \frac{1}{2}x^3; \quad [0,30]$

a) Find $f'(x)$

$$f'(x) = 30x - \frac{3}{2}x^2$$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve
$f'(x)=0$.

$$30x-\frac{3}{2}x^2=0$$
$$60x-3x^2=0$$
$$3x(20-x)=0$$
$$3x=0 \quad \text{or} \quad 20-x=0$$
$$x=0 \quad \text{or} \quad x=20$$

Both critical values are in the interval $[0,30]$.

c) Since the interval is closed and there is more than one critical value, we apply the Max-Min Principle 1.
The critical values and the endpoints are $0,\ 20,$ and 30.
Next, we find the function values.

$$f(0)=15(0)^2-\frac{1}{2}(0)^3=0$$
$$f(20)=15(20)^2-\frac{1}{2}(20)^3=2000$$
$$f(30)=15(30)^2-\frac{1}{2}(30)^3=0$$

The largest of these values, 2000, is the maximum. It occurs at $x=20$. The smallest of these values, 0, is the minimum. It occurs at $x=0$ and $x=30$.
Thus, the absolute maximum over the interval $[0,30]$, is 2000, which occurs at $x=20$, and the absolute minimum over $[0,30]$ is 0, which occurs at $x=0$ and $x=30$.

64. $f(x)=4x^2-\frac{1}{2}x^3;\qquad [0,8]$

a) $f'(x)=8x-\frac{3}{2}x^2$

b) $f'(x)$ exists for all real numbers, Solve:

$$f'(x)=0$$
$$8x-\frac{3}{2}x^2=0$$
$$16x-3x^2=0$$
$$x(16-3x)=0$$
$$x=0 \quad \text{or} \quad 16x-3=0$$
$$x=0 \quad \text{or} \quad x=\frac{3}{16}$$

c) The critical values and the endpoints are $0,\ \frac{3}{16},$ and 8.

d) Find the function values.

$$f(0)=4(0)^2-\frac{1}{2}(0)^3=0$$
$$f\left(\frac{16}{3}\right)=4\left(\frac{16}{3}\right)^2-\frac{1}{2}\left(\frac{16}{3}\right)^3=\frac{1024}{27}\approx 37.93$$
$$f(8)=4(8)^2-\frac{1}{2}(8)^3=0$$

Therefore, the absolute maximum over the interval $[0,8]$ is $\frac{1024}{27}$ or $37\frac{25}{27}$, which occurs at $x=\frac{16}{3}$. The absolute minimum is 0, which occurs at $x=0$ and $x=8$.

65. $f(x)=2x+\frac{72}{x};\qquad (0,\infty)$

$f(x)=2x+72x^{-1}$

a) Find $f'(x)$

$$f'(x)=2-72x^{-2}=2-\frac{72}{x^2}$$

b) Find the critical values. $f'(x)$ does not exist for $x=0$; however, 0 is not in the interval $(0,\infty)$. Therefore, we solve

$$f'(x)=0$$
$$2-\frac{72}{x^2}=0$$
$$2=\frac{72}{x^2}$$
$$2x^2=72 \qquad \text{Multiplying by } x^2,\text{ since } x\neq 0.$$
$$x^2=36$$
$$x=\pm 6$$

c) The interval $(0,\infty)$ is not closed. The only critical value in the interval is $x=6$. Therefore, we can apply Max-Min Principle 2. First, we find the second derivative.

$$f''(x)=144x^{-3}=\frac{144}{x^3}$$

Evaluating the second derivative at $x=6$, we have:

$$f''(6)=\frac{144}{(6)^3}=\frac{2}{3}>0.$$

Since the second derivative is positive when $x=6$, there is a minimum at $x=6$.

Next, find the function value at $x = 6$.

$$f(6) = 2(6) + \frac{72}{6} = 24$$

Thus, the absolute minimum over the interval $(0,\infty)$ is 24, which occurs at $x = 6$. The function has no maximum value over the interval $(0,\infty)$.

66. $f(x) = x + \dfrac{3600}{x}; \qquad (0,\infty)$

a) $f'(x) = 1 - \dfrac{3600}{x^2}$

b) Find the critical values. $f'(x)$ does not exists for $x = 0$; however, 0 is not in the interval $(0,\infty)$. Therefore, we solve

$$\begin{aligned} f'(x) &= 0 \\ 1 - \frac{3600}{x^2} &= 0 \\ x^2 &= 3600 \\ x &= \pm 60 \end{aligned}$$

c) The interval $(0,\infty)$ is not closed. The only critical value in the interval is $x = 60$.

$$f''(x) = \frac{7200}{x^3}$$

$$f''(60) = \frac{7200}{(60)^3} = \frac{1}{30} > 0 .$$

Since the second derivative is positive when $x = 60$, there is a minimum at $x = 60$.

$$f(60) = (60) + \frac{3600}{60} = 120$$

Thus, the absolute minimum over the interval $(0,\infty)$ is 120, which occurs at $x = 60$. The function has no maximum value over the interval $(0,\infty)$.

67. $f(x) = x^2 + \dfrac{432}{x}; \qquad (0,\infty)$

$f(x) = x^2 + 432x^{-1}$

a) Find $f'(x)$

$$f'(x) = 2x - 432x^{-2} = 2x - \frac{432}{x^2}$$

b) Find the critical values. $f'(x)$ does not exists for $x = 0$; however, 0 is not in the interval $(0,\infty)$.

Therefore, we solve

$$\begin{aligned} f'(x) &= 0 \\ 2x - \frac{432}{x^2} &= 0 \\ 2x &= \frac{432}{x^2} \\ 2x^3 &= 432 \qquad \text{Multiplying by } x^2\text{, since } x \neq 0. \\ x^3 &= 216 \\ x &= 6 \end{aligned}$$

c) The interval $(0,\infty)$ is not closed. The only critical value in the interval is $x = 6$. Therefore, we can apply Max-Min Principle 2. First, we find the second derivative.

$$f''(x) = 2 + 864x^{-3} = 2 + \frac{864}{x^3}$$

Evaluating the second derivative at $x = 6$, we have:

$$f''(6) = 2 + \frac{864}{(6)^3} = 6 > 0 .$$

Since the second derivative is positive when $x = 6$, there is a minimum at $x = 6$.
Next, find the function value at $x = 6$.

$$f(6) = (6)^2 + \frac{432}{6} = 108$$

Thus, the absolute minimum over the interval $(0,\infty)$ is 108, which occurs at $x = 6$. The function has no maximum value over the interval $(0,\infty)$.

68. $f(x) = x^2 + \dfrac{250}{x}; \qquad (0,\infty)$

a) $f'(x) = 2x - \dfrac{250}{x^2}$

b) Find the critical values. $f'(x)$ does not exists for $x = 0$; however, 0 is not in the interval $(0,\infty)$. Therefore, we solve

$$\begin{aligned} f'(x) &= 0 \\ 2x - \frac{250}{x^2} &= 0 \\ 2x &= \frac{250}{x^2} \\ x^3 &= 125 \\ x &= 5 \end{aligned}$$

c) The interval $(0,\infty)$ is not closed. The only critical value in the interval is $x=5$.

$f''(x)=2+\frac{500}{x^3}$

$f''(5)=2+\frac{500}{(5)^3}=6>0$.

There is a minimum at $x=5$.

$f(5)=(5)^2+\frac{250}{5}=75$

Thus, the absolute minimum over the interval $(0,\infty)$ is 75, which occurs at $x=5$. The function has no maximum value over the interval $(0,\infty)$.

69. $f(x)=2x^4-x;\qquad[-1,1]$

a) Find $f'(x)$

$f'(x)=8x^3-1$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve $f'(x)=0$.

$8x^3-1=0$

$8x^3=1$

$x^3=\frac{1}{8}$

$x=\frac{1}{2}$

The only critical value $x=\frac{1}{2}$ is in the interval $[-1,1]$.

c) The interval is closed, and we are looking for both the absolute maximum and absolute minimum values, so we use Max-Min Principle 1.
The critical values and the endpoints are $-1,\ \frac{1}{2},$ and 1.
Next, we find the function values at these points.

$f(-1)=2(-1)^4-(-1)=3$

$f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^4-\left(\frac{1}{2}\right)=-\frac{3}{8}$

$f(1)=2(1)^4-(1)=1$

The largest of these values, 3, is the maximum. It occurs at $x=-1$.

The smallest of these values, $-\frac{3}{8}$, is the minimum. It occurs at $x=\frac{1}{2}$.

Thus, the absolute maximum over the interval $[-1,1]$, is 3, which occurs at $x=-1$, and the absolute minimum over $[-1,1]$ is $-\frac{3}{8}$, which occurs at $x=\frac{1}{2}$.

70. $f(x)=2x^4+x;\qquad[-1,1]$

a) $f'(x)=8x^3+1$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve $f'(x)=0$.

$8x^3+1=0$

$8x^3=-1$

$x^3=-\frac{1}{8}$

$x=-\frac{1}{2}$

The only critical value $x=-\frac{1}{2}$ is in the interval $[-1,1]$.

c) The interval is closed, and we are looking for both the absolute maximum and absolute minimum values, so we use Max-Min Principle 1.
The critical points and the endpoints are $-1,\ -\frac{1}{2},$ and 1.
Next, we find the function values at these points.

$f(-1)=2(-1)^4+(-1)=1$

$f\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^4+\left(-\frac{1}{2}\right)=-\frac{3}{8}$

$f(1)=2(1)^4+(1)=3$

The largest of these values, 3, is the maximum. It occurs at $x=1$. The smallest of these values, $-\frac{3}{8}$, is the minimum. It occurs at $x=-\frac{1}{2}$.

Thus, the absolute maximum over the interval $[-1,1]$, is 3, which occurs at $x=1$., and the absolute minimum over $[-1,1]$ is $-\frac{3}{8}$, which occurs at $x=-\frac{1}{2}$.

71. $f(x)=\sqrt[3]{x}=x^{1/3}$; $[0,8]$

a) Find $f'(x)$

$f'(x)=\frac{1}{3}x^{-2/3}=\frac{1}{3\cdot\sqrt[3]{x^2}}$

b) Find the critical values. $f'(x)$ does not exist for $x=0$. The equation $f'(x)=0$ has no solution, so the only critical value is 0, which is also an endpoint.

c) The interval is closed, and we are looking for both the absolute maximum and absolute minimum values, so we use Max-Min Principle 1.
The only critical value is an endpoint. The endpoints are 0 and 8.
Next, we find the function values at these points.

$f(0)=\sqrt[3]{0}=0$

$f(8)=\sqrt[3]{8}=2$

The largest of these values, 2, is the maximum. It occurs at $x=8$. The smallest of these values, 0, is the minimum. It occurs at $x=0$.
Thus, the absolute maximum over the interval $[0,8]$, is 2, which occurs at $x=8$, and the absolute minimum over $[0,8]$ is 0, which occurs at $x=0$.

72. $f(x)=\sqrt{x}=x^{1/2}$; $[0,4]$

a) $f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

b) Find the critical values. $f'(x)$ does not exist for $x=0$. The equation $f'(x)=0$ has no solution, so the only critical value is 0, which is also an endpoint.

c) The interval is closed, and we are looking for both the absolute maximum and absolute minimum values, so we use Max-Min Principle 1.
The only critical value is an endpoint. The endpoints are 0 and 4.
Next, we find the function values at these points.

$f(0)=\sqrt{0}=0$

$f(4)=\sqrt{4}=2$

The largest of these values, 2, is the maximum. It occurs at $x=4$. The smallest of these values, 0, is the minimum. It occurs at $x=0$. Thus, the absolute maximum over the interval $[0,4]$, is 2, which occurs at $x=4$, and the absolute minimum over $[0,4]$ is 0, which occurs at $x=0$.

73. $f(x)=(x+1)^3$

When no interval is specified, we use the real line $(-\infty,\infty)$.

a) Find $f'(x)$

$f'(x)=3(x+1)^2$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve $f'(x)=0$.

$3(x+1)^2=0$

$x+1=0$

$x=-1$

The only critical value is $x=-1$.

c) Since there is only one critical value, we can apply Max-Min Principle 2. First we find $f''(x)$.

$f''(x)=6(x+1)$.

Now,

$f''(-1)=6((-1)+1)=0$. So the Max-Min Principle 2 fails. We cannot use Max-Min Principle 1, because there are no endpoints.
We note that $f'(x)=3(x+1)^2$ is always positive, except at $x=-1$. Thus, $f(x)$ is increasing everywhere except at $x=-1$.
Therefore, the function has no maximum or minimum over the interval $(-\infty,\infty)$.

Notice

$f''(-2)=-6<0$

$f''(0)=6>0$

and

$f(-1)=0$

Therefore, there is a point of inflection at $(-1,0)$.

74. $f(x)=(x-1)^3$

When no interval is specified , we use the real line $(-\infty,\infty)$.

a) $f'(x)=3(x-1)^2$

b) $f'(x)$ exists for all real numbers. Solve:

$$f'(x)=0$$
$$3(x-1)^2=0.$$
$$x=1$$

The only critical value is $x=1$.

c) $f''(x)=6(x-1)$.

$f''(1)=6((1)-1)=0$. So the Max-Min Principle 2 fails. We note that $f'(x)=3(x-1)^2$ is never negative. Thus, $f(x)$ is increasing everywhere except at $x=1$. Therefore, the function has no maximum or minimum over the interval $(-\infty,\infty)$.

Notice

$f''(0)=-6<0$

$f''(2)=6>0$

and

$f(1)=0$

Therefore, there is a point of inflection at $(1,0)$.

75. $f(x)=2x-3;\quad [-1,1]$

a) Find $f'(x)$

$f'(x)=2$

b) and c)

The derivative exists and is 2 for all real numbers. Therefore, $f'(x)$ is never 0. Thus, there are no critical values. We apply the Max-Min Principle 1. The endpoints are -1 and 1. We find the function values at the endpoints.

$$f(-1)=2(-1)-3=-5$$
$$f(1)=2(1)-3=-1$$

Therefore, the absolute maximum over the interval $[-1,1]$ is -1, which occurs at $x=1$, and the absolute minimum over the interval $[-1,1]$ is -5, which occurs at $x=-1$.

76. $f(x)=9-5x;\quad [-10,10]$

a) $f'(x)=-5$

b) and c)

The derivative exists for all real numbers and is never 0. There are no critical values, so the maximum and minimum occur at the endpoints, -10 and 10. We find the function values at the endpoints.

$$f(-10)=9-5(-10)=59$$
$$f(10)=9-5(10)=-41$$

Therefore, the absolute maximum over the interval $[-10,10]$ is 59, which occurs at $x=-10$, and the absolute minimum over the interval $[-10,10]$ is -41, which occurs at $x=10$.

77. $f(x)=2x-3;\quad [-1,5)$

a) Find $f'(x)$

$f'(x)=2$

b) and c)

The derivative exists and is 2 for all real numbers. Therefore, $f'(x)$ is never 0. Thus, there are no critical values. We apply the Max-Min Principle 1. There is only one endpoint, $x=-1$. We find the function value at the endpoint.

$$f(-1)=2(-1)-3=-5$$

We know $f'(x)>0$ over the interval, so the function is increasing over the interval $[-1,5)$. Therefore, the minimum value will be the left hand endpoint. The absolute minimum over the interval $[-1,5)$ is -5, which occurs at $x=-1$. Since the right endpoint is not included in the interval, the function has no maximum value over the interval $[-1,5)$.

78. $f(x)=9-5x;\quad [-2,3)$

a) $f'(x)=-5$

b) and c)

The derivative exists for all real numbers and is never 0. There are no critical values. The only endpoint is the left endpoint -2. $f'(x)<0$ over the interval, so the function is decreasing and a maximum occurs at $x=-2$. We find the function value at $x=-2$.

$f(-2) = 9 - 5(-2) = 19$.

The absolute maximum over the interval $[-2,3)$ is 19, which occurs at $x = -2$. The function has no minimum value over the interval $[-2,3)$.

79. $f(x) = x^{2/3}$; $[-1,1]$

a) Find $f'(x)$

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3 \cdot \sqrt[3]{x}}$$

b) Find the critical values. $f'(x)$ does not exist for $x = 0$. The equation $f'(x) = 0$ has no solution, so the only critical value is 0.

c) The interval is closed, and we are looking for both the absolute maximum and absolute minimum values, so we use Max-Min Principle 1.
The critical value and the endpoints are -1, 0, and 1.
Next, we find the function values at these points.

$$f(-1) = (-1)^{2/3} = 1$$
$$f(0) = (0)^{2/3} = 0$$
$$f(1) = (1)^{2/3} = 1$$

The largest of these values, 1, is the maximum. It occurs at $x = -1$ and $x = 1$. The smallest of these values, 0, is the minimum. It occurs at $x = 0$.
Thus, the absolute maximum over the interval $[-1,1]$, is 1, which occurs at $x = -1$ and $x = 1$, and the absolute minimum over $[-1,1]$ is 0, which occurs at $x = 0$.

80. $g(x) = x^{2/3}$

When no interval is specified , we use the real line $(-\infty,\infty)$.

a) Find $g'(x)$

$$g'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3 \cdot \sqrt[3]{x}}$$

b) Find the critical values. $g'(x)$ does not exist for $x = 0$. The equation $g'(x) = 0$ has no solution, so the only critical value is 0.

c) We apply the Max-Min Principle 2.

$$g''(x) = -\frac{2}{9x^{4/3}}$$

$g''(0)$ does not exist.
Note that $g'(x) < 0$ for $x < 0$ and $g'(x) > 0$ for $x > 0$, so $g(x)$ is decreasing on $(-\infty,0)$ and increasing on $(0,\infty)$.
Therefore, the absolute minimum over the interval $(-\infty,\infty)$ is 0, which occurs at $x = 0$.
The function has no maximum value.

81. $f(x) = \frac{1}{3}x^3 - x + \frac{2}{3}$

When no interval is specified , we use the real line $(-\infty,\infty)$.

a) Find $f'(x)$

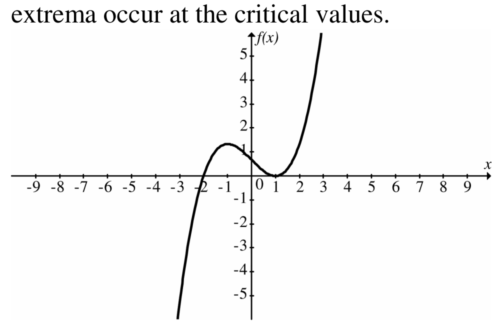
$$f'(x) = x^2 - 1$$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve $f'(x) = 0$.

$$x^2 - 1 = 0$$
$$x^2 = 1$$
$$x = \pm 1$$

There are two critical values -1 and 1.

c) The interval $(-\infty,\infty)$ is not closed, so the Max-Min Principle 1 does not apply. Since there is more than one critical value, the Max-Min Principle 2 does not apply.
A quick sketch of the graph will help us determine whether absolute or relative extrema occur at the critical values.



We determine that the function has no absolute extrema over the interval $(-\infty,\infty)$.

82. $f(x)=\frac{1}{3}x^3-\frac{1}{2}x^2-2x+1$

When no interval is specified , we use the real line $(-\infty,\infty)$.

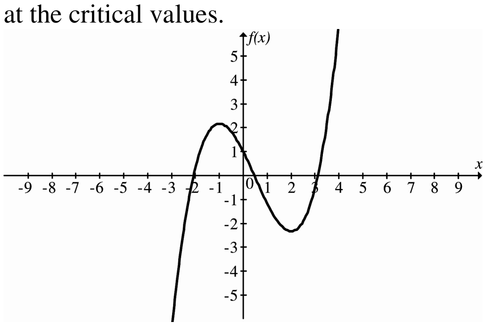
a) $f'(x)=x^2-x-2$

b) $f'(x)$ exists for all real numbers. Solve:

$$f'(x)=0$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$$x=2 \quad \text{or} \quad x=-1$$

There are two critical values -1 and 2.

c) The interval $(-\infty,\infty)$ is not closed, so the Max-Min Principle 1 does not apply. Since there is more than one critical value, the Max-Min Principle 2 does not apply.
A quick sketch of the graph will help us determine absolute or relative extrema occur at the critical values.



We determine that the function has no absolute extrema over the interval $(-\infty,\infty)$.

83. $f(x)=\frac{1}{3}x^3-2x^2+x; \qquad [0,4]$

a) Find $f'(x)$

$f'(x)=x^2-4x+1$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve $f'(x)=0$.

$x^2-4x+1=0$

We use the quadratic formula to solve the equation.

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$$
$$=\frac{4\pm\sqrt{12}}{2}$$
$$=\frac{4\pm2\sqrt{3}}{2}$$
$$=2\pm\sqrt{3}$$

Both critical values $x=2-\sqrt{3}\approx0.27$ and $x=2+\sqrt{3}\approx3.73$ are in the closed interval $[0,4]$.

c) The interval is closed, and there is more than one critical value in the interval, so we use Max-Min Principle 1.
The critical values and the endpoints are $0,\ 2-\sqrt{3},\ 2+\sqrt{3},$ and 4.
Next, we find the function values at these points.

$$f(0)=\frac{1}{3}(0)^3-2(0)^2+(0)=0$$
$$f(2-\sqrt{3})=\frac{1}{3}(2-\sqrt{3})^3-2(2-\sqrt{3})^2+(2-\sqrt{3})$$
$$=-\frac{10}{3}+2\sqrt{3}\approx0.131$$
$$f(2+\sqrt{3})=\frac{1}{3}(2+\sqrt{3})^3-2(2+\sqrt{3})^2+(2+\sqrt{3})$$
$$=-\frac{10}{3}-2\sqrt{3}\approx-6.797$$
$$f(4)=\frac{1}{3}(4)^3-2(4)^2+(4)$$
$$=-\frac{20}{3}\approx-6.66\overline{6}$$

The largest of these values, $-\frac{10}{3}+2\sqrt{3}$, is the maximum. It occurs at $x=2-\sqrt{3}$ The smallest of these values, $-\frac{10}{3}-2\sqrt{3}$, is the minimum. It occurs at $x=2+\sqrt{3}$
Thus, the absolute maximum over the interval $[0,4]$, is $-\frac{10}{3}+2\sqrt{3}$, which occurs at $x=2-\sqrt{3}$, and the absolute minimum over $[0,4]$ is $-\frac{10}{3}-2\sqrt{3}$, which occurs at $x=2+\sqrt{3}$.

84. $g(x)=\frac{1}{3}x^3+2x^2+x; \qquad [-4,0]$

a) $g'(x)=x^2+4x+1$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve $g'(x)=0$.

$x^2+4x+1=0$

We use the quadratic formula to solve the equation.
The solution to the quadratic equation is $x = -2 \pm \sqrt{3}$.
Both critical values $x = -2 - \sqrt{3} \approx -3.73$ and $x = -2 + \sqrt{3} \approx -0.27$ are in the closed interval $[-4, 0]$.

c) The interval is closed, and there is more than one critical value in the interval, so we use Max-Min Principle 1.
The critical values and the endpoints are $-4,\ -2-\sqrt{3},\ -2+\sqrt{3}$, and 0.
Next, we find the function values at these points.

$$g(-4) = \frac{20}{3} \approx 6.66\overline{6}$$
$$g\left(-2-\sqrt{3}\right) = \frac{10}{3} + 2\sqrt{3} \approx 6.797$$
$$g\left(-2+\sqrt{3}\right) = \frac{10}{3} - 2\sqrt{3} \approx -0.131$$
$$g(0) = 0$$

Thus, the absolute maximum over the interval $[-4, 0]$, is $\frac{10}{3} + 2\sqrt{3}$, which occurs at $x = -2 - \sqrt{3}$, and the absolute minimum over $[-4, 0]$ is $\frac{10}{3} - 2\sqrt{3}$, which occurs at $x = -2 + \sqrt{3}$.

85. $t(x) = x^4 - 2x^2$

When no interval is specified, we use the real line $(-\infty, \infty)$.

a) Find $t'(x)$

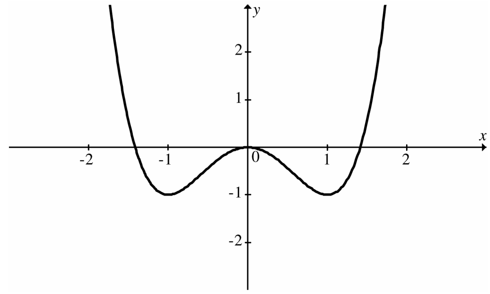
$t'(x) = 4x^3 - 4x$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve $t'(x) = 0$.

$$4x^3 - 4x = 0$$
$$4x(x^2 - 1) = 0$$
$$x = 0 \quad \text{or} \quad x^2 - 1 = 0$$
$$x = 0 \quad \text{or} \quad x = \pm 1$$

There are three critical values -1, 0, and 1.

c) The interval $(-\infty, \infty)$ is not closed, so the Max-Min Principle 1 does not apply. Since there is more than one critical value, the Max-Min Principle 2 does not apply.
A quick sketch of the graph will help us determine whether absolute or relative extrema occur at the critical values.



We determine that the function has no absolute maximum over the interval $(-\infty, \infty)$. The function's absolute minimum is -1, which occurs at $x = -1$ and $x = 1$.

86. $f(x) = 2x^4 - 4x^2 + 2$

When no interval is specified, we use the real line $(-\infty, \infty)$.

a) Find $f'(x)$

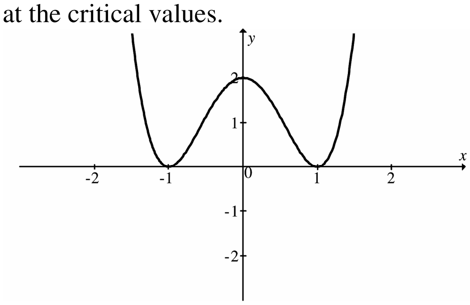
$f'(x) = 8x^3 - 8x$

b) Find the critical values. The derivative exists for all real numbers. Thus, we solve $f'(x) = 0$.

$$8x^3 - 8x = 0$$
$$8x(x^2 - 1) = 0$$
$$x = 0 \quad \text{or} \quad x^2 - 1 = 0$$
$$x = 0 \quad \text{or} \quad x = \pm 1$$

There are three critical values -1, 0, and 1.

c) The interval $(-\infty, \infty)$ is not closed, so the Max-Min Principle 1 does not apply. Since there is more than one critical value, the Max-Min Principle 2 does not apply.
A quick sketch of the graph will help us determine absolute or relative extrema occur at the critical values.



We determine that the function has no absolute maximum over the interval $(-\infty, \infty)$. The function's absolute minimum is 0, which occurs at $x = -1$ and $x = 1$.

87. – 96. Left to the Student.

97. $M(t) = -2t^2 + 100t + 180, \quad 0 \le t \le 40$

a) $M'(t) = -4t + 100$

b) $M'(t)$ exists for all real numbers. We solve

$M'(t) = 0.$

$$-4t + 100 = 0$$
$$4t = 100$$
$$t = 25$$

c) Since there is only one critical value, we apply the Max-Min Principle 2. First, we find the second derivative.

$f''(t) = -4$. The second derivative is negative for all values of t in the interval, therefore, a maximum occurs at $t = 25$.

$$f(25) = -2(25)^2 + 100(25) + 180 = 1430$$

The maximum productivity for $0 \le t \le 40$ is 1430 units per month, which occurs at $t = 25$ years of service.

98. $N(a) = -a^2 + 300a + 6, \quad 0 \le a \le 300$

a) $N'(a) = -2a + 300$

b) $N'(a)$ exists for all real numbers. Solve:

$$N'(a) = 0$$
$$-2a + 300 = 0$$
$$a = 150$$

c) Since there is only one critical value, we apply the Max-Min Principle 2. First, we find the second derivative.

$N''(a) = -2$. The second derivative is negative for all values of t in the interval, therefore, a maximum occurs at $a = 150$.

$$N(150) = -(150)^2 + 300(150) + 6 = 22{,}506$$

The maximum number of units that can be sold is 22,506. In order to achieve this maximum, \$150,000 must be spent on advertising.

99. $p(x) = \dfrac{13x^3 - 240x^2 - 2460x + 585{,}000}{75{,}000}$

$$p(x) = \frac{1}{75{,}000}\left(13x^3 - 240x^2 - 2460x + 585{,}000\right)$$

We restrict our attention to the years 1970 to 2000. That is, we will look at the x-values $0 \le x \le 30$.

a) Find $p'(x)$

$$p'(x) = \frac{1}{75{,}000}\left(39x^2 - 480x - 2460\right)$$

b) Find the critical values. $p'(x)$ exists for all real numbers. Therefore, we solve $p'(x) = 0$.

$$\frac{1}{75{,}000}\left(39x^2 - 480x - 2460\right) = 0$$
$$39x^2 - 480x - 2460 = 0$$

Using the quadratic formula, we have:

$$x = \frac{-(-480) \pm \sqrt{(-480)^2 - 4(39)(-2460)}}{2(39)}$$
$$= \frac{480 \pm \sqrt{614{,}160}}{78}$$

$x \approx -3.89$ or $x \approx 16.20$

Only $x \approx 16.20$ is in the interval $[0,30]$.

c) The critical values and the endpoints are 0, 16.20, and 30.

d) Using a calculator, we find the function values.

$$p(0) \approx 7.8$$
$$p(16.20) \approx 7.17$$
$$p(30) \approx 8.62$$

From 1970 to 2000 the minimum percentage of U.S. national income generated by nonfarm proprietors was 7.17 percent. This occurred about 16.20 years after 1970, or in 1986.

100. $f(x) = 0.025x^2 - 0.71x + 20.44$

We restrict our attention to the years 1970 to 2000. That is, we will look at the x-values $0 \le x \le 30$.

a) $f'(x) = 0.05x - 0.71$

b) $f'(x)$ exists for all real numbers. Solve:

$$f'(x) = 0$$
$$0.05x - 0.71 = 0$$
$$x = 14.2$$

The critical value 14.2 is in the interval.

c) The critical value and the endpoints are 0, 14.2, and 30.

d) Find the function values.

$$f(0)=0.025(0)^2-0.71(0)+20.44 = 20.44$$

$$f(14.2)=0.025(14.2)^2-0.71(14.2)+20.44 = 15.399$$

$$f(30)=0.025(30)^2-0.71(30)+20.44 = 21.64$$

From 1970 through 2000, the minimum percentage the U.S. civilian labor force age 45-54 was about 15.399 percent. This occurred about 14.2 years after 1970, or in 1984.

101. $P(t)=0.000008533t^4-0.001685t^3+0.090t^2-0.687t+4.00,\quad 0\le t\le 90$

a) Find $P'(t)$.

$$P'(t)=0.00003413t^3-0.005055t^2+0.18t-0.687$$

b) $P'(t)$ exists for all real numbers. Solve

$$P'(t)=0$$

$$0.00003413t^3-0.005055t^2+0.18t-0.687=0$$

Using a graphing calculator, we approximate the zeros of $P'(t)$. We find the solutions:

$t\approx 4.3$

$t\approx 49.2$

$t\approx 94.6$

Two of the critical values are in the interval.

c) The critical values and the endpoints are: 0, 4.3, 49.2, and 90.

d) Find the function values.

$P(0)=4.0$

$P(4.3)\approx 2.6$

$P(49.2)\approx 37.4$

$P(90)\approx 2.7$

The absolute maximum production of world wide oil was 37.4 billion barrels. The world achieved this production 49.2 years after 1950, or in the year 1999.

102. $P(x)=\dfrac{1500}{x^2-6x+10}$

We will restrict our analysis to the nonnegative real numbers $[0,\infty)$, since you cannot produce and sell a negative number of amplifiers.

a) $P'(x)=-\dfrac{3000(x-3)}{(x^2-6x+10)^2}$

b) $P'(x)$ exists for all real numbers. Solve

$$P'(x)=0$$

$$-\frac{3000(x-3)}{(x^2-6x+10)^2}=0$$

$$x=3$$

c) Since the interval is open, we apply the Max-Min Principle 2.

$$P''(x)=\frac{3000(3x^2-18x+26)}{(x^2-6x+10)^3}$$

$$P''(3)=-3000$$

Therefore, a maximum occurs at $x=3$. We find the function value at $x=3$.

$$P(3)=\frac{1500}{(3)^2-6(3)+10}=1500$$

Producing and selling 3 amplifiers will result in a maximum weekly profit of $1500.

103. $C(x)=5000+600x$

$R(x)=-\dfrac{1}{2}x^2+1000x,\ 0\le x\le 600$

a) $P(x)=R(x)-C(x)$

$$=-\frac{1}{2}x^2+1000x-(5000+600x)$$

$$=-\frac{1}{2}x^2+400x-5000$$

b) First, we find the critical values.

$P'(x)=-x+400$

$P'(x)$ exists for all real numbers. Solve:

$$P'(x)=0$$

$$-x+400=0$$

$$x=400$$

The critical value is 400 and the endpoints are 0 and 600. Using the Max-Min Principle 1, we have:

$$P(0) = -\frac{1}{2}(0)^2 + 400(0) - 5000$$
$$= -5000$$
$$P(400) = -\frac{1}{2}(400)^2 + 400(400) - 5000$$
$$= 75{,}000$$
$$P(600) = -\frac{1}{2}(600)^2 + 400(600) - 5000$$
$$= 55{,}000$$

The total profit is maximized when 400 items are produced.

104. From Exercise 103, we know that

$$P(x) = -\frac{1}{2}x^2 + 400x - 5000$$

a) $A(x) = \frac{P(x)}{x} = -\frac{1}{2}x + 400 - \frac{5000}{x}$, $0 < x \le 600$.

b) $A'(x) = -\frac{1}{2} + \frac{5000}{x^2}$

$A'(x)$ exists everywhere on $0 < x \le 600$.

Solve $A'(x) = 0$.

$$-\frac{1}{2} + \frac{5000}{x^2} = 0$$
$$x^2 = 10{,}000$$
$$x = \pm 100$$

$x = 100$ is the only critical value in the interval $0 < x \le 600$.

The critical value and the endpoint are 100 and 600.

$A(100) = 300$

$A(600) = 91.67$

The average profit is maximized when 100 items are produced.

105. $B(x) = 305x^2 - 1830x^3, \quad 0 \le x \le 0.16$

a) $B'(x) = 610x - 5490x^2$

b) $B'(x)$ exists for all real numbers. Solve:

$$B'(x) = 0$$
$$610x - 5490x^2 = 0$$
$$610x(1 - 9x) = 0$$
$$x = 0 \quad \text{or} \quad 1 - 9x = 0$$
$$x = 0 \quad \text{or} \quad x = \frac{1}{9} \approx 0.11$$

c) The critical points and the endpoints are $0, \frac{1}{9}$, and 0.16.

d) We find the function values.

$$B(0) = 305(0)^2 - 1830(0)^3 = 0$$
$$B\left(\frac{1}{9}\right) = 305\left(\frac{1}{9}\right)^2 - 1830\left(\frac{1}{9}\right)^3$$
$$= \frac{305}{243} \approx 1.255$$
$$B(0.16) = 305(0.16)^2 - 1830(0.16)^3$$
$$\approx 0.312$$

The maximum blood pressure is approximately 1.255, which occurs at a dose of $x = \frac{1}{9}$ cc, or about 0.11 cc of the drug.

106. $\boxed{tw}$ In finding the absolute extrema of a function, the second derivative is used when there is exactly one critical value interior to an interval. If there is exactly one critical value, the second derivative will determine the concavity of the function on the interval, and thus determine if there is an absolute maximum or absolute minimum on the interval.

107. $g(x) = x\sqrt{x+3}; \qquad [-3,3]$

a) Find $g'(x)$.

$$g'(x) = x\left[\frac{1}{2}(x+3)^{-1/2}(1)\right] + (1)(x+3)^{1/2}$$
$$= \frac{x}{2(x+3)^{1/2}} + (x+3)^{1/2}$$
$$= \frac{x}{2(x+3)^{1/2}} + \frac{(x+3)^{1/2}}{1} \cdot \frac{2(x+3)^{1/2}}{2(x+3)^{1/2}}$$

Multiplying by a form of 1

$$= \frac{x}{2(x+3)^{1/2}} + \frac{2(x+3)}{2(x+3)^{1/2}}$$
$$= \frac{3x+6}{2(x+3)^{1/2}}, \text{ or } \frac{3x+6}{2\sqrt{x+3}}$$

b) Find the critical values. $g'(x)$ exists for all values in $[-3,3]$ except -3. This is a critical value as well as an end point. To find the other critical values, we solve $g'(x)=0$.

$$g'(x)=0$$
$$\frac{3x+6}{2\sqrt{x+3}}=0$$
$$3x+6=0$$
$$3x=-6$$
$$x=-2$$

The second critical value on the interval is -2.

c) On a closed interval, the Max-Min Principle 1 can always be used. The critical values and the endpoints are $-3,\ -2$, and 3.

d) Find the function value at each value in (c).

$$g(-3)=(-3)\sqrt{(-3)+3}=0$$
$$g(-2)=(-2)\sqrt{(-2)+3}=-2$$
$$g(3)=(3)\sqrt{(3)+3}=3\sqrt{6}$$

Thus, the absolute maximum over the interval $[-3,3]$ is $3\sqrt{6}$, which occurs at $x=3$, and the absolute minimum is -2, which occurs at $x=-2$.

108. $h(x)=x\sqrt{1-x};\qquad [0,1]$

a) Find $h'(x)$.

$$h'(x)=x\left(\frac{1}{2}\right)(1-x)^{-1/2}(-1)+(1)(1-x)^{1/2}$$
$$=\frac{-x}{2\sqrt{1-x}}+\sqrt{1-x}$$
$$=\frac{-x}{2\sqrt{1-x}}+\frac{2(1-x)}{2\sqrt{1-x}}$$
$$=\frac{-3x+2}{2\sqrt{1-x}}$$

b) $h'(x)$ does not exist for $x=1$. Solve:

$$h'(x)=0$$
$$\frac{-3x+2}{2\sqrt{1-x}}=0$$
$$-3x+2=0$$
$$x=\frac{2}{3}$$

c) The critical values and the endpoints are $0,\ \frac{2}{3}$, and 1.

d) Find the function values.

$$h(0)=(0)\sqrt{1-(0)}=0$$
$$h\left(\frac{2}{3}\right)=\left(\frac{2}{3}\right)\sqrt{1-\left(\frac{2}{3}\right)}=\frac{2}{3}\sqrt{\frac{1}{3}}=\frac{2}{3\sqrt{3}}$$
$$h(1)=(1)\sqrt{1-(1)}=0$$

Thus, the absolute maximum over the interval $[0,1]$ is $\frac{2}{3\sqrt{3}}$, which occurs at $x=\frac{2}{3}$, and the absolute minimum over the interval is 0, which occurs at $x=0$ and $x=1$.

109. $C(x)=(2x+4)+\left(\frac{2}{x-6}\right),\qquad x>6$

$$=2x+4+2(x-6)^{-1}$$

a) Find $C'(x)$.

$$C'(x)=2-2(x-6)^{-2}(1)$$
$$=2-\frac{2}{(x-6)^2}$$

b) Find the critical values.

$C'(x)$ does not exist for $x=6$; however, this value is not in the domain interval, so it is not a critical value. Solve $C'(x)=0$.

$$2-\frac{2}{(x-6)^2}=0$$
$$2=\frac{2}{(x-6)^2}$$
$$2(x-6)^2=2 \quad \text{Multiplying by } (x-6)^2 \text{ Since } x\neq 6.$$
$$(x-6)^2=1$$
$$x-6=\pm 1 \quad \text{Taking the square root of both sides.}$$
$$x=6\pm 1$$
$$x=5 \quad \text{or} \quad x=7$$

The only critical value in $(6,\infty)$ is 7.

c) Since there is only one critical value, we apply the Max-Min Principle 2.

$$C''(x)=4(x-6)^{-3}=\frac{4}{(x-6)^3}$$
$$C''(7)=\frac{4}{(7-6)^3}=4>0$$

Therefore, since $C''(7) > 0$, there is a minimum at $x = 7$.
The firm should use 7 "quality units" to minimize its total cost of service.

110. $y = (x-a)^2 + (x-b)^2$

a) $\frac{dy}{dx} = 2(x-a) + 2(x-b) = 4x - 2a - 2b$

b) The derivative exists for all real numbers. Solve:

$$\frac{dy}{dx} = 0$$
$$4x - 2a - 2b = 0$$
$$4x = 2a + 2b$$
$$x = \frac{a+b}{2}$$

c) There is only one critical value, we apply the Max-Min Principle 2.

$\frac{d^2y}{dx^2} = 4 > 0$ for all values of *x*.

Thus, *y* is a minimum for $x = \frac{a+b}{2}$.

111. $\boxed{tw}$ The first derivative is used to find the critical values of the function on an interval. For closed intervals, we know that absolute extrema occur at a critical value in the interval, or at an endpoint of the interval. For open intervals, absolute extrema, if they exists, will occur at a critical value in the interval.

112. From Exercise 101, we know

$$P'(t) = 0.00003413t^3 - 0.005055t^2 + 0.18t - 0.687$$

a) We find the maximum rate of change, by finding the critical values of the derivative.

$P''(t) = 0.0001024t^2 - 0.01011t + 0.180$

b) $P''(t)$ exists for all values of *x*. So we use the quadratic formula to solve $P''(t) = 0$.
The solutions are $t \approx 23.3$ or $t \approx 75.4$.
There are two critical values on a closed interval, so we use the Max-Min Principle 1.

c) The critical values and the endpoints are 0, 23.3, 75.4, and 90.

d) Find the function values. Remember, we are finding the maximum of the derivative.

$$P'(0) \approx -0.687$$
$$P'(23.3) \approx 1.194$$
$$P'(75.4) \approx -1.223$$
$$P'(90) \approx -0.552$$

Therefore, the absolute maximum over the interval $[0, 90]$ is 1.194, which occurs when $t = 23.3$.
World wide production of oil was increasing most rapidly 23.3 years after 1950, or in 1973. It was increasing at a rate of 1.194 billion barrels per year.

113. $P(t) = 0.0000000219t^4 - 0.0000167t^3 + 0.00155t^2 + 0.002t + 0.22, \quad 0 \le t \le 110$

a) Find $P'(t)$.

$$P'(t) = 0.0000000876t^3 - 0.0000501t^2 + 0.0031t + 0.002$$

$P'(t)$ exists for all real numbers. Solve $P'(t) = 0$. We use a calculator to find the zeros of $P'(t)$. We estimate the solutions to be:

$$x \approx -0.639$$
$$x \approx 71.333$$
$$x \approx 501.223$$

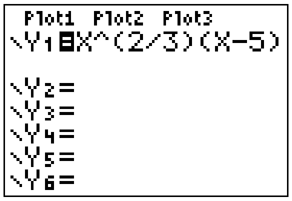
Only one of the critical values, $x \approx 71.333$, is in the interval $[0, 110]$. We apply the Max-Min Principle 1, to find the absolute maximum.
The critical values and the endpoints are 0, 71.333, and 110.
The function values at these points are

$$P(0) = 0.22$$
$$P(71.333) \approx 2.755$$
$$P(110) \approx 0.174$$

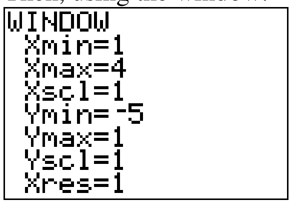
Thus, the absolute maximum oil production for the U.S. after 1910 was 2.755 billion barrels per year. This production level occurred 71.333 year after 1910, or in 1981.

114. $f(x) = x^{2/3}(x-5); \quad [1, 4]$

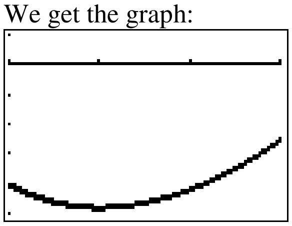
Using a calculator, we enter the equation into the graphing editor:



Then, using the window:



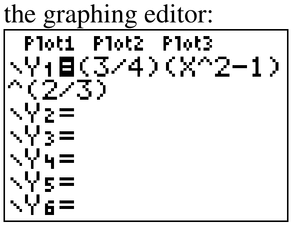
We get the graph:



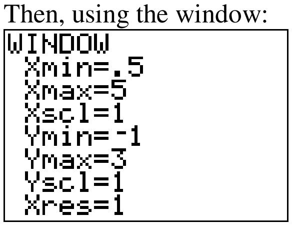
Using the table feature, we locate the extrema. We estimate the absolute maximum to be -2.520, which occurs at $x = 4$, and the absolute minimum to be -4.762, which occurs at $x = 2$.

115. $f(x) = \frac{3}{4}\left(x^2 - 1\right)^{2/3}$; $\left[\frac{1}{2}, \infty\right)$

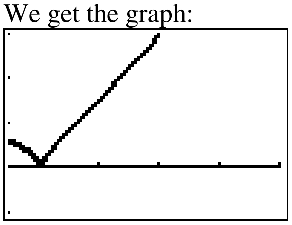
Using a calculator, we enter the equation into the graphing editor:



Then, using the window:



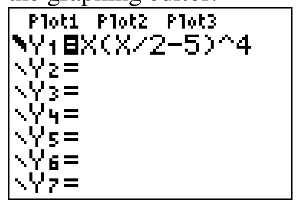
We get the graph:



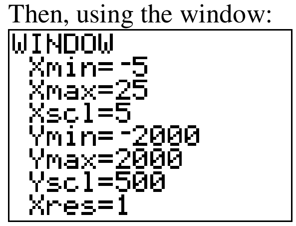
Using the table feature, we locate the extrema. We estimate the absolute minimum to be 0, which occurs at $x = 1$. There is no absolute maximum.

116. $f(x) = x\left(\frac{x}{2} - 5\right)^4$; $\mathbb{R}$

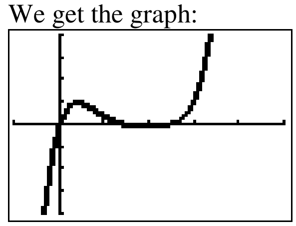
Using a calculator, we enter the equation into the graphing editor:



Then, using the window:



We get the graph:



Looking at the graph, it is clear to see that there are no absolute extrema over the real numbers.

117. a) Using a graphing calculator, we fit the linear equation $y = x + 8.857$. This corresponds to the model $P(t) = t + 8.857$. Where P is the pressure of the contractions and t is the time in minutes. We substitute 7 for t to find the pressure at 7 minutes.

$P(7) = 7 + 8.857 = 15.857$.

The pressure at 7 minutes is 15.857 mm of Hg.

b) Rounding the coefficients to 3 decimal places, we find the quartic regression

$y = 0.117x^4 - 1.520x^3 + 6.193x^2 -$
$7.018x + 10.009$

Changing the variables we get the model

$P(t) = 0.117t^4 - 1.520t^3 + 6.193t^2 -$
$7.018t + 10.009$

Using the table feature, when $x = 7$, $y = 24.857$. So the pressure at 7 minutes is 24.86 mm of mercury. (If we use the rounded coefficients above, we get 23.897 mm of Hg.)

Using the trace feature, we estimate the smallest contraction on the interval $[0,10]$ was about 7.62 mm of Hg. This occurred when $x \approx 0.765$ min.