d) Find the points of inflection. $f''(x)$ exists for all values of x, so the only possible inflection points occur when $f''(x) = 0$.

$18 - 36x + 12x^2 = 0$

$3 - 6x + 2x^2 = 0$ Dividing by 6

Using the quadratic formula we have:

$x = \dfrac{3 \pm \sqrt{3}}{2}$

$x \approx 0.634$ or $x \approx 2.366$

$f(0.634) \approx 2.250$

$f(2.366) \approx 2.250$

The points, $(0.634, 2.250)$ and $(2.366, 2.250)$ are possible inflection points on the graph.

e) To determine concavity we use 0.634 and 2.366 to divide the real number line into three intervals,

$A: (-\infty, 0.634)$, B: $(0.634, 2.366)$, and C: $(2.366, \infty)$

Then test a point in each interval.

A: Test 0,

$f''(0) = 18 - 36(0) + 12(0)^2$
$= 18 > 0$

B: Test 1,

$f''(1) = 18 - 36(1) + 12(1)^2$
$= -6 < 0$

C: Test 3,

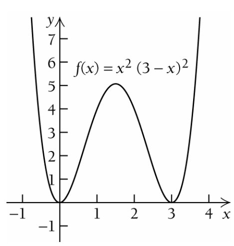
$f''(3) = 18 - 36(3) + 12(3)^2$
$= 18 > 0$

We see that f is concave up on the intervals $(-\infty, 0.634)$ and $(2.366, \infty)$ and concave down on the interval $(0.634, 2.366)$.

Therefore, the points $(0.634, 2.250)$ and $(2.366, 2.250)$ are inflection points.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
-2	100
-1	16
1	4
2	4
4	16



34. $f(x) = x^2(1-x)^2 = x^2 - 2x^3 + x^4$

a) $f'(x) = 2x - 6x^2 + 4x^3$

$f''(x) = 2 - 12x + 12x^2$

b) $f'(x)$ exists for all values of x, so the only critical points of f are where $f'(x) = 0$.

$2x - 6x + 4x^3 = 0$

$2x(1 - 3x + 2x^2) = 0$

$2x(1-x)(1-2x) = 0$

$2x = 0$ or $1 - 2x = 0$ or $1 - x = 0$

$x = 0$ or $x = \dfrac{1}{2}$ or $x = 1$

The critical values are 0, $\dfrac{1}{2}$, and 1.

$f(0) = (0)^2(1-(0))^2 = 0$

$f\left(\dfrac{1}{2}\right) = \left(\dfrac{1}{2}\right)^2\left(1 - \left(\dfrac{1}{2}\right)\right)^2 = \dfrac{1}{16}$

$f(1) = (1)^2(1-(1))^2 = 0$

The critical points $(0,0)$, $\left(\dfrac{1}{2}, \dfrac{1}{16}\right)$, and $(1,0)$ are on the graph.

c) Applying the Second Derivative Test, we have:

$f''(0) = 2 - 12(0) + 12(0)^2 = 2 > 0$

So, the critical point $(0,0)$ is a relative minimum.

$f''\left(\dfrac{1}{2}\right) = 2 - 12\left(\dfrac{1}{2}\right) + 12\left(\dfrac{1}{2}\right)^2 = -1 < 0$

So, the critical point $\left(\dfrac{1}{2}, \dfrac{1}{16}\right)$ is a relative maximum.

$f''(1) = 2 - 12(1) + 12(1)^2 = 2 > 0$

So, the critical point $(1,0)$ is a relative minimum.

We use the points 0, $\frac{1}{2}$, and 1 to divide the real number line into four intervals, $(-\infty,0)$, $\left(0,\frac{1}{2}\right)$, $\left(\frac{1}{2},1\right)$, and $(1,\infty)$, we know that $f(x)$ is decreasing on the intervals $(-\infty,0)$ and $\left(\frac{1}{2},1\right)$, and $f(x)$ is increasing on the intervals $\left(0,\frac{1}{2}\right)$ and $(1,\infty)$.

d) Find the points of inflection. $f''(x)$ exists for all values of *x*, so the only possible inflection points occur when $f''(x)=0$.

$$2-12x+12x^2=0$$
$$1-6x+6x^2=0$$

Using the quadratic formula we have:

$$x=\frac{3\pm\sqrt{3}}{6}$$

$x\approx 0.211$ or $x\approx 0.789$

$f(0.211)\approx 0.028$

$f(0.789)\approx 0.028$

The points, $(0.211,0.028)$ and $(0.789,0.028)$ are possible inflection points on the graph.

e) To determine concavity we use 0.211 and 0.789 to divide the real number line into three intervals, A: $(-\infty,0.211)$, B: $(0.211,0.789)$, and C: $(0.789,\infty)$

Then test a point in each interval.

A: Test 0, $f''(0)=2-12(0)+12(0)^2 = 2>0$

B: Test $\frac{1}{2}$, $f''\left(\frac{1}{2}\right)=2-12\left(\frac{1}{2}\right)+12\left(\frac{1}{2}\right)^2 = -1<0$

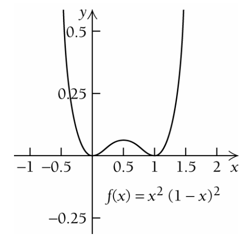
C: Test 1, $f''(1)=2-12(1)+12(1)^2 = 2>0$

We see that *f* is concave up on the intervals $(-\infty,0.211)$ and $(0.789,\infty)$ and concave down on the interval $(0.211,0.789)$.

Therefore, the points $(0.211,0.028)$ and $(0.789,0.028)$ are inflection points.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
–2	36
–1	4
2	4
3	36



35. $f(x)=(x+1)^{2/3}$

a) $f'(x)=\frac{2}{3}(x+1)^{-1/3}=\frac{2}{3(x+1)^{1/3}}$

$$f''(x)=-\frac{2}{9}(x+1)^{-4/3}=-\frac{2}{9(x+1)^{4/3}}$$

The domain of f is $\mathbb{R}$.

b) $f'(x)$ does not exist for $x=-1$. The equation $f'(x)=0$ has no solution, therefore, $x=-1$ is the only critical value.

$f(-1)=(-1+1)^{2/3}=0$.

So, the critical point, $(-1,0)$, is on the graph.

c) We apply the First Derivative Test. We use -1 to divide the real number line into two intervals A: $(-\infty,-1)$ and B: $(-1,\infty)$ and then we test a point in each interval.

A: Test -2,

$$f'(-2)=\frac{2}{3((-2)+1)^{1/3}}=-\frac{2}{3}<0$$

B: Test 0,

$$f'(0)=\frac{2}{3((0)+1)^{1/3}}=\frac{2}{3}>0$$

Thus, $(-1,0)$ is a relative minimum. We also know that $f(x)$ is decreasing on the interval $(-\infty,-1]$ and increasing on the interval $[-1,\infty)$.

d) Find the points of inflection. $f''(x)$ does not exist when $x=-1$. The equation $f''(x)=0$ has no solution, so $x=-1$ is the only possible inflection point. We know that $f(-1)=0$.

e) To determine concavity, we divide the real number line into two intervals, $\text{A}:(-\infty,-1)$ and $\text{B}:(-1,\infty)$ and then we test a point in each interval.

A: Test -2,

$$f''(-2)=-\frac{2}{9((-2)+1)^{4/3}}=-\frac{2}{9}<0$$

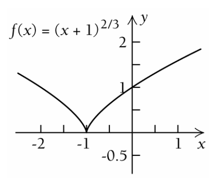
B: Test 0,

$$f''(0)=-\frac{2}{9((0)+1)^{4/3}}=-\frac{2}{9}<0$$

Thus, $f(x)$ is concave down on the interval $(-\infty,-1)$ and on the interval $(-1,\infty)$. Therefore, the point $(-1,0)$ is not an inflection point.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
–9	4
–2	1
0	1
7	4



36. $f(x)=(x-1)^{2/3}$

a) $f'(x)=\frac{2}{3}(x-1)^{-1/3}=\frac{2}{3(x-1)^{1/3}}$

$$f''(x)=-\frac{2}{9}(x-1)^{-4/3}=-\frac{2}{9(x-1)^{4/3}}$$

The domain of f is $\mathbb{R}$.

b) $f'(x)$ does not exist for $x=1$. The equation $f'(x)=0$ has no solution, therefore, $x=1$ is the only critical point.

$f(1)=(1-1)^{2/3}=0$.

So, the critical point, $(1,0)$ is on the graph.

c) We apply the First Derivative Test. We use 1 to divide the real number line into two intervals $\text{A}:(-\infty,1)$ and $\text{B}:(1,\infty)$ and then we test a point in each interval.

A: Test 0, $f'(0)=\frac{2}{3((0)-1)^{1/3}}=-\frac{2}{3}<0$

B: Test 2, $f'(2)=\frac{2}{3((2)-1)^{1/3}}=\frac{2}{3}>0$

$(1,0)$ is a relative minimum. $f(x)$ is decreasing on the interval $(-\infty,1)$ and increasing on the interval $(1,\infty)$.

d) Find the points of inflection. $f''(x)$ does not exist when $x=1$. The equation $f''(x)=0$ has no solution, so $x=1$ is the only possible inflection point. We know that $f(1)=0$.

e) To determine concavity, we divide the real number line into two intervals, $\text{A}:(-\infty,1)$ and $\text{B}:(1,\infty)$ and then we test a point in each interval.

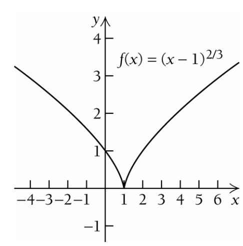
A: Test 0, $f''(0)=-\frac{2}{9((0)-1)^{4/3}}=-\frac{2}{9}<0$

B: Test 2, $f''(2)=-\frac{2}{9((2)-1)^{4/3}}=-\frac{2}{9}<0$

Thus, $f(x)$ is concave down on the intervals $(-\infty,1)$ and $(1,\infty)$. Therefore, the point $(1,0)$ is not an inflection point.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
–7	4
0	1
2	1
9	4



37. $f(x)=(x-3)^{1/3}-1$

a) $f'(x)=\frac{1}{3}(x-3)^{-2/3}=\frac{1}{3(x-3)^{2/3}}$

$$f''(x)=-\frac{2}{9}(x-3)^{-5/3}=-\frac{2}{9(x-3)^{5/3}}$$

The domain of f is $\mathbb{R}$.

b) $f'(x)$ does not exist for $x=3$. The equation $f'(x)=0$ has no solution, therefore, $x=3$ is the only critical value.

$f(3)=((3)-3)^{1/3}-1=-1$.

So, the critical point, $(3,-1)$ is on the graph.

c) We apply the First Derivative Test. We use 3 to divide the real number line into two intervals A: $(-\infty,3)$ and B: $(3,\infty)$ and then we test a point in each interval.

A: Test 2, $f'(2)=\dfrac{1}{3((2)-3)^{2/3}}=\dfrac{1}{3}>0$

B: Test 4, $f'(4)=\dfrac{1}{3((4)-3)^{2/3}}=\dfrac{1}{3}>0$

$f(x)$ is increasing on both intervals $(-\infty,3]$ and $[3,\infty)$, therefore $(3,-1)$ is not a relative extremum.

d) Find the points of inflection. $f''(x)$ does not exist when $x=3$. The equation $f''(x)=0$ has no solution, so at $x=3$ is the only possible inflection point. We know that $f(3)=-1$.

e) To determine concavity, we divide the real number line into two intervals, A: $(-\infty,3)$ and B: $(3,\infty)$ and then we test a point in each interval.

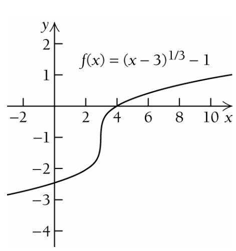
A: Test 2, $f''(2)=-\dfrac{2}{9((2)-3)^{5/3}}=\dfrac{2}{9}>0$

B: Test 4, $f''(4)=-\dfrac{2}{9((4)-3)^{5/3}}=-\dfrac{2}{9}<0$

Thus, $f(x)$ is concave up on the interval $(-\infty,3)$ and $f(x)$ is concave down on the interval $(3,\infty)$. Therefore, the point $(3,-1)$ is an inflection point.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
-5	-3
2	-2
4	0
11	1



38. $f(x)=(x-2)^{1/3}+3$

a) $f'(x)=\dfrac{1}{3}(x-2)^{-2/3}=\dfrac{1}{3(x-2)^{2/3}}$

$f''(x)=-\dfrac{2}{9}(x-2)^{-5/3}=-\dfrac{2}{9(x-2)^{5/3}}$

The domain of f is $\mathbb{R}$.

b) $f'(x)$ does not exist for $x=2$. The equation $f'(x)=0$ has no solution, therefore, $x=2$ is the only critical point.

$f(2)=((2)-2)^{1/3}+3=3$.

So, the critical point, $(2,3)$ is on the graph.

c) We apply the First Derivative Test. We use 3 to divide the real number line into two intervals A: $(-\infty,2)$ and B: $(2,\infty)$ and then we test a point in each interval.

A: Test 1, $f'(1)=\dfrac{1}{3((1)-2)^{2/3}}=\dfrac{1}{3}>0$

B: Test 3, $f'(3)=\dfrac{1}{3((3)-2)^{2/3}}=\dfrac{1}{3}>0$

$f(x)$ is increasing on both intervals $(-\infty,2)$ and $(2,\infty)$, therefore $(2,3)$ is not a relative extremum.

d) Find the points of inflection. $f''(x)$ does not exist when $x=2$. The equation $f''(x)=0$ has no solution, so $x=2$ is the only possible inflection point. We know that $f(2)=3$. To determine concavity, we divide the real number line into two intervals, A: $(-\infty,2)$ and B: $(2,\infty)$ and then we test a point in each interval.

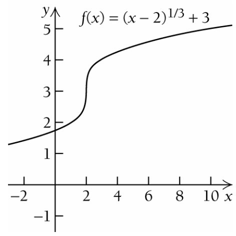
A: Test 1, $f''(1)=-\dfrac{2}{9((1)-2)^{5/3}}=\dfrac{2}{9}>0$

B: Test 3, $f''(3)=-\dfrac{2}{9((3)-2)^{5/3}}=-\dfrac{2}{9}<0$

Thus, $f(x)$ is concave up on the interval $(-\infty,2)$ and $f(x)$ is concave down on the interval $(2,\infty)$. Therefore, the point $(2,3)$ is an inflection point.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
–6	1
1	2
3	4
10	5



39. $f(x) = -2(x-4)^{2/3} + 5$

a) $f'(x) = -\frac{4}{3}(x-4)^{-1/3} = -\frac{4}{3(x-4)^{1/3}}$

$f''(x) = \frac{4}{9}(x-4)^{-4/3} = \frac{4}{9(x-4)^{4/3}}$

The domain of f is $\mathbb{R}$.

b) $f'(x)$ does not exist for $x = 4$. The equation $f'(x) = 0$ has no solution, therefore, $x = 4$ is the only critical point.

$f(4) = -2((4)-4)^{2/3} + 5 = 5$.

So, the critical point $(4,5)$, is on the graph.

c) We apply the First Derivative Test. We use 4 to divide the real number line into two intervals $A: (-\infty, 4)$ and B: $(4, \infty)$ and then we test a point in each interval.

A: Test 3, $f'(3) = -\frac{4}{3((3)-4)^{1/3}} = \frac{4}{3} > 0$

B: Test 5, $f'(5) = -\frac{4}{3((5)-4)^{1/3}} = -\frac{4}{3} < 0$

Thus, $(4,5)$ is a relative maximum. We also know that $f(x)$ is increasing on the interval $(-\infty, 4)$ and decreasing on the interval $(4, \infty)$.

d) Find the points of inflection. $f''(x)$ does not exist when $x = 4$. The equation $f''(x) = 0$ has no solution, so $x = 4$ is the only possible inflection point. We know that $f(4) = 5$.

e) To determine concavity, we divide the real number line into two intervals, $A: (-\infty, 4)$ and B: $(4, \infty)$ and then we test a point in each interval.

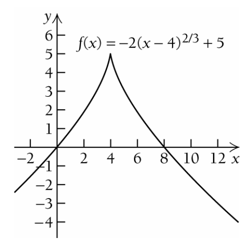
A: Test 3, $f''(3) = \frac{4}{9((3)-4)^{4/3}} = \frac{4}{9} > 0$

B: Test 5, $f''(5) = -\frac{4}{9((5)-4)^{4/3}} = \frac{4}{9} > 0$

Thus, $f(x)$ is concave up on both intervals $(-\infty, 4)$ and $(4, \infty)$ Therefore, the point $(4,5)$ is not an inflection point.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
–4	–3
3	3
5	3
12	–3



40. $f(x) = -3(x-2)^{2/3} + 3$

a) $f'(x) = -\frac{6}{3}(x-2)^{-1/3} = -\frac{2}{(x-2)^{1/3}}$

$f''(x) = \frac{6}{9}(x-2)^{-4/3} = \frac{2}{3(x-2)^{4/3}}$

The domain of f is $\mathbb{R}$.

b) $f'(x)$ does not exist for $x = 2$. The equation $f'(x) = 0$ has no solution, therefore, $x = 2$ is the only critical point.

$f(2) = -3((2)-2)^{2/3} + 3 = 3$.

So, the critical point, $(2,3)$ is on the graph.

c) We apply the First Derivative Test. We use 2 to divide the real number line into two intervals $A: (-\infty, 2)$ and B: $(2, \infty)$ and then we test a point in each interval.

A: Test 1, $f'(1) = -\frac{2}{((1)-2)^{1/3}} = 2 > 0$

B: Test 3, $f'(3) = -\frac{2}{((3)-2)^{1/3}} = -2 < 0$

Thus, $(2,3)$ is a relative maximum. We also know that $f(x)$ is increasing on the interval $(-\infty,2)$ and decreasing on the interval $(2,\infty)$.

d) Find the points of inflection. $f''(x)$ does not exist when $x=2$. The equation $f''(x)=0$ has no solution, so $x=2$ is the only possible inflection point. We know that $f(2)=3$.

e) To determine concavity, we divide the real number line into two intervals, A : $(-\infty,2)$ and B: $(2,\infty)$ and then we test a point in each interval.

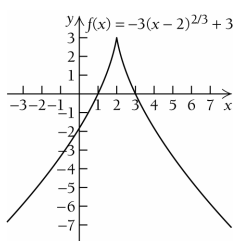
A: Test 1, $f''(1)=\dfrac{2}{3((1)-2)^{4/3}}=\dfrac{2}{3}>0$

B: Test 3, $f''(3)=\dfrac{2}{3((3)-2)^{4/3}}=\dfrac{2}{3}>0$

Thus, $f(x)$ is concave up on both intervals $(-\infty,2)$ and $(2,\infty)$ Therefore, the point $(2,3)$ is not an inflection point.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
-6	-9
1	0
3	0
10	-9



41. $f(x)=x\sqrt{4-x^2}=x\left(4-x^2\right)^{1/2}$

a) $$f'(x)=x\cdot\frac{1}{2}\left(4-x^2\right)^{-1/2}(-2x)+\left(4-x^2\right)^{1/2}\cdot(1)$$

Next, we simplify the derivative.

$$f'(x)=\frac{-x^2}{\left(4-x^2\right)^{1/2}}+\left(4-x^2\right)^{1/2}$$
$$=\frac{-x^2+4-x^2}{\left(4-x^2\right)^{1/2}}$$
$$=\frac{4-2x^2}{\left(4-x^2\right)^{1/2}}$$
$$=\left(4-2x^2\right)\left(4-x^2\right)^{-1/2}$$

$f''(x)$
$$=\left(4-2x^2\right)\left(-\frac{1}{2}\right)\left(4-x^2\right)^{-3/2}(-2x)+\left(4-x^2\right)^{-1/2}(-4x)$$
$$=\frac{x\left(4-2x^2\right)}{\left(4-x^2\right)^{3/2}}-\frac{4x}{\left(4-x^2\right)^{1/2}}$$
$$=\frac{4x-2x^3-4x\left(4-x^2\right)}{\left(4-x^2\right)^{3/2}}$$
$$=\frac{4x-2x^3-16x+4x^3}{\left(4-x^2\right)^{3/2}}$$
$$=\frac{2x^3-12x}{\left(4-x^2\right)^{3/2}}$$

The domain of $f(x)$ is $[-2,2]$.

b) First, we find the critical points. $f'(x)$ does not exist when $4-x^2=0$.

Solve:

$$4-x^2=0$$
$$x^2=4$$
$$x=\pm\sqrt{4}$$
$$x=\pm 2$$

Since the domain of $f(x)$ is $[-2,2]$, relative extrema can not occur at $x=-2$ or $x=2$ because there is not an open interval containing -2 or 2 on which the function is defined. For this reason, we do not consider -2 or 2 in our discussion of relative extrema.

The other critical points occur where $f'(x)=0$.

$$\frac{4-2x^2}{\sqrt{4-x^2}}=0$$

$$4-2x^2=0$$

$$2x^2=4$$

$$x^2=2$$

$$x=\pm\sqrt{2}$$

The critical values are $-\sqrt{2}$ and $\sqrt{2}$.

$$f\left(-\sqrt{2}\right)=-\sqrt{2}\sqrt{4-\left(-\sqrt{2}\right)^2}=-\sqrt{2}\sqrt{2}=-2$$

$$f\left(\sqrt{2}\right)=\sqrt{2}\sqrt{4-\left(\sqrt{2}\right)^2}=\sqrt{2}\sqrt{2}=2$$

Therefore, $\left(-\sqrt{2},-2\right)$ and $\left(\sqrt{2},2\right)$ are critical points on the graph.

c) We use the Second Derivative Test.

$$f''\left(-\sqrt{2}\right)=\frac{2\left(-\sqrt{2}\right)^3-12\left(-\sqrt{2}\right)}{\left[4-\left(-\sqrt{2}\right)^2\right]^{3/2}}$$

$$=\frac{-4\sqrt{2}+12\sqrt{2}}{2^{3/2}}=\frac{8\sqrt{2}}{2\sqrt{2}}=4>0$$

The critical point $\left(-\sqrt{2},-2\right)$ is a relative minimum.

$$f''\left(\sqrt{2}\right)=\frac{2\left(\sqrt{2}\right)^3-12\left(\sqrt{2}\right)}{\left[4-\left(\sqrt{2}\right)^2\right]^{3/2}}$$

$$=\frac{4\sqrt{2}-12\sqrt{2}}{2^{3/2}}=\frac{-8\sqrt{2}}{2\sqrt{2}}=-4<0$$

The critical point $\left(\sqrt{2},2\right)$ is a relative maximum.

If we use the points $-\sqrt{2}$ and $\sqrt{2}$ to divide the interval $[-2,2]$ into three intervals $\left[-2,-\sqrt{2}\right)$, $\left(-\sqrt{2},\sqrt{2}\right)$, and $\left(\sqrt{2},2\right]$, we see that $f(x)$ is decreasing on the intervals $\left[-2,-\sqrt{2}\right]$ and $\left[\sqrt{2},2\right]$ and $f(x)$ is increasing on the interval $\left[-\sqrt{2},\sqrt{2}\right]$.

d) Find the points of inflection. $f''(x)$ does not exist where $4-x^2=0$. We know that this occurs at $x=-2$ and $x=2$. However, just as relative extrema cannot occur at $(-2,0)$ and $(2,0)$, they can not be inflection points either. Inflection points could occur where $f''(x)=0$.

$$\frac{2x^3-12x}{\left(4-x^2\right)^{3/2}}=0$$

$$2x^3-12x=0$$

$$2x\left(x^2-6\right)=0$$

$$2x=0 \quad \text{or} \quad x^2-6=0$$

$$x=0 \quad \text{or} \quad x^2=6$$

$$x=0 \quad \text{or} \quad x=\pm\sqrt{6}$$

Note that $f(x)$ is not defined for $x=\pm\sqrt{6}$. Therefore, the only possible inflection point is $x=0$.

$$f(0)=(0)\sqrt{4-(0)^2}=0.$$

Therefore, $(0,0)$ is a possible inflection point on the graph.

e) To determine concavity, we use 0 to divide the interval $(-2,2)$ into two intervals, A: $(-2,0)$ and B: $(0,2)$ and then we test a point in each interval.

A: Test -1,

$$f''(-1)=\frac{2(-1)^3-12(-1)}{\left[4-(-1)^2\right]^{3/2}}=\frac{10}{3^{3/2}}>0$$

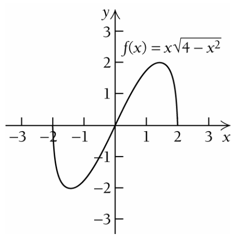
B: Test 1,

$$f''(1)=\frac{2(1)^3-12(1)}{\left[4-(1)^2\right]^{3/2}}=\frac{-10}{3^{3/2}}<0$$

Thus, $f(x)$ is concave up on the interval $(-2,0)$ and $f(x)$ is concave down on the interval $(0,2)$. Therefore, the point $(0,0)$ is an inflection point.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
-1	$-\sqrt{3}$
1	$\sqrt{3}$



42. $f(x) = -x\sqrt{1-x^2} = -x\left(1-x^2\right)^{1/2}$

a) $f'(x) = -x \cdot \frac{1}{2}\left(1-x^2\right)^{-1/2}(-2x) + \left(1-x^2\right)^{1/2} \cdot (-1)$

$= \frac{2x^2-1}{\left(1-x^2\right)^{1/2}} = \frac{2x^2-1}{\sqrt{\left(1-x^2\right)}}$

$f''(x) = \left(2x^2-1\right)\left(-\frac{1}{2}\right)\left(1-x^2\right)^{-3/2}(-2x) + \left(1-x^2\right)^{-1/2}(4x)$

$= \frac{-2x^3+3x}{\left(1-x^2\right)^{3/2}}$

The domain of $f(x)$ is $[-1,1]$.

b) $f'(x)$ does not exist when $x = \pm 1$. However, the domain of $f(x)$ is $[-1,1]$. Therefore, relative extrema can not occur at $x=-1$ or $x=1$ because there is not an open interval containing -1 or 1 on which the function is defined. The other critical points occur where $f'(x)=0$.

$\frac{2x^2-1}{\sqrt{1-x^2}} = 0$

$2x^2-1=0$

$x^2 = \frac{1}{2}$

$x = \pm\frac{1}{\sqrt{2}}$

The critical values are $-\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

$f\left(-\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$

$f\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}$

Therefore, $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ and $\left(\frac{1}{\sqrt{2}}, -\frac{1}{2}\right)$ are critical points on the graph.

c) We use the Second Derivative Test.

$f''\left(-\frac{1}{\sqrt{2}}\right) = -4 < 0$

The critical point $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ is a relative maximum.

$f''\left(\frac{1}{\sqrt{2}}\right) = 4 > 0$

The critical point $\left(\frac{1}{\sqrt{2}}, -\frac{1}{2}\right)$ is a relative minimum.

If we use the points $-\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$ to divide the interval $[-1,1]$ into three intervals $\left[-1,-\frac{1}{\sqrt{2}}\right)$, $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$, and $\left(\frac{1}{\sqrt{2}},1\right]$, we see that $f(x)$ is increasing on the intervals $\left[-1,-\frac{1}{\sqrt{2}}\right]$ and $\left[\frac{1}{\sqrt{2}},1\right]$ and $f(x)$ is decreasing on the interval $\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$.

d) Find the points of inflection. $f''(x)$ does not exist when $x=-1$ and $x=1$. However, inflection points cannot occur at those values because the domain of the function is $[-1,1]$. The remaining possible inflection points occur when $f''(x)=0$.

$\frac{-2x^3+3x}{\left(1-x^2\right)^{3/2}} = 0$

$-2x^3+3x=0$

$x\left(-2x^2+3\right)=0$

$$x = 0 \quad \text{or} \quad 2x^2 - 3 = 0$$

$$x = 0 \quad \text{or} \quad x^2 = \frac{3}{2}$$

$$x = 0 \quad \text{or} \quad x = \frac{\pm\sqrt{6}}{2}$$

Note that $f(x)$ is not defined for $x = \pm\frac{\sqrt{6}}{2}$.

Therefore, the only possible inflection point is $x = 0$.

$f(0) = 0$.

Therefore, $(0,0)$ is a possible inflection point on the graph.

e) To determine concavity, we use 0 to divide the interval $(-1,1)$ into two intervals, A: $(-1,0)$ and B: $(0,1)$ and then we test a point in each interval.

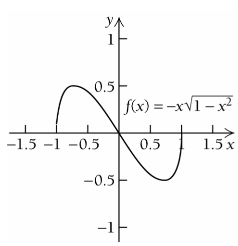
A: Test $-\frac{1}{2}, f''\left(-\frac{1}{2}\right) = \frac{-10}{3^{3/2}} < 0$

B: Test $\frac{1}{2}, \quad f''\left(\frac{1}{2}\right) = \frac{10}{3^{3/2}} > 0$

Thus, $f(x)$ is concave down on the interval $(-1,0)$ and $f(x)$ is concave down on the interval $(0,1)$. Therefore, the point $(0,0)$ is an inflection point.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
-1	0
$-\frac{1}{2}$	$\frac{\sqrt{3}}{4}$
$\frac{1}{2}$	$-\frac{\sqrt{3}}{4}$
1	0



43. $f(x) = \frac{x}{x^2+1}$

a) $f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2}$ Quotient Rule

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2}$$

$f''(x)$

$$= \frac{(x^2+1)^2(-2x) - (1-x^2)\left[2(x^2+1)^1(2x)\right]}{\left((x^2+1)^2\right)^2}$$

$$= \frac{(x^2+1)\left[-2x(x^2+1) - 4x(1-x^2)\right]}{(x^2+1)^4}$$

$$= \frac{-2x^3 - 2x - 4x + 4x^3}{(x^2+1)^3}$$

$$= \frac{2x^3 - 6x}{(x^2+1)^3}$$

The domain of f is $\mathbb{R}$.

b) Since $f'(x)$ exists for all real numbers, the only critical values are where $f'(x) = 0$.

$$\frac{1-x^2}{(x^2+1)^2} = 0$$

$$1 - x^2 = 0 \qquad \text{Multiplying by } (x^2+1)^2$$

$$x^2 = 1$$

$$x = \pm\sqrt{1} = \pm 1$$

The two critical values are $x = -1$ and $x = 1$.

$$f(-1) = \frac{-1}{(-1)^2+1} = -\frac{1}{2}$$

$$f(1) = \frac{1}{(1)^2+1} = \frac{1}{2}$$

The critical points $\left(-1, -\frac{1}{2}\right)$ and $\left(1, \frac{1}{2}\right)$ are on the graph.

c) We use the Second Derivative Test.

$$f''(-1) = \frac{2(-1)^3 - 6(-1)}{\left[(-1)^2+1\right]^3} = \frac{4}{8} = \frac{1}{2} > 0$$

So the point $\left(-1, -\frac{1}{2}\right)$ is a relative minimum.

$$f''(1) = \frac{2(1)^3 - 6(1)}{\left[(1)^2+1\right]^3} = \frac{-4}{8} = -\frac{1}{2} < 0$$

So the point $\left(1, \frac{1}{2}\right)$ is a relative maximum.

We use -1 and 1 to divide the real number line into three intervals $(-\infty,-1)$, $(-1,1)$, and $(1,\infty)$. $f(x)$ is decreasing on the intervals $(-\infty,1]$ and $[1,\infty)$, and $f(x)$ is increasing on the interval $[-1,1]$.

d) Find the points of inflection. $f''(x)$ exists for all real numbers, so the only possible points of inflection occur when $f''(x)=0$.

$$\frac{2x^3-6x}{(x^2+1)^3}=0$$

$$2x^3-6x=0$$

$$2x(x^2-3)=0$$

$$2x=0 \quad \text{or} \quad x^2-3=0$$

$$x=0 \quad \text{or} \quad x^2=3$$

$$x=0 \quad \text{or} \quad x=\pm\sqrt{3}$$

There are three possible inflection points at $x=-\sqrt{3}, 0,$ and $\sqrt{3}$.

$$f(-\sqrt{3})=\frac{-\sqrt{3}}{(-\sqrt{3})^2+1}=-\frac{\sqrt{3}}{4}$$

$$f(0)=\frac{\sqrt{0}}{(\sqrt{0})^2+1}=\frac{0}{1}=0$$

$$f(\sqrt{3})=\frac{\sqrt{3}}{(\sqrt{3})^2+1}=\frac{\sqrt{3}}{4}$$

The points $\left(-\sqrt{3},-\frac{\sqrt{3}}{4}\right)$, $(0,0)$, and $\left(\sqrt{3},\frac{\sqrt{3}}{4}\right)$ are three possible inflection points on the graph.

e) To determine concavity we use $-\sqrt{3}, 0,$ and $\sqrt{3}$ to divide the real number line into four intervals,

A: $(-\infty,-\sqrt{3})$, B: $(-\sqrt{3},0)$, C: $(0,\sqrt{3})$, and D: $(\sqrt{3},\infty)$

Then test a point in each interval.

A: Test -2, $f''(-2)=-\frac{4}{125}<0$

B: Test -1, $f''(-1)=\frac{1}{2}>0$

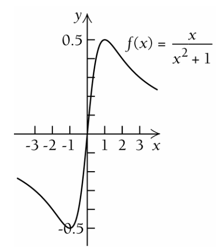
C: Test 1, $f''(1)=-\frac{1}{2}<0$

D: Test 2, $f''(2)=\frac{4}{125}>0$

We see that f is concave down on the intervals $(-\infty,-\sqrt{3})$ and $(0,\sqrt{3})$ and concave up on the intervals $(-\sqrt{3},0)$ and $(\sqrt{3},\infty)$. Therefore the points $\left(-\sqrt{3},-\frac{\sqrt{3}}{4}\right)$, $(0,0)$, and $\left(\sqrt{3},\frac{\sqrt{3}}{4}\right)$ are inflection points.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
-3	$-\frac{3}{10}$
-2	$-\frac{2}{5}$
2	$\frac{2}{5}$
3	$\frac{3}{10}$



44. $f(x)=\frac{8x}{x^2+1}$

a) $f'(x)=\frac{8-8x^2}{(x^2+1)^2}$

$$f''(x)=\frac{16x^3-48x}{(x^2+1)^3}$$

The domain of f is $\mathbb{R}$.

b) $f'(x)$ exists for all real numbers. Solve:

$$f'(x)=0$$

$$\frac{8-8x^2}{(x^2+1)^2}=0$$

$$8-8x^2=0$$

$$x^2=1$$

$$x=\pm1$$

The two critical values are $x=-1$ and $x=1$.

$f(-1)=\frac{8(-1)}{(-1)^2+1}=-\frac{8}{2}=-4$

$f(1)=\frac{8(1)}{(1)^2+1}=\frac{8}{2}=4$

The critical points $(-1,-4)$ and $(1,4)$ are on the graph.

c) We use the Second Derivative Test.
$f''(-1)=4>0$
So the point $(-1,-4)$ is a relative minimum.
$f''(1)=-4<0$
So the point $(1,4)$ is a relative maximum.
$f(x)$ is decreasing on the intervals $(-\infty,1]$ and $[1,\infty)$, and $f(x)$ is increasing on the interval $[-1,1]$.

d) Find the points of inflection. $f''(x)$ exists for all real numbers, so the only possible points of inflection occur when $f''(x)=0$.

$\frac{16x^3-48x}{(x^2+1)^3}=0$

$16x^3-48x=0$

$16x(x^2-3)=0$

$16x=0 \quad \text{or} \quad x^2-3=0$

$x=0 \quad \text{or} \quad x^2=3$

$x=0 \quad \text{or} \quad x=\pm\sqrt{3}$

There are three possible inflection points $-\sqrt{3}$, 0, and $\sqrt{3}$.

$f(-\sqrt{3})=-2\sqrt{3}$

$f(0)=0$

$f(\sqrt{3})=2\sqrt{3}$

The points $(-\sqrt{3},-2\sqrt{3})$, $(0,0)$, and $(\sqrt{3},2\sqrt{3})$ are three possible inflection points on the graph.

e) To determine concavity we use $-\sqrt{3}$, 0, and $\sqrt{3}$ to divide the real number line into four intervals, A: $(-\infty,-\sqrt{3})$, B: $(-\sqrt{3},0)$, C: $(0,\sqrt{3})$, and D: $(\sqrt{3},\infty)$
Then test a point in each interval.

A: Test -2, $f''(-2)=-\frac{32}{125}<0$

B: Test -1, $f''(-1)=4>0$

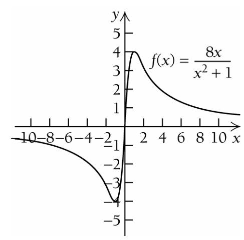
C: Test 1, $f''(1)=-4<0$

D: Test 2, $f''(2)=\frac{32}{125}>0$

We see that *f* is concave down on the intervals $(-\infty,-\sqrt{3})$ and $(0,\sqrt{3})$ and concave up on the intervals $(-\sqrt{3},0)$ and $(\sqrt{3},\infty)$. Therefore the points $(-\sqrt{3},-2\sqrt{3})$, $(0,0)$, and $(\sqrt{3},2\sqrt{3})$ are inflection points.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
-3	$-\frac{12}{5}$
-2	$-\frac{16}{5}$
2	$\frac{16}{5}$
3	$\frac{12}{5}$



45. $f(x)=\frac{3}{x^2+1}=3(x^2+1)^{-1}$

a) $f'(x)=3(-1)(x^2+1)^{-2}(2x)$
$=-6x(x^2+1)^{-2}$
$=\frac{-6x}{(x^2+1)^2}$

$f''(x)$
$=\frac{(x^2+1)^2(-6)-(-6x)(2(x^2+1)(2x))}{((x^2+1)^2)^2}$
$=\frac{(x^2+1)[(x^2+1)(-6)-(-6x)(2)(2x)]}{(x^2+1)^4}$
$=\frac{-6x^2-6+24x^2}{(x^2+1)^3}$
$=\frac{18x^2-6}{(x^2+1)^3}$

The domain of f is $\mathbb{R}$.

b) Since $f'(x)$ exists for all real numbers, the only critical values are where $f'(x)=0$.

$$\frac{-6x}{\left(x^2+1\right)^2}=0$$

$$-6x=0 \quad \text{Multiplying by } \left(x^2+1\right)^2$$

$$x=0$$

The critical value is $x=0$.

$$f(0)=\frac{3}{(0)^2+1}=3$$

The critical point $(0,3)$ is on the graph.

c) We use the Second Derivative Test.

$$f''(0)=\frac{18(0)-6}{\left((0)^2+1\right)^2}=\frac{-6}{1}=-6<0$$

So the point $(0,3)$ is a relative maximum. We use 0 to divide the real number line into two intervals $(-\infty,0)$ and $(0,\infty)$. $f(x)$ is increasing on the interval $(-\infty,0]$, and $f(x)$ is decreasing on the interval $[0,\infty)$.

d) Find the points of inflection. $f''(x)$ exists for all real numbers, so the only possible points of inflection occur when $f''(x)=0$.

$$\frac{18x^2-6}{\left(x^2+1\right)^3}=0$$

$$18x^2-6=0$$

$$18x^2=6$$

$$x^2=\frac{1}{3}$$

$$x=\pm\frac{1}{\sqrt{3}}$$

There are two possible inflection points at $x=-\frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$.

$$f\left(-\frac{1}{\sqrt{3}}\right)=\frac{3}{\left(-\frac{1}{\sqrt{3}}\right)^2+1}$$

$$=\frac{3}{\frac{1}{3}+1}=\frac{3}{\frac{4}{3}}=\frac{9}{4}$$

$$f\left(\frac{1}{\sqrt{3}}\right)=\frac{3}{\left(\frac{1}{\sqrt{3}}\right)^2+1}$$

$$=\frac{3}{\frac{1}{3}+1}=\frac{3}{\frac{4}{3}}=\frac{9}{4}$$

The points $\left(-\frac{1}{\sqrt{3}},\frac{9}{4}\right)$ and $\left(\frac{1}{\sqrt{3}},\frac{9}{4}\right)$ are possible inflection points on the graph.

e) To determine concavity we use $-\frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$ to divide the real number line into three intervals,

A: $\left(-\infty,-\frac{1}{\sqrt{3}}\right)$, B: $\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$, and

C: $\left(\frac{1}{\sqrt{3}},\infty\right)$

Then test a point in each interval.

A: Test -1, $f''(-1)=\dfrac{18(-1)^2-6}{\left((-1)^2+1\right)^3}=\dfrac{3}{2}>0$

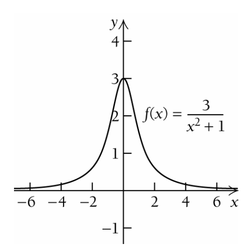
B: Test 0, $f''(0)=\dfrac{18(0)^2-6}{\left((0)^2+1\right)^3}=-6<0$

C: Test 1, $f''(1)=\dfrac{18(1)^2-6}{\left((1)^2+1\right)^3}=\dfrac{3}{2}>0$

We see that f is concave up on the intervals $\left(-\infty,-\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}},\infty\right)$ and concave down on the interval $\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$. Therefore the points $\left(-\frac{1}{\sqrt{3}},\frac{9}{4}\right)$ and $\left(\frac{1}{\sqrt{3}},\frac{9}{4}\right)$ are inflection points.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

x	$f(x)$
-3	$\frac{3}{10}$
-1	$\frac{3}{2}$
1	$\frac{3}{2}$
3	$\frac{3}{10}$



46. $f(x)=\frac{-4}{x^2+1}=-4\left(x^2+1\right)^{-1}$

a) $f'(x)=\frac{8x}{\left(x^2+1\right)^2}$

$f''(x)=\frac{8-24x^2}{\left(x^2+1\right)^3}$

The domain of f is $\mathbb{R}$.

b) $f'(x)$ exists for all real numbers Solve:

$$f'(x)=0$$
$$\frac{8x}{\left(x^2+1\right)^2}=0$$
$$8x=0$$
$$x=0$$

The critical value is $x=0$.

$f(0)=\frac{-4}{(0)^2+1}=-4$

The critical point $(0,-4)$ is on the graph.

c) We use the Second Derivative Test.

$f''(0)=8>0$

So the point $(0,-4)$ is a relative minimum.

$f(x)$ is decreasing on the interval $(-\infty,0]$, and $f(x)$ is increasing on the interval $[0,\infty)$.

d) $f''(x)$ exists for all real numbers. Solve $f''(x)=0$.

$$\frac{8-24x^2}{\left(x^2+1\right)^3}=0$$
$$8-24x^2=0$$
$$x^2=\frac{1}{3}$$
$$x=\pm\frac{1}{\sqrt{3}}$$

There are two possible inflection points $-\frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$.

$$f\left(\pm\frac{1}{\sqrt{3}}\right)=\frac{-4}{\left(\pm\frac{1}{\sqrt{3}}\right)^2+1}=\frac{-4}{\frac{1}{3}+1}=\frac{-4}{\frac{4}{3}}=-3$$

The points $\left(-\frac{1}{\sqrt{3}},-3\right)$ and $\left(\frac{1}{\sqrt{3}},-3\right)$ are possible inflection points on the graph.

e) To determine concavity we use $-\frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$ to divide the real number line into three intervals, A : $\left(-\infty,-\frac{1}{\sqrt{3}}\right)$, B: $\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$, and C: $\left(\frac{1}{\sqrt{3}},\infty\right)$.

Then test a point in each interval.

A: Test -1, $f''(-1)=-2<0$

B: Test 0, $f''(0)=8>0$

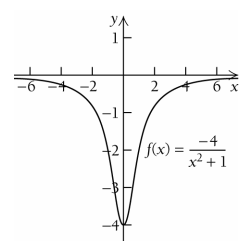
C: Test 1, $f''(1)=-2<0$

We see that f is concave down on the intervals $\left(-\infty,-\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}},\infty\right)$ and concave up on the interval $\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$.

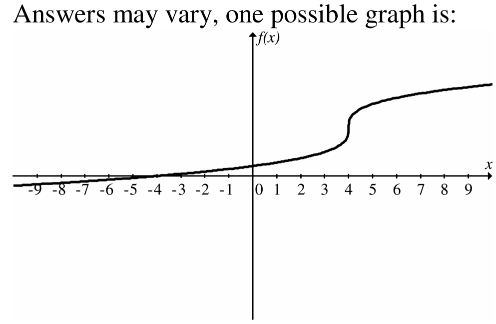
Therefore the points $\left(-\frac{1}{\sqrt{3}},-3\right)$ and $\left(\frac{1}{\sqrt{3}},-3\right)$ are inflection points.

f) We sketch the graph using the preceding information. Additional function values may also be calculated as necessary.

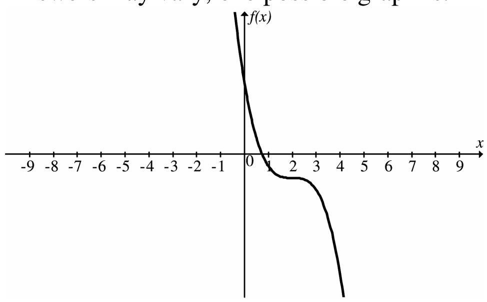
x	$f(x)$
-4	$-\frac{4}{17}$
-2	$-\frac{4}{5}$
-1	-2
1	-2
2	$-\frac{4}{5}$
4	$-\frac{4}{17}$



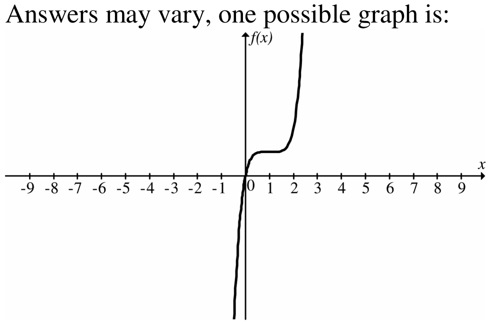
47. Answers may vary, one possible graph is:



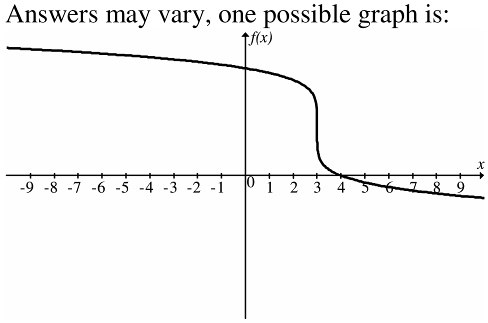
48. Answers may vary, one possible graph is:



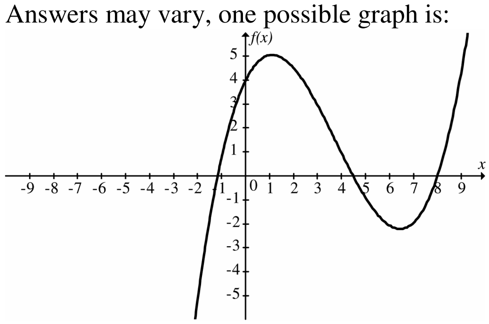
49. Answers may vary, one possible graph is:



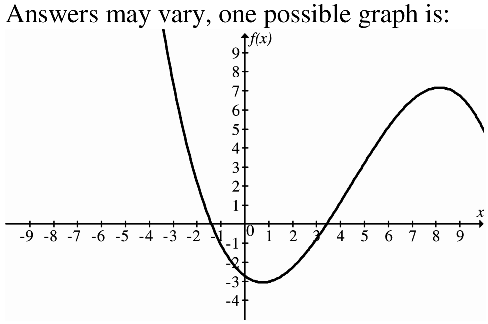
50. Answers may vary, one possible graph is:



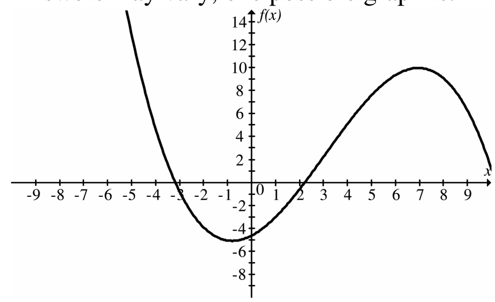
51. Answers may vary, one possible graph is:



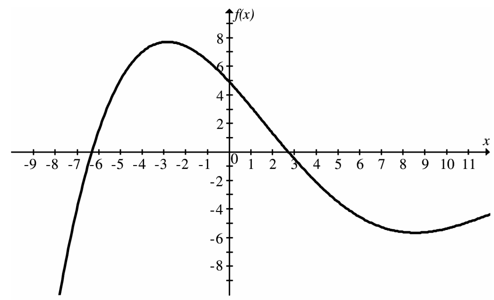
52. Answers may vary, one possible graph is:



53. Answers may vary, one possible graph is:



54. Answers may vary, one possible graph is:



55. – 100. Left to the Student.

101. $R(x) = 50x - 0.5x^2$

$C(x) = 4x + 10$

$$P(x) = R(x) - C(x)$$
$$= (50x - 0.5x^2) - (4x + 10)$$
$$= -0.5x^2 + 46x - 10$$

We will restrict the domains of all three functions to $x \geq 0$ since a negative number of units cannot be produced and sold.

First graph $R(x) = 50x - 0.5x^2$

$R'(x) = 50 - x$

$R''(x) = -1$

Since $R'(x)$ exists for all $x \geq 0$, the only critical points are where $R'(x) = 0$.

$50 - x = 0$

$50 = x$ Critical Value

Find the function value at $x = 50$.

$$R(50) = 50(50) - 0.5(50)^2$$
$$= 2500 - 1250$$
$$= 1250$$

This critical point $(50, 1250)$ is on the graph.

We use the Second Derivative Test:

$R''(50) = -1 < 0$

The point $(50,1250)$ is a relative maximum.
We use 50 to divide the interval $[0,\infty)$ into two intervals, $[0,50)$ and $(50,\infty)$, we know that R is increasing on $[0,50]$ and decreasing on $[50,\infty)$.
Next, find the inflection points. Since $R''(x)$ exists for all $x \ge 0$, and $R''(x) = -1$, there are no possible inflection points.
Furthermore, since $R''(x) < 0$ for all $x \ge 0$, R is concave down over the interval $(0,\infty)$.
Sketch the graph using the preceding information. The x-intercepts of R are found by solving $R(x) = 0$.

$$50x - 0.5x^2 = 0$$
$$0.5x(100 - x) = 0$$
$$0.5x = 0 \quad \text{or} \quad 100 - x = 0$$
$$x = 0 \quad \text{or} \quad 100 = x$$

The x-intercepts are $(0,0)$ and $(100,0)$.

Next, we graph $C(x) = 4x + 10$. This is a linear function with slope 4 and y-intercept $(0,10)$. $C(x)$ is increasing over the entire domain $x \ge 0$ and has no relative extrema or points of inflection.

Finally, we graph $P(x) = -0.5x^2 + 46x - 10$

$$P'(x) = -x + 46$$
$$P''(x) = -1$$

Since $P'(x)$ exists for all $x \ge 0$, the only critical points occur when $P'(x) = 0$.

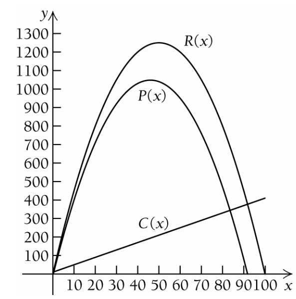
$$-x + 46 = 0$$
$$46 = x \qquad \text{Critical Value}$$

Find the function value at $x = 46$.

$$P(46) = -0.5(46)^2 + 46(46) - 10$$
$$= -1058 + 2116 - 10$$
$$= 1048$$

The critical point $(46,1048)$ is on the graph.
We use the Second Derivative Test:
$P''(46) = -1 < 0$
The point $(46,1048)$ is a relative maximum.

We use 46 to divide the interval $[0,\infty)$ into two intervals, $[0,46)$ and $(46,\infty)$, we know that P is increasing on $[0,46]$ and decreasing on $[46,\infty)$.
Next, find the inflection points. Since $P''(x)$ exists for all $x \ge 0$, and $P''(x) = -1$, there are no possible inflection points. Furthermore, since $P''(x) < 0$ for all $x \ge 0$, P is concave down over the interval $(0,\infty)$.
Sketch the graph using the preceding information.



102. $R(x) = 50x - 0.5x^2$

$$C(x) = 10x + 3$$
$$P(x) = R(x) - C(x)$$
$$= (50x - 0.5x^2) - (10x + 3)$$
$$= -0.5x^2 + 40x - 3$$

We will restrict the domains of all three functions to $x \ge 0$ since a negative number of units cannot be produced and sold.
First graph $R(x) = 50x - 0.5x^2$ as in Exercise 101.

Next, we graph $C(x) = 10x + 3$. This is a linear function with slope 10 and y-intercept $(0,3)$. $C(x)$ is increasing over the entire domain $x \ge 0$ and has no relative extrema or points of inflection.

Finally, we graph $P(x) = -0.5x^2 + 40x - 3$

$$P'(x) = -x + 40$$
$$P''(x) = -1$$

Since $P'(x)$ exists for all $x \geq 0$, the only critical points occur when $P'(x) = 0$.

$-x + 40 = 0$

$40 = x$ Critical Value

$P(40) = 797$

The critical point $(40, 797)$ is on the graph.

We use the Second Derivative Test:

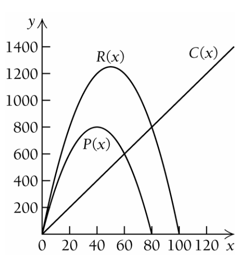
$P''(40) = -1 < 0$

The point $(40, 797)$ is a relative maximum.

We know that P is increasing on $[0, 40]$ and decreasing on $[40, \infty)$.

Next, find the inflection points. Since $P''(x)$ exists for all $x \geq 0$, and $P''(x) = -1$, there are no possible inflection points. Furthermore, since $P''(x) < 0$ for all $x \geq 0$, P is concave down over the interval $(0, \infty)$.

Sketch the graph using the preceding information.



103. $p(x) = \dfrac{13x^3 - 240x^2 - 2460x + 585{,}000}{75{,}000}$

$p'(x) = \dfrac{39x^2 - 480x - 2460}{75{,}000}$

$p''(x) = \dfrac{78x - 480}{75{,}000}$

Since $p'(x)$ exists for all real numbers, the only critical points are where $p'(x) = 0$.

$\dfrac{39x^2 - 480x - 2460}{75{,}000} = 0$

$39x^2 - 480x - 2460 = 0$

We use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-480) \pm \sqrt{(-480)^2 - 4(39)(-2460)}}{2(39)}$$

$$= \frac{480 \pm \sqrt{614{,}160}}{78}$$

$x \approx -3.89$ or $x \approx 16.20$ Critical values

Since the domain of the function is $0 \leq x \leq 40$, we consider only $x \approx 16.20$

$p(16.20)$

$$= \frac{13(16.20)^3 - 240(16.20)^2 - 2460(16.20) - 585{,}000}{75{,}000}$$

≈ 7.17

The critical point $(16.20, 7.17)$ is on the graph.

We use the Second Derivative Test:

$$p''(x) = \frac{78(16.20) - 480}{75{,}000} \approx 0.01 > 0$$

The point $(16.20, 7.17)$ is a relative minimum.

If we use the point 16.20 to divide the domain into two intervals, $[0, 16.20)$ and $(16.20, 40]$, we know that p is decreasing on $[0, 16.20]$ and increasing on $[16.20, 40]$.

Next, we find the inflection points. $p''(x)$ exists for all real numbers, so the only possible inflection points are where $p''(x) = 0$

$\dfrac{78x - 480}{75{,}000} = 0$

$78x - 480 = 0$

$78x = 480$

$x \approx 6.15$

$p(6.15)$

$$= \frac{13(6.15)^3 - 240(6.15)^2 - 2460(6.15) - 585{,}000}{75{,}000}$$

≈ 7.52

The point $(6.15, 7.52)$ is a possible inflection point.

To determine concavity, we use 6.15 to divide the domain into two intervals

A: $[0, 6.15)$ and B: $(6.15, 40]$ and test a point in each interval.

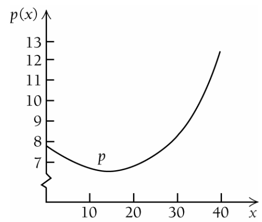
A: Test 1, $p''(1) = \dfrac{78(1) - 480}{75{,}000} = -0.005 < 0$

B: Test 7, $p''(7) = \dfrac{78(7) - 480}{75{,}000} = 0.00088 > 0$

Then p is concave down on $(0, 6.15)$ and concave up on $(6.15, 40)$ and the point $(6.15, 7.52)$ is a point of inflection.

Sketch the graph for $0 \le x \le 40$ using the preceding information. Additional function values may be calculated if necessary.

x	$p(x)$
0	7.8
8	7.42
12	7.25
20	7.25
24	7.57
32	9.15
40	12.46



104. $f(x) = 0.025x^2 - 0.71x + 20.44$

$f'(x) = 0.05x - 0.71$

$f''(x) = 0.05$

$f'(x)$ exists for all real numbers. Solve

$$f'(x) = 0$$
$$0.05x - 0.71 = 0$$
$$0.05x = 0.71$$
$$x = 14.2$$

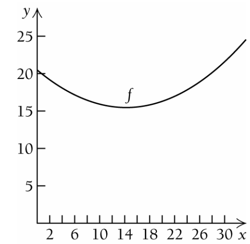
$$f(14.2) = 0.025(14.2)^2 - 0.71(14.2) + 20.44$$
$$= 15.399$$

$f''(14.2) = 0.05$, so $(14.2, 15.399)$ is a relative minimum. Then $f(x)$ is decreasing on $[0, 14.2]$ and increasing on $[14.2, 30]$.

Next, find the points of inflection. Since $f''(x) = 0.05$ exists for all real numbers and is always positive, $f(x)$ is concave up on the interval $(0, 30)$.

Sketch the graph of $f(x)$ using the preceding information. Additional values may be calculated as necessary.

x	$f(x)$
0	20.44
5	17.52
10	15.84
15	15.42
20	16.24
25	18.32
30	21.64



105. $V(r) = k(20r^2 - r^3), \qquad 0 \le r \le 20$

$V'(r) = k(40r - 3r^2)$

$V''(r) = k(40 - 6r)$

$V'(r)$ exists for all r in $[0, 20]$, so the only critical points occur where $V'(r) = 0$.

$$k(40r - 3r^2) = 0$$
$$40r - 3r^2 = 0$$
$$r(40 - 3r) = 0$$
$$r = 0 \quad \text{or} \quad 40 - 3r = 0$$
$$r = 0 \quad \text{or} \quad 40 = 3r$$
$$r = 0 \quad \text{or} \quad \frac{40}{3} = r$$

Using the Second Derivative Test:

$V''(0) = k(40 - 6(0)) = 40k > 0 \qquad [k > 0]$

$V''\left(\dfrac{40}{3}\right) = k\left(40 - 6\left(\dfrac{40}{3}\right)\right) = -40k < 0$

Since $V''\left(\dfrac{40}{3}\right) < 0$, we know that there is a relative maximum at $x = \dfrac{40}{3}$. Thus, for an object whose radius is $\dfrac{40}{3}$ mm or 13.33 mm, the maximum velocity is needed to remove the object.

106. $T(x) = 43.5 - 18.4x + 8.57x^2 - 0.996x^3 + 0.0338x^4$

a) Based on the graph, we would expect the highest temperature to occur in mid to late July.

b) Based on the graph, we would expect the lowest temperature to occur in mid to late January.

c) $\boxed{tw}$ $T''(x) = 0.4056x^2 - 5.976x + 17.14$

Since $T''(x)$ exists for all real values, we solve $T''(x) = 0$ By the quadratic formula, the solutions are $x \approx 3.9$ and $x \approx 10.8$.

$T(3.9) \approx 50.8$

$T(10.8) \approx 49.2$

The two points of inflection are $(3.9, 50.8)$ and $(10.8, 49.2)$.

The left most inflection point $(3.9, 50.8)$ implies that the increase in temperatures will begin to slow down. The rate of increase in temperature will start to slow down until maximum temperature is reached a few months down the road at which point temperatures will begin to fall.. The right most inflection point $(10.8, 49.2)$ signifies the moment when the decrease of temperature begins to slow down. Again, the rate of decrease in temperature will start to slow down, until the minimum temperature is reached at which point the temperature will begin to rise.

107. $\boxed{tw}$ The rate of change is maximized at the points of inflection. Looking at the graph, we estimate the points of inflection to be 75 days after January first, and 270 days after January first. Therefore, the number of hours of daylight are increasing most rapidly approximately 75 days after January 1st or approximately March 16th and the number of hours of daylight are decreasing most rapidly approximately 270 days after January 1st or approximately September 27th.

108. $\boxed{tw}$ Observe that *h* is increasing for all values of *x* for which *g* is positive and *h* is decreasing for all values of *x* for which *g* is negative. Furthermore, for all values of *x* for which *g*=0, *h* has a horizontal tangent. Therefore, *g*=*h'*.

109. $\boxed{tw}$ Observe that *g* is increasing for all values of *x* for which *h* is positive and *g* is decreasing for all values of *x* for which *h* is negative. Furthermore, for all values of *x* for which *h*=0, *g* has a horizontal tangent. Therefore, $h = g'$.

110. $\boxed{tw}$ Answers will vary. The "Passion" graph increases at an increasing rate (concave up) for a while. It passes through an inflection point during the increasing part of the graph, where the level is still increasing, but the rate at which it increases begins to slow. The graph is increasing at a decreasing rate (concave down) The graph continues to increase until it reaches a relative maximum, at which point the passion level begins to fall. The rate at which the level is falling slows passing through one more point of inflection, until the graph becomes horizontal.
The "Intimacy" graph is increasing. It appears to be concave up at first, then passing through an inflection point, it becomes concave down. The Intimacy levels continue to increase over time, so there are no relative extrema.
The "Commitment" graph is increasing and concave up at the beginning. Then, it passes thought an inflection point. The graph continues to increase, but is now concave down. The commitment graph continues to increase over time, so there are no relative extrema.

111. $f(x) = ax^2 + bx + c, \quad a \neq 0$

$f'(x) = 2ax + b$

$f''(x) = 2a$

Since $f'(x)$ exists for all real numbers, the only critical points occur when $f'(x) = 0$. We solve:

$$2ax + b = 0$$
$$2ax = -b$$
$$x = \frac{-b}{2a}$$

So the critical value will occur at $x = \frac{-b}{2a}$.

Applying the second derivative test, we see that

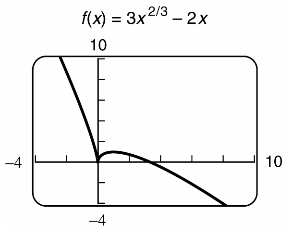
$f''(x) = 2a > 0, \quad$ for $a > 0$

$f''(x) = 2a < 0, \quad$ for $a < 0$

Therefore, a relative maximum occurs at $x = \frac{-b}{2a}$ when $a < 0$ and a relative minimum occurs at $x = \frac{-b}{2a}$ when $a > 0$.

112. $f(x) = 3x^{2/3} - 2x$

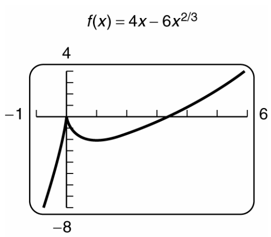
Graphing the function on the calculator we have:



Using the minimum/maximum feature on the calculator, we estimate a relative maximum at $(1,1)$ and a relative minimum at $(0,0)$.

113. $f(x) = 4x - 6x^{2/3}$

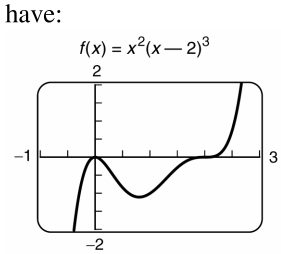
Graphing the function on the calculator we have:



Using the minimum/maximum feature on the calculator, we estimate a relative maximum at $(0,0)$ and a relative minimum at $(1,-2)$.

114. $f(x) = x^2(x-2)^3$

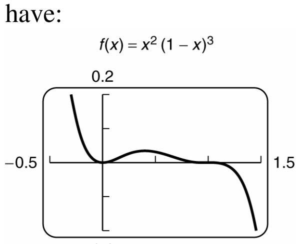
Graphing the function on the calculator we have:



Using the minimum/maximum feature on the calculator, we estimate a relative maximum at $(0,0)$ and a relative minimum at $\left(\frac{4}{5}, -1.106\right)$.

115. $f(x) = x^2(1-x)^3$

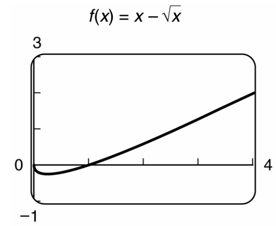
Graphing the function on the calculator we have:



Using the minimum/maximum feature on the calculator, we estimate a relative maximum at $(0.4, 0.035)$ and a relative minimum at $(0,0)$.

116. $f(x) = x - \sqrt{x}$

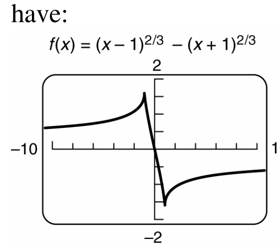
Graphing the function on the calculator we have:



Using the minimum/maximum feature on the calculator, we estimate a relative minimum at $(0.25, -0.25)$.

117. $f(x) = (x-1)^{2/3} - (x+1)^{2/3}$

Graphing the function on the calculator we have:



Using the minimum/maximum feature on the calculator, we estimate a relative maximum at $(-1, 1.587)$ and a relative minimum at $(1, -1.587)$.

118. $\boxed{tw}$

a) The cubic and quartic functions appear to be equally good fits. The cubic function will ease the computations, where as the quartic function will be a little better fit to the data.

b) The domain of is the set of positive real numbers. Realistically, an infant would spend very little time on the computer, and eventually people die, so there is a lower limit and an upper limit on age. We could use the domain $[3, 110]$

c) The graphs show that the cubic function does not have relative extrema on a reasonable domain. The quartic function has a relative minimum around $x = 4.8$. However, judging by the data, it appears that the average use of a home computer is increasing with age.