Chapter 1

Differentiation

Exercise Set 1.1

1. We select x-values closer and closer to 3, and observe the corresponding output.

x	2.99	2.999	3	3.001	3.01
$2x+5$	10.98	10.998	$\to 11 \leftarrow$	11.002	11.02

As x approaches 3, the value of $2x+5$ approaches 11.

2. As x approaches -4, the value of $3x+7$ approaches -5.

3. We solve the equation:

$$-3x = 6$$
$$x = -2$$

Therefore, As x approaches -2, the value of $-3x$ approaches 6.

4. As x approaches 7, the value of $x-2$ approaches 5.

5. The notation $\lim_{x\to 4} f(x)$ is read "the limit, as x approaches 4, of $f(x)$."

6. The notation $\lim_{x\to 1} g(x)$ is read "the limit, as x approaches 1, of $g(x)$."

7. The notation $\lim_{x\to 5^-} F(x)$ is read "the limit, as x approaches 5 from the left, of $F(x)$."

8. The notation $\lim_{x\to 4^+} G(x)$ is read "the limit, as x approaches 4 from the right, of $G(x)$."

9. The notation $\lim_{x\to 2^+}$ is read "the limit, as x approaches 2 from the right."

10. The notation $\lim_{x\to 3^-}$ is read "the limit, as x approaches 3 from the left".

11. As inputs x approach 3 from the right, outputs $f(x)$ approach 2. Thus the limit from the right is 2. That is,

$$\lim_{x\to 3^+} f(x) = 2.$$

12. As inputs x approach 3 from the left, outputs $f(x)$ approach 1. That is,

$$\lim_{x\to 3^-} f(x) = 1.$$

13. As inputs x approach -1 from the left, outputs $f(x)$ approach -3. Thus the limit from the left is -3. That is,

$$\lim_{x\to -1^-} f(x) = -3.$$

14. As inputs x approach -1 from the right, outputs $f(x)$ approach -3. That is,

$$\lim_{x\to -1^+} f(x) = -3.$$

15. From Exercise (11) and (12) we k now that $\lim_{x\to 3^-} f(x) = 1$ and $\lim_{x\to 3^+} f(x) = 2$. Since the limit from the left, 1, is not the same as the limit from the right, 2, $\lim_{x\to 3} f(x)$ *does not exist.*

16. From Exercise (13) and (14) we find $\lim_{x\to -1} f(x) = -3$.

17. As inputs x approach 4 from the left, outputs $f(x)$ approach 3. Thus the limit from the left is 3. That is,

$$\lim_{x\to 4^-} f(x) = 3.$$

As inputs x approach 4 from the right, outputs $f(x)$ approach 3. Thus the limit from the right is 4. That is,

$$\lim_{x\to 4^+} f(x) = 3.$$

Since the limit from the left, 3, is the same as the limit from the right, 3, we have

$$\lim_{x\to 4} f(x) = 3.$$

18. $\lim_{x\to 2} f(x) = 0$.

19. As inputs x approach -2 from the left, outputs $g(x)$ approach 4. Thus the limit from the left is 4. That is,
$\lim_{x\to -2^-} g(x) = 4$.

20. $\lim_{x\to -2^+} g(x) = 2$.

21. As inputs x approach 4 from the right, outputs $g(x)$ approach -1. Thus the limit from the right is -1. That is,
$\lim_{x\to 4^+} g(x) = -1$.

22. $\lim_{x\to 4^-} g(x) = -1$.

23. Since the limit from the left, -1, is the same as the limit from the right, -1, we have
$\lim_{x\to 4} g(x) = -1$.

24. $\lim_{x\to -2} g(x)$ *does not exist.*

25. As inputs x approach 2 from the left, outputs $g(x)$ approach 0. Thus the limit from the left is 0. That is,
$\lim_{x\to 2^-} g(x) = 0$.
As inputs x approach 2 from the right, outputs $g(x)$ approach 0. Thus the limit from the right is 0. That is,
$\lim_{x\to 2^+} g(x) = 0$.
Since the limit from the left, 0, is the same as the limit from the right, 0, we have
$\lim_{x\to 2} g(x) = 0$.

26. $\lim_{x\to -4} g(x) = 2$.

27. As inputs x approach -3 from the left, outputs $F(x)$ approach 5. Thus the limit from the left is 5. That is,
$\lim_{x\to -3^-} F(x) = 5$.
As inputs x approach -3 from the right, outputs $F(x)$ approach 5. Thus the limit from the right is 5. That is,
$\lim_{x\to -3^+} F(x) = 5$.
Since the limit from the left, 5, is the same as the limit from the right, 5, we have
$\lim_{x\to -3} F(x) = 5$.

28. We have $\lim_{x\to 2^-} F(x) = 4$ and $\lim_{x\to 2^+} F(x) = 4$.
Therefore, $\lim_{x\to 2} F(x) = 4$.

29. As inputs x approach -2 from the left, outputs $F(x)$ approach 4. Thus the limit from the left is 4. That is,
$\lim_{x\to -2^-} F(x) = 4$.
As inputs x approach -2 from the right, outputs $F(x)$ approach 2. Thus the limit from the right is 2. That is,
$\lim_{x\to -2^+} F(x) = 2$.
Since the limit from the left, 4, is not the same as the limit from the right, 2, we have
$\lim_{x\to -2} F(x)$ *does not exist.*

30. We have $\lim_{x\to -5^-} F(x) = 0$ and $\lim_{x\to -5^+} F(x) = 0$.
Therefore, $\lim_{x\to -5} F(x) = 0$.

31. As inputs x approach 4 from the left, outputs $F(x)$ approach 2. Thus the limit from the left is 2. That is,
$\lim_{x\to 4^-} F(x) = 2$.
As inputs x approach 4 from the right, outputs $F(x)$ approach 2. Thus the limit from the right is 2. That is,
$\lim_{x\to 4^+} F(x) = 2$.
Since the limit from the left, 2, is the same as the limit from the right, 2, we have
$\lim_{x\to 4} F(x) = 2$.

32. We have $\lim_{x\to 6^-} F(x) = 0$ and $\lim_{x\to 6^+} F(x) = 0$.
Therefore, $\lim_{x\to 6} F(x) = 0$.

33. As inputs x approach -2 from the right, outputs $F(x)$ approach 2. Thus the limit from the right is 2. That is,
$\lim_{x\to -2^+} F(x) = 2$.

34. As inputs x approach -2 from the left, outputs $F(x)$ approach 4. Thus the limit from the left is 4. That is,
$\lim_{x\to -2^-} F(x) = 4$.

35. As inputs x approach -1 from the left, outputs $f(x)$ approach 1. Thus the limit from the left is 1. That is,
$\lim_{x\to -1^-} f(x) = 1$.
As inputs x approach -1 from the right, outputs $f(x)$ approach 1. Thus the limit from the right is 1. That is,
$\lim_{x\to -1^+} f(x) = 1$.
Since the limit from the left, 1, is the same as the limit from the right, 1, we have
$\lim_{x\to -1} f(x) = 1$.

36. We have $\lim_{x\to 2^-} f(x) = -1$ and $\lim_{x\to 2^+} f(x) = -1$.
Therefore, $\lim_{x\to 2} f(x) = -1$.

37. As inputs x approach -3 from the left, outputs $f(x)$ increase without bound. We say that the limit from the left is infinity. That is
$\lim_{x\to -3^-} f(x) = \infty$.
As inputs x approach -3 from the right, outputs $f(x)$ decrease without bound. We say that limit from the right is negative infinity. That is,
$\lim_{x\to -3^+} f(x) = -\infty$.
Since the function values as $x \to 3$ from the left increase without bound, and the function values as $x \to 3$ from the right decrease without bound, the limit does not exist. We have,
$\lim_{x\to -3} f(x)$ *does not exist.*

38. We have $\lim_{x\to 0^-} f(x) = 2$ and $\lim_{x\to 0^+} f(x) = 2$.
Therefore, $\lim_{x\to 0} f(x) = 2$.

39. As inputs x approach 3 from the left, outputs $f(x)$ approach 0. Thus the limit from the left is 0. That is,
$\lim_{x\to 3^-} f(x) = 0$.
As inputs x approach 3 from the right, outputs $f(x)$ approach 0. Thus the limit from the right is 0. That is,
$\lim_{x\to 3^+} f(x) = 0$.
Since the limit from the left, 0, is the same as the limit from the right, 0, we have
$\lim_{x\to 3} f(x) = 0$.

40. We have $\lim_{x\to 1^-} f(x) = \infty$ and $\lim_{x\to 1^+} f(x) = -\infty$.
Therefore, $\lim_{x\to 1} f(x)$ *does not exist.*

41. As inputs x approach -4 from the left, outputs $f(x)$ approach 3. Thus the limit from the left is 3. That is,
$\lim_{x\to -4^-} f(x) = 3$.
As inputs x approach -4 from the right, outputs $f(x)$ approach 3. Thus the limit from the right is 3. That is,
$\lim_{x\to -4^+} f(x) = 3$.
Since the limit from the left, 3, is the same as the limit from the right, 3, we have
$\lim_{x\to -4} f(x) = 3$.

42. We have $\lim_{x\to -2^-} f(x) = 0$ and $\lim_{x\to -2^+} f(x) = 0$.
Therefore, $\lim_{x\to -2} f(x) = 0$.

43. As inputs x get larger and larger, outputs $f(x)$ get closer and closer to 1. We have
$\lim_{x\to \infty} f(x) = 1$.

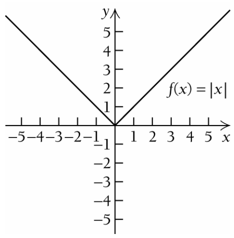
44. As inputs x get more and more negative, output $f(x)$ get closer and closer to 2. $\lim_{x\to -\infty} f(x) = 2$.

45. Defining $f(x) = |x|$ as a piecewise defined function we have:
$$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \ge 0 \end{cases}.$$

We graph the function by creating an input-output table.

x	-2	-1	0	1	2
$f(x)$	2	1	0	1	2

Next, we plot the points from the table and draw the graph at the top of the next page.



Find $\lim_{x\to 0} f(x)$.

As inputs x approach 0 from the left, outputs $f(x)$ approach 0. We have,

$\lim_{x\to 0^-} f(x) = 0$.

As inputs x approach 0 from the right, outputs $f(x)$ approach 0. We have,

$\lim_{x\to 0^+} f(x) = 0$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to 0} f(x) = 0$.

Find $\lim_{x\to -2} f(x)$.

As inputs x approach -2 from the left, outputs $f(x)$ approach 2. We have,

$\lim_{x\to -2^-} f(x) = 2$.

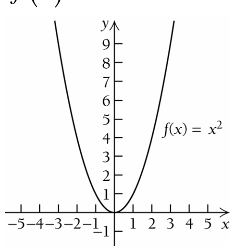
As inputs x approach -2 from the right, outputs $f(x)$ approach 2. We have,

$\lim_{x\to -2^+} f(x) = 2$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to -2} f(x) = 2$.

46. $f(x) = x^2$



Find $\lim_{x\to -1} f(x)$.

We have $\lim_{x\to -1^-} f(x) = 1$ and $\lim_{x\to -1^+} f(x) = 1$.

Therefore, $\lim_{x\to -1} f(x) = 1$.

Find $\lim_{x\to 0} f(x)$.

We have $\lim_{x\to 0^-} f(x) = 0$ and $\lim_{x\to 0^+} f(x) = 0$.

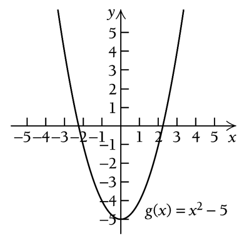
Therefore, $\lim_{x\to 0} f(x) = 0$.

47. $g(x) = x^2 - 5$

We graph the function by creating an input-output table.

x	-2	-1	0	1	2
$g(x)$	-1	-4	-5	-4	-1

Next, we plot the points from the table and draw the graph.



Find $\lim_{x\to 0} g(x)$.

As inputs x approach 0 from the left, outputs $g(x)$ approach -5. We have,

$\lim_{x\to 0^-} g(x) = -5$.

As inputs x approach 0 from the right, outputs $g(x)$ approach -5. We have,

$\lim_{x\to 0^+} g(x) = -5$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to 0} g(x) = -5$.

Find $\lim_{x\to -1} g(x)$.

As inputs x approach -1 from the left, outputs $g(x)$ approach -4. We have,

$\lim_{x\to -1^-} g(x) = -4$.

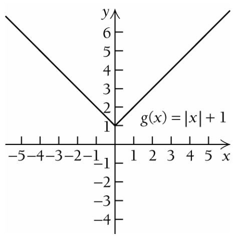
As inputs x approach -1 from the right, outputs $g(x)$ approach -4. We have,

$\lim_{x\to -1^+} g(x) = -4$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to -1} g(x) = -4$.

48. $g(x) = |x| + 1$



Find $\lim\limits_{x \to -3} g(x)$.

We have $\lim\limits_{x \to -3^-} g(x) = 4$ and $\lim\limits_{x \to -3^+} g(x) = 4$.

Therefore, $\lim\limits_{x \to -3} g(x) = 4$.

Find $\lim\limits_{x \to 0} g(x)$.

We have $\lim\limits_{x \to 0^-} g(x) = 1$ and $\lim\limits_{x \to 0^+} g(x) = 1$.

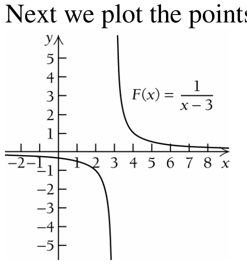
Therefore, $\lim\limits_{x \to 0} g(x) = 1$.

49. $F(x) = \dfrac{1}{x-3}$

Since $x = 3$ makes the denominator zero, we exclude the value 3 from the domain. Creating an input-output table we have

x	1	2	2.5	2.9	3.1	3.5	4	5
$F(x)$	$-\frac{1}{2}$	-1	-2	-10	10	2	1	$\frac{1}{2}$

Next we plot the points and draw the graph.



Find $\lim\limits_{x \to 3} F(x)$.

As inputs x approach 3 from the left, outputs $F(x)$ decrease without bound . We have,

$\lim\limits_{x \to 3^-} F(x) = -\infty$.

As inputs x approach 3 from the right, outputs $F(x)$ increase without bound. We have,

$\lim\limits_{x \to 3^+} F(x) = \infty$

Since the function values as $x \to 3$ from the left decrease without bound, and the function values as $x \to 3$ from the right increase without bound, the limit does not exist. We have,

$\lim\limits_{x \to 3} F(x)$ *does not exist.*

Find $\lim\limits_{x \to 4} F(x)$.

As inputs x approach 4 from the left, outputs $F(x)$ approach 1. We have,

$\lim\limits_{x \to 4^-} F(x) = 1$.

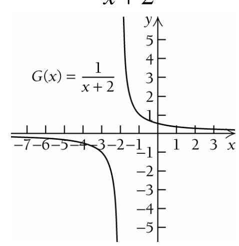
As inputs x approach 4 from the right, outputs $F(x)$ approach 1. We have,

$\lim\limits_{x \to 4^+} F(x) = 1$

Since the limit from the left is the same as the limit from the right, we have

$\lim\limits_{x \to 4} F(x) = 1$.

50. $G(x) = \dfrac{1}{x+2}$



Find $\lim\limits_{x \to -1} G(x)$.

We have $\lim\limits_{x \to -1^-} G(x) = 1$ and $\lim\limits_{x \to -1^+} G(x) = 1$.

Therefore, $\lim\limits_{x \to -1} G(x) = 1$.

Find $\lim\limits_{x \to -2} G(x)$

We have $\lim\limits_{x \to -2^-} G(x) = -\infty$ and $\lim\limits_{x \to -2^+} G(x) = \infty$.

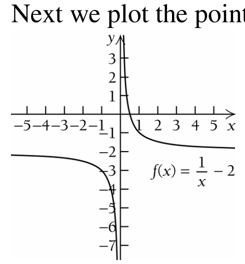
Therefore, $\lim\limits_{x \to -2} G(x)$ *does not exist.*

51. $f(x) = \dfrac{1}{x} - 2$

Since $x = 0$ makes the denominator zero, we exclude the value 0 from the domain. Creating an input-output table we have

x	-1	-0.5	-0.1	0.1	0.5	1
$f(x)$	-3	-4	-12	8	0	-1

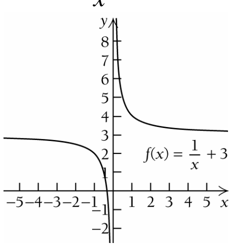
Next we plot the points and draw the graph.



Find $\lim_{x\to\infty} f(x)$.
As inputs x get larger and larger, outputs $f(x)$ get closer and closer to -2. We have $\lim_{x\to\infty} f(x) = -2$.

Find $\lim_{x\to 0} f(x)$.
As inputs x approach 0 from the left, outputs $f(x)$ decrease without bound. We have, $\lim_{x\to 0^-} f(x) = -\infty$.
As inputs x approach 0 from the right, outputs $f(x)$ increase without bound. We have, $\lim_{x\to 0^+} f(x) = \infty$
Since the function values as $x \to 0$ from the left decrease without bound, and the function values as $x \to 0$ from the right increase without bound, the limit does not exist. We have, $\lim_{x\to 0} f(x)$ *does not exist.*

52. $f(x) = \frac{1}{x} + 3$



Find $\lim_{x\to\infty} f(x)$.
As inputs x get larger and larger, outputs $f(x)$ get closer and closer to 3. We have $\lim_{x\to\infty} f(x) = 3$.

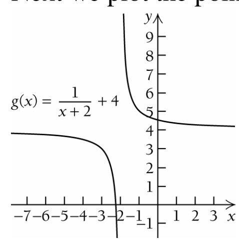
Find $\lim_{x\to 0} f(x)$.
As inputs x approach 0 from the left, outputs $f(x)$ decrease without bound. We have, $\lim_{x\to 0^-} f(x) = -\infty$.
As inputs x approach 0 from the right, outputs $f(x)$ increase without bound. We have, $\lim_{x\to 0^+} f(x) = \infty$

Since the function values as $x \to 0$ from the left decrease without bound, and the function values as $x \to 0$ from the right increase without bound, the limit does not exist. We have, $\lim_{x\to 0} f(x)$ *does not exist.*

53. $g(x) = \frac{1}{x+2} + 4$
Since $x = -2$ makes the denominator zero, we exclude the value -2 from the domain.
Creating an input-output table we have

x	-3	-2.5	-2.1	-1.9	-1.5	-1	0
$g(x)$	3	2	-6	14	6	5	$\frac{9}{2}$

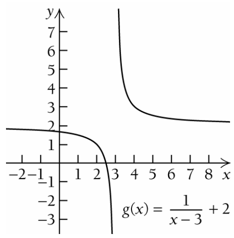
Next we plot the points and draw the graph.



Find $\lim_{x\to\infty} g(x)$.
As inputs x get larger and larger, outputs $g(x)$ get closer and closer to 4. We have $\lim_{x\to\infty} g(x) = 4$.

Find $\lim_{x\to -2} g(x)$.
As inputs x approach -2 from the left, outputs $g(x)$ decrease without bound. We have, $\lim_{x\to -2^-} g(x) = -\infty$.
As inputs x approach -2 from the right, outputs $g(x)$ increase without bound. We have, $\lim_{x\to -2^+} g(x) = \infty$.
Since the function values as $x \to 2$ from the left decrease without bound, and the function values as $x \to 2$ from the right increase without bound, the limit does not exist. We have, $\lim_{x\to -2} g(x)$ *does not exist.*

54. $g(x)=\dfrac{1}{x-3}+2$



Find $\lim\limits_{x\to\infty} g(x)$.

As inputs x get larger and larger, outputs $g(x)$ get closer and closer to 2. We have

$\lim\limits_{x\to\infty} g(x)=2$.

Find $\lim\limits_{x\to 3} g(x)$.

As inputs x approach 3 from the left, outputs $g(x)$ decrease without bound. We have,

$\lim\limits_{x\to 3^-} g(x)=-\infty$.

As inputs x approach 3 from the right, outputs $g(x)$ increase without bound. We have,

$\lim\limits_{x\to 3^+} g(x)=\infty$.

Since the function values as $x\to 3$ from the left decrease without bound, and the function values as $x\to 3$ from the right increase without bound, the limit does not exist. We have,

$\lim\limits_{x\to 3} g(x)$ *does not exist.*

55. $F(x)=\begin{cases} 2x+1, & \text{for } x<1 \\ x, & \text{for } x\ge 1. \end{cases}$

We create an input-output table for each piece of the function.

For $x<1$

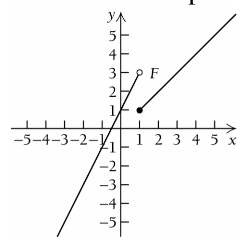
x	-1	0	0.9
$F(x)$	-1	1	2.8

We plot the points and draw the graph. Notice we draw an open circle at the point $(1,3)$ to indicate that the point is not part of the graph.

For $x\ge 1$

x	1	2	3
$F(x)$	1	2	3

We plot the points and draw the graph. Notice we draw a solid circle at the point $(1,1)$ to indicate that the point is part of the graph.



Find $\lim\limits_{x\to 1^-} F(x)$.

As inputs x approach 1 from the left, outputs $F(x)$ approach 3. That is,

$\lim\limits_{x\to 1^-} F(x)=3$.

Find $\lim\limits_{x\to 1^+} F(x)$.

As inputs x approach 1 from the right, outputs $F(x)$ approach 1. That is,

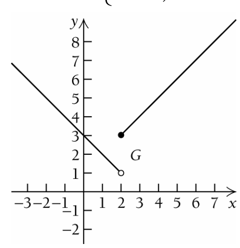
$\lim\limits_{x\to 1^+} F(x)=1$.

Find $\lim\limits_{x\to 1} F(x)$

Since the limit from the left, 3, is not the same as the limit from the right, 1, we have

$\lim\limits_{x\to 1} F(x)$ *does not exist.*

56. $G(x)=\begin{cases} -x+3, & \text{for } x<2 \\ x+1, & \text{for } x\ge 2 \end{cases}$



We have $\lim\limits_{x\to 2^-} G(x)=1$ and $\lim\limits_{x\to 2^+} G(x)=3$.

Therefore, $\lim\limits_{x\to 2} G(x)$ *does not exist.*

57. $g(x)=\begin{cases} -x+4, & \text{for } x<3 \\ x-3, & \text{for } x>3. \end{cases}$

We create an input-output table for each piece of the function.

For $x<3$

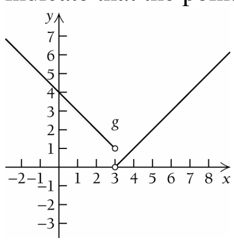
x	0	1	2	2.9
$g(x)$	4	3	2	1.1

We plot the points from the table at the bottom of the previous page and draw the graph. Notice we draw an open circle at the point $(3,1)$ to indicate that the point is not part of the graph.

For $x > 3$

x	3.1	4	5	6
$g(x)$	0.1	1	2	3

We plot the points and draw the graph. Notice we draw an open circle at the point $(3,0)$ to indicate that the point is not part of the graph.



Find $\lim_{x\to 3^-} g(x)$.

As inputs x approach 3 from the left, outputs $g(x)$ approach 1. That is,

$\lim_{x\to 3^-} g(x) = 1$.

Find $\lim_{x\to 3^+} g(x)$.

As inputs x approach 3 from the right, outputs $g(x)$ approach 0. That is,

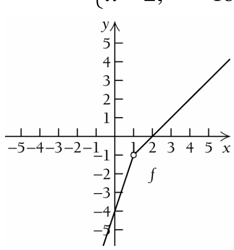
$\lim_{x\to 3^+} g(x) = 0$.

Find $\lim_{x\to 3} g(x)$.

Since the limit from the left, 1, is not the same as the limit from the right, 0, we have

$\lim_{x\to 3} g(x)$ *does not exist.*

58. $f(x) = \begin{cases} 3x-4, & \text{for } x < 1 \\ x-2, & \text{for } x > 1 \end{cases}$



We have $\lim_{x\to 1^-} f(x) = -1$ and $\lim_{x\to 1^+} f(x) = -1$.

Therefore, $\lim_{x\to 1} f(x) = -1$.

59. $G(x) = \begin{cases} x^2, & \text{for } x < -1 \\ x+2, & \text{for } x > -1. \end{cases}$

We create an input-output table for each piece of the function.

For $x < -1$

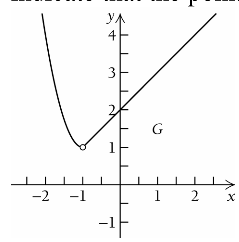
x	-3	-2	-1.1
$G(x)$	9	4	-1.21

We plot the points and draw the graph. Notice we draw an open circle at the point $(-1,1)$ to indicate that the point is not part of the graph.

For $x > -1$

x	-0.9	0	1
$G(x)$	1.1	2	3

We plot the points and draw the graph. Notice we draw an open circle at the point $(-1,1)$ to indicate that the point is not part of the graph.



Find $\lim_{x\to -1} G(x)$.

As inputs x approach -1 from the left, outputs $G(x)$ approach 1. We have,

$\lim_{x\to -1^-} G(x) = 1$.

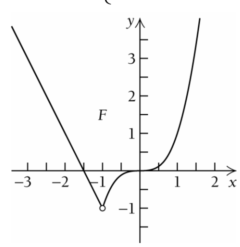
As inputs x approach -1 from the right, outputs $G(x)$ approach 1. We have,

$\lim_{x\to -1^+} G(x) = 1$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to -1} G(x) = 1$.

60. $F(x) = \begin{cases} -2x-3, & \text{for } x < -1 \\ x^3, & \text{for } x > -1 \end{cases}$



We have $\lim_{x\to -1^-} F(x) = -1$ and $\lim_{x\to -1^+} F(x) = -1$.

Therefore, $\lim_{x\to -1} F(x) = -1$.

61. $\lim_{x \to 0.25^-} C(x) = 3.30$

$\lim_{x \to 0.25^+} C(x) = 3.30$

$\lim_{x \to 0.25} C(x) = 3.30$

62. $\lim_{x \to 0.2^-} C(x) = 2.90$

$\lim_{x \to 0.2^+} C(x) = 3.30$

$\lim_{x \to 0.2} C(x)$ *does not exist.*

63. $\lim_{x \to 0.6^-} C(x) = 3.70$

$\lim_{x \to 0.6^+} C(x) = 4.10$

$\lim_{x \to 0.6} C(x)$ *does not exist.*

64. $\lim_{x \to 1^-} p(x) = 0.39$

$\lim_{x \to 1^+} p(x) = 0.63$

$\lim_{x \to 1} p(x)$ *does not exist.*

65. $\lim_{x \to 2^-} p(x) = 0.63$

$\lim_{x \to 2^+} p(x) = 0.87$

$\lim_{x \to 2} p(x)$ *does not exist.*

66. $\lim_{x \to 2.6^-} p(x) = 0.87$

$\lim_{x \to 2.6^+} p(x) = 0.87$

$\lim_{x \to 2.6} p(x) = 0.87$

67. $\lim_{x \to 3^-} p(x) = 0.87$

$\lim_{x \to 3^+} p(x) = 1.11 \qquad (0.87 + 0.24 = 1.11)$

$\lim_{x \to 3} p(x)$ *does not exist.*

68. $\lim_{x \to 3.4^-} p(x) = 1.11$

$\lim_{x \to 3.4^+} p(x) = 1.11$

$\lim_{x \to 3.4} p(x) = 1.11.$

69. $\lim_{t \to 1.5^-} p(t) = 11.0$ hundred

$\lim_{t \to 1.5^+} p(t) = 12.0$ hundred

$\lim_{t \to 1.5} p(t)$ *does not exist.*

70. $\lim_{t \to 1.75^-} p(t) = 12.0$ hundred

$\lim_{t \to 1.75^+} p(t) = 13.0$ hundred

$\lim_{t \to 1.75} p(t)$ *does not exist.*

71. $\boxed{tw}$ Answers will vary. At the values $t = 0.5, 0.75, 1.5, 1.75$ the population could be jumping up because of a large number of births. At the values $t = 1.25$ the population could be dropping because of a large number of deaths among the deer population.

72. $\lim_{t \to 0.6^-} p(t) = 33$

$\lim_{t \to 0.6^+} p(t) = 35$

$\lim_{t \to 0.6} p(t)$ *does not exist.*

73. $\lim_{t \to 0.8^-} p(t) = 35$

$\lim_{t \to 0.8^+} p(t) = 34$

$\lim_{t \to 0.8} p(t)$ *does not exist.*

74. Answers will vary. At the values $t = 0.1, 0.4, 0.5, 0.6$ the population increase could be contributed to births in the bear population.
At the values $t = 0.3, 0.8$ the population could be dropping because of deaths in the bear population.

75. As inputs *x* approach 2 from the right, outputs $f(x)$ approach 4. We have, $\lim_{x \to 2^+} f(x) = 4$. In order for $\lim_{x \to 2} f(x)$ to exist we need $\lim_{x \to 2^-} f(x) = 4$. We will use the letter *c* for the unknown in the equation and this gives us

$$\lim_{x \to 2^-} \frac{1}{2}(x) + c = 4$$

Substitute 2 in for x to get the equation:

$\frac{1}{2}(2)+c=4$ and solving for c we get

$1+c=4$

$c=3.$

Therefore, in order for the limit to exist as x approaches 2, the function must be:

$$f(x)=\begin{cases}\frac{1}{2}x+\underline{3} & \text{for } x<2\\ -x+6 & \text{for } x>2.\end{cases}$$

76. As inputs x approach 2 from the left, outputs $f(x)$ approach 0. We have, $\lim_{x\to 2^-} f(x)=0$. In order for $\lim_{x\to 2} f(x)$ to exist we need $\lim_{x\to 2^+} f(x)=0$. We will use the letter c for the unknown in the equation and this gives us

$\lim_{x\to 2^+} \frac{3}{2}(x)+c=0$

Substitute 2 in for x to get the equation:

$\frac{3}{2}(2)+c=0$ and solving for c we get

$3+c=0$

$c=-3.$

Therefore, in order for the limit to exist as x approaches 2, the function must be:

$$f(x)=\begin{cases}-\frac{1}{2}x+1 & \text{for } x<2\\ \frac{3}{2}x+\underline{-3} & \text{for } x>2.\end{cases}$$

77. As inputs x approach 2 from the left, outputs $f(x)$ approach -5. We have, $\lim_{x\to 2^-} f(x)=-5$. In order for $\lim_{x\to 2} f(x)$ to exist we need $\lim_{x\to 2^+} f(x)=-5$. We will use the letter c for the unknown in the equation and this gives us

$\lim_{x\to 2^+}(-x^2+c)=-5$

Substitute 2 in for x to get the equation:

$-(2)^2+c=-5$ and solving for c we get

$-(4)+c=-5$

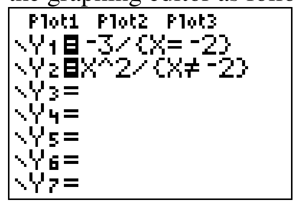
$c=-1.$

Therefore, in order for the limit to exist as x approaches 2, the function must be:

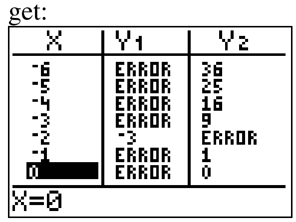
$$f(x)=\begin{cases}x^2-9 & \text{for } x<2\\ -x^2+\underline{-1} & \text{for } x>2.\end{cases}$$

78. Graph $f(x)=\begin{cases}-3, & \text{for } x=-2\\ x^2, & \text{for } x\neq -2.\end{cases}$

Using the calculator we enter the function into the graphing editor as follows:



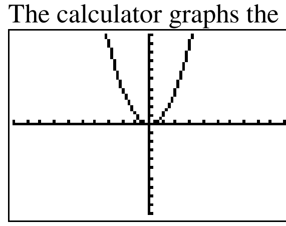
Notice, when you select the table feature you get:



X	Y1	Y2
-6	ERROR	36
-5	ERROR	25
-4	ERROR	16
-3	ERROR	9
-2	-3	ERROR
-1	ERROR	1
0	ERROR	0

X=0

The calculator graphs the function:



Using the trace feature, we find the limits.

a) $\lim_{x\to -2^+} f(x)=4$

b) $\lim_{x\to -2^-} f(x)=4$

c) $\lim_{x\to -2} f(x)=4$

d) $\lim_{x\to 2^+} f(x)=4$

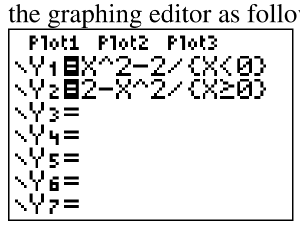
e) $\lim_{x\to 2^-} f(x)=4$

f) $\lim_{x\to -2} f(x)=4\neq -3=f(-2)$

g) $\lim_{x\to 2} f(x)=4=f(2)$

79. Graph $f(x)=\begin{cases}x^2-2, & \text{for } x<0\\ 2-x^2, & \text{for } x\geq 0.\end{cases}$

Using the calculator we enter the function into the graphing editor as follows:

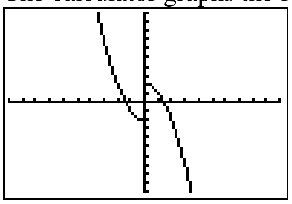


When you select the table feature you get:

X	Y1	Y2
-3	7	ERROR
-2	2	ERROR
-1	-1	ERROR
0	ERROR	2
1	ERROR	1
2	ERROR	-2
3	ERROR	-7

X=3

The calculator graphs the function:



Using the trace feature, we find the limits.

Find $\lim_{x\to 0} f(x)$.

As inputs x approach 0 from the left, outputs $f(x)$ approach -2. We have,

$\lim_{x\to 0^-} f(x) = -2$.

As inputs x approach 0 from the right, outputs $f(x)$ approach 2. We have,

$\lim_{x\to 0^+} f(x) = 2$

Since the limit from the left, -2, is not the same as the limit from the right, 2, we have $\lim_{x\to 0} f(x)$ *does not exist.*

Find $\lim_{x\to -2} f(x)$.

As inputs x approach -2 from the left, outputs $f(x)$ approach 2. We have,

$\lim_{x\to -2^-} f(x) = 2$.

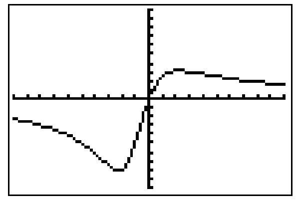
As inputs x approach -2 from the right, outputs $f(x)$ approach 2. We have,

$\lim_{x\to -2^+} f(x) = 2$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to -2} f(x) = 2$.

80. Graph $g(x) = \dfrac{20x^2}{x^3 + 2x^2 + 5x}$



Using the trace feature on the graph, we have:

$\lim_{x\to \infty} g(x) = 0$

$\lim_{x\to -\infty} g(x) = 0$.

81. Graph $f(x) = \dfrac{1}{x^2 - 4x - 5}$

Using the calculator we enter the function into the graphing editor as follows:

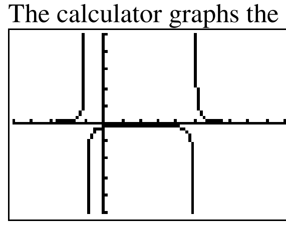
```
Plot1 Plot2 Plot3
\Y1=1/(X^2-4X-5)

\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

Using the following window:

```
WINDOW
 Xmin=-5
 Xmax=10
 Xscl=1
 Ymin=-5
 Ymax=5
 Yscl=1
 Xres=1
```

The calculator graphs the function:



Using the trace feature on the calculator we find the limits.

Find $\lim_{x\to -1} f(x)$.

As inputs x approach -1 from the left, outputs $f(x)$ increase without bound. We have,

$\lim_{x\to -1^-} f(x) = \infty$.

As inputs x approach -1 from the right, outputs $f(x)$ decrease without bound. We have,

$\lim_{x\to -1^+} f(x) = -\infty$

Since the function values as $x \to -1$ from the left increase without bound, and the function values as $x \to -1$ from the right decrease without bound, the limit does not exist. We have,
$\lim_{x\to -1} f(x)$ *does not exist.*

Find $\lim_{x\to 5} f(x)$.
As inputs x approach 5 from the left, outputs $f(x)$ decrease without bound. We have,
$\lim_{x\to 5^-} f(x) = -\infty$.
As inputs x approach 5 from the right, outputs $f(x)$ increase without bound. We have,
$\lim_{x\to 5^+} f(x) = \infty$
Since the function values as $x \to 5$ from the left decrease without bound, and the function values as $x \to 5$ from the right increase without bound, the limit does not exist. We have,
$\lim_{x\to 5} f(x)$ *does not exist.*

Exercise Set 1.2

1. By limit principle L1,
$\lim_{x\to 3} 7 = 7$
Therefore, The statement is a true.

2. By limit principle L2, the statement is true.

3. By limit principle L2,
$\lim_{x\to 1}\left[g(x)\right]^2 = \left[\lim_{x\to 1} g(x)\right]^2 = [5]^2 = 25.$
Therefore, the statement is false.

4. By limit principle L6, the statement is true.

5. By the definition of continuity, in order for *f* to be continuous at $x = 2$, $f(2)$ must exist. Therefore, the statement is true.

6. The statement is false.

7. This statement is false. If $\lim_{x\to 4} F(x)$ exists but is not equal to $F(4)$, then F is not continuous.

8. By the definition of continuity, the statement is true.

9. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\begin{aligned}\lim_{x\to 1}(3x+2) &= 3(1)+2\\ &= 5.\end{aligned}$$

10. $\lim_{x\to 2}(4x-5) = 4(2)-5 = 3$.

11. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\begin{aligned}\lim_{x\to -1}\left(x^2-4\right) &= (-1)^2-4\\ &= 1-4\\ &= -3.\end{aligned}$$

12. $\lim_{x\to -2}\left(x^2+3\right) = (-2)^2+3 = 7$

13. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\begin{aligned}\lim_{x\to 3}\left(x^2-4x+7\right) &= (3)^2-4(3)+7\\ &= 9-12+7\\ &= 4.\end{aligned}$$

14. $\lim_{x\to 5}\left(x^2-6x+9\right) = (5)^2-6(5)+9 = 4$.

15. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\begin{aligned}&\lim_{x\to 2}\left(2x^4-3x^3+4x-1\right)\\ &= 2(2)^4-3(2)^3+4(2)-1\\ &= 2(16)-3(8)+8-1\\ &= 32-24+8-1\\ &= 15.\end{aligned}$$

16. $\lim_{x\to -1}\left(3x^5+4x^4-3x+6\right)$
$$\begin{aligned}&= 3(-1)^5+4(-1)^4-3(-1)+6\\ &= 10.\end{aligned}$$

17. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:

$$\begin{aligned}\lim_{x\to 3}\left(\frac{x^2-8}{x-2}\right) &= \frac{(3)^2-8}{3-2}\\ &= \frac{9-8}{3-2}\\ &= 1.\end{aligned}$$

18. $\lim_{x\to 3}\frac{x^2-25}{x^2-5} = \frac{(3)^2-25}{(3)^2-5} = \frac{9-25}{9-5} = \frac{-16}{-4} = 4$.

19. We attempt to find the limit by substitution:

$$\begin{aligned}\lim_{x\to 3}\frac{x^2-9}{x-3} &= \frac{(3)^2-9}{(3)-3}\\ &= \frac{0}{0}\end{aligned}$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the numerator and dividing out common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\begin{aligned}\lim_{x\to 3}\frac{x^2-9}{x-3} &= \lim_{x\to 3}\frac{(x-3)(x+3)}{x-3}\\ &= \lim_{x\to 3}(x+3) && \text{simplifying, assuming } x\neq -3\\ &= 3+3 && \text{substitution}\\ &= 6.\end{aligned}$$

20. $\lim_{x\to 5}\frac{x^2-25}{x-5}$

First we will simplify the function by factoring the numerator and dividing out common factors.

$$\begin{aligned}\lim_{x\to 5}\frac{x^2-25}{x-5} &= \lim_{x\to 5}\frac{(x-5)(x+5)}{x-5}\\ &= \lim_{x\to 5}(x+5)\\ &= 5+5\\ &= 10.\end{aligned}$$

21. $\lim_{x\to 4}\sqrt{x^2-9}$

By limit principle L2,

$$\begin{aligned}\lim_{x\to 4}\sqrt{x^2-9} &= \sqrt{\lim_{x\to 4}(x^2-9)}\\ &= \sqrt{\lim_{x\to 4}x^2-\lim_{x\to 4}9} && \text{By L3}\\ &= \sqrt{\left(\lim_{x\to 4}x\right)^2-9} && \text{By L2 and L1}\\ &= \sqrt{(4)^2-9}\\ &= \sqrt{16-9}\\ &= \sqrt{7}.\end{aligned}$$

22.

$$\begin{aligned}\lim_{x\to 5}\sqrt{x^2-16} &= \sqrt{\lim_{x\to 5}(x^2-16)} && \text{By L2}\\ &= \sqrt{\lim_{x\to 5}x^2-\lim_{x\to 5}16} && \text{By L3}\\ &= \sqrt{\left(\lim_{x\to 5}x\right)^2-16} && \text{By L2 and L1}\\ &= \sqrt{(5)^2-16}\\ &= \sqrt{25-16}\\ &= \sqrt{9}\\ &= 3.\end{aligned}$$

23. $\lim_{x\to 2}\sqrt{x^2-9}$

By limit principle L2,

$$\begin{aligned}\lim_{x\to 2}\sqrt{x^2-9} &= \sqrt{\lim_{x\to 2}(x^2-9)}\\ &= \sqrt{\lim_{x\to 2}x^2-\lim_{x\to 2}9} && \text{By L3}\\ &= \sqrt{\left(\lim_{x\to 2}x\right)^2-9} && \text{By L2 and L1}\\ &= \sqrt{(2)^2-9}\\ &= \sqrt{4-9}\\ &= \sqrt{-5}.\end{aligned}$$

Therefore, $\lim_{x\to 2}\sqrt{x^2-9}$ *does not exist.*

24. $\lim\limits_{x\to 3}\sqrt{x^2-16}=\sqrt{\lim\limits_{x\to 3}\left(x^2-16\right)}$ By L2

$=\sqrt{\lim\limits_{x\to 3}x^2-\lim\limits_{x\to 3}16}$ By L3

$=\sqrt{\left(\lim\limits_{x\to 3}x\right)^2-16}$ By L2 and L1

$=\sqrt{(3)^2-16}$

$=\sqrt{9-16}$

$=\sqrt{-7}.$

Therefore, $\lim\limits_{x\to 3}\sqrt{x^2-16}$ *does not exist.*

25. $\lim\limits_{x\to 3^+}\sqrt{x^2-9}$

By limit principle L2,

$\lim\limits_{x\to 3^+}\sqrt{x^2-9}=\sqrt{\lim\limits_{x\to 3^+}\left(x^2-9\right)}$

$=\sqrt{\lim\limits_{x\to 3^+}x^2-\lim\limits_{x\to 3^+}9}$ By L3

$=\sqrt{\left(\lim\limits_{x\to 3^+}x\right)^2-9}$ By L2 and L1

$=\sqrt{(3)^2-9}$

$=\sqrt{9-9}$

$=\sqrt{0}$

$=0.$

26. $\lim\limits_{x\to -4^-}\sqrt{x^2-16}=\sqrt{\lim\limits_{x\to -4^-}\left(x^2-16\right)}$ By L2

$=\sqrt{\lim\limits_{x\to -4^-}x^2-\lim\limits_{x\to -4^-}16}$ By L3

$=\sqrt{\left(\lim\limits_{x\to -4^-}x\right)^2-16}$ By L2 and L1

$=\sqrt{(4)^2-16}$

$=\sqrt{16-16}$

$=\sqrt{0}$

$=0.$

27. The function is not continuous over the interval, because $f(x)$ is not continuous at $x=1$. As x approaches 1 from the left, $f(x)$ approaches 2. However, as x approaches 1 from the right $f(x)$ approaches -1. Therefore, $f(x)$ is not continuous at 1.

28. The function is not continuous over the interval, because it is not continuous at $x=-2$.

29. The function is not continuous over the interval, because $k(x)$ is not continuous at $x=-1$. The function is not defined at $x=-1$, in other words $k(-1)$ does not exist. Therefore, $k(x)$ is not continuous at -1.

30. The function is continuous over the interval.

31. The function is not continuous over the interval, because it is not continuous at $x=-2$. The limit does not exist as x approaches -2, furthermore, $t(-2)$ does not exist. Therefore the function is not continuous at $x=-2$.

32. a) $\lim\limits_{x\to 1^+}f(x)=-1$ and $\lim\limits_{x\to 1^-}f(x)=2$.

Therefore, $\lim\limits_{x\to 1}f(x)$ *does not exist*

b) $f(1)=-1$

c) Since the limit of $f(x)$ as x approaches 1 does not exist, the function is not continuous at $x=1$.

d) $\lim\limits_{x\to -2^+}f(x)=3$ and $\lim\limits_{x\to -2^-}f(x)=3$

Therefore, $\lim\limits_{x\to -2}f(x)=3$

e) $f(-2)=3$

f) The function $f(x)$ is continuous at $x=-2$, because

1) $f(-2)$ exists, $f(-2)=3$,

2) $\lim\limits_{x\to -2}f(x)$ exists, $\lim\limits_{x\to -2}f(x)=3$, and

3) $\lim\limits_{x\to -2}f(x)=3=f(-2)$.

33. a) As inputs x approach 1 from the right, outputs $g(x)$ approach -2. Thus, the limit from the right is -2. $\lim\limits_{x\to 1^+}g(x)=-2$.

As inputs x approach 1 from the left, outputs $g(x)$ approach -2. Thus, the limit from the left is -2. $\lim\limits_{x\to 1^-}g(x)=-2$.

Since the limit from the left, -2, is the same as the limit from the right, -2, we have. $\lim\limits_{x\to 1}g(x)=-2$.

b) When the input is 1, the output $g(1)$ is -2. That is $g(1)=-2$.

c) The function $g(x)$ is continuous at $x=1$, because
1) $g(1)$ exists, $g(1)=-2$
2) $\lim_{x\to 1} g(x)$ exists, $\lim_{x\to 1} g(x)=-2$, and
3) $\lim_{x\to 1} g(x)=-2=g(1)$.

d) As inputs x approach -2 from the right, outputs $g(x)$ approach -3. Thus, the limit from the right is -3. $\lim_{x\to -2^+} g(x)=-3$.
As inputs x approach -2 from the left, outputs $g(x)$ approach 4. Thus, the limit from the left is 4. $\lim_{x\to -2^-} g(x)=4$.
Since the limit from the left, 4, is not the same as the limit from the right, -3, we say $\lim_{x\to -2} g(x)$ does not exist.

e) When the input is -2, the output $g(-2)$ is -3. That is $g(-2)=-3$.

f) Since the limit of $g(x)$ as x approaches -2 does not exist, the function is not continuous at $x=-2$.

34. a) $\lim_{x\to -1^+} k(x)=2$ and $\lim_{x\to -1^-} k(x)=2$.
Therefore, $\lim_{x\to -1} k(x)=2$.

b) $k(-1)$ is not defined, or does not exist.

c) Since the value of $k(-1)$ does not exist, the function is not continuous at $x=-1$.

d) $\lim_{x\to 3^+} k(x)=-2$ and $\lim_{x\to 3^-} k(x)=-2$
Therefore, $\lim_{x\to 3} k(x)=-2$

e) $k(3)=-2$

f) The function $k(x)$ is continuous at $x=3$, because
1) $k(3)$ exists,
2) $\lim_{x\to 3} k(x)$ exists,, and
3) $\lim_{x\to 3} k(x)=-2=k(3)$.

35. a) As inputs x approach 1 from the right, outputs $h(x)$ approach 2. Thus, the limit from the right is 2. $\lim_{x\to 1^+} h(x)=2$.
As inputs x approach 1 from the left, outputs $h(x)$ approach 2. Thus, the limit from the left is 2. $\lim_{x\to 1^-} h(x)=2$.
Since the limit from the left, 2, is the same as the limit from the right, 2, we have. $\lim_{x\to 1} h(x)=2$.

b) When the input is 1, the output $h(1)$ is 2. That is $h(1)=2$.

c) The function $h(x)$ is continuous at $x=1$, because
1) $h(1)$ exists, $\mathrm{h}(1)=2$
2) $\lim_{x\to 1} h(x)$ exists, $\lim_{x\to 1} h(x)=2$, and
3) $\lim_{x\to 1} h(x)=2=h(1)$.

d) As inputs x approach -2 from the right, outputs $h(x)$ approach 0. Thus, the limit from the right is 0. $\lim_{x\to -2^+} h(x)=0$.
As inputs x approach -2 from the left, outputs $h(x)$ approach 0. Thus, the limit from the left is 0. $\lim_{x\to -2^-} h(x)=0$.
Since the limit from the left, 0, is the same as the limit from the right, 0, we say $\lim_{x\to -2} h(x)=0$.

e) When the input is -2, the output $h(-2)$ is 0. That is $h(-2)=0$.

f) The function $h(x)$ is continuous at $x=-2$, because
1) $h(-2)$ exists, $h(-2)=0$
2) $\lim_{x\to -2} h(x)$ exists, $\lim_{x\to -2} h(x)=0$, and
3) $\lim_{x\to -2} h(x)=0=h(-2)$.

36. a) $\lim_{x\to 1^+} t(x)=0.25$ and $\lim_{x\to 1^-} t(x)=0.25$.
Therefore, $\lim_{x\to 1} t(x)=0.25$.

b) $t(1)=0.25$

c) The function $t(x)$ is continuous at $x=1$, because
1) $t(1)$ exists,
2) $\lim_{x\to 1} t(x)$ exists, and
3) $\lim_{x\to 1} t(x) = 0.25 = t(1)$.

d) $\lim_{x\to -2^+} t(x) = \infty$ and $\lim_{x\to -2^-} f(x) = \infty$
Therefore, $\lim_{x\to -2} f(x) = \infty$, which means that the limit does not exist because as x gets closer to -2 the function values increase without bound

e) $t(-2)$ is undefined or does not exist.

f) Since $t(-2)$ is undefined and the limit of $t(x)$ as x approaches -2 does not exist, the function is not continuous at $x=-2$.

37. a) As inputs x approach 3 from the right, outputs $G(x)$ approach 3. Thus, $\lim_{x\to 3^+} G(x) = 3$.

b) As inputs x approach 3 from the left, outputs $G(x)$ approach 1. Thus, $\lim_{x\to 3^-} G(x) = 1$.

c) Since the limit from the left, 1, is not the same as the limit from the right, 3, the limit does not exist. $\lim_{x\to 3} G(x)$ does not exist.

d) $G(3) = 1$

e) The function $G(x)$ is not continuous at $x=3$ because the limit does not exist as x approaches 3.

f) The function $G(x)$ is continuous at $x=0$, because
1) $G(0)$ exists,
2) $\lim_{x\to 0} G(x)$ exists, , and
3) $\lim_{x\to 0} G(x) = 0 = G(0)$.

g) The function $G(x)$ is continuous at $x=2.9$, because
1) $G(2.9)$ exists,
2) $\lim_{x\to 2.9} G(x)$ exists, , and
3) $\lim_{x\to 2.9} G(x) = G(2.9)$.

38. a) $\lim_{x\to 2^+} C(x) = 1$

b) $\lim_{x\to 2^-} C(x) = -1$

c) $\lim_{x\to 2} C(x)$ does not exist.

d) $C(2) = 1$

e) The function $C(x)$ is not continuous at $x=2$ because the limit does not exist as x approaches 2.

f) The function $C(x)$ is continuous at $x=1.95$, because $\lim_{x\to 1.95} C(x) = -1 = C(1.95)$.

39. First we find the function value when $x=5$. $f(5) = 3(5)-2 = 13$. Hence, $f(5)$ exists.
Next, we find the limit as x approaches 5. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution: $\lim_{x\to 5} f(x) = 3(5)-2 = 13$
Therefore, $\lim_{x\to 5} f(x) = 13 = f(5)$ and the function is continuous at $x=5$.

40. The function $g(x)$ is continuous at $x=4$, because:
1) $g(4)$ exists, $g(4) = 4$
2) $\lim_{x\to 4} g(x)$ exists, $\lim_{x\to 4} g(x) = 4$, and
3) $\lim_{x\to 4} g(x) = 4 = g(4)$.

41. The function $G(x) = \frac{1}{x}$ is not continuous at $x=0$ because $G(0) = \frac{1}{0}$ is undefined.

42. The function $F(x) = \sqrt{x}$ is not continuous at $x=-1$ because $F(-1) = \sqrt{-1}$ is undefined on the real numbers.

43. First we find the function value when $x=3$.
$g(3) = \frac{1}{3}(3)+4 = 1+4 = 5$. Hence, $g(3)$ exists.
Next, we find the limit as x approaches 3. As the inputs x approach 3 from the right, the outputs $g(x)$ approach 5, that is,
$\lim_{x\to 3^-} g(x) = \frac{1}{3}(3)+4 = 5$.

As the inputs x approach 3 from the left, the outputs $g(x)$ approach 5, that is,
$\lim_{x\to 3^+} g(x) = 2(3)-1=5$.
Since the limit from the left, 5, is the same as the limit from the right, 5. The limit exists. We have:
$\lim_{x\to 3} g(x) = 5$.
Therefore, we have
$\lim_{x\to 3} g(x) = 5 = g(3)$.
Thus the function is continuous at $x=3$.

44. The function $f(x)$ is continuous at $x=4$, because:
1) $f(4)$ exists,
2) $\lim_{x\to 4} f(x)$ exists, and
3) $\lim_{x\to 4} f(x) = 3 = f(4)$.

45. The function is not continuous at $x=3$ because the limit does not exist as x approaches 3. To verify this we take the limit as x approaches 3 from the left and the limit as x approaches 3 from the right.
As x approaches 3 from the left we have
$\lim_{x\to 3^-} F(x) = \frac{1}{3}(3)+4=5$.
As x approaches 3 from the right we have
$\lim_{x\to 3^+} F(x) = 2(3)-5=1$.
Since the limit from the left, 5, is not the same as the limit from the right, 1, the limit does not exist. $\lim_{x\to 3} F(x)$ does not exist.

46. The function is not continuous at $x=4$ because the limit does not exist as x approaches 4.
$\lim_{x\to 4^-} G(x) = \frac{1}{2}(4)+1=3$
$\lim_{x\to 4^+} G(x) = -(4)+5=1$
$\lim_{x\to 4^-} G(x) \neq \lim_{x\to 4^+} G(x)$
Therefore,
$\lim_{x\to 4} G(x)$ does not exist.
Furthermore, the function is not defined at $x=4$, so $G(4)$ does not exist.

47. First we find the function value when $x=3$.
$f(3)=2(3)-1=5$. Hence, $f(3)$ exists.
Next, we find the limit as x approaches 3. As the inputs x approach 3 from the left, the outputs $f(x)$ approach 5, that is,
$\lim_{x\to 3^-} f(x) = \frac{1}{3}(3)+4=5$.
As the inputs x approach 3 from the left, the outputs $f(x)$ approach 5, that is,
$\lim_{x\to 3^+} f(x) = 2(3)-1=5$.
Since the limit from the left, 5, is the same as the limit from the right, 5. The limit exists. We have:
$\lim_{x\to 3} f(x) = 5$.
Therefore, we have
$\lim_{x\to 3} f(x) = 5 = f(3)$.
Thus the function is continuous at $x=3$.

48. The function is not continuous at $x=4$ because the function is not defined at $x=4$. Therefore, $g(4)$ does not exist.

49. The function is not continuous at $x=2$. To verify this, we take the limit as x approaches 2. Using the Theorem on Limits of Rational Functions, we simplify the function near 2 by factoring the numerator and dividing out common factors.

$$\begin{aligned}\lim_{x\to 2} G(x) &= \lim_{x\to 2}\frac{x^2-4}{x-2}\\ &= \lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}\\ &= \lim_{x\to 2}(x+2)\\ &= 2+2=4\end{aligned}$$

Therefore,
$\lim_{x\to 2} G(x) = 4$.
However, when $x=2$, the output $G(2)$ is defined to be 5. That is, $G(2)=5$. Therefore, $\lim_{x\to 2} G(x) = 4 \neq 5 = G(2)$. Thus the function is not continuous at $x=2$.

50. The function is not continuous at $x=1$ because $\lim_{x\to 1} F(x) = 2 \neq 4 = F(1)$.

51. First we find the function value when $x = 5$.
$f(5) = (5) + 1 = 6,\ f(5)$ exists.
Next we find the limit as *x* approaches 5. To find the limit as *x* approaches 5 from the left, we first simplify the rational function by factoring the numerator and canceling common factors.

$$\begin{aligned}\lim_{x\to5^-} f(x) &= \lim_{x\to5^-} \frac{x^2-4x-5}{x-5}\\ &= \lim_{x\to5^-} \frac{(x-5)(x+1)}{x-5}\\ &= \lim_{x\to5^-}(x+1)\\ &= 5+1\\ &= 6\end{aligned}$$

To find the limit as *x* approaches 5 from the right, we can use substitution.

$$\begin{aligned}\lim_{x\to5^+} f(x) &= \lim_{x\to5^+} x+1\\ &= 5+1\\ &= 6\end{aligned}$$

Therefore, the limit exists.
$\lim_{x\to5} f(x) = 6$.
Thus we have,
$\lim_{x\to5} f(x) = 6 = f(5)$.
Therefore, the function in continuous at $x = 5$.

52. The function $G(x)$ is continuous at $x = 4$, because:
1) $G(4)$ exists,
2) $\lim_{x\to4} G(x)$ exists, and
3) $\lim_{x\to4} G(x) = 5 = G(4)$.

53. The function is not continuous at $x = 5$ because $g(5)$ does not exist.

$$\begin{aligned}g(5) &= \frac{1}{(5)^2-7(5)+10}\\ &= \frac{1}{25-35+10}\\ &= \frac{1}{0}\end{aligned}$$

54. The function $f(x)$ is continuous at $x = 3$, because:
1) $f(3)$ exists,
2) $\lim_{x\to3} f(x)$ exists, and
3) $\lim_{x\to3} f(x) = -1 = f(3)$.

55. First we find the function value when $x = 4$.

$$\begin{aligned}F(4) &= \frac{1}{(4)^2-7(4)+10}\\ &= \frac{1}{16-28+10}\\ &= \frac{1}{-2}\\ &= -\frac{1}{2}\end{aligned}$$

Hence, $F(4)$ exists.
Next we find the limit. Applying the Theorem on Limits of Rational Functions we have:

$$\lim_{x\to4} F(x) = \frac{1}{(4)^2-7(4)+10} = -\frac{1}{2}.$$

We now have,

$$\lim_{x\to4} F(x) = \frac{1}{(4)^2-7(4)+10} = -\frac{1}{2} = F(4).$$

Therefore, the function is continuous at $x = 4$.

56. The function is not continuous at $x = 2$, because $G(2)$ does not exist.

$$\begin{aligned}G(2) &= \frac{1}{(2)^2-6(2)+8}\\ &= \frac{1}{0}\end{aligned}$$

57. Yes, the function is continuous over the interval $(-4,4)$. Since the function is defined for every value in the interval, the Theorem on Limits of Rational Functions tells us $\lim_{x\to a} g(x) = g(a)$ for all values *a* in the interval. Thus $g(x)$ is continuous over the interval.

58. Yes, the function is continuous over the interval $(-5,5)$. Since the function is defined for every value in the interval, the Theorem on Limits of Rational Functions tells us $\lim_{x\to a} F(x) = F(a)$ for all values *a* in the interval. Thus $F(x)$ is continuous over the interval.

59. No, the function is not continuous over the interval $(-7,7)$ because the function does not exist at $x = 0$. $f(0) = \frac{1}{0} + 3$, which is undefined.

60. No, the function is not Continuous over the interval $(0,\infty)$ because the function does not exist at $x=1$. $G(1)=\frac{1}{1-1}=\frac{1}{0}$, which is undefined.

61. Yes, the function is continuous on $\mathbb{R}$. The function is defined for all real numbers, so by the Theorem on Limits of Rational Functions, $\lim\limits_{x\to a} g(x)=g(a)$ for all a in $\mathbb{R}$.

62. No, the function is not continuous over $\mathbb{R}$, because the function does not exist at $x=5$. $F(5)=\frac{3}{x-5}=\frac{3}{0}$, which is undefined.

63. As the inputs x approach 0 from the left, the outputs approach -1. We see this by looking at a table:

x	-0.1	-0.01	-0.001
$\frac{\lvert x\rvert}{x}$	-1	-1	-1

$$\lim_{x\to 0^-}\frac{|x|}{x}=-1.$$

As the inputs x approach 0 from the right, the outputs approach 1. We see this by looking at a table:

x	0.001	0.01	0.1
$\frac{\lvert x\rvert}{x}$	1	1	1

$$\lim_{x\to 0^+}\frac{|x|}{x}=1$$

Since the limit from the left, -1, is not the same as the limit from the right, 1, the limit does not exist. $\lim\limits_{x\to 0}\frac{|x|}{x}$ does not exist.

64. The limit as x approaches -2 from the left is:

$$\lim_{x\to -2^-}\frac{x^3+8}{x^2-4}=-3$$

The limit as x approaches -2 from the right is:

$$\lim_{x\to -2^+}\frac{x^3+8}{x^2-4}=-3$$

Since the limit from the left is the same as the limit from the right, the limit exists and is:

$$\lim_{x\to -2}\frac{x^3+8}{x^2-4}=-3.$$

Another approach would be to simplify the function, by factoring the numerator and denominator and dividing out common factors.

$$\begin{aligned}\lim_{x\to -2}\frac{x^3+8}{x^2-4}&=\lim_{x\to -2}\frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)}\\&=\lim_{x\to -2}\frac{x^2-2x+4}{x-2}\quad \text{assuming } x\neq -2\\&=\frac{(-2)^2-2(-2)+4}{(-2)-2}\\&=\frac{12}{-4}\\&=-3\end{aligned}$$

65. 6

66. 0.5, or $\frac{1}{2}$

67. -0.2887, or $-\frac{1}{2\sqrt{3}}$

68. 0.5, or $\frac{1}{2}$

69. 0.75, or $\frac{3}{4}$

70. 0.378, or $\frac{1}{\sqrt{7}}$

71. 0.25, or $\frac{1}{4}$

72. 0