# Chapter 5

# Applications of Integration

## Exercise Set 5.1

**1.** $D(x) = -\frac{5}{6}x + 9, \quad S(x) = \frac{1}{2}x + 1$

a) To find the equilibrium point we set $D(x) = S(x)$ and solve.

$$-\frac{5}{6}x + 9 = \frac{1}{2}x + 1$$

$$9 - 1 = \frac{1}{2}x + \frac{5}{6}x$$

$$8 = \frac{4}{3}x \qquad \left(\frac{1}{2} + \frac{5}{6} = \frac{8}{6} = \frac{4}{3}\right)$$

$$\frac{3}{4} \cdot 8 = x \qquad \text{Multiplying by } \frac{3}{4}$$

$$6 = x$$

Thus $x_E = 6$ units. To find $p_E$ we substitute $x_E$ into $D(x)$ or $S(x)$. Here we use $D(x)$.

$$\begin{aligned} p_E &= D(x_E) \\ &= D(6) \\ &= -\frac{5}{6}(6) + 9 \\ &= -5 + 9 \\ &= 4 \end{aligned}$$

When 6 units are sold the equilibrium price is \$4; therefore, the equilibrium point is $(6, \$4)$.

Notice, we could have used $S(x)$ to find the equilibrium point as well. Substituting, we have:

$$\begin{aligned} p_E &= S(x_E) \\ &= S(6) \\ &= \frac{1}{2}(6) + 1 \\ &= 3 + 1 \\ &= 4 \end{aligned}$$

This results in the same equilibrium point $(6, \$4)$.

b) The consumer surplus is $\int_0^{x_E} D(x)\,dx - x_E p_E$.

Substituting $-\frac{5}{6}x + 9$ for $D(x)$, 6 for $x_E$, and 4 for $p_E$ we have:

$$\begin{aligned} &\int_0^6 \left(-\frac{5}{6}x + 9\right) dx - 6 \cdot 4 \\ &= \left[-\frac{5x^2}{12} + 9x\right]_0^6 - 24 \\ &= \left[\left(-\frac{5(6)^2}{12} + 9 \cdot 6\right) - \left(-\frac{5(0)^2}{12} + 9 \cdot 0\right)\right] - 24 \\ &= \left[(-15 + 54) - (0)\right] - 24 \\ &= 39 - 24 \\ &= 15 \end{aligned}$$

The consumer surplus at the equilibrium point is \$15.

c) The producer surplus is $x_E p_E - \int_0^{x_E} S(x)\,dx$.

Substituting $\frac{1}{2}x + 1$ for $S(x)$, 6 for $x_E$, and 4 for $p_E$ we have:

$$\begin{aligned} &6 \cdot 4 - \int_0^6 \left(\frac{1}{2}x + 1\right) dx \\ &= 24 - \left[\frac{x^2}{4} + x\right]_0^6 \\ &= 24 - \left[\left(\frac{6^2}{4} + 6\right) - \left(0^2 + 0\right)\right] \\ &= 24 - \left[\frac{36}{4} + 6\right] \\ &= 24 - [9 + 6] \\ &= 24 - 15 \\ &= 9 \end{aligned}$$

The producer surplus at the equilibrium point is \$9.

**2.** $D(x) = -3x+7, \qquad S(x) = 2x+2$

a) $D(x) = S(x)$

$$-3x+7 = 2x+2$$
$$5 = 5x$$
$$1 = x$$
$$p_E = S(x_E) = 2(1)+2 = 4$$

When 1 unit is sold, equilibrium price is \$4; therefore, the equilibrium point is $(1, \$4)$.

b) $\int_0^1 (-3x+7)\,dx - 1\cdot 4$

$$= \left[-\frac{3x^2}{2} + 7x\right]_0^1 - 4$$
$$= \left[\left(-\frac{3(1)^2}{2} + 7(1)\right) - \left(-\frac{3(0)^2}{2} + 7(0)\right)\right] - 4$$
$$= \left[\left(-\frac{3}{2} + 7\right) - (0)\right] - 4$$
$$= \left[\frac{11}{2}\right] - 4$$
$$= \frac{3}{2} = 1.50$$

The consumer surplus at the equilibrium point is \$1.50.

c) $1\cdot 4 - \int_0^1 (2x+2)\,dx$

$$= 4 - \left[x^2 + 2x\right]_0^1$$
$$= 4 - \left[\left(1^2 + 2\cdot 1\right) - \left(0^2 + 2(0)\right)\right]$$
$$= 4 - \left[(1+2) - (0)\right]$$
$$= 4 - [3]$$
$$= 1$$

The producer surplus at the equilibrium point is \$1.

**3.** $D(x) = (x-4)^2, \;\; S(x) = x^2 + 2x + 6$

a) To find the equilibrium point we set $D(x) = S(x)$ and solve.

$$(x-4)^2 = x^2 + 2x + 6$$
$$x^2 - 8x + 16 = x^2 + 2x + 6$$
$$-8x + 16 = 2x + 6$$
$$10 = 10x$$
$$1 = x$$

Thus $x_E = 1$ unit. To find $p_E$ we substitute $x_E$ into $D(x)$ or $S(x)$. Here we use $D(x)$.

$$p_E = D(x_E)$$
$$= D(1)$$
$$= \left((1)-4\right)^2$$
$$= (-3)^2$$
$$= 9$$

When 1 unit is sold the equilibrium price is \$9; therefore, the equilibrium point is $(1, \$9)$.

b) The consumer surplus is

$\int_0^{x_E} D(x)\,dx - x_E p_E$.

Substituting $(x-4)^2$ for $D(x)$, 1 for $x_E$, and 9 for $p_E$ we have:

$$\int_0^1 (x-4)^2\,dx - 1\cdot 9$$
$$= \int_0^1 \left(x^2 - 8x + 16\right)dx - 9$$
$$= \left[\frac{x^3}{3} - 4x^2 + 16x\right]_0^1 - 9$$
$$= \left[\left(\frac{1}{3} - 4 + 16\right) - (0 - 0 + 0)\right] - 9$$
$$= \frac{37}{3} - 9$$
$$= \frac{10}{3} \approx 3.33$$

The consumer surplus at the equilibrium point is \$3.33.

c) The producer surplus is

$x_E p_E - \int_0^{x_E} S(x)\,dx$.

Substituting $x^2 + 2x + 6$ for $S(x)$, 1 for $x_E$, and 9 for $p_E$ we have:

$$1\cdot 9 - \int_0^1 \left(x^2 + 2x + 6\right)dx$$
$$= 9 - \left[\frac{x^3}{3} + x^2 + 6x\right]_0^1$$
$$= 9 - \left[\left(\frac{1}{3} + 1 + 6\right) - (0 + 0 + 0)\right]$$
$$= 9 - \left[\frac{22}{3}\right]$$
$$= \frac{5}{3} \approx 1.67$$

The producer surplus at the equilibrium point is \$1.67.

**4.** $D(x)=(x-3)^2, \quad S(x)=x^2+2x+1$

a) $D(x)=S(x)$

$$(x-3)^2 = x^2+2x+1$$
$$x^2-6x+9 = x^2+2x+1$$
$$8 = 8x$$
$$1 = x$$
$$p_E = D(x_E) = (1-3)^2 = 4$$

When 1 unit is sold, equilibrium price is \$4; therefore, the equilibrium point is $(1, \$4)$.

b) $\int_0^1 (x-3)^2\,dx - 1\cdot 4$

$$= \int_0^1 (x^2-6x+9)\,dx - 4$$
$$= \left[\frac{x^3}{3} - 3x^2 + 9x\right]_0^1 - 4$$
$$= \left[\left(\frac{1}{3} - 3 + 9\right) - (0-0+0)\right] - 4$$
$$= \left[\left(\frac{19}{3}\right) - (0)\right] - 4$$
$$= \frac{19}{3} - 4$$
$$= \frac{7}{3} \approx 2.33$$

The consumer surplus at the equilibrium point is \$2.33.

c) $1\cdot 4 - \int_0^1 (x^2+2x+1)\,dx$

$$= 4 - \left[\frac{x^3}{3} + x^2 + x\right]_0^1$$
$$= 4 - \left[\left(\frac{1}{3}+1+1\right) - (0+0+0)\right]$$
$$= 4 - \left[\left(\frac{7}{3}\right) - (0)\right]$$
$$= 4 - \frac{7}{3}$$
$$= \frac{5}{3} \approx 1.67$$

The producer surplus at the equilibrium point is \$1.67.

**5.** $D(x)=(x-6)^2, \; S(x)=x^2$

a) To find the equilibrium point we set $D(x)=S(x)$ and solve.

$$(x-6)^2 = x^2$$
$$x^2-12x+36 = x^2$$
$$-12x+36 = 0$$
$$36 = 12x$$
$$3 = x$$

Thus $x_E = 3$ units. To find $p_E$ we substitute $x_E$ into $D(x)$ or $S(x)$. Here we use $S(x)$.

$$p_E = S(x_E) = S(3) = (3)^2 = 9$$

When 3 units are sold the equilibrium price is \$9; therefore, the equilibrium point is $(3, \$9)$.

b) The consumer surplus is

$\int_0^{x_E} D(x)\,dx - x_E p_E$.

Substituting $(x-6)^2$ for $D(x)$, 3 for $x_E$, and 9 for $p_E$ we have:

$$\int_0^3 (x-6)^2\,dx - 3\cdot 9$$
$$= \int_0^3 (x^2-12x+36)\,dx - 27$$
$$= \left[\frac{x^3}{3} - 6x^2 + 36x\right]_0^3 - 27$$
$$= \left[\left(\frac{3^3}{3} - 6\cdot 3^2 + 36\cdot 3\right) - (0-0+0)\right] - 27$$
$$= \left[(9-54+108) - (0)\right] - 27$$
$$= 63 - 27$$
$$= 36$$

The consumer surplus at the equilibrium point is \$36.

c) The producer surplus is

$x_E p_E - \int_0^{x_E} S(x)\,dx$.

Substituting $x^2$ for $S(x)$, 3 for $x_E$, and 9 for $p_E$ we have:

$$3\cdot 9 - \int_0^3 x^2 dx$$
$$= 27 - \left[\frac{x^3}{3}\right]_0^3$$
$$= 27 - \left[\left(\frac{3^3}{3}\right) - \left(\frac{0^3}{3}\right)\right]$$
$$= 27 - [9]$$
$$= 18$$

The producer surplus at the equilibrium point is \$18.

**6.** $D(x) = (x-8)^2, \quad S(x) = x^2$

a) $D(x) = S(x)$

$$(x-8)^2 = x^2$$
$$x^2 - 16x + 64 = x^2$$
$$64 = 16x$$
$$4 = x$$
$$p_E = S(x_E) = (4)^2 = 16$$

When 4 units are sold, equilibrium price is \$16; therefore, the equilibrium point is $(4, \$16)$.

b) $\int_0^4 (x-8)^2\,dx - 4\cdot 16$

$$= \left[\frac{(x-8)^3}{3}\right]_0^4 - 64$$
$$= \left[\frac{(4-8)^3}{3} - \frac{(0-8)^3}{3}\right] - 64$$
$$= \left[\frac{-64}{3} - \frac{-512}{3}\right] - 64$$
$$= -\frac{64}{3} + \frac{512}{3} - 64$$
$$= \frac{256}{3} \approx 85.33$$

The consumer surplus at the equilibrium point is \$85.33.

c) $4\cdot 16 - \int_0^4 (x^2)\,dx$

$$= 64 - \left[\frac{x^3}{3}\right]_0^4$$
$$= 64 - \left[\left(\frac{4^3}{3}\right) - \left(\frac{0^3}{3}\right)\right]$$
$$= 64 - \frac{64}{3}$$
$$= \frac{128}{3} \approx 42.67$$

The producer surplus at the equilibrium point is \$42.67.

**7.** $D(x) = 1000 - 10x, \quad S(x) = 250 + 5x$

a) To find the equilibrium point we set $D(x) = S(x)$ and solve.

$$1000 - 10x = 250 + 5x$$
$$750 = 15x$$
$$50 = x$$

Thus $x_E = 50$ units. To find $p_E$ we substitute $x_E$ into $D(x)$ or $S(x)$. Here we use $D(x)$.

$$p_E = D(x_E)$$
$$= D(50)$$
$$= 1000 - 10\cdot 50$$
$$= 500$$

When 50 units are sold the equilibrium price is \$500; therefore, the equilibrium point is $(50, \$500)$.

b) The consumer surplus is

$\int_0^{x_E} D(x)\,dx - x_E p_E$.

Substituting $1000 - 10x$ for $D(x)$, 50 for $x_E$, and 500 for $p_E$ we have:

$$\int_0^{50} (1000 - 10x)\,dx - 50\cdot 500$$
$$= \left[1000x - 5x^2\right]_0^{50} - 25{,}000$$
$$= \left[\left(1000\cdot 50 - 5(50)^2\right) - \left(1000\cdot 0 - 5\cdot 0^2\right)\right] - 25{,}000$$
$$= [50{,}000 - 12{,}500] - 25{,}000$$
$$= 12{,}500$$

The consumer surplus at the equilibrium point is \$12,500.

c) The producer surplus is

$x_E p_E - \int_0^{x_E} S(x)\,dx$.

Substituting $250+5x$ for $S(x)$, 50 for $x_E$, and 500 for $p_E$ we have:

$$50 \cdot 500 - \int_0^{50} (250+5x)\,dx$$
$$= 25{,}000 - \left[250x + \frac{5x^2}{2}\right]_0^{50}$$
$$= 25{,}000 - \left[\left(250 \cdot 50 + \frac{5(50)^2}{2}\right) - \left(250 \cdot 0 - \frac{5(0)^2}{2}\right)\right]$$
$$= 25{,}000 - [12{,}500 + 6250]$$
$$= 6250$$

The producer surplus at the equilibrium point is \$6250.

**8.** $D(x) = 8800 - 30x, \qquad S(x) = 7000 + 15x$

a) $D(x) = S(x)$

$$8800 - 30x = 7000 + 15x$$
$$1800 = 45x$$
$$40 = x$$
$$p_E = D(x_E) = 8800 - 30 \cdot 40 = 7600$$

When 40 units are sold, equilibrium price is \$7600; therefore, the equilibrium point is $(40, \$7600)$.

b) $\int_0^{40} (8800 - 30x)\,dx - 40 \cdot 7600$

$$= \left[8800x - 15x^2\right]_0^{40} - 304{,}000$$
$$= \left[\left(8800 \cdot 40 - 15(40)^2\right) - \left(8800 \cdot 0 - 15(0^2)\right)\right] - 304{,}000$$
$$= [352{,}000 - 24{,}000] - 304{,}000$$
$$= 24{,}000$$

The consumer surplus at the equilibrium point is \$24,000.

c) $40 \cdot 7600 - \int_0^{40} (7000 + 15x)\,dx$

$$= 304{,}000 - \left[7000x + \frac{15x^2}{2}\right]_0^{40}$$
$$= 304{,}000 - \left[\left(7000 \cdot 40 + \frac{15(40)^2}{2}\right) - \left(7000 \cdot 0 + \frac{15(0)^2}{2}\right)\right]$$
$$= 304{,}000 - [280{,}000 + 12{,}000]$$
$$= 12{,}000$$

The producer surplus at the equilibrium point is \$12,000.

**9.** $D(x) = 5 - x$, for $0 \le x \le 5$;

$S(x) = \sqrt{x+7}$

a) To find the equilibrium point we set $D(x) = S(x)$ and solve.

$$5 - x = \sqrt{x+7}$$
$$(5-x)^2 = \left(\sqrt{x+7}\right)^2$$
$$25 - 10x + x^2 = x + 7$$
$$x^2 - 11x + 18 = 0$$
$$(x-2)(x-9) = 0$$
$$x - 2 = 0 \quad \text{or} \quad x - 9 = 0$$
$$x = 2 \quad \text{or} \quad x = 9$$

Note, $x = 9$ is not a solution to the equation. Only $x = 2$ is in the domain of $D(x)$, thus $x_E = 2$ units. To find $p_E$ we substitute $x_E$ into $D(x)$ or $S(x)$. Here we use $D(x)$.

$$p_E = D(x_E) = D(2) = 5 - 2 = 3$$

When 2 units are sold the equilibrium price is \$3; therefore, the equilibrium point is $(2, \$3)$.

b) The consumer surplus is

$\int_0^{x_E} D(x)\,dx - x_E p_E$.

Substituting $5 - x$ for $D(x)$, 2 for $x_E$, and 3 for $p_E$ we have:

$\int_0^2 (5-x)\,dx - 2\cdot 3$

$= \left[5x - \frac{x^2}{2}\right]_0^2 - 6$

$= \left[\left(5\cdot 2 - \frac{(2)^2}{2}\right) - \left(5\cdot 0 - \frac{(0)^2}{0}\right)\right] - 6$

$= [10-2] - 6$

$= 2$

The consumer surplus at the equilibrium point is \$2.

c) The producer surplus is

$x_E p_E - \int_0^{x_E} S(x)\,dx$.

Substituting $\sqrt{7+x}$ for $S(x)$, 2 for $x_E$, and 3 for $p_E$ we have:

$2\cdot 3 - \int_0^2 \left(\sqrt{x+7}\right) dx$

$= 6 - \int_0^2 (x+7)^{1/2}\,dx$

$= 6 - \left[\frac{2}{3}(x+7)^{3/2}\right]_0^2$

$= 6 - \left[\left(\frac{2}{3}(2+7)^{3/2}\right) - \left(\frac{2}{3}(0+7)^{3/2}\right)\right]$

$= 6 - \left[\frac{2}{3}(9)^{3/2} - \frac{2}{3}(7)^{3/2}\right]$

$= 6 - \left[\frac{2}{3}\cdot(27) - \frac{2}{3}(7)^{3/2}\right]$

$\approx 0.35$ Using a calculator

The producer surplus at the equilibrium point is \$0.35.

**10.** $D(x) = 7 - x$, for $0 \le x \le 7$;

$S(x) = 2\sqrt{x+1}$

a) $D(x) = S(x)$

$7 - x = 2\sqrt{x+1}$

$(7-x)^2 = \left(2\sqrt{x+1}\right)^2$

$49 - 14x + x^2 = 4(x+1)$

$x^2 - 18x + 45 = 0$

$(x-3)(x-15) = 0$

$x = 3$ or $x = 15$

Only 3 is in the domain of $D(x)$, thus $x_E = 3$ units.

$p_E = D(x_E) = 7 - 3 = 4$

When 3 units are sold, equilibrium price is \$4; therefore, the equilibrium point is $(3, \$4)$.

b) $\int_0^3 (7-x)\,dx - 3\cdot 4$

$= \left[7x - \frac{x^2}{2}\right]_0^3 - 12$

$= \left[\left(7\cdot 3 - \frac{(3)^2}{2}\right) - \left(7\cdot 0 - \frac{(0)^2}{2}\right)\right] - 12$

$= \left[\frac{33}{2}\right] - 12$

$= \frac{9}{2} = 4.50$

The consumer surplus at the equilibrium point is \$4.50.

c) $3\cdot 4 - \int_0^3 \left(2\sqrt{x+1}\right) dx$

$= 12 - \int_0^3 \left(2(x+1)^{1/2}\right) dx$

$= 12 - \left[\frac{4}{3}(x+1)^{3/2}\right]_0^3$

$= 12 - \left[\left(\frac{4}{3}(3+1)^{3/2}\right) - \left(\frac{4}{3}(0+1)^{3/2}\right)\right]$

$= 12 - \left[\frac{32}{3} - \frac{4}{3}\right]$

$= \frac{8}{3} \approx 2.67$

The producer surplus at the equilibrium point is \$2.67.

**11.** $D(x) = \frac{100}{\sqrt{x}}$, $S(x) = \sqrt{x}$

a) To find the equilibrium point we set $D(x) = S(x)$ and solve.

$\frac{100}{\sqrt{x}} = \sqrt{x}$

$100 = \sqrt{x}\cdot\sqrt{x}$

$100 = x$

Thus $x_E = 100$ units. To find $p_E$ we substitute $x_E$ into $D(x)$ or $S(x)$. Here we use $S(x)$.

$p_E = S(x_E) = S(100) = \sqrt{100} = 10$

When 100 units are sold the equilibrium price is \$10; therefore, the equilibrium point is $(100, \$10)$.

b) The consumer surplus is

$\int_0^{x_E} D(x)\,dx - x_E p_E$.

Substituting $\frac{100}{\sqrt{x}} = 100x^{-1/2}$ for $D(x)$, 100 for $x_E$, and 10 for $p_E$ we have:

$$\int_0^{100}\left(100x^{-1/2}\right)dx - 100\cdot 10$$
$$= \left[\frac{100x^{1/2}}{\frac{1}{2}}\right]_0^{100} - 1000$$
$$= \left[\left(200(100)^{1/2}\right) - \left(200(0)^{1/2}\right)\right] - 1000$$
$$= [2000] - 1000$$
$$= 1000$$

The consumer surplus at the equilibrium point is \$1000.

c) The producer surplus is

$x_E p_E - \int_0^{x_E} S(x)\,dx$.

Substituting $\sqrt{x}$ for $S(x)$, 100 for $x_E$, and 10 for $p_E$ we have:

$$100\cdot 10 - \int_0^{100}\left(x^{1/2}\right)dx$$
$$= 1000 - \left[\frac{2}{3}(x)^{3/2}\right]_0^{100}$$
$$= 1000 - \left[\left(\frac{2}{3}(100)^{3/2}\right) - \left(\frac{2}{3}(0)^{3/2}\right)\right]$$
$$= 1000 - \left[\frac{2000}{3} - 0\right]$$
$$= 1000 - \frac{2000}{3}$$
$$= \frac{1000}{3} \approx 333.33$$

The producer surplus at the equilibrium point is \$333.33.

**12.** $D(x) = \frac{1800}{\sqrt{x+1}}, \qquad S(x) = 2\sqrt{x+1}$

a) $D(x) = S(x)$

$$\frac{1800}{\sqrt{x+1}} = 2\sqrt{x+1}$$
$$1800 = \left(2\sqrt{x+1}\right)\sqrt{x+1}$$
$$1800 = 2(x+1)$$
$$900 = x+1$$
$$899 = x$$

$$p_E = S(x_E) = 2\sqrt{899+1} = 2\sqrt{900} = 60$$

When 899 units are sold, equilibrium price is \$60; therefore, the equilibrium point is $(899, \$60)$.

b) $\int_0^{899}\left(1800x+1^{-1/2}\right)dx - 899\cdot 60$

$$= 1800\left[2(x+1)^{1/2}\right]_0^{899} - 53{,}940$$
$$= 1800\left[\left(2\sqrt{899+1}\right) - \left(2\sqrt{0+1}\right)\right] - 53{,}940$$
$$= 1800[2(30) - 2(1)] - 53{,}940$$
$$= 1800(58) - 53{,}940$$
$$= 104{,}400 - 53{,}940$$
$$= 50{,}460$$

The consumer surplus at the equilibrium point is \$50,460.

c) $899\cdot 60 - \int_0^{899}\left(2\sqrt{x+1}\right)dx$

$$= 53{,}940 - \int_0^{899}\left(2(x+1)^{1/2}\right)dx$$
$$= 53{,}940 - 2\left[\frac{2}{3}(x+1)^{3/2}\right]_0^{899}$$
$$= 53{,}940 - \frac{4}{3}\left[\left((899+1)^{3/2}\right) - \left((0+1)^{3/2}\right)\right]$$
$$= 53{,}940 - \frac{4}{3}[27{,}000 - 1]$$
$$= 53{,}940 - \frac{4}{3}(26{,}999)$$
$$\approx 17{,}941.33$$

The producer surplus at the equilibrium point is \$17,941.33.

**13.** $D(x) = (x-4)^2, \quad S(x) = x^2 + 2x + 8$

a) To find the equilibrium point we set $D(x) = S(x)$ and solve.

$$(x-4)^2 = x^2 + 2x + 8$$
$$x^2 - 8x + 16 = x^2 + 2x + 8$$
$$-8x + 16 = 2x + 8$$
$$8 = 10x$$
$$\frac{8}{10} = x$$
$$\frac{4}{5} = x$$

Thus $x_E = \frac{4}{5}$=0.8 units. Assuming partial units are acceptable, to find $p_E$ we substitute $x_E$ into $D(x)$ or $S(x)$. Here we use $D(x)$.

$$p_E = D(x_E)$$
$$= D\left(\frac{4}{5}\right)$$
$$= \left(\frac{4}{5} - 4\right)^2$$
$$= \left(-\frac{16}{5}\right)^2$$
$$= \frac{256}{25}$$
$$= 10.24$$

When 0.8 units are sold the equilibrium price is \$10.24; therefore, the equilibrium point is $(0.8, \$10.24)$.

b) The consumer surplus is

$\int_0^{x_E} D(x)\,dx - x_E p_E$.

Substituting $(x-4)^2$ for $D(x)$, 0.8 for $x_E$, and 10.24 for $p_E$ we have:

$$\int_0^{0.8} (x-4)^2\,dx - 0.8 \cdot 10.24$$
$$= \left[\frac{(x-4)^3}{3}\right]_0^{0.8} - 8.192$$
$$= \left[\left(\frac{(0.8-4)^3}{3}\right) - \left(\frac{(0-4)^3}{3}\right)\right] - 8.192$$
$$= \left[\left(\frac{-32.768}{3}\right) - \left(\frac{-64}{3}\right)\right] - 8.192$$
$$\approx 2.22 \qquad \text{Using a calculator}$$

The consumer surplus at the equilibrium point is \$2.22.

c) The producer surplus is

$x_E p_E - \int_0^{x_E} S(x)\,dx$.

Substituting $x^2 + 2x + 8$ for $S(x)$, 0.8 for $x_E$, and 10.24 for $p_E$ we have:

$$0.8 \cdot 10.24 - \int_0^{0.8} (x^2 + 2x + 8)\,dx$$
$$= 8.192 - \left[\frac{x^3}{3} + x^2 + 8x\right]_0^{0.8}$$
$$= 8.192 - \left[\left(\frac{(0.8)^3}{3} + (0.8)^2 + 8(0.8)\right) - (0)\right]$$
$$\approx 8.192 - 7.21066667$$
$$\approx 0.98$$

The producer surplus at the equilibrium point is \$0.98.

**14.** $D(x) = 13 - x$, for $0 \le x \le 13$;
$S(x) = \sqrt{x+17}$

a) $D(x) = S(x)$

$$13 - x = \sqrt{x+17}$$
$$(13-x)^2 = \left(\sqrt{x+17}\right)^2$$
$$169 - 26x + x^2 = (x+17)$$
$$x^2 - 27x + 152 = 0$$
$$(x-8)(x-19) = 0$$
$$x - 8 = 0 \quad \text{or} \quad x - 19 = 0$$
$$x = 8 \quad \text{or} \quad x = 19$$

Only 8 is in the domain of $D(x)$, thus $x_E = 8$ units.

$p_E = D(x_E) = 13 - 8 = 5$

When 8 units are sold, equilibrium price is \$5; therefore, the equilibrium point is $(8, \$5)$.

b) $\int_0^8 (13 - x)\,dx - 8 \cdot 5$

$$= \left[13x - \frac{x^2}{2}\right]_0^8 - 40$$
$$= \left[\left(13 \cdot 8 - \frac{(8)^2}{2}\right) - \left(13 \cdot 0 - \frac{(0)^2}{2}\right)\right] - 40$$
$$= [(104 - 32) - 0] - 40$$
$$= 32$$

The consumer surplus at the equilibrium point is \$32.

c) $8\cdot 5-\int_0^8\left(\sqrt{x+17}\right)dx$

$= 40-\int_0^8\left(x+17\right)^{1/2}dx$

$= 40-\left[\frac{2}{3}\left(x+17\right)^{3/2}\right]_0^8$

$= 40-\frac{2}{3}\left[\left(\left(8+17\right)^{3/2}\right)-\left(\left(0+17\right)^{3/2}\right)\right]$

$= 40-\frac{2}{3}\left[\left(25\right)^{3/2}-\left(17\right)^{3/2}\right]$

$\approx 3.39519709$

$\approx 3.40$

The producer surplus at the equilibrium point is \$3.40.

**15.** $D(x)=e^{-x+4.5}, \qquad S(x)=e^{x-5.5}$

a) To find the equilibrium point we set $D(x)=S(x)$ and solve.

$$\begin{aligned} e^{-x+4.5} &= e^{x-5.5} \\ \ln\left(e^{-x+4.5}\right) &= \ln\left(e^{x-5.5}\right) \\ -x+4.5 &= x-5.5 \\ 10 &= 2x \\ 5 &= x \end{aligned}$$

Thus $x_E=5$ units. To find $p_E$ we substitute $x_E$ into $D(x)$ or $S(x)$. Here we use $D(x)$.

$p_E = D(x_E)$

$= D(5)=e^{-5+4.5}=e^{-0.5}\approx 0.61$

When 5 units are sold the equilibrium price is \$0.61; therefore, the equilibrium point is $(5,\$0.61)$.

b) The consumer surplus is

$\int_0^{x_E} D(x)\,dx - x_E p_E$.

Substituting $e^{-x+4.5}$ for $D(x)$, 5 for $x_E$, and 0.61 for $p_E$ we have:

$\int_0^5 e^{-x+4.5}dx-5\cdot 0.61$

$=\left[-e^{-x+4.5}\right]_0^5-3.05$

$=\left[\left(-e^{-5+4.5}\right)-\left(-e^{-0+4.5}\right)\right]-3.05$

$=\left[-e^{-0.5}+e^{4.5}\right]-3.05$

$\approx 86.36$ Using a calculator

The consumer surplus at the equilibrium point is \$86.36.

c) The producer surplus is

$x_E p_E-\int_0^{x_E} S(x)\,dx$.

Substituting $e^{x-5.5}$ for $S(x)$, 5 for $x_E$, and 0.61 for $p_E$ we have:

$5\cdot 0.61-\int_0^5\left(e^{x-5.5}\right)dx$

$=3.05-\left[e^{x-5.5}\right]_0^5$

$=3.05-\left[\left(e^{5-5.5}\right)-\left(e^{0-5.5}\right)\right]$

$=3.05-\left[e^{-0.5}-e^{-5.5}\right]$

$\approx 2.45$

The producer surplus at the equilibrium point is \$2.45.

**16.** $D(x)=\sqrt{56-x}, \qquad S(x)=x$

a) $D(x)=S(x)$

$$\begin{aligned} \sqrt{56-x} &= x \\ \left(\sqrt{56-x}\right)^2 &= (x)^2 \\ 56-x &= x^2 \\ 0 &= x^2+x-56 \\ 0 &= (x-7)(x+8) \end{aligned}$$

$x-7=0 \quad \text{or} \quad x+8=0$

$x=7 \quad \text{or} \quad x=-8$

A negative quantity is not possible, thus $x_E=7$ units. To find $p_E$ we substitute $x_E$ into $D(x)$ or $S(x)$. Here we use $S(x)$.

$p_E=S(x_E)=7$

When 7 units are sold the equilibrium price is \$7; therefore, the equilibrium point is $(7,\$7)$.

b) $\int_0^7\sqrt{56-x}\,dx-7\cdot 7$

$=\int_0^7\left(56-x\right)^{1/2}dx-49$

$=\left[-\frac{2}{3}\left(56-x\right)^{3/2}\right]_0^7-49$

$=-\frac{2}{3}\left[\left(\left(56-7\right)^{3/2}\right)-\left(\left(56-0\right)^{3/2}\right)\right]-49$

$=-\frac{2}{3}\left[\left(343\right)-\left(56\right)^{3/2}\right]-49$

$\approx 1.71$

The consumer surplus at the equilibrium point is \$1.71.

c) $7 \cdot 7 - \int_0^7 x dx$

$= 49 - \left[\frac{x^2}{2}\right]_0^7$

$= 49 - \left[\left(\frac{(7)^2}{2}\right) - \left(\frac{(0)^2}{2}\right)\right]$

$= 49 - \left[\frac{49}{2}\right]$

$= \frac{49}{2} = 24.50$

The producer surplus at the equilibrium point is \$24.50.

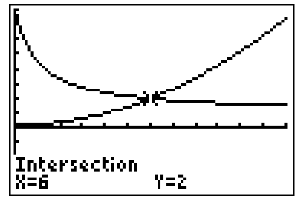
**17.** $\boxed{tw}$ Answers will vary. When a surplus exists, consumers are purchasing goods or services at a price lower than what they are willing to pay, and producers are selling goods or services at a price above which they are willing to provide the good or service. By purchasing a good or service at a price below the perceived value, consumers feel like they get a deal and are happy. Likewise for the producers.

**18.** $\boxed{tw}$ Answers will vary. Consumer surplus is the net benefit, in aggregate, consumers receive by purchasing goods or services at a market price below what they are willing to pay. In other words, it is the difference in what the consumers are willing to pay and what they have to pay for a certain good or service. On a hot summer day, a consumer might be willing to pay \$5 for a bottle of water. If the market price of the water is \$1, then the consumer will receive a \$4 surplus for the first bottle of water.

Producer surplus is the net benefit, in aggregate, producers receive by selling goods or services at a market price above what they are willing to provide the good or service. In other words, it is the difference in the price that producers receive in the market and the price that producers would be willing to provide the good or service. It is important to note that producer surplus is not the profit producers earn.

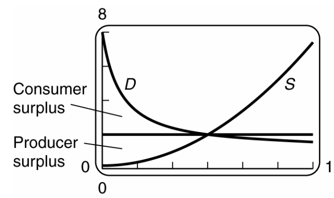
**19.** $D(x) = \frac{x+8}{x+1}, \quad S(x) = \frac{x^2+4}{20}$

a) Graphing the equations and using the INTERSECT feature we have:




The equilibrium point is $(6, \$2)$.

b)




c) The consumer surplus is

$\int_0^{x_E} D(x)\,dx - x_E p_E$.

Substituting $\frac{x+8}{x+1}$ for $D(x)$, 6 for $x_E$, and 2 for $p_E$ we have:

$\int_0^6 \left(\frac{x+8}{x+1}\right) dx - 6 \cdot 2$

$\approx 19.62137104 - 12$ Using a calculator

$\approx 7.62$

The consumer surplus at the equilibrium point is \$7.62.

d) The producer surplus is

$x_E p_E - \int_0^{x_E} S(x)\,dx$.

Substituting $\frac{x^2+4}{20}$ for $S(x)$, 6 for $x_E$, and 2 for $p_E$ we have:

$6 \cdot 2 - \int_0^6 \left(\frac{x^2+4}{20}\right) dx$

$= 12 - \left[\frac{x^3}{60} + \frac{x}{5}\right]_0^6$

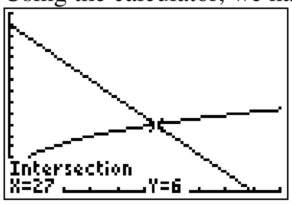
$= 12 - \frac{24}{5}$

$= \frac{36}{5} = 7.20$

The producer surplus at the equilibrium point is \$7.20.

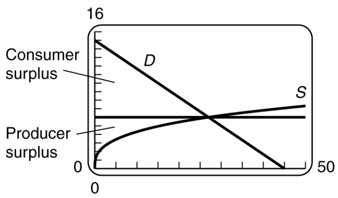
**20.** $D(x)=15-\frac{1}{3}x, \quad S(x)=2\sqrt[3]{x}$

a) Using the calculator, we have:




The equilibrium point is $(27, \$6)$.

b)



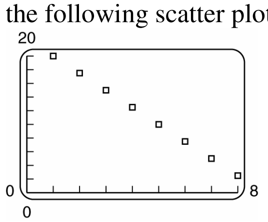

c) $\int_0^{27}\left(15-\frac{1}{3}x\right)dx-27\cdot 6=121.50$

The consumer surplus is \$121.50.

d) $27\cdot 6-\int_0^{27} 2\cdot\sqrt[3]{x}dx\approx 40.50$

The producer surplus is \$40.50.

**21.** a) Entering the data into the STAT editor on the calculator and plotting the points, we get the following scatter plot:




A linear function appears to be the best fit for the data.

b) Using the linear regression feature on the calculator, we get the equation:
$y=-2.5x+22.5$.

c) The consumer surplus is

$\int_0^{x} D(x)\,dx-x\cdot p$.

Substituting $-2.5x+22.5$ for $D(x)$, 6 for $x$, and 7.50 for $p$ we have:

$\int_0^{6}(-2.5x+22.5)\,dx-6(7.50)=45$

The consumer surplus is \$45.

d) First find the value for $x$ when $y$=11.50.

$$11.50=-2.5x+22.5$$
$$-11=-2.5x$$
$$4.4=x$$

Next, find the consumer surplus.

$\int_0^{4.4}(-2.5x+22.5)\,dx-4.4(11.50)=24.20$

When the price of bungee jumping is \$11.50 per half hour, Reggie's consumer surplus is \$24.20.

## Exercise Set 5.2

**1.** $P(t)=P_0e^{kt}$

Substituting 3 for $t$, 700 for $P_0$ and 0.09 for $k$ we have:

$$\begin{aligned}P(3)&=700e^{0.09(3)}\\&=700e^{0.27}\\&\approx 700(1.309964)\\&\approx 916.98\end{aligned}$$

The amount in the account is \$916.98 after 3 years.

**2.** $P(t)=P_0e^{kt}$

$$\begin{aligned}P(4)&=700e^{0.10(4)}\\&=700e^{0.40}\\&\approx 1044.28\end{aligned}$$

The amount in the account is \$1044.28 after 4 years.

**3.** $\int_0^{T} R(t)e^{kt}\,dt$

Substituting 2000 for $R(t)$, 0.06 for $k$, and 8 for $T$ we have

$$\begin{aligned}\int_0^{8}(2000)e^{0.06t}\,dt&=\left[\frac{2000}{0.06}e^{0.06t}\right]_0^8\\&=\frac{2000}{0.06}\left(e^{0.48}-1\right)\\&\approx 20{,}535.81\end{aligned}$$

The future value of the continuous money flow is \$20,535.81.

**4.** $\int_0^T R(t)e^{kt}\,dt$

Substituting 800 for $R(t)$, 0.07 for *k,* and 10 for *T* we have

$$\int_0^{10}(800)e^{0.07t}\,dt = \left[\frac{800}{0.07}e^{0.07t}\right]_0^{10}$$
$$= \frac{800}{0.07}\left(e^{0.7}-1\right)$$
$$= 11{,}585.75$$

The future value of the continuous money flow is \$11,585.75.

**5.** $\int_0^T R(t)e^{kt}\,dt$

Substituting 1500 for $R(t)$, 0.05 for *k,* and 20 for *T* we have

$$\int_0^{20}(1500)e^{0.05t}\,dt = \left[\frac{1500}{0.05}e^{0.05t}\right]_0^{20}$$
$$= \frac{1500}{0.05}\left(e^{1}-1\right)$$
$$\approx 51{,}548.45$$

The future value of the continuous money flow is \$51,548.45.

**6.** $\int_0^T R(t)e^{kt}\,dt$

Substituting 1200 for $R(t)$, 0.06 for *k,* and 15 for *T* we have

$$\int_0^{15}(1200)e^{0.06t}\,dt = \left[\frac{1200}{0.06}e^{0.06t}\right]_0^{15}$$
$$= \frac{1200}{0.06}\left(e^{0.9}-1\right)$$
$$\approx 29{,}192.06$$

The future value of the continuous money flow is \$29,192.06.

**7.** We find $P_0$ such that

$$50{,}000 = \int_0^{20} P_0 e^{0.085t}\,dt$$
$$50{,}000 = P_0\left[\frac{1}{0.085}e^{0.085t}\right]_0^{20}$$
$$50{,}000 = P_0\left[\frac{1}{0.085}e^{1.7} - \frac{1}{0.085}e^{0}\right]$$
$$50{,}000 = P_0\left[\frac{1}{0.085}\left(e^{1.7}-1\right)\right]$$
$$50{,}000 \approx P_0\left[52.63467520\right]$$
$$\frac{50{,}000}{52.63467520} \approx P_0$$
$$949.94 \approx P_0$$

A continuous money flow of \$949.94 per year, invested at 8.5%, compounded continuously for 20 years, will yield \$50,000.

**8.** We find $P_0$ such that

$$50{,}000 = \int_0^{20} P_0 e^{0.075t}\,dt$$
$$50{,}000 = P_0\left[\frac{1}{0.075}e^{0.075t}\right]_0^{20}$$
$$50{,}000 = P_0\left[\frac{1}{0.075}\left(e^{1.5}-1\right)\right]$$
$$50{,}000 \approx P_0\left[46.42252094\right]$$
$$1077.06 \approx P_0$$

A continuous money flow of \$1077.06 per year, invested at 7.5%, compounded continuously for 20 years, will yield \$50,000.

**9.** We find $P_0$ such that

$$40{,}000 = \int_0^{30} P_0 e^{0.09t}\,dt$$
$$40{,}000 = P_0\left[\frac{1}{0.09}e^{0.09t}\right]_0^{30}$$
$$40{,}000 = P_0\left[\frac{1}{0.09}e^{2.7} - \frac{1}{0.09}e^{0}\right]$$
$$40{,}000 = P_0\left[\frac{1}{0.09}\left(e^{2.7}-1\right)\right]$$
$$40{,}000 \approx P_0\left[154.21924139\right]$$
$$\frac{40{,}000}{154.21924139} \approx P_0$$
$$259.37 \approx P_0$$

A continuous money flow of \$259.37 per year, invested at 9%, compounded continuously for 30 years, will yield \$40,000.

**10.** We find $P_0$ such that

$$40{,}000 = \int_0^{30} P_0 e^{0.1t}\,dt$$
$$40{,}000 = P_0\left[\frac{1}{0.1}e^{0.1t}\right]_0^{30}$$
$$40{,}000 = P_0\left[\frac{1}{0.1}\left(e^3 - 1\right)\right]$$
$$40{,}000 \approx P_0\left[190.85536923\right]$$
$$209.58 \approx P_0$$

A continuous money flow of \$209.58 per year, invested at 10%, compounded continuously for 30 years, will yield \$40,000.

**11.** $P = P_0 e^{kt}$

Therefore,

$$P_0 = \frac{P}{e^{kt}} = Pe^{-kt}$$

Substituting 50,000 for $P$, 0.07 for $k$ and 20 for $t$, we have:

$$P_0 = 50{,}000e^{-0.07(20)}$$
$$P_0 = 50{,}000e^{-1.4}$$
$$P_0 \approx 50{,}000(0.24659696)$$
$$P_0 \approx 12{,}329.85$$

Dori and Dwayne should make an initial investment \$12,329.85.

**12.** $P_0 = \dfrac{P}{e^{kt}} = Pe^{-kt}$

$$P_0 = 60{,}000e^{-0.06(20)}$$
$$P_0 = 60{,}000e^{-1.2}$$
$$P_0 \approx 18{,}071.65$$

Addie and Wesley should make an initial investment of \$18,071.65.

**13.** $P = P_0 e^{kt}$

Therefore,

$$P_0 = \frac{P}{e^{kt}} = Pe^{-kt}$$

Substituting 100,000 for $P$, 0.05 for $k$ and 10 for $t$, we have:

$$P_0 = 100{,}000e^{-0.05(10)}$$
$$P_0 = 100{,}000e^{-0.5}$$
$$P_0 \approx 100{,}000(0.60653066)$$
$$P_0 \approx 60{,}653.07$$

The present value of \$100,000 due in 10 years, at 5%, compounded continuously is \$60,653.07.

**14.** $P_0 = \dfrac{P}{e^{kt}} = Pe^{-kt}$

$$P_0 = 50{,}000e^{-0.074(16)}$$
$$P_0 = 50{,}000e^{-1.184}$$
$$P_0 \approx 15{,}302.60$$

The present value of \$50,000 due in 16 years, at 7.4%, compounded continuously is \$15,302.60.

**15.** The accumulated present value is

$$A = \int_0^T R(t)e^{-kt}\,dt\,.$$

Substituting 2700 for $R(t)$, 0.09 for $k$, and 10 for $T$, we have:

$$A = \int_0^{10} 2700e^{-0.09t}\,dt$$
$$= \left[\frac{2700}{-0.09}e^{-0.09t}\right]_0^{10}$$
$$= \left[-30{,}000\left(e^{-0.9} - e^0\right)\right]$$
$$\approx -30{,}000(-0.59343034)$$
$$\approx 17{,}802.91$$

The accumulated present value is \$17,802.91.

**16.** $A = \int_0^T R(t)e^{-kt}\,dt\,.$

Substituting 2700 for $R(t)$, 0.10 for $k$, and 10 for $T$, we have:

$$A = \int_0^{10} 2700e^{-0.1t}\,dt$$
$$= \left[\frac{2700}{-0.1}e^{-0.1t}\right]_0^{10}$$
$$= \left[-27{,}000\left(e^{-1} - e^0\right)\right]$$
$$\approx -27{,}000(-0.63212056)$$
$$\approx 17{,}067.26$$

The accumulated present value is \$17,067.26.

**17.** $A = \int_0^T R(t)e^{-kt}\,dt\,.$

Substituting 95,000 for $R(t)$, 0.06 for $k$, and 30 [65-35=30] for $T$, we have:

$$A = \int_0^{30} 95{,}000 e^{-0.06t}\,dt$$
$$= \left[\frac{95{,}000}{-0.06} e^{-0.06t}\right]_0^{30}$$
$$= \left[\frac{95{,}000}{-0.06}\left(e^{-1.8} - e^{0}\right)\right]$$
$$\approx \frac{95{,}000}{-0.06}(-0.83470111)$$
$$\approx 1{,}321{,}610.09$$

The accumulated present value is \$1,321.610.09.

**18.** The accumulated present value is $A = \int_0^{T} R(t) e^{-kt}\,dt$ .

$$A = \int_0^{40} 41{,}000 e^{-0.07t}\,dt$$
$$= \left[\frac{41{,}000}{-0.07} e^{-0.07t}\right]_0^{40}$$
$$= \left[\frac{41{,}000}{-0.07}\left(e^{-2.8} - e^{0}\right)\right]$$
$$\approx 550{,}096.96$$

The accumulated present value is \$550,096.96.

**19.** $c = c_0 + \int_0^{L} m(t) e^{-rt}\,dt$

Substituting 500,000 for $c_0$, 20,000 for $m(t)$, 0.05 for $r$, and 20 for $L$, we have:

$$c = 500{,}000 + \int_0^{20} 20{,}000 e^{-0.05t}\,dt$$
$$= 500{,}000 + \left[\frac{20{,}000}{-0.05} e^{-0.05t}\right]_0^{20}$$
$$= 500{,}000 - 400{,}000\left[e^{-0.05(20)} - e^{-0.05(0)}\right]$$
$$= 500{,}000 - 400{,}000\left[e^{-1} - e^{0}\right]$$
$$\approx 752{,}848$$

The capitalized cost under the given assumptions is \$752,848.

**20.** $c = c_0 + \int_0^{L} m(t) e^{-rt}\,dt$

$$c = 400{,}000 + \int_0^{25} 10{,}000 e^{-0.055t}\,dt$$
$$= 400{,}000 + \left[\frac{10{,}000}{-0.055} e^{-0.055t}\right]_0^{25}$$
$$= 400{,}000 - \frac{10{,}000}{0.055}\left[e^{-1.375} - e^{0}\right]$$
$$\approx 535{,}847$$

The capitalized cost under the given assumptions is \$535,847.

**21.** $c = c_0 + \int_0^{L} m(t) e^{-rt}\,dt$

Substituting 600,000 for $c_0$, $40{,}000 + 1000e^{0.01t}$ for $m(t)$, 0.04 for $r$, and 40 for $L$, we have:

$$c = 600{,}000 + \int_0^{40}\left(40{,}000 + 1000e^{0.01t}\right) e^{-0.04t}\,dt$$
$$= 600{,}000 + \int_0^{40}\left(40{,}000 e^{-0.04t} + 1000 e^{-0.03t}\right)dt$$
$$= 600{,}000 + \left[\frac{40{,}000}{-0.04} e^{-0.04t} + \frac{1000}{-0.03} e^{-0.03t}\right]_0^{40}$$
$$= 600{,}000 + \left[-1{,}000{,}000 e^{-0.04t} - \frac{100{,}000}{3} e^{-0.03t}\right]_0^{40}$$
$$= 600{,}000 + \left[-1{,}000{,}000 e^{-0.04(40)} - \frac{100{,}000}{3} e^{-0.03(40)} - \left(-1{,}000{,}000 e^{-0.04(0)} - \frac{100{,}000}{3} e^{-0.03(0)}\right)\right]$$
$$\approx 1{,}421{,}397$$

The capitalized cost under the given assumptions is \$1,421,397.

**22.** $c = c_0 + \int_0^{L} m(t) e^{-rt}\,dt$

$$c = 300{,}000 + \int_0^{20}\left(30{,}000 + 500t\right) e^{-0.05t}\,dt$$
$$= 300{,}000 + \int_0^{20}\left(30{,}000 e^{-0.05t} + 500t e^{-0.05t}\right)dt$$
$$= 300{,}000 + \left[\frac{30{,}000}{-0.05} e^{-0.05t} + 500\frac{1}{(-0.05)^2} e^{-0.05t}(-0.05t - 1)\right]_0^{20}$$
$$= 300{,}000 + \left[-600{,}000 e^{-0.05t} + 200{,}000 e^{-0.05t}(-0.05t - 1)\right]_0^{20}$$
$$= 300{,}000 + \left[-600{,}000 e^{-0.05(20)} + 200{,}000 e^{-0.05(20)}(-0.05(20) - 1) - \left(-600{,}000 e^{-0.05(0)} + 200{,}000 e^{-0.05(0)}(-0.05(0) - 1)\right)\right]$$
$$\approx 732{,}121$$

The capitalized cost under the given assumptions is \$732,121.

**23.** The equation for the consumption of a natural resource is $\int_0^T P_0 e^{kt}\,dt = \frac{P_0}{k}\left(e^{kT}-1\right)$.

Substituting 101.4 for $P_0$, 0.026 for $k$, and 14 $[2020-2006=14]$ for $t$, we have

$$\begin{aligned}\int_0^{14} 101.4e^{0.026t} &= \frac{101.4}{0.026}\left(e^{0.026(14)}-1\right)\\ &= \frac{101.4}{0.026}\left(e^{0.364}-1\right)\\ &\approx \frac{101.4}{0.026}(0.43907421)\\ &\approx 1712.4\end{aligned}$$

If demand continues to grow exponentially at 2.6% per year, the world will consume approximately 1712.4 trillion cubic feet of natural gas from 2006 to 2020.

**24.** $\int_0^T P_0 e^{kt}\,dt = \frac{P_0}{k}\left(e^{kT}-1\right)$.

$$\begin{aligned}\int_0^{25} 153e^{0.025t} &= \frac{153}{0.025}\left(e^{0.025(25)}-1\right)\\ &= 6120\left(e^{0.625}-1\right)\\ &\approx 6120(0.86824596)\\ &\approx 5313.7\end{aligned}$$

If demand continues to grow exponentially at 2.5% per year, the world will consume approximately 5313.7 million metric tons of bauxite from 2005 to 2030.

**25.** The equation for the consumption of a natural resource is $\int_0^T P_0 e^{kt}\,dt = \frac{P_0}{k}\left(e^{kT}-1\right)$.

Using the information from Exercise 23, we want to find $T$, such that

$$6112 = \frac{101.4}{0.026}\left(e^{0.026T}-1\right)$$

We solve for $T$ as follows:

$$\begin{aligned}6112 &= 3900\left(e^{0.026T}-1\right)\\ 1.5672 &= e^{0.026T}-1 && \text{Dividing both sides by 3900.}\\ 2.5672 &\approx e^{0.026T} && \text{Adding 1 to both sides.}\\ \ln(2.5672) &\approx \ln\left(e^{0.026T}\right) && \text{Taking the natural logarithm of each side.}\\ \ln(2.5672) &\approx 0.026T && \text{Recall that } \ln e^k = k.\\ \frac{\ln(2.5672)}{0.026} &\approx T && \text{Dividing both sides by 0.026.}\\ 36.3 &\approx T\end{aligned}$$

Assuming the world consumption of natural gas continues to grow at 2.6% per year, and no new reserves are found, the world reserves of natural gas will be depleted 36 years after 2006, in 2042.

**26.** Using the information from Exercise 24, we want to find $T$, such that

$$23{,}000 = \frac{153}{0.025}\left(e^{0.025T}-1\right)$$

(23 billion is 23,000 million.)

We solve for $T$ as follows:

$$\begin{aligned}23{,}000 &= 6120\left(e^{0.025T}-1\right)\\ 3.7582 &\approx e^{0.025T}-1\\ 4.7582 &\approx e^{0.025T}\\ \ln(4.7582) &\approx \ln\left(e^{0.025T}\right)\\ \ln(4.7582) &\approx 0.025T\\ \frac{\ln(4.7582)}{0.025} &\approx T\\ 62.4 &\approx T\end{aligned}$$

Assuming the world consumption of bauxite continues to grow at 2.5% per year, and no new reserves are found, the world reserves of bauxite will be depleted 62 years after 2005, in 2067.

**27.** We will use the equation

$$\int_0^T Pe^{-kt}\,dt = \frac{P}{-k}\left(e^{-kT}-1\right) = \frac{P}{k}\left(1-e^{-kT}\right).$$

Substituting 0.00003 for $k$, 1 for $P$, and 20 for $T$, we have:

$$\begin{aligned}\int_0^{20} 1\cdot e^{-0.00003t}\,dt &= \frac{1}{0.00003}\left(1-e^{-0.00003(20)}\right)\\ &\approx 33{,}333.33\left(1-e^{-0.0006}\right)\\ &\approx 19.994 && \text{Using a calculator.}\end{aligned}$$

After 20 years, approximately 19.994 lbs of Plutonium-239 will remain in the atmosphere.

**28.** $\int_0^T Pe^{-kt}\,dt = \frac{P}{-k}\left(e^{-kT}-1\right) = \frac{P}{k}\left(1-e^{-kT}\right)$.

$$\begin{aligned}\int_0^{20} 1\cdot e^{-0.023t}\,dt &= \frac{1}{0.023}\left(1-e^{-0.023(20)}\right)\\ &\approx 43.47826\left(1-e^{-0.46}\right)\\ &\approx 16.031\end{aligned}$$

After 20 years, approximately 16.031 lbs of Cesium-237 will remain in the atmosphere.

**29.** a) $P = P_0 e^{kt}$

We will substitute the known information and solve for $k$.

$$34.5 = 30.8e^{k(2010-2006)}$$
$$34.5 = 30.8e^{k\cdot 4}$$
$$\frac{34.5}{30.8} = e^{4k}$$
$$\ln\left(\frac{34.5}{30.8}\right) = \ln\left(e^{4k}\right)$$
$$\ln\left(\frac{34.5}{30.8}\right) = 4k$$
$$\frac{\ln\left(\frac{34.5}{30.8}\right)}{4} = k$$
$$0.02836 \approx k$$

The exponential growth rate of demand for oil is 0.0284 or 2.84% per year.

b) Using the growth rate and the initial demand from Part (a) we have the exponential demand function $P = 30.8e^{0.02836t}$.

Substituting 9 $[2015 - 2006 = 9]$ for $t$, we have:

$$P = 30.8e^{0.02836(9)}$$
$$= 30.8e^{0.25524}$$
$$\approx 30.8(1.29077)$$
$$\approx 39.76$$

In 2015, the world demand for oil will be approximately 39.76 billion barrels.

c) The equation for the consumption of a natural resource is $\int_0^T P_0 e^{kt}\,dt = \frac{P_0}{k}\left(e^{kT} - 1\right)$.

We want to find $T$, such that

$$1293 = \frac{30.8}{0.02836}\left(e^{0.02836T} - 1\right)$$
$$\frac{1293(0.02836)}{30.8} = e^{0.02836T} - 1$$
$$1.1906 \approx e^{0.02836T} - 1$$
$$2.1906 \approx e^{0.02836T}$$
$$\ln(2.1906) \approx \ln e^{0.02836T}$$
$$\ln(2.1906) \approx 0.02836T$$
$$\frac{\ln(2.1906)}{0.02836} \approx T$$
$$27.7 \approx T$$

Assuming the world consumption of oil continues to grow at 2.84% per year, and no new reserves are found, the world reserves of oil will be depleted 27.7 years after 2006, in 2033.

**30.** $\int_0^T R(t)e^{k(T-t)}\,dt$

Substituting, we have:

$\int_0^{40} t^2 e^{0.07(40-t)}\,dt$

Letting $f(t) = t^2$ and $g(t) = e^{0.07(40-t)}$

We use tabular integration by parts to integrate.

| $f(t)$ and repeated derivatives | Sign of Product | $g(t)$ and repeated integrals |
|---|---|---|
| $t^2$ | + | $e^{0.07(40-t)}$ |
| $2t$ | − | $-\frac{1}{0.07}e^{0.07(40-t)}$ |
| $2$ | + | $\frac{1}{(0.07)^2}e^{0.07(40-t)}$ |
| $0$ | | $-\frac{1}{(0.07)^3}e^{0.07(40-t)}$ |

$$\int_0^{40} t^2 e^{0.07(40-t)}\,dt$$
$$= \left[-\frac{1}{0.07}t^2 e^{0.07(40-t)} - \frac{2}{(0.07)^2}te^{0.07(40-t)} - \frac{2}{(0.07)^3}e^{0.07(40-t)}\right]_0^{40}$$
$$= \left[-\frac{1}{0.07}(40)^2 e^{0.07(40-40)} - \frac{2}{(0.07)^2}(40)e^{0.07(40-40)} - \frac{2}{(0.07)^3}e^{0.07(40-40)} - \left(-\frac{1}{0.07}(0)^2 e^{0.07(40-0)} - \frac{2}{(0.07)^2}(0)e^{0.07(40-0)} - \frac{2}{(0.07)^3}e^{0.07(40-0)}\right)\right]$$
$$\approx 50{,}872.58$$

The future value of the continuous money flow is approximately \$50,872.58.

**31.** $\int_0^T R(t)e^{k(T-t)}dt$

Substituting, we have:

$$\int_0^{30}(2000t+7)e^{0.08(30-t)}dt$$
$$=\int_0^{30}2000te^{0.08(30-t)}dt+\int_0^{30}7e^{0.08(30-t)}dt$$
$$=2000\int_0^{30}te^{0.08(30-t)}dt+7\int_0^{30}e^{0.08(30-t)}dt$$

We will use integration by parts to evaluate the first integral.

$$\int te^{0.08(30-t)}dt$$

Let

$u=t$ and $dv=e^{0.08(30-t)}dt$

Then

$du=dt$ and $v=-\frac{1}{0.08}e^{0.08(30-t)}$

Therefore, we have:

$$\int te^{0.08(30-t)}dt$$
$$=-\frac{1}{0.08}te^{0.08(30-t)}-\int -\frac{1}{0.08}e^{0.08(30-t)}dt$$
$$=-\frac{1}{0.08}te^{0.08(30-t)}-\frac{1}{0.08}\cdot\frac{1}{0.08}e^{0.08(30-t)}+C$$
$$=-12.5te^{0.08(30-t)}-156.25e^{0.08(30-t)}+C$$

Next, we find the definite integral

$$\int_0^{30}te^{0.08(30-t)}dt$$
$$=\left[-12.5te^{0.08(30-t)}-156.25e^{0.08(30-t)}\right]_0^{30}$$
$$=\Big[-12.5(30)e^{0.08(30-30)}-156.25e^{0.08(30-30)}-$$
$$\left(-12.5(0)e^{0.08(30-0)}-156.25e^{0.08(30-0)}\right)\Big]$$
$$=\left(-375e^{0}-156.25e^{0}\right)-\left(0-156.25e^{2.4}\right)$$
$$\approx -375-156.25-0+1722.37$$
$$\approx 1191.12$$

Next, we integrate the second part of the original integral. From the process of integration by parts we know:

$$\int e^{0.08(30-t)}dt=-\frac{1}{0.08}e^{0.08(30-t)}+C$$
$$=-12.5e^{0.08(30-t)}+C$$

Finding the definite integral, we have:

$$\int_0^{30}e^{0.08(30-t)}dt$$
$$=\left[-12.5e^{0.08(30-t)}\right]_0^{30}$$
$$=\left[-12.5e^{0.08(30-30)}-\left(-12.5e^{0.08(30-0)}\right)\right]$$
$$=-12.5e^{0}+12.5e^{2.4}$$
$$\approx -12.5+137.79$$
$$\approx 125.29$$

Finally, we have:

$$2000\int_0^{30}te^{0.08(30-t)}dt+7\int_0^{30}e^{0.08(30-t)}dt$$
$$\approx 2000(1191.12)+7(125.29)$$
$$\approx 2{,}383{,}117$$

The future value of the continuous money flow is approximately \$2,383,117.

Note, if we had done all the calculations before rounding, the result would have been \$2,383,119.65.

**32.** $\int_0^T R(t)e^{-k(T-t)}dt$

$$\int_0^{10}e^{t}e^{-0.07(10-t)}dt$$
$$=\int_0^{10}e^{t}e^{-0.7+0.07t}dt$$
$$=\int_0^{10}e^{-0.7+1.07t}dt$$
$$=\left[\frac{1}{1.07}e^{-0.7+1.07t}\right]_0^{10}$$
$$=\frac{1}{1.07}e^{-0.7+1.07(10)}-\frac{1}{1.07}e^{-0.7+1.07(0)}$$
$$=\frac{1}{1.07}e^{10}-\frac{1}{1.07}e^{-0.7}$$
$$\approx 20{,}585.02$$

The accumulated present value is approximately \$20,585.02.

**33.** $\int_0^T R(t)e^{-k(T-t)}dt$

$$\int_0^{20}te^{-0.08(20-t)}dt$$

Use integration by parts.

Let

$u=t$ and $dv=e^{-0.08(20-t)}dt$ .

Then,

$du=dt$ and $v=\frac{1}{0.08}e^{0.08(20-t)}$

Finding the indefinite integral we have:

$$\int te^{-0.08(20-t)}dt$$
$$= \frac{1}{0.08}te^{-0.08(20-t)} - \int \frac{1}{0.08}e^{-0.08(20-t)}dt$$
$$= \frac{1}{0.08}te^{-0.08(20-t)} - \frac{1}{0.08}\cdot\frac{1}{0.08}e^{-0.08(20-t)} + C$$
$$= 12.5te^{-0.08(20-t)} - 156.25e^{-0.08(20-t)} + C$$

Next, find the definite integral.

$$\int_0^{20} te^{-0.08(20-t)}dt$$
$$= \left[12.5te^{-0.08(20-t)} - 156.25e^{-0.08(20-t)}\right]_0^{20}$$
$$= \Big[\left(12.5(20)e^{-0.08(20-20)} - 156.25e^{-0.08(20-20)}\right) - \left(12.5(0)e^{-0.08(20-0)} - 156.25e^{-0.08(20-0)}\right)\Big]$$
$$= \left(250e^0 - 156.25e^0\right) - \left(0 - 156.25e^{-1.6}\right)$$
$$\approx 250 - 156.25 - 0 + 31.55$$
$$\approx 125.30$$

The accumulated present value is approximately \$125.30.

**34.** $\int_0^T R(t)e^{-k(T-t)}dt$

$$\int_0^{15}(2t+7)e^{-0.06(15-t)}dt$$
$$= \int_0^{15} 2te^{-0.06(15-t)}dt + \int_0^{15} 7e^{-0.06(15-t)}dt$$
$$= 2\int_0^{15} te^{-0.06(15-t)}dt + 7\int_0^{15} e^{-0.06(15-t)}dt$$

We will use integration by parts to evaluate the first integral.

$$\int te^{-0.06(15-t)}dt$$

Let

$u = t$ and $dv = e^{-0.06(15-t)}dt$

Then

$du = dt$ and $v = \frac{1}{0.06}e^{-0.06(15-t)}$

Therefore, we have:

$$\int te^{-0.06(15-t)}dt$$
$$= \frac{1}{0.06}te^{-0.06(15-t)} - \int \frac{1}{0.06}e^{-0.06(15-t)}dt$$
$$= \frac{1}{0.06}te^{-0.06(15-t)} - \frac{1}{0.06}\cdot\frac{1}{0.06}e^{-0.06(15-t)} + C$$
$$= 16.667te^{-0.06(15-t)} - 277.778e^{-0.06(15-t)} + C$$

Next, we find the definite integral

$$\int_0^{15} te^{-0.06(15-t)}dt$$
$$\approx \left[16.667te^{-0.06(15-t)} - 277.778e^{-0.06(15-t)}\right]_0^{15}$$
$$\approx \Big[16.667(15)e^{-0.06(15-15)} - 277.778e^{-0.06(15-15)} \left(16.667(0)e^{-0.06(15-0)} - 277.778e^{-0.06(15-0)}\right)\Big]$$
$$\approx \left(250e^0 - 277.778e^0\right) - \left(0 - 277.778e^{-0.9}\right)$$
$$\approx 250 - 277.778 - 0 + 112.936$$
$$\approx 85.158$$

Next, we integrate the second part of the original integral. From the process of integration by parts we know:

$$\int e^{-0.06(15-t)}dt = \frac{1}{0.06}e^{-0.06(15-t)} + C$$
$$= 16.667e^{-0.06(15-t)} + C$$

Finding the definite integral, we have:

$$\int_0^{15} e^{-0.06(15-t)}dt$$
$$= \left[16.667e^{-0.06(15-t)}\right]_0^{15}$$
$$= \left[16.667e^{-0.06(15-15)} - 16.667e^{-0.06(15-0)}\right]$$
$$= 16.667e^0 - 16.667e^{-0.9}$$
$$\approx 9.891$$

Putting it all together we have:

$$2\int_0^{15} te^{-0.06(15-t)}dt + 7\int_0^{15} e^{-0.06(15-t)}dt$$
$$\approx 2(85.158) + 7(9.891)$$
$$\approx 239.55$$

The accumulated present value is approximately \$239.55.

**35.** A continuous investment \$2500 per year at 6% will yield:

$$\int_0^{15} 2500e^{0.06t}dt = \left[\frac{2500}{0.06}e^{0.06t}\right]_0^{15}$$
$$= \frac{2500}{0.06}\left(e^{0.06(15)} - e^{0.06(0)}\right)$$
$$= \frac{2500}{0.06}\left(e^{0.9} - 1\right)$$
$$\approx 60{,}816.80$$

A single initial investment of \$20,000 at 6% will yield:

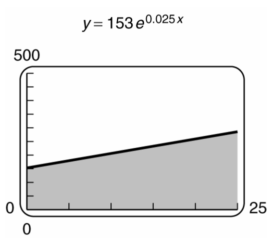
$$20{,}000e^{0.06(15)} = 20{,}000e^{0.9}$$
$$\approx 49{,}192.06$$

Therefore, the account in which \$2500 per year is continuously invested will have the greatest future value of \$60,816.80.

**36.** $\boxed{tw}$ The present value of an amount $A$ due $n$ years from now is the amount that must be invested today at a given interest rate, compounded continuously, to grow to $A$ in $n$ years.
Suppose an amount of money flows into an investment continuously at a given rate until some time in the future. Then the sum of all the present values of the amounts of money that flow continuously into the account is the accumulated present value of the continuous money flow.

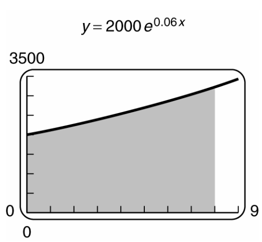
**37.** $\boxed{tw}$ Answers will vary.

**38.**




The area and the number of tons of bauxite are the same. Both are 5313.7 million metric tons.

**39.**




The area and the future value of the continuous money flow are the same. Both are $20,535.81.

## Exercise Set 5.3

**1.** $\int_2^\infty \frac{dx}{x^2}$

$= \lim_{b\to\infty} \int_2^b x^{-2}dx$

$= \lim_{b\to\infty} \left[\frac{x^{-1}}{-1}\right]_2^b$

$= \lim_{b\to\infty} \left[-\frac{1}{x}\right]_2^b$

$= \lim_{b\to\infty} \left[-\frac{1}{b}-\left(-\frac{1}{2}\right)\right]$

$= 0+\frac{1}{2}$ $\quad\left(\text{As } b\to\infty, -\frac{1}{b}\to 0\right)$

$= \frac{1}{2}$

The limit does exist. The improper integral is convergent.

**2.** $\int_4^\infty \frac{dx}{x^2}$

$= \lim_{b\to\infty} \int_4^b x^{-2}dx$

$= \lim_{b\to\infty} \left[\frac{x^{-1}}{-1}\right]_4^b$

$= \lim_{b\to\infty} \left[-\frac{1}{x}\right]_4^b$

$= \lim_{b\to\infty} \left[-\frac{1}{b}-\left(-\frac{1}{4}\right)\right]$

$= \frac{1}{4}$

The limit does exist. The improper integral is convergent.

**3.** $\int_3^\infty \frac{dx}{x}$

$= \lim_{b\to\infty} \int_3^b x^{-1}dx$

$= \lim_{b\to\infty} \left[\ln(x)\right]_3^b$

$= \lim_{b\to\infty} \left[\ln(b)-\ln(3)\right]$

Note that $\ln b$ increases without bound as $b$ increases. Therefore, the limit does not exist. If the limit does not exist, we say the improper integral is divergent.

**4.** $\int_4^\infty \frac{dx}{x}$

$= \lim_{b\to\infty}\int_4^b x^{-1}dx$

$= \lim_{b\to\infty}\left[\ln(x)\right]_4^b$

$= \lim_{b\to\infty}\left[\ln(b)-\ln(4)\right]$

The limit does not exist. The integral is divergent.

**5.** $\int_0^\infty 3e^{-3x}dx$

$= \lim_{b\to\infty}\int_0^b 3e^{-3x}dx$

$= \lim_{b\to\infty}\left[\frac{3}{-3}e^{-3x}\right]_0^b$

$= \lim_{b\to\infty}\left[-e^{-3x}\right]_0^b$

$= \lim_{b\to\infty}\left[-e^{-3\cdot b}-\left(-e^{-3\cdot 0}\right)\right]$

$= \lim_{b\to\infty}\left[-e^{-3b}+1\right]$

$= \lim_{b\to\infty}\left[1-\frac{1}{e^{3b}}\right]$

$= 1-0 \quad \left[\text{As } b\to\infty, e^{3b}\to\infty, \text{so } \frac{1}{e^{3b}}\to 0\right]$

$= 1$

The limit does exist. The improper integral is convergent.

**6.** $\int_0^\infty 4e^{-4x}dx$

$= \lim_{b\to\infty}\int_0^b 4e^{-4x}dx$

$= \lim_{b\to\infty}\left[\frac{4}{-4}e^{-4x}\right]_0^b$

$= \lim_{b\to\infty}\left[-e^{-4x}\right]_0^b$

$= \lim_{b\to\infty}\left[-e^{-4b}-\left(-e^{0}\right)\right]$

$= \lim_{b\to\infty}\left[-e^{-4b}+1\right]$

$= 1$

The limit does exist. The improper integral is convergent.

**7.** $\int_1^\infty \frac{dx}{x^3}$

$= \lim_{b\to\infty}\int_1^b x^{-3}dx$

$= \lim_{b\to\infty}\left[\frac{x^{-2}}{-2}\right]_1^b$

$= \lim_{b\to\infty}\left[-\frac{1}{2x^2}\right]_1^b$

$= \lim_{b\to\infty}\left[-\frac{1}{2b^2}-\left(-\frac{1}{2(1)^2}\right)\right]$

$= 0+\frac{1}{2} \qquad \left(\text{As } b\to\infty, -\frac{1}{2b^2}\to 0\right)$

$= \frac{1}{2}$

The limit does exist. The improper integral is convergent.

**8.** $\int_1^\infty \frac{dx}{x^4}$

$= \lim_{b\to\infty}\int_1^b x^{-4}dx$

$= \lim_{b\to\infty}\left[\frac{x^{-3}}{-3}\right]_1^b$

$= \lim_{b\to\infty}\left[-\frac{1}{3x^3}\right]_1^b$

$= \lim_{b\to\infty}\left[-\frac{1}{3b^3}-\left(-\frac{1}{3(1)^3}\right)\right]$

$= \frac{1}{3}$

The limit does exist. The improper integral is convergent.

**9.** $\int_0^\infty \frac{dx}{2+x}$

$= \lim_{b\to\infty}\int_0^b \frac{1}{2+x}dx$

$= \lim_{b\to\infty}\left[\ln(2+x)\right]_0^b$

$= \lim_{b\to\infty}\left[\ln(2+b)-\ln(2+0)\right]$

$= \lim_{b\to\infty}\left[\ln(2+b)-\ln(2)\right]$

Note that $\ln(2+b)$ increases without bound as $b$ increases. Therefore, the limit does not exist. If the limit does not exist, we say the improper integral is divergent.

**10.** $\int_0^\infty \frac{4dx}{3+x}$

$= \lim_{b\to\infty} \int_0^b \frac{4}{3+x}dx$

$= \lim_{b\to\infty} \left[4\ln(3+x)\right]_0^b$

$= \lim_{b\to\infty} \left[4\ln(3+b) - 4\ln(3+0)\right]$

$= \lim_{b\to\infty} \left[4\ln(3+b) - 4\ln(3)\right]$

Note that $\ln(3+b)$ increases without bound as $b$ increases. Therefore, the limit does not exist. If the limit does not exist, we say the improper integral is divergent.

**11.** $\int_2^\infty 4x^{-2}dx$

$= \lim_{b\to\infty} \int_2^b 4x^{-2}dx$

$= \lim_{b\to\infty} \left[\frac{4x^{-1}}{-1}\right]_2^b$

$= \lim_{b\to\infty} \left[-\frac{4}{x}\right]_2^b$

$= \lim_{b\to\infty} \left[-\frac{4}{b} - \left(-\frac{4}{2}\right)\right]$

$= 0 + 2 \qquad \left(\text{As } b\to\infty, -\frac{4}{b}\to 0\right)$

$= 2$

The limit does exist. The improper integral is convergent.

**12.** $\int_2^\infty 7x^{-2}dx$

$= \lim_{b\to\infty} \int_2^b 7x^{-2}dx$

$= \lim_{b\to\infty} \left[\frac{7x^{-1}}{-1}\right]_2^b$

$= \lim_{b\to\infty} \left[-\frac{7}{x}\right]_2^b$

$= \lim_{b\to\infty} \left[-\frac{7}{b} - \left(-\frac{7}{2}\right)\right]$

$= \frac{7}{2}$

The limit does exist. The improper integral is convergent.

**13.** $\int_0^\infty e^x dx$

$= \lim_{b\to\infty} \int_0^b e^x dx$

$= \lim_{b\to\infty} \left[e^x\right]_0^b$

$= \lim_{b\to\infty} \left[e^b - \left(e^0\right)\right]$

$= \lim_{b\to\infty} \left[e^b - 1\right]$

As $b\to\infty, e^b\to\infty$; thus, the limit does not exist. The improper integral is divergent.

**14.** $\int_0^\infty e^{2x} dx$

$= \lim_{b\to\infty} \int_0^b e^{2x} dx$

$= \lim_{b\to\infty} \left[\frac{1}{2}e^{2x}\right]_0^b$

$= \lim_{b\to\infty} \left[\frac{1}{2}e^{2b} - \left(\frac{1}{2}e^0\right)\right]$

$= \lim_{b\to\infty} \left[\frac{1}{2}e^{2b} - \frac{1}{2}\right]$

As $b\to\infty, e^{2b}\to\infty$; thus, the limit does not exist. The improper integral is divergent.

**15.** $\int_3^\infty x^2 dx$

$= \lim_{b\to\infty} \int_3^b x^2 dx$

$= \lim_{b\to\infty} \left[\frac{x^3}{3}\right]_3^b$

$= \lim_{b\to\infty} \left[\frac{(b)^3}{3} - \frac{(3)^3}{3}\right]$

$= \lim_{b\to\infty} \left[\frac{b^3}{3} - 9\right]$

As $b\to\infty, \frac{b^3}{3}\to\infty$; thus, the limit does not exist. The improper integral is divergent.

**16.** $\int_5^\infty x^4 dx$

$= \lim_{b\to\infty} \int_5^b x^4 dx$

$= \lim_{b\to\infty} \left[\frac{x^5}{5}\right]_5^b$

$= \lim_{b\to\infty} \left[\frac{(b)^5}{5} - \frac{(5)^5}{5}\right]$

$= \lim_{b\to\infty} \left[\frac{b^5}{5} - 625\right]$

As $b \to \infty, \frac{b^5}{5} \to \infty$; thus, the limit does not exist. The improper integral is divergent.

**17.** $\int_0^\infty xe^x dx$

$= \lim_{b\to\infty} \int_0^b xe^x dx$

$= \lim_{b\to\infty} \left[e^x(x-1)\right]_0^b$ Using integration by parts.

$= \lim_{b\to\infty} \left[e^b(b-1) - \left(e^0(0-1)\right)\right]$

$= \lim_{b\to\infty} \left[e^b(b-1) + 1\right]$

As $b \to \infty, e^b(b-1) \to \infty$; thus, the limit does not exist. The improper integral is divergent.

**18.** $\int_1^\infty \ln x\, dx$

$= \lim_{b\to\infty} \int_1^b \ln x\, dx$

$= \lim_{b\to\infty} [x\ln x - x]_1^b$

$= \lim_{b\to\infty} \left[(b\ln b - b) - (1\ln 1 - 1)\right]$

$= \lim_{b\to\infty} \left[\left(b(\ln b - 1)\right) + 1\right]$

As $b \to \infty, b(\ln b - 1) \to \infty$; thus, the limit does not exist. The improper integral is divergent.

**19.** $\int_0^\infty me^{-mx} dx, \quad m > 0$

$= \lim_{b\to\infty} \int_0^b me^{-mx} dx$

$= \lim_{b\to\infty} \left[\frac{m}{-m} e^{-mx}\right]_0^b$

$= \lim_{b\to\infty} \left[-e^{-mx}\right]_0^b$

$= \lim_{b\to\infty} \left[-e^{-m\cdot b} - \left(-e^{-m\cdot 0}\right)\right]$

$= \lim_{b\to\infty} \left[-e^{-mb} + 1\right]$

$= \lim_{b\to\infty} \left[1 - \frac{1}{e^{mb}}\right]$

$= 1 - 0 \quad \left[\text{As } b \to \infty, e^{mb} \to \infty, \text{so } \frac{1}{e^{mb}} \to 0\right]$

$= 1$

The limit does exist. The improper integral is convergent.

**20.** $\int_0^\infty Qe^{-kt} dt, \quad k > 0$

$= \lim_{b\to\infty} \int_0^b Qe^{-kt} dt$

$= \lim_{b\to\infty} \left[\frac{Q}{-k} e^{-kt}\right]_0^b$

$= \lim_{b\to\infty} \left[-\frac{Q}{k} e^{-kt}\right]_0^b$

$= \lim_{b\to\infty} \left[-\frac{Q}{k} e^{-k\cdot b} - \left(-\frac{Q}{k} e^{-k\cdot 0}\right)\right]$

$= \lim_{b\to\infty} \left[-\frac{Q}{k} e^{-k\cdot b} + \frac{Q}{k} e^0\right]$

$= \lim_{b\to\infty} \left[\frac{Q}{k}\left(1 - \frac{1}{e^{kb}}\right)\right]$

$= \frac{Q}{k}(1 - 0) \quad \left[\text{As } b \to \infty, e^{kb} \to \infty, \text{so } \frac{1}{e^{kb}} \to 0\right]$

$= \frac{Q}{k}$

The limit does exist. The improper integral is convergent.

**21.** $\int_{\pi}^{\infty} \frac{dt}{t^{1.001}}$

$$= \lim_{b\to\infty} \int_{\pi}^{b} t^{-1.001} dt$$
$$= \lim_{b\to\infty} \left[\frac{t^{-0.001}}{-0.001}\right]_{\pi}^{b}$$
$$= \lim_{b\to\infty} \left[-\frac{1000}{t^{0.001}}\right]_{\pi}^{b}$$
$$= \lim_{b\to\infty} \left[-\frac{1000}{b^{0.001}} - \left(-\frac{1000}{(\pi)^{0.001}}\right)\right]$$
$$= 0 + \frac{1000}{\pi^{0.001}} \qquad \left(\text{As } b\to\infty, -\tfrac{1000}{b^{0.001}} \to 0\right)$$
$$= \frac{1000}{\pi^{0.001}}$$
$$\approx 998.86$$

The limit does exist. The improper integral is convergent.

**22.** $\int_{1}^{\infty} \frac{2tdt}{t^2+1}$

$$= \lim_{b\to\infty} \int_{1}^{b} \frac{2t}{t^2+1} dt$$
$$= \lim_{b\to\infty} \left[\ln\left(t^2+1\right)\right]_{1}^{b}$$
$$= \lim_{b\to\infty} \left[\ln\left(b^2+1\right) - \ln\left(1^2+1\right)\right]$$
$$= \lim_{b\to\infty} \left[\ln\left(b^2+1\right) - \ln(2)\right]$$

Note that $\ln\left(b^2+1\right)$ increases without bound as $b$ increases. Therefore, the limit does not exist. Thus, the improper integral is divergent.

**23.** $\int_{-\infty}^{\infty} t\ dt$

$$= \int_{-\infty}^{0} t\ dt + \int_{0}^{\infty} t\ dt \qquad \text{Using Definition 2 with } c = 0.$$
$$= \lim_{a\to-\infty} \int_{a}^{0} t\ dt + \lim_{b\to\infty} \int_{0}^{b} t\ dt$$
$$= \lim_{a\to-\infty} \left[\frac{t^2}{2}\right]_{a}^{0} + \lim_{b\to\infty} \left[\frac{t^2}{2}\right]_{0}^{b}$$
$$= \lim_{a\to-\infty} \left[\frac{0^2}{2} - \frac{a^2}{2}\right] + \lim_{b\to\infty} \left[\frac{b^2}{2} - \frac{0^2}{2}\right]$$

Neither $\lim_{a\to-\infty} \frac{a^2}{2}$ nor $\lim_{b\to\infty} \frac{b^2}{2}$ exists, so the integral is divergent.

**24.** $\int_{1}^{\infty} \frac{3x^2}{\left(x^3+1\right)^2} dx$

$$= \lim_{b\to\infty} \int_{1}^{b} \frac{3x^2}{\left(x^3+1\right)^2} dx$$
$$= \lim_{b\to\infty} \left[-\frac{1}{x^3+1}\right]_{1}^{b} \qquad \left[u = x^3+1, du = 3x^2 dx\right]$$
$$= \lim_{b\to\infty} \left[-\frac{1}{b^3+1} - \left(-\frac{1}{1^3+1}\right)\right]$$
$$= 0 + \frac{1}{2} \qquad \left(\text{As } b\to\infty, -\tfrac{1}{b^3+1} \to 0\right)$$
$$= \frac{1}{2}$$

The limit does exist. The improper integral is convergent.

**25.** The area is given by

$$\int_{2}^{\infty} \frac{1}{x^2} dx = \lim_{b\to\infty} \int_{2}^{b} x^{-2} dx$$
$$= \lim_{b\to\infty} \left[\frac{x^{-1}}{-1}\right]_{2}^{b}$$
$$= \lim_{b\to\infty} \left[-\frac{1}{x}\right]_{2}^{b}$$
$$= \lim_{b\to\infty} \left[-\frac{1}{b} - \left(-\frac{1}{2}\right)\right]$$
$$= 0 + \frac{1}{2} \qquad \left(\text{As } b\to\infty, -\tfrac{1}{b} \to 0\right)$$
$$= \frac{1}{2}$$

The area of the region is $\frac{1}{2}$.

**26.** The area is given by

$$\int_{2}^{\infty} \frac{1}{x} dx = \lim_{b\to\infty} \int_{2}^{b} x^{-1} dx$$
$$= \lim_{b\to\infty} \left[\ln(x)\right]_{2}^{b}$$
$$= \lim_{b\to\infty} \left[\ln(b) - \ln(2)\right]$$

The limit does not exist. As $b$ increases, $\ln b$ increases without bound. The area is infinite.

**27.** The area is given by

$$\int_0^{\infty} 2xe^{-x^2}\,dx$$
$$= \lim_{b\to\infty}\int_0^{b} 2xe^{-x^2}\,dx$$
$$= \lim_{b\to\infty}\left[-e^{-x^2}\right]_0^b \qquad \left[u=-x^2, du=-2xdx\right]$$
$$= \lim_{b\to\infty}\left[-e^{-b^2}-\left(-e^{-0^2}\right)\right]$$
$$= \lim_{b\to\infty}\left[-e^{-b^2}+1\right]$$
$$= \lim_{b\to\infty}\left[-\frac{1}{e^{b^2}}+1\right]$$
$$= -0+1 \qquad \left[\text{As } b\to\infty, -\frac{1}{e^{b^2}}\to 0\right]$$
$$=1$$

The area of the region is 1.

**28.** The area is given by:

$$\int_6^{\infty}\frac{1}{\sqrt{(3x-2)^3}}\,dx$$
$$= \lim_{b\to\infty}\int_6^{b}(3x-2)^{-3/2}\,dx$$
$$= \lim_{b\to\infty}\left[-\frac{2}{3}(3x-2)^{-1/2}\right]_6^b \qquad \left[u=3x-2, du=3dx\right]$$
$$= \lim_{b\to\infty}\left[-\frac{2}{3}(3b-2)^{-1/2}-\left(-\frac{2}{3}(3\cdot 6-2)^{-1/2}\right)\right]$$
$$= \lim_{b\to\infty}\left[\frac{-2}{3\sqrt{3b-2}}+\frac{2}{3\sqrt{16}}\right]$$
$$=\frac{1}{6}$$

The area of the region is $\frac{1}{6}$.

**29.** From Theorem 2, The accumulated present value is given by

$$\int_0^{\infty} Pe^{-kt}\,dt=\frac{P}{k}$$

Substituting 3600 for $P$ and 0.07 for $k$, we have:

$$\int_0^{\infty} 3600e^{-0.07t}\,dt=\frac{3600}{0.07}\approx 51{,}428.57\,.$$

The accumulated present value is approximately $51,428.57.

**30.** $\int_0^{\infty} Pe^{-kt}\,dt=\frac{P}{k}$

$$\int_0^{\infty} 3500e^{-0.06t}\,dt=\frac{3500}{0.06}\approx 58{,}333.33\,.$$

The accumulated present value is approximately $58,333.33.

**31.** Total profit is given by:

$$P(x)=\int_0^{\infty}200e^{-0.032x}\,dx$$
$$= \lim_{b\to\infty}\int_0^{b}200e^{-0.032x}\,dx$$
$$= \lim_{b\to\infty}\left[\frac{200}{-0.032}e^{-0.032x}\right]_0^b$$
$$= \lim_{b\to\infty}\left[-6250e^{-0.032x}\right]_0^b$$
$$= \lim_{b\to\infty}\left[-6250e^{-0.032b}-\left(-6250e^{-0.032(0)}\right)\right]$$
$$= \lim_{b\to\infty}\left[-\frac{6250}{e^{0.032b}}+6250\right]$$
$$=6250$$

The total profit if it were possible to produce an infinite number of units is $6250.

**32.** Total profit is given by:

$$P(x)=\int_1^{\infty}200x^{-1.032}\,dx$$
$$= \lim_{b\to\infty}\int_1^{b}200x^{-1.032}\,dx$$
$$= \lim_{b\to\infty}\left[\frac{200}{-0.032}x^{-0.032}\right]_1^b$$
$$= \lim_{b\to\infty}\left[-6250x^{-0.032}\right]_1^b$$
$$= \lim_{b\to\infty}\left[-6250b^{-0.032}-\left(-6250(1)^{-0.032}\right)\right]$$
$$= \lim_{b\to\infty}\left[-\frac{6250}{b^{0.032}}+6250\right]$$
$$=6250$$

The total profit if it were possible to produce an infinite number of units is $6250.

**33.** The total cost is given by:

$$C(x)=\int_1^{\infty}3600x^{-1.8}\,dx$$
$$= \lim_{b\to\infty}\int_1^{b}3600x^{-1.8}\,dx$$
$$= \lim_{b\to\infty}\left[\frac{3600}{-0.8}x^{-0.8}\right]_1^b$$
$$= \lim_{b\to\infty}\left[-4500x^{-0.8}\right]_1^b$$
$$= \lim_{b\to\infty}\left[-\frac{4500}{b^{0.8}}-\left(-\frac{4500}{1^{0.8}}\right)\right]$$
$$=0+4500 \qquad \left(\text{As } b\to\infty, -\frac{4500}{b^{0.8}}\to 0\right)$$
$$=4500$$

The total cost would be $4500.

**34.** Total production is given by:

$$\int_0^{\infty} 2000e^{-0.42t}\,dt$$
$$= \lim_{b\to\infty}\int_0^{b} 2000e^{-0.42t}\,dt$$
$$= \lim_{b\to\infty}\left[\frac{2000}{-0.42}e^{-0.42t}\right]_0^b$$
$$= \lim_{b\to\infty}\left[\frac{2000}{-0.42}e^{-0.42b} - \left(\frac{2000}{-0.42}e^{-0.42(0)}\right)\right]$$
$$= \lim_{b\to\infty}\left[\frac{2000}{-0.42e^{0.42b}} + \frac{2000}{0.42}\right]$$
$$\approx 0 + 4762$$
$$\approx 4762$$

The firm can produce approximately 4762 tires.

**35.** From Theorem 2, The accumulated present value is given by

$$\int_0^{\infty} Pe^{-kt}\,dt = \frac{P}{k}$$

Substituting 5000 for $P$ and 0.08 for $k$, we have:

$$\int_0^{\infty} 5000e^{-0.08t}\,dt = \frac{5000}{0.08} \approx 62,500\,.$$

The accumulated present value is \$62,500.

**36.** $$\int_0^{\infty} Pe^{-kt}\,dt = \frac{P}{k}$$
$$\int_0^{\infty} 2000e^{-0.01t}\cdot e^{-0.07t}\,dt$$
$$= \lim_{b\to\infty}\int_0^{b} 2000e^{-0.08t}\,dt$$
$$= \lim_{b\to\infty}\left[\frac{2000}{-0.08}e^{-0.08t}\right]_0^b\,.$$
$$= \lim_{b\to\infty}\left[-25,000e^{-0.08b} - \left(-25,000e^{-0.08(0)}\right)\right]$$
$$= \lim_{b\to\infty}\left[-\frac{25,000}{e^{0.08b}} + 25,000\right]$$
$$= 25,000$$

The accumulated present value is \$25,000.

**37.** $$c = c_0 + \int_0^{\infty} m(t)e^{-rt}\,dt$$

Substituting 500,000 for $c_0$, 0.05 for $r$, and 20,000 for $m(t)$, we have:

$$c = 500,000 + \int_0^{\infty} 20,000e^{-0.05t}\,dt$$
$$= 500,000 + \lim_{b\to\infty}\int_0^{b} 20,000e^{-0.05t}\,dt$$
$$= 500,000 + \lim_{b\to\infty}\left[\frac{20,000}{-0.05}e^{-0.05t}\right]_0^b$$
$$= 500,000 + \lim_{b\to\infty}\left[-400,000\left(e^{-0.05b} - e^{-0.05(0)}\right)\right]$$
$$= 500,000 + \left[-400,000(0-1)\right]$$
$$= 500,000 + 400,000$$
$$= 900,000$$

The capitalized cost is \$900,000.

**38.** $$c = c_0 + \int_0^{\infty} m(t)e^{-rt}\,dt$$
$$c = 700,000 + \int_0^{\infty} 30,000e^{-0.05t}\,dt$$
$$= 700,000 + \lim_{b\to\infty}\int_0^{b} 30,000e^{-0.05t}\,dt$$
$$= 700,000 + \lim_{b\to\infty}\left[\frac{30,000}{-0.05}e^{-0.05t}\right]_0^b$$
$$= 700,000 + \lim_{b\to\infty}\left[-600,000\left(e^{-0.05b} - e^{-0.05(0)}\right)\right]$$
$$= 700,000 + \left[-600,000(0-1)\right]$$
$$= 700,000 + 600,000$$
$$= 1,300,000$$

The capitalized cost is \$1,300,000.

**39.** $$\int_0^{T} Pe^{-kt}\,dt = \frac{P}{k}\left(1 - e^{-kT}\right)$$

As $T \to \infty$, we have:

$$\lim_{T\to\infty}\int_0^{T} P(t)e^{-kt}\,dt$$
$$= \lim_{T\to\infty}\left[\frac{P}{-k}e^{-kt}\right]_0^T$$
$$= \lim_{T\to\infty}\left[\frac{P}{-k}e^{-kT} - \left(\frac{P}{-k}e^{-k\cdot 0}\right)\right]$$
$$= \lim_{T\to\infty}\left[\frac{P}{k}\left(1 - e^{-k\cdot T}\right)\right]$$
$$= \frac{P}{k}$$

Substituting 0.00003 for $k$, and 1 for $P$, we have

$$\frac{P}{k} = \frac{1}{0.00003} \approx 33,333\tfrac{1}{3}\,.$$

The limiting value of the radioactive buildup is $33,333\frac{1}{3}$ pounds.

**40.** $\int_0^{\infty} Pe^{-kt}\,dt = \frac{P}{k}$

$\int_0^{\infty} 1 \cdot e^{-0.023t}\,dt = \frac{1}{0.023} \approx 43.5$

The limiting value of the radioactive buildup is about 43.5 pounds.

**41.** $E = \int_0^a P_0 e^{-kt}\,dt$

a) Note that 60.1 days is $\frac{60.1}{365}$ yr $\approx 0.16465753$ yr .

Using the half-life, we find $k$ as follows:

$$\frac{1}{2}P_0 = P_0 e^{-k(0.16465753)}$$

$$\frac{1}{2} = e^{-k(0.16465753)} \qquad \text{Dividing by } P_0$$

$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-0.16465753k}\right)$$

$$\ln\left(\frac{1}{2}\right) = -0.16465753k$$

$$\frac{\ln\left(\frac{1}{2}\right)}{-0.16465753} = k$$

$$4.20963 \approx k$$

The decay rate is 420.963% per year.

b) The first month is $\frac{1}{12}$ yr.

$$E = \int_0^{1/12} 10e^{-4.20963t}\,dt$$

$$= \frac{10}{-4.20963}\left[e^{-4.20963t}\right]_0^{1/12}$$

$$= \frac{10}{-4.20963}\left[e^{-4.20963(1/12)} - e^{-4.20963(0)}\right]$$

$$= \frac{10}{-4.20963}\left[e^{-0.3508025} - 1\right]$$

$$\approx 0.702858$$

In the first month, 0.702858 rems of energy is transmitted.

c) $E = \int_0^{\infty} 10e^{-4.20963t}\,dt$

$$E = \lim_{b\to\infty}\int_0^b 10e^{-4.20963t}\,dt$$

$$= \frac{10}{4.20963} \qquad \left[\int_0^{\infty} Pe^{-kt} = \frac{P}{k}\right]$$

$$\approx 2.37551$$

The total amount of energy transmitted is 2.37551 rems.

**42.** $E = \int_0^a P_0 e^{-kt}\,dt$

a) Note that 16.99 days is approximately 0.046548 yr.

$$\frac{1}{2}P_0 = P_0 e^{-k(0.046548)}$$

$$\frac{1}{2} = e^{-k(0.046548)}$$

$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-0.046548k}\right)$$

$$\ln\left(\frac{1}{2}\right) = -0.046548k$$

$$14.891 \approx k$$

The decay rate is 1489.1% per year.

b) The first month is $\frac{1}{12}$ yr.

$$E = \int_0^{1/12} 10e^{-14.891t}\,dt$$

$$= \frac{10}{-14.891}\left[e^{-14.891t}\right]_0^{1/12}$$

$$\approx 0.47739$$

In the first month, 0.47739 rems of energy is transmitted.

c) $E = \int_0^{\infty} 10e^{-14.891t}\,dt$

$$E = \lim_{b\to\infty}\int_0^b 10e^{-14.891t}\,dt$$

$$= \frac{10}{14.891}$$

$$\approx 0.67155$$

The total amount of energy transmitted is 0.67155 rems.

**43.** $\int_0^{\infty} \frac{dx}{x^{2/3}}$

$$= \lim_{b\to\infty}\int_0^b x^{-2/3}\,dx$$

$$= \lim_{b\to\infty}\left[\frac{x^{1/3}}{1/3}\right]_0^b$$

$$= \lim_{b\to\infty}\left[3x^{1/3}\right]_0^b$$

$$= \lim_{b\to\infty}\left[3b^{1/3} - 3(0)^{1/3}\right]$$

$$= \lim_{b\to\infty}\left[3\cdot\sqrt[3]{b}\right]$$

As $b \to \infty$, $\sqrt[3]{b} \to \infty$. Therefore, the limit does not exist. The improper integral is divergent.

**44.** $\int_1^{\infty} \frac{dx}{\sqrt{x}}$

$= \lim_{b\to\infty} \int_1^b x^{-1/2} dx$

$= \lim_{b\to\infty} \left[ \frac{x^{1/2}}{1/2} \right]_1^b$

$= \lim_{b\to\infty} \left[ 2x^{1/2} \right]_1^b$

$= \lim_{b\to\infty} \left[ 2b^{1/2} - 2(1)^{1/2} \right]$

$= \lim_{b\to\infty} \left[ 2\cdot\sqrt{b} - 2 \right]$

As $b \to \infty$, $\sqrt{b} \to \infty$. Therefore, the limit does not exist. The improper integral is divergent.

**45.** $\int_0^{\infty} \frac{dx}{(x+1)^{3/2}}$

$= \lim_{b\to\infty} \int_0^b (x+1)^{-3/2} dx$

$= \lim_{b\to\infty} \left[ \frac{(x+1)^{-1/2}}{-1/2} \right]_0^b$

$= \lim_{b\to\infty} \left[ -\frac{2}{\sqrt{x+1}} \right]_0^b$

$= \lim_{b\to\infty} \left[ -\frac{2}{\sqrt{b+1}} - \left( -\frac{2}{\sqrt{0+1}} \right) \right]$

$= \lim_{b\to\infty} \left[ -\frac{2}{\sqrt{b+1}} + 2 \right]$

$= 0 + 2 \qquad \left[ \text{As } b \to \infty, -\frac{2}{\sqrt{b+1}} \to 0 \right]$

$= 2$

Therefore, the limit exists. The improper integral is convergent.

**46.** $\int_{-\infty}^{0} e^{2x} dx$

$= \lim_{a\to-\infty} \int_a^0 e^{2x} dx$

$= \lim_{a\to-\infty} \left[ \frac{1}{2} e^{2x} \right]_a^0$

$= \lim_{a\to-\infty} \left[ \frac{1}{2} e^{2(0)} - \left( \frac{1}{2} e^{2(a)} \right) \right]$

$= \lim_{a\to-\infty} \left[ \frac{1}{2} e^{0} - \frac{1}{2} e^{2a} \right]$

$= \frac{1}{2} - 0$

$= \frac{1}{2}$

Therefore, the limit exists. The improper integral is convergent.

**47.** $\int_0^{\infty} xe^{-x^2} dx$

$= \lim_{b\to\infty} \int_0^b xe^{-x^2} dx$

$= \lim_{b\to\infty} \int_0^b -\frac{1}{2}\cdot(-2)xe^{-x^2} dx$ Multiplying by 1.

$= \lim_{b\to\infty} \left[ -\frac{1}{2} e^{-x^2} \right]_0^b$ Using substitution where $u=-x^2$ and $du=-2x\,dx$.

$= \lim_{b\to\infty} \left[ -\frac{1}{2} e^{-b^2} - \left( -\frac{1}{2} e^{-(0)^2} \right) \right]$

$= \lim_{b\to\infty} \left[ -\frac{1}{2e^{b^2}} + \frac{1}{2} \right]$

$= 0 + \frac{1}{2} \qquad \left[ \text{As } b \to \infty, \frac{1}{e^{b^2}} \to 0 \right]$

$= \frac{1}{2}$

Therefore, the limit exists. The improper integral is convergent.

**48.** $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$

$= \int_{-\infty}^{0} xe^{-x^2}\,dx + \int_{0}^{\infty} xe^{-x^2}\,dx$

$= \lim_{a\to-\infty}\int_a^0 xe^{-x^2}\,dx + \lim_{b\to\infty}\int_0^b xe^{-x^2}\,dx$

$= \lim_{a\to-\infty}\left[-\frac{1}{2}e^{-x^2}\right]_a^0 + \lim_{b\to\infty}\left[-\frac{1}{2}e^{-x^2}\right]_0^b$

See Exercise 47.

$= -\frac{1}{2}\lim_{a\to\infty}\left[e^{-(0)^2} - e^{-(a)^2}\right] - \frac{1}{2}\lim_{b\to\infty}\left[e^{-b^2} - e^{-(0)^2}\right]$

$= -\frac{1}{2}(1-0) - \frac{1}{2}(0-1)$

$= -\frac{1}{2} + \frac{1}{2}$

$= 0$

Therefore, the limit exists. The improper integral is convergent.

**49.** $\int_0^\infty E(t)\,dt$

$= \int_0^\infty te^{-kt}\,dt$

$= \lim_{b\to\infty}\int_0^b te^{-kt}\,dt$

$= \lim_{b\to\infty}\left[\frac{1}{(-k)^2}\cdot e^{-kt}(-kt-1)\right]_o^b$

$= \lim_{b\to\infty}\left[-\frac{kt+1}{k^2e^{kt}}\right]_0^b$

$= \lim_{b\to\infty}\left[-\frac{kb+1}{k^2e^{kb}} - \left(-\frac{k(0)+1}{k^2e^{k(0)}}\right)\right]$

$= \lim_{b\to\infty}\left[-\frac{kb+1}{k^2e^{kb}} + \frac{1}{k^2}\right]$

$= \frac{1}{k^2}$ $\qquad \left[\text{As } b\to\infty, -\frac{kb+1}{k^2e^{kb}}\to 0\right]$

The integral represents the total dose of the drug.

**50.** $100 = \frac{1}{k^2}$

$k^2 = \frac{1}{100}$

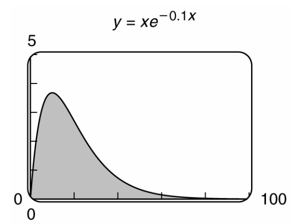
$k = \frac{1}{10}$ $\qquad (k>0)$

$k$ is 0.1 mg/hr.

**51.** $\boxed{tw}$ Since the area of the region under the graph of $y=\frac{1}{x^2}$ on $[1,\infty)$ is finite, you could buy paint to cover it. The area of the region under the graph of $y=\frac{1}{x}$ on $[1,\infty)$ is infinite, so you could not buy enough paint to cover the total area.

**52.** $\boxed{tw}$ You would use the concept of the accumulated present value of a perpetual continuous money flow to determine the present value of the entire stream of future rental income.

**53.**




**54.** Using the fnInt feature on a graphing calculator with a large value for the upper limit, we find $\int_1^\infty \frac{4}{1+x^2}\,dx = \pi \approx 3.14.$

**55.** Using the fnInt feature on a graphing calculator with a large value for the upper limit, we find $\int_1^\infty \frac{6}{5+e^x}\,dx \approx 1.2523.$

## Exercise Set 5.4

**1.** $f(x)=\frac{1}{4}x,\quad [1,3]$

$$\begin{aligned}P([1,3])&=\int_1^3 f(x)\,dx\\&=\int_1^3 \frac{1}{4}x\,dx\\&=\left[\frac{x^2}{8}\right]_1^3\\&=\frac{(3)^2}{8}-\frac{(1)^2}{8}\\&=\frac{9}{8}-\frac{1}{8}\\&=1\end{aligned}$$

**2.** $f(x)=2x,\quad [0,1]$

$$\begin{aligned}P([0,1])&=\int_0^1 f(x)\,dx\\&=\int_0^1 2x\,dx\\&=\left[x^2\right]_0^1\\&=(1)^2-(0)^2\\&=1\end{aligned}$$

**3.** $f(x)=3,\quad \left[0,\frac{1}{3}\right]$

$$\begin{aligned}P\left(\left[0,\frac{1}{3}\right]\right)&=\int_0^{1/3} f(x)\,dx\\&=\int_0^{1/3} 3\,dx\\&=[3x]_0^{1/3}\\&=3\left(\frac{1}{3}\right)-3(0)\\&=1\end{aligned}$$

**4.** $f(x)=\frac{1}{5},\quad [3,8]$

$$\begin{aligned}P([3,8])&=\int_3^8 f(x)\,dx\\&=\int_3^8 \frac{1}{5}\,dx\\&=\left[\frac{1}{5}x\right]_3^8\\&=\frac{1}{5}(8)-\frac{1}{5}(3)\\&=\frac{8}{5}-\frac{3}{5}\\&=1\end{aligned}$$

**5.** $f(x)=\frac{3}{64}x^2,\quad [0,4]$

$$\begin{aligned}P([0,4])&=\int_0^4 f(x)\,dx\\&=\int_0^4 \frac{3}{64}x^2\,dx\\&=\left[\frac{x^3}{64}\right]_0^4\\&=\frac{(4)^3}{64}-\frac{(0)^3}{64}\\&=\frac{64}{64}-0\\&=1\end{aligned}$$

**6.** $f(x)=\frac{3}{26}x^2,\quad [1,3]$

$$\begin{aligned}P([1,3])&=\int_1^3 f(x)\,dx\\&=\int_1^3 \frac{3}{26}x^2\,dx\\&=\left[\frac{x^3}{26}\right]_1^3\\&=\frac{(3)^3}{26}-\frac{(1)^3}{26}\\&=\frac{27}{26}-\frac{1}{26}\\&=1\end{aligned}$$

**7.** $f(x)=\frac{1}{x}, \quad [1,e]$

$$\begin{aligned}P([1,e]) &= \int_1^e f(x)\,dx \\ &= \int_1^e \frac{1}{x}\,dx \\ &= [\ln x]_1^e \\ &= \ln(e)-\ln(1) \\ &= 1-0 \\ &= 1\end{aligned}$$

**8.** $f(x)=\frac{1}{e-1}e^x, \quad [0,1]$

$$\begin{aligned}P([0,1]) &= \int_0^1 f(x)\,dx \\ &= \int_0^1 \frac{1}{e-1}e^x\,dx \\ &= \left[\frac{e^x}{e-1}\right]_0^1 \\ &= \frac{e^1}{e-1}-\frac{e^0}{e-1} \\ &= \frac{e}{e-1}-\frac{1}{e-1} \\ &= \frac{e-1}{e-1} \\ &= 1\end{aligned}$$

**9.** $f(x)=\frac{3}{2}x^2, \quad [-1,1]$

$$\begin{aligned}P([-1,1]) &= \int_{-1}^1 f(x)\,dx \\ &= \int_{-1}^1 \frac{3}{2}x^2\,dx \\ &= \left[\frac{x^3}{2}\right]_{-1}^1 \\ &= \frac{(1)^3}{2}-\frac{(-1)^3}{2} \\ &= \frac{1}{2}-\frac{-1}{2} \\ &= 1\end{aligned}$$

**10.** $f(x)=\frac{1}{3}x^2, \quad [-2,1]$

$$\begin{aligned}P([-2,1]) &= \int_{-2}^1 f(x)\,dx \\ &= \int_{-2}^1 \frac{1}{3}x^2\,dx \\ &= \left[\frac{x^3}{9}\right]_{-2}^1 \\ &= \frac{(1)^3}{9}-\frac{(-2)^3}{9} \\ &= \frac{1}{9}-\frac{-8}{9} \\ &= 1\end{aligned}$$

**11.** $f(x)=3e^{-3x}, \quad [0,\infty)$

$$\begin{aligned}P([0,\infty)) &= \int_0^\infty f(x)\,dx \\ &= \lim_{b\to\infty}\int_0^b 3e^{-3x}\,dx \\ &= \lim_{b\to\infty}\left[\frac{3}{-3}e^{-3x}\right]_0^b \\ &= \lim_{b\to\infty}\left[-e^{-3x}\right]_0^b \\ &= \lim_{b\to\infty}\left[-e^{-3\cdot b}-\left(-e^{-3(0)}\right)\right] \\ &= \lim_{b\to\infty}\left[-\frac{1}{e^{3\cdot b}}+1\right] \\ &= 0+1 \qquad \left[\text{As } b\to\infty, -\frac{1}{e^{3b}}\to 0\right] \\ &= 1\end{aligned}$$

**12.** $f(x)=4e^{-4x}, \quad [0,\infty)$

$$\begin{aligned}P([0,\infty)) &= \int_0^\infty f(x)\,dx \\ &= \lim_{b\to\infty}\int_0^b 4e^{-4x}\,dx \\ &= \lim_{b\to\infty}\left[\frac{4}{-4}e^{-4x}\right]_0^b \\ &= \lim_{b\to\infty}\left[-e^{-4\cdot b}-\left(-e^{-4(0)}\right)\right] \\ &= \lim_{b\to\infty}\left[-\frac{1}{e^{4\cdot b}}+1\right] \\ &= 1\end{aligned}$$

**13.** $f(x) = kx, \quad [2,5]$

Find $k$ such that $\int_2^5 kxdx = 1$

We have

$$\int_2^5 xdx = \left[\frac{x^2}{2}\right]_2^5$$
$$= \left[\frac{(5)^2}{2} - \frac{(2)^2}{2}\right]$$
$$= \frac{25}{2} - \frac{4}{2} = \frac{21}{2}$$

Thus $k = \frac{1}{\frac{21}{2}} = \frac{2}{21}$ and the probability density function is $f(x) = \frac{2}{21}x$.

**14.** $f(x) = kx, \quad [1,4]$

Find $k$ such that $\int_1^4 kxdx = 1$

We have

$$\int_1^4 xdx = \left[\frac{x^2}{2}\right]_1^4$$
$$= \left[\frac{(4)^2}{2} - \frac{(1)^2}{2}\right]$$
$$= \frac{16}{2} - \frac{1}{2} = \frac{15}{2}$$

Thus $k = \frac{1}{\frac{15}{2}} = \frac{2}{15}$ and the probability density function is $f(x) = \frac{2}{15}x$.

**15.** $f(x) = kx^2, \quad [-1,1]$

Find $k$ such that $\int_{-1}^1 kx^2dx = 1$

We have

$$\int_{-1}^1 x^2dx = \left[\frac{x^3}{3}\right]_{-1}^1$$
$$= \left[\frac{(1)^3}{3} - \frac{(-1)^3}{3}\right]$$
$$= \frac{1}{3} - \frac{-1}{3} = \frac{2}{3}$$

Thus $k = \frac{1}{\frac{2}{3}} = \frac{3}{2}$ and the probability density function is $f(x) = \frac{3}{2}x^2$.

**16.** $f(x) = kx^2, \quad [-2,2]$

Find $k$ such that $\int_{-2}^2 kx^2dx = 1$

We have

$$\int_{-2}^2 x^2dx = \left[\frac{x^3}{3}\right]_{-2}^2$$
$$= \left[\frac{(2)^3}{3} - \frac{(-2)^3}{3}\right]$$
$$= \frac{8}{3} - \frac{-8}{3} = \frac{16}{3}$$

Thus $k = \frac{1}{\frac{16}{3}} = \frac{3}{16}$ and the probability density function is $f(x) = \frac{3}{16}x^2$.

**17.** $f(x) = k, \quad [1,7]$

Find $k$ such that $\int_1^7 kdx = 1$

We have

$$\int_1^7 dx = [x]_1^7$$
$$= [7-1]$$
$$= 6$$

Thus $k = 6 = \frac{1}{6}$ and the probability density function is $f(x) = \frac{1}{6}$.

**18.** $f(x) = k, \quad [3,9]$

Find $k$ such that $\int_3^9 kdx = 1$

We have

$$\int_3^9 dx = [x]_3^9$$
$$= [9-3]$$
$$= 6$$

Thus $k = 6 = \frac{1}{6}$ and the probability density function is $f(x) = \frac{1}{6}$.

**19.** $f(x)=k(2-x), \quad [0,2]$

Find $k$ such that $\int_0^2 k(2-x)\,dx=1$

We have

$$\int_0^2 (2-x)\,dx=\left[2x-\frac{x^2}{2}\right]_0^2$$
$$=\left[\left(2(2)-\frac{(2)^2}{2}\right)-\left(2(0)-\frac{0^2}{2}\right)\right]$$
$$=4-\frac{4}{2}-0$$
$$=2$$

Thus $k=\frac{1}{2}$ and the probability density function is $f(x)=\frac{1}{2}(2-x)=\frac{2-x}{2}$.

**20.** $f(x)=k(4-x), \quad [0,4]$

Find $k$ such that $\int_0^4 k(4-x)\,dx=1$

We have

$$\int_0^4 (4-x)\,dx=\left[4x-\frac{x^2}{2}\right]_0^4$$
$$=\left[\left(4(4)-\frac{(4)^2}{2}\right)-\left(4(0)-\frac{0^2}{2}\right)\right]$$
$$=16-\frac{16}{2}-0$$
$$=8$$

Thus $k=\frac{1}{8}$ and the probability density function is $f(x)=\frac{1}{8}(4-x)=\frac{4-x}{8}$.

**21.** $f(x)=\frac{k}{x}, \quad [1,3]$

Find $k$ such that $\int_1^3 \frac{k}{x}\,dx=1$

We have

$$\int_1^3 \frac{1}{x}\,dx=\left[\ln(x)\right]_1^3$$
$$=\left[\ln(3)-\ln(1)\right]$$
$$=\ln 3-0=\ln 3$$

Thus $k=\frac{1}{\ln 3}$ and the probability density function is $f(x)=\frac{1}{\ln 3}\cdot\frac{1}{x}=\frac{1}{x\ln 3}$.

**22.** $f(x)=\frac{k}{x}, \quad [1,2]$

Find $k$ such that $\int_1^2 \frac{k}{x}\,dx=1$

We have

$$\int_1^2 \frac{1}{x}\,dx=\left[\ln(x)\right]_1^2$$
$$=\left[\ln(2)-\ln(1)\right]$$
$$=\ln 2-0=\ln 2$$

Thus $k=\frac{1}{\ln 2}$ and the probability density function is $f(x)=\frac{1}{\ln 2}\cdot\frac{1}{x}=\frac{1}{x\ln 2}$.

**23.** $f(x)=ke^x, \quad [0,3]$

Find $k$ such that $\int_0^3 ke^x\,dx=1$

We have

$$\int_0^3 e^x\,dx=\left[e^x\right]_0^3$$
$$=\left[e^3-e^0\right]$$
$$=e^3-1$$

Thus $k=\frac{1}{e^3-1}$ and the probability density function is $f(x)=\frac{1}{e^3-1}\cdot e^x=\frac{e^x}{e^3-1}$.

**24.** $f(x)=ke^x, \quad [0,2]$

Find $k$ such that $\int_0^2 ke^x\,dx=1$

We have

$$\int_0^2 e^x\,dx=\left[e^x\right]_0^2$$
$$=\left[e^2-e^0\right]$$
$$=e^2-1$$

Thus $k=\frac{1}{e^2-1}$ and the probability density function is $f(x)=\frac{1}{e^2-1}\cdot e^x=\frac{e^x}{e^2-1}$.

**25.** a) $f(x)=\frac{1}{50}x,$ for $0\le x\le 10$

$$P(2\le x\le 6)=\int_2^6 \frac{1}{50}x\,dx$$
$$=\left[\frac{1}{50}\cdot\frac{x^2}{2}\right]_2^6$$
$$=\left[\frac{x^2}{100}\right]_2^6$$
$$=\frac{6^2}{100}-\frac{2^2}{100}$$
$$=\frac{36-4}{100}$$
$$=\frac{32}{100}$$
$$=\frac{8}{25}=0.32$$

The probability that the dart lands in the interval $[2,6]$ is $\frac{8}{25}$, or 0.32.

b) $\boxed{tw}$ The answer to part (a) is the area under the graph of *f* over a subinterval from 2 to 6.

**26.** a) $f(x)=\frac{3}{125}x^2,$ for $0\le x\le 5$

$$P(1\le x\le 4)=\int_1^4 \frac{3}{125}x^2\,dx$$
$$=\left[\frac{x^3}{125}\right]_1^4$$
$$=\frac{4^3}{125}-\frac{1^3}{125}$$
$$=\frac{63}{125}=0.504$$

The probability that the dart lands in the interval $[1,4]$ is $\frac{63}{125}$, or 0.504.

b) $\boxed{tw}$ The answer to part (a) is the area under the graph of *f* over a subinterval from 1 to 4.

**27.** $f(x)=\frac{1}{16},$ for $4\le x\le 20$

$$P(9\le x\le 20)=\int_9^{20} \frac{1}{16}\,dx$$
$$=\left[\frac{1}{16}x\right]_9^{20}$$
$$=\frac{1}{16}\cdot 20-\frac{1}{16}\cdot 9$$
$$=\frac{20-9}{16}$$
$$=\frac{11}{16}=0.6875$$

The probability that the number selected is in the subinterval $[9,20]$ is $\frac{11}{16}$, or 0.6875.

**28.** $f(x)=\frac{1}{24},$ for $5\le x\le 29$

$$P(14\le x\le 29)=\int_{14}^{29} \frac{1}{24}\,dx$$
$$=\left[\frac{1}{24}x\right]_{14}^{29}$$
$$=\frac{1}{24}\cdot 29-\frac{1}{24}\cdot 14$$
$$=\frac{15}{24}$$
$$=\frac{5}{8}=0.625$$

The probability that the number selected is in the subinterval $[14,29]$ is $\frac{5}{8}$, or 0.625.

**29.** From Example 10, we know

$f(x)=ke^{-kx},$ for $0\le x<\infty$, where $k=\frac{1}{a}$ and $a$ is the average distance between successive cars over some period of time.

First, we determine *k*:

$k=\frac{1}{100}=0.01$.

The probability that the distance between cars is 40 feet or less is calculated as follows:

$$P(0 \le x \le 40) = \int_0^{40} 0.01e^{-0.01x}\,dx$$
$$= \left[\frac{0.01}{-0.01}e^{-0.01x}\right]_0^{40}$$
$$= \left[-e^{-0.01x}\right]_0^{40}$$
$$= -e^{-0.01(40)} - \left(-e^{-0.01(0)}\right)$$
$$= -e^{-0.4} + e^0$$
$$= -e^{-0.4} + 1$$
$$\approx -0.670320 + 1$$
$$\approx 0.329680$$
$$\approx 0.3297$$

The probability that the distance between two successive cars, chosen at random, is 40 feet or less is 0.3297.

**30.** $f(x) = ke^{-kx}$, for $0 \le x < \infty$

$k = \dfrac{1}{200} = 0.005$.

$$P(0 \le x \le 10) = \int_0^{10} 0.005e^{-0.005x}\,dx$$
$$= \left[\frac{0.005}{-0.005}e^{-0.005x}\right]_0^{10}$$
$$= \left[-e^{-0.005x}\right]_0^{10}$$
$$= -e^{-0.005(10)} - \left(-e^{-0.005(0)}\right)$$
$$= -e^{-0.05} + e^0$$
$$\approx 0.0488$$

The probability that the distance between two successive cars, chosen at random, is 10 feet or less is 0.0488.

**31.** $f(t) = 2e^{-2t}$, for $0 \le t < \infty$

$$P(0 \le t \le 5) = \int_0^{5} 2e^{-2t}\,dt$$
$$= \left[\frac{2}{-2}e^{-2t}\right]_0^{5}$$
$$= \left[-e^{-2t}\right]_0^{5}$$
$$= -e^{-2(5)} - \left(-e^{-2(0)}\right)$$
$$= -e^{-10} + e^0$$
$$= -e^{-10} + 1$$
$$\approx -0.0000454 + 1$$
$$\approx 0.9999546$$
$$\approx 0.999955$$

The probability that a phone call will last no more than 5 minutes is 0.999955.

**32.** $f(t) = 2e^{-2t}$, for $0 \le t < \infty$

$$P(0 \le t \le 2) = \int_0^{2} 2e^{-2t}\,dt$$
$$= \left[\frac{2}{-2}e^{-2t}\right]_0^{2}$$
$$= -e^{-2(2)} - \left(-e^{-2(0)}\right)$$
$$= -e^{-4} + e^0$$
$$\approx 0.9817$$

The probability that a phone call will last no more than 2 minutes is 0.9817.

**33.** $f(t) = ke^{-kt}$, for $0 \le t < \infty$, where $k = \dfrac{1}{a}$ and $a$ is the average amount of time that will pass before a failure occurs.

First, we determine $k$:

$k = \dfrac{1}{100} = 0.01$.

The probability that a failure will occur in 50 hours or less is

$$P(0 \le x \le 50) = \int_0^{50} 0.01e^{-0.01x}\,dx$$
$$= \left[\frac{0.01}{-0.01}e^{-0.01x}\right]_0^{50}$$
$$= \left[-e^{-0.01x}\right]_0^{50}$$
$$= -e^{-0.01(50)} - \left(-e^{-0.01(0)}\right)$$
$$= -e^{-0.5} + e^0$$
$$= -e^{-0.5} + 1$$
$$\approx -0.606531 + 1$$
$$\approx 0.393469$$
$$\approx 0.3935$$

The probability that a failure will occur in 50 hours or less is 0.3935.

**34.** $R(t) = 1 - \int_0^{T} 0.01e^{-0.01t}\,dt$

Integrating we have:

$$R(t) = 1 - \left[\frac{0.01}{-0.01}e^{-0.01t}\right]_0^{T}$$
$$R(t) = 1 - \left[-e^{-0.01t}\right]_0^{T}$$
$$R(t) = 1 - \left[-e^{-0.01(T)} - \left(-e^{-0.01(0)}\right)\right]$$
$$R(t) = 1 + e^{-0.01(T)} - e^0$$
$$R(t) = 1 + e^{-0.01(T)} - 1$$
$$R(t) = e^{-0.01T}$$

**35.** $f(x) = 8.1305e^{0.074x}$, for $1 \le x \le 85$

a) Find $k$ such that $\int_1^{85} kf(x)\,dx = 1$

We have

$$\int_1^{85} 8.1305e^{0.074x}dx$$
$$= \left[\frac{8.1305}{0.074}e^{0.074x}\right]_1^{85}$$
$$\approx 109.8716216\left[e^{0.074(85)} - e^{0.074(1)}\right]$$
$$\approx 59{,}119.34$$

Thus,

$$k = \frac{1}{59{,}119.34} \approx 0.000017\,.$$

b) The probability density function is

$$kf(x) = (0.000017)8.1305e^{0.074x}, \text{ for } [1,85]$$
$$= \left(1.382\times10^{-4}\right)e^{0.074x}$$

The probability that a female who died in 2003 was between 25 and 40 years old is:

$$P([25,40]) = \int_{25}^{40}\left(1.382\times10^{-4}\right)e^{0.074x}dx$$
$$= \left[\frac{1.382\times10^{-4}}{0.074}e^{0.074x}\right]_{25}^{40}$$
$$\approx \left[0.0018676e^{0.074x}\right]_{25}^{40}$$
$$\approx 0.0018676\left(e^{0.074(40)} - e^{0.074(25)}\right)$$
$$\approx 0.024$$

The probability that a female who died in 2003 was between 25 and 40 years of age was approximately 0.024.
Note: if we waited until the end of all calculations to round, the probability would be 0.02404.

**36.** $f(x) = 17.359e^{0.067x}$, for $1 \le x \le 85$

a) Find $k$ such that $\int_1^{85} kf(x)\,dx = 1$

We have

$$\int_1^{85} 17.359e^{0.067x}dx$$
$$= \left[\frac{17.359}{0.067}e^{0.067x}\right]_1^{85}$$
$$\approx 259.0896\left[e^{0.067(85)} - e^{0.067(1)}\right]$$
$$\approx 76{,}770.19$$

Thus,

$$k = \frac{1}{76{,}770.19} \approx 0.000013 = 1.3\times10^{-5}\,.$$

b) The probability that a male who died in 2003 was between 20 and 30 years old is:

$$P([20,30]) = \int_{20}^{30}\left(1.3\times10^{-5}\right)17.359e^{0.067x}dx$$
$$= \left[0.003368e^{0.067x}\right]_{20}^{30}$$
$$\approx 0.003368\left(e^{0.067(30)} - e^{0.067(20)}\right)$$
$$\approx 0.012274$$

The probability that a male who died in 2003 was between 20 and 30 years of age was approximately 0.01227.
Note: if we waited until the end of all calculations to round, the probability would be 0.012299.

**37.** $f(t) = 0.02e^{-0.02t}$, for $0 \le t < \infty$

$$P(0 \le t \le 150) = \int_0^{150} 0.02e^{-0.02t}dt$$
$$= \left[\frac{0.02}{-0.02}e^{-0.02t}\right]_0^{150}$$
$$= \left[-e^{-0.02}\right]_0^{150}$$
$$= -e^{-0.02(150)} - \left(-e^{-0.02(0)}\right)$$
$$= -e^{-3} + e^0$$
$$= -e^{-3} + 1$$
$$\approx -0.049787 + 1$$
$$\approx 0.950213$$

The probability that the rat will learn its way through a maze in 150 seconds, or less, is approximately 0.950213.

**38.** $f(t) = 0.02e^{-0.02t}$, for $0 \le t < \infty$

$$P(0 \le t \le 50) = \int_0^{50} 0.02e^{-0.02t}dt$$
$$= \left[-e^{-0.02}\right]_0^{50}$$
$$= -e^{-0.02(50)} - \left(-e^{-0.02(0)}\right)$$
$$= -e^{-1} + 1$$
$$\approx 0.632121$$

The probability that the rat will learn its way through a maze in 50 seconds, or less, is approximately 0.632121.

**39.** $f(x)=x^3$ is a probability density function over $[0,b]$. Thus,

$$\int_0^b f(x)\,dx=1$$
$$\int_0^b x^3\,dx=1$$
$$\left[\frac{x^4}{4}\right]_0^b=1$$
$$\frac{b^4}{4}-\frac{0^4}{4}=1$$
$$\frac{b^4}{4}=1$$
$$b^4=4$$
$$b=\sqrt[4]{4}$$
$$b=\sqrt[4]{2^2}$$
$$b=\sqrt{2}$$

$f(x)=x^3$ is a probability density function over $\left[0,\sqrt{2}\right]$.

**40.**
$$\int_{-a}^{a}12x^2\,dx=1$$
$$\left[4x^3\right]_{-a}^{a}=1$$
$$4a^3-4(-a)^3=1$$
$$4a^3+4a^3=1$$
$$8a^3=1$$
$$a^3=\frac{1}{8}$$
$$a=\frac{1}{2}$$

$f(x)=12x^2$ is a probability density function over the interval $\left[-\frac{1}{2},\frac{1}{2}\right]$.

**41.** In Exercise 37 we found that $P(0\le t\le 150)\approx 0.950213$. Since this is a probability density function, we know

$$\int_0^\infty f(x)\,dx=\int_0^{150} f(x)\,dx+\int_{150}^\infty f(x)\,dx=1$$

Thus,

$$\int_{150}^\infty f(x)\,dx=1-\int_0^{150} f(x)\,dx$$
$$\approx 1-0.950213$$
$$=0.049787$$

Thus, the probability that a rat requires more than 150 seconds to learn its way through the maze is 0.049787.

**42.** $\boxed{tw}$ Answers will vary. Examples include the probability of precipitation in a weather forecast, the probability that a particular candidate will win an election, the probability that a flight will arrive on time, the probability that particular treatment will heal a sick patient, and the probability that a particular team will win an athletic competition.

**43 – 54.** Check your answers using the solutions to Exercises 1-12.

## Exercise Set 5.5

**1.** $f(x)=\frac{1}{4},\quad [3,7]$

$$E(x)=\int_a^b x\cdot f(x)\,dx \qquad \text{Expected value of } x.$$
$$E(x)=\int_3^7 x\cdot\frac{1}{4}\,dx$$
$$=\frac{1}{4}\left[\frac{x^2}{2}\right]_3^7$$
$$=\frac{1}{4}\left[\frac{7^2}{2}-\frac{3^2}{2}\right]$$
$$=\frac{1}{4}\left[\frac{49}{2}-\frac{9}{2}\right]$$
$$=\frac{1}{4}\cdot\frac{40}{2}$$
$$=5$$
$$E(x^2)=\int_a^b x^2\cdot f(x)\,dx \qquad \text{Expected value of } x^2.$$
$$E(x^2)=\int_3^7 x^2\cdot\frac{1}{4}\,dx$$
$$=\frac{1}{4}\left[\frac{x^3}{3}\right]_3^7$$
$$=\frac{1}{4}\left[\frac{7^3}{3}-\frac{3^3}{3}\right]$$
$$=\frac{1}{4}\left[\frac{343}{3}-\frac{27}{3}\right]$$
$$=\frac{1}{4}\cdot\frac{316}{3}$$
$$=\frac{79}{3}$$

$\mu = E(x) = 5$ Mean

$\sigma^2 = E(x^2) - [E(x)]^2$

$= \frac{79}{3} - [5]^2$ Substituting 79/3 for $E(x^2)$ and 5 for $E(x)$

$= \frac{79}{3} - 25$

$= \frac{79}{3} - \frac{75}{3}$

$= \frac{4}{3}$ Variance

$\sigma = \sqrt{\sigma^2}$

$= \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$ Standard deviation

**2.** $f(x) = \frac{1}{5}, \quad [3,8]$

$E(x) = \int_3^8 x \cdot \frac{1}{5}\,dx$

$= \frac{1}{5}\left[\frac{x^2}{2}\right]_3^8$

$= \frac{1}{5}\left[\frac{8^2}{2} - \frac{3^2}{2}\right] = \frac{1}{5} \cdot \frac{55}{2} = \frac{11}{2}$

$E(x^2) = \int_3^8 x^2 \cdot \frac{1}{5}\,dx$

$= \frac{1}{5}\left[\frac{x^3}{3}\right]_3^8$

$= \frac{1}{5}\left[\frac{8^3}{3} - \frac{3^3}{3}\right] = \frac{1}{5} \cdot \frac{485}{3} = \frac{97}{3}$

$\mu = E(x) = \frac{11}{2}$

$\sigma^2 = E(x^2) - [E(x)]^2$

$= \frac{97}{3} - \left[\frac{11}{2}\right]^2 = \frac{25}{12}$

$\sigma = \sqrt{\frac{25}{12}} = \frac{5}{2\sqrt{3}}$

**3.** $f(x) = \frac{1}{8}x, \quad [0,4]$

$E(x) = \int_a^b x \cdot f(x)\,dx$ Expected value of $x$.

$E(x) = \int_0^4 x \cdot \frac{1}{8}x\,dx$

$= \int_0^4 \frac{1}{8}x^2\,dx$

$= \frac{1}{8}\left[\frac{x^3}{3}\right]_0^4$

$= \frac{1}{8}\left[\frac{4^3}{3} - \frac{0^3}{3}\right]$

$= \frac{1}{8} \cdot \frac{64}{3}$

$= \frac{8}{3}$

$E(x^2) = \int_a^b x^2 \cdot f(x)\,dx$ Expected value of $x^2$.

$E(x^2) = \int_0^4 x^2 \cdot \frac{1}{8}x\,dx$

$= \int_0^4 \frac{1}{8}x^3\,dx$

$= \frac{1}{8}\left[\frac{x^4}{4}\right]_0^4$

$= \frac{1}{8}\left[\frac{4^4}{4} - \frac{0^4}{4}\right]$

$= \frac{1}{8} \cdot \frac{256}{4}$

$= 8$

$\mu = E(x) = \frac{8}{3}$ Mean

$\sigma^2 = E(x^2) - [E(x)]^2$

$= 8 - \left[\frac{8}{3}\right]^2$ Substituting 8 for $E(x^2)$ and 8/3 for $E(x)$

$= 8 - \frac{64}{9}$

$= \frac{72}{9} - \frac{64}{9}$

$= \frac{8}{9}$ Variance

$\sigma = \sqrt{\sigma^2}$

$= \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$ Standard deviation

**4.** $f(x)=\frac{2}{9}x,\quad [0,3]$

$$E(x)=\int_0^3 x\cdot\frac{2}{9}x\,dx=\int_0^3\frac{2}{9}x^2dx$$
$$=\frac{2}{9}\left[\frac{x^3}{3}\right]_0^3$$
$$=\frac{2}{9}\left[\frac{3^3}{3}-\frac{0^3}{3}\right]=\frac{2}{9}\cdot 9=2$$
$$E(x^2)=\int_0^3 x^2\cdot\frac{2}{9}x\,dx=\int_0^3\frac{2}{9}x^3dx$$
$$=\frac{2}{9}\left[\frac{x^4}{4}\right]_0^3$$
$$=\frac{2}{9}\left[\frac{3^4}{4}-\frac{0^4}{4}\right]=\frac{2}{9}\cdot\frac{81}{4}=\frac{9}{2}$$
$$\mu=E(x)=2$$
$$\sigma^2=E(x^2)-[E(x)]^2$$
$$=\frac{9}{2}-[2]^2=\frac{1}{2}$$
$$\sigma=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

**5.** $f(x)=\frac{1}{4}x,\quad [1,3]$

$$E(x)=\int_a^b x\cdot f(x)\,dx$$ Expected value of $x$.
$$E(x)=\int_1^3 x\cdot\frac{1}{4}x\,dx$$
$$=\int_1^3\frac{x^2}{4}dx$$
$$=\left[\frac{x^3}{12}\right]_1^3$$
$$=\left[\frac{3^3}{12}-\frac{1^3}{12}\right]$$
$$=\frac{26}{12}$$
$$=\frac{13}{6}$$

$$E(x^2)=\int_a^b x^2\cdot f(x)\,dx$$ Expected value of $x^2$.
$$E(x^2)=\int_1^3 x^2\cdot\frac{1}{4}x\,dx$$
$$=\int_1^3\frac{x^3}{4}dx$$
$$=\left[\frac{x^4}{16}\right]_1^3$$
$$=\left[\frac{3^4}{16}-\frac{1^4}{16}\right]$$
$$=\frac{80}{16}$$
$$=5$$
$$\mu=E(x)=\frac{13}{6}$$ Mean
$$\sigma^2=E(x^2)-[E(x)]^2$$
$$=5-\left[\frac{13}{6}\right]^2$$ Substituting 5 for $E(x^2)$ and 13/6 for $E(x)$
$$=5-\frac{169}{36}$$
$$=\frac{180}{36}-\frac{169}{36}$$
$$=\frac{11}{36}$$ Variance
$$\sigma=\sqrt{\sigma^2}$$
$$=\sqrt{\frac{11}{36}}=\frac{\sqrt{11}}{6}$$ Standard deviation

**6.** $f(x)=\frac{2}{3}x,\quad [1,2]$

$$E(x)=\int_1^2 x\cdot\frac{2}{3}x\,dx=\int_1^2\frac{2}{3}x^2dx$$
$$=\frac{2}{3}\left[\frac{x^3}{3}\right]_1^2$$
$$=\frac{2}{3}\left[\frac{2^3}{3}-\frac{1^3}{3}\right]=\frac{2}{3}\cdot\frac{7}{3}=\frac{14}{9}$$
$$E(x^2)=\int_1^2 x^2\cdot\frac{2}{3}x\,dx=\int_1^2\frac{2}{3}x^3dx$$
$$=\frac{2}{3}\left[\frac{x^4}{4}\right]_1^2$$
$$=\frac{2}{3}\left[\frac{2^4}{4}-\frac{1^4}{4}\right]=\frac{2}{3}\cdot\frac{15}{4}=\frac{5}{2}$$

$$\mu = E(x) = \frac{14}{9}$$

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$$= \frac{5}{2} - \left[\frac{14}{9}\right]^2 = \frac{13}{162}$$

$$\sigma = \sqrt{\frac{13}{162}} = \frac{1}{9}\sqrt{\frac{13}{2}}$$

**7.** $f(x) = \frac{3}{2}x^2, \quad [-1,1]$

$E(x) = \int_a^b x \cdot f(x)\,dx$ Expected value of $x$.

$$E(x) = \int_{-1}^{1} x \cdot \frac{3}{2}x^2 dx$$

$$= \int_{-1}^{1} \frac{3}{2}x^3 dx$$

$$= \left[\frac{3x^4}{8}\right]_{-1}^{1}$$

$$= \left[\frac{3(1)^4}{8} - \frac{3(-1)^4}{8}\right]$$

$$= \frac{3}{8} - \frac{3}{8}$$

$$= 0$$

$E(x^2) = \int_a^b x^2 \cdot f(x)\,dx$ Expected value of $x^2$.

$$E(x^2) = \int_{-1}^{1} x^2 \cdot \frac{3}{2}x^2 dx$$

$$= \int_{-1}^{1} \frac{3}{2}x^4 dx$$

$$= \left[\frac{3x^5}{10}\right]_{-1}^{1}$$

$$= \left[\frac{3(1)^5}{10} - \frac{3(-1)^5}{10}\right]$$

$$= \frac{3}{10} + \frac{3}{10}$$

$$= \frac{6}{10} = \frac{3}{5}$$

$\mu = E(x) = 0$ Mean

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$= \frac{3}{5} - [0]^2$ Substituting 3/5 for $E(x^2)$ and 0 for $E(x)$

$= \frac{3}{5}$ Variance

$$\sigma = \sqrt{\sigma^2}$$

$= \sqrt{\frac{3}{5}}$ Standard deviation

**8.** $f(x) = \frac{1}{3}x^2, \quad [-2,1]$

$$E(x) = \int_{-2}^{1} x \cdot \frac{1}{3}x^2 dx = \int_{-2}^{1} \frac{1}{3}x^3 dx$$

$$= \left[\frac{x^4}{12}\right]_{-2}^{1}$$

$$= \left[\frac{(1)^4}{12} - \frac{(-2)^4}{12}\right] = -\frac{15}{12} = -\frac{5}{4}$$

$$E(x^2) = \int_{-2}^{1} x^2 \cdot \frac{1}{3}x^2 dx = \int_{-2}^{1} \frac{1}{3}x^4 dx$$

$$= \left[\frac{x^5}{15}\right]_{-2}^{1}$$

$$= \left[\frac{(1)^5}{15} - \frac{(-2)^5}{15}\right] = \frac{33}{15} = \frac{11}{5}$$

$$\mu = E(x) = -\frac{5}{4}$$

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$$= \frac{11}{5} - \left[-\frac{5}{4}\right]^2 = \frac{51}{80}$$

$$\sigma = \sqrt{\frac{51}{80}} = \frac{1}{4}\sqrt{\frac{51}{5}}$$

**9.** $f(x) = \frac{1}{\ln 5} \cdot \frac{1}{x}, \quad [1.5, 7.5]$

$E(x) = \int_a^b x \cdot f(x)\,dx$ Expected value of $x$.

$$E(x) = \int_{1.5}^{7.5} x \cdot \frac{1}{\ln 5} \cdot \frac{1}{x} dx$$

$$= \int_{1.5}^{7.5} \frac{1}{\ln 5} dx$$

$$= \frac{1}{\ln 5}[x]_{1.5}^{7.5}$$

$$= \frac{1}{\ln 5}[7.5 - 1.5]$$

$$= \frac{6}{\ln 5}$$

$$E\left(x^2\right)=\int_a^b x^2\cdot f\left(x\right)dx \qquad \text{Expected value of } x^2.$$

$$E\left(x^2\right)=\int_{1.5}^{7.5} x^2\cdot\frac{1}{\ln 5}\cdot\frac{1}{x}\,dx$$

$$=\int_{1.5}^{7.5}\frac{1}{\ln 5}x\,dx$$

$$=\frac{1}{\ln 5}\left[\frac{x^2}{2}\right]_{1.5}^{7.5}$$

$$=\frac{1}{\ln 5}\left[\frac{(7.5)^2}{2}-\frac{(1.5)^2}{2}\right]$$

$$=\frac{1}{\ln 5}\left[\frac{56.25}{2}-\frac{2.25}{2}\right]$$

$$=\frac{1}{\ln 5}\left[\frac{54}{2}\right]$$

$$=\frac{27}{\ln 5}$$

$$\mu=E\left(x\right)=\frac{6}{\ln 5} \qquad \text{Mean}$$

$$\sigma^2=E\left(x^2\right)-\left[E\left(x\right)\right]^2$$

$$=\frac{27}{\ln 5}-\left[\frac{6}{\ln 5}\right]^2 \qquad \text{Substituting } 27/\ln 5 \text{ for } E\left(x^2\right) \text{ and } 6/\ln 5 \text{ for } E(x)$$

$$=\frac{27}{\ln 5}-\frac{36}{(\ln 5)^2}$$

$$=\frac{27\ln 5}{(\ln 5)^2}-\frac{36}{(\ln 5)^2}$$

$$=\frac{27\ln 5-36}{(\ln 5)^2} \qquad \text{Variance}$$

$$\sigma=\sqrt{\sigma^2}$$

$$=\sqrt{\frac{27\ln 5-36}{(\ln 5)^2}}$$

$$=\frac{\sqrt{27\ln 5-36}}{\ln 5} \qquad \text{Standard deviation}$$

**10.** $f\left(x\right)=\frac{1}{\ln 4}\cdot\frac{1}{x},\quad [0.8,3.2]$

$$E\left(x\right)=\int_{0.8}^{3.2} x\cdot\frac{1}{\ln 4}\cdot\frac{1}{x}\,dx=\int_{0.8}^{3.2}\frac{1}{\ln 4}\,dx$$

$$=\frac{1}{\ln 4}\left[x\right]_{0.8}^{3.2}=\frac{1}{\ln 4}\left[3.2-0.8\right]=\frac{2.4}{\ln 4}$$

$$E\left(x^2\right)=\int_{0.8}^{3.2} x^2\cdot\frac{1}{\ln 4}\cdot\frac{1}{x}\,dx=\int_{0.8}^{3.2}\frac{1}{\ln 4}x\,dx$$

$$=\frac{1}{\ln 4}\left[\frac{x^2}{2}\right]_{0.8}^{3.2}$$

$$=\frac{1}{\ln 4}\left[\frac{(3.2)^2}{2}-\frac{(0.8)^2}{2}\right]$$

$$=\frac{4.8}{\ln 4}$$

$$\mu=E\left(x\right)=\frac{2.4}{\ln 4}$$

$$\sigma^2=E\left(x^2\right)-\left[E\left(x\right)\right]^2$$

$$=\frac{4.8}{\ln 4}-\left[\frac{2.4}{\ln 4}\right]^2$$

$$=\frac{4.8}{\ln 4}-\frac{5.76}{(\ln 4)^2}$$

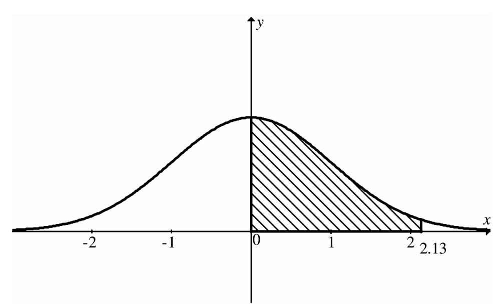
$$=\frac{4.8\ln 4-5.76}{(\ln 4)^2}$$

$$\sigma=\sqrt{\sigma^2}$$

$$=\sqrt{\frac{4.8\ln 4-5.76}{(\ln 4)^2}}$$

$$=\frac{\sqrt{4.8\ln 4-5.76}}{\ln 4}$$

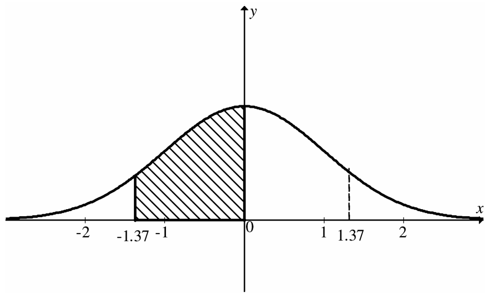
**11.**




Using Table 2, we have
$P\left(0\le x\le 2.13\right)=0.4834$

**12.** $P\left(0\le x\le 0.36\right)=0.1406$

**13.**



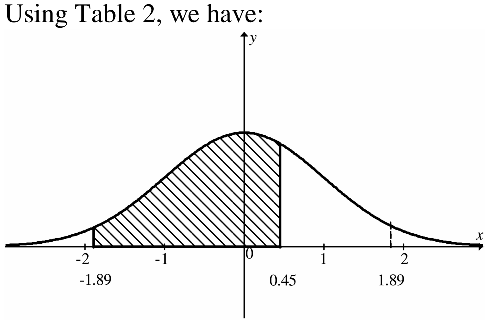

$P(-1.37 \le x \le 0)$

$= P(0 \le x \le 1.37)$ Symmetry of the graph.

$= 0.4147$ Using Table 2.

**14.** $P(-2.01 \le x \le 0) = P(0 \le x \le 2.01) = 0.4778$

**15.** Using Table 2, we have:




$P(-1.89 \le x \le 0.45)$

$= P(-1.89 \le x \le 0) + P(0 \le x \le 0.45)$

$= P(0 \le x \le 1.89) + P(0 \le x \le 0.45)$

Symmetry of the graph

$= 0.4706 + 0.1736$ Using Table 2

$= 0.6442$

**16.** $P(-2.94 \le x \le 2.00)$

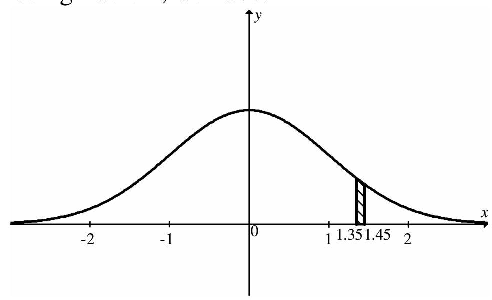
$= P(-2.94 \le x \le 0) + P(0 \le x \le 2.00)$

$= P(0 \le x \le 2.94) + P(0 \le x \le 2.00)$

$= 0.4984 + 0.4772$

$= 0.9756$

**17.** Using Table 2, we have:




$P(1.35 \le x \le 1.45)$

$= P(0 \le x \le 1.45) - P(0 \le x \le 1.35)$

$= 0.4265 - 0.4115$

$= 0.0150$

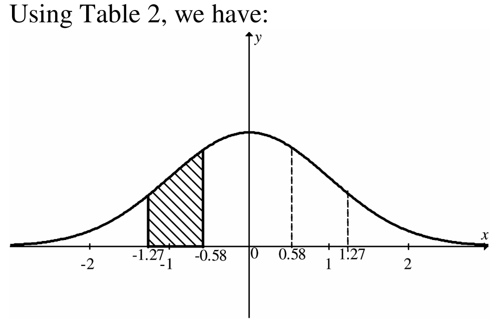
**18.** $P(0.76 \le x \le 1.45)$

$= P(0 \le x \le 1.45) - P(0 \le x \le 0.76)$

$= 0.4265 - 0.2764$

$= 0.1501$

**19.** Using Table 2, we have:




$P(-1.27 \le x \le -0.58)$

$= P(0.58 \le x \le 1.27)$ Symmetry of the graph.

$= P(0 \le x \le 1.27) - P(0 \le x \le 0.58)$

$= 0.3980 - 0.2190$

$= 0.1790$

**20.** $P(-2.45 \le x \le -1.24)$

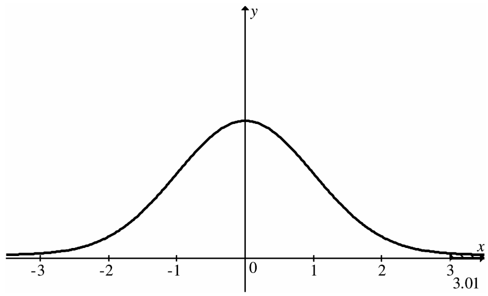
$= P(1.24 \le x \le 2.45)$

$= P(0 \le x \le 2.45) - P(0 \le x \le 1.24)$

$= 0.4929 - 0.3925$

$= 0.1004$

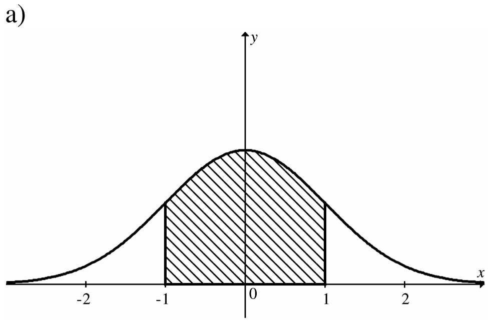
**21.**




$P(x \ge 3.01)$
$= 0.5 - P(0 \le x \le 3.01)$
$= 0.5 - 0.4987$
$= 0.0013$

**22.** $P(x \ge 1.01)$
$= 0.5 - P(0 \le x \le 1.01)$
$= 0.5 - 0.3438$
$= 0.1562$

**23.** a)




$P(-1 \le x \le 1)$
$= P(-1 \le x \le 0) + P(0 \le x \le 1)$
$= P(0 \le x \le 1) + P(0 \le x \le 1)$
Symmetry of the graph
$= 2[P(0 \le x \le 1)]$
$= 2[0.3413]$ Using Table 2
$= 0.6826$

b) $0.6826 = 68.26\%$

**24.** a) $P(-2 \le x \le 2)$
$= P(-2 \le x \le 0) + P(0 \le x \le 2)$
$= P(0 \le x \le 2) + P(0 \le x \le 2)$
$= 2[P(0 \le x \le 2)]$
$= 2[0.4772]$
$= 0.9544$

b) $0.9544 = 95.44\%$

**25.** $P(24 \le x \le 30)$
Mean $\mu = 22$
Standard deviation $\sigma = 5$
First, we standardize the numbers 24 and 30.
30 is standardized to $\frac{b-\mu}{\sigma} = \frac{30-22}{5} = \frac{8}{5} = 1.6$
24 is standardized to $\frac{a-\mu}{\sigma} = \frac{24-22}{5} = \frac{2}{5} = 0.4$
Then,
$P(24 \le x \le 30)$
$= P(0.4 \le z \le 1.6)$
$= P(0 \le z \le 1.6) - P(0 \le z \le 0.4)$
$= 0.4452 - 0.1554$
$= 0.2898$

**26.** $P(22 \le x \le 27)$
We standardize 22 and 27.
27 is standardized to $\frac{b-\mu}{\sigma} = \frac{27-22}{5} = \frac{5}{5} = 1.0$
22 is standardized to $\frac{a-\mu}{\sigma} = \frac{22-22}{5} = \frac{0}{5} = 0$
Then,
$P(22 \le x \le 27)$
$= P(0 \le z \le 1.0)$
$= 0.3413$

**27.** $P(19 \le x \le 25)$
Mean $\mu = 22$
Standard deviation $\sigma = 5$
First, we standardize the numbers 19 and 25.
25 is standardized to $\frac{b-\mu}{\sigma} = \frac{25-22}{5} = \frac{3}{5} = 0.6$
19 is standardized to $\frac{a-\mu}{\sigma} = \frac{19-22}{5} = \frac{-3}{5} = -0.6$
Then,
$P(19 \le x \le 25)$
$= P(-0.6 \le z \le 0.6)$
$= P(-0.6 \le z \le 0) + P(0 \le z \le 0.6)$
$= P(0 \le z \le 0.6) + P(0 \le z \le 0.6)$
Symmetry of the graph
$= 2[P(0 \le z \le 0.6)]$
$= 2[0.2257]$
$= 0.4514$

**28.** $P(18 \le x \le 26)$

First, we standardize the numbers 18 and 26.

26: $\dfrac{b-\mu}{\sigma} = \dfrac{26-22}{5} = \dfrac{4}{5} = 0.8$

18: $\dfrac{a-\mu}{\sigma} = \dfrac{18-22}{5} = \dfrac{-4}{5} = -0.8$

Then,

$$\begin{aligned}
&P(18 \le x \le 26)\\
&= P(-0.8 \le z \le 0.8)\\
&= P(-0.8 \le z \le 0) + P(0 \le z \le 0.8)\\
&= P(0 \le z \le 0.8) + P(0 \le z \le 0.8)\\
&= 2\left[P(0 \le z \le 0.8)\right]\\
&= 2[0.2881]\\
&= 0.5762
\end{aligned}$$

**29 – 46.** Check answers using solutions to Exercises 11-28.

**47.** We first standardize 300 orders. The mean is 250 and the standard deviation is 20, so 300 is standardized to

$$\frac{b-\mu}{\sigma} = \frac{300-250}{20} = \frac{50}{20} = \frac{5}{2} = 2.5$$

Therefore,

$$\begin{aligned}
&P(x \ge 300)\\
&= P(z \ge 2.5)\\
&= 0.5 - P(z \le 2.5)\\
&= 0.5 - 0.4938 \qquad \text{Using Table 2}\\
&= 0.0062\\
&= 0.62\%
\end{aligned}$$

The company will have to hire extra help or pay overtime 0.62% of the days.

**48.** We first standardize 1100 loaves. The mean is 1000 and the standard deviation is 50, so 1100 is standardized to

$$\frac{b-\mu}{\sigma} = \frac{1100-1000}{50} = \frac{100}{50} = 2$$

Therefore,

$$\begin{aligned}
P(x \ge 1100) &= P(z \ge 2)\\
&= 0.5 - P(z \le 2)\\
&= 0.5 - 0.4772\\
&= 0.0228\\
&= 2.28\%
\end{aligned}$$

The company will have to pay bonuses 2.28% of the days.

**49.** We first standardize 40 seconds. The mean is 38.6 and the standard deviation is 1.729, so 40 is standardized to

$$\frac{b-\mu}{\sigma} = \frac{40-38.6}{1.729} \approx 0.81$$

Therefore,

$$\begin{aligned}
&P(x \le 40)\\
&= P(z \le 0.81)\\
&= 0.5 + P(0 \le z \le 0.81)\\
&= 0.5 + 0.2910 \qquad \text{Using Table 2}\\
&= 0.7910
\end{aligned}$$

The probability that the next operation of the robogate will take 40 seconds, or less, is 0.7910.

**50.** Standardize 35 seconds and 40 seconds.

$\mu = 36.2$

$\sigma = 2.108$

40 is standardized to $\dfrac{b-\mu}{\sigma} = \dfrac{40-36.2}{2.108} \approx 1.80$

35 is standardized to $\dfrac{a-\mu}{\sigma} = \dfrac{35-36.2}{2.108} \approx -0.57$

Therefore,

$$\begin{aligned}
&P(35 \le x \le 40)\\
&\approx P(-0.57 \le z \le 1.80)\\
&\approx P(-0.57 \le z \le 0) + P(0 \le z \le 1.80)\\
&\approx P(0 \le z \le 0.57) + P(0 \le z \le 1.80)\\
&\approx 0.2157 + 0.4641\\
&\approx 0.6798
\end{aligned}$$

The probability that the next operation will take between 35 and 40 seconds is 0.6798.

**51.** We first standardize the scores 80 and 89. The mean is 65 and the standard deviation is 20.

89 is standardized to

$$\frac{b-\mu}{\sigma} = \frac{89-65}{20} = \frac{24}{20} = 1.2$$

80 is standardized to

$$\frac{a-\mu}{\sigma} = \frac{80-65}{20} = \frac{15}{20} = 0.75$$

Therefore,

$$\begin{aligned}
&P(80 \le x \le 89)\\
&= P(0.75 \le z \le 1.2)\\
&= P(0 \le z \le 1.2) - P(0 \le z \le 0.75)\\
&= 0.3849 - 0.2734 \qquad \text{Using Table 2}\\
&= 0.1115
\end{aligned}$$

The probability of making a B on the biology exam is 0.1115.

**52.** a) $P(x \le z) = \dfrac{30}{100} = 0.3$

$0.3 = 0.5 - 0.2$ and $0.2$ corresponds to $z \approx 0.52$ in Table 2, so we have $z \approx -0.52$.

b) $P(x \le z) = \dfrac{50}{100} = 0.5$

$0.5 = 0.5 + 0$ and 0 corresponds to $z = 0$ in Table 2, so we have $z = 0$.

c) $P(x \le z) = \dfrac{95}{100} = 0.95$

$0.95 = 0.5 + 0.45$ and from Table 2 we see that $0.45$ corresponds to a value of $z$ halfway between 1.64 and 1.65, or 1.645. Thus, $z = 1.645$

**53.** a) Find $z$ such that $P(x \le z) = 0.35$

$0.35 = 0.5 - 0.15$ and 0.15 corresponds to $z \approx 0.39$ in Table 2. Then the score that corresponds to the 35th percentile is 0.39 standard deviations less than the mean, or $1020 - 0.39(140) \approx 965$.

b) Find $z$ such that $P(x \le z) = 0.60$

$0.60 = 0.5 + 0.1$ and 0.1 corresponds to $z \approx 0.25$ in Table 2. Then the score that corresponds to the 60th percentile is 0.25 standard deviations more than the mean, or $1020 + 0.25(140) \approx 1055$.

c) Find $z$ such that $P(x \le z) = 0.92$

$0.92 = 0.5 + 0.42$ and 0.42 corresponds to $z \approx 1.41$ in Table 2. Then the score that corresponds to the 92nd percentile is 1.41 standard deviations more than the mean, or $1020 + 1.41(140) \approx 1217$.

**54.** $\mu = 201; \sigma = 23$

a) Standardize 185 and 215.

$$215: \frac{b-\mu}{\sigma} = \frac{215-201}{23} = \frac{14}{23} \approx 0.61$$

$$185: \frac{a-\mu}{\sigma} = \frac{185-201}{23} = \frac{-16}{23} \approx -0.70$$

$$\begin{aligned} &P(185 \le x \le 215) \\ &= P(-0.70 \le z \le 0.61) \\ &= P(-0.70 \le z \le 0) + P(0 \le z \le 0.61) \\ &= P(0 \le z \le 0.70) + P(0 \le z \le 0.61) \\ &= 0.2580 + 0.2291 \\ &= 0.4871 \end{aligned}$$

The authors score will be between 185 and 215 about 48.71% of the time.

b) Standardize 160 and 175.

$$175: \frac{b-\mu}{\sigma} = \frac{175-201}{23} = \frac{-26}{23} \approx -1.13$$

$$160: \frac{a-\mu}{\sigma} = \frac{160-201}{23} = \frac{-41}{23} \approx -1.78$$

$$\begin{aligned} &P(160 \le x \le 175) \\ &= P(-1.78 \le z \le -1.13) \\ &= P(1.13 \le z \le 1.78) \\ &= P(0 \le z \le 1.78) - P(0 \le z \le 1.13) \\ &= 0.4625 - 0.3708 \\ &= 0.0917 \end{aligned}$$

The authors score will be between 160 and 175 about 9.17% of the time.

c) Standardize 200.

$$200: \frac{b-\mu}{\sigma} = \frac{200-201}{23} = \frac{-1}{23} \approx -0.04$$

$$\begin{aligned} &P(x > 200) \\ &= P(z > -0.04) \\ &= P(-0.04 < z \le 0) + P(0 \le z) \\ &= P(0 \le z < 0.04) + P(0 \le z) \\ &= 0.0160 + 0.5 \\ &= 0.5160 \end{aligned}$$

The authors score will be between greater than 200 about 51.6% of the time.

**55.** $f(x) = \dfrac{1}{b-a}, \quad [a,b]$

$E(x) = \int_a^b x \cdot f(x)\,dx \qquad$ Expected value of $x$.

$$\begin{aligned} E(x) &= \int_a^b x \cdot \frac{1}{b-a}\,dx \\ &= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b \\ &= \frac{1}{b-a}\left[\frac{b^2}{2} - \frac{a^2}{2}\right] \\ &= \frac{1}{b-a}\left[\frac{b^2-a^2}{2}\right] \\ &= \frac{1}{b-a}\cdot\frac{(b-a)(b+a)}{2} \\ &= \frac{b+a}{2} \end{aligned}$$

$$E\left(x^2\right)=\int_a^b x^2\cdot f(x)\,dx \qquad \text{Expected value of } x^2.$$

$$E\left(x^2\right)=\int_a^b x^2\cdot\frac{1}{b-a}\,dx$$

$$=\frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$=\frac{1}{b-a}\left[\frac{b^3}{3}-\frac{a^3}{3}\right]$$

$$=\frac{1}{b-a}\left[\frac{b^3-a^3}{3}\right]$$

$$=\frac{1}{b-a}\cdot\frac{(b-a)\left(b^2+ab+a^2\right)}{3}$$

$$=\frac{b^2+ab+a^2}{3}$$

$$\mu=E(x)=\frac{b+a}{2} \qquad \text{Mean}$$

$$\sigma^2=E\left(x^2\right)-\left[E(x)\right]^2$$

$$=\frac{b^2+ab+a^2}{3}-\left[\frac{b+a}{2}\right]^2 \qquad \text{Substituting}$$

$$=\frac{b^2+ab+a^2}{3}-\left[\frac{b^2+2ba+a^2}{4}\right]$$

$$=\frac{4b^2+4ab+4a^2}{12}-\frac{3b^2+6ba+3a^2}{12}$$

$$=\frac{4b^2+4ab+4a^2-3b^2-6ba-3a^2}{12}$$

$$=\frac{b^2-2ba+a^2}{12}$$

$$=\frac{(b-a)^2}{12} \qquad \text{Variance}$$

$$\sigma=\sqrt{\sigma^2}$$

$$=\sqrt{\frac{(b-a)^2}{12}}$$

$$=\frac{b-a}{2\sqrt{3}} \qquad \text{Standard deviation}$$

**56.** $f(x)=\dfrac{3a^3}{x^4},\quad [a,\infty]$

$$E(x)=\int_a^\infty x\cdot\frac{3a^3}{x^4}\,dx$$

$$=\lim_{b\to\infty}\int_a^b\frac{3a^3}{x^3}\,dx$$

$$=\lim_{b\to\infty}\int_a^b 3a^3x^{-3}\,dx$$

$$=\lim_{b\to\infty}3a^3\left[-\frac{1}{2x^2}\right]_a^b$$

$$=\lim_{b\to\infty}3a^3\left[-\frac{1}{2b^2}-\left(-\frac{1}{2a^2}\right)\right]$$

$$=3a^3\left[0+\frac{1}{2a^2}\right]$$

$$=\frac{3a^3}{2a^2}$$

$$=\frac{3a}{2}$$

$$E\left(x^2\right)=\int_a^\infty x^2\cdot\frac{3a^3}{x^4}\,dx$$

$$=\lim_{b\to\infty}\int_a^b\frac{3a^3}{x^2}\,dx$$

$$=\lim_{b\to\infty}\int_a^b 3a^3x^{-2}\,dx$$

$$=\lim_{b\to\infty}3a^3\left[-\frac{1}{x}\right]_a^b$$

$$=\lim_{b\to\infty}3a^3\left[-\frac{1}{b}-\left(-\frac{1}{a}\right)\right]$$

$$=3a^3\left[0+\frac{1}{a}\right]$$

$$=\frac{3a^3}{a}$$

$$=3a^2$$

$$\mu=E(x)=\frac{3a}{2} \qquad \text{Mean}$$

$$\sigma^2=E\left(x^2\right)-\left[E(x)\right]^2$$

$$=3a^2-\left[\frac{3a}{2}\right]^2$$

$$=3a^2-\frac{9a^2}{4}$$

$$=\frac{12a^2}{4}-\frac{9a^2}{4}$$

$$=\frac{3a^2}{4} \qquad \text{Variance}$$

$$\sigma = \sqrt{\sigma^2}$$
$$= \sqrt{\frac{3a^2}{4}} = \frac{a\sqrt{3}}{2} \quad \text{Standard deviation}$$

**57.** $f(x) = \frac{1}{2}x, \quad [0,2]$

$$\int_0^m f(x)\,dx = \frac{1}{2}$$
$$\int_0^m \frac{1}{2}x\,dx = \frac{1}{2}$$
$$\frac{1}{2}\int_0^m x\,dx = \frac{1}{2}$$
$$\int_0^m x\,dx = 1$$
$$\left[\frac{x^2}{2}\right]_0^m = 1$$
$$\frac{m^2}{2} - \frac{0^2}{2} = 1$$
$$\frac{m^2}{2} = 1$$
$$m^2 = 2$$
$$m = \sqrt{2}$$

**58.** $f(x) = \frac{3}{2}x^2, \quad [-1,1]$

$$\int_{-1}^m f(x)\,dx = \frac{1}{2}$$
$$\int_{-1}^m \frac{3}{2}x^2\,dx = \frac{1}{2}$$
$$\left[\frac{x^3}{2}\right]_{-1}^m = \frac{1}{2}$$
$$\frac{m^3}{2} - \frac{(-1)^3}{2} = \frac{1}{2}$$
$$\frac{m^3}{2} + \frac{1}{2} = \frac{1}{2}$$
$$\frac{m^3}{2} = 0$$
$$m = 0$$

**59.** $f(x) = ke^{-kx}, \quad [0,\infty)$

$$\int_0^m f(x)\,dx = \frac{1}{2}$$
$$\int_0^m ke^{-kx}\,dx = \frac{1}{2}$$
$$\left[\frac{k}{-k}e^{-kx}\right]_0^m = \frac{1}{2}$$
$$\left[-e^{-kx}\right]_0^m = \frac{1}{2}$$
$$-e^{-k(m)} - \left(-e^{-k(0)}\right) = \frac{1}{2}$$
$$-e^{-k\cdot m} + e^0 = \frac{1}{2}$$
$$-e^{-k\cdot m} + 1 = \frac{1}{2}$$
$$\frac{1}{2} = e^{-k\cdot m}$$
$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-k\cdot m}\right)$$
$$\ln(1) - \ln(2) = -k\cdot m \qquad [\ln 1 = 0]$$
$$\frac{-\ln 2}{-k} = m$$
$$\frac{\ln 2}{k} = m$$

**60.** Standardize 16.

$$z = \frac{16-\mu}{0.2}$$

We are looking for a value of $c$ for which $P(z < c) = \frac{1}{50} = 0.02$ . Now, if $c \geq 0$ , then

$$P(z<c) = P(z \leq 0) + P(0 \leq z < c)$$
$$= 0.5 + P(0 \leq z < c)$$

Then,

$$P(0 \leq z < c) = P(z < c) - 0.5$$
$$= 0.02 - 0.5$$
$$= -0.48$$

Which is not possible. Therefore, $c \leq 0$ , and

$$P(0 \leq z < c) = 0.5 - P(z < c)$$
$$= 0.5 - 0.02$$
$$= 0.48$$

Looking in Table 2 for the number closest to 0.48, we find that 0.4798 and 0.4803 are the two closest numbers. Since we want to ensure that only 1 bag in 50 will have less than 16oz we select the larger value 0.4803.

This value corresponds to $c = 2.06$, but in our case $c \le 0$, so we use $c = -2.06$. We set $z = -2.06$ and solve for $\mu$.

$$-2.06 = \frac{16-\mu}{0.2}$$
$$(-2.06)(0.2) = 16-\mu$$
$$-0.412 = 16-\mu$$
$$\mu - 0.412 = 16$$
$$\mu = 16+0.412$$
$$\mu = 16.412$$

The mean weight should be adjusted to 16.412 oz to ensure that only 1 bag in 50 will have less than 16 oz.

**61.** Standardize 8.5.

$$z = \frac{8.5-\mu}{0.3}$$

We are looking for a value of $c$ for which $P(z > c) = \frac{1}{100} = 0.01$. Now, since we are guarding against overflow we will have $c \ge 0$. We have:

$$P(z > c) = P(z \ge 0) - P(0 \le z \le c)$$
$$= 0.5 - P(0 \le z \le c)$$

Then,

$$P(0 \le z \le c) = 0.5 - P(z > c)$$
$$= 0.5 - 0.01$$
$$= 0.4900$$

Looking in Table 2 for the number closest to 0.49, we find that 0.4901 and 0.4898 are the two closest numbers. Since we want to ensure that only 1 cup in 100 will overflow we select the larger value 0.4901. This value corresponds to $c = 2.33$. We set $z = 2.33$ and solve for $\mu$.

$$2.33 = \frac{8.5-\mu}{0.3}$$
$$(2.33)(0.3) = 8.5-\mu$$
$$0.699 = 8.5-\mu$$
$$\mu + 0.699 = 8.5$$
$$\mu = 8.5 - 0.699$$
$$\mu = 7.801$$

The mean volume should be adjusted to 7.801 oz to ensure that only 1 cup in 100 will overflow.

**62.** $\boxed{tw}$ Answers will vary. The should include finding the expected value, the mean, the variance, and the standard deviation of a continuous random variable. The areas for a standard normal distribution are also defined in terms of an integral.

**63.** $\boxed{tw}$ Answers will vary. To show that a particular function $g(x)$ is not the antiderivative of $f(x)$, find $g'(x)$ and show that $g'(x) \ne f(x)$.

**64.** Using the fnInt feature on a calculator, we have $\int_{-\infty}^{\infty} e^{-x^2}\,dx \approx 1.772$.

## Exercise Set 5.6

**1.** Find the volume of the solid of revolution generated by rotating about the $x$-axis the region under the graph of

$$y = x$$

from $x = 0$ to $x = 1$

$$V = \int_a^b \pi\left[f(x)\right]^2 dx \quad \text{Volume of a solid of revolution}$$
$$V = \int_0^1 \pi[x]^2 dx \quad \text{Substituting 0 for } a\text{, 1 for } b\text{, and } x \text{ for } f(x).$$
$$V = \int_0^1 \pi x^2 dx$$
$$= \left[\pi \cdot \frac{x^3}{3}\right]_0^1$$
$$= \frac{\pi}{3}\left[1^3 - 0^3\right]$$
$$= \frac{\pi}{3}[1]$$
$$= \frac{\pi}{3}, \text{ or about } 1.05$$

**2.** $V = \int_a^b \pi\left[f(x)\right]^2 dx$

$V = \int_0^2 \pi[x]^2 dx$

$V = \int_0^2 \pi x^2 dx$

$= \left[\pi \cdot \frac{x^3}{3}\right]_0^2$

$= \frac{\pi}{3}\left[2^3 - 0^3\right]$

$= \frac{8\pi}{3}$, or about 8.38

**3.** Find the volume of the solid of revolution generated by rotating about the $x$-axis the region under the graph of

$y = \sqrt{x}$

from $x = 1$ to $x = 4$

$V = \int_a^b \pi\left[f(x)\right]^2 dx$ Volume of a solid of revolution

$V = \int_1^4 \pi\left[\sqrt{x}\right]^2 dx$ Substituting 1 for $a$, 4 for $b$, and $\sqrt{x}$ for $f(x)$.

$V = \int_1^4 \pi x\, dx$

$= \left[\pi \cdot \frac{x^2}{2}\right]_1^4$

$= \frac{\pi}{2}\left[4^2 - 1^2\right]$

$= \frac{\pi}{2}[15]$

$= \frac{15\pi}{2}$, or about 23.56

**4.** $V = \int_a^b \pi\left[f(x)\right]^2 dx$

$V = \int_1^3 \pi[2x]^2 dx$

$V = \int_1^3 \pi \cdot 4x^2 dx$

$= \left[4\pi \cdot \frac{x^3}{3}\right]_1^3$

$= \frac{4\pi}{3}\left[3^3 - 1^3\right]$

$= \frac{4\pi}{3} \cdot 26$

$= \frac{104\pi}{3}$, or about 108.91

**5.** Find the volume of the solid of revolution generated by rotating about the $x$-axis the region under the graph of

$y = e^x$

from $x = -2$ to $x = 5$

$V = \int_a^b \pi\left[f(x)\right]^2 dx$ Volume of a solid of revolution

$V = \int_{-2}^5 \pi\left[e^x\right]^2 dx$ Substituting $-2$ for $a$, 5 for $b$, and $e^x$ for $f(x)$.

$V = \int_{-2}^5 \pi e^{2x} dx$

$= \left[\pi \cdot \frac{1}{2} e^{2x}\right]_{-2}^5$

$= \frac{\pi}{2}\left[e^{2(5)} - e^{2(-2)}\right]$

$= \frac{\pi}{2}\left[e^{10} - e^{-4}\right]$, or about 34,599.06

**6.** $V = \int_a^b \pi\left[f(x)\right]^2 dx$

$V = \int_{-3}^2 \pi\left[e^x\right]^2 dx$

$V = \int_{-3}^2 \pi \cdot e^{2x} dx$

$= \left[\pi \cdot \frac{1}{2} e^{2x}\right]_{-3}^2$

$= \frac{\pi}{2}\left[e^{2(2)} - e^{2(-3)}\right]$

$= \frac{\pi}{2}\left[e^4 - e^{-6}\right]$, or about 85.76

**7.** Find the volume of the solid of revolution generated by rotating about the $x$-axis the region under the graph of

$$y=\frac{1}{x}$$

from $x=1$ to $x=3$

$V=\int_a^b \pi\left[f(x)\right]^2 dx$ Volume of a solid of revolution

$V=\int_1^3 \pi\left[\frac{1}{x}\right]^2 dx$ Substituting 1 for $a$, 3 for $b$, and $1/x$ for $f(x)$.

$$V=\int_1^3 \pi\cdot\frac{1}{x^2}dx$$

$$V=\int_1^3 \pi x^{-2}dx$$

$$=\left[\pi\cdot\frac{x^{-1}}{-1}\right]_1^3$$

$$=-\pi\left[\frac{1}{x}\right]_1^3$$

$$=-\pi\left[\frac{1}{3}-\frac{1}{1}\right]$$

$$=-\pi\left[-\frac{2}{3}\right]$$

$=\frac{2\pi}{3}$, or about 2.09

**8.** $V=\int_a^b \pi\left[f(x)\right]^2 dx$

$$V=\int_1^4 \pi\left[\frac{1}{x}\right]^2 dx$$

$$V=\int_1^4 \pi\cdot\frac{1}{x^2}dx$$

$$=\left[\pi\cdot\frac{x^{-1}}{-1}\right]_1^4$$

$$=-\pi\left[\frac{1}{x}\right]_1^4$$

$$=-\pi\left[\frac{1}{4}-\frac{1}{1}\right]$$

$$=-\pi\left[-\frac{3}{4}\right]$$

$=\frac{3\pi}{4}$, or about 2.36

**9.** Find the volume of the solid of revolution generated by rotating about the $x$-axis the region under the graph of

$$y=\frac{2}{\sqrt{x}}$$

from $x=4$ to $x=9$

$V=\int_a^b \pi\left[f(x)\right]^2 dx$ Volume of a solid of revolution

$V=\int_4^9 \pi\left[\frac{2}{\sqrt{x}}\right]^2 dx$ Substituting 4 for $a$, 9 for $b$, and $2/\sqrt{x}$ for $f(x)$.

$$V=\int_4^9 \pi\cdot\frac{4}{x}dx$$

$$V=4\pi\int_4^9 \frac{1}{x}dx$$

$$=4\pi\left[\ln x\right]_4^9$$

$$=4\pi\left[\ln 9-\ln 4\right]$$

$=4\pi\ln\left(\frac{9}{4}\right)$, or about 10.19

**10.** $V=\int_a^b \pi\left[f(x)\right]^2 dx$

$$V=\int_1^4 \pi\left[\frac{1}{\sqrt{x}}\right]^2 dx$$

$$V=\int_1^4 \pi\cdot\frac{1}{x}dx$$

$$V=\pi\int_1^4 \frac{1}{x}dx$$

$$=\pi\left[\ln x\right]_1^4$$

$$=\pi\left[\ln 4-\ln 1\right]$$

$=\pi\ln 4$, or about 4.36

**11.** Find the volume of the solid of revolution generated by rotating about the $x$-axis the region under the graph of

$$y=4$$

from $x=1$ to $x=3$

$V=\int_a^b \pi\left[f(x)\right]^2 dx$ Volume of a solid of revolution

$V=\int_1^3 \pi\left[4\right]^2 dx$ Substituting 1 for $a$, 3 for $b$, and 4 for $f(x)$.

$$V=\int_1^3 16\pi dx$$

$$=16\pi\left[x\right]_1^3$$

$$=16\pi\left[3-1\right]$$

$$=16\pi\left[2\right]$$

$=32\pi$, or about 100.53

**12.** $V=\int_a^b \pi\left[f(x)\right]^2 dx$

$V=\int_1^3 \pi[5]^2\,dx$

$V=\int_1^3 25\pi dx$

$=25\pi[x]_1^3$

$=25\pi[3-1]$

$=50\pi$, or about 157.08

**13.** Find the volume of the solid of revolution generated by rotating about the $x$-axis the region under the graph of

$y=x^2$

from $x=0$ to $x=2$

$V=\int_a^b \pi\left[f(x)\right]^2 dx$ Volume of a solid of revolution

$V=\int_0^2 \pi\left[x^2\right]^2 dx$ Substituting 0 for $a$, 2 for $b$, and $x^2$ for $f(x)$.

$V=\int_0^2 \pi x^4 dx$

$=\left[\pi\cdot\frac{x^5}{5}\right]_0^2$

$=\frac{\pi}{5}\left[2^5-0^5\right]$

$=\frac{\pi}{5}[32]$

$=\frac{32\pi}{5}$, or about 20.11

**14.** $V=\int_a^b \pi\left[f(x)\right]^2 dx$

$V=\int_{-1}^2 \pi[x+1]^2\,dx$

$=\left[\frac{\pi(x+1)^3}{3}\right]_{-1}^2$

$=\frac{\pi}{3}\left[(x+1)^3\right]_{-1}^2$

$=\frac{\pi}{3}\left[(2+1)^3-(-1+1)^3\right]$

$=\frac{\pi}{3}[27]$

$=9\pi$, or about 28.27

**15.** Find the volume of the solid of revolution generated by rotating about the $x$-axis the region under the graph of

$y=\sqrt{1+x}$

from $x=2$ to $x=10$

$V=\int_a^b \pi\left[f(x)\right]^2 dx$ Volume of a solid of revolution

$V=\int_2^{10} \pi\left[\sqrt{1+x}\right]^2 dx$ Substituting 2 for $a$, 10 for $b$, and $\sqrt{1+x}$ for $f(x)$.

$V=\int_2^{10} \pi(1+x)dx$

$=\pi\left[\frac{(1+x)^2}{2}\right]_2^{10}$

$=\frac{\pi}{2}\left[(1+10)^2-(1+2)^2\right]$

$=\frac{\pi}{2}\left[(11)^2-(3)^2\right]$

$=\frac{\pi}{2}[121-9]$

$=\frac{\pi}{2}\cdot 112$

$=56\pi$, or about 175.93

**16.** $V=\int_a^b \pi\left[f(x)\right]^2 dx$

$V=\int_1^2 \pi\left[2\sqrt{x}\right]^2 dx$

$V=\int_1^2 4\pi x\,dx$

$=\left[4\pi\cdot\frac{x^2}{2}\right]_1^2$

$=2\pi\left[2^2-1^2\right]$

$=2\pi[3]$

$=6\pi$, or about 18.85

**17.** Find the volume of the solid of revolution generated by rotating about the $x$-axis the region under the graph of

$$y=\sqrt{4-x^2}$$

from $x=-2$ to $x=2$

$$V=\int_a^b \pi\left[f(x)\right]^2 dx \quad \text{Volume of a solid of revolution}$$

$$V=\int_{-2}^{2} \pi\left[\sqrt{4-x^2}\right]^2 dx \quad \text{Substituting } -2 \text{ for } a,\ 2 \text{ for } b, \text{ and } \sqrt{4-x^2} \text{ for } f(x).$$

$$V=\int_{-2}^{2} \pi\left(4-x^2\right)dx$$

$$=\pi\left[4x-\frac{x^3}{3}\right]_{-2}^{2}$$

$$=\pi\left[\left(4(2)-\frac{(2)^3}{3}\right)-\left(4(-2)-\frac{(-2)^3}{3}\right)\right]$$

$$=\pi\left[\left(8-\frac{8}{3}\right)-\left(-8+\frac{8}{3}\right)\right]$$

$$=\pi\left[\frac{16}{3}-\left(-\frac{16}{3}\right)\right]$$

$$=\pi\left(\frac{32}{3}\right)$$

$$=\frac{32\pi}{3}, \text{ or about } 33.51$$

**18.** Find the volume of the solid of revolution generated by rotating about the $x$-axis the region under the graph of

$$y=\sqrt{r^2-x^2}$$

from $x=-r$ to $x=r$

$$V=\int_a^b \pi\left[f(x)\right]^2 dx \quad \text{Volume of a solid of revolution}$$

$$V=\int_{-r}^{r} \pi\left[\sqrt{r^2-x^2}\right]^2 dx \quad \text{Substituting } -r \text{ for } a,\ \text{r for } b, \text{ and } \sqrt{r^2-x^2} \text{ for } f(x).$$

$$V=\int_{-r}^{r} \pi\left(r^2-x^2\right)dx$$

$$=\pi\left[r^2x-\frac{x^3}{3}\right]_{-r}^{r}$$

$$=\pi\left[\left(r^2(r)-\frac{(r)^3}{3}\right)-\left(r^2(-r)-\frac{(-r)^3}{3}\right)\right]$$

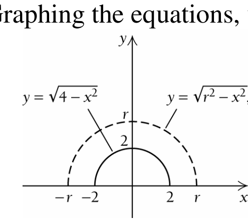
$$=\pi\left[\left(r^3-\frac{r^3}{3}\right)-\left(-r^3+\frac{r^3}{3}\right)\right]$$

$$=\pi\left[\frac{2}{3}r^3-\left(-\frac{2}{3}r^3\right)\right]$$

$$=\pi\left(\frac{4}{3}r^3\right)$$

$$=\frac{4}{3}\pi r^3$$

**19.** Graphing the equations, we have




The graphs are semicircles. Their rotation about the $x$-axis creates spheres of radius 2 and radius $r$ respectively. In Exercise 18, by finding the volume of the solid of revolution created by rotating $y=\sqrt{r^2-x^2}$ , we actually derived the general formula for finding the volume of a sphere with radius $r$.

**20.** $V = \int_a^b \pi\left[f(x)\right]^2 dx$

$V = \int_e^{e^3} \pi\left[\sqrt{\ln x}\right]^2 dx$

$= \int_e^{e^3} \pi \cdot \ln x dx$

$= \pi\left[x\ln x - x\right]_e^{e^3}$ Using Formula 8

$= \pi\left[\left(e^3 \ln e^3 - e^3\right) - \left(e\ln e - e\right)\right]$

$= \pi\left[\left(3e^3 - e^3\right) - \left(e - e\right)\right]$

$= \pi\left[2e^3\right]$

$= 2\pi e^3$, or about 126.20

**21.** Find the volume of the solid of revolution generated by rotating about the *x*-axis the region under the graph of

$y = \sqrt{xe^{-x}}$

from $x = 1$ to $x = 2$

$V = \int_a^b \pi\left[f(x)\right]^2 dx$ Volume of a solid of revolution

$V = \int_1^2 \pi\left[\sqrt{xe^{-x}}\right]^2 dx$ Substituting 1 for $a$, 2 for $b$, and $\sqrt{xe^{-x}}$ for $f(x)$.

$V = \int_1^2 \pi xe^{-x} dx$

$= \pi\left[\frac{1}{(-1)^2} e^{-x}\left(-x-1\right)\right]_1^2$ Using formula 6

$== \pi\left[\frac{-x-1}{e^x}\right]_1^2$

$= \pi\left[\left(\frac{-2-1}{e^2}\right) - \left(\frac{-1-1}{e^1}\right)\right]$

$= \pi\left[\left(\frac{-3}{e^2}\right) - \left(\frac{-2}{e}\right)\right]$

$= \pi\left[\frac{-3}{e^2} + \frac{2}{e}\right]$

$= \pi\left(\frac{-3+2e}{e^2}\right)$, or about 1.04

**22.** $V = \int_1^\infty \pi\left[\frac{1}{x}\right]^2 dx$

$V = \int_1^\infty \pi \frac{1}{x^2} dx$

$= \int_1^\infty \pi x^{-2} dx$

$= \lim_{b\to\infty} \int_1^b \pi x^{-2} dx$

$= \lim_{b\to\infty}\left[\pi \frac{x^{-1}}{-1}\right]_1^b$

$= \lim_{b\to\infty}\left[-\frac{\pi}{x}\right]_1^b$

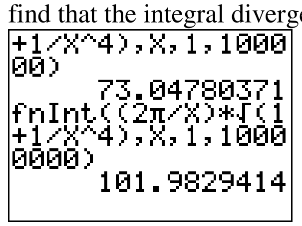
$= \lim_{b\to\infty}\left[-\frac{\pi}{b} - \left(-\frac{\pi}{1}\right)\right]$

$= \lim_{b\to\infty}\left[-\frac{\pi}{b} + \frac{\pi}{1}\right]$

$= \left[0 + \pi\right]$

$= \pi$

**23.** Using the fnInt feature on a calculator with successively larger values of the upper limit, we find that the integral diverges.




Therefore, the surface area of Gabriel's horn does not exist.

## Exercise Set 5.7

**1.** Solve: $y' = 5x^4$

$$y = \int 5x^4 dx$$
$$= 5\left(\frac{x^5}{5}\right) + C$$
$$= x^5 + C$$

This solution is called a general solution because taking all values of *C* gives all the solutions. Taking specific values of *C* gives particular solutions. The following are particular solutions of $y' = 5x^4$. Answers may vary depending on the choice of *C*.

$y = x^5$

$y = x^5 - 1$

$y = x^5 + \pi$

**2.** Solve: $y' = 6x^5$

$$y = \int 6x^5 dx$$
$$= 6\left(\frac{x^6}{6}\right) + C$$
$$= x^6 + C \qquad \text{General Solution}$$

Three particular solutions follow. Answers may vary.

$y = x^6$

$y = x^6 - 1$

$y = x^6 + \pi$

**3.** Solve: $y' = e^{2x} + x$

$$y = \int \left(e^{2x} + x\right) dx$$
$$= \frac{1}{2}e^{2x} + \frac{1}{2}x^2 + C$$

This solution is called a general solution because taking all values of *C* gives all the solutions. Taking specific values of *C* gives particular solutions. The following are particular solutions of $y' = e^{2x} + x$. Answers may vary depending on the choice of *C*.

$y = \frac{1}{2}e^{2x} + \frac{1}{2}x^2$

$y = \frac{1}{2}e^{2x} + \frac{1}{2}x^2 - 3$

$y = \frac{1}{2}e^{2x} + \frac{1}{2}x^2 + 3$

**4.** Solve: $y' = e^{4x} - x + 2$

$$y = \int \left(e^{4x} - x + 2\right) dx$$
$$= \frac{1}{4}e^{4x} - \frac{1}{2}x^2 + 2x + C \qquad \text{General Solution}$$

Three particular solutions follow. Answers may vary.

$y = \frac{1}{4}e^{4x} - \frac{1}{2}x^2 + 2x$

$y = \frac{1}{4}e^{4x} - \frac{1}{2}x^2 + 2x - 4$

$y = \frac{1}{4}e^{4x} - \frac{1}{2}x^2 + 2x + 57.2$

**5.** Solve: $y' = \frac{8}{x} - x^2 + x^5$

$$y = \int \left(\frac{8}{x} - x^2 + x^5\right) dx$$
$$= 8\ln x - \frac{1}{3}x^3 + \frac{1}{6}x^6 + C$$

This solution is called a general solution because taking all values of *C* gives all the solutions. Taking specific values of *C* gives particular solutions. The following are particular solutions of $y' = \frac{8}{x} - x^2 + x^5$. Answers may vary depending on the choice of *C*.

$y = 8\ln x - \frac{1}{3}x^3 + \frac{1}{6}x^6$

$y = 8\ln x - \frac{1}{3}x^3 + \frac{1}{6}x^6 + 5$

$y = 8\ln x - \frac{1}{3}x^3 + \frac{1}{6}x^6 - 17$

**6.** Solve: $y' = \frac{3}{x} + x^2 - x^4$

$$y = \int \left(\frac{3}{x} + x^2 - x^4\right) dx$$
$$= 3\ln x + \frac{1}{3}x^3 - \frac{1}{5}x^5 + C$$

Three particular solutions follow. Answers may vary.

$y = 3\ln x + \frac{1}{3}x^3 - \frac{1}{5}x^5$

$y = 3\ln x + \frac{1}{3}x^3 - \frac{1}{5}x^5 - 7$

$y = 3\ln x + \frac{1}{3}x^3 - \frac{1}{5}x^5 + \frac{5}{3}$

**7.** Solve $y' = x^2 + 2x - 3$, given that $y = 4$ when $x = 0$.

First, find the general solution.

$$y = \int y'\,dx$$
$$= \int \left(x^2 + 2x - 3\right) dx$$
$$y = \frac{1}{3}x^3 + x^2 - 3x + C$$

We substitute $y = 4$ and $x = 0$ into the general solution to find *C*.

$$4 = \frac{1}{3}(0)^3 + (0)^2 - 3(0) + C$$
$$4 = C$$

Thus the particular solution determined by the given condition is

$$y = \frac{1}{3}x^3 + x^2 - 3x + 4.$$

**8.** Solve $y' = 3x^2 - x + 5$, given that $y = 6$ when $x = 0$.

First, find the general solution.

$$y = \int y'\,dx$$
$$= \int \left(3x^2 - x + 5\right) dx$$
$$y = x^3 - \frac{1}{2}x^2 + 5x + C$$

Substitute to find *C*.

$$6 = (0)^3 - \frac{1}{2}(0)^2 + 5(0) + C$$
$$6 = C$$

Thus the particular solution determined by the given condition is

$$y = x^3 - \frac{1}{2}x^2 + 5x + 6.$$

**9.** Solve $f'(x) = x^{2/3} - x$, given $f(1) = -6$.

First, find the general solution.

$$f(x) = \int f'(x)\,dx$$
$$= \int \left(x^{2/3} - x\right) dx$$
$$f(x) = \frac{3}{5}x^{5/3} - \frac{1}{2}x^2 + C$$

We substitute $-6$ for $f(x)$ and 1 for *x* into the general solution to find *C*.

$$-6 = \frac{3}{5}(1)^{5/3} - \frac{1}{2}(1)^2 + C$$
$$-6 = \frac{3}{5} - \frac{1}{2} + C$$
$$-6 - \frac{3}{5} + \frac{1}{2} = C$$
$$\frac{-60}{10} - \frac{6}{10} + \frac{5}{10} = C$$
$$-\frac{61}{10} = C$$

Thus the particular solution determined by the given condition is

$$f(x) = \frac{3}{5}x^{5/3} - \frac{1}{2}x^2 - \frac{61}{10}.$$

**10.** Solve $f'(x) = x^{2/5} + x$, given $f(1) = -7$.

First, find the general solution.

$$f(x) = \int f'(x)\,dx$$
$$= \int \left(x^{2/5} + x\right) dx$$
$$f(x) = \frac{5}{7}x^{7/5} + \frac{1}{2}x^2 + C$$

We substitute $-7$ for $f(x)$ and 1 for *x* into the general solution to find *C*.

$$-7 = \frac{5}{7}(1)^{7/5} + \frac{1}{2}(1)^2 + C$$
$$-7 = \frac{5}{7} + \frac{1}{2} + C$$
$$-\frac{115}{14} = C$$

Thus the particular solution determined by the given condition is

$$f(x) = \frac{5}{7}x^{7/5} + \frac{1}{2}x^2 - \frac{115}{14}.$$

**11.** Show that $y = x\ln x + 3x - 2$ is a solution to $y'' - \frac{1}{x} = 0$.

First find $y'$ and $y''$.

$$y = x\ln x + 3x - 2$$
$$y' = x \cdot \frac{1}{x} + 1 \cdot \ln x + 3$$
$$= \ln x + 4$$
$$y'' = \frac{1}{x}$$

Then we substitute into the differential equation, as follows

$$\begin{array}{c|c} y''-\frac{1}{x}=0 & \\ \hline \frac{1}{x}-\frac{1}{x} & 0 \\ 0 & 0 \end{array}$$

Since a true equation $0=0$ results, we know that $y = x\ln x + 3x - 2$ is a solution of the differential equation.

**12.** Show that $y = x\ln x - 5x + 7$ is a solution to $y''-\frac{1}{x}=0$.

First find $y'$ and $y''$.

$$y = x\ln x - 5x + 7$$
$$y' = x\cdot\frac{1}{x}+1\cdot\ln x - 5$$
$$= \ln x - 4$$
$$y'' = \frac{1}{x}$$

Then we substitute into the differential equation, as follows

$$\begin{array}{c|c} y''-\frac{1}{x}=0 & \\ \hline \frac{1}{x}-\frac{1}{x} & 0 \\ 0 & 0 \end{array}$$

Since a true equation $0=0$ results, we know that $y = x\ln x - 5x + 7$ is a solution of the differential equation.

**13.** Show that $y = e^x + 3xe^x$ is a solution to $y''-2y'+y=0$.

First find $y'$ and $y''$.

$$y = e^x + 3xe^x$$
$$y' = e^x + 3\left(xe^x + e^x\right)$$
$$= e^x + 3xe^x + 3e^x$$
$$= 4e^x + 3xe^x$$
$$y'' = 4e^x + 3\left(xe^x + e^x\right)$$
$$= 4e^x + 3xe^x + 3e^x$$
$$= 7e^x + 3xe^x$$

Then we substitute into the differential equation, as follows

$$\begin{array}{r|c} y''-2y'+y=0 & \\ \hline \left(7e^x+3xe^x\right)-2\left(4e^x+3xe^x\right)+\left(e^x+3xe^x\right) & 0 \\ 7e^x+3xe^x-8e^x-6xe^x+e^x+3xe^x & 0 \\ (7-8+1)e^x+(3-6+3)xe^x & 0 \\ 0+0 & 0 \\ 0 & 0 \end{array}$$

Since a true equation $0=0$ results, we know that $y = e^x + 3xe^x$ is a solution of the differential equation.

**14.** Show that $y = -2e^x + xe^x$ is a solution to $y''-2y'+y=0$.

First find $y'$ and $y''$.

$$y = -2e^x + xe^x$$
$$y' = -2e^x + xe^x + e^x$$
$$= -e^x + xe^x$$
$$y'' = -e^x + xe^x + e^x$$
$$= xe^x$$

Then we substitute into the differential equation, as follows

$$\begin{array}{r|c} y''-2y'+y=0 & \\ \hline \left(xe^x\right)-2\left(-e^x+xe^x\right)+\left(-2e^x+xe^x\right) & 0 \\ xe^x+2e^x-2xe^x-2e^x+xe^x & 0 \\ (2-2)e^x+(1-2+1)xe^x & 0 \\ 0+0 & 0 \\ 0 & 0 \end{array}$$

Since a true equation $0=0$ results, we know that $y = -2e^x + xe^x$ is a solution of the differential equation.

**15.** Solve $\frac{dy}{dx} = 4x^3y$

First, we separate the variables, as follows:

$dy = 4x^3ydx$ Multiplying by $dx$.

$\frac{dy}{y} = 4x^3dx$ Dividing by $y$.

We then integrate both sides:

$\int \frac{dy}{y} = \int 4x^3 dx$

$\ln|y| = x^4 + C$

$|y| = e^{x^4+C}$ Definition of logarithms.

$y = \pm e^{x^4+C}$

$y = \pm e^{x^4} \cdot e^C$ Properties of exponents.

$y = C_1 e^{x^4}$ $[C_1 = \pm e^C]$

Thus, the general solution to the differential equation is $y = C_1 e^{x^4}$, where $C_1 = \pm e^C$.

**16.** Solve $\frac{dy}{dx} = 5x^4 y$

First, we separate the variables, as follows

$\frac{dy}{y} = 5x^4 dx.$

We then integrate both sides:

$\int \frac{dy}{y} = \int 5x^4 dx$

$\ln|y| = x^5 + C$

$|y| = e^{x^5+C}$

$|y| = e^{x^5} \cdot e^C$

$y = \pm e^{x^5} \cdot e^C$

$y = C_1 e^{x^5}$ $[C_1 = \pm e^C]$

Thus, the general solution to the differential equation is $y = C_1 e^{x^5}$, where $C_1 = \pm e^C$.

**17.** Solve $3y^2 \frac{dy}{dx} = 8x$

First, we separate the variables, as follows:

$3y^2 dy = 8x dx$ Multiplying by $dx$.

We then integrate both sides:

$\int 3y^2 dy = \int 8x dx$

$y^3 = 4x^2 + C$

$y = \sqrt[3]{4x^2 + C}$ Taking the cubed root of both sides.

Thus, the general solution to the differential equation is $y = \sqrt[3]{4x^2 + C}$.

**18.** Solve $3y^2 \frac{dy}{dx} = 5x$

$3y^2 dy = 5x dx$

$\int 3y^2 dy = \int 5x dx$

$y^3 = \frac{5}{2}x^2 + C$

$y = \sqrt[3]{\frac{5}{2}x^2 + C}$

Thus, the general solution to the differential equation is $y = \sqrt[3]{\frac{5}{2}x^2 + C}$.

**19.** Solve $\frac{dy}{dx} = \frac{2x}{y}$

First, we separate the variables, as follows:

$dy = \frac{2x}{y} dx$ Multiplying by $dx$.

$y dy = 2x dx$ Multiplying by $y$.

We then integrate both sides:

$\int y dy = \int 2x dx$

$\frac{1}{2} y^2 = x^2 + C$

$y^2 = 2x^2 + 2C$

$y = \pm\sqrt{2x^2 + C_1}$, $[C_1 = 2C]$

Thus, the general solutions to the differential equation are

$y = \sqrt{2x^2 + C_1}$ and $y = -\sqrt{2x^2 + C_1}$, where $C_1 = 2C$.

**20.** Solve $\frac{dy}{dx} = \frac{x}{2y}$

$2y dy = x dx$

$\int 2y dy = \int x dx$

$y^2 = \frac{1}{2}x^2 + C$

$y^2 = \frac{1}{2}x^2 + C$

$y = \pm\sqrt{\frac{1}{2}x^2 + C}$

Thus, the general solutions to the differential equation are

$y = \sqrt{\frac{1}{2}x^2 + C}$ and $y = -\sqrt{\frac{1}{2}x^2 + C}$.

**21.** Solve $\frac{dy}{dx}=\frac{6}{y}$

First, we separate the variables, as follows:

$dy=\frac{6}{y}dx$ Multiplying by $dx$.

$ydy=6dx$ Multiplying by $y$.

We then integrate both sides:

$$\int ydy=\int 6dx$$
$$\frac{1}{2}y^2=6x+C$$
$$y^2=12x+2C$$
$$y=\pm\sqrt{12x+C_1},\qquad [C_1=2C]$$

Thus, the general solutions to the differential equation are

$y=\sqrt{12x+C_1}$ and $y=-\sqrt{12x+C_1}$, where $C_1=2C$.

**22.** Solve $\frac{dy}{dx}=\frac{7}{y^2}$

$$y^2dy=7dx$$
$$\int y^2dy=\int 7dx$$
$$\frac{1}{3}y^3=7x+C$$
$$y^3=21x+3C$$
$$y^3=21x+C_1\qquad [C_1=3C]$$
$$y=\sqrt[3]{21x+C_1}$$

Thus, the general solution to the differential equation is $y=\sqrt[3]{21x+C_1}$, where $C_1=3C$.

**23.** Solve $y'=3x+xy$; $y=5$, when $x=0$.

First, we separate the variables, as follows:

$\frac{dy}{dx}=3x+xy$ Replacing $y'$ with $\frac{dy}{dx}$.

$\frac{dy}{dx}=x(3+y)$ Factoring.

$dy=x(3+y)dx$ Multiplying by $dx$.

$\frac{dy}{3+y}=xdx$ Dividing by $(3+y)$.

We then integrate both sides:

$$\int\frac{dy}{3+y}=\int xdx$$
$$\ln(3+y)=\frac{1}{2}x^2+C\qquad [3+y>0]$$
$$3+y=e^{\frac{x^2}{2}+C}$$
$$3+y=e^{\frac{x^2}{2}}\cdot e^C$$
$$y=e^{\frac{x^2}{2}}\cdot e^C-3$$
$$y=C_1e^{\frac{x^2}{2}}-3\qquad [C_1=e^C]$$

Thus, the general solution to the differential equation is $y=C_1e^{\frac{x^2}{2}}-3$ where $C_1=e^C$.

We substitute 0 for $x$ and 5 for $y$ to find $C_1$.

$$5=C_1e^{\frac{0^2}{2}}-3$$
$$8=C_1e^0$$
$$8=C_1\cdot 1$$
$$8=C_1$$

Therefore, the particular solution is

$$y=8e^{\frac{x^2}{2}}-3.$$

**24.** Solve $y'=2x-xy$; $y=9$, when $x=0$.

First, we separate the variables, as follows:

$$\frac{dy}{dx}=2x-xy$$
$$\frac{dy}{dx}=x(2-y)$$
$$\frac{dy}{2-y}=xdx\ .$$

We then integrate both sides:

$$\int \frac{dy}{2-y} = \int x\,dx$$

$$-\ln|2-y| = \frac{1}{2}x^2 + C$$

$$\ln|2-y| = -\frac{1}{2}x^2 - C$$

$$|2-y| = e^{-\frac{x^2}{2}-C}$$

$$2-y = \pm e^{-\frac{x^2}{2}} \cdot e^{-C}$$

$$-y = \pm e^{-\frac{x^2}{2}} \cdot e^{-C} - 2$$

$$-y = C_1 e^{-\frac{x^2}{2}} - 2 \qquad \left[C_1 = \pm e^{-C}\right]$$

$$y = 2 - C_1 e^{-\frac{x^2}{2}}$$

$$y = 2 + C_2 e^{-\frac{x^2}{2}} \qquad \left[C_2 = -C_1\right]$$

Thus, the general solution to the differential equation is $y = C_2 e^{-\frac{x^2}{2}} + 2$ where $C_2 = \pm e^{-C}$.

We substitute 0 for $x$ and 9 for $y$ to find $C_2$.

$$9 = C_2 e^{-\frac{0^2}{2}} + 2$$

$$7 = C_2 e^0$$

$$7 = C_2$$

Therefore, the particular solution is

$$y = 7e^{-\frac{x^2}{2}} + 2.$$

**25.** Solve $y' = 5y^{-2}$; $y = 3$, when $x = 2$.

First, we separate the variables, as follows:

$$\frac{dy}{dx} = 5y^{-2} \qquad \text{Replacing } y' \text{ with } \frac{dy}{dx}.$$

$$dy = 5y^{-2}dx \qquad \text{Multiplying by } dx.$$

$$y^2 dy = 5dx \qquad \text{Multiplying by } y^2.$$

We then integrate both sides:

$$\int y^2 dy = \int 5dx$$

$$\frac{1}{3}y^3 = 5x + C$$

$$y^3 = 15x + 3C$$

$$y = \sqrt[3]{15x + C_1} \qquad \left[C_1 = 3C\right]$$

Thus, the general solution to the differential equation is $y = \sqrt[3]{15x + C_1}$ where $C_1 = 3C$.

We substitute 2 for $x$ and 3 for $y$ to find $C_1$.

$$3 = \sqrt[3]{15(2) + C_1}$$

$$3 = \sqrt[3]{30 + C_1}$$

$$27 = 30 + C_1$$

$$-3 = C_1$$

Therefore, the particular solution is

$$y = \sqrt[3]{15x - 3}.$$

**26.** Solve $y' = 7y^{-2}$; $y = 3$, when $x = 1$.

$$\frac{dy}{dx} = 7y^{-2}$$

$$y^2 dy = 7dx$$

We then integrate both sides:

$$\int y^2 dy = \int 7dx$$

$$\frac{1}{3}y^3 = 7x + C$$

$$y^3 = 21x + 3C$$

$$y = \sqrt[3]{21x + C_1} \qquad \left[C_1 = 3C\right]$$

We substitute 1 for $x$ and 3 for $y$ to find $C_1$.

$$3 = \sqrt[3]{21(1) + C_1}$$

$$27 = 21 + C_1$$

$$6 = C_1$$

Therefore, the particular solution is

$$y = \sqrt[3]{21x + 6}.$$

**27.** Solve $\frac{dy}{dx} = 3y$

First, we separate the variables, as follows:

$$dy = 3y\,dx \qquad \text{Multiplying by } dx.$$

$$\frac{dy}{y} = 3dx \qquad \text{Dividing by } y.$$

We then integrate both sides:

$$\int \frac{dy}{y} = \int 3dx$$

$$\ln|y| = 3x + C$$

$$|y| = e^{3x+C}$$

$$y = \pm e^{3x+C}$$

$$y = \pm e^{3x} \cdot e^C$$

$$y = C_1 e^{3x} \qquad \left[C_1 = \pm e^C\right]$$

Thus, the general solution to the differential equation is $y = C_1 e^{3x}$, where $C_1 = \pm e^C$.

**28.** Solve $\frac{dy}{dx}=4y$

$$\frac{dy}{y}=4dx$$
$$\int\frac{dy}{y}=\int 4dx$$
$$\ln|y|=4x+C$$
$$|y|=e^{4x+C}$$
$$y=\pm e^{4x+C}$$
$$y=\pm e^{4x}\cdot e^{C}$$
$$y=C_1e^{4x}\qquad\left[C_1=\pm e^{C}\right]$$

Thus, the general solution to the differential equation is $y=C_1e^{4x}$, where $C_1=\pm e^{C}$.

**29.** Solve $\frac{dP}{dt}=2P$

First, we separate the variables, as follows:

$dP=2Pdt$ Multiplying by $dt$.

$\frac{dP}{P}=2dt$ Dividing by $P$.

We then integrate both sides:

$$\int\frac{dP}{P}=\int 2dt$$
$$\ln|P|=2t+C$$
$$|P|=e^{2t+C}$$
$$P=\pm e^{2t+C}$$
$$P=\pm e^{2t}\cdot e^{C}$$
$$P=C_1e^{2t}\qquad\left[C_1=\pm e^{C}\right]$$

Thus, the general solution to the differential equation is $P=C_1e^{2t}$, where $C_1=\pm e^{C}$.

**30.** Solve $\frac{dP}{dt}=4P$

$$\frac{dP}{P}=4dt$$
$$\int\frac{dP}{P}=\int 4dt$$
$$\ln|P|=4t+C$$
$$|P|=e^{4t+C}$$
$$P=\pm e^{4t+C}$$
$$P=\pm e^{4t}\cdot e^{C}$$
$$P=C_1e^{4t}\qquad\left[C_1=\pm e^{C}\right]$$

Thus, the general solution to the differential equation is $P=C_1e^{4t}$, where $C_1=\pm e^{C}$.

**31.** Solve $f'(x)=\frac{1}{x}-4x+\sqrt{x}$, given that $f(1)=\frac{23}{3}$.

Integrating, we have:

$$f(x)=\int f'(x)\,dx$$
$$=\int\left(\frac{1}{x}-4x+\sqrt{x}\right)dx$$
$$=\ln x-2x^2+\frac{2}{3}x^{3/2}+C$$

We substitute 1 for $x$ and $\frac{23}{3}$ for $f(1)$ to find $C$.

$$\frac{23}{3}=\ln(1)-2(1)^2+\frac{2}{3}(1)^{3/2}+C$$
$$\frac{23}{3}=0-2+\frac{2}{3}+C$$
$$\frac{23}{3}=-\frac{6}{3}+\frac{2}{3}+C$$
$$\frac{23}{3}=-\frac{4}{3}+C$$
$$\frac{23}{3}+\frac{4}{3}=C$$
$$\frac{27}{3}=C$$
$$9=C$$

Therefore, the particular solution is

$$f(x)=\ln x-2x^2+\frac{2}{3}x^{3/2}+9.$$

**32.** $R'(x)=300-2x$

$$R(x)=\int R'(x)\,dx$$
$$R(x)=\int(300-2x)\,dx$$
$$=300x-x^2+C$$

Substitute to find $C$.

$$0=300(0)-(0)^2+C$$
$$0=C$$

Therefore, the total revenue function is $R(x)=300x-x^2$.

**33.** $C'(x) = 2.6 - 0.02x$

$C(x) = \int C'(x)\,dx$

$C(x) = \int (2.6 - 0.02x)\,dx$

$= 2.6x - 0.01x^2 + K$

We use $K$ as the constant of integration so we do not confuse it with total cost.
Next, we substitute to find $K$.

$$C(0) = 120$$
$$2.6(0) - 0.01(0)^2 + K = 120$$
$$K = 120$$

Thus, the total cost function is
$C(x) = 2.6x - 0.01x^2 + 120$.

The average cost function is given by:

$$A(x) = \frac{C(x)}{x}$$
$$= \frac{2.6x - 0.01x^2 + 120}{x}$$
$$= 2.6 - 0.01x + \frac{120}{x}.$$

**34.** $\dfrac{dI}{dt} = hkI$

a) $\dfrac{dI}{I} = hkdt$

$$\int \frac{dI}{I} = \int hkdt$$
$$\ln I = hkt + C \qquad I > 0$$
$$I = e^{hkt+C}$$
$$I = e^{hkt} \cdot e^{C}$$
$$I = C_1 e^{hkt} \qquad [C_1 = e^C]$$

The general solution to the differential equation is $I = C_1 e^{hkt}$, where $C_1 = e^C$.

b) Substituting the initial condition $I_0 = I(0)$, we have

$$I_0 = C_1 e^{hk(0)}$$
$$I_0 = C_1$$

So the particular solution is $I = I_0 e^{hkt}$.

**35.** $\dfrac{dP}{dC} = \dfrac{-200}{(C+3)^{3/2}}$

a) Integrating, we have

$$P(C) = \int \frac{-200}{(C+3)^{3/2}}\,dC$$
$$= -200\int (C+3)^{-3/2}\,dC$$
$$= -200\left[\frac{(C+3)^{-1/2}}{-1/2} + K\right]$$
$$= \frac{400}{(C+3)^{1/2}} + K_1 \qquad [K_1 = -200K]$$
$$= \frac{400}{\sqrt{C+3}} + K_1$$

We substitute 10 for $P$ and 61 for $C$ to find $K_1$.

$$10 = \frac{400}{\sqrt{61+3}} + K_1$$
$$10 = \frac{400}{8} + K_1$$
$$10 = 50 + K_1$$
$$-40 = K_1$$

Thus, the particular solution, given the conditions is $P(C) = \dfrac{400}{\sqrt{C+3}} - 40.$

b) The firm will break even when $P = 0$. Substituting, we have:

$$0 = \frac{400}{\sqrt{C+3}} - 40$$
$$40 = \frac{400}{\sqrt{C+3}}$$
$$40\sqrt{C+3} = 400$$
$$\sqrt{C+3} = 10$$
$$C + 3 = 100$$
$$C = 97$$

When total cost is \$97, the firm will break even.

**36.** Substituting the information given in the problem, we have

$$\frac{dV}{dt} = k(24.81 - V)$$

We solve the equation by separation of variables.

$$\frac{dV}{24.81-V} =$$

$$\int \frac{dV}{24.81-V} = \int k dt$$

$$-\ln(24.81-V) = kt + C \qquad [24.81-V>0]$$

$$\ln(24.81-V) = -kt - C$$

$$24.81-V = e^{-kt-C}$$

$$24.81-V = e^{-kt}\cdot e^{-C}$$

$$-V = e^{-kt}\cdot e^{-C} - 24.81$$

$$V = 24.81 - e^{-kt}\cdot e^{-C}$$

$$V = 24.81 + C_1 e^{-kt} \ \left[C_1 = -e^{-C}\right]$$

Substitute to find $C_1$.

$$20 = 24.81 + C_1 e^{-k(0)}$$

$$-4.81 = C_1$$

Therefore, the particular solution to the differential equation is

$V(t) = -4.81e^{-kt} + 24.81$.

**37.** a) $\dfrac{dR}{dS} = \dfrac{k}{S+1}$

First, we separate the variables, as follows:

$dR = \dfrac{k}{S+1}dS$ Multiplying by $dS$.

We then integrate both sides:

$$\int dR = \int \frac{k}{S+1} dS$$

$$R = k\ln(S+1) + C \qquad [S+1>0]$$

The general solution is $R = k\ln(S+1)+C$.

b) We substitute the initial conditions to find $C$.

$$R = k\ln(S+1)+C$$

$$0 = k\ln(0+1)+C$$

$$0 = k\ln(1)+C$$

$$0 = k\cdot 0 + C$$

$$0 = C$$

Thus the particular solution given the initial conditions is $R = k\ln(S+1)$.

c) $\boxed{tw}$ It is reasonable to assume that a consumer will receive 0 pleasure units by consuming 0 units of a product.

**38.** $E(x) = \dfrac{4}{x}$; $q = e$ when $x = 4$

Elasticity is given by

$$E(x) = -\frac{x}{q}\cdot\frac{dq}{dx}$$

Therefore,

$$-\frac{x}{q}\cdot\frac{dq}{dx} = \frac{4}{x}.$$

Solving the differential equation by separation of variables, we have:

$$-\frac{x}{q}\cdot\frac{dq}{dx} = \frac{4}{x}$$

$$\frac{dq}{q} = -\frac{4}{x^2}dx$$

$$\int \frac{dq}{q} = \int -4x^{-2}dx$$

$$\ln q = \frac{4}{x} + C \qquad [q>0]$$

$$q = e^{\frac{4}{x}+C}$$

$$q = e^{\frac{4}{x}}\cdot e^{C}$$

$$q = C_1 e^{\frac{4}{x}} \qquad \left[C_1 = e^{C}\right]$$

Substitute to find $C_1$.

$$e = C_1 e^{\frac{4}{4}}$$

$$e = C_1 e$$

$$1 = C_1$$

Therefore, the demand function is $q = e^{\frac{4}{x}}$.

**39.** $E(x) = \dfrac{x}{200-x}$; $q = 190$ when $x = 10$

Elasticity is given by

$$E(x) = -\frac{x}{q}\cdot\frac{dq}{dx}$$

Therefore,

$$-\frac{x}{q}\cdot\frac{dq}{dx} = \frac{x}{200-x}.$$

Solving the differential equation by separation of variables, we have:

$$-\frac{x}{q}\cdot\frac{dq}{dx}=\frac{x}{200-x}$$
$$\frac{dq}{q}=-\frac{1}{200-x}dx$$
$$\int\frac{dq}{q}=-\int\frac{1}{200-x}dx$$
$$\ln q=\ln(200-x)+C \qquad \left[\begin{smallmatrix}q>0,\\ 200-x>0\end{smallmatrix}\right]$$
$$q=e^{\ln(200-x)+C}$$
$$q=e^{\ln(200-x)}\cdot e^{C}$$
$$q=C_1(200-x) \qquad \left[C_1=e^C\right]$$

Substitute to find $C_1$.

$$190=C_1(200-10)$$
$$190=C_1(190)$$
$$1=C_1$$

Therefore, the demand function is $q=200-x$.

**40.** $E(x)=2;$ for all $x>0$

Elasticity is given by

$$E(x)=-\frac{x}{q}\cdot\frac{dq}{dx}$$

Therefore,

$$-\frac{x}{q}\cdot\frac{dq}{dx}=2.$$

Solving the differential equation by separation of variables, we have:

$$-\frac{x}{q}\cdot\frac{dq}{dx}=2$$
$$\frac{dq}{q}=-\frac{2}{x}dx$$
$$\int\frac{dq}{q}=-\int\frac{2}{x}dx$$
$$\ln q=-2\ln(x)+C \qquad [q>0,x>0]$$
$$\ln q=\ln x^{-2}+C$$
$$q=e^{\ln\left(x^{-2}\right)+C}$$
$$q=e^{\ln x^{-2}}\cdot e^{C}$$
$$q=C_1e^{\ln x^{-2}} \qquad \left[\mathrm{C}_1=e^C\right]$$
$$q=C_1\cdot x^{-2}$$
$$q=\frac{C_1}{x^2}$$

Therefore, the demand function is $q=\dfrac{C_1}{x^2}$.

**41.** $E(x)=n;$ for some constant $n$ and all $x>0$.

Elasticity is given by

$$E(x)=-\frac{x}{q}\cdot\frac{dq}{dx}$$

Therefore,

$$-\frac{x}{q}\cdot\frac{dq}{dx}=n.$$

Solving the differential equation by separation of variables, we have:

$$-\frac{x}{q}\cdot\frac{dq}{dx}=n$$
$$\frac{dq}{q}=-\frac{n}{x}dx$$
$$\int\frac{dq}{q}=-\int\frac{n}{x}dx$$
$$\ln q=-n\ln(x)+C \qquad [q>0,x>0]$$
$$\ln q=\ln x^{-n}+C$$
$$q=e^{\ln\left(x^{-n}\right)+C}$$
$$q=e^{\ln x^{-n}}\cdot e^{C}$$
$$q=C_1e^{\ln x^{-n}} \qquad \left[C_1=e^C\right]$$
$$q=C_1\cdot x^{-n}$$
$$q=\frac{C_1}{x^n}$$

Therefore, the demand function is $q=\dfrac{C_1}{x^n}$.

**42.** a) Solve $\dfrac{dP}{dt}=kP$

$$\frac{dP}{P}=kdt$$
$$\int\frac{dP}{P}=\int kdt$$
$$\ln P=kt+C \qquad [P>0]$$
$$P=e^{kt+C}$$
$$P=e^{kt+C}$$
$$P=e^{kt}\cdot e^{C}$$
$$P=C_1e^{kt} \qquad \left[C_1=e^C\right]$$

Thus, the general solution to the differential equation is $P=C_1e^{kt}$, where $C_1=e^C$.

b) Substituting the initial condition we have

$$P_0=C_1e^{k(0)}$$
$$P_0=C_1$$

Therefore, the solution is $P=P_0e^{kt}$.

**43.** $\dfrac{dR}{dS} = k \cdot \dfrac{R}{S}$

First, we separate the variables

$\dfrac{dR}{R} = k \cdot \dfrac{dS}{S}$  Multiplying by $\dfrac{dS}{R}$.

Then we integrate both sides.

$$\int \frac{dR}{R} = \int k \cdot \frac{dS}{S}$$

$$\ln R = k \ln S + C \qquad [S, R > 0]$$

$$\ln R = \ln S^k + C$$

$$R = e^{\ln S^k + C}$$

$$R = e^{\ln S^k} \cdot e^C$$

$$R = C_1 e^{\ln S^k} \qquad \left[C_1 = e^C\right]$$

$$R = C_1 \cdot S^k$$

Thus the solution to the differential equation is $R = C_1 \cdot S^k$ where $C_1 = e^C$.

**44.** Solve $\dfrac{dy}{dx} = 5x^4y^2 + x^3y^2$

First, we separate the variables, as follows

$$\frac{dy}{dx} = \left(5x^4 + x^3\right)y^2$$

$$\frac{dy}{y^2} = \left(5x^4 + x^3\right)dx.$$

We then integrate both sides:

$$\int \frac{dy}{y^2} = \int \left(5x^4 + x^3\right)dx$$

$$-\frac{1}{y} = x^5 + \frac{1}{4}x^4 + C$$

$$-\frac{4}{y} = 4x^5 + x^4 + 4C$$

$$-4 = \left(4x^5 + x^4 + C_1\right)y \quad \left[C_1 = 4C\right]$$

$$\frac{-4}{4x^5 + x^4 + C_1} = y$$

Thus, the general solution to the differential equation is $y = \dfrac{-4}{4x^5 + x^4 + C_1}$, where $C_1 = 4C$.

**45.** Solve $e^{-1/x} \cdot \dfrac{dy}{dx} = x^{-2} \cdot y^2$

First, we separate the variables, as follows

$$\frac{dy}{y^2} = \frac{e^{1/x}}{x^2}dx.$$

We then integrate both sides:

$$\int \frac{dy}{y^2} = \int x^{-2}e^{1/x}dx$$

$$-\frac{1}{y} = -e^{1/x} + C \quad \left[u = x^{-1}, du = -x^{-2}\right]$$

$$\frac{1}{y} = e^{1/x} - C$$

$$1 = \left(e^{1/x} - C\right)y$$

$$\frac{1}{e^{1/x} - C} = y$$

Thus, the general solution to the differential equation is $y = \dfrac{1}{e^{1/x} + C_1}$, where $C_1 = -C$.

**46.** $\boxed{tw}$ Answers will include finding consumer and producer surplus, the many applications of exponential growth and decay, applications of probability, finding the volume of a solid of revolution and the many applications of differential equations.

**47.** $\boxed{tw}$ It is necessary to use the $\pm$ because of the absolute value in the equation.

**48.** $\dfrac{dy}{dx} = \dfrac{5}{y}$

$$y\,dy = 5dx$$

$$\int y\,dy = \int 5dx$$

$$\frac{1}{2}y^2 = 5x + C$$

$$y^2 = 10x + 2C$$

$$y^2 = 10x + C_1 \qquad \left[C_1 = 2C\right]$$

$$y = \pm\sqrt{10x + C_1}$$

Thus, the general solution is $y = \sqrt{10x + C_1}$ and $y = -\sqrt{10x + C_1}$, where $C_1 = 2C$.