89. $f(t)=\ln\left(t^2-t\right)^7$

$f(t)=7\ln\left(t^2-t\right)$ (P3)

$f'(t)=7\left(\frac{(2t-1)}{t^2-t}\right)$ By Theorem 7

$f'(t)=\frac{7(2t-1)}{t^2-t}$ Simplifying

90. $g(x)=\left[\ln(x+5)\right]^4$

$g'(x)=4\left[\ln(x+5)\right]^3\cdot\frac{1}{x+5}$

$g'(x)=\frac{4\left[\ln(x+5)\right]^3}{x+5}$

91. $f(x)=\ln\left[\ln(\ln(3x))\right]$

Differentiating this function will require several applications of the Chain Rule. We will start on the outside and work our way in to the middle.

$$f'(x)=\left(\frac{1}{\ln(\ln(3x))}\right)\cdot\frac{d}{dx}\left(\ln(\ln(3x))\right)$$
$$=\left(\frac{1}{\ln(\ln(3x))}\right)\cdot\left[\left(\frac{1}{\ln(3x)}\right)\cdot\frac{d}{dx}\left(\ln(3x)\right)\right]$$
$$=\left(\frac{1}{\ln(\ln(3x))}\right)\cdot\left(\frac{1}{\ln(3x)}\right)\left[\frac{1}{3x}\cdot 3\right]$$
$$=\frac{1}{x\ln(3x)\cdot\ln(\ln(3x))}$$

92. $f(t)=\ln\left[\left(t^3+3\right)\left(t^2-1\right)\right]$

$f(t)=\ln\left(t^3+3\right)+\ln\left(t^2-1\right)$

$$f'(t)=\frac{3t^2}{t^3+3}+\frac{2t}{t^2-1}$$
$$=\frac{3t^2\left(t^2-1\right)}{\left(t^3+3\right)\left(t^2-1\right)}+\frac{2t\left(t^3+3\right)}{\left(t^3+3\right)\left(t^2-1\right)}$$
$$=\frac{5t^4-3t^2+6t}{\left(t^3+3\right)\left(t^2-1\right)}$$
$$=\frac{t\left(5t^3-3t+6\right)}{\left(t^3+3\right)\left(t^2-1\right)}$$

93. $f(t)=\ln\frac{1-t}{1+t}$

$f(t)=\ln(1-t)-\ln(1+t)$ (P2)

$f'(t)=\frac{(-1)}{1-t}-\frac{(1)}{1+t}$ By Theorem 7

$=\frac{-1}{1-t}-\frac{1}{1+t}$

$=\frac{1}{(-1)(1-t)}-\frac{1}{1+t}$ Properties of fractions

$=\frac{1}{(t-1)}-\frac{1}{t+1}$ Distribution

Next, we find a common denominator.

$=\frac{t+1}{(t-1)(t+1)}-\frac{t-1}{(t-1)(t+1)}$

$f'(t)=\frac{2}{t^2-1}$

94. $y=\ln\frac{x^5}{(8x+5)^2}$

$y=5\ln(x)-2\ln(8x+5)$

$$\frac{dy}{dx}=5\cdot\frac{1}{x}-2\cdot\frac{8}{8x+5}$$
$$=\frac{5}{x}-\frac{16}{8x+5}$$
$$=\frac{5(8x+5)}{x(8x+5)}-\frac{16x}{x(8x+5)}$$
$$=\frac{24x+25}{x(8x+5)}$$

95. $f(x)=\log_5 x$

Using (P7), the change-of-base formula.

$f(x)=\frac{\ln x}{\ln 5}$

Remember that $\ln 5$ is a constant.

$$f'(x)=\frac{1}{\ln 5}\cdot\frac{1}{x}$$
$$=\frac{1}{x\ln 5}$$

96. $f(x) = \log_7 x$

Using (P7), the change-of-base formula.

$$f(x) = \frac{\ln x}{\ln 7}$$

$$f'(x) = \frac{1}{\ln 7} \cdot \frac{1}{x} = \frac{1}{x \ln 7}$$

97. $y = \ln\sqrt{5+x^2}$

$$y = \ln\left(5+x^2\right)^{1/2}$$

$$y = \tfrac{1}{2}\ln\left(5+x^2\right) \qquad \text{(P3)}$$

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{2x}{5+x^2} \qquad \text{Theorem 7}$$

$$\frac{dy}{dx} = \frac{x}{5+x^2}$$

98. $f(t) = \dfrac{\ln t^2}{t^2}$

$$f'(t) = \frac{t^2\left(\frac{1}{t^2}\right) \cdot 2t - 2t \cdot \ln\left(t^2\right)}{\left(t^2\right)^2}$$

$$= \frac{2t - 2t\ln\left(t^2\right)}{t^4}$$

$$= \frac{2t\left(1-\ln\left(t^2\right)\right)}{t^4}$$

$$= \frac{2\left(1-\ln\left(t^2\right)\right)}{t^3}$$

99. $f(x) = \frac{1}{5}x^5\left(\ln x - \frac{1}{5}\right)$

Using the Product Rule.

$$f'(x) = \frac{1}{5}x^5\left(\frac{1}{x} - 0\right) + \frac{1}{5}\left(5x^4\right)\left(\ln x - \frac{1}{5}\right)$$

$$= \frac{1}{5}x^4 + x^4\ln x - \frac{1}{5}x^4$$

$$= x^4 \ln x$$

100. $y = \dfrac{x^{n+1}}{n+1}\left(\ln x - \dfrac{1}{n+1}\right)$

$$\frac{dy}{dx} = \frac{x^{n+1}}{n+1}\left(\frac{1}{x}\right) + x^n\left(\ln x - \frac{1}{n+1}\right)$$

$$= \frac{x^n}{n+1} + x^n \ln x - \frac{x^n}{n+1}$$

$$= x^n \ln x$$

101. $f(x) = \ln\dfrac{1+\sqrt{x}}{1-\sqrt{x}}$

$$f(x) = \ln\left(1+\sqrt{x}\right) - \ln\left(1-\sqrt{x}\right) \qquad \text{(P2)}$$

We use theorem 7, and the Chain Rule to differentiate both logarithms.

Note: $\dfrac{d}{dx}\left(1+\sqrt{x}\right) = \dfrac{1}{2}x^{-1/2}$

$\dfrac{d}{dx}\left(1-\sqrt{x}\right) = -\dfrac{1}{2}x^{-1/2}$

$$f'(x) = \frac{\left(\frac{1}{2}\right)x^{-1/2}}{1+\sqrt{x}} - \frac{\left(-\frac{1}{2}\right)x^{-1/2}}{1-\sqrt{x}}$$

Simplifying

$$f'(x) = \frac{1}{1+\sqrt{x}} \cdot \frac{1}{2x^{1/2}} + \frac{1}{1-\sqrt{x}} \cdot \frac{1}{2x^{1/2}}$$

$$= \frac{1}{2\sqrt{x}\left(1+\sqrt{x}\right)} + \frac{1}{2\sqrt{x}\left(1-\sqrt{x}\right)}$$

Now, we combine the fractions.

$f'(x)$

$$= \frac{1}{2\sqrt{x}\left(1+\sqrt{x}\right)} \cdot \frac{1-\sqrt{x}}{1-\sqrt{x}} + \frac{1}{2\sqrt{x}\left(1-\sqrt{x}\right)} \cdot \frac{1+\sqrt{x}}{1+\sqrt{x}}$$

$$= \frac{1-\sqrt{x}}{2\sqrt{x}\left(1+\sqrt{x}\right)\left(1-\sqrt{x}\right)} + \frac{1+\sqrt{x}}{2\sqrt{x}\left(1+\sqrt{x}\right)\left(1-\sqrt{x}\right)}$$

$$= \frac{2}{2\sqrt{x}\left(1+\sqrt{x}\right)\left(1-\sqrt{x}\right)}$$

$$= \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)\left(1-\sqrt{x}\right)}$$

or, if we multiply the two binomials

$$f'(x) = \frac{1}{\sqrt{x}\,(1-x)}.$$

102. $f(x) = \ln[\ln x]^3$

$$f'(x) = \frac{1}{[\ln x]^3} \cdot 3[\ln x]^2 \cdot \frac{1}{x}$$

$$= \frac{3}{x \ln x}$$

103. Let $X = \log_a M$ and $Y = \log_a N$
Proof of Property 1:

$M = a^X$ and $N = a^Y$ Definition of Logarithm

So,

$MN = a^X a^Y = a^{X+Y}$ Product Rule for Exponents

Thus,

$\log_a(MN) = X + Y$ Definition of Logarithm

$= \log_a M + \log_a N$ Substitution

104. Proof of Property 2:

$M = a^X$ and $N = a^Y$ Definition of Logarithm

So,

$\frac{M}{N} = \frac{a^X}{a^Y} = a^{X-Y}$ Quotient rule for Exponents

Thus,

$\log_a\left(\frac{M}{N}\right) = X - Y$ Definition of Logarithm

$= \log_a M - \log_a N$ Substitution

105. Proof of Property 3:

$M = a^X$ Definition of Logarithm

So,

$M^k = \left(a^X\right)^k$ $\left[a = b \Rightarrow a^c = b^c\right]$

$= a^{X\cdot k}$ Power Rule for Exponents

Thus,

$\log_a\left(M^k\right) = X \cdot k$ Definition of Logarithm

$= k \cdot \log_a M$ Substitution

106. Proof of Property 7:

Let $\log_b M = R$.

Then,

$b^R = M$ Definition of Logarithm

and

$\log_a\left(b^R\right) = \log_a M$ $\left[a = b \Rightarrow \log_c a = \log_c b\right]$

Thus,

$R \cdot \log_a b = \log_a M$ Property (P3)

and

$R = \frac{\log_a M}{\log_a b}$ $\left[a = b; c \neq 0 \Rightarrow \frac{a}{c} = \frac{b}{c}\right]$

It follows that

$\log_b M = \frac{\log_a M}{\log_a b}$ Substitution

107. $\lim\limits_{h\to 0} \frac{\ln(1+h)}{h}$

Note: $\ln(1+h)$ exists only for $h > -1$.

Since the function is not continuous at $h = 0$, we will use input-output tables.

First, we look as h approaches 0 from the left.

h	$\frac{\ln(1+h)}{h}$
-0.9	2.56
-0.5	1.39
-0.1	1.05
-0.01	1.01
-0.001	1.001

From the table we observe that

$\lim\limits_{h\to 0^-} \frac{\ln(1+h)}{h} = 1$.

Now, we look as h approaches 0 from the right.

h	$\frac{\ln(1+h)}{h}$
0.9	0.71
0.5	0.81
0.1	0.95
0.01	0.995
0.001	0.9995

From the table we observe that

$\lim\limits_{h\to 0^+} \frac{\ln(1+h)}{h} = 1$.

Thus,

$\lim\limits_{h\to 0} \frac{\ln(1+h)}{h} = 1$.

108. $\ln(kx) = \ln k + \ln x$

Since $\ln k$ is a constant, we see that the graph of $\ln(kx)$ is the graph of $\ln(x)$ shifted vertically $\ln(k)$ units. The vertical shift does not change the slope of the graph at x, therefore

$\frac{d}{dx}\left[\ln(kx)\right] = \frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$.

109. We consider the function $y = \dfrac{\ln x}{x}$. It can be shown that this function has a maximum at $x = e$. Thus:

$$\frac{\ln e}{e} > \frac{\ln \pi}{\pi} \qquad \text{Definition of Maximum}$$
$$\pi \cdot \ln e > e \cdot \ln \pi$$
$$\ln e^{\pi} > \ln \pi^{e} \qquad (\text{P3})$$
$$e^{\pi} > \pi^{e}.$$

Therefore; e^{π} is larger.

110. $\sqrt[e]{e} = e^{1/e} \approx 1.444667861$

$e \approx 2.71828183$

For comparison select values of x $(x > 0)$ less than e and greater than e and compute $\sqrt[x]{x}$

$x = 0.5,\quad \sqrt[0.5]{0.5} = 0.25$

$x = 0.8,\quad \sqrt[0.8]{0.8} \approx 0.756593$

$x = 1.5,\quad \sqrt[1.5]{1.5} \approx 1.310371$

$x = 2.0,\quad \sqrt[2.0]{2.0} \approx 1.414214$

$x = 2.5,\quad \sqrt[2.5]{2.5} \approx 1.442700$

$x = 2.7,\quad \sqrt[2.7]{2.7} \approx 1.444656$

$x = 2.71,\quad \sqrt[2.71]{2.71} \approx 1.44466538$

$x = 2.72,\quad \sqrt[2.72]{2.72} \approx 1.44466775$

$x = 3.0,\quad \sqrt[3.0]{3.0} \approx 1.442250$

$x = 3.5,\quad \sqrt[3.5]{3.5} \approx 1.430369$

For any $x > 0$ such that $x \neq e$, $\sqrt[e]{e} > \sqrt[x]{x}$.

111. Find $\lim\limits_{x \to 1} \ln x$

First, we observe as x approaches 1 from the left.

x	$\ln x$
0.1	-2.30
0.4	-0.92
0.7	-0.36
0.9	-0.11
0.99	-0.01
0.999	-0.001

$\lim\limits_{x \to 1^-} \ln x = 0$

Now, we observe as x approaches 1 from the right.

x	$\ln x$
1.9	0.64
1.7	0.53
1.4	0.34
1.1	0.10
1.01	0.01
1.001	0.001

$\lim\limits_{x \to 1^+} \ln x = 0$

Thus, $\lim\limits_{x \to 1} \ln x = 0$

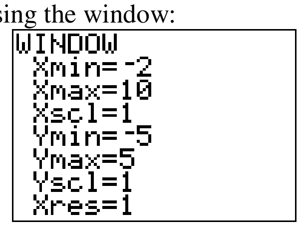
112. Find $\lim\limits_{x \to \infty} \ln x$

$\ln x$ exist only for $x > 0$.

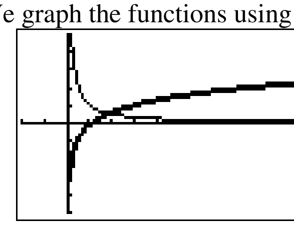
x	$\ln x$
1	0
10	2.3
100	4.6
1000	6.9
10,000	9.2
100,000	11.5
1,000,000	13.8

Thus, $\lim\limits_{x \to \infty} \ln x = \infty$, the limit does not exist.

113. Using the window:

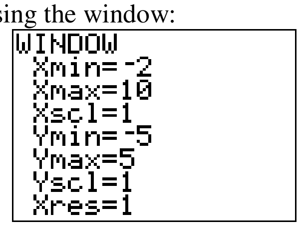


We graph the functions using a calculator.

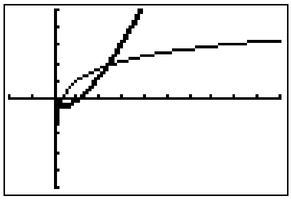


Notice the function is the thicker graph.

114. Using the window:



We graph the functions using a calculator.

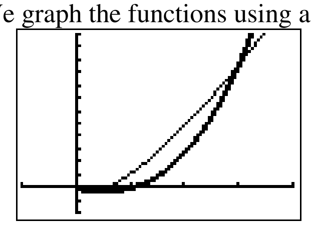


Notice, the function is the thicker graph.

115. Using the window:

```
WINDOW
 Xmin=-1
 Xmax=4
 Xscl=1
 Ymin=-2
 Ymax=12
 Yscl=1
 Xres=1
```

We graph the functions using a calculator.

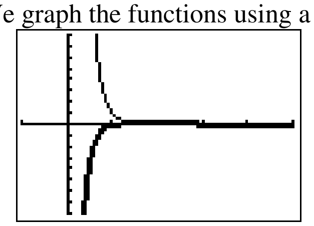


Notice, the function is the thicker graph.

116. Using the window:

```
WINDOW
 Xmin=-1
 Xmax=5
 Xscl=1
 Ymin=-8
 Ymax=8
 Yscl=1
 Xres=1
```

We graph the functions using a calculator.



Notice, the function is the thicker graph.

117. $f(x) = x\ln x$

$f'(x) = x\cdot\frac{1}{x} + \ln x$

$\quad = 1 + \ln x$

$f'(x)$ exists for all x in the domain of $f(x)$.

Solve:

$f'(x) = 0$

$1 + \ln x = 0$

$\ln x = -1$

$x = e^{-1}$

The critical value occurs when $x = e^{-1}$.

Use the Second-Derivative test to determine if the critical value represents a relative maximum or a relative minimum.

$f''(x) = \frac{1}{x}$

$f''(e^{-1}) = \frac{1}{e^{-1}} = e > 0$

Therefore the function is concave up and there is a relative minimum at the critical value $x = e^{-1}$.

$f(e^{-1}) = e^{-1}\ln e^{-1}$

$\quad = e^{-1}\cdot(-1)$

$\quad = -\frac{1}{e} \approx -0.368$

The minimum value that occurs at $x = e^{-1}$ is $-\frac{1}{e} \approx -0.368$

118. $f(x) = x^2\ln x$

$f'(x) = x^2\cdot\frac{1}{x} + 2x\ln x$

$\quad = x + 2x\ln x$

$f'(x)$ exists for all x in the domain of $f(x)$.

Solve:

$f'(x) = 0$

$x + 2x\ln x = 0$

$x(2 + \ln x) = 0$

$x = 0 \quad or \quad 1 + 2\ln x = 0$

$x = 0 \quad or \quad 2\ln x = -1$

$x = 0 \quad or \quad \ln x = -\frac{1}{2}$

$x = 0 \quad or \quad x = e^{-1/2}$

Only $x = e^{-1/2}$ is in the interval $(0,\infty)$.

Use the Second-Derivative test to determine if the critical value represents a relative maximum or a relative minimum.

$f''(x) = 1 + 2x\cdot\frac{1}{x}2\ln x$

$\quad = 1 + 2 + 2\ln x$

$\quad = 3 + 2\ln x$

$f''\left(e^{-1/2}\right) = 3 + 2\ln e^{-1/2} = 3 - 1 = 2$

Therefore the function is concave up and there is a relative minimum at the critical value $x = e^{-1/2}$.

Finding the function value at $x = e^{-1/2}$, we have:

$$f\left(e^{-1/2}\right) = \left(e^{-1/2}\right)^2 \ln e^{-1/2}$$
$$= e^{-1}\left(-\frac{1}{2}\right)$$
$$= -\frac{1}{2e} \approx -0.184$$

The minimum value that occurs at $x = e^{-1/2}$ is -0.184.

Exercise Set 3.3

1. Using Theorem 8, the general form of f that satisfies the equation $f'(x) = 4 \cdot f(x)$ is $f(x) = ce^{4x}$ for some constant *c*.

2. Using Theorem 8, the general form of f that satisfies the equation $g'(x) = 6 \cdot g(x)$ is $g(x) = ce^{6x}$ for some constant *c*.

3. Using Theorem 8, the general form of A that satisfies the equation $\frac{dA}{dt} = -9 \cdot A$ is $A = ce^{-9t}$, or $A(t) = ce^{-9t}$ for some constant *c*.
Note: If we were to let the initial population, the population when $t = 0$, be represented by $A(0) = A_0$, then we have $A_0 = A(0) = ce^{-9 \cdot 0} = ce^0 = c$. Thus, $A_0 = c$, and we can express $A(t) = A_0 e^{-9t}$.

4. Using Theorem 8, the general form of P that satisfies the equation $\frac{dP}{dt} = -3 \cdot P$ is $P = ce^{-3t}$, or $P(t) = ce^{-3t}$ for some constant *c*.

Note: If we were to let the initial population, the population when $t = 0$, be represented by $P(0) = P_0$, then we have $P_0 = P(0) = ce^{-3 \cdot 0} = ce^0 = c$. Thus, $P_0 = c$, and we can express $P(t) = P_0 e^{-3t}$.

5. Using Theorem 8, the general form of Q that satisfies the equation $\frac{dQ}{dt} = k \cdot Q$ is $Q = ce^{kt}$, or $Q(t) = ce^{kt}$ for some constant *c*.
Note: If we were to let the initial population, the population when $t = 0$, be represented by $Q(0) = Q_0$, then we have $Q_0 = Q(0) = ce^{k \cdot 0} = ce^0 = c$. Thus, $Q_0 = c$, and we can express $Q(t) = Q_0 e^{kt}$.

6. Using Theorem 8, the general form of R that satisfies the equation $\frac{dR}{dt} = k \cdot R$ is $R = ce^{kt}$, or $R(t) = ce^{kt}$ for some constant *c*.
Note: If we were to let the initial population, the population when $t = 0$, be represented by $R(0) = R_0$, then we have $R_0 = R(0) = ce^{k \cdot 0} = ce^0 = c$. Thus, $R_0 = c$, and we can express $R(t) = R_0 e^{kt}$.

7. a) Using Theorem 8, the general form of N that satisfies the equation $N'(t) = 0.046 \cdot N(t)$ is $N(t) = ce^{0.046t}$ for some constant *c*.
Allowing $t = 0$ to correspond to 1980 when approximately 112,000 patent applications were received, gives us the initial condition $N(0) = 112{,}000$. Therefore:

$$112{,}000 = ce^{0.046 \cdot 0}$$
$$112{,}000 = ce^0$$
$$112{,}000 = c$$

Substituting this value for *c*. We get: $N(t) = 112{,}000e^{0.046t}$

b) In 2010, $t = 2010 - 1980 = 30$.

$$N(30) = 112{,}000e^{0.046(30)}$$
$$= 112{,}000e^{1.38}$$
$$= 112{,}000(3.974902)$$
$$= 445{,}188.98$$
$$\approx 445{,}189$$

There will be approximately 445,189 patent applications received in 2010.

c) From Theorem 9, the doubling time T is given by $T = \frac{\ln 2}{k}$.

$T = \frac{\ln 2}{0.046} \approx 15.1$

It will take approximately 15.1 years for the number of patent applications to double. This means that in 1995, the number of patents will have doubled.

8. $\frac{dN}{dt} = 0.10N$

a) Substituting $N_0 = 50$ and $k = 0.10$

$N(t) = N_0 e^{kt}$

$N(t) = 50e^{0.1t}$.

b) $N(20) = 50e^{0.1(20)} \approx 369$

In 20 years, there will be 369 franchises.

c) $T = \frac{\ln 2}{k}$

$T = \frac{\ln 2}{0.10}$

≈ 6.9

The initial number of franchises will double in 6.9 years.

9. a) Using Theorem 8, the general form of P that satisfies the equation

$\frac{dP}{dt} = 0.065 \cdot P(t)$ is

$P(t) = P_0 e^{0.065t}$ for some initial principal P_0.

b) If \$1000 is invested, then $P_0 = 1000$ and

$P(t) = 1000e^{0.065t}$

$P(1) = 1000e^{0.065(1)}$

$= 1000e^{0.065}$

$= 1000(1.067159)$

≈ 1067.16

The balance after 1 year is \$1067.16.

$P(2) = 1000e^{0.065(2)}$

$= 1000e^{0.13}$

$= 1000(1.138828)$

≈ 1138.83

The balance after 2 years is \$1138.83

c) From Theorem 9, the doubling time T is given by $T = \frac{\ln 2}{k}$.

$T = \frac{\ln 2}{0.065} \approx 10.7$

It will take approximately 10.7 years for the balance to double.

10. a) Using Theorem 8, the general form of P that satisfies the equation

$\frac{dP}{dt} = 0.08 \cdot P(t)$ is

$P(t) = P_0 e^{0.08t}$ for some initial principal P_0.

b) If \$20,000 is invested, then $P_0 = 20{,}000$ and

$P(t) = 20{,}000e^{0.08t}$

$P(1) = 20{,}000e^{0.08(1)}$

$= 20{,}000e^{0.08}$

$\approx 21{,}655.74$

The balance after 1 year is \$21,655.74.

$P(2) = 20{,}000e^{0.08(2)}$

$= 20{,}000e^{0.16}$

$\approx 23{,}470.22$

The balance after 2 years is \$23,470.22

c) From Theorem 9, the doubling time T is given by $T = \frac{\ln 2}{k}$.

$T = \frac{\ln 2}{0.08} \approx 8.7$

It will take approximately 8.7 years for the balance to double.

11. a) Using Theorem 8, the general form of G that satisfies the equation

$\frac{dG}{dt} = 0.093 \cdot G(t)$ is

$G(t) = ce^{0.093t}$ for some constant c.

Allowing $t = 0$ to correspond to 2000 when approximately 4.7 billion gallons of bottled water were sold, gives us the initial condition $G(0) = 4.7$. Therefore:

$4.7 = ce^{0.093 \cdot 0}$

$4.7 = ce^{0}$

$4.7 = c$

Substituting this value for c. We get:

$G(t) = 4.7e^{0.093t}$

Where $G(t)$ is in billions of gallons sold and t is the number of years since 2000.

b) In 2010, $t = 2010 - 2000 = 10$.

$$\begin{aligned} N(10) &= 4.7e^{0.093(10)} \\ &= 4.7e^{0.93} \\ &= 4.7(2.534509) \\ &= 11.912193 \\ &\approx 11.91 \end{aligned}$$

There will be approximately 11.91 billion gallons of bottle water sold in 2010.

c) From Theorem 9, the doubling time T is given by $T = \frac{\ln 2}{k}$.

$$T = \frac{\ln 2}{0.093} \approx 7.5$$

It will take approximately 7.5 years for the amount of bottled water sold to double. Sometime in the middle of 2007 according to our model.

12. a) Using Theorem 8, the general form of G that satisfies the equation

$$\frac{dP}{dt} = 0.182 \cdot P(t) \text{ is}$$

$$P(t) = P_0 e^{0.182t}$$

Let $t = 0$ correspond to 1994 and use the initial condition $P(0) = 6100$

$$P(t) = 6100e^{0.182t}$$

b) In 2007, $t = 2007 - 1994 = 13$.

$$\begin{aligned} N(13) &= 6100e^{0.182(13)} \\ &= 6100e^{2.366} \\ &= 6100(10.654688) \\ &= 64{,}993.60 \\ &\approx 64{,}994 \end{aligned}$$

The number of immigration cases prosecuted in 2007 will be approximately 64,994.

c) From Theorem 9, the doubling time T is given by $T = \frac{\ln 2}{k}$.

$$T = \frac{\ln 2}{0.182} \approx 3.8$$

The doubling time is approximately 3.8 years.

13. From Theorem 9, the doubling time T is given by $T = \frac{\ln 2}{k}$. Substituting $T = 15$ we get:

$$15 = \frac{\ln 2}{k}$$

Solve for k to find the interest rate.

$$\begin{aligned} 15 \cdot k &= \ln 2 \\ k &= \frac{\ln 2}{15} \\ k &\approx 0.046210 \end{aligned}$$

The annual interest rate is 4.62%.

14. $T = \frac{\ln 2}{k}$

$$\begin{aligned} 12 &= \frac{\ln 2}{k} \\ 12 \cdot k &= \ln 2 \\ k &= \frac{\ln 2}{12} \\ k &\approx 0.0578 \end{aligned}$$

The annual interest rate is 5.78%.

15. From Theorem 9, the doubling time T is given by $T = \frac{\ln 2}{k}$. Substituting the growth rate $k = 0.10$ into the formula.

$$\begin{aligned} T &= \frac{\ln 2}{0.10} \\ &\approx 6.9 \end{aligned}$$

The doubling time for the demand for oil is 6.9 years; therefore, at the end of the year 2012 the demand for oil will be double the demand for oil in 2006.

16. $T = \frac{\ln 2}{k}$

$$\begin{aligned} T &= \frac{\ln 2}{0.04} \\ &\approx 17.3 \end{aligned}$$

The doubling time for the demand for coal is 17.3 years; therefore, in the year 2023 the demand for coal will be double the demand for coal in 2006.

17. Find the doubling time:

$$T=\frac{\ln 2}{k}=\frac{\ln 2}{0.062}\approx 11.2$$

The doubling time is 11.2 years.

$$P(t)=P_0e^{kt}$$

$$P(t)=75{,}000e^{0.062t}$$

We substitute $t=5$ to find the amount after five years:

$$\begin{aligned}P(5)&=75{,}000e^{0.062(5)}\\&=75{,}000e^{0.31}\\&\approx 102{,}256.88\end{aligned}$$

In five years the account will have \$102,256.88.

18. Find the interest rate.

$$7130.90=5000e^{k(5)}$$

$$\frac{7130.90}{5000}=e^{5k}$$

$$\ln\left(\frac{7130.90}{5000}\right)=5k$$

$$\frac{\ln\left(\frac{7130.90}{5000}\right)}{5}=k$$

$$0.071\approx k$$

The interest rate is 7.1%.
Find the doubling time:

$$T=\frac{\ln 2}{k}=\frac{\ln 2}{0.071}\approx 9.8$$

The doubling time is 9.8 years.

19. First, we find the initial investment:

$$P(t)=P_0e^{kt}$$

Substituting, we have

$$11{,}414.71=P_0e^{0.084(5)}$$

$$11{,}414.71=P_0e^{0.42}$$

$$\frac{11{,}414.71}{e^{0.42}}=P_0$$

$$7499.9989\approx P_0$$

$$7500.00\approx P_0$$

The initial investment is \$7500.
Next, we find the doubling time:

$$T=\frac{\ln 2}{k}=\frac{\ln 2}{0.084}\approx 8.3$$

The doubling time is 8.3 years.

20. Find the interest rate:

$$11=\frac{\ln 2}{k}$$

$$k=\frac{\ln 2}{11}\approx 0.063$$

The interest rate is 6.3%.
Find the initial investment:

$$17{,}539.32=P_0e^{0.063(5)}$$

$$17{,}539.32=P_0e^{0.315}$$

$$\frac{17{,}539.32}{e^{0.315}}=P_0$$

$$12{,}800.0006\approx P_0$$

$$12{,}800\approx P_0$$

The initial investment is \$12,800.

21. a) The exponential growth function is $V(t)=V_0e^{kt}$. We will express $V(t)$ in dollars and t as the number of years since 1950. This set up gives us the initial value of $V_0=30{,}000$ dollars.
Substituting this into the function, we have:
$V(t)=30{,}000e^{kt}$
In 2004, $t=2004-1950=54$, and the painting sold for 104,168,000 so we know $V(54)=104{,}168{,}000$ million dollars.
Substitute this information into the function and solve for k.

$$104{,}168{,}000=30{,}000e^{k(54)}$$

$$\frac{104{,}168{,}000}{30{,}000}=e^{54k}$$

$$\ln\left(\frac{104{,}168{,}000}{30{,}000}\right)=\ln e^{54k}$$

$$\ln\left(\frac{104{,}168{,}000}{30{,}000}\right)=54k$$

$$\frac{\ln\left(\frac{104{,}168{,}000}{30{,}000}\right)}{54}=k$$

$$0.151\approx k$$

The exponential growth rate is approximately 15.1%. Substituting this back into the function, we find the exponential growth function is
$V(t)=30{,}000e^{0.151t}$.

b) In 2010, $t = 2010 - 1950 = 60$. We evaluate the exponential growth function found in part 'a' to get:

$$\begin{aligned} V(60) &= 30{,}000e^{0.151(60)} \\ &= 30{,}000e^{9.06} \\ &= 30{,}000(8604.150654) \\ &\approx 258{,}124{,}520 \end{aligned}$$

In the year 2010, the value of the painting will be approximately \$258,124,520.

c) From Theorem 9, the doubling time T is given by $T = \frac{\ln 2}{k}$.

$$T = \frac{\ln 2}{0.151} \approx 4.59 \approx 4.6$$

The painting doubles in value approximately every 4.6 years.

d) We set $V(t) = 1{,}000{,}000{,}000$ and solve for t.

$$\begin{aligned} 30{,}000e^{0.151t} &= 1{,}000{,}000{,}000 \\ e^{0.151t} &= \frac{1{,}000{,}000{,}000}{30{,}000} \\ \ln e^{0.151t} &= \ln\left(\frac{1{,}000{,}000{,}000}{30{,}000}\right) \\ 0.151t &= \ln\left(\frac{1{,}000{,}000{,}000}{30{,}000}\right) \\ t &= \frac{\ln\left(\frac{1{,}000{,}000{,}000}{30{,}000}\right)}{0.151} \\ t &\approx 68.969 \\ t &\approx 69 \end{aligned}$$

It will take approximately 69 years for the value of the painting to reach 1 billion dollars. This will occur in the year 2019. $[1950 + 69 = 2019]$

22. a) $B(t) = B_0e^{kt}$

$$\begin{aligned} B(t) &= 39.4e^{kt} \\ 2153.9 &= 39.4e^{k(56)} \\ \frac{2153.9}{39.4} &= e^{56k} \\ \ln\left(\frac{2153.9}{39.4}\right) &= 56k \\ \frac{\ln\left(\frac{2153.9}{39.4}\right)}{56} &= k \\ 0.07145 &\approx k \end{aligned}$$

The growth rate is 0.07145 or 7.145%. Substituting k back into the exponential growth function we get:

$B(t) = 39.4e^{0.07145t}$

b) In 2010, $t = 2010 - 1950 = 60$.

$B(60) = 39.4e^{0.07145(60)} \approx 2866.3$

In 2010, the U.S. federal budget will be approximately 2866.3 billion dollars.

c) 10 trillion is 10,000 billion. Solve:

$$\begin{aligned} B(t) &= 10{,}000 \\ 39.4e^{0.07145t} &= 10{,}000 \\ e^{0.07145t} &= \frac{10{,}000}{39.4} \\ 0.07145t &= \ln\left(\frac{10{,}000}{39.4}\right) \\ t &= \frac{\ln\left(\frac{10{,}000}{39.4}\right)}{0.07145} \\ t &\approx 77.489 \\ t &\approx 77.5 \end{aligned}$$

The federal budget will reach 10 trillion dollars 77.5 years after 1950, or in 2027.

23. a) Letting $t = 0$ correspond to 1960, means that $P_0 = 2277$. Therefore, the exponential growth function will be $P(t) = 2277e^{kt}$.

In 2004, $t = 2004 - 1960 = 44$, the per capital income was \$32,907. Using this information we find k.

$$\begin{aligned} 32{,}907 &= 2277e^{k(44)} \\ \frac{32{,}907}{2277} &= e^{44k} \\ \ln\left(\frac{32{,}907}{2277}\right) &= \ln e^{44k} \\ \ln\left(\frac{32{,}907}{2277}\right) &= 44k \\ \frac{\ln\left(\frac{32{,}907}{2277}\right)}{44} &= k \\ 0.0607 &\approx k \end{aligned}$$

The exponential growth rate is 0.0607 or 6.07%.

Substituting the growth rate back into the function, we find the exponential growth function.

$P(t) = 2277e^{0.0607t}$.

b) In 2020, $t = 2020 - 1960 = 60$.
$P(60) = 2277e^{0.0607(60)} \approx 86,908.76$
In 2020, the U.S. per capita personal income will be approximately $86,908.76.

c) We need to solve the equation $P(t) = 100,000$ for *t*.

$$2277e^{0.0607t} = 100,000$$
$$e^{0.0607t} = \frac{100,000}{2277}$$
$$\ln e^{0.0607t} = \ln\left(\frac{100,000}{2277}\right)$$
$$0.0607t = \ln\left(\frac{100,000}{2277}\right)$$
$$t = \frac{\ln\left(\frac{100,000}{2277}\right)}{0.0607}$$
$$t \approx 62.3$$

The U.S. per capita income will be $100,000 approximately 62.3 years after 1960, or in 2022.

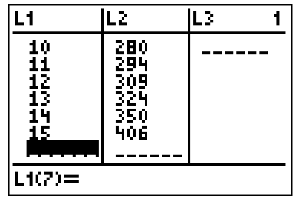
d) From Theorem 9, the doubling time *T* is given by $T = \frac{\ln 2}{k}$.
$$T = \frac{\ln 2}{0.0607} \approx 11.419 \approx 11.4$$
The U.S. per capita income doubles approximately every 11.4 years. Therefore, during 1971 the U.S. per capita income doubled that of the U.S. per capita income in 1960.

24. a) $P(t) = P_0e^{kt}$
Substitute 100 for P_0, 184.50 for $P(10)$, and 10 for *t*.
$$184.50 = 100e^{k(10)}$$
$$\frac{184.50}{100} = e^{10k}$$
$$\ln(1.845) = 10k$$
$$\frac{\ln 1.845}{10} = k$$
$$0.0612 \approx k$$
Thus, $P(t) = 100e^{0.0612t}$

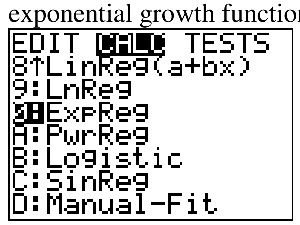
b) In 2010, $t = 2010 - 1967 = 43$.
$P(43) = 100e^{0.0612(43)} \approx 1389.60$
The same goods and services that cost $100 in 1967 will cost $1389.60 in 2010.

c) $T = \frac{\ln 2}{k} = \frac{\ln 2}{0.0612} \approx 11.3$
It took 11.3 years for 1967 prices to double. Which means in 1978 the same goods and services that cost $100 in 1967 will cost $200.

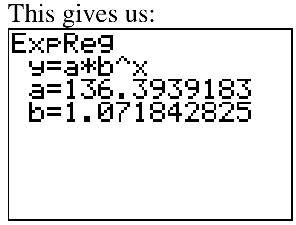
25. a) Enter the data into your calculator statistics editor.



Now use the regression command to find the exponential growth function.



This gives us:



The calculator models the function.
$y = 136.3939183(1.071842825)^x$, where *y* is in millions of dollars and *x* is the number of years after 1990.
From the technology connection section, we know that $b^x = e^{x(\ln b)}$. Apply this to the model our calculator found to get:
$$1.071842825^x = e^{x(\ln 1.071842825)}$$
$$= e^{0.0693794334x}$$
Rounding the values and substituting, we get the exponential growth function:
$P(t) = 136.4e^{0.0694t}$, where t is years since 1990, $k \approx 0.069$, or 6.9%, and $P_0 \approx 136.4$ million.

b) Using the model in part ‘a’.
In 2007, $t = 2007 - 1990 = 17$
$P(17) = 136.4e^{0.0964(17)} \approx 443.8$
In 2007, paper shredder sales were approximately $444 million.

In 2012, $t = 2012 - 1990 = 22$

$P(22) = 136.4e^{0.0694(22)} \approx 627.9$

In 2012, paper shredder sales were approximately \$628 million.

c) We need to solve the equation $P(t) = 500$ for t.

$$136.4e^{0.0694t} = 500$$
$$e^{0.0694t} = \frac{500}{136.4}$$
$$\ln e^{0.0694t} = \ln\left(\frac{500}{136.4}\right)$$
$$0.0694t = \ln\left(\frac{500}{136.4}\right)$$
$$t = \frac{\ln\left(\frac{500}{136.4}\right)}{0.0694}$$
$$t \approx 18.7$$

It will take approximately 18.7 years for the total sales of paper shredders to reach \$500 million. This should occur in the year 2008.

d) From Theorem 9, the doubling time T is given by $T = \frac{\ln 2}{k}$.

$$T = \frac{\ln 2}{0.0694} \approx 9.9877 \approx 10.0$$

The doubling time for sales of paper shredders is approximately 10 years.

26. a) Using the points $(10, 280)$ and $(15, 406)$ we can construct the following system of equations.

$$280 = P_0 e^{k(10)}$$
$$406 = P_0 e^{k(15)}$$

Solve the system for P_0 *and* k.

Solving the first equation for P_0 we get

$$P_0 = 280e^{-10k}$$

Substituting into the second equation we get

$$406 = \left(280e^{-10k}\right)e^{15k}$$
$$406 = 280e^{15k-10k}$$
$$406 = 280e^{5k}$$

Now solve for k.

$$\frac{406}{280} = e^{5k}$$
$$\ln\left(\frac{406}{280}\right) = 5k$$
$$0.0743127113 \approx k$$

Now go back and find the value of P_0.

$$P_0 = 280e^{-10k}$$
$$= 280e^{-10(0.0743127113)}$$
$$\approx 133.1747919$$

Substituting the values of P_0 *and* k into the exponential growth function we get:

$$P(t) = 133.1747919e^{0.0743127113t}$$

b) In 2007, $t = 2007 - 1990 = 17$

$$P(17) = 133.1747919e^{0.0743127113(17)}$$
$$\approx 471.057$$

In 2007, paper shredder sales were approximately \$471 million.

In 2012, $t = 2012 - 1990 = 22$

$$P(22) = 133.1747919e^{0.0743127113(22)} \approx 683.032$$

In 2012, paper shredder sales were approximately \$683 million.

c) $P(t) = 500$

$$133.1747919e^{0.0743127113t} = 500$$
$$e^{0.0743127113t} = \frac{500}{133.1747919}$$
$$\ln e^{0.0743127113t} = \ln\left(\frac{500}{133.1747919}\right)$$
$$0.0743127113t = \ln\left(\frac{500}{133.1747919}\right)$$
$$t = \frac{\ln\left(\frac{500}{133.1747919}\right)}{0.0743127113}$$
$$t \approx 17.8$$

It will take approximately 17.8 years for the total sales of paper shredders to reach \$500 million. This should occur in the year 2007.

d) $T = \frac{\ln 2}{k}$

$$T = \frac{\ln 2}{0.0743127113}$$
$$\approx 9.3$$

The sales of paper shredders will double approximately every 9.3 years.

e) $\boxed{tw}$ The function in exercise 25 gives a lower sales estimate and a longer doubling time then the function in exercise 26. Answers will vary as to which one of these functions is "better".

27. If we let $t = 0$ correspond to the year 1626 the initial value of Manhattan is $V_0 = 24$ Using the exponential growth function $V(t) = V_0 e^{kt}$.
Assuming an exponential rate of inflation of 5% means that $k = 0.05$. Substituting these values into the growth function gives us:
$V(t) = 24e^{0.05t}$.
In 2010, $t = 2010 - 1626 = 384$

$$\begin{aligned} V(384) &= 24e^{0.05(384)} \\ &= 24e^{19.2} \\ &\approx 5{,}231{,}970{,}592 \end{aligned}$$

Manhattan Island will be worth approximately $5,231,970,592 or $5.23 billion.

28. Let $R(t)$ be the revenue of Intel in billions of dollars. If $t = 0$ corresponds to 1986, $R_0 = 1.265$. The exponential growth model is:
$R(t) = 1.265e^{kt}$
In 2005, $t = 2005 - 1986 = 19$
$R(19) = 38.8$

$$1.265e^{k(19)} = 38.8$$
$$e^{19k} = \frac{38.8}{1.265}$$
$$19k = \ln\left(\frac{38.8}{1.265}\right)$$
$$k = \frac{\ln\left(\frac{38.8}{1.265}\right)}{19}$$
$$k \approx 0.1802$$

$R(t) = 1.265e^{0.1802t}$
In 2012, $t = 2012 - 1986 = 26$
$R(26) = 1.265e^{0.1802(26)} \approx 137.04$
Intel's total revenue will be $137.04 billion in 2012 according to the model.

29. Let $S(t)$ be the average salary in dollars t years after 1970. Therefore $S_0 = 29{,}303$. Therefore the exponential growth model is $S(t) = 29{,}303e^{kt}$. Use the average salary in 2005 to find the growth rate.

In 2005, $t = 2005 - 1970 = 35$,
$S(35) = 2{,}632{,}655$, so:

$$2{,}632{,}655 = 29{,}303e^{k(35)}$$
$$\frac{2{,}632{,}655}{29{,}303} = e^{35k}$$
$$\ln\left(\frac{2{,}632{,}655}{29{,}303}\right) = \ln e^{35k}$$
$$\ln\left(\frac{2{,}632{,}655}{29{,}303}\right) = 35k$$
$$\frac{\ln\left(\frac{2{,}632{,}655}{29{,}303}\right)}{35} = k$$
$$0.1285 \approx k$$

The growth rate is 12.85% per year.
Thus, the exponential growth model is
$S(t) = 29{,}303e^{.1285t}$
In 2010, $t = 2010 - 1970 = 40$
$S(40) = 29{,}303e^{0.1285(40)} \approx 5{,}002{,}484.16$
The average salary will be $5,002,484 in 2010.
In 2020, $t = 2020 - 1970 = 50$
$S(50) = 29{,}303e^{0.1285(50)} \approx 18{,}082{,}319.20$
The average salary will be $18,082,319 in 2020.

30. If we let $t = 0$ correspond to 1962, then $P_0 = 4$, the exponential growth function is given by $P(t) = P_0 e^{kt}$ so:
$P(t) = 4e^{kt}$.
To find the growth rate, use the fact that in 2007, the price of a stamp was 39 cents.
$t = 2007 - 1962 = 45$ so

$$39 = 4e^{k(45)}$$
$$\frac{39}{4} = e^{45k}$$
$$\ln\left(\frac{39}{4}\right) = 45k$$
$$\frac{\ln\left(\frac{39}{4}\right)}{45} = k$$
$$0.0506 \approx k$$

The growth rate will be approximately 5.06% per year.
Therefore, $P(t) = 4e^{0.0506t}$.
In 2010, $t = 2010 - 1962 = 48$
$P(48) = 4e^{0.0506(48)} \approx 45.4$
A stamp will cost approximately 45 cents in 2010.

In 2020, $t = 2020 - 1962 = 58$

$P(58) = 4e^{0.0506(58)} \approx 75.3$

A stamp will cost approximately 75 cents in 2020.

31. $P(x) = \dfrac{100}{1+49e^{-0.13x}}$

a) $$P(0) = \frac{100}{1+49e^{-0.13(0)}} = \frac{100}{1+49e^{0}} = \frac{100}{1+49} = \frac{100}{50} = 2$$

About 2% purchased the game without seeing the advertisement.

b) $$P(5) = \frac{100}{1+49e^{-0.13(5)}} = \frac{100}{1+49e^{-0.65}} \approx 3.8$$

About 3.8% will purchase the game after the advertisement runs 5 times.

$$P(10) = \frac{100}{1+49e^{-0.13(10)}} = \frac{100}{1+49e^{-1.3}} \approx 7.0$$

About 7.0% will purchase the game after the advertisement runs 10 times.

$$P(20) = \frac{100}{1+49e^{-0.13(20)}} = \frac{100}{1+49e^{-2.6}} \approx 21.6$$

About 21.6% will purchase the game after the advertisement runs 20 times.

$$P(30) = \frac{100}{1+49e^{-0.13(30)}} = \frac{100}{1+49e^{-3.9}} \approx 50.2$$

About 50.2% will purchase the game after the advertisement runs 30 times.

$$P(50) = \frac{100}{1+49e^{-0.13(50)}} = \frac{100}{1+49e^{-6.5}} \approx 93.1$$

About 93.1% will purchase the game after the advertisement runs 50 times.

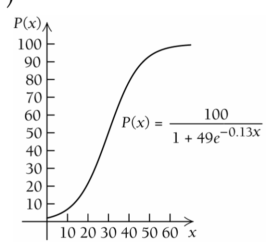
$$P(60) = \frac{100}{1+49e^{-0.13(60)}} = \frac{100}{1+49e^{-7.8}} \approx 98.0$$

About 98.0% will purchase the game after the advertisement runs 60 times.

c) We apply the Quotient Rule to take the derivative.

$$P'(x) = \frac{\left(1+49e^{-0.13x}\right)(0) - 49(-0.13)e^{-0.13x}\cdot 100}{\left(1+49e^{-0.13x}\right)^2} = \frac{637e^{-0.13x}}{\left(1+49e^{-0.13x}\right)^2}$$

d) The derivative $P'(x)$ exists for all real numbers. The equation $P'(x) = 0$ has no solution. Thus, the function has no critical points and hence, no relative extrema. $P'(x) > 0$ for all real numbers, so $P(x)$ is increasing on $[0, \infty)$. The second derivative can be used to show that the graph has an inflection point at $(29.9, 50)$. The function is concave up on the interval $(0, 29.9)$ and concave down on the interval $(29.9, \infty)$



32. Use the information in the problem to develop the exponential growth function $P(t)=0.05e^{0.097t}$. Where $P(t)$ is price in dollars, and t is time since 1962.

a) In 2010, $t=2010-1962=48$

$P(48)=0.05e^{0.097(48)}\approx 5.26$

A Hershey bar will cost \$5.26 in 2010 according to this model.

In 2015, $t=2015-1962=53$

$P(53)=0.05e^{0.097(53)}\approx 8.54$

A Hershey bar will cost \$8.54 in 2015 according to this model.

b) $\boxed{tw}$ No, the prices are not realistic given the current price of a Hershey bar. The rate of growth in the price of a Hershey bar has more than likely leveled off between 1962 and today.

33. $T=\frac{\ln 2}{k}=\frac{\ln 2}{0.035}\approx 19.8$

The doubling time T is 19.8 years.

34. $k=\frac{\ln 2}{T}=\frac{\ln 2}{69.31}\approx 0.01$

The growth rate k is 0.01 or 1% per year.

35. $k=\frac{\ln 2}{T}=\frac{\ln 2}{6.931}\approx 0.10$

The growth rate k is 0.10 or 10% per year.

36. $k=\frac{\ln 2}{T}=\frac{\ln 2}{17.3}\approx 0.04$

The growth rate k is 0.04 or 4% per year.

37. $T=\frac{\ln 2}{k}=\frac{\ln 2}{0.02794}\approx 24.8$

The doubling time T is 24.8 years.

38. $k=\frac{\ln 2}{T}=\frac{\ln 2}{19.8}\approx 0.035$

The growth rate k is 0.035 or 3.5% per year.

39. Let $t=0$ correspond to 1972. Then the initial population of grizzly bears is $P_0=190$. The set up of the exponential growth function is $P(t)=190e^{kt}$. In 2005, $t=2005-1972=33$. The population of grizzly bears had grown to 610, hence $P(33)=610$. Use this information to find the growth rate k.

$$610=190e^{k(33)}$$
$$\frac{610}{190}=e^{33k}$$
$$\ln\left(\frac{610}{190}\right)=33k$$
$$\frac{\ln\left(\frac{610}{190}\right)}{33}=k$$
$$0.035\approx k$$

The growth rate is approximately 3.5%. Now we can use the information to find the exponential growth function.

$P(t)=190e^{0.035t}$

In 2012, $t=2012-1972=40$

$P(40)=190e^{0.035(40)}\approx 770$

Yellowstone National Park will be home to approximately 770 grizzly bears in 2012.

40. Let $t=0$ correspond to 1776.

a) $P(t)=2{,}508{,}000e^{kt}$

$$216{,}000{,}000=2{,}508{,}000e^{k(200)}$$
$$\frac{216{,}000{,}000}{2{,}508{,}000}=e^{200k}$$
$$\ln\left(\frac{216{,}000{,}000}{2{,}508{,}000}\right)=200k$$
$$\frac{\ln\left(\frac{216{,}000{,}000}{2{,}508{,}000}\right)}{200}=k$$
$$0.0223$$

The U.S. population was growing at a rate of 2.23% per year. Therefore, the exponential growth model is $P(t)=2{,}508{,}000e^{0.0223t}$.

b) $\boxed{tw}$ It is reasonable to assume that the population of the United States grew exponentially between 1776 and 1976, rather than linearly or in some other pattern.

41. From Example 7 we know: $R(b)=e^{21.4b}$

For the risk of an accident to be 80% we have

$$80=e^{21.4b}$$
$$\ln 80=\ln e^{21.4b}$$
$$\ln 80=21.4b$$
$$\frac{\ln 80}{21.4}=b$$
$$0.205\approx b$$

When the blood alcohol level is 0.205%, the risk of having an accident is 80%.

42. From Example 7 we know: $R(b)=e^{21.4b}$
For the risk of and accident to be 90% we have

$$90=e^{21.4b}$$
$$\ln 90=21.4b$$
$$\frac{\ln 90}{21.4}=b$$
$$0.21\approx b$$

When the blood alcohol level is 0.21%, the risk of having an accident is 80%.

43. $P(t)=\dfrac{5780}{1+4.78e^{-0.4t}}$

a) $P(0)=\dfrac{5780}{1+4.78e^{-0.4(0)}}=\dfrac{5780}{1+4.78e^{0}}=\dfrac{5780}{1+4.78}=1000$

The island population in year zero is 1000.

$$P(1)=\frac{5780}{1+4.78e^{-0.4(1)}}=\frac{5780}{1+4.78e^{-0.4}}\approx 1375$$

The island population after 1 year is 1375.

$$P(2)=\frac{5780}{1+4.78e^{-0.4(2)}}=\frac{5780}{1+4.78e^{-0.8}}\approx 1836$$

The island population after 2 years is 1836.

$$P(5)=\frac{5780}{1+4.78e^{-0.4(5)}}=\frac{5780}{1+4.78e^{-2}}\approx 3510$$

The island population after 5 years is 3510.

$$P(10)=\frac{5780}{1+4.78e^{-0.4(10)}}=\frac{5780}{1+4.78e^{-4}}\approx 5315$$

The island population after 10 years is 5315.

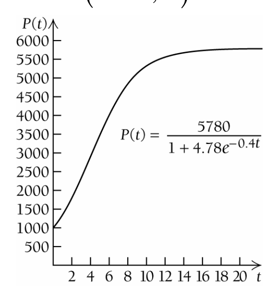
$$P(20)=\frac{5780}{1+4.78e^{-0.4(20)}}=\frac{5780}{1+4.78e^{-8}}\approx 5771$$

The island population after 20 years is 5771.

b) We apply the Quotient Rule to take the derivative.

$$P'(t)=\frac{\left(1+4.78e^{-0.4t}\right)(0)-4.78(-0.4)e^{-0.4t}\cdot 5780}{\left(1+4.78e^{-0.4t}\right)^2}=\frac{11,051.36e^{-0.4t}}{\left(1+4.78e^{-0.4t}\right)^2}$$

c) The derivative $P'(t)$ exists for all real numbers. The equation $P'(t)=0$ has no solution. Thus, the function has no critical points and hence, no relative extrema. $P'(t)>0$ for all real numbers, so $P(t)$ is increasing on $[0,\infty)$. The second derivative can be used to show that the graph has an inflection point at $(3.911, 2890)$. The function is concave up on the interval $(0, 3.911)$ and concave down on the interval $(3.911,\infty)$



44. $P(t)=\dfrac{2500}{1+5.25e^{-0.32t}}$

a) $P(0)=\dfrac{2500}{1+5.25e^{-0.32(0)}}=400$

The initial trout population is 400.

$$P(1)=\frac{2500}{1+5.25e^{-0.32(1)}}\approx 520$$

The trout population after 1 month is 520.

$$P(5)=\frac{2500}{1+5.25e^{-0.32(5)}}\approx 1214$$

The trout population after 5 months is 1214.

$P(10) = \dfrac{2500}{1+5.25e^{-0.32(10)}} \approx 2059$

The trout population after 10 months is 2059.

$P(15) = \dfrac{2500}{1+5.25e^{-0.32(15)}} \approx 2396$

The trout population after 15 months is 2396.

$P(20) = \dfrac{2500}{1+5.25e^{-0.32(20)}} \approx 2478$

The trout population after 20 months is 2478.

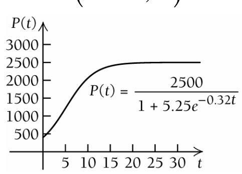
b) We apply the Quotient Rule to take the derivative.

$P'(t)$

$= \dfrac{\left(1+5.25e^{-0.32t}\right)(0) - 5.25(-0.32)e^{-0.32t} \cdot 2500}{\left(1+5.25e^{-0.32t}\right)^2}$

$= \dfrac{4200e^{-0.32t}}{\left(1+5.25e^{-0.32t}\right)^2}$

c) The derivative $P'(t)$ exists for all real numbers. The equation $P'(t) = 0$ has no solution. Thus, the function has no critical points and hence, no relative extrema. $P'(t) > 0$ for all real numbers, so $P(t)$ is increasing on $[0,\infty)$. The second derivative can be used to show that the graph has an inflection point at $(5.182, 1250)$. The function is concave up on the interval $(0, 5.182)$ and concave down on the interval $(5.182, \infty)$



45. Let $t = 0$ correspond to 1930. Thus the initial number of women earning bachelor's degrees is $P_0 = 48{,}869$. So the exponential growth function is $P(t) = 48{,}869e^{kt}$. In 2005, $t = 2005 - 1930 = 75$, approximately 832,000 women received a degree. Using this information we can find the exponential growth rate k. Substitute the information into the growth function.

$832{,}000 = 48{,}869e^{k(75)}$

$\dfrac{832{,}000}{48{,}869} = e^{75k}$

$\ln\left(\dfrac{832{,}000}{48{,}869}\right) = \ln e^{75k}$

$\ln\left(\dfrac{832{,}000}{48{,}869}\right) = 75k$

$\dfrac{\ln\left(\dfrac{832{,}000}{48{,}869}\right)}{75} = k$

$0.0378 \approx k$

The exponential growth rate is 0.0378 or 3.78%. Therefore the exponential growth function is: $P(t) = 48{,}869e^{0.0378t}$, where t is time in years since 1930 and $P(t)$ is number of women earning bachelor's degrees.

46. $p(t) = 1 - e^{-0.28t}$

a) $p(1) = 1 - e^{-0.28(1)} \approx 0.244$

$p(2) = 1 - e^{-0.28(2)} \approx 0.429$

$p(5) = 1 - e^{-0.28(5)} \approx 0.753$

$p(11) = 1 - e^{-0.28(11)} \approx 0.954$

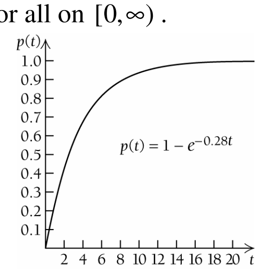
$p(16) = 1 - e^{-0.28(16)} \approx 0.989$

$p(20) = 1 - e^{-0.28(20)} \approx 0.996$

b) $p'(t) = 0.28e^{-0.28t}$

c) The derivative $p'(t)$ exists for all real numbers. The equation $p'(t) = 0$ has no solution. Thus, the function has no critical points and hence, no relative extrema. $p'(t) > 0$ for all real numbers, so $p(t)$ is increasing on $[0,\infty)$.

$p''(t) = -0.0784e^{-0.28t}$, so $p''(t) < 0$ for all real numbers and hence $p(t)$ is concave down for all on $[0,\infty)$.



47. $P(t)=100\left(1-e^{-0.4t}\right)$

a) $P(0)=100\left(1-e^{-0.4(0)}\right)$

$=100(1-1)$

$=0$

0% of doctors are prescribing this medication after zero months.

$P(1)=100\left(1-e^{-0.4(1)}\right)$

$=100\left(1-e^{-0.4}\right)$

≈ 33.0

33% of doctors are prescribing this medication after 1 month.

$P(2)=100\left(1-e^{-0.4(2)}\right)$

$=100\left(1-e^{-0.8}\right)$

≈ 55.1

55% of doctors are prescribing this medication after 2 months.

$P(3)=100\left(1-e^{-0.4(3)}\right)$

$=100\left(1-e^{-1.2}\right)$

≈ 69.9

70% of doctors are prescribing this medication after 3 months.

$P(5)=100\left(1-e^{-0.4(5)}\right)$

$=100\left(1-e^{-2}\right)$

≈ 86.47

86% of doctors are prescribing this medication after 5 months.

$P(12)=100\left(1-e^{-0.4(12)}\right)$

$=100\left(1-e^{-4.8}\right)$

≈ 99.2

99.2% of doctors are prescribing this medication after 12 months.

$P(16)=100\left(1-e^{-0.4(16)}\right)$

$=100\left(1-e^{-6.4}\right)$

≈ 99.8

99.8% of doctors are prescribing this medication after 16 months.

b) $P'(t)=100\left[0-(-0.4)e^{-0.4t}\right]$

$=100\left(0.4e^{-0.4t}\right)$

$=40e^{-0.4t}$

Therefore,

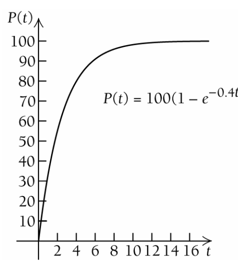
$P'(7)=40e^{-0.4(7)}$

$=40e^{-2.8}$

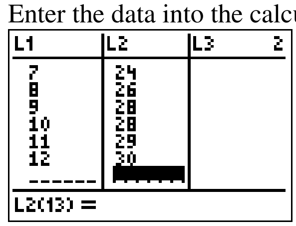
≈ 2.432

At 7 months, the percentage of doctors who are prescribing the medicine is growing at a rate of 2.4% per month.

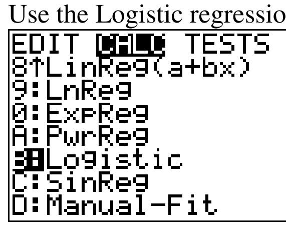
c) The derivative $P'(t)$ exists for all real numbers. The equation $P'(t)=0$ has no solution. Thus, the function has no critical points and hence, no relative extrema. $P'(t)>0$ for all real numbers, so $P(t)$ is increasing on $[0,\infty)$. $P''(t)=-16e^{-0.4t}$,so $P''(t)<0$ for all real numbers and hence $P(t)$ is concave down for all on $[0,\infty)$.



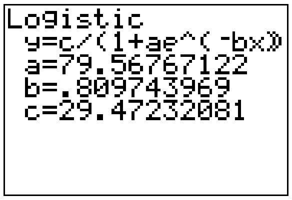
48. a) Enter the data into the calculator.



Use the Logistic regression.



We have:



The calculator determined the logistic growth function to be:

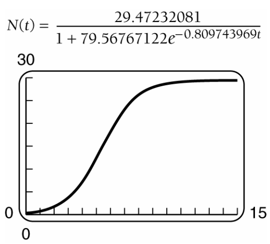
$$y=\frac{29.47232081}{1+79.56767122e^{-0.809743969x}}$$

Converting the calculator to function notation we get:

$$N(t)=\frac{29.47232081}{1+79.56767122e^{-0.809743969t}}$$

b) We round down since the model deals with people. The limiting value appears to be 29 people.

c) The graph is:



d) Use the quotient rule to find $N'(t)$.

$$N'(t)=\frac{1898.885181e^{-0.809743969t}}{\left(1+79.56767122e^{-0.809743969t}\right)^2}$$

e) $\boxed{tw}$ $\lim_{t\to\infty} N'(t)=0$; this indicates that eventually the population does not change. That is, it reaches and remains at the limiting value.

Note: In Exercises 49 – 56, it is a good idea to look at the data before you make assumptions about a particular model. Given a data set, all of these solutions could change.

49. $\boxed{tw}$ The growth in value of a U.S. savings bond would be considered exponential. Although the maturity date is set, the interest still grows exponentially. The domain is the length of time you own the bond and would have to be positive and less than the number of years to the maturity date.

50. $\boxed{tw}$ The growth of Zachary's hair following a haircut would tend to be linear. Hair grows at a constant rate, so exponential growth is out of the question. The domain is the length of time after the haircut and would have to be positive.

51. $\boxed{tw}$ At first this might appear to be exponential, but since there is a finite number of computers on the campus, the growth would be logistic. Eventually there would be a limiting value. The domain is the time since the first computer was infected and must be positive.

52. $\boxed{tw}$ Since there is a drop and then a rise in the water level, we would tend to think the model might be quadratic. The domain is time and should be positive.

53. $\boxed{tw}$ Sales of organic foods is probably best modeled as exponential growth since there does not have to be a limiting value. Sales can increase without bound. The domain would be positive.

54. $\boxed{tw}$ Since we are talking about the percentage of produce, the best model would be logistic growth. There will be a limiting value. You can not genetically modify more than 100% of the produce. The domain would need to be positive.

55. $\boxed{tw}$ Because there is a limiting value, the logistic growth model is the best model. The domain would be years since 1995 and need to be positive.

56. $\boxed{tw}$ It could be that there is a limiting value and logistic growth is the way to approach the model. The domain is positive.