105. $y=(x-3)^2$

First square the binomial.

$$\begin{aligned} y&=(x-3)^2\\ &=(x-3)(x-3)\\ &=x^2-6x+9 \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{dy}{dx}&=\frac{d}{dx}\left(x^2-6x+9\right)\\ &=\frac{d}{dx}\left(x^2\right)-\frac{d}{dx}(6x)+\frac{d}{dx}(9)\\ &=2x-6 \end{aligned}$$

106. $y=\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)^2$

$$\begin{aligned} y&=\left(x^{1/2}-x^{-1/2}\right)^2\\ &=\left(x^{1/2}-x^{-1/2}\right)\left(x^{1/2}-x^{-1/2}\right)\\ &=x-2x^0+x^{-1}\\ &=x+x^{-1}-2 \end{aligned}$$

$$\frac{dy}{dx}=1-2x^{-2}$$

107. $y=\left(\sqrt{x}+\sqrt[3]{x}\right)^2$

First rewrite the radicals as expressions with rational exponents, and then square the binomial.

$$\begin{aligned} y&=\left(x^{1/2}+x^{1/3}\right)^2\\ &=\left(x^{1/2}+x^{1/3}\right)\left(x^{1/2}+x^{1/3}\right)\\ &=x^{1/2+1/2}+2x^{1/2+1/3}+x^{1/3+1/3}\\ &=x+2x^{5/6}+x^{2/3} \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{dy}{dx}&=\frac{d}{dx}\left(x+2x^{5/6}+x^{2/3}\right)\\ &=\frac{d}{dx}(x)+\frac{d}{dx}\left(2x^{5/6}\right)+\frac{d}{dx}\left(x^{2/3}\right)\\ &=1+2\left(\frac{5}{6}x^{5/6-1}\right)+\frac{2}{3}x^{2/3-1}\\ &=1+\frac{5}{3}x^{-1/6}+\frac{2}{3}x^{-1/3}. \end{aligned}$$

108. $y=(x+1)^3=x^3+3x^2+3x+1$

$$\frac{dy}{dx}=3x^2+6x+3$$

109. First, we rewrite 1 using properties of exponents. We know $x^0=1$, therefore, $\frac{d}{dx}(1)=\frac{d}{dx}\left(x^0\right)$. Applying theorem 1, we have

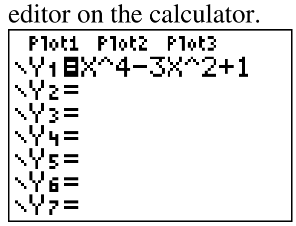
$$\begin{aligned} \frac{d}{dx}(1)&=\frac{d}{dx}\left(x^0\right)\\ &=0x^{0-1}\\ &=0x^{-1}\\ &=0. \end{aligned}$$

110. $\boxed{tw}$ Answers will vary. Leibniz notation is more convenient when the variables are named to identify what they are. For example, marginal revenue using Leibniz notation is $\frac{dR}{dp}$. This notation allows the reader to determine that marginal revenue is the rate of change of revenue with respect to price. Thus Leibniz notation give the reader more information about the problem then the function notation $R'(p)$.

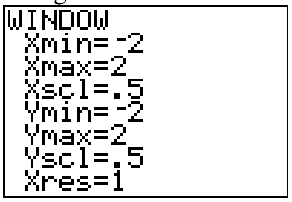
111. $\boxed{tw}$ Answers will vary. Papers on the German mathematician Leibniz (1646-1716) should include discussions on his independent discovery of calculus in 1675, as well as his invention of the binary number system and that of an early calculating machine. Papers on the English mathematician Newton (1642-1727) should include discussions on his discovery of the principles of calculus, theory of equations, and his generalizations of the binomial theorem; as well as his work on universal gravitation and optics.

112. $f(x)=x^4-3x^2+1$

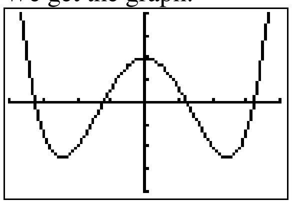
First we enter the equation into the graphing editor on the calculator.



Using the window:



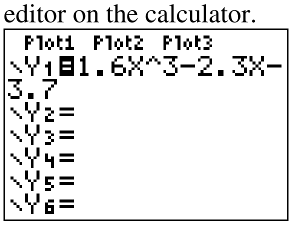
We get the graph:



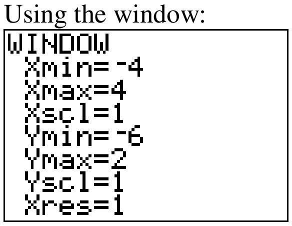
We estimate the x-values at which the tangent lines are horizontal are $x = -1.225,\ x = 0,$ and, $x = 1.225$.

113. $f(x) = 1.6x^3 - 2.3x - 3.7$

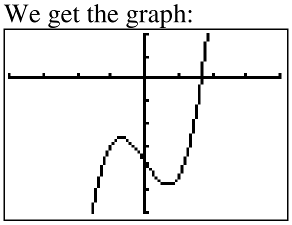
First we enter the equation into the graphing editor on the calculator.



Using the window:



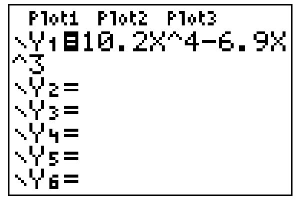
We get the graph:



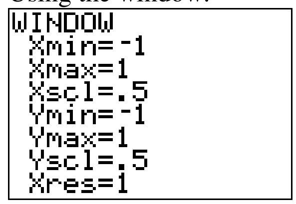
The horizontal tangents occur at the turning points of this function. Using the trace feature, or the minimum/maximum feature on the calculator, we find the turning points.
We estimate the x-values at which the tangent lines are horizontal are $x = -0.692$ and $x = 0.692$.

114. $f(x) = 10.2x^4 - 6.9x^3$

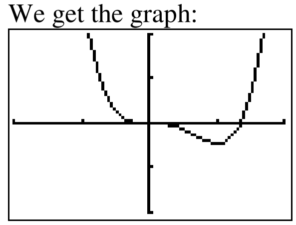
First we enter the equation into the graphing editor on the calculator.



Using the window:



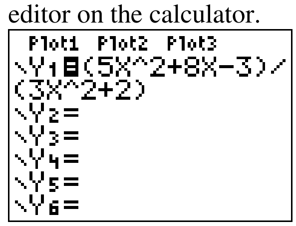
We get the graph:



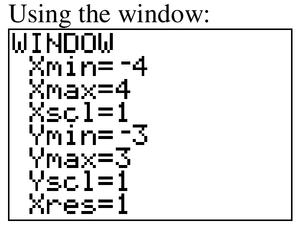
We estimate the x-values at which the tangent lines are horizontal are $x = 0$ and $x = 0.507$.

115. $f(x) = \dfrac{5x^2 + 8x - 3}{3x^2 + 2}$

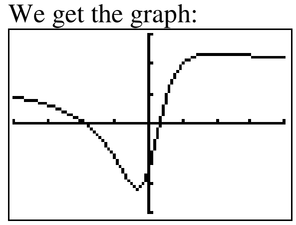
First we enter the equation into the graphing editor on the calculator.



Using the window:



We get the graph:



The horizontal tangents occur at the turning points of this function. Using the trace feature, or the minimum/maximum feature on the calculator, we find the turning points.
We estimate the x-values at which the tangent lines are horizontal are
$x = -0.346$ and $x = 1.929$.

116. $f(x) = 20x^3 - 3x^5$

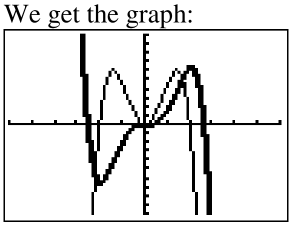
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.

```
 Plot1  Plot2  Plot3
\Y1=20X^3-3X^5
\Y2=nDeriv(Y1,X,
X)
\Y3=
\Y4=
\Y5=
\Y6=
```

Using the window:

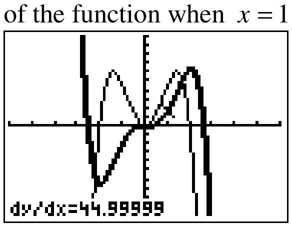
```
WINDOW
 Xmin=-6
 Xmax=6
 Xscl=1
 Ymin=-100
 Ymax=100
 Yscl=10
 Xres=1
```

We get the graph:



Note, the function $f(x)$ is the thicker graph.

Using the calculator, we can find the derivative of the function when $x = 1$.



We have $f'(1) = 45$.

117. $f(x) = x^4 - 3x^2 + 1$

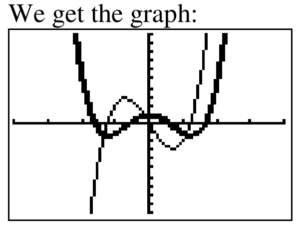
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative. The syntax is shown in the screen shot at the top of the next column.

```
 Plot1  Plot2  Plot3
\Y1=X^4-3X^2+1
\Y2=nDeriv(Y1,X,
X)
\Y3=
\Y4=
\Y5=
\Y6=
```

Using the window:

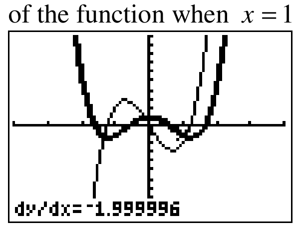
```
WINDOW
 Xmin=-4
 Xmax=4
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

We get the graph:



Note, the function $f(x)$ is the thicker graph.

Using the calculator, we can find the derivative of the function when $x = 1$.



We have $f'(1) = -2$.

118. $f(x) = x^3 - 2x - 2$

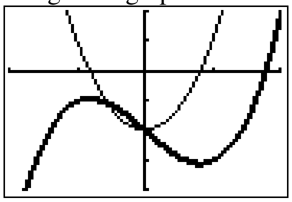
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.

```
 Plot1  Plot2  Plot3
\Y1=X^3-2X-2
\Y2=nDeriv(Y1,X,
X)
\Y3=
\Y4=
\Y5=
\Y6=
```

Using the window:

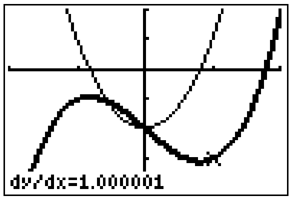
```
WINDOW
 Xmin=-2
 Xmax=2
 Xscl=1
 Ymin=-4
 Ymax=2
 Yscl=1
 Xres=1
```

We get the graph:



Note, the function $f(x)$ is the thicker graph.

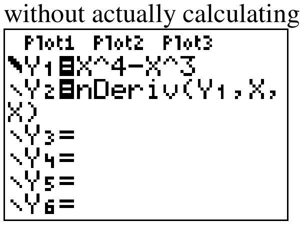
Using the calculator, we can find the derivative of the function when $x = 1$.



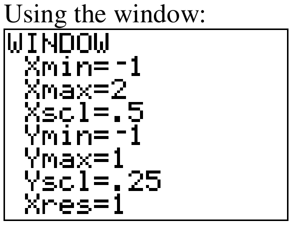
We have $f'(1) = 1$.

119. $f(x) = x^4 - x^3$

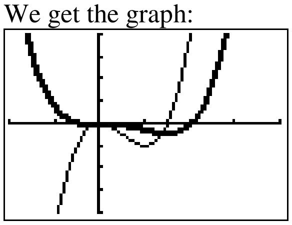
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:

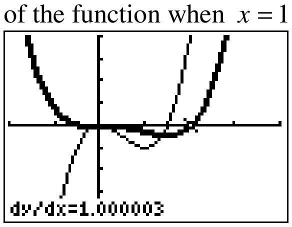


We get the graph:



Note, the function $f(x)$ is the thicker graph.

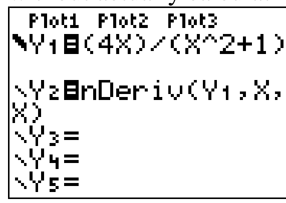
Using the calculator, we can find the derivative of the function when $x = 1$.



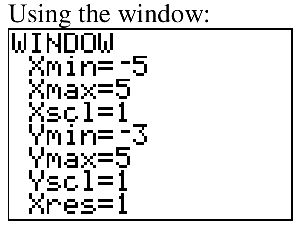
We have $f'(1) = 1$.

120. $f(x) = \frac{4x}{x^2 + 1}$

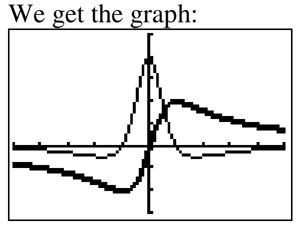
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:

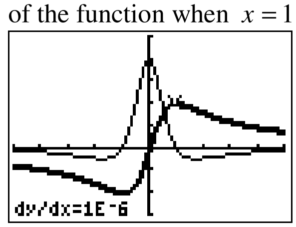


We get the graph:



Note, the function $f(x)$ is the thicker graph.

Using the calculator, we can find the derivative of the function when $x = 1$.



We have $f'(1) = 0$.

121. $f(x) = \frac{5x^2 + 8x - 3}{3x^2 + 2}$

Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.