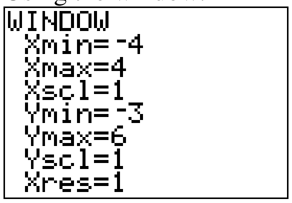
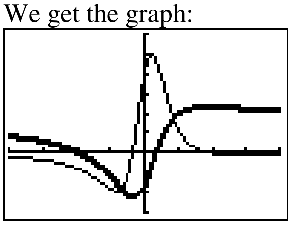
Using the window:

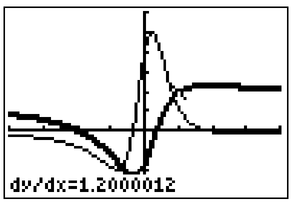


We get the graph:



Note, the function $f(x)$ is the thicker graph.

Using the calculator, we can find the derivative of the function when $x = 1$.



We have $f'(1) = 1.2$.

Exercise Set 1.6

1. Differentiate $y = x^5 \cdot x^6$ using the Product Rule (Theorem 5).

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^5 \cdot x^6\right)\\ &= x^5 \cdot \frac{d}{dx}\left(x^6\right) + x^6 \frac{d}{dx}\left(x^5\right)\\ &= x^5 \cdot 6x^5 + x^6 \cdot 5x^4\\ &= 6x^{10} + 5x^{10}\\ &= 11x^{10}\end{aligned}$$

Differentiate $y = x^5 \cdot x^6 = x^{11}$ using the Power Rule (Theorem 1).

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}x^{11}\\ &= 11x^{11-1}\\ &= 11x^{10}\end{aligned}$$

The two results are equivalent.

2. Using the Product Rule:

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^9 \cdot x^4\right)\\ &= x^9 \cdot 4x^3 + x^4 \cdot 9x^8\\ &= 4x^{12} + 9x^{12}\\ &= 13x^{12}\end{aligned}$$

Using the Power Rule:

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^4 \cdot x^9\right)\\ &= \frac{d}{dx}\left(x^{13}\right)\\ &= 13x^{12}\end{aligned}$$

The two results are equivalent.

3. Differentiate $f(x) = (2x+5)(3x-4)$ using the Product Rule (Theorem 5).

$$\begin{aligned}f'(x) &= \frac{d}{dx}\left[(2x+5)(3x-4)\right]\\ &= (2x+5)\cdot\frac{d}{dx}(3x-4) +\\ &\qquad (3x-4)\cdot\frac{d}{dx}(2x+5)\\ &= (2x+5)\cdot 3 + (3x-4)\cdot 2\\ &= 6x+15+6x-8\\ &= 12x+7\end{aligned}$$

Differentiate $f(x) = (2x+5)(3x-4)$ using the Power Rule (Theorem 1). First, we multiply the binomial terms in the function.

$$\begin{aligned}f(x) &= (2x+5)(3x-4)\\ &= 6x^2+7x-20\end{aligned}$$

Therefore, by Theorem 1 and Theorem 4 we have:

$$\begin{aligned}f'(x) &= \frac{d}{dx}\left(6x^2+7x-20\right)\\ &= \frac{d}{dx}\left(6x^2\right) + \frac{d}{dx}(7x) - \frac{d}{dx}(20) && \text{Theorem 4}\\ &= 12x+7 && \text{Theorem 1}\end{aligned}$$

The two results are equivalent.

4. Using the Product Rule:

$$\begin{aligned}g'(x) &= \frac{d}{dx}\left[(3x-2)(4x+1)\right]\\ &= (3x-2)\cdot 4 + (4x+1)\cdot 3\\ &= 12x-8+12x+3\\ &= 24x-5\end{aligned}$$

Using the Power Rule:

$g(x)=(3x-2)(4x+1)=12x^2-5x-2$

$$g'(x)=\frac{d}{dx}(12x^2-5x-2)$$
$$=24x-5$$

The two results are equivalent.

5. Differentiate $G(x)=4x^2(x^3+5x)$ using the Product Rule.

$$G'(x)=\frac{d}{dx}\left[4x^2(x^3+5x)\right]$$
$$=4x^2\cdot\frac{d}{dx}(x^3+5x)+(x^3+5x)\cdot\frac{d}{dx}(4x^2)$$
$$=4x^2\cdot(3x^2+5)+(x^3+5x)\cdot(8x)$$
$$=12x^4+20x^2+8x^4+40x^2$$
$$=20x^4+60x^2$$

Differentiate $G(x)=4x^2(x^3+5x)$ using the Power Rule. First, we multiply the function.

$$G(x)=4x^2(x^3+5x)$$
$$=4x^5+20x^3$$

Therefore, we have:

$$G'(x)=\frac{d}{dx}(4x^5+20x^3)$$
$$=\frac{d}{dx}(4x^5)+\frac{d}{dx}(20x^3) \quad \text{Theorem 4}$$
$$=4(5x^4)+20(3x^2) \quad \text{Theorem 1, Theorem 3}$$
$$=20x^4+60x^2$$

The two results are equivalent.

6. Using the Product Rule:

$$F'(x)=\frac{d}{dx}\left[3x^4(x^2-4x)\right]$$
$$=3x^4\cdot(2x-4)+(x^2-4x)\cdot12x^3$$
$$=6x^5-12x^4+12x^5-48x^4$$
$$=18x^5-60x^4$$

Using the Power Rule:

$F(x)=3x^4(x^2-4x)=3x^6-12x^5$

$$F'(x)=\frac{d}{dx}(3x^6-12x^5)$$
$$=18x^5-60x^4$$

The two results are equivalent.

7. Differentiate $y=(3\sqrt{x}+2)x^2$ using the Product Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left[(3\sqrt{x}+2)x^2\right]$$
$$=(3\sqrt{x}+2)\cdot\frac{d}{dx}(x^2)+x^2\cdot\frac{d}{dx}(3\sqrt{x}+2)$$
$$=(3x^{1/2}+2)\cdot\frac{d}{dx}(x^2)+x^2\cdot\frac{d}{dx}(3x^{1/2}+2)$$
$$=(3x^{1/2}+2)\cdot2x+x^2\cdot\frac{3}{2}x^{-1/2} \quad \text{Theorem 1}$$
$$=6x^{3/2}+4x+\frac{3}{2}x^{3/2}$$
$$=\frac{15}{2}x^{3/2}+4x$$

Differentiate $y=(3\sqrt{x}+2)x^2$ using the Power Rule. First, we multiply the function.

$$y=(3\sqrt{x}+2)x^2$$
$$=3x^{1/2+2}+2x^2$$
$$=3x^{5/2}+2x^2$$

Therefore, we have:

$$\frac{dy}{dx}=\frac{d}{dx}\left(3x^{5/2}+2x^2\right)$$
$$=\frac{d}{dx}\left(3x^{5/2}\right)+\frac{d}{dx}(2x^2) \quad \text{Theorem 4}$$
$$=3\left(\frac{5}{2}x^{5/2-1}\right)+2(2x^1) \quad \text{Theorem 1, Theorem 3}$$
$$=\frac{15}{2}x^{3/2}+4x$$

The two results are equivalent.

8. Using the Product Rule:

$$\frac{dy}{dx}=\frac{d}{dx}\left[(4\sqrt{x}+3)x^3\right]$$
$$=(4x^{1/2}+3)\cdot\frac{d}{dx}(x^3)+x^3\cdot\frac{d}{dx}(4x^{1/2}+3)$$
$$=(4x^{1/2}+3)\cdot3x^2+x^2\cdot\frac{4}{2}x^{-1/2}$$
$$=14x^{5/2}+9x^2$$

Using the Power Rule.

$$y = \left(4\sqrt{x}+3\right)x^3 = 4x^{7/2}+3x^3$$

Therefore, we have:

$$\frac{dy}{dx} = \frac{d}{dx}\left(4x^{7/2}+3x^3\right)$$
$$= 4\left(\frac{7}{2}x^{7/2-1}\right)+3\left(3x^2\right)$$
$$= 14x^{5/2}+9x^2$$

The two results are equivalent.

9. Differentiate $g(x) = (4x-3)\left(2x^2+3x+5\right)$ using the Product Rule.

$$g'(x) = \frac{d}{dx}\left[(4x-3)\left(2x^2+3x+5\right)\right]$$
$$= (4x-3)\cdot\frac{d}{dx}\left(2x^2+3x+5\right)+$$
$$\left(2x^2+3x+5\right)\cdot\frac{d}{dx}(4x-3)$$
$$= (4x-3)\cdot(4x+3)+\left(2x^2+3x+5\right)\cdot 4$$
$$= 16x^2-9+8x^2+12x+20$$
$$= 24x^2+12x+11$$

Differentiate $g(x) = (4x-3)\left(2x^2+3x+5\right)$ using the Power Rule. First, we multiply the terms in the function.

$$g(x) = (4x-3)\left(2x^2+3x+5\right)$$
$$= 8x^3+6x^2+11x-15$$

Therefore, we have:

$$g'(x) = \frac{d}{dx}\left(8x^3+6x^2+11x-15\right)$$
$$= \frac{d}{dx}\left(8x^3\right)+\frac{d}{dx}\left(6x^2\right)+$$
$$\frac{d}{dx}(11x)-\frac{d}{dx}(15)$$
$$= 24x^2+12x+11$$

The two results are equivalent.

10. Using the Product Rule:

$$f'(x) = \frac{d}{dx}\left[(2x+5)\left(3x^2-4x+1\right)\right]$$
$$f'(x) = (2x+5)\cdot(6x-4)+\left(3x^2-4x+1\right)\cdot 2$$
$$= 12x^2+22x-20+6x^2-8x+2$$
$$= 18x^2+14x-18$$

Using the Power Rule:

$$f(x) = \left[(2x+5)\left(3x^2-4x+1\right)\right]$$
$$= 6x^3+7x^2-18x+5$$
$$f'(x) = \frac{d}{dx}\left(6x^3+7x^2-18x+5\right)$$
$$= 18x^2+14x-18$$

The two results are equivalent.

11. Differentiate $F(t) = \left(\sqrt{t}+2\right)\left(3t-4\sqrt{t}+7\right)$ using the Product Rule.

$$F'(t) = \frac{d}{dt}\left[\left(\sqrt{t}+2\right)\left(3t-4\sqrt{t}+7\right)\right]$$
$$= \left(t^{1/2}+2\right)\cdot\frac{d}{dt}\left(3t-4t^{1/2}+7\right)+$$
$$\left(3t-4t^{1/2}+7\right)\cdot\frac{d}{dt}\left(t^{1/2}+2\right) \qquad \left[\sqrt{t}=t^{1/2}\right]$$
$$= \left(t^{1/2}+2\right)\cdot\left(3-4\left(\frac{1}{2}t^{-1/2}\right)\right)+$$
$$\left(3t-4t^{1/2}+7\right)\cdot\left(\frac{1}{2}t^{-1/2}\right)$$
$$= \left(t^{1/2}+2\right)\cdot\left(3-2t^{-1/2}\right)+$$
$$\left(3t-4t^{1/2}+7\right)\cdot\left(\frac{1}{2}t^{-1/2}\right)$$
$$= 3t^{1/2}-2+6-4t^{-1/2}+\frac{3}{2}t^{1/2}-2+\frac{7}{2}t^{-1/2}$$
$$= \frac{9}{2}t^{1/2}-\frac{1}{2}t^{-1/2}+2$$
$$= \frac{9\sqrt{t}}{2}-\frac{1}{2\sqrt{t}}+2$$

Differentiate $F(t) = \left(\sqrt{t}+2\right)\left(3t-4\sqrt{t}+7\right)$ using the Power Rule.

$$F(t) = \left(\sqrt{t}+2\right)\left(3t-4\sqrt{t}+7\right)$$
$$= 3t^{3/2}-4t+7t^{1/2}+6t-8t^{1/2}+14$$
$$= 3t^{3/2}-t^{1/2}+2t+14$$

Therefore, we have:

$$F'(t) = \frac{d}{dt}\left(3t^{3/2}-t^{1/2}+2t+14\right)$$
$$= \frac{d}{dt}\left(3t^{3/2}\right)-\frac{d}{dt}\left(t^{1/2}\right)+\frac{d}{dt}(2t)+\frac{d}{dt}(14)$$
$$= \frac{9t^{1/2}}{2}-\frac{1}{2}t^{-1/2}+2$$
$$= \frac{9\sqrt{t}}{2}-\frac{1}{2\sqrt{t}}+2.$$

The two results are equivalent.

12. Using the Product Rule:

$$G'(t) = \frac{d}{dt}\left[\left(2t+3\sqrt{t}+5\right)\left(\sqrt{t}+4\right)\right]$$
$$= \left(2t+3t^{1/2}+5\right)\cdot\left(\frac{1}{2}t^{-1/2}\right)+$$
$$\left(t^{1/2}+4\right)\cdot\left(2+\frac{3}{2}t^{-1/2}\right)$$
$$= t^{1/2}+\frac{3}{2}+\frac{5}{2}t^{-1/2}+2t^{1/2}+\frac{3}{2}+8+6t^{-1/2}$$
$$= 3t^{1/2}+\frac{17}{2}t^{-1/2}+11$$
$$= 3\sqrt{t}+\frac{17}{2\sqrt{t}}+11$$

Using the Power Rule:

$$G(x) = \left[\left(2t+3\sqrt{t}+5\right)\left(\sqrt{t}+4\right)\right]$$
$$= 2t^{3/2}+17t^{1/2}+11t+20$$
$$G'(t) = \frac{d}{dt}\left(2t^{3/2}+17t^{1/2}+11t+20\right)$$
$$= 3t^{1/2}+\frac{17}{2}t^{-1/2}+11$$
$$= 3\sqrt{t}+\frac{17}{2\sqrt{t}}+11$$

The two results are equivalent.

13. Differentiate $y = \frac{x^7}{x^3}$ using the Quotient Rule (Theorem 6).

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^7}{x^3}\right)$$
$$= \frac{x^3\frac{d}{dx}\left(x^7\right)-x^7\frac{d}{dx}\left(x^3\right)}{\left(x^3\right)^2}$$
$$= \frac{x^3\left(7x^6\right)-x^7\left(3x^2\right)}{x^6}$$
$$\frac{dy}{dx} = \frac{7x^9-3x^9}{x^6}$$
$$= \frac{4x^9}{x^6}$$
$$= 4x^3, \quad \text{for } x \neq 0$$

Differentiate $y = \frac{x^7}{x^3} = x^4$ using the Power Rule.

$$\frac{dy}{dx} = \frac{d}{dx}x^4$$
$$= 4x^{4-1}$$
$$= 4x^3, \quad \text{for } x \neq 0$$

The two results are equivalent.

14. Using the Quotient Rule.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^6}{x^4}\right)$$
$$= \frac{x^4\frac{d}{dx}\left(x^6\right)-x^6\frac{d}{dx}\left(x^4\right)}{\left(x^4\right)^2}$$
$$= \frac{x^4\left(6x^5\right)-x^6\left(4x^3\right)}{x^8}$$
$$= \frac{6x^9-4x^9}{x^8}$$
$$= \frac{2x^9}{x^8}$$
$$= 2x, \quad \text{for } x \neq 0$$

Using the Power Rule.

$$y = \frac{x^6}{x^4} = x^2$$
$$\frac{dy}{dx} = \frac{d}{dx}x^2$$
$$= 2x, \quad \text{for } x \neq 0$$

The two results are equivalent.

15. Differentiate $f(x) = \frac{2x^5+x^2}{x}$ using the Quotient Rule.

$$f'(x) = \frac{d}{dx}\left(\frac{2x^5+x^2}{x}\right)$$
$$= \frac{x\frac{d}{dx}\left(2x^5+x^2\right)-\left(2x^5+x^2\right)\frac{d}{dx}(x)}{(x)^2}$$
$$f'(x) = \frac{x\left(10x^4+2x\right)-\left(2x^5+x^2\right)(1)}{x^2}$$
$$= \frac{10x^5+2x^2-2x^5-x^2}{x^2}$$
$$= \frac{8x^5+x^2}{x^2}$$
$$= \frac{x^2\left(8x^3+1\right)}{x^2}$$
$$= 8x^3+1, \quad \text{for } x \neq 0$$

Differentiate $f(x)=\dfrac{2x^5+x^2}{x}$ using the Power Rule.

First, factor the numerator and divide the common factors.

$$\begin{aligned}f(x)&=\frac{2x^5+x^2}{x}\\&=\frac{x\left(2x^4+x\right)}{x}\\&=2x^4+x\end{aligned}$$

$$\begin{aligned}f'(x)&=\frac{d}{dx}\left(2x^4+x\right)\\&=8x^3+1, \qquad \text{for } x\neq 0\end{aligned}$$

The two results are equivalent.

16. Using the Quotient Rule.

$$\begin{aligned}g'(x)&=\frac{d}{dx}\left(\frac{3x^7-x^3}{x}\right)\\&=\frac{x\left(21x^6-3x^2\right)-\left(3x^7-x^3\right)(1)}{x^2}\\&=\frac{18x^7-2x^3}{x^2}\\&=\frac{x^2\left(18x^5-2x\right)}{x^2}\\&=18x^5-2x, \qquad \text{for } x\neq 0\end{aligned}$$

Using the Power Rule.

$$g(x)=\frac{3x^7-x^3}{x}=\frac{x\left(3x^6-x^2\right)}{x}=3x^6-x^2$$

$$\begin{aligned}g'(x)&=\frac{d}{dx}\left(3x^6-x^2\right)\\&=18x^5-2x, \qquad \text{for } x\neq 0\end{aligned}$$

The two results are equivalent.

17. Differentiate $G(x)=\dfrac{8x^3-1}{2x-1}$ using the Quotient Rule.

$$\begin{aligned}G'(x)&=\frac{d}{dx}\left(\frac{8x^3-1}{2x-1}\right)\\&=\frac{(2x-1)\frac{d}{dx}\left(8x^3-1\right)-\left(8x^3-1\right)\frac{d}{dx}(2x-1)}{(2x-1)^2}\\&=\frac{(2x-1)\left(24x^2\right)-\left(8x^3-1\right)(2)}{(2x-1)^2}\\&=\frac{(2x-1)\left(24x^2\right)-(2x-1)\left(4x^2+2x+1\right)(2)}{(2x-1)^2}\\&=\frac{(2x-1)\left[\left(24x^2\right)-\left(4x^2+2x+1\right)(2)\right]}{(2x-1)^2}\\&=\frac{(2x-1)\left[\left(24x^2\right)-\left(8x^2+4x+2\right)\right]}{(2x-1)^2}\\&=\frac{\left[16x^2-4x-2\right]}{(2x-1)}\\&=\frac{(2x-1)(8x+2)}{(2x-1)}\\&=8x+2; \qquad x\neq \tfrac{1}{2}\end{aligned}$$

Differentiate $G(x)=\dfrac{8x^3-1}{2x-1}$ using the Power Rule.

First, factor the numerator and divide the common factors.

$$\begin{aligned}G(x)&=\frac{8x^3-1}{2x-1}\\&=\frac{(2x-1)\left(4x^2+2x+1\right)}{2x-1} \qquad \text{Difference of cubes}\\&=4x^2+2x+1\end{aligned}$$

$$\begin{aligned}G'(x)&=\frac{d}{dx}\left(4x^2+2x+1\right)\\&=8x+2; \qquad x\neq \tfrac{1}{2}\end{aligned}$$

The two results are equivalent.

18. Using the Quotient Rule.

$$F'(x) = \frac{d}{dx}\left(\frac{x^3+27}{x+3}\right)$$
$$= \frac{(x+3)(3x^2)-(x^3+27)(1)}{(x+3)^2}$$
$$= \frac{(x+3)(3x^2)-(x+3)(x^2-3x+9)}{(x+3)^2}$$
$$= \frac{(x+3)\left(3x^2-(x^2-3x+9)\right)}{(x+3)^2}$$
$$= \frac{(2x^2+3x-9)}{(x+3)}$$
$$= \frac{(x+3)(2x-3)}{(x+3)}$$
$$= 2x-3, \qquad \text{for } x \neq -3$$

Using the Power Rule.

$$F(x) = \frac{x^3+27}{x+3}$$
$$= \frac{(x+3)(x^2-3x+9)}{x+3}$$
$$= x^2-3x+9$$
$$F'(x) = \frac{d}{dx}(x^2-3x+9)$$
$$= 2x-3, \qquad \text{for } x \neq -3$$

The two results are equivalent.

19. Differentiate $y = \frac{t^2-16}{t+4}$ using the Quotient Rule.

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{t^2-16}{t+4}\right)$$
$$= \frac{(t+4)\frac{d}{dt}(t^2-16)-(t^2-16)\frac{d}{dt}(t+4)}{(t+4)^2}$$
$$= \frac{(t+4)(2t)-(t^2-16)(1)}{(t+4)^2}$$
$$= \frac{2t^2+8t-t^2+16}{(t+4)^2}$$
$$= \frac{t^2+8t+16}{(t+4)^2}$$
$$\frac{dy}{dt} = \frac{(t+4)^2}{(t+4)^2}$$
$$= 1; \qquad t \neq -4$$

Differentiate $y = \frac{t^2-16}{t+4}$ using the Power Rule.

First, factor the numerator and divide the common factors.

$$y = \frac{t^2-16}{t+4}$$
$$= \frac{(t+4)(t-4)}{t+4} \qquad \text{Difference of squares}$$
$$= t-4$$
$$\frac{dy}{dt} = \frac{d}{dt}(x-4)$$
$$= 1, \qquad \text{for } \ t \neq -4$$

The two results are equivalent.

20. Using the Quotient Rule.

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{t^2-25}{t-5}\right)$$
$$= \frac{(t-5)(2t)-(t^2-25)(1)}{(t-5)^2}$$
$$= \frac{2t^2-10t-(t^2-25)}{(t-5)^2}$$
$$= \frac{t^2-10t+25}{(t-5)^2}$$
$$= \frac{(t-5)^2}{(t-5)^2}$$
$$= 1, \qquad \text{for } t \neq 5$$

Using the Power Rule.

$$y = \frac{t^2-25}{t-5}$$
$$= \frac{(t-5)(t+5)}{t-5}$$
$$= t+5$$
$$\frac{dy}{dt} = \frac{d}{dt}(t+5)$$
$$= 1, \qquad \text{for } t \neq 5$$

The two results are equivalent.

21. $f(x)=(3x^2-2x+5)(4x^2+3x-1)$

Using the Product Rule, we have:

$$\begin{aligned}
f'(x)&=\frac{d}{dx}\left[(3x^2-2x+5)(4x^2+3x-1)\right]\\
&=(3x^2-2x+5)\cdot\frac{d}{dx}(4x^2+3x-1)+\\
&\quad(4x^2+3x-1)\cdot\frac{d}{dx}(3x^2-2x+5)\\
&=(3x^2-2x+5)\cdot(8x+3)+\\
&\quad(4x^2+3x-1)\cdot(6x-2)
\end{aligned}$$

Simplifying, we get

$$\begin{aligned}
&=(24x^3-7x^2+34x+15)+\\
&\quad(24x^3+10x^2-12x+2)\\
&=48x^3+3x^2+22x+17
\end{aligned}$$

22. $g(x)=(5x^2+4x-3)(2x^2-3x+1)$

Using the Product Rule, we have:

$$\begin{aligned}
g'(x)&=\frac{d}{dx}\left[(5x^2+4x-3)(2x^2-3x+1)\right]\\
&=(5x^2+4x-3)\cdot(4x-3)+\\
&\quad(2x^2-3x+1)\cdot(10x+4)\\
&=(20x^3+x^2-24x+9)+\\
&\quad(20x^3-22x^2-2x+4)\\
&=40x^3-21x^2-26x+13
\end{aligned}$$

23. $y=\dfrac{5x^2-1}{2x^3+3}$

Using the Quotient Rule.

$$\begin{aligned}
\frac{dy}{dx}&=\frac{d}{dx}\left(\frac{5x^2-1}{2x^3+3}\right)\\
&=\frac{(2x^3+3)\frac{d}{dx}(5x^2-1)-(5x^2-1)\frac{d}{dx}(2x^3+3)}{(2x^3+3)^2}\\
&=\frac{(2x^3+3)(10x)-(5x^2-1)(6x^2)}{(2x^3+3)^2}\\
&=\frac{20x^4+30x-30x^4+6x^2}{(2x^3+3)^2}\\
&=\frac{-10x^4+6x^2+30x}{(2x^3+3)^2}\\
&=\frac{-2x(5x^3-3x-15)}{(2x^3+3)^2}
\end{aligned}$$

24. $y=\dfrac{3x^4+2x}{x^3-1}$

Using the Quotient Rule.

$$\begin{aligned}
\frac{dy}{dx}&=\frac{d}{dx}\left(\frac{3x^4+2x}{x^3-1}\right)\\
&=\frac{(x^3-1)\frac{d}{dx}(3x^4+2x)-(3x^4+2x)\frac{d}{dx}(x^3-1)}{(x^3-1)^2}\\
&=\frac{(x^3-1)(12x^3+2)-(3x^4+2x)(3x^2)}{(x^3-1)^2}\\
&=\frac{12x^6-10x^3-2-(9x^6+6x^3)}{(x^3-1)^2}\\
&=\frac{3x^6-16x^3-2}{(x^3-1)^2}
\end{aligned}$$

25. $G(x)=(8x+\sqrt{x})(5x^2+3)$

$G(x)=(8x+x^{1/2})(5x^2+3)\qquad\left[\sqrt{x}=x^{1/2}\right]$

Using the Product Rule, we have:

$$\begin{aligned}
G'(x)&=\frac{d}{dx}\left[(8x+x^{1/2})(5x^2+3)\right]\\
&=(8x+x^{1/2})\cdot\frac{d}{dx}(5x^2+3)+\\
&\quad(5x^2+3)\cdot\frac{d}{dx}(8x+x^{1/2})\\
&=(8x+x^{1/2})\cdot(10x)+\\
&\quad(5x^2+3)\cdot\left(8+\frac{1}{2}x^{-1/2}\right)
\end{aligned}$$

Simplifying, we get

$$\begin{aligned}
&=(80x^2+10x^{3/2})+\\
&\quad\left(40x^2+\frac{5}{2}x^{3/2}+\frac{3}{2}x^{-1/2}+24\right)\\
&=120x^2+\frac{25x^{3/2}}{2}+\frac{3}{2x^{1/2}}+24
\end{aligned}$$

26. $F(x)=\left(-3x^2+4x\right)\left(7\sqrt{x}+1\right)$

Using the Product Rule, we have:

$$F(x)=\frac{d}{dx}\left[\left(-3x^2+4x\right)\left(7\sqrt{x}+1\right)\right]$$
$$=\left(-3x^2+4x\right)\cdot\frac{d}{dx}\left(7\sqrt{x}+1\right)+\left(7\sqrt{x}+1\right)\cdot\frac{d}{dx}\left(-3x^2+4x\right)$$
$$=\left(-3x^2+4x\right)\cdot\left(\frac{7}{2}x^{-1/2}\right)+\left(7x^{1/2}+1\right)\cdot\left(-6x+4\right)$$
$$=\left(-\frac{21}{2}x^{3/2}+14x^{1/2}\right)+\left(-42x^{3/2}-6x+28x^{1/2}+4\right)$$
$$=-\frac{105x^{3/2}}{2}-6x+42x^{1/2}+4$$

27. $g(t)=\frac{t}{3-t}+5t^3$

Differentiating we have:

$$g'(t)=\frac{d}{dt}\left(\frac{t}{3-t}+5t^3\right)$$
$$=\frac{d}{dt}\left(\frac{t}{3-t}\right)+\frac{d}{dt}\left(5t^3\right)$$

We will apply the Quotient Rule to the first term, and the Power Rule to the second term.

$$g'(t)=\underbrace{\frac{(3-t)\cdot\frac{d}{dt}(t)-t\cdot\frac{d}{dt}(3-t)}{(3-t)^2}}_{\text{Quotient Rule}}+15t^2$$
$$=\frac{(3-t)(1)-t(-1)}{(3-t)^2}+15t^2$$
$$=\frac{3}{(3-t)^2}+15t^2$$

28. $f(t)=\frac{t}{5+2t}-2t^4$

$$f'(t)=\frac{d}{dt}\left(\frac{t}{5+2t}-2t^4\right)$$
$$=\underbrace{\frac{(5+2t)(1)-t(2)}{(5+2t)^2}}_{\text{Quotient Rule}}-8t^3$$
$$f'(t)=\frac{5}{(5+2t)^2}-8t^3$$

29. $F(x)=(x+3)^2=(x+3)(x+3)$

Using the Product Rule, we have

$$F'(x)=\frac{d}{dx}\left[(x+3)(x+3)\right]$$
$$=(x+3)\cdot\frac{d}{dx}(x+3)+(x+3)\cdot\frac{d}{dx}(x+3)$$
$$=(x+3)\cdot(1)+(x+3)\cdot(1)$$
$$=2x+6$$
$$=2(x+3)$$

30. $G(x)=(5x-4)^2=(5x-4)(5x-4)$

Using the Product Rule, we have

$$G'(x)=\frac{d}{dx}\left[(5x-4)(5x-4)\right]$$
$$=(5x-4)\cdot\frac{d}{dx}(5x-4)+(5x-4)\cdot\frac{d}{dx}(5x-4)$$
$$=(5x-4)\cdot(5)+(5x-4)\cdot(5)$$
$$=50x-40$$

31. $y=\left(x^3-4x\right)^2=\left(x^3-4x\right)\left(x^3-4x\right)$

Using the Product Rule, we have

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(x^3-4x\right)\left(x^3-4x\right)\right]$$
$$=\left(x^3-4x\right)\cdot\frac{d}{dx}\left(x^3-4x\right)+\left(x^3-4x\right)\cdot\frac{d}{dx}\left(x^3-4x\right)$$
$$=\left(x^3-4x\right)\cdot\left(3x^2-4\right)+\left(x^3-4x\right)\cdot\left(3x^2-4\right)$$
$$=2\left(x^3-4x\right)\left(3x^2-4\right)$$

32. $y=\left(3x^2-4x+5\right)^2$

$$=\left(3x^2-4x+5\right)\left(3x^2-4x+5\right)$$

Using the Product Rule, we have

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(3x^2-4x+5\right)\left(3x^2-4x+5\right)\right]$$
$$=\left(3x^2-4x+5\right)\cdot\frac{d}{dx}\left(3x^2-4x+5\right)+\left(3x^2-4x+5\right)\cdot\frac{d}{dx}\left(3x^2-4x+5\right)$$
$$=\left(3x^2-4x+5\right)\cdot(6x-4)+\left(3x^2-4x+5\right)\cdot(6x-4)$$
$$=2\left(3x^2-4x+5\right)(6x-4)$$

33. $g(x)=5x^{-3}\left(x^4-5x^3+10x-2\right)$

Using the Product Rule:

$$\begin{aligned}g'(x)&=5x^{-3}\frac{d}{dx}\left(x^4-5x^3+10x-2\right)+\\&\quad\left(x^4-5x^3+10x-2\right)\frac{d}{dx}\left(5x^{-3}\right)\\&=\left(5x^{-3}\right)\left(4x^3-15x^2+10\right)+\\&\quad\left(x^4-5x^3+10x-2\right)\left(-15x^{-4}\right)\end{aligned}$$

Simplifying, we get

$$\begin{aligned}&=20-75x^{-1}+50x^{-3}-15+75x^{-1}-\\&\quad150x^{-3}+30x^{-4}\\&=5-100x^{-3}+30x^{-4}\end{aligned}$$

34. $f(x)=6x^{-4}\left(6x^3+10x^2-8x+3\right)$

Using the Product Rule:

$$\begin{aligned}f'(x)&=\left(6x^{-4}\right)\left(18x^2+20x-8\right)+\\&\quad\left(6x^3+10x^2-8x+3\right)\left(-24x^{-5}\right)\\&=108x^{-2}+120x^{-3}-48x^{-4}-\\&\quad144x^{-2}-240x^{-3}+192x^{-4}-72x^{-5}\\&=-36x^{-2}-120x^{-3}+144x^{-4}-72x^{-5}\end{aligned}$$

35. $F(t)=\left(t+\frac{2}{t}\right)\left(t^2-3\right)=\left(t+2t^{-1}\right)\left(t^2-3\right)$

Using the Product Rule, we have:

$$\begin{aligned}f'(t)&=\frac{d}{dt}\left[\left(t+2t^{-1}\right)\left(t^2-3\right)\right]\\&=\left(t+2t^{-1}\right)\cdot\frac{d}{dt}\left(t^2-3\right)+\\&\quad\left(t^2-3\right)\cdot\frac{d}{dt}\left(t+2t^{-1}\right)\end{aligned}$$

$$f'(t)=\left(t+2t^{-1}\right)\cdot(2t)+\left(t^2-3\right)\cdot\left(1-2t^{-2}\right)$$

Simplifying, we get

$$\begin{aligned}&=2t^2+4+\left(t^2-2-3+6t^{-2}\right)\\&=3t^2-1+6t^{-2}\\&=3t^2-1+\frac{6}{t^2}\end{aligned}$$

36. $G(t)=\left(3t^5-t^2\right)\left(t-\frac{5}{t}\right)=\left(3t^5-t^2\right)\left(t-5t^{-1}\right)$

Using the Product Rule, we have:

$$\begin{aligned}G'(t)&=\frac{d}{dt}\left[\left(3t^5-t^2\right)\left(t-5t^{-1}\right)\right]\\&=\left(3t^5-t^2\right)\left(1+5t^{-2}\right)+\\&\quad\left(t-5t^{-1}\right)\left(15t^4-2t\right)\end{aligned}$$

Simplifying, we get

$$\begin{aligned}&=\left(3t^5+15t^3-t^2-5\right)+\\&\quad\left(15t^5-75t^3-2t^2+10\right)\\&=18t^5-60t^3-3t^2+5,\quad\text{for } t\neq 0\end{aligned}$$

37. $y=\frac{x^2+1}{x^3-1}-5x^2$

Differentiating we have:

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left(\frac{x^2+1}{x^3-1}-5x^2\right)\\&=\frac{d}{dx}\left(\frac{x^2+1}{x^3-1}\right)-\frac{d}{dx}\left(5x^2\right)\end{aligned}$$

We will apply the Quotient Rule to the first term, and the Power Rule to the second term.

$$\frac{dy}{dx}=\underbrace{\frac{\left(x^3-1\right)\cdot(2x)-\left(x^2+1\right)\cdot\left(3x^2\right)}{\left(x^3-1\right)^2}}_{\text{Quotient Rule}}-10x$$

Simplifying, we get

$$\begin{aligned}&=\frac{2x^4-2x-\left(3x^4+3x^2\right)}{\left(x^3-1\right)^2}-10x\\&=\frac{-x^4-3x^2-2x}{\left(x^3-1\right)^2}-10x\end{aligned}$$

38. $y=\frac{x^3-1}{x^2+1}+4x^3$

Differentiating we have:

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^3-1}{x^2+1}+4x^3\right)$$

$$\begin{aligned}\frac{dy}{dx}&=\frac{\left(x^2+1\right)\cdot\left(3x^2\right)-\left(x^3-1\right)\cdot(2x)}{\left(x^2+1\right)^2}+12x^2\\&=\frac{\left(3x^4+3x^2\right)-\left(2x^4-2x\right)}{\left(x^2+1\right)^2}+12x^2\\&=\frac{x^4+3x^2+2x}{\left(x^2+1\right)^2}+12x^2\end{aligned}$$

39. $y=\dfrac{\sqrt[3]{x}-7}{\sqrt{x}+3}=\dfrac{x^{1/3}-7}{x^{1/2}+3}$

Using the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^{1/3}-7}{x^{1/2}+3}\right)$$

$$=\frac{\left(x^{1/2}+3\right)\frac{d}{dx}\left(x^{1/3}-7\right)-\left(x^{1/3}-7\right)\frac{d}{dx}\left(x^{1/2}+3\right)}{\left(x^{1/2}+3\right)^2}$$

$$=\frac{\left(x^{1/2}+3\right)\left(\frac{1}{3}x^{-2/3}\right)-\left(x^{1/3}-7\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}+3\right)^2}$$

Note, the previous derivative can be simplified as follows

$$\frac{dy}{dx}=\frac{\left(x^{1/2}+3\right)\left(\frac{1}{3}x^{-2/3}\right)-\left(x^{1/3}-7\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}+3\right)^2}$$

$$=\frac{\frac{1}{3}x^{-1/6}+x^{-2/3}-\frac{1}{2}x^{-1/6}+\frac{7}{2}x^{-1/2}}{\left(x^{1/2}+3\right)^2}$$

$$=\frac{\frac{1}{3}x^{-1/6}+x^{-2/3}-\frac{1}{2}x^{-1/6}+\frac{7}{2}x^{-1/2}}{\left(x^{1/2}+3\right)^2}$$

$$=\frac{x^{-2/3}-\frac{1}{6}x^{-1/6}+\frac{7}{2}x^{-1/2}}{\left(x^{1/2}+3\right)^2}\cdot\frac{6x^{2/3}}{6x^{2/3}}$$

$$=\frac{6-\sqrt{x}+21x^{1/6}}{6x^{2/3}\left(\sqrt{x}+3\right)^2}$$

40. $y=\dfrac{\sqrt{x}+4}{\sqrt[3]{x}-5}=\dfrac{x^{1/2}+4}{x^{1/3}-5}$

Using the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^{1/2}+4}{x^{1/3}-5}\right)$$

$$=\frac{\left(x^{1/3}-5\right)\left(\frac{1}{2}x^{-1/2}\right)-\left(x^{1/2}+4\right)\left(\frac{1}{3}x^{-2/3}\right)}{\left(x^{1/3}-5\right)^2}$$

Simplifying the derivative, we get

$$\frac{dy}{dx}==\frac{-8+\sqrt{x}-15x^{1/6}}{6x^{2/3}\left(x^{1/3}-5\right)^2}$$

41. $f(x)=\dfrac{x}{x^{-1}+1}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{x}{x^{-1}+1}\right)$$

$$=\frac{\left(x^{-1}+1\right)\frac{d}{dx}(x)-(x)\frac{d}{dx}\left(x^{-1}+1\right)}{\left(x^{-1}+1\right)^2}$$

$$=\frac{\left(x^{-1}+1\right)(1)-(x)\left(-1x^{-2}\right)}{\left(x^{-1}+1\right)^2}$$

$$=\frac{x^{-1}+1+x^{-1}}{\left(x^{-1}+1\right)^2}$$

$$=\frac{2x^{-1}+1}{\left(x^{-1}+1\right)^2},\qquad \text{for } x\neq 0 \text{ and } x\neq -1$$

Note, the previous derivative could be simplified as follows:

$$f'(x)=\frac{2x^{-1}+1}{\left(x^{-1}+1\right)^2}$$

$$=\frac{\frac{2}{x}+1}{\left(\frac{1}{x}+1\right)^2}$$

$$=\frac{\frac{2+x}{x}}{\left(\frac{1+x}{x}\right)^2}$$

$$f'(x)=\frac{2+x}{x}\cdot\frac{x^2}{(1+x)^2}$$

$$=\frac{x(x+2)}{(1+x)^2},\qquad \text{for } x\neq 0 \text{ and } x\neq -1$$

42. $f(x)=\dfrac{x^{-1}}{x+x^{-1}}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{x^{-1}}{x+x^{-1}}\right)$$
$$=\frac{(x+x^{-1})(-1x^{-2})-(x^{-1})(1-x^{-2})}{(x+x^{-1})^2}$$
$$=\frac{-x^{-1}-x^{-3}-x^{-1}+x^{-3}}{(x^{-1}+1)^2}$$
$$=\frac{-2x^{-1}}{(x^{-1}+1)^2},\qquad \text{for } x\neq 0$$

The previous derivative can be simplified as follows:

$$f'(x)=\frac{-2x}{(x^2+1)^2},\qquad \text{for } x\neq 0$$

43. $F(t)=\dfrac{1}{t-4}$

Using the Quotient Rule, we have

$$F'(t)=\frac{d}{dt}\left(\frac{1}{t-4}\right)$$
$$=\frac{(t-4)\frac{d}{dt}(1)-(1)\frac{d}{dt}(t-4)}{(t-4)^2}$$
$$=\frac{(t-4)(0)-(1)(1)}{(t-4)^2}$$
$$=\frac{-1}{(t-4)^2}$$

44. $G(t)=\dfrac{1}{t+2}$

Using the Quotient Rule, we have

$$G'(t)=\frac{d}{dt}\left(\frac{1}{t+2}\right)$$
$$=\frac{(t+2)(0)-(1)(1)}{(t+2)^2}$$
$$=\frac{-1}{(t+2)^2}$$

45. $f(x)=\dfrac{3x^2+2x}{x^2+1}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{3x^2+2x}{x^2+1}\right)$$
$$=\frac{(x^2+1)\frac{d}{dx}(3x^2+2x)-(3x^2+2x)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$
$$=\frac{(x^2+1)(6x+2)-(3x^2+2x)(2x)}{(x^2+1)^2}$$
$$=\frac{6x^3+2x^2+6x+2-(6x^3+4x^2)}{(x^2+1)^2}$$
$$=\frac{-2x^2+6x+2}{(x^2+1)^2}$$
$$=\frac{-2(x^2-3x-1)}{(x^2+1)^2}$$

46. $f(x)=\dfrac{3x^2-5x}{x^2-1}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{3x^2-5x}{x^2-1}\right)$$
$$=\frac{(x^2-1)(6x-5)-(3x^2-5x)(2x)}{(x^2-1)^2}$$
$$=\frac{6x^3-5x^2-6x+5-(6x^3-10x^2)}{(x^2-1)^2}$$
$$=\frac{5x^2-6x+5}{(x^2-1)^2}$$

47. $g(t)=\dfrac{-t^2+3t+5}{t^2-2t+4}$

the Quotient Rule, we have

$$g'(t)=\frac{d}{dt}\left(\frac{-t^2+3t+5}{t^2-2t+4}\right)$$

$$=\frac{(t^2-2t+4)\frac{d}{dt}(-t^2+3t+5)}{(t^2-2t+4)^2}-\frac{(-t^2+3t+5)\frac{d}{dt}(t^2-2t+4)}{(t^2-2t+4)^2}$$

$$=\frac{(t^2-2t+4)(-2t+3)-(-t^2+3t+5)(2t-2)}{(t^2-2t+4)^2}$$

Note, the previous derivative could be simplified as follows:

$$g'(t)=\frac{-2t^3+7t^2-14t+12-(-2t^3+8t^2+4t-10)}{(t^2-2t+4)^2}$$

$$=\frac{-t^2-18t+22}{(t^2-2t+4)^2}$$

48. $f(t)=\dfrac{3t^2+2t-1}{-t^2+4t+1}$

Using the Quotient Rule, we have

$$f'(t)=\frac{d}{dt}\left(\frac{3t^2+2t-1}{-t^2+4t+1}\right)$$

$$=\frac{(-t^2+4t+1)(6t+2)-(3t^2+2t-1)(-2t+4)}{(-t^2+4t+1)^2}$$

The previous derivative can be simplified as follows:

$$f'(t)=\frac{14t^2+4t+6}{(t^2-4t-1)^2}.$$

49. – 96. Left to the student.

97. $y=\dfrac{8}{x^2+4}$

$$\frac{dy}{dx}=\frac{(x^2+4)(0)-8(2x)}{(x^2+4)^2}$$

$$\frac{dy}{dx}=\frac{-16x}{(x^2+4)^2}$$

When $x=0$, $\dfrac{dy}{dx}=\dfrac{-16(0)}{(0^2+4)^2}=0,$ so the slope of the tangent line at $(0,2)$ is 0. The equation of the horizontal line passing through $(0,2)$ is $y=2$.

When $x=-2$, $\dfrac{dy}{dx}=\dfrac{-16(-2)}{((-2)^2+4)^2}=\dfrac{32}{64}=\dfrac{1}{2}$, so the slope of the tangent line at $(-2,1)$ is $\dfrac{1}{2}$.

Using the point-slope equation, we have:

$$y-y_1=m(x-x_1)$$

$$y-1=\frac{1}{2}(x-(-2))$$

$$y-1=\frac{1}{2}x+1$$

$$y=\frac{1}{2}x+2$$

98. $y=\dfrac{\sqrt{x}}{x+1}=\dfrac{x^{1/2}}{x+1}$

$$\frac{dy}{dx}=\frac{(x+1)\cdot\left(\frac{1}{2}x^{-1/2}\right)-x^{1/2}\cdot(1)}{(x+1)^2}$$

$$=\frac{\frac{1}{2}x^{1/2}+\frac{1}{2}x^{-1/2}-x^{1/2}}{(x+1)^2}$$

$$=\frac{-x^{1/2}+x^{-1/2}}{2(x+1)^2}$$

When $x=1$, $y=\dfrac{\sqrt{1}}{(1+1)}=\dfrac{1}{2}$.

$$\frac{dy}{dx}=\frac{-(1)^{1/2}+(1)^{-1/2}}{2(1+1)^2}=0$$

Therefore, the slope of the tangent line at $\left(1,\dfrac{1}{2}\right)$ is 0. The equation of the horizontal line passing through $\left(1,\dfrac{1}{2}\right)$ is $y=\dfrac{1}{2}$.

When $x=\dfrac{1}{4}$, $y=\dfrac{\sqrt{\frac{1}{4}}}{\left(\frac{1}{4}\right)+1}=\dfrac{\frac{1}{2}}{\frac{5}{4}}=\dfrac{1}{2}\cdot\dfrac{4}{5}=\dfrac{2}{5}$

$$\frac{dy}{dx}=\frac{-\left(\frac{1}{4}\right)^{1/2}+\left(\frac{1}{4}\right)^{-1/2}}{2\left(\frac{1}{4}+1\right)^2}=\frac{-\frac{1}{2}+2}{\frac{25}{8}}=\frac{3}{2}\cdot\frac{8}{25}=\frac{12}{25}$$

Therefore, the slope of the tangent line at $\left(\frac{1}{4},\frac{2}{5}\right)$ is $\frac{12}{25}$. Using the point-slope equation, we have:

$$y-y_1=m\left(x-x_1\right)$$
$$y-\frac{2}{5}=\frac{12}{25}\left(x-\frac{1}{4}\right)$$
$$y-\frac{2}{5}=\frac{12}{25}x-\frac{3}{25}$$
$$y=\frac{12}{25}x+\frac{7}{25}$$

99. $y=x^2+\dfrac{3}{x-1}$

$$\frac{dy}{dx}=2x+\frac{(x-1)(0)-3(1)}{(x-1)^2}$$
$$=2x-\frac{3}{(x-1)^2}$$

When $x=2$, $y=(2)^2+\dfrac{3}{2-1}=4+3=7$, and

$$\frac{dy}{dx}=2(2)-\frac{3}{(2-1)^2}=4-3=1.$$

Therefore, the slope of the tangent line at $(2,7)$ is 1.

Using the point-slope equation, we have:

$$y-y_1=m\left(x-x_1\right)$$
$$y-7=1(x-2)$$
$$y-7=x-2$$
$$y=x+5$$

When $x=3$, $y=(3)^2+\dfrac{3}{3-1}=9+\dfrac{3}{2}=\dfrac{21}{2}$, and

$$\frac{dy}{dx}=2(3)-\frac{3}{(3-1)^2}=6-\frac{3}{4}=\frac{21}{4}.$$

Therefore, the slope of the tangent line at $\left(3,\frac{21}{2}\right)$ is $\frac{21}{4}$.

Using the point-slope equation, we have:

$$y-y_1=m\left(x-x_1\right)$$
$$y-\frac{21}{2}=\frac{21}{4}(x-3)$$
$$y-\frac{21}{2}=\frac{21}{4}x-\frac{63}{4}$$
$$y=\frac{21}{4}x-\frac{21}{4}$$

100. $y=\dfrac{4x}{1+x^2}$

$$\frac{dy}{dx}=\frac{\left(1+x^2\right)(4)-4x(2x)}{\left(1+x^2\right)^2}$$
$$=\frac{4+4x^2-8x^2}{\left(1+x^2\right)^2}$$
$$=\frac{4-4x^2}{\left(1+x^2\right)^2}$$

When $x=0$, $\dfrac{dy}{dx}=\dfrac{4-4(0)^2}{\left(1+(0)^2\right)^2}=4$.

Therefore, the slope of the tangent line at $(0,0)$ is 4.

Using the point-slope equation, we have:

$$y-y_1=m\left(x-x_1\right)$$
$$y-0=4(x-0)$$
$$y=4x$$

When $x=-1$, $\dfrac{dy}{dx}=\dfrac{4-4(-1)^2}{\left(1+(-1)^2\right)^2}=\dfrac{0}{4}=0$.

Therefore, the slope of the tangent line at $(-1,-2)$ is 0. The equation of the horizontal line passing through $(-1,-2)$ is $y=-2$.

101. The average cost of producing x items is $A_C(x)=\dfrac{C(x)}{x}$. Therefore,

$$A_C(x)=\frac{950+15\sqrt{x}}{x}.$$

Next, we take the derivative using the Quotient Rule to find the rate at which average cost is changing.

$$A_C'(x) = \frac{d}{dx}\left(\frac{950 + 15x^{1/2}}{x}\right)$$

$$A_C'(x) = \frac{x\left(\frac{15}{2}x^{-1/2}\right) - \left(950 + 15x^{1/2}\right)(1)}{(x)^2}$$

$$= \frac{\frac{15}{2}x^{1/2} - 950 - 15x^{1/2}}{x^2}$$

$$= \frac{-\frac{15}{2}x^{1/2} - 950}{x^2}$$

$$= \frac{-\frac{15}{2}\sqrt{x} - 950}{x^2}$$

Substituting 400 for x, we have

$$A_C'(400) = \frac{-\frac{15}{2}\sqrt{400} - 950}{(400)^2}$$

$$= \frac{-150 - 950}{160,000}$$

$$\approx -0.006875$$

Therefore, when 400 jackets have been produced, average cost is changing at a rate of -0.0069 dollars per jacket.

102. $A_C(x) = \dfrac{C(x)}{x}$

$$A_C(x) = \frac{375 + 0.75x^{3/4}}{x}$$

$$A_C'(x) = \frac{x\left(0.75\left(\frac{3}{4}x^{-1/4}\right)\right) - \left(375 + 0.75x^{3/4}\right)(1)}{x^2}$$

$$= \frac{\frac{9}{16}x^{3/4} - 375 - \frac{3}{4}x^{3/4}}{x^2}$$

$$= \frac{-\frac{3}{16}x^{3/4} - 375}{x^2}$$

Substituting 81 for x, we have:

$$A_C'(81) = \frac{\frac{-3}{16}(81)^{3/4} - 375}{(81)^2}$$

$$= -0.05792753$$

$$\approx -0.0579$$

Therefore, when 81 bottles of barbecue sauce have been produced, the average cost is changing at a rate of -0.0579 dollars per bottle.

103. The average revenue of producing x items is $A_R(x) = \dfrac{R(x)}{x}$. Therefore,

$$A_R(x) = \frac{85\sqrt{x}}{x} = \frac{85}{x^{1/2}}.$$

Next, we take the derivative using the Quotient Rule to find the rate at which average revenue is changing.

$$A_R'(x) = \frac{d}{dx}\left(\frac{85}{\sqrt{x}}\right)$$

$$= \frac{x^{1/2}(0) - (85)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}\right)^2}$$

$$= \frac{-\frac{85}{2}x^{-1/2}}{x}$$

$$= -\frac{85}{2x^{3/2}}$$

Substituting 400 for x, we have

$$A_R'(400) = -\frac{85}{2(400)^{3/2}}$$

$$= -\frac{85}{16000}$$

$$= -0.00531250$$

Therefore, when 400 jackets have been produced, average revenue is changing at a rate of -0.0053 dollars per jacket.

104. $A_R(x) = \dfrac{R(x)}{x}$

$$A_R(x) = \frac{7.5x^{0.7}}{x} = \frac{7.5}{x^{0.3}}$$

$$A_R'(x) = \frac{x^{0.3}(0) - (7.5)(0.3x^{-0.7})}{\left(x^{0.3}\right)^2}$$

$$= \frac{-2.25x^{-0.7}}{x^{0.6}}$$

$$= -\frac{2.25}{x^{1.3}}$$

Substituting 81 for x, we have:

$$A_R'(81) = -\frac{2.25}{(81)^{1.3}}$$

$$= -0.007432792$$

$$\approx -0.0074$$

Therefore, when 81 bottles of barbecue sauce have been produce, the average revenue is changing at rate of -0.0074 dollars per bottle.

105. $A_P(x) = \dfrac{P(x)}{x} = \dfrac{R(x) - C(x)}{x}$

From Exercises 101 and 103, we know that

$$A_P(x) = \frac{85x^{1/2} - \left(950 + 15x^{1/2}\right)}{x} = \frac{70x^{1/2} - 950}{x}$$

Using the Quotient Rule to take the derivative, we have:

$$A_P'(x) = \frac{x\left(\frac{70}{2}x^{-1/2}\right) - \left(70x^{1/2} - 950\right)(1)}{(x)^2}$$

$$= \frac{35x^{1/2} - 70x^{1/2} + 950}{x^2}$$

$$= \frac{-35x^{1/2} + 950}{x^2}$$

Substituting 400 for x, we have:

$$A_P'(400) = \frac{-35(400)^{1/2} + 950}{(400)^2}$$

$$= \frac{-700 + 950}{16{,}000}$$

$$= \frac{250}{16{,}000}$$

$$\approx 0.015625$$

When 400 jackets have been produced and sold, the average profit is changing at a rate of 0.0156 dollars per jacket.

Alternatively, we could have used the information in Exercises 101 and 103 to find the rate of change of average profit when 400 jackets are produced and sold. Notice that

$$A_P'(x) = A_R'(x) - A_C'(x)$$

$$= -0.0053125 - (-0.006875)$$

$$= 0.00156250$$

106. $A_P'(x) = A_R'(x) - A_C'(x)$

$$= -0.0074 - (-0.0579)$$

$$= 0.0505$$

Therefore, when 81 bottles where produced and sold, average profit is changing at rate of 0.050 dollars per bottle.

107. The average profit of producing x items is

$A_P(x) = \dfrac{R(x) - C(x)}{x}$. Therefore,

$$A_P(x) = \frac{65x^{0.9} - \left(4300 + 2.1x^{0.6}\right)}{x}$$

$$= \frac{65x^{0.9} - 2.1x^{0.6} - 4300}{x}.$$

Using the Quotient Rule to take the derivative, we have

$A_P'(x)$

$$= \frac{x\left(65\left(0.9x^{-0.1}\right) - 2.1\left(0.6x^{-0.4}\right)\right) - \left(65x^{0.9} - 2.1x^{0.6} - 4300\right)(1)}{(x)^2}$$

$$= \frac{58.5x^{0.9} - 1.26x^{0.6} - \left(65x^{0.9} - 2.1x^{0.6} - 4300\right)}{x^2}$$

$$= \frac{-6.5x^{0.9} + 0.84x^{0.6} + 4300}{x^2}$$

Substituting 50 for x, we have

$$A_P'(50) = \frac{-6.5(50)^{0.9} + 0.84(50)^{0.6} + 4300}{(50)^2}$$

$$= \frac{4089.00428745}{2500}$$

$$= 1.63560171$$

$$\approx 1.64$$

Therefore, when 50 vases have been produced and sold, the average profit is changing at rate of 1.64 dollars per vase.

108. $A_P(x) = \dfrac{R(x) - C(x)}{x}$.

Therefore,

$$A_P(x) = \frac{75x^{0.8} - \left(900 + 18x^{0.7}\right)}{x}$$

$$= \frac{75x^{0.8} - 18x^{0.7} - 900}{x}.$$

$A_P'(x)$

$$= \frac{x\left(60x^{-0.2} - 12.6x^{-0.3}\right) - \left(75x^{0.8} - 18x^{0.7} - 900\right)(1)}{(x)^2}$$

$$= \frac{-15x^{0.8} + 5.4x^{0.7} + 900}{x^2}$$

Substituting 20 for *x,* we have

$$A_P'(20) = \frac{-15(20)^{0.8} + 5.4(20)^{0.7} + 900}{(20)^2}$$

$$= \frac{779.18169591}{400}$$

$$= 1.94795424$$

$$\approx 1.95$$

Therefore, when 20 skateboards have been produced and sold, the average profit is changing at rate of 1.95 dollars per skateboard.

109. $P(t) = 567 + t(36t^{0.6} - 104)$

a) Using the Product Rule and remembering that the derivative of a constant is 0, we have

$$P'(t) = 0 + t\left(36\left(0.6t^{-0.4}\right)\right) + \left(36t^{0.6} - 104\right)(1)$$

$$= 21.6t^{0.6} + 36t^{0.6} - 104$$

$$= 57.6t^{0.6} - 104$$

b) Substituting 45 for *t*, we have:

$$P'(45) = 57.6(45)^{0.6} - 104$$

$$= 565.39243291 - 104$$

$$= 461.39243291$$

c) $\boxed{tw}$ $P'(45)$ is the rate of change of the gross domestic product 45 years after 1960, or in 2005. In other words, the U.S. Gross domestic product was increasing at a rate of 461.39243291 billion dollars per year in 2005.

110. $P(t) = \dfrac{500t}{2t^2 + 9}$

a) $$P'(t) = \frac{(2t^2 + 9)(500) - (500t)(4t)}{(2t^2 + 9)^2}$$

$$= \frac{1000t^2 + 4500 - 2000t^2}{(2t^2 + 9)^2}$$

$$= \frac{-1000t^2 + 4500}{(2t^2 + 9)^2}$$

b) $$P(12) = \frac{500(12)}{2(12)^2 + 9} = \frac{6000}{297} = 20.2020$$

After 12 months, the city's population is approximately 20.2020 thousand people or 20,202 people.

c) $$P'(12) = \frac{-1000(12)^2 + 4500}{\left(2(12)^2 + 9\right)^2}$$

$$P'(12) = \frac{-139{,}500}{88209}$$

$$= -1.58147128$$

The city population is growing at rate of -1.581 thousand people per month. In other words, the city population is declining at a rate of 1,581 people per month.

111. $T(t) = \dfrac{4t}{t^2 + 1} + 98.6$

a) $$T'(t) = \frac{(t^2 + 1)(4) - (4t)(2t)}{(t^2 + 1)^2} + 0$$

$$= \frac{4t^2 + 4 - 8t^2}{(t^2 + 1)^2}$$

$$= \frac{-4t^2 + 4}{(t^2 + 1)^2}$$

b) $$T(2) = \frac{4(2)}{(2)^2 + 1} + 98.6 = \frac{8}{5} + 98.6 = 100.2$$

After 2 hours, the temperature of the ill person is approximately 100.2 degrees Fahrenheit.

c) $$T'(2) = \frac{-4(2)^2 + 4}{\left((2)^2 + 1\right)^2} = \frac{-12}{25} = -0.48$$

After 2 hours, the person's temperature is changing at rate of -0.48 degrees per hour.

112. $f(x) = \dfrac{7 - \frac{3}{2x}}{\frac{4}{x^2} + 5}$

Simplifying the function we have:

$$f(x) = \frac{7 - \frac{3}{2x}}{\frac{4}{x^2} + 5} \cdot \frac{2x^2}{2x^2}$$

$$= \frac{7(2x^2) - 3(x)}{4(2) + 5(2x^2)}$$

$$= \frac{14x^2 - 3x}{8 + 10x^2}$$

We apply the quotient rule to take the derivative.

$$f'(x)=\frac{(8+10x^2)(28x-3)-(14x^2-3x)(20x)}{(8+10x^2)^2}$$
$$=\frac{280x^3-30x^2+224x-24-280x^3+60x^2}{(8+10x^2)^2}$$
$$=\frac{30x^2+224x-24}{(8+10x^2)^2}$$

The previous derivative can be simplified as follows:

$$f'(x)=\frac{15x^2+112x-12}{2(5x^2+4)^2}.$$

113. $y(t)=5t(t-1)(2t+3)$

First, group the factors of $y(t)$ in order to apply the product rule.

$y(t)=[5t(t-1)]\cdot(2t+3)$

Now we can take the derivative. Notice that when we take the derivative of the first term, $[5t(t-1)]$ we will have to apply the Product Rule again.

$$y'(t)=[5t(t-1)](2)+(2t+3)\left[\underbrace{(5t)(1)+(t-1)(5)}_{\text{Product Rule for }[5t(t-1)]}\right]$$
$$=10t(t-1)+(2t+3)[5t+5t-5]$$
$$=10t(t-1)+(2t+3)(10t-5)$$

The previous derivative can be simplified as follows:

$$y'(t)=10t^2-10t+20t^2+30t-10t-15$$
$$=30t^2+10t-15$$

114. $f(x)=[x(3x^3+6x-2)](3x^4+7)$

$$f'(x)=[x(3x^3+6x-2)](12x^3)+$$
$$(3x^4+7)[x(9x^2+6)+(3x^3+6x-2)(1)]$$
$$=36x^7+72x^5-24x^4+$$
$$(3x^4+7)[9x^3+6x+3x^3+6x-2]$$
$$=36x^7+72x^5-24x^4+$$
$$(3x^4+7)[12x^3+12x-2]$$
$$=36x^7+72x^5-24x^4+36x^7+$$
$$36x^5-6x^4+84x^3+84x-14$$
$$=72x^7+108x^5-30x^4+84x^3+84x-14$$

115. $g(x)=(x^3-8)\cdot\dfrac{x^2+1}{x^2-1}$

We will begin by applying the Product Rule.

$$g'(x)=(x^3-8)\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)+\frac{x^2+1}{x^2-1}\cdot\frac{d}{dx}(x^3-8)$$

Notice, that we will have to apply the Quotient Rule to take the derivative of $\dfrac{x^2+1}{x^2-1}$.

$$g'(x)=(x^3-8)\frac{(x^2-1)(2x)-(x^2+1)(2x)}{(x^2-1)^2}+$$
$$\left(\frac{x^2+1}{x^2-1}\right)\cdot(3x^2)$$
$$=(x^3-8)\frac{-4x}{(x^2-1)^2}+\left(\frac{x^2+1}{x^2-1}\right)\cdot(3x^2)$$

The following derivative can be simplified as follows.

$$g'(x)=\frac{-4x(x^3-8)}{(x^2-1)^2}+\frac{3x^2(x^2+1)}{x^2-1}$$
$$=\frac{-4x^4+32x}{(x^2-1)^2}+\frac{3x^4+3x^2}{x^2-1}\cdot\frac{x^2-1}{x^2-1}$$
$$=\frac{-4x^4+32x+3x^6-3x^2}{(x^2-1)^2}$$
$$=\frac{3x^6-4x^4-3x^2+32x}{(x^2-1)^2}$$

116. $f(t)=(t^5+3)\cdot\dfrac{t^3-1}{t^3+1}$

$$f'(t)=(t^5+3)\left[\frac{(t^3+1)(3t^2)-(t^3-1)(3t^2)}{(t^3+1)^2}\right]+$$
$$\left(\frac{t^3-1}{t^3+1}\right)(5t^4)$$
$$=(t^5+3)\left[\frac{6t^2}{(t^3+1)^2}\right]+\left(\frac{t^3-1}{t^3+1}\right)5t^4$$
$$=\frac{6t^2(t^5+3)}{(t^3+1)^2}+\frac{5t^4(t^3-1)}{t^3+1}$$

117. $f(x)=\dfrac{(x-1)(x^2+x+1)}{x^4-3x^3-5}$

First we will group the numerator, to apply the Quotient Rule. Remember that we will have to apply the Product Rule when taking the derivative of the numerator.

$$f(x)=\frac{\left[(x-1)(x^2+x+1)\right]}{x^4-3x^3-5}$$

$$f'(x)$$
$$=\frac{(x^4-3x^3-5)\left[(x-1)(2x+1)+(x^2+x+1)(1)\right]}{(x^4-3x^3-5)^2}-\frac{\left[(x-1)(x^2+x+1)\right](4x^3-9x^2)}{(x^4-3x^3-5)^2}$$
$$=\frac{(x^4-3x^3-5)\left[2x^2-x-1+x^2+x+1\right]}{(x^4-3x^3-5)^2}-\frac{\left[x^3-1\right](4x^3-9x^2)}{(x^4-3x^3-5)^2}$$
$$=\frac{(x^4-3x^3-5)\left[3x^2\right]-\left[x^3-1\right](4x^3-9x^2)}{(x^4-3x^3-5)^2}$$
$$=\frac{3x^6-9x^5-15x^2-4x^6+9x^5+4x^3-9x^2}{(x^4-3x^3-5)^2}$$
$$=\frac{-x^6+4x^3-24x^2}{(x^4-3x^3-5)^2}$$

118. $f(x)=\dfrac{x}{x+1}$ and $g(x)=\dfrac{-1}{x+1}$

a) $f'(x)=\dfrac{(x+1)(1)-x(1)}{(x+1)^2}=\dfrac{1}{(x+1)^2}$

b) $g'(x)=\dfrac{(x+1)(0)-(-1)(1)}{(x+1)^2}=\dfrac{1}{(x+1)^2}$

c) $\boxed{tw}$ The two functions have the same rate of change.

119. $f(x)=\dfrac{x^2}{x^2-1}$ and $g(x)=\dfrac{1}{x^2-1}$

a) $f'(x)=\dfrac{(x^2-1)(2x)-x^2(2x)}{(x^2-1)^2}=\dfrac{-2x}{(x^2-1)^2}$

b) $g'(x)=\dfrac{(x^2-1)(0)-1(2x)}{(x^2-1)^2}=\dfrac{-2x}{(x^2-1)^2}$

c) $\boxed{tw}$ The two functions have the same rate of change.

120. Suppose $F(x)=f(x)\cdot g(x)\cdot h(x)$, where $f(x)$ is the "first" function, $g(x)$ is the "second" function and $h(x)$ is the third function. Using the associative law of multiplication, we can write $F(x)=\left[f(x)\cdot g(x)\right]\cdot h(x)$. Then using the Product Rule we have:

$$F'(x)=\frac{d}{dx}\left(\left[f(x)\cdot g(x)\right]\cdot h(x)\right)$$
$$=\left[f(x)g(x)\right]h'(x)+h(x)\left[f(x)g(x)\right]'$$
$$=\left[f(x)g(x)\right]h'(x)+h(x)\left[f(x)g'(x)+g(x)f'(x)\right]$$
$$=f(x)g(x)h'(x)+f(x)g'(x)h(x)+f'(x)g(x)h(x)$$

The derivative of the product of three functions is the first function times the second function times the derivative of the third functions plus the first function times the derivative of the second function times the third function plus the derivative of the first function times the second function times the third function.

121. $\boxed{tw}$ In general the derivative of the reciprocal of a function is not the reciprocal of the derivative. Let $f'(x)$ be the derivative of $f(x)$. Therefore, the reciprocal of the derivative is $\dfrac{1}{f'(x)}$.

The reciprocal of the function is $\dfrac{1}{f(x)}$, using the Quotient Rule, we find the derivative of the reciprocal.

$$\frac{d}{dx}\left(\frac{1}{f(x)}\right)=\frac{f(x)(0)-(1)f'(x)}{\left(f(x)\right)^2}=-\frac{f'(x)}{\left(f(x)\right)^2}$$

Clearly, the derivative of the reciprocal is not equal to the reciprocal of the derivative.

122. $R(Q)=Q^2\left(\dfrac{k}{2}-\dfrac{Q}{3}\right)$

a) $$\frac{dR}{dQ}=Q^2\left(-\frac{1}{3}\right)+\left(\frac{k}{2}-\frac{Q}{3}\right)(2Q)$$
$$=-\frac{1}{3}Q^2+\frac{k}{2}2Q-\frac{2Q^2}{3}$$
$$=-Q^2+kQ$$

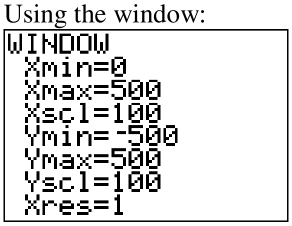
b) $\boxed{tw}$ The derivative found in part (a) tells us the rate of change of the reaction with respect to a small change in the quantity of the dose. If the reaction is measured in blood pressure, then the derivative tells us at a dosage Q, how much change in millimeters of mercury is seen due to a small change in the dosage. If the reaction is measured in temperature, the derivative tells us at a dosage Q, how much change in degrees Fahrenheit is seen due to a small change in the dosage.

123. a) Definition of the derivative.
b) Adding and subtracting the same quantity is the same as adding 0.
c) The limit of a sum is the sum of the limits.
d) Factoring common factors.
e) The limit of a product is the product of the limits and $\lim_{h\to 0} f(x+h)=f(x)$.
f) Definition of the derivative.
g) Using Leibniz's notation.

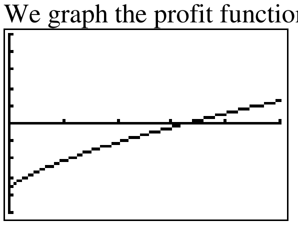
124. The break even point occurs when $P(x)=0$.

$$P(x)=R(x)-C(x)$$
$$=7.5x^{0.7}-375-0.75x^{3/4}$$

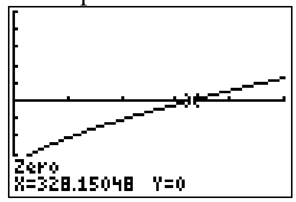
Using the window:



We graph the profit function on the calculator.



The break even point will be the zero or x-intercept of the function.



We see that the break-even point occurs at $x=328$ bottles.

The profit is changing at rate of

$$P'(x)=5.25x^{-0.3}-0.5625x^{-1/4}$$

Substituting 328 for x we have:

$$P'(328)=5.25(328)^{-0.3}-0.5625(328)^{-1/4}$$
$$=0.791239$$
$$\approx 0.79$$

Therefore, at the break-even point, profit is increasing at a rate of 0.79 dollars per day, or 79 cents per day.

From Exercises 102, 104 and 106 we know that:

$$A_P'(x)=A_R'(x)-A_C'(x)$$
$$=-\frac{2.25}{x^{1.3}}-\frac{-\frac{3}{16}x^{3/4}-375}{x^2}$$
$$=\frac{3x^{3/4}-36x^{0.7}+6000}{16x^2}$$

Substituting 328 for x we get:

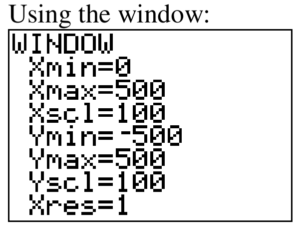
$$A_P'(328)=\frac{3(328)^{3/4}-36(328)^{0.7}+6000}{16(328)^2}$$
$$\approx 0.00241342$$
$$\approx 0.0024$$

At the break-even point, average profit is changing at a rate of 0.0024 dollars per day.

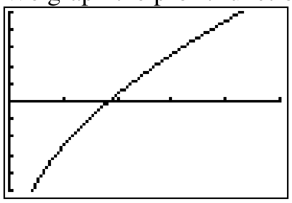
125. The break even point occurs when $P(x)=0$.

$$P(x)=R(x)-C(x)$$
$$=85x^{1/2}-\left(950+15x^{1/2}\right)$$
$$=70x^{1/2}-950$$

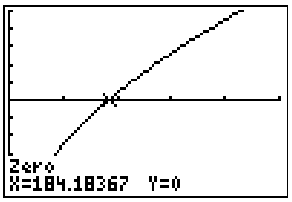
Using the window:



We graph the profit function on the calculator.



The break even point will be the zero of the function. Using the zero finder on the calculator we have:



We see that the break-even point occurs at $x = 184$ jackets.

The profit is changing at rate of

$$P'(x) = 70\left(\frac{1}{2}x^{-1/2}\right) = \frac{70}{2x^{1/2}} = \frac{35}{\sqrt{x}}.$$

Substituting 184 for x we have:

$$P'(184) = \frac{35}{\sqrt{184}}$$
$$= 2.580234233$$
$$\approx 2.58$$

Therefore, at the break-even point, profit is increasing at a rate of 2.58 dollars per jacket.

From Exercise 105 we know that:

$$A_P'(x) = \frac{-35x^{1/2} + 950}{x^2}.$$

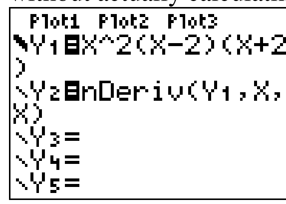
Substituting 184 for x we get:

$$A_P'(184) = \frac{-35(184)^{1/2} + 950}{(184)^2}$$
$$\approx 0.014037$$
$$\approx 0.014$$

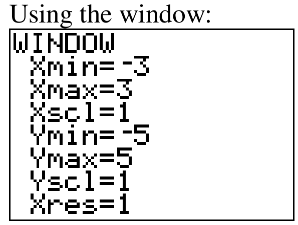
At the break-even point, average profit is changing at a rate of 0.014 dollars per jacket.

126. $f(x) = x^2(x-2)(x+2)$

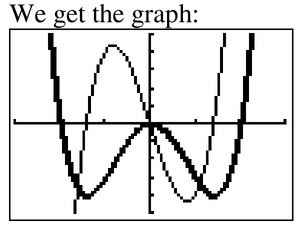
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



We get the graph:



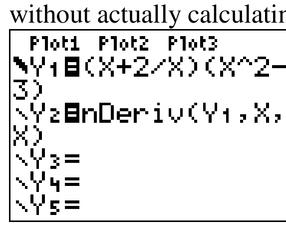
Note, the function $f(x)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

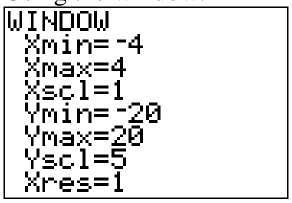
We estimate the points at which the tangent lines are horizontal are $(0,0), (-1.414, -4)$, and $(1.414, -4)$.

127. $f(x) = \left(x + \frac{2}{x}\right)(x^2 - 3)$

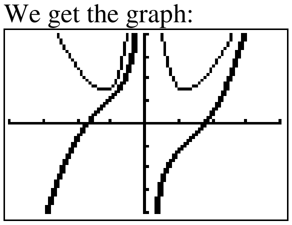
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



We get the graph:

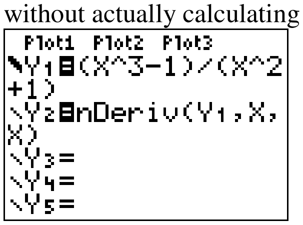


Note, the function $f(x)$ is the thicker graph.

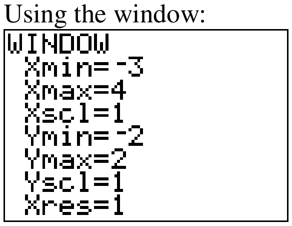
The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. We can see that the derivative never intersects the x-axis, therefore, there are no points at which the tangent line is horizontal.

128. $f(x)=\dfrac{x^3-1}{x^2+1}$

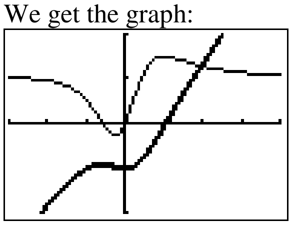
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



We get the graph:



Note, the function $f(x)$ is the thicker graph.

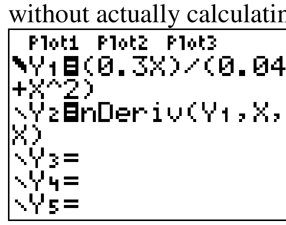
The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative.

Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

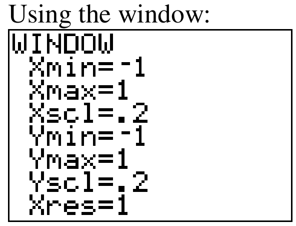
We estimate the points at which the tangent lines are horizontal are $(-0.596,-0.894)$ and $(0,-1)$.

129. $f(x)=\dfrac{0.3x}{0.04+x^2}$

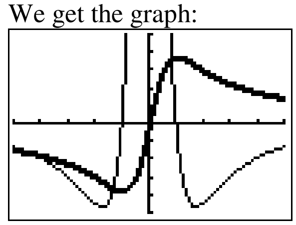
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



We get the graph:



Note, the function $f(x)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

We estimate the points at which the tangent lines are horizontal are $(-0.2,-0.75)$ and $(0.2,0.75)$.

130. $f(x)=\dfrac{0.01x^2}{x^4+0.0256}$

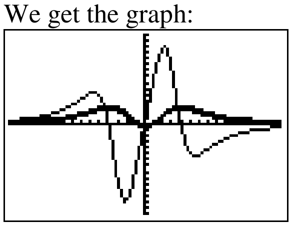
Using the calculator, we graph the function and the derivative in the same window.
We can use the nDeriv feature to graph the derivative without actually calculating the derivative.

```
Plot1 Plot2 Plot3
\Y1=(0.01X^2)/(X
^4+0.0256)
\Y2=nDeriv(Y1,X,
X)
\Y3=
\Y4=
\Y5=
```

Using the window:

```
WINDOW
 Xmin=-1.5
 Xmax=1.5
 Xscl=.1
 Ymin=-.15
 Ymax=.15
 Yscl=.01
 Xres=1
```

We get the graph:



Note, the function $f(x)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

We estimate the points at which the tangent lines are horizontal are $(-0.4,0.03125)$, $(0,0)$, and $(0.4,0.03125)$.

131. $f(x)=\dfrac{4x}{x^2+1}$

Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.

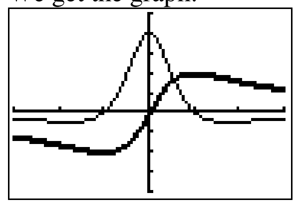
```
Plot1 Plot2 Plot3
\Y1=(4X)/(X^2+1)

\Y2=nDeriv(Y1,X,
X)
\Y3=
\Y4=
\Y5=
```

Using the window:

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-4
 Ymax=5
 Yscl=1
 Xres=1
```

We get the graph:

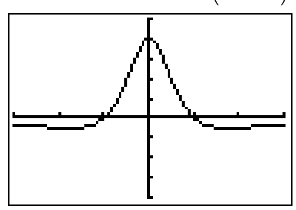


Note, the function $f(x)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

We estimate the points at which the tangent lines are horizontal are $(-1,-2)$ and $(1,2)$.

132. The graph of $y_3=\dfrac{4-4x^2}{(x^2+1)^2}$ is:



This graph appears to be the correct derivative of the function in Exercise 131. Using the Quotient Rule we can verify this result.

$$f(x)=\frac{4x}{x^2+1}$$

$$f'(x)=\frac{4-4x^2}{(x^2+1)^2}$$

Exercise Set 1.7

1. $y=(2x+1)^2$

Using the Extended Power Rule:

$$\frac{dy}{dx}=\frac{d}{dx}\left[(2x+1)^2\right]$$
$$=2(2x+1)^{2-1}\cdot\frac{d}{dx}(2x+1)$$
$$=2(2x+1)(2)$$
$$=8x+4$$

Simplifying the function first, we have:

$$y=(2x+1)^2$$
$$=(2x+1)(2x+1)$$
$$=4x^2+4x+1$$

Now we take the derivative using the Power Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(4x^2+4x+1\right)$$
$$=\frac{d}{dx}\left(4x^2\right)+\frac{d}{dx}(4x)+\frac{d}{dx}(1)$$
$$=8x+4$$

The results are the same.

2. $y=(3-2x)^2$

Using the Extended Power Rule:

$$\frac{dy}{dx}=\frac{d}{dx}\left[(3-2x)^2\right]$$
$$=2(3-2x)^{2-1}(-2)$$
$$=8x-12$$

Simplifying the function first, we have:

$$y=(3-2x)^2=4x^2-12x+9$$

We take the derivative using the Power Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(4x^2-12x+9\right)$$
$$=8x-12$$

The results are the same.

3. $y=(7-x)^{55}$

Using the Extended Power Rule:

$$\frac{dy}{dx}=\frac{d}{dx}\left[(7-x)^{55}\right]$$
$$=55(7-x)^{55-1}\cdot\frac{d}{dx}(7-x)$$
$$\frac{dy}{dx}=55(7-x)^{54}(-1)$$
$$=-55(7-x)^{54}$$

4. $y=(8-x)^{100}$

$$\frac{dy}{dx}=100(8-x)^{99}(-1)$$
$$=-100\left((8-x)^{99}\right)$$

5. $y=\sqrt{1+8x}=(1+8x)^{1/2}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[(1+8x)^{1/2}\right]$$
$$=\frac{1}{2}(1+8x)^{-1/2}\frac{d}{dx}(1+8x)$$
$$=\frac{1}{2(1+8x)^{1/2}}\cdot(8)$$
$$=\frac{4}{\sqrt{1+8x}}$$

6. $y=\sqrt{1-x}=(1-x)^{1/2}$

$$\frac{dy}{dx}=\frac{1}{2}(1-x)^{-1/2}(-1)$$
$$=\frac{-1}{2(1-x)^{1/2}}$$
$$=\frac{-1}{2\sqrt{1-x}}$$

7. $y=\sqrt{3x^2-4}=\left(3x^2-4\right)^{1/2}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(3x^2-4\right)^{1/2}\right]$$
$$=\frac{1}{2}\left(3x^2-4\right)^{-1/2}\frac{d}{dx}\left(3x^2-4\right)$$
$$=\frac{1}{2\left(3x^2-4\right)^{1/2}}\cdot(6x)$$
$$=\frac{3x}{\sqrt{3x^2-4}}$$

8. $y=\sqrt{4x^2+1}=\left(4x^2+1\right)^{1/2}$

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(4x^2+1\right)^{1/2}\right]$$
$$\frac{dy}{dx}=\frac{1}{2}\left(4x^2+1\right)^{-1/2}(8x)$$
$$=\frac{8x}{2\left(4x^2+1\right)^{1/2}}$$
$$=\frac{4x}{\sqrt{4x^2+1}}$$

9. $y=\left(8x^2-6\right)^{-40}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(8x^2-6\right)^{-40}\right]$$
$$=-40\left(8x^2-6\right)^{-40-1}\frac{d}{dx}\left(8x^2-6\right)$$
$$=-40\left(8x^2-6\right)^{-41}\cdot(16x)$$
$$=-640x\left(8x^2-6\right)^{-41}$$
$$=\frac{-640x}{\left(8x^2-6\right)^{41}}$$

10. $y=\left(4x^2+1\right)^{-50}$

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(4x^2+1\right)^{-50}\right]$$
$$=-50\left(4x^2+1\right)^{-51}\cdot(8x)$$
$$=-400x\left(4x^2+1\right)^{-51}$$
$$=\frac{-400x}{\left(4x^2+1\right)^{51}}$$

11. $y=(x-4)^8(2x+3)^6$

Using the Product Rule, we have

$$\frac{dy}{dx}=\frac{d}{dx}\left[(x-4)^8(2x+3)^6\right]$$
$$=(x-4)^8\frac{d}{dx}(2x+3)^6+(2x+3)^6\frac{d}{dx}(x-4)^8$$

Next, we will apply the Extended Power Rule.

$$\frac{dy}{dx}=(x-4)^8\left[6(2x+3)^{6-1}\frac{d}{dx}(2x+3)\right]+$$
$$(2x+3)^6\left[8(x-4)^{8-1}\frac{d}{dx}(x-4)\right]$$
$$=(x-4)^8\left[6(2x+3)^5(2)\right]+$$
$$(2x+3)^6\left[8(x-4)^7(1)\right]$$
$$=12(x-4)^8(2x+3)^5+8(2x+3)^6(x-4)^7$$

Factoring out common factors, we have:

$$\frac{dy}{dx}=4(x-4)^7(2x+3)^5\left[3(x-4)+2(2x+3)\right]$$
$$=4(x-4)^7(2x+3)^5\left[3x-12+4x+6\right]$$
$$=4(x-4)^7(2x+3)^5(7x-6)$$

12. $y=(x+5)^7(4x-1)^{10}$

$$\frac{dy}{dx}=(x+5)^7\left[10(4x-1)^9(4)\right]+$$
$$(4x-1)^{10}\left[7(x+5)^6(1)\right]$$
$$=40(x+5)^7(4x-1)^9+7(4x-1)^{10}(x+5)^6$$
$$=(x+5)^6(4x-1)^9\left[40(x+5)+7(4x-1)\right]$$
$$=(x+5)^6(4x-1)^9\left[40x+200+28x-7\right]$$
$$=(x+5)^6(4x-1)^9(68x+193)$$

13. $y=\dfrac{1}{(3x+8)^2}=(3x+8)^{-2}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[(3x+8)^{-2}\right]$$
$$=-2(3x+8)^{-2-1}\frac{d}{dx}(3x+8)$$
$$=-2(3x+8)^{-3}\cdot(3)$$
$$=-6(3x+8)^{-3}$$
$$=\frac{-6}{(3x+8)^3}$$

14. $y=\dfrac{1}{(4x+5)^2}=(4x+5)^{-2}$

$$\frac{dy}{dx}=-2(4x+5)^{-3}(4)$$
$$\frac{dy}{dx}=-8(4x+5)^{-3}$$
$$=\frac{-8}{(4x+5)^3}$$

15. $y=\dfrac{4x^2}{(7-5x)^3}$

First, we use the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{4x^2}{(7-5x)^3}\right]$$
$$=\frac{(7-5x)^3\frac{d}{dx}\left(4x^2\right)-4x^2\frac{d}{dx}(7-5x)^3}{\left((7-5x)^3\right)^2}$$

Next, using the Extended Power Rule, we have:

$$\frac{dy}{dx} = \frac{(7-5x)^3(8x) - 4x^2\left[3(7-5x)^2(-5)\right]}{(7-5x)^6}$$

$$= \frac{8x(7-5x)^3 + 60x^2(7-5x)^2}{\left((7-5x)^3\right)^2}$$

$$= \frac{(7-5x)^2\left[8x(7-5x) + 60x^2\right]}{(7-5x)^6} \quad \text{Factoring}$$

$$= \frac{56x - 40x^2 + 60x^2}{(7-5x)^4} \quad \text{Dividing common factors}$$

$$= \frac{20x^2 + 56x}{(7-5x)^4}$$

$$= \frac{4x(5x+14)}{(7-5x)^4}$$

16. $y = \dfrac{7x^3}{(4-9x)^5}$

$$\frac{dy}{dx} = \frac{(4-9x)^5(21x^2) - 7x^3\left[5(4-9x)^4(-9)\right]}{\left((4-9x)^5\right)^2}$$

$$= \frac{21x^2(4-9x)^5 + 315x^3(4-9x)^4}{(4-9x)^{10}}$$

$$= \frac{21x^2(4-9x)^4\left[(4-9x) + 15x\right]}{(4-9x)^{10}}$$

$$\frac{dy}{dx} = \frac{21x^2(4+6x)}{(4-9x)^6}$$

$$= \frac{42x^2(3x+2)}{(4-9x)^6}$$

17. $f(x) = (1+x^3)^3 - (2+x^8)^4$

Using the Difference Rule and then the Extended Power Rule we have:

$$f'(x) = \frac{d}{dx}\left[(1+x^3)^3 - (2+x^8)^4\right]$$

$$= \frac{d}{dx}(1+x^3)^3 - \frac{d}{dx}(2+x^8)^4$$

$$= 3(1+x^3)^{3-1}\left(\frac{d}{dx}(1+x^3)\right) -$$

$$4(2+x^8)^{4-1}\left(\frac{d}{dx}(2+x^8)\right)$$

$$= 3(1+x^3)^2(3x^2) - 4(2+x^8)^3(8x^7)$$

$$= 9x^2(1+x^3)^2 - 32x^7(2+x^8)^3$$

18. $f(x) = (3+x^3)^5 - (1+x^7)^4$

$$f'(x) = 5(3+x^3)^4(3x^2) - 4(1+x^7)^3(7x^6)$$

$$= 15x^2(3+x^3)^4 - 28x^6(1+x^7)^3$$

19. $f(x) = x^2 + (200-x)^2$

Using the Sum Rule and the Extended Power Rule, we have:

$$f'(x) = \frac{d}{dx}\left[x^2 + (200-x)^2\right]$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(200-x)^2$$

$$= 2x + 2(200-x)^{2-1}\left[\frac{d}{dx}(200-x)\right]$$

$$= 2x + 2(200-x)(-1)$$

$$= 2x + 2x - 400$$

$$= 4x - 400$$

20. $f(x) = x^2 + (100-x)^2$

$$f'(x) = 2x + 2(100-x)^{2-1}(-1)$$

$$= 2x + 2x - 200$$

$$= 4x - 200$$

21. $g(x)=\sqrt{x}+(x-3)^3=x^{1/2}+(x-3)^3$

Using the Sum Rule and the Extended Power Rule, we have:

$$g'(x)=\frac{d}{dx}\left[x^{1/2}+(x-3)^3\right]$$
$$=\frac{d}{dx}\left(x^{1/2}\right)+\frac{d}{dx}(x-3)^3$$
$$=\frac{1}{2}x^{1/2-1}+3(x-3)^{3-1}\left[\frac{d}{dx}(x-3)\right]$$
$$=\frac{1}{2}x^{-1/2}+3(x-3)^2(1)$$
$$=\frac{1}{2x^{1/2}}+3(x-3)^2$$
$$=\frac{1}{2\sqrt{x}}+3(x-3)^2$$

22.

$$G(x)=\sqrt[3]{2x-1}+(4-x)^2=(2x-1)^{1/3}+(4-x)^2$$
$$G'(x)=\frac{1}{3}(2x-1)^{1/3-1}(2)+2(4-x)^{2-1}(-1)$$
$$=\frac{2}{3}(2x-1)^{-2/3}-2(4-x)$$
$$=\frac{2}{3(2x-1)^{2/3}}-8+2x$$
$$=\frac{2}{3\sqrt[3]{(2x-1)^2}}-8+2x$$

23. $f(x)=-5x(2x-3)^4$

Using the Product Rule, we have

$$f'(x)=\frac{d}{dx}\left[-5x(2x-3)^4\right]$$
$$=-5x\frac{d}{dx}\left[(2x-3)^4\right]+(2x-3)^4\frac{d}{dx}(-5x)$$

Using the Extended Power Rule, we have

$$f'(x)=-5x\left[4(2x-3)^3\left(\frac{d}{dx}(2x-3)\right)\right]+(2x-3)^4(-5)$$
$$=-5x\left[4(2x-3)^3(2)\right]+(2x-3)^4(-5)$$
$$=-40x(2x-3)^3-5(2x-3)^4$$
$$=-5(2x-3)^3\left[8x+(2x-3)\right] \quad \text{Factoring}$$
$$=-5(2x-3)^3(10x-3)$$

24. $f(x)=-3x(5x+4)^6$

$$f'(x)=-3x\left[6(5x+4)^{6-1}(5)\right]+(5x+4)^6(-3)$$
$$=-90x(5x+4)^5-3(5x+4)^6$$
$$=-3(5x+4)^5\left[30x+5x+4\right]$$
$$=-3(5x+4)^5(35x+4)$$

25. $g(x)=(3x-1)^7(2x+1)^5$

Using the Product Rule and the Extended Power Rule, we have

$$g'(x)=\frac{d}{dx}\left[(3x-1)^7(2x+1)^5\right]$$
$$=(3x-1)^7\frac{d}{dx}(2x+1)^5+(2x+1)^5\frac{d}{dx}(3x-1)^7$$
$$=(3x-1)^7\left[5(2x+1)^4(2)\right]+(2x+1)^5\left[7(3x-1)^6(3)\right]$$
$$=10(3x-1)^7(2x+1)^4+21(2x+1)^5(3x-1)^6$$
$$=(3x-1)^6(2x+1)^4\left[10(3x-1)+21(2x+1)\right]$$
$$=(3x-1)^6(2x+1)^4\left[30x-10+42x+21\right]$$
$$=(3x-1)^6(2x+1)^4(72x+11)$$

26. $F(x)=(5x+2)^4(2x-3)^8$

$$F'(x)=(5x+2)^4\left[8(2x-3)^7(2)\right]+(2x-3)^8\left[4(5x+2)^3(5)\right]$$
$$=16(5x+2)^4(2x-3)^7+20(2x-3)^8(5x+2)^3$$
$$=4(5x+2)^3(2x-3)^7\left[4(5x+2)+5(2x-3)\right]$$
$$=4(5x+2)^3(2x-3)^7(30x-7)$$

27. $f(x)=x^2\sqrt{4x-1}=x^2(4x-1)^{1/2}$

Using the Product Rule and the Extended Power Rule, we have

$$f'(x)=\frac{d}{dx}\left[x^2(4x-1)^{1/2}\right]$$
$$=x^2\left[\frac{1}{2}(4x-1)^{-1/2}(4)\right]+(4x-1)^{1/2}(2x)$$
$$f'(x)=\frac{2x^2}{(4x-1)^{1/2}}+2x(4x-1)^{1/2}$$
$$=\frac{2x^2}{\sqrt{(4x-1)}}+2x\sqrt{(4x-1)}$$

The derivative on the previous page can be simplified as follows:

$$f'(x)=\frac{2x^2}{\sqrt{4x-1}}+\frac{2x\sqrt{4x-1}}{1}\cdot\frac{\sqrt{4x-1}}{\sqrt{4x-1}}$$
$$=\frac{2x^2}{\sqrt{4x-1}}+\frac{2x(4x-1)}{\sqrt{4x-1}}$$
$$=\frac{2x^2}{\sqrt{4x-1}}+\frac{8x^2-2x}{\sqrt{4x-1}}$$
$$=\frac{10x^2-2x}{\sqrt{4x-1}}$$
$$=\frac{2x(5x-1)}{\sqrt{4x-1}}$$

28. $f(x)=x^3\sqrt{5x+2}=x^3(5x+2)^{1/2}$

$$f'(x)=x^3\left[\frac{1}{2}(5x+2)^{-1/2}(5)\right]+(5x+2)^{1/2}(3x^2)$$
$$=\frac{5}{2}x^3(5x+2)^{-1/2}+3x^2(5x+2)^{1/2}$$
$$=\frac{5x^3}{2\sqrt{5x+2}}+3x^2\sqrt{5x+2}$$

The previous derivative can be simplified as follows:

$$f'(x)=\frac{x^2(35x+12)}{2\sqrt{5x+2}}$$

29. $G(x)=\sqrt[3]{x^5+6x}=(x^5+6x)^{1/3}$

Using the Extended Power Rule, we have

$$G'(x)=\frac{d}{dx}\left[(x^5+6x)^{1/3}\right]$$
$$=\frac{1}{3}(x^5+6x)^{1/3-1}\frac{d}{dx}(x^5+6x)$$
$$=\frac{1}{3}(x^5+6x)^{-2/3}(5x^4+6)$$
$$=\frac{5x^4+6}{3(x^5+6x)^{2/3}}$$
$$=\frac{5x^4+6}{3\cdot\sqrt[3]{(x^5+6x)^2}}$$

30. $F(x)=\sqrt[4]{x^2-5x+2}=(x^2-5x+2)^{1/4}$

$$F'(x)=\frac{1}{4}(x^2-5x+2)^{1/4-1}(2x-5)$$
$$=\frac{1}{4}(x^2-5x+2)^{-3/4}(2x-5)$$
$$=\frac{2x-5}{4(x^2-5x+2)^{3/4}}$$

31. $f(x)=\left(\frac{3x-1}{5x+2}\right)^4$

Using the Extended Power Rule, we have

$$f'(x)=4\left(\frac{3x-1}{5x+2}\right)^3\frac{d}{dx}\left[\frac{3x-1}{5x+2}\right]$$

Using the Quotient Rule, we have

$$f'(x)=4\left(\frac{3x-1}{5x+2}\right)^3\left[\frac{(5x+2)(3)-(3x-1)(5)}{(5x+2)^2}\right]$$
$$=4\left(\frac{3x-1}{5x+2}\right)^3\left[\frac{15x+6-15x+5}{(5x+2)^2}\right]$$
$$=4\left(\frac{3x-1}{5x+2}\right)^3\left[\frac{11}{(5x+2)^2}\right]$$
$$=\frac{44(3x-1)^3}{(5x+2)^5}$$

32. $f(x)=\left(\frac{2x}{x^2+1}\right)^3$

$$f'(x)=3\left(\frac{2x}{x^2+1}\right)^2\left[\frac{(x^2+1)(2)-2x(2x)}{(x^2+1)^2}\right]$$
$$=3\left(\frac{2x}{x^2+1}\right)^2\left[\frac{2x^2+2-4x^2}{(x^2+1)^2}\right]$$
$$=3\left(\frac{2x}{x^2+1}\right)^2\left[\frac{2-2x^2}{(x^2+1)^2}\right]$$
$$=\frac{3(2x)^2(2-2x^2)}{(x^2+1)^3}$$
$$f'(x)=\frac{24x^2(1-x^2)}{(x^2+1)^3}$$
$$=\frac{-24x^2(x^2-1)}{(x^2+1)^3}$$

33. $g(x)=\sqrt{\frac{4-x}{3+x}}=\left(\frac{4-x}{3+x}\right)^{1/2}$

Using the Extended Power Rule, we have

$$g'(x)=\frac{1}{2}\left(\frac{4-x}{3+x}\right)^{1/2-1}\frac{d}{dx}\left[\frac{4-x}{3+x}\right]$$

Using the Quotient Rule, we have

$$g'(x)=\frac{1}{2}\left(\frac{4-x}{3+x}\right)^{-1/2}\left[\frac{(3+x)(-1)-(4-x)(1)}{(3+x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{4-x}{3+x}\right)^{-1/2}\left[\frac{-3-x-4+x}{(3+x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{3+x}{4-x}\right)^{1/2}\left[\frac{-7}{(3+x)^2}\right]$$
$$=\frac{-7}{2(3+x)^{3/2}(4-x)^{1/2}}$$
$$=\frac{-7}{2\sqrt{(3+x)^3}\cdot\sqrt{4-x}}$$

34. $g(x)=\sqrt{\frac{3+2x}{5-x}}=\left(\frac{3+2x}{5-x}\right)^{1/2}$

$g'(x)$
$$=\frac{1}{2}\left(\frac{3+2x}{5-x}\right)^{-1/2}\left[\frac{(5-x)(2)-(3+2x)(-1)}{(5-x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{5-x}{3+2x}\right)^{1/2}\left[\frac{10-2x+3+2x}{(5-x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{5-x}{3+2x}\right)^{1/2}\left[\frac{13}{(5-x)^2}\right]$$
$$=\frac{13}{2(5-x)^{3/2}(3+2x)^{1/2}}$$
$$=\frac{13}{2\sqrt{(5-x)^3}\cdot\sqrt{(3+2x)}}$$

35. $f(x)=\left(2x^3-3x^2+4x+1\right)^{100}$

Using the Extended Power Rule, we have

$$f'(x)=100\left(2x^3-3x^2+4x+1\right)^{99}\left(6x^2-6x+4\right)$$
$$=200\left(3x^2-3x+2\right)\left(2x^3-3x^2+4x+1\right)^{99}$$

36. $f(x)=\left(7x^4+6x^3-x\right)^{204}$

$$f'(x)=204\left(7x^4+6x^3-x\right)^{203}\left(28x^3+18x^2-1\right)$$

37. $g(x)=\left(\frac{2x+3}{5x-1}\right)^{-4}=\left(\frac{5x-1}{2x+3}\right)^{4}$

Using the Extended Power Rule, we have

$$g'(x)=\frac{d}{dx}\left[\left(\frac{5x-1}{2x+3}\right)^4\right]$$
$$=4\left(\frac{5x-1}{2x+3}\right)^{4-1}\left[\frac{d}{dx}\left(\frac{5x-1}{2x+3}\right)\right]$$

Next, using the Quotient Rule, we have

$$g'(x)=4\left(\frac{5x-1}{2x+3}\right)^3\left[\frac{(2x+3)(5)-(5x-1)(2)}{(2x+3)^2}\right]$$
$$=4\left(\frac{5x-1}{2x+3}\right)^3\left[\frac{10x+15-10x+2}{(2x+3)^2}\right]$$
$$=4\left(\frac{5x-1}{2x+3}\right)^3\left[\frac{17}{(2x+3)^2}\right]$$
$$=\frac{68(5x-1)^3}{(2x+3)^5}$$

38. $h(x)=\left(\frac{1-3x}{2-7x}\right)^{-5}=\left(\frac{2-7x}{1-3x}\right)^{5}$

$h'(x)$
$$=5\left(\frac{2-7x}{1-3x}\right)^4\left(\frac{(1-3x)(-7)-(2-7x)(-3)}{(1-3x)^2}\right)$$
$$=5\left(\frac{2-7x}{1-3x}\right)^4\left(\frac{-7+21x+6-21x}{(1-3x)^2}\right)$$
$$=5\left(\frac{2-7x}{1-3x}\right)^4\left(\frac{-1}{(1-3x)^2}\right)$$
$$=\frac{-5(2-7x)^4}{(1-3x)^6}$$

39. $f(x)=\sqrt{\frac{x^2+x}{x^2-x}}=\left(\frac{x^2+x}{x^2-x}\right)^{1/2}$

Using the Extended Power Rule, we have

$$f'(x)=\frac{1}{2}\left(\frac{x^2+x}{x^2-x}\right)^{1/2-1}\frac{d}{dx}\left[\frac{x^2+x}{x^2-x}\right]$$

Using the Quotient Rule, we have

$$f'(x)$$
$$=\frac{1}{2}\left(\frac{x^2+x}{x^2-x}\right)^{-1/2}\left[\frac{(x^2-x)(2x+1)-(x^2+x)(2x-1)}{(x^2-x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{x^2+x}{x^2-x}\right)^{-1/2}\left[\frac{2x^3-x^2-x-2x^3-x^2+x}{(x^2-x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{x^2-x}{x^2+x}\right)^{1/2}\left(\frac{-2x^2}{(x^2-x)^2}\right)$$
$$=\frac{-x^2}{(x^2-x)^{3/2}(x^2+x)^{1/2}}$$

The previous derivative can be simplified as follows:

$$f'(x)=\frac{-x^2}{(x^2-x)^{3/2}(x^2+x)^{1/2}}$$
$$=\frac{-x^2}{x^{3/2}(x-1)^{3/2}x^{1/2}(x+1)^{1/2}} \qquad \text{Factoring}$$
$$=\frac{-x^2}{x^2(x-1)^{3/2}(x+1)^{1/2}}$$
$$=\frac{-1}{(x-1)^{3/2}(x+1)^{1/2}}.$$

40. $f(x)=\sqrt[3]{\frac{4-x^3}{x-x^2}}=\left(\frac{4-x^3}{x-x^2}\right)^{1/3}$

$$f'(x)$$
$$=\frac{1}{3}\left(\frac{4-x^3}{x-x^2}\right)^{-2/3}\left[\frac{(x-x^2)(-3x^2)-(4-x^3)(1-2x)}{(x-x^2)^2}\right]$$
$$=\frac{1}{3}\left(\frac{x-x^2}{4-x^3}\right)^{2/3}\left[\frac{-3x^3+3x^4-4+8x+x^3-2x^4}{(x-x^2)^2}\right]$$

Next, we simplify the derivative.

$$f'(x)=\frac{1}{3}\left(\frac{x-x^2}{4-x^3}\right)^{2/3}\left[\frac{x^4-2x^2+8x-4}{(x-x^2)^2}\right]$$
$$=\frac{x^4-2x^2+8x-4}{3(4-x^3)^{2/3}(x-x^2)^{4/3}}$$
$$f'(x)=\frac{x^4-2x^2+8x-4}{3(4-x^3)^{2/3}(x-x^2)^{4/3}}$$

41. $f(x)=\frac{(2x+3)^4}{(3x-2)^5}$

Using the Quotient Rule and the Extended Power Rule, we have:

$$f'(x)=\frac{d}{dx}\left[\frac{(2x+3)^4}{(3x-2)^5}\right]$$
$$=\frac{(3x-2)^5\left[4(2x+3)^3(2)\right]-(2x+3)^4\left[5(3x-2)^4(3)\right]}{\left((3x-2)^5\right)^2}$$
$$=\frac{8(2x+3)^3(3x-2)^5-15(2x+3)^4(3x-2)^4}{(3x-2)^{10}}$$
$$=\frac{(2x+3)^3(3x-2)^4\left[8(3x-2)-15(2x+3)\right]}{(3x-2)^{10}}$$
$$=\frac{(2x+3)^3\left[24x-16-30x-45\right]}{(3x-2)^6}$$
$$=\frac{(2x+3)^3\left[-6x-61\right]}{(3x-2)^6}$$
$$=\frac{-(2x+3)^3\left[6x+61\right]}{(3x-2)^6}$$
$$f'(x)=\frac{-(2x+3)^3\left[6x+61\right]}{(3x-2)^6}$$

42. $f(x)=\dfrac{(5x-4)^7}{(6x+1)^3}$

$$f'(x)=\frac{(6x+1)^3\left[7(5x-4)^6(5)\right]-(5x-4)^7\left[3(6x+1)^2(6)\right]}{\left((6x+1)^3\right)^2}$$

$$f'(x)=\frac{35(5x-4)^6(6x+1)^3-18(5x-4)^7(6x+1)^2}{(6x+1)^6}$$
$$=\frac{(5x-4)^6(6x+1)^2\left[35(6x+1)-18(5x-4)\right]}{(6x+1)^6}$$
$$=\frac{(5x-4)^6\left[210x+35-90x+72\right]}{(6x+1)^4}$$
$$=\frac{(5x-4)^6\left[120x+107\right]}{(6x+1)^4}$$
$$f'(x)=\frac{(5x-4)^6\left[120x+107\right]}{(6x+1)^4}$$

43. $f(x)=12(2x+1)^{2/3}(3x-4)^{5/4}$

Using the Product Rule and the Extended Power Rule, we have:

$$f'(x)=12\frac{d}{dx}\left[(2x+1)^{2/3}(3x-4)^{5/4}\right]$$
$$=12\left[(2x+1)^{2/3}\left[\frac{5}{4}(3x-4)^{1/4}(3)\right]\right]+12\left[(3x-4)^{5/4}\left[\frac{2}{3}(2x+1)^{-1/3}(2)\right]\right]$$
$$=12\left[\frac{15}{4}(2x+1)^{2/3}(3x-4)^{1/4}\right]+12\left[\frac{4}{3}(3x-4)^{5/4}(2x+1)^{-1/3}\right]$$
$$=45(2x+1)^{2/3}(3x-4)^{1/4}+\frac{16(3x-4)^{5/4}}{(2x+1)^{1/3}}.$$

Simplifying, we have

$$f'(x)$$
$$=\frac{45(2x+1)^{2/3}(3x-4)^{1/4}}{1}\cdot\frac{(2x+1)^{1/3}}{(2x+1)^{1/3}}+\frac{16(3x-4)^{5/4}}{(2x+1)^{1/3}}$$
$$=\frac{45(2x+1)(3x-4)^{1/4}}{(2x+1)^{1/3}}+\frac{16(3x-4)^{5/4}}{(2x+1)^{1/3}}$$
$$=\frac{45(2x+1)(3x-4)^{1/4}+16(3x-4)^{5/4}}{(2x+1)^{1/3}}$$
$$=\frac{(3x-4)^{1/4}\left[45(2x+1)+16(3x-4)^{4/4}\right]}{(2x+1)^{1/3}}$$
$$=\frac{(3x-4)^{1/4}\left[45(2x+1)+16(3x-4)\right]}{(2x+1)^{1/3}}$$
$$=\frac{(3x-4)^{1/4}\left[90x+45+48x-64\right]}{(2x+1)^{1/3}}$$
$$=\frac{(3x-4)^{1/4}\left[138x-19\right]}{(2x+1)^{1/3}}.$$
$$f'(x)=\frac{(3x-4)^{1/4}\left[138x-19\right]}{(2x+1)^{1/3}}$$

44. $y=6\sqrt[3]{x^2+x}\left(x^4-6x\right)^3$

$$y=6\left(x^2+x\right)^{1/3}\left(x^4-6x\right)^3$$
$$\frac{dy}{dx}$$
$$=6\left[\left(x^2+x\right)^{1/3}\left[3\left(x^4-6x\right)^2\left(4x^3-6\right)\right]\right]+6\left[\left(x^4-6x\right)^3\left[\frac{1}{3}\left(x^2+x\right)^{-2/3}(2x+1)\right]\right]$$
$$=18\left(x^2+x\right)^{1/3}\left(x^4-6x\right)^2\left(4x^3-6\right)+2\left(x^4-6x\right)^3\left(x^2+x\right)^{-2/3}(2x+1)$$

45. $y=\sqrt{u}=u^{1/2}$ and $u=x^2-1$

$$\frac{dy}{du}=\frac{1}{2}u^{1/2-1}=\frac{1}{2}u^{-1/2}=\frac{1}{2\sqrt{u}}$$
$$\frac{du}{dx}=2x^{2-1}=2x$$

Applying the Chain Rule, we have:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= \frac{1}{2\sqrt{u}} \cdot 2x$$
$$= \frac{2x}{2\sqrt{x^2-1}} \quad \text{Subtituting } x^2-1 \text{ for } u.$$
$$= \frac{x}{\sqrt{x^2-1}} \quad \text{Simplifying}$$

46. $y = \dfrac{15}{u^3} = 15u^{-3}$ and $u = 2x+1$

$$\frac{dy}{du} = 15\left(-3u^{-3-1}\right) = -45u^{-4} = \frac{-45}{u^4}$$
$$\frac{du}{dx} = 2$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= \frac{-45}{u^4} \cdot 2$$
$$= \frac{-90}{(2x+1)^4}$$

47. $y = u^{50}$ and $u = 4x^3 - 2x^2$

$$\frac{dy}{du} = 50u^{50-1} = 50u^{49}$$
$$\frac{du}{dx} = 4\left(3x^{3-1}\right) - 2\left(2x^{2-1}\right) = 12x^2 - 4x$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= 50u^{49} \cdot \left(12x^2 - 4x\right)$$

Subtituting $4x^3 - 2x^2$ for *u*.

$$= 50\left(4x^3 - 2x^2\right)^{49} \cdot \left(12x^2 - 4x\right)$$
$$= 200x(3x-1)\left(4x^3 - 2x^2\right)^{49} \quad \text{Simplifying}$$

48. $y = \dfrac{u+1}{u-1}$ and $u = 1+\sqrt{x} = 1+x^{1/2}$

$$\frac{dy}{du} = \frac{(u-1)(1)-(u+1)(1)}{(u-1)^2} = \frac{-2}{(u-1)^2}$$
$$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= \frac{-2}{(u-1)^2} \cdot \frac{1}{2\sqrt{x}}$$
$$= \frac{-1}{\sqrt{x}\left(\left(1+\sqrt{x}\right)-1\right)^2} \quad \text{Subtituting } 1+\sqrt{x} \text{ for } u.$$
$$= \frac{-1}{\sqrt{x}\left(\sqrt{x}\right)^2} \quad \text{Simplifying}$$
$$= \frac{-1}{x^{3/2}}$$

49. $y = u(u+1)$ and $u = x^3 - 2x$

$$\frac{dy}{du} = u(1) + (u+1)(1) \quad \text{Product Rule}$$
$$= 2u+1$$
$$\frac{du}{dx} = 3x^2 - 2$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= (2u+1) \cdot \left(3x^2 - 2\right)$$

Subtituting $x^3 - 2x$ for *u*.

$$= \left(2\left(x^3 - 2x\right)+1\right) \cdot \left(3x^2 - 2\right)$$
$$= \left(2x^3 - 4x + 1\right) \cdot \left(3x^2 - 2\right) \quad \text{Simplifying}$$

50. $y = (u+1)(u-1)$ and $u = x^3 + 1$

$$\frac{dy}{du} = (u+1)(1) + (u-1)(1) = 2u$$
$$\frac{du}{dx} = 3x^2$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= (2u) \cdot \left(3x^2\right)$$
$$= \left(2\left(x^3+1\right)\right) \cdot \left(3x^2\right)$$
$$= 6x^2\left(x^3+1\right)$$

51. $y = 5u^2 + 3u$ and $u = x^3 + 1$

$$\frac{dy}{du} = 10u + 3$$
$$\frac{du}{dx} = 3x^2$$

Applying the Chain Rule, we have:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= (10u+3) \cdot (3x^2)$$
$$= (10(x^3+1)+3) \cdot (3x^2) \quad \text{Substituting for } u.$$
$$= 3x^2((10x^3+10)+3)$$
$$= 3x^2(10x^3+13)$$

52. $y = u^3 - 7u^2$ and $u = x^2 + 3$

$$\frac{dy}{du} = 3u^2 - 14u$$
$$\frac{du}{dx} = 2x$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= (3u^2 - 14u) \cdot (2x)$$
$$= \left(3(x^2+3)^2 - 14(x^2+3)\right) \cdot (2x)$$
$$\frac{dy}{dx} = (3x^4 + 18x^2 + 27 - 14x^2 - 42)(2x)$$
$$= (3x^4 + 4x^2 - 15)(2x)$$
$$= 2x(x^2+3)(3x^2-5)$$

53. $y = \sqrt[3]{2u+5} = (2u+5)^{1/3}$ and $u = x^2 - x$

$$\frac{dy}{du} = \frac{1}{3}(2u+5)^{-2/3}(2) \quad \text{Extended Power Rule}$$
$$= \frac{2}{3(2u+5)^{2/3}}$$
$$\frac{du}{dx} = 2x - 1$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= \left(\frac{2}{3(2u+5)^{2/3}}\right) \cdot (2x-1)$$
$$= \left(\frac{2}{3(2(x^2-x)+5)^{2/3}}\right) \cdot (2x-1) \quad \text{Substituting}$$
$$= \frac{2(2x-1)}{3(2x^2-2x+5)^{2/3}}$$

54. $y = \sqrt{7-3u} = (7-3u)^{1/2}$ and $u = x^2 - 9$

$$\frac{dy}{du} = \frac{1}{2}(7-3u)^{-1/2}(-3) = \frac{-3}{2(7-3u)^{1/2}}$$
$$\frac{du}{dx} = 2x$$
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$
$$= \left(\frac{-3}{2(7-3u)^{1/2}}\right) \cdot (2x)$$
$$= \left(\frac{-3}{2(7-3(x^2-9))^{1/2}}\right) \cdot (2x)$$
$$= \frac{-3x}{(34-3x^2)^{1/2}}$$
$$= \frac{-3x}{\sqrt{34-3x^2}}$$

55. $y = \dfrac{1}{u^2+u}$ and $u = 5 + 3t$

$$\frac{dy}{du} = \frac{(u^2+u)(0) - (1)(2u+1)}{(u^2+u)^2} \quad \text{Quotient Rule}$$
$$= \frac{-(2u+1)}{(u^2+u)^2}$$
$$\frac{du}{dt} = 3$$
$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}$$
$$= \left(\frac{-(2u+1)}{(u^2+u)^2}\right) \cdot (3)$$
$$= \left(\frac{-(2(5+3t)+1)}{\left((5+3t)^2 + (5+3t)\right)^2}\right) \cdot (3) \quad \text{Substituting}$$
$$= \frac{-3(10+6t+1)}{\left((5+3t)^2 + (5+3t)\right)^2}$$
$$= \frac{-3(6t+11)}{(5+3t)^2\left((5+3t)+1\right)^2} \quad \text{Factoring}$$
$$= \frac{-3(6t+11)}{(5+3t)^2(6+3t)^2}$$

56. $y=\dfrac{1}{3u^5-7}$ and $u=7t^2+1$

$$\frac{dy}{du}=\frac{(3u^5-7)(0)-(1)(15u^4)}{(3u^5-7)^2} \quad \text{Quotient Rule}$$
$$=\frac{-15u^4}{(3u^5-7)^2}$$
$$\frac{du}{dt}=14t$$
$$\frac{dy}{dt}=\frac{dy}{du}\cdot\frac{du}{dt}$$
$$=\left(\frac{-15u^4}{(3u^5-7)^2}\right)\cdot(14t)$$
$$=\left(\frac{-15(7t^2+1)^4}{\left(3(7t^2+1)^5-7\right)^2}\right)\cdot(14t)$$
$$=\frac{-210t(7t^2+1)^4}{\left(3(7t^2+1)^5-7\right)^2}$$

57. $y=\sqrt{x^2+3x}=(x^2+3x)^{1/2}$

First, we find the derivative using the Extended Power Rule.

$$\frac{dy}{dx}=\frac{1}{2}(x^2+3x)^{1/2-1}(2x+3)$$
$$=\frac{2x+3}{2\sqrt{x^2+3x}}$$

When $x=1$,

$$\frac{dy}{dx}=\frac{2(1)+3}{2\sqrt{(1)^2+3(1)}}=\frac{5}{2\sqrt{4}}=\frac{5}{2\cdot 2}=\frac{5}{4}$$

Thus, the slope of the tangent line at $(1,2)$ is $\dfrac{5}{4}$.

Using the point-slope equation, we find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-2=\frac{5}{4}(x-1)$$
$$y-2=\frac{5}{4}x-\frac{5}{4}$$
$$y=\frac{5}{4}x+\frac{3}{4}$$

58. $y=(x^3-4x)^{10}$

$$\frac{dy}{dx}=10(x^3-4x)^9(3x^2-4)$$

When $x=2$,

$$\frac{dy}{dx}=10\left((2)^3-4(2)\right)^9\left(3(2)^2-4\right)$$
$$=10(0)^9(8)$$
$$=0$$

Thus, the slope of the tangent line at the point, $(2,0)$ is 0. The equation of the horizontal line passing through the point $(2,0)$ is $y=0$.

59. $y=x\sqrt{2x+3}=x(2x+3)^{1/2}$

First, we find the derivative using the Product Rule and the Extended Power Rule.

$$\frac{dy}{dx}=x\left(\frac{1}{2}(2x+3)^{1/2-1}(2)\right)+(2x+3)^{1/2}(1)$$
$$=\frac{x}{\sqrt{2x+3}}+\sqrt{2x+3}$$

When $x=3$,

$$\frac{dy}{dx}=\frac{(3)}{\sqrt{2(3)+3}}+\sqrt{2(3)+3}$$
$$=\frac{3}{\sqrt{9}}+\sqrt{9}$$
$$=\frac{3}{3}+3$$
$$=1+3$$
$$=4.$$

Thus, the slope of the tangent line at $(3,9)$ is 4.

Using the point-slope equation, we find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-9=4(x-3)$$
$$y-9=4x-12$$
$$y=4x-3$$

60. $y=\left(\frac{2x+3}{x-1}\right)^3$

$$\frac{dy}{dx}=3\left(\frac{2x+3}{x-1}\right)^2\left[\frac{(x-1)(2)-(2x+3)(1)}{(x-1)^2}\right]$$
$$=3\left(\frac{2x+3}{x-1}\right)^2\left[\frac{2x-2-2x-3}{(x-1)^2}\right]$$
$$=3\left(\frac{2x+3}{x-1}\right)^2\left[\frac{-5}{(x-1)^2}\right]$$
$$=\frac{-15(2x+3)^2}{(x-1)^4}$$

When $x=2$,

$$\frac{dy}{dx}=\frac{-15(2(2)+3)^2}{((2)-1)^4}$$
$$=\frac{-15(7)^2}{(1)^4}$$
$$=-15\cdot 49$$
$$=-735$$

Thus, the slope of the tangent line at $(2,343)$ is -735.

Using the point-slope equation, we find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-343=-735(x-2)$$
$$y-343=-735x+1470$$
$$y=-735x+1813$$

61. $f(x)=\frac{x^2}{(1+x)^5}$

a) Using the Quotient Rule and the Extended Power Rule, we have:

$$f'(x)=\frac{(1+x)^5(2x)-x^2\left[5(1+x)^4(1)\right]}{\left((1+x)^5\right)^2}$$
$$=\frac{2x(1+x)^5-5x^2(1+x)^4}{(1+x)^{10}}$$
$$=\frac{(1+x)^4\left(2x(1+x)-5x^2\right)}{(1+x)^{10}} \quad \text{Factoring}$$
$$=\frac{2x+2x^2-5x^2}{(1+x)^6}$$
$$=\frac{2x-3x^2}{(1+x)^6}$$
$$=\frac{x(2-3x)}{(1+x)^6}$$

b) Using the Product Rule and the Extended Power Rule on $f(x)=x^2(1+x)^{-5}$, we have

$$f'(x)=x^2\left(-5(1+x)^{-6}(1)\right)+(1+x)^{-5}(2x)$$
$$=\frac{-5x^2}{(1+x)^6}+\frac{2x}{(1+x)^5}$$
$$=\frac{-5x^2}{(1+x)^6}+\frac{2x}{(1+x)^5}\cdot\frac{(1+x)}{(1+x)}$$
$$=\frac{-5x^2+2x+2x^2}{(1+x)^6}$$
$$=\frac{2x-3x^2}{(1+x)^6}$$
$$=\frac{x(2-3x)}{(1+x)^6}$$

c) The results are the same.

62. $g(x)=\left(\frac{6x+1}{2x-5}\right)^2$

a) Using Extended Power Rule, we have

$$g'(x)=2\left(\frac{6x+1}{2x-5}\right)\left[\frac{(2x-5)(6)-(6x+1)(2)}{(2x-5)^2}\right]$$
$$=\frac{2(6x+1)}{2x-5}\left[\frac{12x-30-12x-2}{(2x-5)^2}\right]$$
$$=\frac{2(6x+1)}{2x-5}\left[\frac{-32}{(2x-5)^2}\right]$$
$$=\frac{-64(6x+1)}{(2x-5)^3}$$

b) Using the Quotient Rule on

$$g(x)=\frac{36x^2+12x+1}{4x^2-20x+25}\text{, we have}$$

$$g'(x)=\frac{(4x^2-20x+25)(72x+12)}{(4x^2-20x+25)^2}-\frac{(36x^2+12x+1)(8x-20)}{4x^2-20x+25}$$

$$=\frac{288x^3-1392x^2+1560x+300}{(4x^2-20x+25)^2}-\frac{288x^3-624x^2-232x-20}{(4x^2-20x+25)^2}$$

$$g'(x)=\frac{-768x^2+1792x+320}{(4x^2-20x+25)^2}$$

$$=\frac{-64(2x-5)(6x+1)}{(2x-5)^4}$$

$$=\frac{-64(6x+1)}{(2x-5)^3}$$

c) $\boxed{tw}$ The results are the same. Which method is easier depends on the student. We believe that the Extended Power rule offers us a more efficient approach. It takes too much time to expand the function, and then factor it back binomials.

63. $h(x)=(3x^2-7)^5$

Let $f(x)=x^5$ and $g(x)=3x^2-7$.

$(f\circ g)(x)=f(g(x))=f(3x^2-7)=(3x^2-7)^5$

Thus, $h(x)=(f\circ g)(x)$.

Answers may vary.

64. $h(x)=\dfrac{1}{\sqrt{7x+2}}$

Let $f(x)=\dfrac{1}{\sqrt{x}}$ and $g(x)=7x+2$.

Answers may vary.

65. $h(x)=\dfrac{x^3+1}{x^3-1}$

Let $f(x)=\dfrac{x+1}{x-1}$ and $g(x)=x^3$.

$(f\circ g)(x)=f(g(x))=f(x^3)=\dfrac{x^3+1}{x^3-1}$

Thus, $h(x)=(f\circ g)(x)$.

Answers may vary.

66. $h(x)=(\sqrt{x}+5)^4$

Let $f(x)=x^4$ and $g(x)=\sqrt{x}+5$.

Answers may vary.

67. Using the Chain Rule:

$f(u)=u^3$, $g(x)=u=2x^4+1$

First find $f'(u)$ and $g'(x)$.

$f'(u)=3u^2$

$f'(g(x))=3(2x^4+1)^2$ Substituting $g(x)$ for u.

$g'(x)=8x^3$

The Chain Rule states

$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$.

Substituting, we have:

$$(f\circ g)'(x)=3(2x^4+1)^2\cdot(8x^3)$$
$$=24x^3(2x^4+1)^2.$$

Therefore,

$$(f\circ g)'(-1)=24(-1)^3(2(-1)^4+1)^2$$
$$=-24(2+1)^2$$
$$=-24\cdot 9$$
$$=-216.$$

Finding $f(g(x))$ first, we have:

$f(g(x))=f(2x^4+1)=(2x^4+1)^3$.

By the Extended Power Rule:

$$f'(g(x))=3(2x^4+1)^2(8x^3)$$
$$=24x^3(2x^4+1)^2.$$

Therefore, $f'(g(-1))=-216$ as above.

68. Using the Chain Rule:

$f(u)=\dfrac{u+1}{u-1}, g(x)=u=\sqrt{x}=x^{1/2}$

$$f'(u)=\frac{(u-1)(1)-(u+1)(1)}{(u-1)^2}=\frac{-2}{(u-1)^2}$$

$$f'(g(x))=\frac{-2}{(\sqrt{x}-1)^2}$$

$$g'(x)=\frac{1}{2\sqrt{x}}$$

$$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$$
$$=\frac{-2}{(\sqrt{x}-1)^2}\cdot\frac{1}{2\sqrt{x}}$$
$$=\frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$$

Therefore,

$$(f\circ g)'(4)=\frac{-1}{\sqrt{4}(\sqrt{4}-1)^2}=\frac{-1}{2(1)^2}=-\frac{1}{2}.$$

Finding $f(g(x))$ first, we have:

$$f(g(x))=f(\sqrt{x})=\frac{\sqrt{x}+1}{\sqrt{x}-1}$$

Using the Quotient Rule:

$$f'(g(x))=\frac{(\sqrt{x}-1)\left(\frac{1}{2}x^{-1/2}\right)-(\sqrt{x}+1)\frac{1}{2}x^{-1/2}}{(\sqrt{x}-1)^2}$$
$$=\frac{\frac{1}{2}-\frac{1}{2}x^{-1/2}-\frac{1}{2}-\frac{1}{2}x^{-1/2}}{(\sqrt{x}-1)^2}$$
$$=\frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$$

Therefore, $(f\circ g)'(4)=-\frac{1}{2}$ as above.

69. Using the Chain Rule:

$f(u)=\sqrt[3]{u}=u^{1/3}$, $g(x)=u=1+3x^2$.

First find $f'(u)$ and $g'(x)$.

$$f'(u)=\frac{1}{3}u^{-2/3}=\frac{1}{3\cdot\sqrt[3]{u^2}}$$

$$f'(g(x))=\frac{1}{3\cdot\sqrt[3]{(1+3x^2)^2}}$$ Substituting $g(x)$ for u.

$$g'(x)=6x$$

The Chain Rule states

$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$.

Substituting, we have:

$$(f\circ g)'(x)=\frac{1}{3\cdot\sqrt[3]{(1+3x^2)^2}}\cdot(6x)$$
$$=\frac{2x}{\sqrt[3]{(1+3x^2)^2}}.$$

Therefore,

$$(f\circ g)'(2)=\frac{2(2)}{\sqrt[3]{(1+3(2)^2)^2}}$$
$$=\frac{4}{\sqrt[3]{(13)^2}}$$
$$\approx 0.72348760$$

Finding $f(g(x))$ first, we have:

$$f(g(x))=f(1+3x^2)=(1+3x^2)^{1/3}$$

By the Extended Power Rule:

$$f'(g(x))=\frac{1}{3}(1+3x^2)^{-2/3}(6x)$$
$$=\frac{2x}{(1+3x^2)^{2/3}}.$$

Therefore $(f\circ g)'(2)=\frac{4}{\sqrt[3]{(13)^2}}\approx 0.72348760$

as above.

70. Using the Chain Rule:

$$f(u)=2u^5, g(x)=u=\frac{3-x}{4+x}$$
$$f'(u)=10u^4$$
$$f'(g(x))=10\left(\frac{3-x}{4+x}\right)^4$$
$$g'(x)=\frac{(4+x)(-1)-(3-x)(1)}{(4+x)^2}$$
$$=\frac{-7}{(4+x)^2}$$
$$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$$
$$=10\left(\frac{3-x}{4+x}\right)^4\cdot\left(\frac{-7}{(4+x)^2}\right)$$
$$=\frac{-70(3-x)^4}{(4+x)^6}$$

Therefore,

$$(f\circ g)'(-10)=\frac{-70(3-(-10))^4}{(4+(-10))^6}$$
$$=\frac{-70(13)^4}{(-6)^6}.$$
$$=-42.85129458$$

Finding $f\left(g\left(x\right)\right)$ first, we have:

$$f\left(g\left(x\right)\right)=f\left(\frac{3-x}{4+x}\right)=2\left(\frac{3-x}{4+x}\right)^5$$

Using the Extended Power Rule:

$$f'\left(g\left(x\right)\right)=2\cdot 5\left(\frac{3-x}{4+x}\right)^4\left[\frac{(4+x)(-1)-(3-x)(1)}{(4+x)^2}\right]$$

$$=10\left(\frac{3-x}{4+x}\right)^4\left[\frac{-7}{(4+x)^2}\right]$$

$$f'\left(g\left(x\right)\right)=\frac{-70(3-x)^4}{(4+x)^6}$$

Therefore, $(f\circ g)'(-10)=-42.85129458$ as before.

71. $R(x)=1000\sqrt{x^2-0.1x}=1000\left(x^2-0.1x\right)^{1/2}$

Using the Extended Power Rule, we have

$$R'(x)=1000\left[\frac{1}{2}\left(x^2-0.1x\right)^{-1/2}(2x-0.1)\right]$$

$$=500\left[\frac{2x-0.1}{\left(x^2-0.1x\right)^{1/2}}\right]$$

$$=\frac{500(2x-0.1)}{\sqrt{x^2-0.1x}}$$

Substituting 20 for *x*, we have

$$R'(20)=\frac{500\left(2(20)-0.1\right)}{\sqrt{(20)^2-0.1(20)}}$$

$$=1000.00314070$$

$$\approx 1000$$

When 20 items have been sold, revenue is changing at a rate of 1,000 thousand of dollars per item, or 1,000,000 dollars per item.

72. $C(x)=2000\left(x^2+2\right)^{1/3}+700$

$$C'(x)=2000\left[\frac{1}{3}\left(x^2+2\right)^{-2/3}(2x)\right]$$

$$=\frac{4000x}{3}\left(x^2+2\right)^{-2/3}$$

$$=\frac{4000x}{3\left(x^2+2\right)^{2/3}}$$

$$C'(20)=\frac{4000(20)}{3\left((20)^2+2\right)^{2/3}}\approx 489.57364458$$

When 20 items are produced, the total cost is changing at a rate of 489.574 thousand of dollars per item, or 489,574 dollars per item.

73. $P(x)=R(x)-C(x)$ and

$P'(x)=R'(x)-C'(x)$

Since we are trying to find the rate at which total profit is changing as a function of *x*, we can use the derivatives found in Exercise 71 and 72 to find the derivative of the profit function. There is no need to find the Profit function first and then take the derivative.

$$P'(x)=R'(x)-C'(x)$$

$$=\frac{500(2x-0.1)}{\sqrt{x^2-0.1x}}-\frac{4000x}{3\left(x^2+2\right)^{2/3}}$$

74. $C(x)=\sqrt{5x^2+60}$ and $x(t)=20t+40$. To find the rate of change of the cost function with respect to the number of months, we must apply the Chain Rule.

$$C'(x)=\frac{1}{2}\left(5x^2+60\right)^{-1/2}(10x)=5x\left(5x^2+60\right)^{-1/2}$$

$$C'\left(x(t)\right)=5(20t+40)\left(5(20t+40)^2+60\right)^{-1/2}$$

$$x'(t)=20$$

Using the Chain Rule, we have:

$$\frac{dC}{dt}=C'\left(x(t)\right)\cdot x'(t)$$

$$=5(20t+40)\left(5(20t+40)^2+60\right)^{-1/2}\cdot(20)$$

$$=100(20t+40)\left(5(20t+40)^2+60\right)^{-1/2}$$

$$=\frac{100(20t+40)}{\left(5(20t+40)^2+60\right)^{1/2}}$$

Substituting 4 in for *t*, we have:

$$\frac{dC}{dt}=\frac{100\left(20(4)+40\right)}{\left(5\left(20(4)+40\right)^2+60\right)^{1/2}}$$

$$\approx 44.70273729$$

After 4 months, the total cost is changing at a rate of 44.7 thousand of dollars per month, or 44,700 dollars per month.

75. Let x be the number of years since 1995.

$$C(x) = 0.21x^4 - 5.92x^3 + 50.53x^2 - 18.92x + 1114.93$$

a) $$\frac{dC}{dx} = 0.21(4x^3) - 5.92(3x^2) + 50.53(2x) - 18.92$$
$$= 0.84x^3 - 17.76x^2 + 101.06x - 18.92$$

b) $\boxed{tw}$ Answers will vary. $\frac{dC}{dx}$ represents the rate at which consumer credit is changing with respect to time.

c) Substitute 15 for x. $[2010 - 1995 = 15]$

$$\frac{dC}{dx} = 0.84(15)^3 - 17.76(15)^2 + 101.06(15) - 18.92$$
$$= 2835 - 3996 + 1515.9 - 18.92$$
$$= 335.98$$

Outstanding consumer credit will be rising at a rate of 335.98 billion of dollars per year in 2010.

76. $$U(x) = 80\sqrt{\frac{2x+1}{3x+4}} = 80\left(\frac{2x+1}{3x+4}\right)^{1/2}$$

$$U'(x) = 40\left(\frac{2x+1}{3x+4}\right)^{-1/2} \cdot \left[\frac{(3x+4)(2) - (2x+1)(3)}{(3x+4)^2}\right]$$
$$= 40\left(\frac{3x+4}{2x+1}\right)^{1/2}\left[\frac{5}{(3x+4)^2}\right]$$
$$= \frac{200}{(2x+1)^{1/2}(3x+4)^{3/2}}$$

77. $A = 1000(1+i)^3$

a) Using the Extended Power Rule, we have

$$\frac{dA}{di} = 1000(3)(1+i)^2(1)$$
$$= 3000(1+i)^2.$$

b) $\boxed{tw}$ $\frac{dA}{di}$ is the rate at which the amount in the account is growing with respect to the interest rate three years after it was invested.

78. $$A = 1000\left(1 + \frac{i}{4}\right)^{20}$$

a) Using the Extended Power Rule, we have

$$\frac{dA}{di} = 1000(20)\left(1 + \frac{i}{4}\right)^{19}\left(\frac{1}{4}\right)$$
$$= 5000\left(1 + \frac{i}{4}\right)^{19}$$

b) $\boxed{tw}$ $\frac{dA}{di}$ is the rate at which the amount in the account is growing with respect to the interest rate five years after it was invested..

79. $D(p) = \frac{80,000}{p}$ and $p = 1.6t + 9$

a) Substitute $1.6t + 9$ in for p in the demand function.

$$D(t) = \frac{80,000}{1.6t + 9}$$

b) Using the Quotient Rule, we have

$$D'(t) = \frac{(1.6t+9)(0) - (80,000)(1.6)}{(1.6t+9)^2}$$
$$= \frac{-128,000}{(1.6t+9)^2}$$

Therefore,

$$D'(100) = \frac{-128,000}{(1.6(100)+9)^2}$$
$$= \frac{-128,000}{28,561}$$
$$\approx -4.48163580$$

After 100 days, quantity demanded is changing -4.482 units per day.

80. $P(x) = 0.08x^2 + 80x$ and $x = 5t + 1$

a) Substituting $5t + 1$ for x, we have

$$P(t) = 0.08(5t+1)^2 + 80(5t+1)$$
$$= 0.08(25t^2 + 10t + 1) + 400t + 80$$
$$= 2t^2 + 400.8t + 80.08$$

b) $P'(t) = 4t + 400.8$

Therefore,

$$P'(48) = 4(48) + 400.8 = 592.80$$

After 48 months, profit is increasing at rate of 592.80 dollars per month.

81. $D = 0.85A(c+25)$ and $c = (140-y)\dfrac{w}{72x}$

a) Substituting 5 for A we have:

$$D(c) = 0.85(5)(c+25)$$
$$= 4.25(c+25)$$
$$= 4.25c + 106.25.$$

Substituting 0.6 for x, 45 for y, we have:

$$c(w) = (140-45)\frac{w}{72(0.6)}$$
$$= 95\frac{w}{43.2}$$
$$= \frac{95w}{43.2} \approx 2.199w.$$

b) $\dfrac{dD}{dc} = \dfrac{d}{dc}(4.25c + 106.25) = 4.25$

The dosage changes at a rate of 4.25 mg per unit of creatine clearance.

c) $\dfrac{dc}{dw} = \dfrac{d}{dw}\left(\dfrac{95w}{43.2}\right) = \dfrac{95}{43.2} \approx 2.199$

The creatine clearance changes at a rate of 2.199 unit of creatine clearance per kilogram.

d) By the Chain Rule:

$$\frac{dD}{dw} = \frac{dD}{dc}\cdot\frac{dc}{dw} = (4.25)\left(\frac{95}{43.2}\right) \approx 9.346\ .$$

The dosage changes at a rate of 9.35 milligrams per kilogram.

e) $\boxed{tw}$ Answers will vary. $\dfrac{dD}{dw}$ represents the rate of change of the dosage with respect to the patient's weight. For each additional kilogram of weight, the dosage is increased by about 9.35 milligrams.

82. $y = \sqrt{(2x-3)^2+1} = \left((2x-3)^2+1\right)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}\left((2x-3)^2+1\right)^{-1/2}\left(2(2x-3)(2)\right)$$
$$= \frac{2(2x-3)}{\left((2x-3)^2+1\right)^{1/2}}$$
$$= \frac{2(2x-3)}{\sqrt{(2x-3)^2+1}}$$

83. $y = \sqrt[3]{x^3+6x+1}\cdot x^5 = \left(x^3+6x+1\right)^{1/3}\cdot x^5$

Using the Product Rule and the Extended Power Rule, we have

$$\frac{dy}{dx} = \left(x^3+6x+1\right)^{1/3}\cdot\left(5x^4\right) + x^5\left[\frac{1}{3}\left(x^3+6x+1\right)^{-2/3}\left(3x^2+6\right)\right]$$
$$= 5x^4\left(x^3+6x+1\right)^{1/3} + \frac{3x^5\left(x^2+2\right)}{3\left(x^3+6x+1\right)^{2/3}}$$

The derivative can be further simplified by finding a common denominator and combining the fractions.

$$\frac{dy}{dx} = \frac{\left(x^3+6x+1\right)^{2/3}}{\left(x^3+6x+1\right)^{2/3}}\cdot\frac{5x^4\left(x^3+6x+1\right)^{1/3}}{1} + \frac{x^5\left(x^2+2\right)}{\left(x^3+6x+1\right)^{2/3}}$$
$$= \frac{5x^4\left(x^3+6x+1\right)}{\left(x^3+6x+1\right)^{2/3}} + \frac{x^5\left(x^2+2\right)}{\left(x^3+6x+1\right)^{2/3}}$$
$$= \frac{5x^7+30x^5+5x^4+x^7+2x^5}{\left(x^3+6x+1\right)^{2/3}}$$
$$= \frac{6x^7+32x^5+5x^4}{\left(x^3+6x+1\right)^{2/3}}$$

84. $s = \sqrt[4]{t^4+3t^2+8}\cdot 3t = \left(t^4+3t^2+8\right)^{1/4}\cdot 3t$

$$\frac{ds}{dt} = \left(t^4+3t^2+8\right)^{1/4}(3) + 3t\left[\frac{1}{4}\left(t^4+3t^2+8\right)^{-3/4}\left(4t^3+6t\right)\right]$$
$$= 3\left(t^4+3t^2+8\right)^{1/4} + \frac{6t^2\left(2t^2+3\right)}{4\left(t^4+3t^2+8\right)^{3/4}}$$
$$= \frac{6\left(t^4+3t^2+8\right)}{2\left(t^4+3t^2+8\right)^{3/4}} + \frac{3t^2\left(2t^2+3\right)}{2\left(t^4+3t^2+8\right)^{3/4}}$$
$$= \frac{6t^4+18t^2+48+6t^4+9t^2}{2\left(t^4+3t^2+8\right)^{3/4}}$$
$$= \frac{12t^4+27t^2+48}{2\left(t^4+3t^2+8\right)^{3/4}}$$

85. $y=\left(\dfrac{x}{\sqrt{x-1}}\right)^3$

Using the Extended Power Rule and the Quotient Rule, we have:

$$\frac{dy}{dx}=3\left(\frac{x}{(x-1)^{1/2}}\right)^{3-1}\frac{d}{dx}\left(\frac{x}{(x-1)^{1/2}}\right)$$

$$=3\left(\frac{x}{\sqrt{x-1}}\right)^2\cdot\left(\frac{(x-1)^{1/2}(1)-x\left(\frac{1}{2}(x-1)^{-1/2}\cdot 1\right)}{\left(\sqrt{x-1}\right)^2}\right)$$

$$=\frac{3x^2}{x-1}\left(\frac{(x-1)^{1/2}-\dfrac{x}{2(x-1)^{1/2}}}{x-1}\right)$$

$$=\frac{3x^2}{x-1}\left(\frac{\dfrac{2x-2}{2(x-1)^{1/2}}-\dfrac{x}{2(x-1)^{1/2}}}{x-1}\right)$$

$$=\frac{3x^2}{(x-1)^2}\left(\frac{x-2}{2(x-1)^{1/2}}\right)$$

$$=\frac{3x^2(x-2)}{2(x-1)^{5/2}}.$$

86. $y=\left(x\sqrt{1+x^2}\right)^3=\left(x(1+x^2)^{1/2}\right)^3$

$$\frac{dy}{dx}=3\left(x(1+x^2)^{1/2}\right)^2\cdot$$

$$\left(x\left(\frac{1}{2}(1+x^2)^{-1/2}\cdot 2x\right)+1(1+x^2)^{1/2}\right)$$

$$=3x^2(1+x^2)\cdot\left(\frac{x^2}{(1+x^2)^{1/2}}+(1+x^2)^{1/2}\right)$$

$$=3x^2(1+x^2)\cdot\left(\frac{x^2}{(1+x^2)^{1/2}}+\frac{1+x^2}{(1+x^2)^{1/2}}\right)$$

$$=3x^2(1+x^2)\cdot\left(\frac{1+2x^2}{(1+x^2)^{1/2}}\right)$$

$$=(3x^2+6x^4)(1+x^2)^{1/2}$$

$$=(3x^2+6x^4)\sqrt{(1+x^2)}$$

87. $y=\dfrac{\sqrt{1-x^2}}{1-x}=\dfrac{(1-x^2)^{1/2}}{1-x}$

Using the Quotient Rule and the Extended Power Rule, we have:

$$\frac{dy}{dx}=\frac{(1-x)\left(\frac{1}{2}(1-x^2)^{-1/2}(-2x)\right)-(1-x^2)^{1/2}(-1)}{(1-x)^2}$$

$$\frac{dy}{dx}=\frac{\dfrac{x(x-1)}{(1-x^2)^{1/2}}+(1-x^2)^{1/2}}{(1-x)^2}$$

$$=\frac{\dfrac{x^2-x}{(1-x^2)^{1/2}}+\dfrac{(1-x^2)^{1/2}}{1}\cdot\dfrac{(1-x^2)^{1/2}}{(1-x^2)^{1/2}}}{(1-x)^2}$$

$$=\frac{\dfrac{x^2-x}{(1-x^2)^{1/2}}+\dfrac{(1-x^2)}{(1-x^2)^{1/2}}}{(1-x)^2}$$

$$=\frac{1-x}{(1-x^2)^{1/2}(1-x)^2}$$

$$=\frac{1}{(1-x^2)^{1/2}(1-x)}$$

88. $w=\dfrac{u}{\sqrt{1+u^2}}=\dfrac{u}{(1+u^2)^{1/2}}$

$$\frac{dw}{du}=\frac{(1+u^2)^{1/2}(1)-u\left[\frac{1}{2}(1+u^2)^{-1/2}(2u)\right]}{\left((1+u^2)^{1/2}\right)^2}$$

$$=\frac{(1+u^2)^{1/2}-\dfrac{u^2}{(1+u^2)^{1/2}}}{(1+u^2)}$$

$$=\frac{\dfrac{(1+u^2)}{(1+u^2)^{1/2}}-\dfrac{u^2}{(1+u^2)^{1/2}}}{(1+u^2)}$$

$$=\frac{1}{(1+u^2)(1+u^2)^{1/2}}$$

$$=\frac{1}{(1+u^2)^{3/2}}$$

89. $y=\left(\frac{x^2-x-1}{x^2+1}\right)^3$

Using the Extended Power Rule and the Quotient Rule, we have

$$\frac{dy}{dx}$$
$$=3\left(\frac{x^2-x-1}{x^2+1}\right)^2\left[\frac{(x^2+1)(2x-1)-(x^2-x-1)(2x)}{(x^2+1)^2}\right]$$
$$=3\left(\frac{x^2-x-1}{x^2+1}\right)^2\left[\frac{x^2+4x-1}{(x^2+1)^2}\right]$$
$$=\frac{3(x^2-x-1)^2(x^2+4x-1)}{(x^2+1)^4}.$$

90. $g(x)=\sqrt{\frac{x^2-4x}{2x+1}}=\left(\frac{x^2-4x}{2x+1}\right)^{1/2}$

$$g'(x)$$
$$=\frac{1}{2}\left(\frac{x^2-4x}{2x+1}\right)^{-1/2}\left[\frac{(2x+1)(2x-4)-(x^2-4x)(2)}{(2x+1)^2}\right]$$
$$=\frac{1}{2}\left(\frac{2x+1}{x^2-4x}\right)^{1/2}\left[\frac{4x^2-6x-4-2x^2+8x}{(2x+1)^2}\right]$$
$$=\frac{1}{2}\left(\frac{2x+1}{x^2-4x}\right)^{1/2}\left[\frac{2x^2+2x-4}{(2x+1)^2}\right]$$
$$=\frac{1}{2}\left(\frac{2x+1}{x^2-4x}\right)^{1/2}\left[\frac{2(x^2+x-2)}{(2x+1)^2}\right]$$
$$=\frac{x^2+x-2}{(2x+1)^{3/2}(x^2-4x)^{1/2}}$$

91. $f(t)=\sqrt{3t+\sqrt{t}}=\left(3t+t^{1/2}\right)^{1/2}$

Using the Extended Power Rule, we have

$$f'(t)=\frac{1}{2}\left(3t+t^{1/2}\right)^{-1/2}\left(3+\frac{1}{2}t^{-1/2}\right)$$
$$=\frac{3+\frac{1}{2\sqrt{t}}}{2\sqrt{3t+\sqrt{t}}}$$
$$=\frac{\frac{3}{1}\cdot\frac{2\sqrt{t}}{2\sqrt{t}}+\frac{1}{2\sqrt{t}}}{2\sqrt{3t+\sqrt{t}}}.$$

Simplifying, we have:

$$f'(t)=\frac{\frac{6\sqrt{t}+1}{2\sqrt{t}}}{2\sqrt{3t+\sqrt{t}}}$$
$$=\frac{6\sqrt{t}+1}{4\sqrt{t}\sqrt{3t+\sqrt{t}}}.$$

92. $F(x)=\left(6x(3-x)^5+2\right)^4$

$$F'(x)$$
$$=4\left(6x(3-x)^5+2\right)^3\left[6x\left(5(3-x)^4(-1)\right)+(3-x)^5(6)\right]$$
$$=4\left(6x(3-x)^5+2\right)^3\left[-30x(3-x)^4+6(3-x)^5\right]$$
$$=4\left(6x(3-x)^5+2\right)^3\left[6(3-x)^4\left(-5x+(3-x)\right)\right]$$
$$=24(3-x)^4(3-6x)\left(6x(3-x)^5+2\right)^3$$
$$=72(3-x)^4(1-2x)\left(6x(3-x)^5+2\right)^3$$

93. $\boxed{tw}$ Let $Q(x)=\frac{N(x)}{D(x)}$.

Then $Q(x)=N(x)\left[D(x)\right]^{-1}$

Therefore,

$$Q'(x)=N(x)\frac{d}{dx}\left[D(x)\right]^{-1}+\left[D(x)\right]^{-1}\frac{d}{dx}N(x)$$

Product Rule

$$=N(x)(-1)\left[D(x)\right]^{-2}\cdot D'(x)+\left[D(x)\right]^{-1}N'(x)$$

Extended Power Rule

$$=\frac{-N(x)\cdot D'(x)}{\left[D(x)\right]^2}+\frac{N'(x)}{D(x)}$$
$$=\frac{-N(x)\cdot D'(x)}{\left[D(x)\right]^2}+\frac{N'(x)\cdot D(x)}{\left[D(x)\right]^2}$$
$$=\frac{D(x)\cdot N'(x)-N(x)\cdot D'(x)}{\left[D(x)\right]^2}$$

94. $f(x)=1.68x\sqrt{9.2-x^2}; \quad [-3,3]$

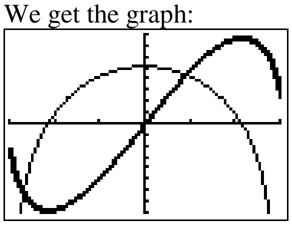
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.

```
 Plot1 Plot2 Plot3
\Y1=1.68X√(9.2-X
^2)
\Y2=nDeriv(Y1,X,
X)
\Y3=
\Y4=
\Y5=
```

Using the window:

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-8
 Ymax=8
 Yscl=1
 Xres=1
```

We get the graph:



Note, the function $f(x)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

We estimate the points at which the tangent lines are horizontal are $(-2.14476,-7.728)$ and $(2.14476,7.728)$.

95. $f(x)=\sqrt{6x^3-3x^2-48x+45}, \qquad [-5,5]$

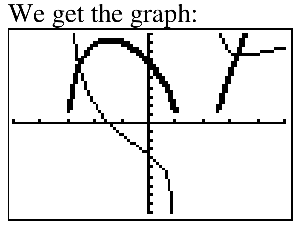
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.

```
 Plot1 Plot2 Plot3
\Y1=√(6X^3-3X^2-
48X+45)
\Y2=nDeriv(Y1,X,
X)
\Y3=
\Y4=
\Y5=
```

Using the window:

```
WINDOW
 Xmin=-5
 Xmax=5
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

We get the graph:



Note, the function $f(x)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

We estimate the point at which the tangent line is horizontal to be $(-1.47481,9.4878)$.

96. $f(x)=x\sqrt{4-x^2}=x\left(4-x^2\right)^{1/2}$

$$f'(x)=x\left[\frac{1}{2}\left(4-x^2\right)^{-1/2}(-2x)\right]+\left(4-x^2\right)^{1/2}(1)$$

$$=\frac{-x^2}{\left(4-x^2\right)^{1/2}}+\left(4-x^2\right)^{1/2}$$

$$=\frac{-x^2}{\left(4-x^2\right)^{1/2}}+\frac{\left(4-x^2\right)}{\left(4-x^2\right)^{1/2}}$$

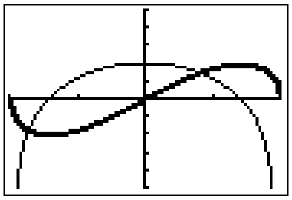
$$=\frac{4-2x^2}{\left(4-x^2\right)^{1/2}}$$

$$=\frac{4-2x^2}{\sqrt{4-x^2}}$$

Using the window:

```
WINDOW
 Xmin=-2
 Xmax=2
 Xscl=1
 Ymin=-5
 Ymax=5
 Yscl=1
 Xres=1
```

The graph of the function and the derivative are shown below.



Note: The graph of the function is the thicker graph.

97. $g(x)=\dfrac{4x}{\sqrt{x-10}}=\dfrac{4x}{(x-10)^{1/2}}$

Using the Quotient Rule and the Extended Power Rule, we have

$$g'(x)=\frac{(x-10)^{1/2}(4)-4x\left(\frac{1}{2}(x-10)^{-1/2}(1)\right)}{\left((x-10)^{1/2}\right)^2}$$

$$=\frac{4(x-10)^{1/2}-\dfrac{2x}{(x-10)^{1/2}}}{(x-10)}$$

$$=\frac{\dfrac{4(x-10)}{(x-10)^{1/2}}-\dfrac{2x}{(x-10)^{1/2}}}{(x-10)}$$

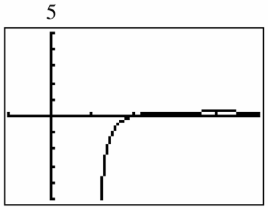
$$=\frac{2x-40}{(x-10)^{3/2}}$$

As a check, we first graph $y_2=\text{nDeriv}(y_1,x,x)$, after entering

$y_1=\dfrac{4x}{(x-10)^{0.5}}$. We then graph

$y_3=\dfrac{(2x-40)}{(x-10)^{1.5}}$, the result of our differentiation. Note that the graphs coincide.

$y_2=\text{nDeriv}(y_1,x,x)$, $y_3=\dfrac{(2x-40)}{(x-10)^{1.5}}$



98. $f(x)=\left(\sqrt{2x-1}+x^3\right)^5=\left((2x-1)^{1/2}+x^3\right)^5$

$$f'(x)=5\left((2x-1)^{1/2}+x^3\right)^4\left[\frac{1}{2}(2x-1)^{-1/2}(2)+3x^2\right]$$

$$=5\left((2x-1)^{1/2}+x^3\right)^4\left[\frac{1}{(2x-1)^{1/2}}+3x^2\right]$$

$$f'(x)=5\left((2x-1)^{1/2}+x^3\right)^4\left[\frac{3x^2(2x-1)^{1/2}+1}{(2x-1)^{1/2}}\right]$$

$$=\frac{5\left(\sqrt{2x-1}+x^3\right)^4\left(3x^2\sqrt{2x-1}+1\right)}{\sqrt{2x-1}}$$

As a check, we first graph $y_2=\text{nDeriv}(y_1,x,x)$ after entering $y_1=((2x-1)^{0.5}+x^3)^5$.

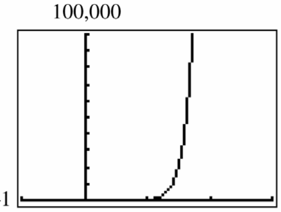
We then graph

$y_3=\dfrac{5((2x-1)^{0.5}+x^3)^4(3x^2(2x-1)^{0.5}+1)}{(2x-1)^{0.5}}$,

the result of our differentiation. Note that the graphs coincide.

$y_2=\text{nDeriv}(y_1,x,x)$,

$y_3=\dfrac{5((2x-1)^{0.5}+x^3)^4(3x^2(2x-1)^{0.5}+1)}{(2x-1)^{0.5}}$



Exercise Set 1.8

1. $y=x^5+9$

$\dfrac{dy}{dx}=5x^{5-1}=5x^4$ First Derivative

$\dfrac{d^2y}{dx^2}=5\left(4x^{4-1}\right)=20x^3$ Second Derivative

2. $y=x^4-7$

$\dfrac{dy}{dx}=4x^3$

$\dfrac{d^2y}{dx^2}=12x^2$

3. $y = 2x^4 - 5x$

$$\frac{dy}{dx} = 2\left(4x^3\right) - 5 =$$

$$= 8x^3 - 5 \qquad \text{First Derivative}$$

$$\frac{d^2y}{dx^2} = 8\left(3x^2\right)$$

$$= 24x^2 \qquad \text{Second Derivative}$$

4. $y = 5x^3 + 4x$

$$\frac{dy}{dx} = 15x^2 + 4$$

$$\frac{d^2y}{dx^2} = 30x$$

5. $y = 4x^2 + 3x - 1$

$$\frac{dy}{dx} = 4\left(2x^{2-1}\right) + 3 - 0$$

$$= 8x + 3 \qquad \text{First Derivative}$$

$$\frac{d^2y}{dx^2} = 8 \qquad \text{Second Derivative}$$

6. $y = 4x^2 - 5x + 7$

$$\frac{dy}{dx} = 8x - 5$$

$$\frac{d^2y}{dx^2} = 8$$

7. $y = 7x + 2$

$$\frac{dy}{dx} = 7 \qquad \text{First Derivative}$$

$$\frac{d^2y}{dx^2} = 0 \qquad \text{Second Derivative}$$

8. $y = 6x - 3$

$$\frac{dy}{dx} = 6$$

$$\frac{d^2y}{dx^2} = 0$$

9. $y = \frac{1}{x^2} = x^{-2}$

$$\frac{dy}{dx} = -2x^{-2-1}$$

$$= -2x^{-3}$$

$$= \frac{-2}{x^3} \qquad \text{First Derivative}$$

$$\frac{d^2y}{dx^2} = -2\left(-3x^{-3-1}\right)$$

$$= 6x^{-4}$$

$$= \frac{6}{x^4} \qquad \text{Second Derivative}$$

10. $y = \frac{1}{x^3} = x^{-3}$

$$\frac{dy}{dx} = -3x^{-4} = \frac{-3}{x^4}$$

$$\frac{d^2y}{dx^2} = 12x^{-5} = \frac{12}{x^5}$$

11. $y = \sqrt{x} = x^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}x^{1/2-1}$$

$$= \frac{1}{2}x^{-1/2}$$

$$= \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}} \qquad \text{First Derivative}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}\cdot\left(-\frac{1}{2}x^{-1/2-1}\right)$$

$$= -\frac{1}{4}x^{-3/2}$$

$$= -\frac{1}{4x^{3/2}} = -\frac{1}{4\sqrt{x^3}} \qquad \text{Second Derivative}$$

12. $y = \sqrt[4]{x} = x^{1/4}$

$$\frac{dy}{dx} = \frac{1}{4}x^{1/4-1} = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}} = \frac{1}{4\cdot\sqrt[4]{x^3}}$$

$$\frac{d^2y}{dx^2} = \frac{1}{4}\left(-\frac{3}{4}\right)x^{-3/4-1}$$

$$= -\frac{3}{16}x^{-7/4}$$

$$= -\frac{3}{16x^{7/4}} = -\frac{3}{16\sqrt[4]{x^7}}$$

13. $f(x) = x^4 + \frac{3}{x}$

$$f'(x) = 4x^{4-1} + 3\left(-1x^{-1-1}\right)$$

$$= 4x^3 - 3x^{-2}$$

$$= 4x^3 - \frac{3}{x^2} \qquad \text{First Derivative}$$

$$f''(x) = 4\left(3x^{3-1}\right) - 3\left(-2x^{-2-1}\right)$$
$$= 12x^2 + 6x^{-3}$$
$$= 12x^2 + \frac{6}{x^3} \quad \text{Second Derivative}$$

14. $f(x) = x^3 - \dfrac{5}{x} = x^3 - 5x^{-1}$

$$f'(x) = 3x^2 + 5x^{-2}$$
$$= 3x^2 + \frac{5}{x^2}$$
$$f''(x) = 6x - 10x^{-3}$$
$$= 6x - \frac{10}{x^3}$$

15. $f(x) = x^{1/5}$

$$f'(x) = \frac{1}{5}x^{1/5-1}$$
$$= \frac{1}{5}x^{-4/5}$$
$$= \frac{1}{5x^{4/5}} \quad \text{First Derivative}$$
$$f''(x) = \frac{1}{5}\left(-\frac{4}{5}\right)x^{-4/5-1}$$
$$= -\frac{4}{25}x^{-9/5}$$
$$= -\frac{4}{25x^{9/5}} \quad \text{Second Derivative}$$

16. $f(x) = x^{1/3}$

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$
$$f''(x) = \frac{1}{3}\left(-\frac{2}{3}\right)x^{-5/3} = -\frac{2}{9x^{5/3}}$$

17. $f(x) = 4x^{-3}$

$$f'(x) = 4\left(-3x^{-3-1}\right)$$
$$= -12x^{-4}$$
$$= -\frac{12}{x^4} \quad \text{First Derivative}$$
$$f''(x) = -12\left(-4x^{-4-1}\right)$$
$$= 48x^{-5}$$
$$= \frac{48}{x^5} \quad \text{Second Derivative}$$

18. $f(x) = 2x^{-2}$

$$f'(x) = -4x^{-3} = -\frac{4}{x^3}$$
$$f''(x) = 12x^{-4} = \frac{12}{x^4}$$

19. $f(x) = \left(x^2 + 3x\right)^7$

$$f'(x) = 7\left(x^2 + 3x\right)^{7-1}(2x+3) \quad \text{Theorem 7}$$
$$= 7(2x+3)\left(x^2+3x\right)^6 \quad \text{First Derivative}$$
$$f''(x) = 7(2x+3)\left(6\left(x^2+3x\right)^{6-1}(2x+3)\right) +$$
$$7\left(x^2+6x\right)^6(2) \quad \text{Theorem 5}$$
$$= 42(2x+3)^2\left(x^2+3x\right)^5 + 14\left(x^2+3x\right)^6$$

We can simplify the second derivative by factoring out common factors.

$$f''(x)$$
$$= 14\left(x^2+3x\right)^5\left[3(2x+3)^2 + \left(x^2+3x\right)\right]$$
$$= 14\left(x^2+3x\right)^5\left[3\left(4x^2+12x+9\right) + \left(x^2+3x\right)\right]$$
$$= 14\left(x^2+3x\right)^5\left[12x^2+36x+27+x^2+3x\right]$$
$$= 14\left(x^2+3x\right)^5\left(13x^2+39x+27\right) \quad \text{Second Derivative}$$

20. $f(x) = \left(x^3 + 2x\right)^6$

$$f'(x) = 6\left(x^3+2x\right)^5\left(3x^2+2\right)$$
$$f''(x) = 6\left[\left(x^3+2x\right)^5(6x)\right] +$$
$$6\left[\left(3x^2+2\right)\cdot 5\left(x^3+2x\right)^4\left(3x^2+2\right)\right]$$
$$= 36x\left(x^3+2x\right)^5 + 30\left(3x^2+2\right)^2\left(x^3+2x\right)^4$$
$$= 6\left(x^3+2x\right)^4\left[6x\left(x^3+2x\right) + 5\left(3x^2+2\right)^2\right]$$
$$= 6\left(x^3+2x\right)^4\left[6x^4+12x^2+45x^4+60x^2+20\right]$$
$$= 6\left(x^3+2x\right)^4\left(51x^4+72x^2+20\right)$$

21. $f(x) = \left(2x^2 - 3x + 1\right)^{10}$

$$f'(x) = 10\left(2x^2-3x+1\right)^{10-1}(4x-3) \quad \text{Theorem 7}$$
$$= 10\left(2x^2-3x+1\right)^9(4x-3) \quad \text{First Derivative}$$

$$f''(x) = 10\left(2x^2-3x+1\right)^9(4) + \quad \text{Theorem 5}$$
$$10(4x-3)\cdot 9\left(2x^2-3x+1\right)^8(4x-3)$$
$$= 40\left(2x^2-3x+1\right)^9 +$$
$$90(4x-3)^2\left(2x^2-3x+1\right)^8$$
$$= 10\left(2x^2-3x+1\right)^8\left(4\left(2x^2-3x+1\right)+ 9\left(16x^2-24x+9\right)\right)$$
$$= 10\left(2x^2-3x+1\right)^8\left(8x^2-12x+4+ 144x^2-216x+81\right)$$
$$= 10\left(2x^2-3x+1\right)^8\left(152x^2-228x+85\right)$$

Second Derivative

22. $f(x) = \left(3x^2+2x+1\right)^5$

$$f'(x) = 5\left(3x^2+2x+1\right)^4(6x+2)$$
$$f''(x) = 5\left(3x^2+2x+1\right)^4(6) +$$
$$5(6x+2)\cdot 4\left(3x^2+2x+1\right)^3(6x+2)$$
$$= 30\left(3x^2+2x+1\right)^4 +$$
$$20(6x+2)^2\left(3x^2+2x+1\right)^3$$
$$= 10\left(3x^2+2x+1\right)^3\left(3\left(3x^2+2x+1\right)+ 2\left(36x^2+24x+4\right)\right)$$
$$= 10\left(3x^2+2x+1\right)^3\left(9x^2+6x+3+ 72x^2+48x+8\right)$$
$$= 10\left(3x^2+2x+1\right)^3\left(81x^2+54x+11\right)$$

23. $f(x) = \sqrt[4]{\left(x^2+1\right)^3} = \left(x^2+1\right)^{3/4}$

$$f'(x) = \frac{3}{4}\left(x^2+1\right)^{-1/4}(2x) \quad \text{Theorem 7}$$
$$= \frac{3}{2}x\left(x^2+1\right)^{-1/4} \quad \text{First Derivative}$$

$$f''(x) = \frac{3}{2}x\cdot\frac{-1}{4}\left(x^2+1\right)^{-5/4}(2x) +$$
$$\frac{3}{2}\left(x^2+1\right)^{-1/4}(1) \quad \text{Theorem 5}$$
$$= -\frac{3}{4}x^2\left(x^2+1\right)^{-5/4} + \frac{3}{2}\left(x^2+1\right)^{-1/4}$$
$$= \frac{-3x^2}{4\left(x^2+1\right)^{5/4}} + \frac{3}{2\left(x^2+1\right)^{1/4}}$$

We can simplify the second derivative by finding a common denominator and combining the fractions.

$$f''(x) = \frac{-3x^2}{4\left(x^2+1\right)^{5/4}} + \frac{3}{2\left(x^2+1\right)^{1/4}}\cdot\frac{2\left(x^2+1\right)}{2\left(x^2+1\right)}$$
$$= \frac{-3x^2}{4\left(x^2+1\right)^{5/4}} + \frac{6\left(x^2+1\right)}{4\left(x^2+1\right)^{5/4}}$$
$$= \frac{3x^2+6}{4\left(x^2+1\right)^{5/4}}$$
$$= \frac{3\left(x^2+2\right)}{4\left(x^2+1\right)^{5/4}}$$

24. $f(x) = \sqrt[3]{\left(x^2-1\right)^2} = \left(x^2-1\right)^{2/3}$

$$f'(x) = \frac{2}{3}\left(x^2-1\right)^{-1/3}(2x)$$
$$= \frac{4}{3}x\left(x^2-1\right)^{-1/3}$$
$$f''(x) = \frac{4}{3}x\cdot\frac{-1}{3}\left(x^2-1\right)^{-4/3}(2x)$$
$$+\frac{4}{3}\left(x^2-1\right)^{-1/3}(1)$$
$$= \frac{-8}{9}x^2\left(x^2-1\right)^{-4/3} + \frac{4}{3}\left(x^2-1\right)^{-1/3}$$
$$= \frac{-8x^2}{9\left(x^2-1\right)^{4/3}} + \frac{4}{3\left(x^2-1\right)^{1/3}}$$
$$= \frac{-8x^2}{9\left(x^2-1\right)^{4/3}} + \frac{12\left(x^2-1\right)}{9\left(x^2-1\right)^{4/3}}$$
$$= \frac{4x^2-12}{9\left(x^2-1\right)^{4/3}}$$
$$= \frac{4\left(x^2-3\right)}{9\left(x^2-1\right)^{4/3}}$$

25. $$y = x^{2/3} + 4x$$

$$y' = \frac{2}{3}x^{2/3-1} + 4$$

$$= \frac{2}{3}x^{-1/3} + 4 \qquad \text{First Derivative}$$

$$y'' = \frac{2}{3}\cdot\frac{-1}{3}x^{-1/3-1}$$

$$= -\frac{2}{9}x^{-4/3}$$

$$= -\frac{2}{9x^{4/3}} \qquad \text{Second Derivative}$$

26. $$y = x^{3/2} - 5x$$

$$y' = \frac{3}{2}x^{1/2} - 5$$

$$y'' = \frac{3}{4}x^{-1/2} = \frac{3}{4\sqrt{x}}$$

27. $$y = \left(x^3 - x\right)^{3/4}$$

$$y' = \frac{3}{4}\left(x^3 - x\right)^{3/4-1}\left(3x^2 - 1\right) \qquad \text{Theorem 7}$$

$$= \frac{3}{4}\left(x^3 - x\right)^{-1/4}\left(3x^2 - 1\right) \qquad \text{First Derivative}$$

$$y'' = \frac{3}{4}\left(x^3 - x\right)^{-1/4}\left(6x\right) + \qquad \text{Theorem 5}$$

$$\frac{3}{4}\left(3x^2 - 1\right)\cdot\frac{-1}{4}\left(x^3 - x\right)^{-1/4-1}\left(3x^2 - 1\right)$$

$$= \frac{9}{2}x\left(x^3 - x\right)^{-1/4} +$$

$$\frac{-3}{16}\left(3x^2 - 1\right)^2\left(x^3 - x\right)^{-5/4}$$

$$= \frac{9x}{2\left(x^3 - x\right)^{1/4}} - \frac{3\left(3x^2 - 1\right)^2}{16\left(x^3 - x\right)^{5/4}}$$

The second derivative can be simplified by finding a common denominator and combining the fractions.

$$y'' = \frac{9x}{2\left(x^3 - x\right)^{1/4}}\cdot\frac{8\left(x^3 - x\right)}{8\left(x^3 - x\right)} - \frac{3\left(3x^2 - 1\right)^2}{16\left(x^3 - x\right)^{5/4}}$$

$$= \frac{72x^4 - 72x^2}{16\left(x^3 - x\right)^{5/4}} - \frac{3\left(9x^4 - 6x^2 + 1\right)}{16\left(x^3 - x\right)^{5/4}}$$

$$= \frac{72x^4 - 72x^2 - 27x^4 + 18x^2 - 3}{16\left(x^3 - x\right)^{5/4}}$$

$$= \frac{45x^4 - 54x^2 - 3}{16\left(x^3 - x\right)^{5/4}} \qquad \text{Second Derivative}$$

28. $$y = \left(x^4 + x\right)^{2/3}$$

$$y' = \frac{2}{3}\left(x^4 + x\right)^{-1/3}\left(4x^3 + 1\right)$$

$$y'' = \frac{2}{3}\Bigg[\left(x^4 + x\right)^{-1/3}\left(12x^2\right) +$$

$$\left(4x^3 + 1\right)\left(-\frac{1}{3}\left(x^4 + x\right)^{-4/3}\left(4x^3 + 1\right)\right)\Bigg]$$

$$= \frac{2}{3}\left[\frac{12x^2}{\left(x^4 + x\right)^{1/3}} - \frac{\left(4x^3 + 1\right)^2}{3\left(x^4 + x\right)^{4/3}}\right]$$

$$= \frac{2}{3}\left[\frac{36x^2\left(x^4 + x\right)}{3\left(x^4 + x\right)^{4/3}} - \frac{16x^6 + 8x^3 + 1}{3\left(x^4 + x\right)^{4/3}}\right]$$

$$= \frac{2}{3}\left[\frac{20x^6 + 28x^3 - 1}{3\left(x^4 + x\right)^{4/3}}\right]$$

$$= \frac{40x^6 + 56x^3 - 2}{9\left(x^4 + x\right)^{4/3}}$$

29. $$y = 2x^{5/4} + x^{1/2}$$

$$y' = 2\cdot\frac{5}{4}x^{5/4-1} + \frac{1}{2}x^{1/2-1}$$

$$= \frac{5}{2}x^{1/4} + \frac{1}{2}x^{-1/2} \qquad \text{First Derivative}$$

$$y'' = \frac{5}{2}\cdot\frac{1}{4}x^{1/4-1} + \frac{1}{2}\cdot\frac{-1}{2}x^{-1/2-1}$$

$$= \frac{5}{8}x^{-3/4} - \frac{1}{4}x^{-3/2}$$

$$= \frac{5}{8x^{3/4}} - \frac{1}{4x^{3/2}} \qquad \text{Second Derivative}$$

30. $y = 3x^{4/3} - x^{1/2}$

$y' = 3\cdot\frac{4}{3}x^{4/3-1} - \frac{1}{2}x^{1/2-1}$

$= 4x^{1/3} - \frac{1}{2}x^{-1/2}$

$y'' = 4\cdot\frac{1}{3}x^{1/3-1} - \frac{1}{2}\cdot\frac{-1}{2}x^{-1/2-1}$

$= \frac{4}{3}x^{-2/3} + \frac{1}{4}x^{-3/2}$

$= \frac{4}{3x^{2/3}} + \frac{1}{4x^{3/2}}$

31. $y = \frac{2}{x^3} + \frac{1}{x^2} = 2x^{-3} + x^{-2}$

$y' = 2\left(-3x^{-3-1}\right) + \left(-2x^{-2-1}\right)$

$= -6x^{-4} - 2x^{-3}$ First Derivative

$y'' = -6\left(-4x^{-4-1}\right) - 2\left(-3x^{-3-1}\right)$

$= 24x^{-5} + 6x^{-4}$

$= \frac{24}{x^5} + \frac{6}{x^4}$ Second Derivative

32. $y = \frac{3}{x^4} - \frac{1}{x} = 3x^{-4} - x^{-1}$

$y' = -12x^{-5} + x^{-2}$

$y'' = 60x^{-6} - 2x^{-3} = \frac{60}{x^6} - \frac{2}{x^3}$

33. $y = \left(x^2+3\right)\left(4x-1\right)$

$y' = \left(x^2+3\right)\left(4\right) + \left(4x-1\right)\left(2x\right)$ Theorem 5

$= 4x^2 + 12 + 8x^2 - 2x$

$= 12x^2 - 2x + 12$ First Derivative

$y'' = 12\left(2x^{2-1}\right) - 2$

$= 24x - 2$ Second Derivative

34. $y = \left(x^3-2\right)\left(5x+1\right)$

$y' = \left(x^3-2\right)\left(5\right) + \left(5x+1\right)\left(3x^2\right)$

$= 5x^3 - 10 + 15x^3 + 3x^2$

$= 20x^3 + 3x^2 - 10$

$y'' = 60x^2 + 6x$

35. $y = \frac{3x+1}{2x-3}$

$y' = \frac{(2x-3)(3)-(3x+1)(2)}{(2x-3)^2}$ Theorem 6

$= \frac{6x-9-6x-2}{(2x-3)^2}$

$= \frac{-11}{(2x-3)^2}$ First Derivative

$y'' = \frac{(2x-3)^2(0)-(-11)\left(2(2x-3)^{2-1}(2)\right)}{\left((2x-3)^2\right)^2}$

Theorem 6 and Theorem 7

$= \frac{44(2x-3)}{(2x-3)^4}$

$= \frac{44}{(2x-3)^3}$ Second Derivative

36. $y = \frac{2x+3}{5x-1}$

$y' = \frac{(5x-1)(2)-(2x+3)(5)}{(5x-1)^2}$

$= \frac{-17}{(5x-1)^2} = -17(5x-1)^{-2}$

$y'' = -17\left(-2(5x-1)^{-3}(5)\right)$

$= 170(5x-1)^{-3}$

$= \frac{170}{(5x-1)^3}$

37. $y = x^4$

$\frac{dy}{dx} = 4x^{4-1} = 4x^3$ First Derivative

$\frac{d^2y}{dx^2} = 4\left(3x^{3-1}\right) = 12x^2$ Second Derivative

$\frac{d^3y}{dx^3} = 12\left(2x^{2-1}\right) = 24x$ Third Derivative

$\frac{d^4y}{dx^4} = 24$ Fourth Derivative

38. $y = x^5$

$$\frac{dy}{dx} = 5x^4$$

$$\frac{d^2y}{dx^2} = 20x^3$$

$$\frac{d^3y}{dx^3} = 60x^2$$

$$\frac{d^4y}{dx^4} = 120x$$

39. $y = x^6 - x^3 + 2x$

$$\frac{dy}{dx} = 6x^{6-1} - 3x^{3-1} + 2$$

$= 6x^5 - 3x^2 + 2$ First Derivative

$$\frac{d^2y}{dx^2} = 6\left(5x^{5-1}\right) - 3\left(2x^{2-1}\right)$$

$= 30x^4 - 6x$ Second Derivative

$$\frac{d^3y}{dx^3} = 30\left(4x^{4-1}\right) - 6$$

$= 120x^3 - 6$ Third Derivative

$$\frac{d^4y}{dx^4} = 120\left(3x^{3-1}\right)$$

$= 360x^2$ Fourth Derivative

$$\frac{d^5y}{dx^5} = 360\left(2x^{2-1}\right)$$

$= 720x$ Fifth Derivative

40. $y = x^7 - 8x^2 + 2$

$$\frac{dy}{dx} = 7x^6 - 16x$$

$$\frac{d^2y}{dx^2} = 42x^5 - 16$$

$$\frac{d^3y}{dx^3} = 210x^4$$

$$\frac{d^4y}{dx^4} = 840x^3$$

$$\frac{d^5y}{dx^5} = 2520x^2$$

$$\frac{d^6y}{dx^6} = 5040x$$

41. $f(x) = x^{-2} - x^{1/2}$

$$f'(x) = -2x^{-2-1} - \frac{1}{2}x^{1/2-1}$$

$= -2x^{-3} - \frac{1}{2}x^{-1/2}$ First Derivative

$$f''(x) = -2\left(-3x^{-3-1}\right) - \frac{1}{2}\cdot\frac{-1}{2}x^{-1/2-1}$$

$= 6x^{-4} + \frac{1}{4}x^{-3/2}$ Second Derivative

$$f'''(x) = 6\left(-4x^{-4-1}\right) + \frac{1}{4}\cdot\frac{-3}{2}x^{-3/2-1}$$

$= -24x^{-5} - \frac{3}{8}x^{-5/2}$ Third Derivative

$$f^{(4)}(x) = -24\left(-5x^{-5-1}\right) - \frac{3}{8}\cdot\frac{-5}{2}x^{-5/2-1}$$

$= 120x^{-6} + \frac{15}{16}x^{-7/2}$ Fourth Derivative

42. $f(x) = x^{-3} + 2x^{1/3}$

$$f'(x) = -3x^{-4} + \frac{2}{3}x^{-2/3}$$

$$f''(x) = 12x^{-5} - \frac{4}{9}x^{-5/3}$$

$$f'''(x) = -60x^{-6} + \frac{20}{27}x^{-8/3}$$

$$f^{(4)}(x) = 360x^{-7} - \frac{160}{81}x^{-11/3}$$

$$f^{(5)}(x) = -2520x^{-8} + \frac{1760}{243}x^{-14/3}$$

43. $g(x) = x^4 - 3x^3 - 7x^2 - 6x + 9$

$$g'(x) = 4x^{4-1} - 3\left(3x^{3-1}\right) - 7\left(2x^{2-1}\right) - 6$$

$= 4x^3 - 9x^2 - 14x - 6$ First Derivative

$$g''(x) = 4\left(3x^{3-1}\right) - 9\left(2x^{2-1}\right) - 14$$

$= 12x^2 - 18x - 14$ Second Derivative

$$g'''(x) = 12\left(2x^{2-1}\right) - 18$$

$= 24x - 18$ Third Derivative

$g^{(4)}(x) = 24$ Fourth Derivative

$g^{(5)}(x) = 0$ Fifth Derivative

$g^{(6)}(x) = 0$ Sixth Derivative

44. $g(x)=6x^5+2x^4-4x^3+7x^2-8x+3$

$g'(x)=30x^4+8x^3-12x^2+14x-8$

$g''(x)=120x^3+24x^2-24x+14$

$g'''(x)=360x^2+48x-24$

$g^{(4)}(x)=720x+48$

$g^{(5)}(x)=720$

$g^{(6)}(x)=0$

$g^{(7)}(x)=0$

45. $s(t)=t^3+t$

a) $v(t)=s'(t)=3t^2+1$

b) $a(t)=v'(t)=s''(t)=6t$

c) When $t=4$

$v(4)=3(4)^2+1=49$

$a(4)=6(4)=24$

After 4 seconds, the velocity is 49 feet per second, and the acceleration is 24 feet per second squared.

46. $s(t)=-10t^2+2t+5$

a) $v(t)=s'(t)=-20t+2$

b) $a(t)=v'(t)=s''(t)=-20$

c) When $t=1$,

$v(1)=-20(1)+2=-18\,\frac{\text{m}}{\text{sec}}$

$a(1)=-20\,\frac{\text{m}}{\text{sec}^2}$

47. $s(t)=3t+10$

a) $v(t)=s'(t)=3$

b) $a(t)=v'(t)=s''(t)=0$

c) When $t=2$

$v(2)=3$

$a(2)=0$

After 2 hours, the velocity is 3 miles per hour, and the acceleration is 0 miles per hour squared.

d) $\boxed{tw}$ Answers will vary. Uniform motion means that the object is moving with a constant velocity. Since the objects velocity is not changing, the objects acceleration will be zero.

48. $s(t)=t^2-\frac{1}{2}t+3$

a) $v(t)=s'(t)=2t-\frac{1}{2}$

b) $a(t)=v'(t)=s''(t)=2$

c) When $t=1$,

$v(1)=2(1)-\frac{1}{2}=\frac{3}{2}=1.5\,\frac{\text{m}}{\text{sec}}$

$a(1)=2\,\frac{\text{m}}{\text{sec}^2}$

49. $s(t)=16t^2$

a) When $t=3$, $s(3)=16(3)^2=144$.

The hammer falls 144 feet in 3 seconds.

b) $v(t)=s'(t)=32t$

When $t=3$, $v(3)=32(3)=96$

the hammer is falling at 96 feet per second after 3 seconds.

c) $a(t)=v'(t)=s''(t)=32$

When $t=3$, $a(3)=32$

the hammer is accelerating at 32 feet per second squared after 3 seconds.

50. $s(t)=16t^2$

a) When $t=2$, $s(2)=16(2)^2=64$.

The bolt falls 64 feet in 2 seconds.

b) $v(t)=s'(t)=32t$

When $t=2$, $v(2)=32(2)=64$

the bolt is falling at 64 feet per second after 2 seconds.

c) $a(t)=v'(t)=s''(t)=32$

When $t=2$, $a(3)=32$

the bolt is accelerating at 32 feet per second squared after 2 seconds.

51. $s(t)=4.905t^2$

The velocity and acceleration are given by:

$v(t)=s'(t)=9.81t$

$a(t)=v'(t)=s''(t)=9.81$

After 2 seconds, we have

$v(2)=9.81(2)=19.62$

The stone is falling at 19.62 meters per second.

$a(2)=9.81$

The stone is accelerating at 9.81 meters per second squared.

52. $s(t)=4.905t^2$

$v(t)=s'(t)=9.81t$

$a(t)=v'(t)=s''(t)=9.81$

After 3 seconds, we have

$v(3)=9.81(3)=29.43$

The stone is falling at 29.43 meters per second.

$a(3)=9.81$

The stone is accelerating at 9.81 meters per second squared.

53. a) The plane's velocity is greater at $t=20$ seconds. We know this because the slope of the tangent line is greater at $t=20$ then it is at $t=6$.

b) The plane's acceleration is positive, since the velocity (slope of the tangent lines) is increasing over time.

54. a) The bicyclist's velocity is the greatest at time $t=0$. The tangent line at $t=0$ has the greatest slope.

b) The bicyclist's acceleration is negative, since the slopes of the tangent line are decreasing with time.

55. $S(t)=2t^3-40t^2+220t+160$

a) $S'(t)=6t^2-80t+220$

When $t=1$,

$S'(1)=6(1)^2-80(1)+220=146$.

After 1 month, sales are increasing at 146 thousand (146,000) dollars per month.

When $t=2$,

$S'(2)=6(2)^2-80(2)+220=84$.

After 2 month, sales are increasing at 84 thousand (84,000) dollars per month.

When $t=4$,

$S'(4)=6(4)^2-80(4)+220=-4$.

After 4 months, sales are changing at a rate of -4 thousand (-4000) dollars per month.

b) $S''(t)=12t-80$

When $t=1$, $S''(1)=12(1)-80=-68$.

After 1 month, the rate of change of sales are changing at a rate of -68 thousand $(-68,000)$ dollars per month squared.

When $t=2$,

$S''(2)=12(2)-80=-56$.

After 2 months, the rate of change of sales are changing at a rate of -56 thousand $(-56,000)$ dollars per month squared.

When $t=4$, $S''(4)=12(4)-80=-32$.

After 4 months, the rate of change of sales are changing at a rate of -32 thousand $(-32,000)$ dollars per month squared.

c) $\boxed{tw}$ Answers will vary. The first derivative found in part (a) determined the rate at which sales were changing *t* months after the product was marketed. We saw that for the first 2 months, sales were increasing. However, in the 4th month, sales had started to decrease. The second derivative found in part (b) determined how fast the rate of change was changing. We saw that the rate at which sales were changing was negative. Which means that in the first two months when sales were increasing, they were doing so at a decreasing rate.

56. $N(t)=2t^3-3t^2+2t$

a) $N'(t)=6t^2-6t+2$

When $t=1$, $N'(1)=6(1)^2-6(1)+2=2$

After 1 day, the number of items sold was increasing by 2 items per day.

When $t=2$, $N'(2)=6(2)^2-6(2)+2=14$

After 2 days, the number of items sold was increasing by 14 items per day.

When $t=4$, $N'(4)=6(4)^2-6(4)+2=74$

After 4 days, the number of items sold was increasing by 74 items per day.

b) $N''(t)=12t-6$

When $t=1$, $N''(1)=12(1)-6=6$

After 1 day, the rate of change of the number of items sold was increasing by 6 items per day squared.

When $t=2$, $N''(2)=12(2)-6=18$

After 2 days, the rate of change of the number of items sold was increasing by 18 items per day squared.

When $t=4$, $N''(4)=12(4)-6=42$

After 4 days, the rate of change of the number of items sold was increasing by 42 items per day squared.

c) $\boxed{tw}$ Answers will vary. The first derivative found in part (a) determined the rate at which items were sold t days after the new sales promotion was launched. The second derivative found in part (b) determined the rate at which the rates in part (a) where changing. The information tells us that after the new sales promotion was launched, the number of items sold were increasing at a increasing rate with respect to time.

57. $y = \dfrac{1}{(1-x)} = (1-x)^{-1}$

$y' = -1(1-x)^{-1-1}(-1) = 1(1-x)^{-2}$

$y'' = -2(1-x)^{-2-1}(-1) = 2(1-x)^{-3}$

$y''' = 2(-3)(1-x)^{-3-1}(-1) = 6(1-x)^{-4}$

Therefore,

$y''' = \dfrac{6}{(1-x)^4}$.

58. $y = x\sqrt{1+x^2} = x(1+x^2)^{1/2}$

$$y' = x\cdot\frac{1}{2}(1+x^2)^{-1/2}(2x) + (1+x^2)^{1/2}(1)$$
$$= x^2(1+x^2)^{-1/2} + (1+x^2)^{1/2}$$
$$= \frac{x^2}{(1+x^2)^{1/2}} + (1+x^2)^{1/2}$$
$$= \frac{x^2}{(1+x^2)^{1/2}} + \frac{(1+x^2)}{(1+x^2)^{1/2}}$$
$$= \frac{2x^2+1}{(1+x^2)^{1/2}}$$

$$y'' = \frac{(1+x^2)^{1/2}(4x) - (2x^2+1)\left(\frac{1}{2}(1+x^2)^{-1/2}(2x)\right)}{\left((1+x^2)^{1/2}\right)^2}$$
$$= \frac{4x(1+x^2)^{1/2} - (2x^3+x)(1+x^2)^{-1/2}}{(1+x^2)}$$
$$= \frac{\dfrac{4x(1+x^2)}{(1+x^2)^{1/2}} - \dfrac{2x^3+x}{(1+x^2)^{1/2}}}{(1+x^2)}$$
$$= \frac{2x^3+3x}{(1+x^2)^{1/2}(1+x^2)}$$
$$= \frac{2x^3+3x}{(1+x^2)^{3/2}}$$

$$y''' = \frac{(1+x^2)^{3/2}(6x^2+3) - (2x^3+3x)\left(\frac{3}{2}(1+x^2)^{1/2}(2x)\right)}{\left((1+x^2)^{3/2}\right)^2}$$
$$= \frac{(6x^2+3)(1+x^2)^{3/2} - 3x(2x^3+3x)(1+x^2)^{1/2}}{(1+x^2)^3}$$
$$= \frac{(1+x^2)^{1/2}\left[(6x^2+3)(1+x^2) - 3x(2x^3+3x)\right]}{(1+x^2)^3}$$
$$= \frac{6x^4+9x^2+3-6x^4-9x^2}{(1+x^2)^{5/2}}$$
$$= \frac{3}{(1+x^2)^{5/2}}$$

59. $y = \dfrac{1}{\sqrt{2x+1}} = (2x+1)^{-1/2}$

$$y' = \frac{-1}{2}(2x+1)^{-1/2-1}(2)$$
$$= -(2x+1)^{-3/2}$$
$$y'' = -\frac{-3}{2}(2x+1)^{-3/2-1}(2)$$
$$= 3(2x+1)^{-5/2}$$

$$y''' = 3\left(\frac{-5}{2}\right)(2x+1)^{-5/2-1}(2)$$
$$= -15(2x+1)^{-7/2}$$
$$= -\frac{15}{(2x+1)^{7/2}}$$

60. $y = \dfrac{3x-1}{2x+3}$

$$y' = \frac{(2x+3)(3)-(3x-1)(2)}{(2x+3)^2}$$
$$= \frac{11}{(2x+3)^2} = 11(2x+3)^{-2}$$
$$y'' = 11(-2)(2x+3)^{-2-1}(2)$$
$$= -44(2x+3)^{-3}$$
$$y''' = -44(-3)(2x+3)^{-3-1}(2)$$
$$= 264(2x+3)^{-4}$$
$$= \frac{264}{(2x+3)^4}$$

61. $y = \dfrac{\sqrt{x}+1}{\sqrt{x}-1} = \dfrac{x^{1/2}+1}{x^{1/2}-1}$

$$y' = \frac{\left(x^{1/2}-1\right)\left(\frac{1}{2}x^{-1/2}\right)-\left(x^{1/2}+1\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}-1\right)^2}$$
$$= \frac{\frac{1}{2}-\frac{1}{2}x^{-1/2}-\frac{1}{2}-\frac{1}{2}x^{-1/2}}{\left(x^{1/2}-1\right)^2}$$
$$= \frac{-x^{-1/2}}{\left(x^{1/2}-1\right)^2}$$

$$y'' = \frac{\left(x^{1/2}-1\right)^2\left(\frac{1}{2}x^{-3/2}\right)-\left(-x^{-1/2}\right)\left[2\left(x^{1/2}-1\right)\frac{1}{2}x^{-1/2}\right]}{\left(\left(x^{1/2}-1\right)^2\right)^2}$$
$$= \frac{\frac{1}{2}x^{-3/2}\left(x^{1/2}-1\right)^2+x^{-1}\left(x^{1/2}-1\right)}{\left(x^{1/2}-1\right)^4}$$
$$= \frac{\left(x^{1/2}-1\right)\left(\frac{1}{2}x^{-3/2}\left(x^{1/2}-1\right)+x^{-1}\right)}{\left(x^{1/2}-1\right)^4}$$
$$= \frac{\frac{1}{2}x^{-1}-\frac{1}{2}x^{-3/2}+x^{-1}}{\left(x^{1/2}-1\right)^3}$$
$$= \frac{\left(\frac{3}{2}x^{-1}-\frac{1}{2}x^{-3/2}\right)}{\left(x^{1/2}-1\right)^3}$$
$$= \frac{\frac{3}{2x}-\frac{1}{2x^{3/2}}}{\left(x^{1/2}-1\right)^3}$$
$$= \frac{\frac{3x^{1/2}}{2x^{3/2}}-\frac{1}{2x^{3/2}}}{\left(x^{1/2}-1\right)^3}$$
$$= \frac{3x^{1/2}-1}{2x^{3/2}\left(x^{1/2}-1\right)^3}$$

62. $y=\dfrac{x}{\sqrt{x-1}}=\dfrac{x}{(x-1)^{1/2}}$

$$y'=\frac{(x-1)^{1/2}(1)-x\left(\frac{1}{2}(x-1)^{-1/2}(1)\right)}{\left((x-1)^{1/2}\right)^2}$$

$$=\frac{(x-1)^{1/2}-\dfrac{x}{2(x-1)^{1/2}}}{x-1}$$

$$=\frac{\dfrac{2(x-1)}{2(x-1)^{1/2}}-\dfrac{x}{2(x-1)^{1/2}}}{x-1}$$

$$=\frac{\dfrac{x-2}{2(x-1)^{1/2}}}{x-1}$$

$$=\frac{x-2}{2(x-1)^{3/2}}$$

$$y''=\frac{1}{2}\cdot\frac{(x-1)^{3/2}(1)-(x-2)\left(\frac{3}{2}(x-1)^{1/2}(1)\right)}{\left((x-1)^{3/2}\right)^2}$$

$$=\frac{1}{2}\cdot\frac{(x-1)^{1/2}\left[(x-1)-\frac{3}{2}(x-2)\right]}{(x-1)^3}$$

$$=\frac{1}{2}\cdot\frac{x-1-\frac{3}{2}x+3}{(x-1)^{5/2}}$$

$$=\frac{1}{2}\cdot\frac{2-\frac{1}{2}x}{(x-1)^{5/2}}$$

$$=\frac{1-\frac{1}{4}x}{(x-1)^{5/2}}$$

63. $y=x^k$

$$\frac{dy}{dx}=kx^{k-1}$$

$$\frac{d^2y}{dx^2}=k(k-1)x^{k-2}$$

$$\frac{d^3y}{dx^3}=k(k-1)(k-2)x^{k-3}$$

$$\frac{d^4y}{dx^4}=k(k-1)(k-2)(k-3)x^{k-4}$$

$$\frac{d^5y}{dx^5}=k(k-1)(k-2)(k-3)(k-4)x^{k-5}$$

64. $y=ax^3+bx^2+cx+d$

$$\frac{dy}{dx}=3ax^2+2bx+c$$

$$\frac{d^2y}{dx^2}=6ax+2b$$

$$\frac{d^3y}{dx^3}=6a$$

65. $f(x)=\dfrac{x-1}{x+2}$

$$f'(x)=\frac{(x+2)(1)-(x-1)(1)}{(x+2)^2}$$

$$=\frac{3}{(x+2)^2}$$

Notice, $f'(x)=\dfrac{3}{(x+2)^2}=3(x+2)^{-2}$.

$$f''(x)=3(-2)(x+2)^{-2-1}$$

$$=-6(x+2)^{-3}$$

$$=-\frac{6}{(x+2)^3}$$

Notice, $f''(x)=-\dfrac{6}{(x+2)^3}=-6(x+2)^{-3}$.

$$f'''(x)=-6(-3)(x+2)^{-3-1}(1)$$

$$=18(x+2)^{-4}$$

$$=\frac{18}{(x+2)^4}$$

Notice, $f'''(x)=\dfrac{18}{(x+2)^4}=18(x+2)^{-4}$.

$$f^{(4)}(x)=18(-4)(x+2)^{-4-1}(1)$$

$$=-72(x+2)^{-5}$$

$$=-\frac{72}{(x+2)^5}$$

66. $f(x)=\dfrac{x+3}{x-2}$

$$f'(x)=\frac{(x-2)(1)-(x+3)(1)}{(x-2)^2}$$
$$=\frac{-5}{(x-2)^2}=-5(x-2)^{-2}$$
$$f''(x)=-5(-2)(x-2)^{-3}(1)$$
$$=10(x-2)^{-3}$$
$$=\frac{10}{(x-2)^3}$$
$$f'''(x)=10(-3)(x-2)^{-3-1}(1)$$
$$=-30(x-2)^{-4}$$
$$=-\frac{30}{(x-2)^4}$$
$$f^{(4)}(x)=-30(-4)(x-2)^{-4-1}(1)$$
$$=120(x-2)^{-5}$$
$$=\frac{120}{(x-2)^5}$$

67. $s(t)=16t^2$

Find the velocity function, $v(t)=s'(t)=32t$.

This velocity function is in feet per second, we need to convert 50 miles per hour to feet per second. There are 5280 feet in one mile and 3600 seconds in one hour. Therefore,

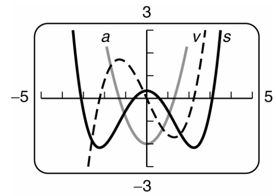
$$\left(50\frac{\text{mi}}{\text{hr}}\right)\left(\frac{5280\text{ft}}{1\text{ mi}}\right)\left(\frac{1\text{ hr}}{3600\text{ sec}}\right)=\frac{220}{3}\frac{\text{ft}}{\text{sec}}$$

Now we solve the equation:

$$v(t)=\frac{220}{3}$$
$$32t=\frac{220}{3}$$
$$t=\frac{220}{3}\cdot\frac{1}{32}$$
$$t=\frac{55}{24}\approx 2.29.$$

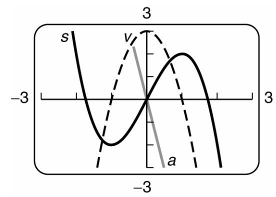
The ball will have to fall approximately 2.29 seconds to reach a speed of 50 mi/hr.

68. $s(t)=0.1t^4-t^2+0.4;\qquad[-5,5]$



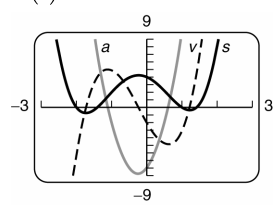
From the graph we see that $v(t)$ switches at $t=-1.29$ and $t=1.29$.

69. $s(t)=-t^3+3t;\qquad[-3,3]$



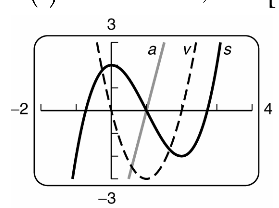
From the graph we see that $v(t)$ switches at $t=0$.

70. $s(t)=t^4+t^3-4t^2-2t+4;\qquad[-3,3]$



From the graph we see that $v(t)$ switches at $t=0.604$ and $t=-1.104$.

71. $s(t)=t^3-3t^2+2;\qquad[-2,4]$



From the graph we see that $v(t)$ switches at $t=1$.