Chapter 2

Applications of Differentiation

Exercise Set 2.1

1. $f(x) = x^2 + 4x + 5$

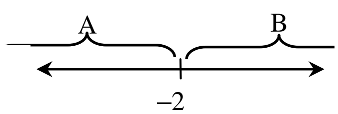
First, find the critical points.

$f'(x) = 2x + 4$

$f'(x)$ exists for all real numbers. We solve

$$\begin{aligned} f'(x) &= 0 \\ 2x + 4 &= 0 \\ 2x &= -4 \\ x &= -2 \end{aligned}$$

The only critical value is -2. We use -2 to divide the real number line into two intervals, A: $(-\infty, -2)$ and B: $(-2, \infty)$:



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test $-3, f'(-3) = 2(-3) + 4 = -2 < 0$

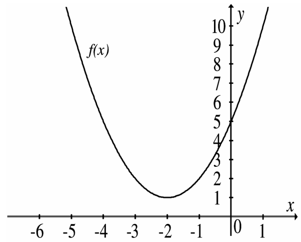
B: Test 0, $\quad f'(0) = 2(0) + 4 = 4 > 0$

We see that $f(x)$ is decreasing on $(-\infty, -2)$ and increasing on $(-2, \infty)$, and the change from decreasing to increasing indicates that a relative minimum occurs at $x = -2$. We substitute into the original equation to find $f(-2)$:

$f(-2) = (-2)^2 + 4(-2) + 5 = 1$

Thus, there is a relative minimum at $(-2, 1)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-5	10
-4	5
-3	2
-2	1
-1	2
0	5
1	10



2. $f(x) = x^2 + 6x - 3$

$f'(x) = 2x + 6$

$f'(x)$ exists for all real numbers. Solve

$$\begin{aligned} f'(x) &= 0 \\ 2x + 6 &= 0 \\ 2x &= -6 \\ x &= -3 \end{aligned}$$

The only critical value is -3. We use -3 to divide the real number line into two intervals, A: $(-\infty, -3)$ and B: $(-3, \infty)$.

A: Test $-4, f'(-4) = 2(-4) + 6 = -2 < 0$

B: Test 0, $\quad f'(0) = 2(0) + 6 = 6 > 0$

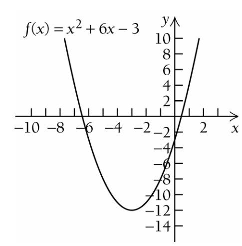
We see that $f(x)$ is decreasing on $(-\infty, -3)$ and increasing on $(-3, \infty)$, there is a relative minimum at $x = -3$.

$f(-3) = (-3)^2 + 6(-3) - 3 = -12$

Thus, there is a relative minimum at $(-3, -12)$.

We sketch the graph.

x	$f(x)$
-6	-3
-5	-8
-4	-11
-3	-12
-2	-11
-1	-8
0	-3



3. $f(x) = 5 - x - x^2$

First, find the critical points.

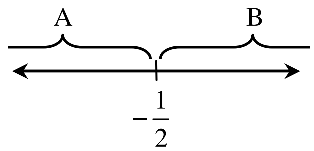
$f'(x) = -1 - 2x$

$f'(x)$ exists for all real numbers. We solve

$$\begin{aligned} f'(x) &= 0 \\ -1 - 2x &= 0 \\ -2x &= 1 \\ x &= -\frac{1}{2} \end{aligned}$$

The only critical value is $-\frac{1}{2}$. We use $-\frac{1}{2}$ to divide the real number line into two intervals,

A: $\left(-\infty,-\frac{1}{2}\right)$ and B: $\left(-\frac{1}{2},\infty\right)$:



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test $-1, f'(-1)=-1-2(-1)=1>0$

B: Test 0, $\quad f'(0)=-1-2(0)=-1<0$

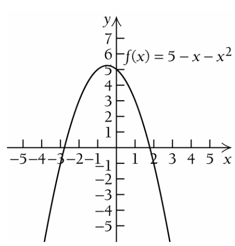
We see that $f(x)$ is increasing on $\left(-\infty,-\frac{1}{2}\right)$ and decreasing on $\left(-\frac{1}{2},\infty\right)$, and the change from increasing to decreasing indicates that a relative maximum occurs at $x=-\frac{1}{2}$. We substitute into the original equation to find $f\left(-\frac{1}{2}\right)$:

$$f\left(-\frac{1}{2}\right)=5-\left(-\frac{1}{2}\right)-\left(-\frac{1}{2}\right)^2=\frac{21}{4}$$

Thus, there is a relative maximum at $\left(-\frac{1}{2},\frac{21}{4}\right)$.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-3	-1
-2	3
-1	5
$-\frac{1}{2}$	$\frac{21}{4}$
0	5
1	3
2	-1



4. $f(x)=2-3x-2x^2$

$f'(x)=-3-4x$

$f'(x)$ exists for all real numbers. Solve

$$f'(x)=0$$
$$-3-4x=0$$
$$x=-\frac{3}{4}$$

The only critical value is $-\frac{3}{4}$. We use $-\frac{3}{4}$ to divide the real number line into two intervals,

A: $\left(-\infty,-\frac{3}{4}\right)$ and B: $\left(-\frac{3}{4},\infty\right)$.

A: Test $-1, f'(-1)=-3-4(-1)=1>0$

B: Test 0, $\quad f'(0)=-3-4(0)=-3<0$

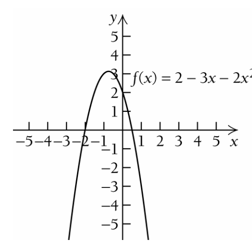
We see that $f(x)$ is increasing on $\left(-\infty,-\frac{3}{4}\right)$ and decreasing on $\left(-\frac{3}{4},\infty\right)$, there is a relative maximum at $x=-\frac{3}{4}$.

$$f\left(-\frac{3}{4}\right)=2-3\left(-\frac{3}{4}\right)-2\left(-\frac{3}{4}\right)^2=\frac{25}{8}$$

Thus, there is a relative maximum at $\left(-\frac{3}{4},\frac{25}{8}\right)$.

We sketch the graph.

x	$f(x)$
-3	-7
-2	0
-1	3
$-\frac{3}{4}$	$\frac{25}{8}$
0	2
1	-3
2	-12



5. $g(x)=1+6x+3x^2$

First, find the critical points.

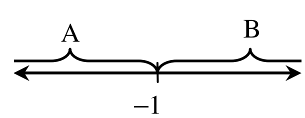
$g'(x)=6+6x$

$g'(x)$ exists for all real numbers. We solve:

$$g'(x)=0$$
$$6+6x=0$$
$$6x=-6$$
$$x=-1$$

The only critical value is -1. We use -1 to divide the real number line into two intervals,

A: $(-\infty,-1)$ and B: $(-1,\infty)$:



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test $-2, g'(-2) = 6 + 6(-2) = -6 < 0$

B: Test 0, $g'(0) = 6 + 6(0) = 6 > 0$

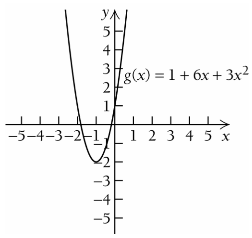
We see that $g'(x)$ is decreasing on $(-\infty, -1)$ and increasing on $(-1, \infty)$, and the change from decreasing to increasing indicates that a relative minimum occurs at $x = -1$. We substitute into the original equation to find $g(-1)$:

$g(-1) = 1 + 6(-1) + 3(-1)^2 = -2$

Thus, there is a relative minimum at $(-1, -2)$.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$g(x)$
–4	25
–3	10
–2	1
–1	–2
0	1
1	10
2	25



6. $F(x) = 0.5x^2 + 2x - 11$

$F'(x) = x + 2$

$F'(x)$ exists for all real numbers. Solve

$F'(x) = 0$

$x + 2 = 0$

$x = -2$

The only critical value is $(-1, 8)$. We use -2 to divide the real number line into two intervals, A: $(-\infty, -2)$ and B: $(-2, \infty)$.

A: Test $-3, F'(-3) = (-3) + 2 = -1 < 0$

B: Test 0, $F'(0) = (0) + 2 = 2 > 0$

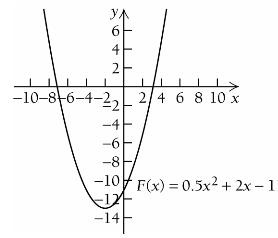
We see that $F(x)$ is decreasing on $(-\infty, -2)$ and increasing on $(-2, \infty)$, there is a relative minimum at $x = -2$.

$F(-2) = 0.5(-2)^2 + 2(-2) + -11 = -13$

Thus, there is a relative minimum at $(-2, -13)$.

We sketch the graph.

x	$F(x)$
–5	$-\frac{17}{2}$
–4	–11
–3	$-\frac{25}{2}$
–2	–13
–1	$-\frac{25}{2}$
0	–11
1	$-\frac{17}{2}$



7. $G(x) = x^3 - x^2 - x + 2$

First, find the critical points.

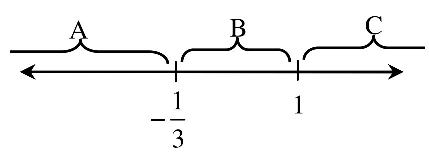
$G'(x) = 3x^2 - 2x - 1$

$G'(x)$ exists for all real numbers. We solve

$$G'(x) = 0$$
$$3x^2 - 2x - 1 = 0$$
$$(3x + 1)(x - 1) = 0$$
$$3x + 1 = 0 \quad \text{or} \quad x - 1 = 0$$
$$3x = -1 \quad \text{or} \quad x = 1$$
$$x = -\frac{1}{3} \quad \text{or} \quad x = 1$$

The critical values are $-\frac{1}{3}$ and 1. We use them to divide the real number line into three intervals,

A: $\left(-\infty, -\frac{1}{3}\right)$, B: $\left(-\frac{1}{3}, 1\right)$, and C: $(1, \infty)$.



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test -1,

$G'(-1) = 3(-1)^2 - 2(-1) - 1 = 4 > 0$

B: Test 0,

$G'(0) = 3(0)^2 - 2(0) - 1 = -1 < 0$

C: Test 2,

$G'(2) = 3(2)^2 - 2(2) - 1 = 7 > 0$

We see that $G(x)$ is increasing on $\left(-\infty,-\frac{1}{3}\right)$, decreasing on $\left(-\frac{1}{3},1\right)$, and increasing on $(1,\infty)$. So there is a relative maximum at $x=-\frac{1}{3}$ and a relative minimum at $x=1$.

We find $G\left(-\frac{1}{3}\right)$:

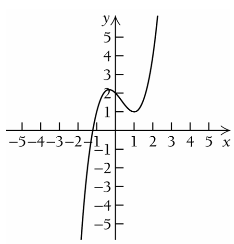
$$G\left(-\frac{1}{3}\right)=\left(-\frac{1}{3}\right)^3-\left(-\frac{1}{3}\right)^2-\left(-\frac{1}{3}\right)+2$$
$$=-\frac{1}{27}-\frac{1}{9}+\frac{1}{3}+2$$
$$=\frac{59}{27}$$

Then we find $G(1)$:

$$G(1)=(1)^3-(1)^2-(1)+2$$
$$=1-1-1+2$$
$$=1$$

There is a relative maximum at $\left(-\frac{1}{3},\frac{59}{27}\right)$, and there is a relative minimum at $(1,1)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$G(x)$
–2	–8
–1	1
0	2
2	4
3	17



$G(x)=x^3-x^2-x+2$

8. $g(x)=x^3+\frac{1}{2}x^2-2x+5$

$g'(x)=3x^2+x-2$

$g'(x)$ exists for all real numbers. We solve

$$g'(x)=0$$
$$3x^2+x-2=0$$
$$(3x-2)(x+1)=0$$
$$3x-2=0 \quad \text{or} \quad x+1=0$$
$$x=\frac{2}{3} \quad \text{or} \quad x=-1$$

The critical values are -1 and $\frac{2}{3}$. We use them to divide the real number line into three intervals,

A: $(-\infty,-1)$, B: $\left(-1,\frac{2}{3}\right)$, and C: $\left(\frac{2}{3},\infty\right)$.

A: Test -2,

$g'(-2)=3(-2)^2+(-2)-2=8>0$

B: Test 0,

$g'(0)=3(0)^2+(0)-2=-2<0$

C: Test 1,

$g'(1)=3(1)^2+(1)-2=2>0$

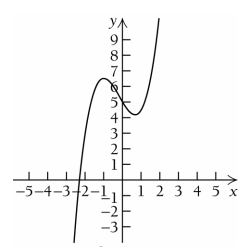
We see that $g(x)$ is increasing on $(-\infty,-1)$, decreasing on $\left(-1,\frac{2}{3}\right)$, and increasing on $\left(\frac{2}{3},\infty\right)$. So there is a relative maximum at $x=-1$ and a relative minimum at $x=\frac{2}{3}$.

$$g(-1)=(-1)^3+\frac{1}{2}(-1)^2-2(-1)+5=\frac{13}{2}$$

$$g\left(\frac{2}{3}\right)=\left(\frac{2}{3}\right)^3+\frac{1}{2}\left(\frac{2}{3}\right)^2-2\left(\frac{2}{3}\right)+5=\frac{113}{27}$$

There is a relative maximum at $\left(-1,\frac{13}{2}\right)$, and there is a relative minimum at $\left(\frac{2}{3},\frac{113}{27}\right)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$g(x)$
–2	3
0	5
1	$\frac{9}{2}$
2	11



$g(x)=x^3+\frac{1}{2}x^2-2x+5$

9. $f(x)=x^3-3x+6$

First, find the critical points.

$f'(x)=3x^2-3$

$f'(x)$ exists for all real numbers. We solve

$f'(x)=0$

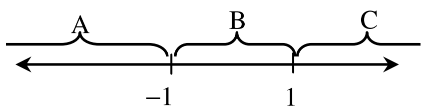
$3x^2-3=0$

$3x^2=3$

$x^2=1$

$x=\pm1$

The critical values are -1 and 1. We use them to divide the real number line into three intervals, A: $(-\infty,-1)$, B: $(-1,1)$, and C:$(1,\infty)$.



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test $-3, f'(-3)=3(-3)^2-3=24>0$

B: Test 0, $\quad f'(0)=3(0)^2-3=-3<0$

C: Test 2, $\quad f'(2)=3(2)^2-3=9>0$

We see that $f(x)$ is increasing on $(-\infty,-1)$, decreasing on $(-1,1)$, and increasing on $(1,\infty)$. So there is a relative maximum at $x=-1$ and a relative minimum at $x=1$.

We find $f(-1)$:

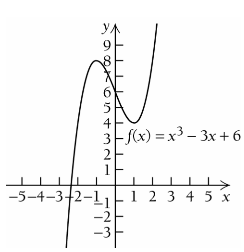
$f(-1)=(-1)^3-3(-1)+6=-1+3+6=8$

Then we find $f(1)$:

$f(1)=(1)^3-3(1)+6=1-3+6=4$

There is a relative maximum at $(-1,8)$, and there is a relative minimum at $(1,4)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
–3	–12
–2	4
0	6
2	8
3	24



10. $f(x)=x^3-3x^2$

$f'(x)=3x^2-6x$

$f'(x)$ exists for all real numbers. We solve

$f'(x)=0$

$3x^2-6x=0$

$3x(x-2)=0$

$x=0 \quad$ or $\quad x=2$

The critical values are 0 and 2. We use them to divide the real number line into three intervals, A: $(-\infty,0)$, B: $(0,2)$, and C:$(2,\infty)$.

A: Test $-1, f'(-1)=3(-1)^2-6(-1)=9>0$

B: Test 1, $\quad f'(1)=3(1)^2-6(1)=-3<0$

C: Test 3, $\quad f'(3)=3(3)^2-6(3)=9>0$

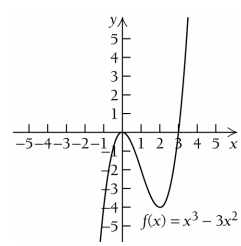
We see that $f(x)$ is increasing on $(-\infty,0)$, decreasing on $(0,2)$, and increasing on $(2,\infty)$. So there is a relative maximum at $x=0$ and a relative minimum at $x=2$.

$f(0)=(0)^3-3(0)^2=0$

$f(2)=(2)^3-3(2)^2=-4$

There is a relative maximum at $(0,0)$, and there is a relative minimum at $(2,-4)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
–2	–20
–1	–4
1	–2
3	0
4	16



11. $f(x)=3x^2+2x^3$

First, find the critical points.

$f'(x)=6x+6x^2$

$f'(x)$ exists for all real numbers. We solve

$f'(x)=0$

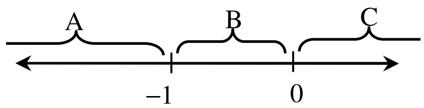
$6x+6x^2=0$

$6x(1+x)=0$

$6x=0 \quad$ or $\quad x+1=0$

$x=0 \quad$ or $\quad x=-1$

The critical values are -1 and 0. We use them to divide the real number line into three intervals,
A: $(-\infty,-1)$, B: $(-1,0)$, and C: $(0,\infty)$.



We use a test value in each interval to determine the sign of the derivative in each interval.
A: Test -2,

$$f'(-2)=6(-2)+6(-2)^2=12>0$$

B: Test $-\frac{1}{2}$,

$$f'\left(-\frac{1}{2}\right)=6\left(-\frac{1}{2}\right)+6\left(-\frac{1}{2}\right)^2=-\frac{3}{2}<0$$

C: Test 1,

$$f'(1)=6(1)+6(1)^2=12>0$$

We see that $f(x)$ is increasing on $(-\infty,-1)$, decreasing on $(-1,0)$, and increasing on $(0,\infty)$. So there is a relative maximum at $x=-1$ and a relative minimum at $x=0$.
We find $f(-1)$:

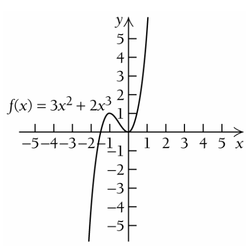
$$f(-1)=3(-1)^2+2(-1)^3=1$$

Then we find $f(0)$:

$$f(0)=3(0)^2+2(0)^3=0$$

There is a relative maximum at $(-1,1)$, and there is a relative minimum at $(0,0)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-3	-27
-2	-4
$\frac{1}{2}$	1
2	28



12. $f(x)=x^3+3x$

$f'(x)=3x^2+3$

$f'(x)$ exists for all real numbers. We solve

$f'(x)=0$

$3x^2+3=0$

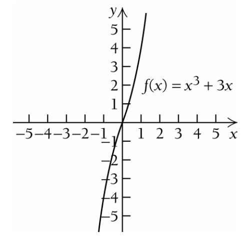
$x^2=-1$

There are no real solutions to this equation. Therefore, the function does not have any critical values.
We test a point

$f'(0)=3(0)^2+3=3>0$

We see that $f(x)$ is increasing on $(-\infty,\infty)$, and that there are no relative extrema. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-2	-14
-1	-4
0	0
1	4
2	14



13. $g(x)=2x^3-16$

First, find the critical points.

$g'(x)=6x^2$

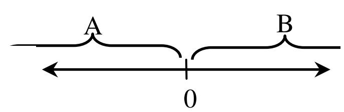
$g'(x)$ exists for all real numbers. We solve

$g'(x)=0$

$6x^2=0$

$x=0$

The only critical value is 0. We use 0 to divide the real number line into two intervals,
A: $(-\infty,0)$, and B: $(0,\infty)$.

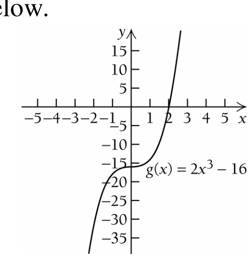


We use a test value in each interval to determine the sign of the derivative in each interval.
A: Test -1, $g'(-1)=6(-1)^2=6>0$

B: Test 1, $g'(1)=6(1)^2=6>0$

We see that $g(x)$ is increasing on $(-\infty,0)$ and increasing on $(0,\infty)$, so the function has no relative extema. We use the information obtained to sketch the graph. Other function values are listed below.

x	$g(x)$
-2	-32
-1	-18
0	-16
1	-14
2	0
3	38



14. $F(x) = 1 - x^3$

First, find the critical points.

$F'(x) = -3x^2$

$F'(x)$ exists for all real numbers. We solve

$F'(x) = 0$

$-3x^2 = 0$

$x = 0$

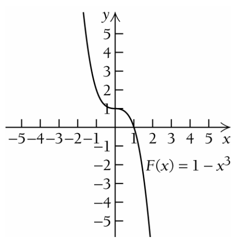
The only critical value is 0. We use 0 to divide the real number line into two intervals, A: $(-\infty, 0)$, and B: $(0, \infty)$.

A: Test -1, $F'(-1) = -3(-1)^2 = -3 < 0$

B: Test 1, $F'(1) = -3(1)^2 = -3 < 0$

We see that $F(x)$ is decreasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$, so the function has no relative extema. We use the information obtained to sketch the graph. Other function values are listed below.

x	$F(x)$
-2	9
-1	2
0	1
1	0
2	-7



15. $G(x) = x^3 - 6x^2 + 10$

First, find the critical points.

$G'(x) = 3x^2 - 12x$

$G'(x)$ exists for all real numbers. We solve

$G'(x) = 0$

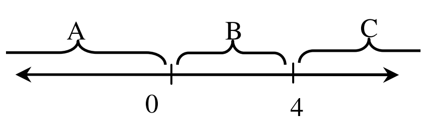
$x^2 - 4x = 0$ Dividing by 3

$x(x - 4) = 0$

$x = 0$ or $x - 4 = 0$

$x = 0$ or $x = 4$

The critical values are 0 and 4. We use them to divide the real number line into three intervals, A: $(-\infty, 0)$, B: $(0, 4)$, and C: $(4, \infty)$.



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test -1, $G'(-1) = 3(-1)^2 - 12(-1) = 15 > 0$

B: Test 1, $G'(1) = 3(1)^2 - 12(1) = -9 < 0$

C: Test 5, $G'(5) = 3(5)^2 - 12(5) = 15 > 0$

We see that $G(x)$ is increasing on $(-\infty, 0)$, decreasing on $(0, 4)$, and increasing on $(4, \infty)$. So there is a relative maximum at $x = 0$ and a relative minimum at $x = 4$.

We find $G(0)$:

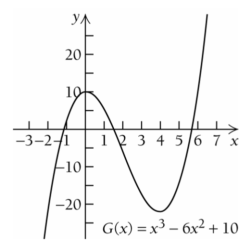
$$G(0) = (0)^3 - 6(0)^2 + 10$$
$$= 10$$

Then we find $G(4)$:

$$G(4) = (4)^3 - 6(4)^2 + 10$$
$$= 64 - 96 + 10$$
$$= -22$$

There is a relative maximum at $(0, 10)$, and there is a relative minimum at $(4, -22)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$G(x)$
-2	-22
-1	3
1	5
2	-6
3	-17



16. $f(x) = 12 + 9x - 3x^2 - x^3$

$f'(x) = 9 - 6x - 3x^2$

$f'(x)$ exists for all real numbers. Solve

$f'(x) = 0$

$9 - 6x - 3x^2 = 0$

$x^2 + 2x - 3 = 0$ Dividing by -3

$(x + 3)(x - 1) = 0$

$x + 3 = 0$ or $x - 1 = 0$

$x = -3$ or $x = 1$

The critical values are -3 and 1. We use them to divide the real number line into three intervals, A: $(-\infty, -3)$, B: $(-3, 1)$, and C: $(1, \infty)$.

A: Test -4,

$f'(-4) = 9 - 6(-4) - 3(-4)^2 = -15 < 0$

B: Test 0,

$f'(0) = 9 - 6(0) - 3(0)^2 = 9 > 0$

C: Test 2,

$f'(2) = 9 - 6(2) - 3(2)^2 = -15 < 0$

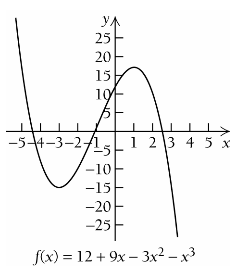
We see that $f(x)$ is decreasing on $(-\infty, -3)$, increasing on $(-3, 1)$, and increasing on $(1, \infty)$. So there is a relative minimum at $x = -3$ and a relative maximum at $x = 1$.

$f(-3) = 12 + 9(-3) - 3(-3)^2 - (-3)^3 = -15$

$f(1) = 12 + 9(1) - 3(1)^2 - (1)^3 = 17$

There is a relative minimum at $(-3, -15)$, and there is a relative maximum at $(1, 17)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-5	17
-4	-8
-2	-10
-1	1
0	12
2	10
3	-15



17. $g(x) = x^3 - x^4$

First, find the critical points.

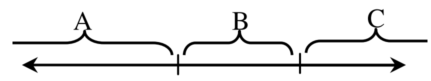
$g'(x) = 3x^2 - 4x^3$

$g'(x)$ exists for all real numbers. We solve

$$g'(x) = 0$$
$$3x^2 - 4x^3 = 0$$
$$x^2(3 - 4x) = 0$$
$$x^2 = 0 \quad \text{or} \quad 3 - 4x = 0$$
$$x = 0 \quad \text{or} \quad -4x = -3$$
$$x = 0 \quad \text{or} \quad x = \frac{3}{4}$$

The critical values are 0 and $\frac{3}{4}$. We use them to divide the real number line into three intervals,

A: $(-\infty, 0)$, B: $\left(0, \frac{3}{4}\right)$, and C: $\left(\frac{3}{4}, \infty\right)$.



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test -1, $g'(-1) = 3(-1)^2 - 4(-1)^3 = 7 > 0$

B: Test $\frac{1}{2}$, $g'\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right)^3$

$= 3\left(\frac{1}{4}\right) - 4\left(\frac{1}{8}\right) = \frac{1}{4} > 0$

C: Test 1, $g'(1) = 3(1)^2 - 4(1)^3 = -1 < 0$

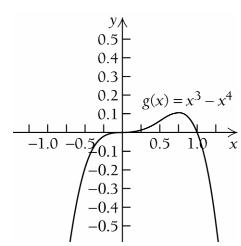
We see that $g(x)$ is increasing on $(-\infty, 0)$ and $\left(0, \frac{3}{4}\right)$, and is decreasing on $\left(\frac{3}{4}, \infty\right)$. So there is no relative extrema at $x = 0$ but there is a relative maximum at $x = \frac{3}{4}$.

We find $g\left(\frac{3}{4}\right)$:

$$g\left(\frac{3}{4}\right) = \left(\frac{3}{4}\right)^3 - \left(\frac{3}{4}\right)^4 = \frac{27}{64} - \frac{81}{256} = \frac{27}{256}$$

There is a relative maximum at $\left(\frac{3}{4}, \frac{27}{256}\right)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$g(x)$
-2	-24
-1	-2
0	0
$\frac{1}{2}$	$\frac{1}{16}$
1	0
2	-8



18. $f(x) = x^4 - 2x^3$

$f'(x) = 4x^3 - 6x^2$

$f'(x)$ exists for all real numbers. Solve

$$f'(x) = 0$$
$$4x^3 - 6x^2 = 0$$
$$2x^2(2x - 3) = 0$$

$x^2 = 0$ or $2x - 3 = 0$

$x = 0$ or $x = \frac{3}{2}$

The critical values are 0 and $\frac{3}{2}$. We use them to divide the real number line into three intervals,

A: $(-\infty, 0)$, B: $\left(0, \frac{3}{2}\right)$, and C: $\left(\frac{3}{2}, \infty\right)$.

A: Test -1, $f'(-1) = 4(-1)^3 - 6(-1)^2 = -10 < 0$

B: Test 1, $f'(1) = 4(1)^3 - 6(1)^2 = -2 < 0$

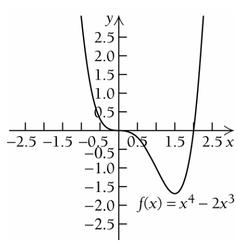
C: Test 2, $f'(2) = 4(2)^3 - 6(2)^2 = 8 > 0$

Since $f(x)$ is decreasing on both $(-\infty, 0)$ and $\left(0, \frac{3}{2}\right)$, and increasing on $\left(\frac{3}{2}, \infty\right)$, there is no relative extrema at $x = 0$ but there is a relative minimum at $x = \frac{3}{2}$.

$$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^4 - 2\left(\frac{3}{2}\right)^3 = -\frac{27}{16}$$

There is a relative minimum at $\left(\frac{3}{2}, -\frac{27}{16}\right)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-2	32
-1	3
0	0
1	-1
2	0
3	27



19. $f(x) = \frac{1}{3}x^3 - 2x^2 + 4x - 1$

First, find the critical points.

$f'(x) = x^2 - 4x + 4$

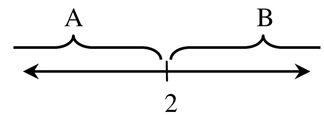
$f'(x)$ exists for all real numbers. We solve

$$f'(x) = 0$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$
$$x - 2 = 0$$
$$x = 2$$

The only critical value is 2.

We divide the real number line into two intervals,

A: $(-\infty, 2)$ and B: $(2, \infty)$.



We use a test value in each interval to determine the sign of the derivative in each interval.

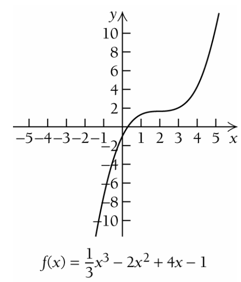
A: Test 0, $f'(0) = (0)^2 - 4(0) + 4 = 4 > 0$

B: Test 3, $f'(3) = (3)^2 - 4(3) + 4 = 1 > 0$

We see that $f(x)$ is increasing on both $(-\infty, 2)$ and $(2, \infty)$. Therefore, there are no relative extrema.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-3	-40
-2	$-\frac{59}{3}$
-1	$-\frac{22}{3}$
0	-1
1	$\frac{4}{3}$
2	$\frac{5}{3}$
3	2



20. $F(x) = -\frac{1}{3}x^3 + 3x^2 - 9x + 2$

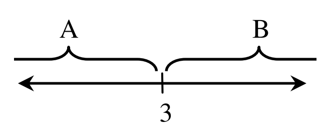
$F'(x) = -x^2 + 6x - 9$

$F'(x)$ exists for all real numbers. Solve

$$F'(x) = 0$$
$$-x^2 + 6x - 9 = 0$$
$$x^2 - 6x + 9 = 0$$
$$(x-3)^2 = 0$$
$$x - 3 = 0$$
$$x = 3$$

The only critical value is 3. We divide the real number line into two intervals,

A: $(-\infty, 3)$ and B: $(3, \infty)$.



We use a test value in each interval to determine the sign of the derivative in each interval.

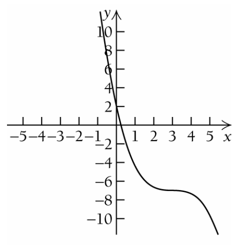
A: Test 0, $F'(0) = -(0)^2 + 6(0) - 9 = -9 < 0$

B: Test 4, $F'(4) = -(4)^2 + 6(4) - 9 = -1 < 0$

We see that $F(x)$ is decreasing on both $(-\infty, 3)$ and $(3, \infty)$. Therefore, there are no relative extrmea.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$F(x)$
-3	65
-2	$\frac{104}{3}$
-1	$\frac{43}{3}$
0	2
1	$-\frac{13}{3}$
2	$-\frac{20}{3}$
3	-7



$F(x) = -\frac{1}{3}x^3 + 3x^2 - 9x + 2$

21. $g(x) = 2x^4 - 20x^2 + 18$

First, find the critical points.

$g'(x) = 8x^3 - 40x$

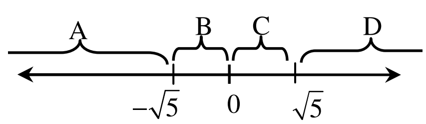
$g'(x)$ exists for all real numbers. We solve

$$g'(x) = 0$$
$$8x^3 - 40x = 0$$
$$8x(x^2 - 5) = 0$$
$$8x = 0 \quad \text{or} \quad x^2 - 5 = 0$$
$$x = 0 \quad \text{or} \quad x^2 = 5$$
$$x = 0 \quad \text{or} \quad x = \pm\sqrt{5}$$

The critical values are 0, $\sqrt{5}$ and $-\sqrt{5}$. We use them to divide the real number line into four intervals,

A: $(-\infty, -\sqrt{5})$, B: $(-\sqrt{5}, 0)$,

C: $(0, \sqrt{5})$, and D: $(\sqrt{5}, \infty)$.



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test -3,

$g'(-3) = 8(-3)^3 - 40(-3) = -96 < 0$

B: Test -1,

$g'(-1) = 8(-1)^3 - 40(-1) = 32 > 0$

C: Test 1,

$g'(1) = 8(1)^3 - 40(1) = -32 < 0$

D: Test 3,

$g'(3) = 8(3)^3 - 40(3) = 96 > 0$

We see that $g(x)$ is decreasing on $(-\infty, -\sqrt{5}]$, increasing on $[-\sqrt{5}, 0]$, decreasing again on $[0, \sqrt{5}]$, and increasing again on $[\sqrt{5}, \infty)$. Thus, there is a relative minimum at $x = -\sqrt{5}$, a relative maximum at $x = 0$, and another relative minimum at $x = \sqrt{5}$.

We find $g(-\sqrt{5})$:

$g(-\sqrt{5}) = 2(-\sqrt{5})^4 - 20(-\sqrt{5})^2 + 18 = -32$

Then we find $g(0)$:

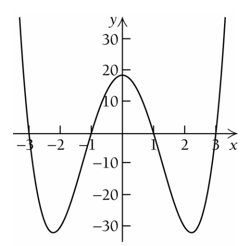
$g(0) = 2(0)^4 - 20(0)^2 + 18 = 18$

Then we find $g(\sqrt{5})$

$g(\sqrt{5}) = 2(\sqrt{5})^4 - 20(\sqrt{5})^2 + 18 = -32$

There are relative minima at $(-\sqrt{5}, -32)$ and $(\sqrt{5}, -32)$. There is a relative maximum at $(0, 18)$ We use the information obtained to sketch the graph. Other function values are listed below.

x	$g(x)$
-4	210
-3	0
-1	0
1	0
3	0
4	210



$g(x) = 2x^4 - 20x^2 + 18$

22. $f(x)=3x^4-15x^2+12$

$f'(x)=12x^3-30x$

$f'(x)$ exists for all real numbers. We solve

$$f'(x)=0$$
$$12x^3-30x=0$$
$$6x(2x^2-5)=0$$
$$6x=0 \quad \text{or} \quad 2x^2-5=0$$
$$x=0 \quad \text{or} \quad x^2=\frac{5}{2}$$
$$x=0 \quad \text{or} \quad x=\pm\frac{\sqrt{10}}{2}$$

The critical values are 0, $\frac{\sqrt{10}}{2}$ and $-\frac{\sqrt{10}}{2}$. We use them to divide the real number line into four intervals,

A: $\left(-\infty,-\frac{\sqrt{10}}{2}\right)$, B: $\left(-\frac{\sqrt{10}}{2},0\right)$,

C: $\left(0,\frac{\sqrt{10}}{2}\right)$, and D: $\left(\frac{\sqrt{10}}{2},\infty\right)$.

A: Test -2,

$$f'(-2)=12(-2)^3-30(-2)=-36<0$$

B: Test -1,

$$f'(-1)=12(-1)^3-30(-1)=18>0$$

C: Test 1,

$$f'(1)=12(1)^3-30(1)=-18<0$$

D: Test 2,

$$f'(2)=12(2)^3-30(2)=36>0$$

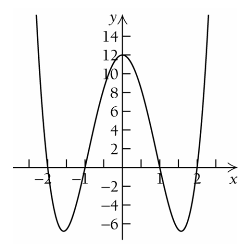
We see that $f(x)$ is decreasing on $\left(-\infty,-\frac{\sqrt{10}}{2}\right)$, increasing on $\left(-\frac{\sqrt{10}}{2},0\right)$, decreasing again on $\left(0,\frac{\sqrt{10}}{2}\right)$, and increasing again on $\left(\frac{\sqrt{10}}{2},\infty\right)$. Thus, there is a relative minimum at $x=-\frac{\sqrt{10}}{2}$, a relative maximum at $x=0$, and another relative minimum at $x=\frac{\sqrt{10}}{2}$.

$$f\left(-\frac{\sqrt{10}}{2}\right)=3\left(-\frac{\sqrt{10}}{2}\right)^4-15\left(-\frac{\sqrt{10}}{2}\right)^2+12$$
$$=-\frac{27}{4}$$
$$f(0)=3(0)^4-15(0)^2+12=12$$
$$f\left(\frac{\sqrt{10}}{2}\right)=3\left(\frac{\sqrt{10}}{2}\right)^4-15\left(\frac{\sqrt{10}}{2}\right)^2+12$$
$$=-\frac{27}{4}$$

There are relative minima at $\left(-\frac{\sqrt{10}}{2},-\frac{27}{4}\right)$ and $\left(\frac{\sqrt{10}}{2},-\frac{27}{4}\right)$.

There is a relative maximum at $(0,12)$ We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-3	120
-2	0
-1	0
1	0
2	0
3	120



$f(x)=3x^4-15x^2+12$

23. $F(x)=\sqrt[3]{x-1}=(x-1)^{1/3}$

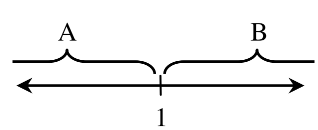
First, find the critical points.

$$F'(x)=\frac{1}{3}(x-1)^{-2/3}(1)$$
$$=\frac{1}{3(x-1)^{2/3}}$$

$F'(x)$ does not exist when $3(x-1)^{2/3}=0$, which means that $F'(x)$ does not exist when $x=1$. The equation $F'(x)=0$ has no solution, therefore, the only critical value is $x=1$.

We use 1 to divide the real number line into two intervals,

A: $(-\infty,1)$ and B: $(1,\infty)$:



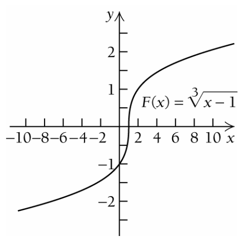
We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 0, $F'(0)=\dfrac{1}{3(0-1)^{2/3}}=\dfrac{1}{3}>0$

B: Test 2, $F'(2)=\dfrac{1}{3(2-1)^{2/3}}=\dfrac{1}{3}>0$

We see that $F(x)$ is increasing on both $(-\infty,1]$ and $[1,\infty)$. Thus, there are no relative extrema for $F(x)$. We use the information obtained to sketch the graph. Other function values are listed.

x	$F(x)$
-7	-2
0	-1
1	0
2	1
9	2



24. $G(x)=\sqrt[3]{x+2}=(x+2)^{1/3}$

$G'(x)=\dfrac{1}{3}(x+2)^{-2/3}(1)$

$=\dfrac{1}{3(x+2)^{2/3}}$

$G'(x)$ does not exist when $x=-2$. The equation $G'(x)=0$ has no solution, therefore, the only critical value is $x=-2$.
We use -2 to divide the real number line into two intervals,
A: $(-\infty,-2)$ and B: $(-2,\infty)$:

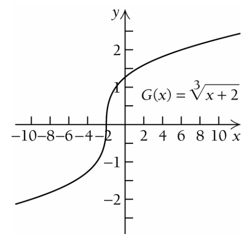
A: Test -3, $G'(-3)=\dfrac{1}{3(-3+2)^{2/3}}=\dfrac{1}{3}>0$

B: Test -1, $G'(-1)=\dfrac{1}{3(-1+2)^{2/3}}=\dfrac{1}{3}>0$

We see that $G(x)$ is increasing on both $(-\infty,-2)$ and $(-2,\infty)$. Thus, there are no relative extrema for $G(x)$.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$G(x)$
-10	-2
-3	-1
-2	0
-1	1
6	2

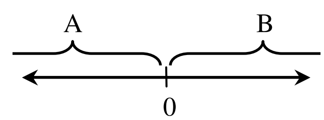


25. $f(x)=1-x^{2/3}$
First, find the critical points.

$f'(x)=\dfrac{-2}{3}x^{-1/3}$

$=\dfrac{-2}{3\sqrt[3]{x}}$

$f'(x)$ does not exist when $3\sqrt[3]{x}=0$, which means that $f'(x)$ does not exist when $x=0$. The equation $f'(x)=0$ has no solution, therefore, the only critical value is $x=0$.
We use 0 to divide the real number line into two intervals,
A: $(-\infty,0)$ and B: $(0,\infty)$:



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test -1, $f'(-1)=-\dfrac{2}{3\sqrt[3]{-1}}=\dfrac{2}{3}>0$

B: Test 1, $f'(1)=-\dfrac{2}{3\sqrt[3]{1}}=-\dfrac{2}{3}<0$

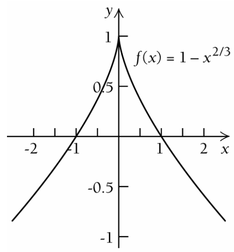
We see that $f(x)$ is increasing on $(-\infty,0]$ and decreasing on $[0,\infty)$. Thus, there is a relative maximum at $x=0$.
We find $f(0)$:

$f(0)=1-(0)^{2/3}=1$.

Therefore, there is a relative maximum at $(0,1)$.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-8	-3
-1	0
1	0
8	-3



26. $f(x) = (x+3)^{2/3} - 5$

$$f'(x) = \frac{2}{3}(x+3)^{-1/3}$$
$$= \frac{2}{3(x+3)^{1/3}}$$

$f'(x)$ does not exist when $x = -3$. The equation $f'(x) = 0$ has no solution, therefore, the only critical value is $x = -3$.

We use -3 to divide the real number line into two intervals, A: $(-\infty, -3)$ and B: $(-3, \infty)$:

A: Test $-4, f'(-4) = \dfrac{2}{3(-4+3)^{1/3}} = -\dfrac{2}{3} < 0$

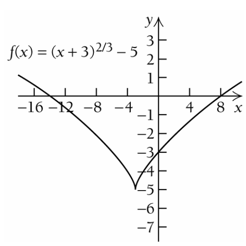
B: Test $-2, f'(-2) = \dfrac{2}{3(-2+3)^{1/3}} = \dfrac{2}{3} > 0$

We see that $f(x)$ is decreasing on $(-\infty, -3)$ and increasing on $(-3, \infty)$. Thus, there is a relative minimum at $x = -3$.

$f(-3) = (-3+3)^{2/3} - 5 = -5$:

Therefore, there is a relative minimum at $(-3, -5)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-11	-1
-4	-4
-2	-4
5	-1



27. $G(x) = \dfrac{-8}{x^2+1} = -8(x^2+1)^{-1}$

First, find the critical points.

$$G'(x) = -8(-1)(x^2+1)^{-2}(2x)$$
$$= \frac{16x}{(x^2+1)^2}$$

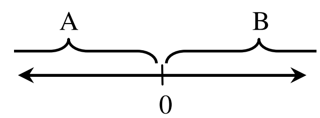
$G'(x)$ exists for all real numbers. We solve

$$G'(x) = 0$$
$$\frac{16x}{(x^2+1)^2} = 0$$
$$16x = 0$$
$$x = 0$$

The only critical value is 0.

We use 0 to divide the real number line into two intervals,

A: $(-\infty, 0)$ and B: $(0, \infty)$:



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test $-1, G'(-1) = \dfrac{16(-1)}{((-1)^2+1)^2} = \dfrac{-16}{4} = -4 < 0$

B: Test 1, $G'(1) = \dfrac{16(1)}{((1)^2+1)^2} = \dfrac{16}{4} = 4 > 0$

We see that $G(x)$ is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$. Thus, a relative minimum occurs at $x = 0$.

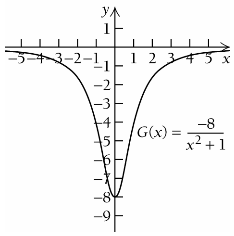
We find $G(0)$:

$$G(0) = \frac{-8}{(0)^2+1} = -8$$

Thus, there is a relative minimum at $(0, -8)$.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$G(x)$
-3	$-\frac{4}{5}$
-2	$-\frac{8}{5}$
-1	-4
1	-4
2	$-\frac{8}{5}$
3	$-\frac{4}{5}$



28. $F(x) = \dfrac{5}{x^2+1} = 5\left(x^2+1\right)^{-1}$

$$F'(x) = 5(-1)\left(x^2+1\right)^{-2}(2x)$$
$$= \frac{-10x}{\left(x^2+1\right)^2}$$

$F'(x)$ exists for all real numbers. We solve

$$F'(x) = 0$$
$$\frac{-10x}{\left(x^2+1\right)^2} = 0$$
$$x = 0$$

The only critical values is 0.
We use 0 to divide the real number line into two intervals,
A: $(-\infty,0)$ and B: $(0,\infty)$:
A: Test -1,

$$F'(-1) = \frac{-10(-1)}{\left((-1)^2+1\right)^2} = \frac{10}{4} = \frac{5}{2} > 0$$

B: Test 1,

$$F'(1) = \frac{-10(1)}{\left((1)^2+1\right)^2} = \frac{-10}{4} = -\frac{5}{2} < 0$$

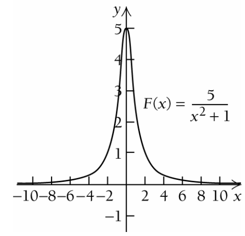
We see that $F(x)$ is increasing on $(-\infty,0)$ and decreasing on $(0,\infty)$. Thus, a relative maximum occurs at $x=0$.
We find $F(0)$:

$$F(0) = \frac{5}{(0)^2+1} = 5$$

Thus, there is a relative maximum at $(0,5)$.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$F(x)$
-3	$\frac{1}{2}$
-2	1
-1	$\frac{5}{2}$
1	$\frac{5}{2}$
2	1
3	$\frac{1}{2}$



29. $g(x) = \dfrac{4x}{x^2+1}$

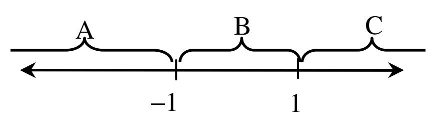
First, find the critical points.

$$g'(x) = \frac{\left(x^2+1\right)(4) - 4x(2x)}{\left(x^2+1\right)^2} \qquad \text{Quotient Rule}$$
$$= \frac{4x^2+4-8x^2}{\left(x^2+1\right)^2}$$
$$= \frac{4-4x^2}{\left(x^2+1\right)^2}$$

$g'(x)$ exists for all real numbers. We solve

$$g'(x) = 0$$
$$\frac{4-4x^2}{\left(x^2+1\right)^2} = 0$$
$$4-4x^2 = 0 \qquad \text{Multiplying by } \left(x^2+1\right)^2$$
$$x^2-1 = 0 \qquad \text{Dividing by } -4$$
$$x^2 = 1$$
$$x = \pm\sqrt{1}$$
$$x = \pm 1$$

The critical values are -1 and 1. We use them to divide the real number line into three intervals,
A: $(-\infty,-1)$, B: $(-1,1)$, and C: $(1,\infty)$.



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test -2, $g'(-2)=\dfrac{4-4(-2)^2}{\left((-2)^2+1\right)^2}=-\dfrac{12}{25}<0$

B: Test 0, $g'(0)=\dfrac{4-4(0)^2}{\left((0)^2+1\right)^2}=4>0$

C: Test 2, $g'(2)=\dfrac{4-4(2)^2}{\left((2)^2+1\right)^2}=-\dfrac{12}{25}<0$

We see that $g(x)$ is decreasing on $(-\infty,-1]$, increasing on $[-1,1]$, and decreasing again on $[1,\infty)$. So there is a relative minimum at $x=-1$ and a relative maximum at $x=1$.

We find $g(-1)$:

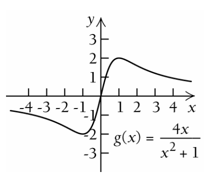
$$g(-1)=\frac{4(-1)}{(-1)^2+1}=\frac{-4}{2}=-2$$

Then we find $g(1)$:

$$g(1)=\frac{4(1)}{(1)^2+1}=\frac{4}{2}=2$$

There is a relative minimum at $(-1,-2)$, and there is a relative maximum at $(1,2)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$g(x)$
-3	$-\frac{6}{5}$
-2	$-\frac{8}{5}$
0	0
2	$\frac{8}{5}$
3	$\frac{6}{5}$



30. $g(x)=\dfrac{x^2}{x^2+1}$

$$g'(x)=\frac{(x^2+1)(2x)-x^2(2x)}{(x^2+1)^2}$$

$$g'(x)=\frac{2x}{(x^2+1)^2}$$

$g'(x)$ exists for all real numbers. We solve

$$g'(x)=0$$

$$\frac{2x}{(x^2+1)^2}=0$$

$$x=0$$

The only critical values is 0.
We use 0 to divide the real number line into two intervals,
A: $(-\infty,0)$ and B: $(0,\infty)$:

A: Test -1,

$$g'(-1)=\frac{2(-1)}{\left((-1)^2+1\right)^2}=\frac{-2}{4}=-\frac{1}{2}<0$$

B: Test 1,

$$g'(1)=\frac{2(1)}{\left((1)^2+1\right)^2}=\frac{2}{4}=\frac{1}{2}>0$$

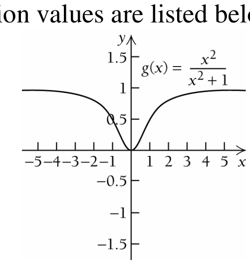
We see that $g(x)$ is decreasing on $(-\infty,0)$ and increasing on $(0,\infty)$. Thus, a relative minimum occurs at $x=0$.

We find $g(0)$:

$$g(0)=\frac{(0)^2}{(0)^2+1}=0$$

Thus, there is a relative minimum at $(0,0)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$g(x)$
-3	$\frac{9}{10}$
-2	$\frac{4}{5}$
-1	$\frac{1}{2}$
1	$\frac{1}{2}$
2	$\frac{4}{5}$
3	$\frac{9}{10}$



31. $f(x)=\sqrt[3]{x}=(x)^{1/3}$

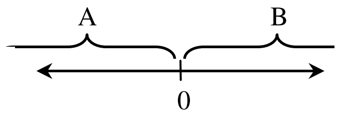
First, find the critical points.

$$f'(x)=\frac{1}{3}(x)^{-2/3}$$

$$=\frac{1}{3(x)^{2/3}}=\frac{1}{3\cdot\sqrt[3]{x^2}}$$

$f'(x)$ does not exist when $x=0$. The equation $f'(x)=0$ has no solution, therefore, the only critical value is $x=0$.

We use 0 to divide the real number line into two intervals,
A: $(-\infty,0)$ and B: $(0,\infty)$:



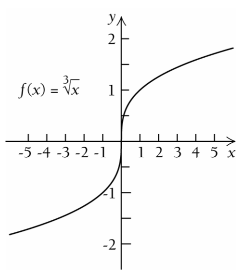
We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test $-1, f'(-1)=\dfrac{1}{3\sqrt[3]{(-1)^2}}=\dfrac{1}{3}>0$

B: Test $1, f'(1)=\dfrac{1}{3\left(\sqrt[3]{(1)^2}\right)}=\dfrac{1}{3}>0$

We see that $f(x)$ is increasing on both $(-\infty,0]$ and $[0,\infty)$. Thus, there are no relative extrema for $f(x)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-8	-2
-1	-1
0	0
1	1
8	2



32. $f(x)=(x+1)^{1/3}$

$$f'(x)=\frac{1}{3}(x+1)^{-2/3}(1)$$
$$=\frac{1}{3(x+1)^{2/3}}$$

$f'(x)$ does not exist when $x=-1$. The equation $f'(x)=0$ has no solution, therefore, the only critical value is $x=-1$.

We use -1 to divide the real number line into two intervals,

A: $(-\infty,-1)$ and B: $(-1,\infty)$:

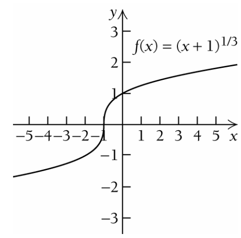
A: Test $-2, f'(-2)=\dfrac{1}{3(-2+1)^{2/3}}=\dfrac{1}{3}>0$

B: Test $0, f'(0)=\dfrac{1}{3(0+1)^{2/3}}=\dfrac{1}{3}>0$

We see that $f(x)$ is increasing on both intervals, Thus, there are no relative extrema for $f(x)$.

We use the information obtained to sketch the graph. Other function values are listed below.

x	$f(x)$
-9	-2
-2	-1
-1	0
0	1
7	2



33. $g(x)=\sqrt{x^2+2x+5}=\left(x^2+2x+5\right)^{1/2}$

First, find the critical points.

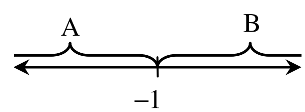
$$g'(x)=\frac{1}{2}\left(x^2+2x+5\right)^{-1/2}(2x+2)$$
$$=\frac{2(x+1)}{2\left(x^2+2x+5\right)^{1/2}}$$
$$=\frac{x+1}{\sqrt{x^2+2x+5}}$$

The equation $x^2+2x+5=0$ has no real-number solution, so $g'(x)$ exists for all real numbers. Next we find out where the derivative is zero. We solve

$$g'(x)=0$$
$$\frac{x+1}{\sqrt{x^2+2x+5}}=0$$
$$x+1=0$$
$$x=-1$$

The only critical value is -1. We use -1 to divide the real number line into two intervals,

A: $(-\infty,-1)$ and B: $(-1,\infty)$:



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test -2,

$$g'(-2)=\frac{(-2)+1}{\sqrt{(-2)^2+2(-2)+5}}=\frac{-1}{\sqrt{5}}<0$$

B: Test 0,

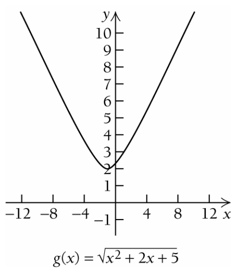
$$g'(0)=\frac{(0)+1}{\sqrt{(0)^2+2(0)+5}}=\frac{1}{\sqrt{5}}>0$$

We see that $g(x)$ is decreasing on $(-\infty,-1]$ and increasing on $[-1,\infty)$, and the change from decreasing to increasing indicates that a relative minimum occurs at $x=-1$. We substitute into the original equation to find $g(-1)$:

$$g(-1)=\sqrt{(-1)^2+2(-1)+5}=\sqrt{4}=2$$

Thus, there is a relative minimum at $(-1,2)$.
We use the information obtained to sketch the graph. Other function values are listed below.

x	$g(x)$
–4	3.61
–2	2.24
0	2.24
1	2.83
3	4.47



34. $F(x)=\dfrac{1}{\sqrt{x^2+1}}=\left(x^2+1\right)^{-1/2}$

$$F'(x)=\left(-\frac{1}{2}\right)\left(x^2+1\right)^{-3/2}(2x)$$

$$=\frac{-x}{\left(x^2+1\right)^{3/2}}$$

$F'(x)$ exists for all real numbers. We solve

$$F'(x)=0$$

$$\frac{-x}{\left(x^2+1\right)^{3/2}}=0$$

$$x=0$$

The only critical value is 0.
We use 0 to divide the real number line into two intervals,
A: $(-\infty,0)$ and B: $(0,\infty)$:
A: Test -1,

$$F'(-1)=\frac{-(-1)}{\left((-1)^2+1\right)^{3/2}}=\frac{1}{\sqrt{8}}>0$$

B: Test 1,

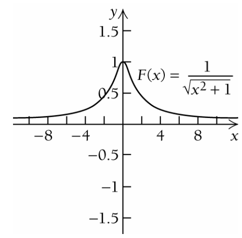
$$F'(1)=\frac{-1}{\left((1)^2+1\right)^{3/2}}=\frac{-1}{\sqrt{8}}<0$$

We see that $F(x)$ is increasing on $(-\infty,0)$ and decreasing on $(0,\infty)$. Thus, a relative maximum occurs at $x=0$.

$$F(0)=\frac{1}{\sqrt{(0)^2+1}}=1$$

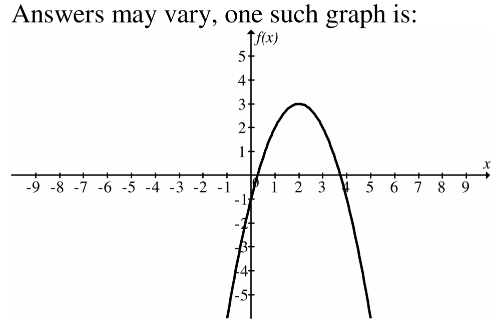
Thus, there is a relative maximum at $(0,1)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$F(x)$
–3	0.32
–2	0.45
–1	0.71
1	0.71
2	0.45
3	0.32

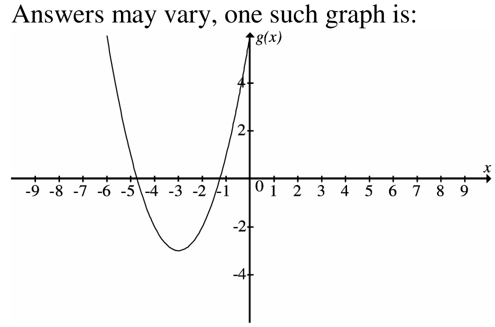


35. – 68. Left to the student.

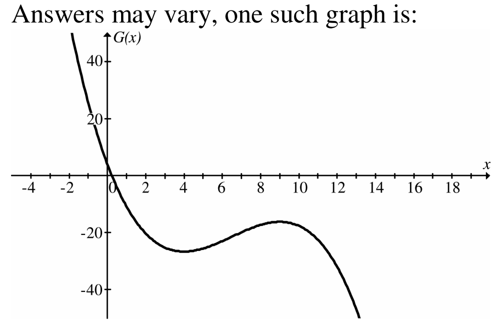
69. Answers may vary, one such graph is:



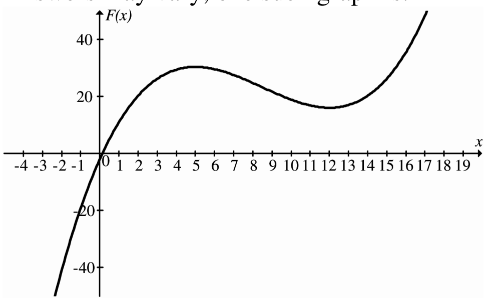
70. Answers may vary, one such graph is:



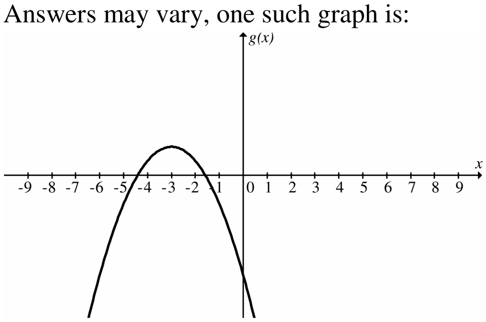
71. Answers may vary, one such graph is:



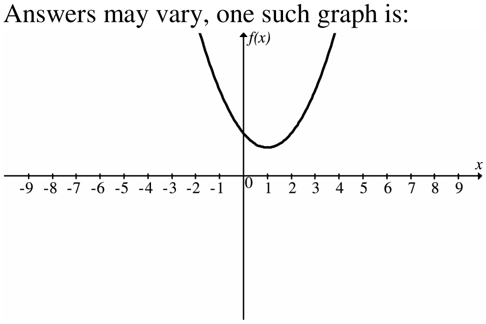
72. Answers may vary, one such graph is:



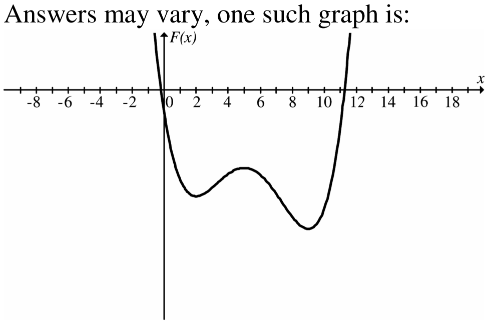
73. Answers may vary, one such graph is:



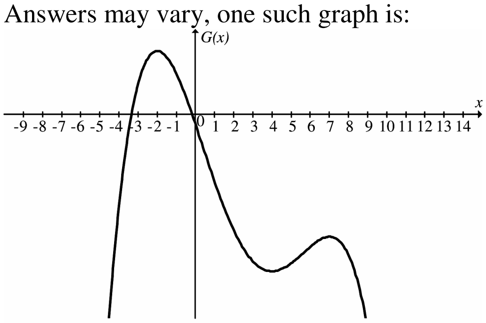
74. Answers may vary, one such graph is:



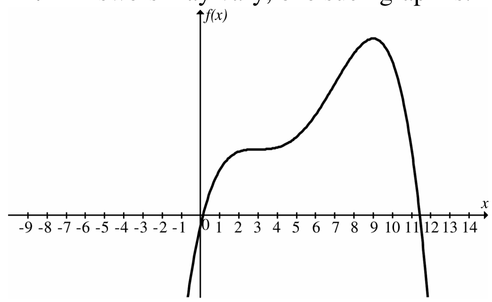
75. Answers may vary, one such graph is:



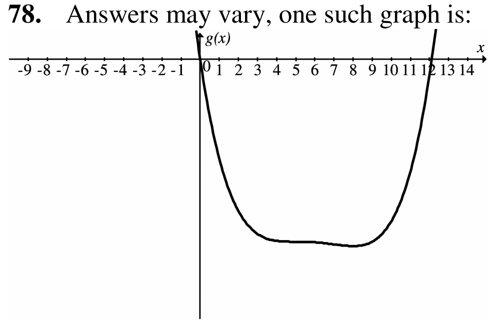
76. Answers may vary, one such graph is:



77. Answers may vary, one such graph is:



78. Answers may vary, one such graph is:



79. $T(t) = -0.1t^2 + 1.2t + 98.6, \quad 0 \le t \le 12$

First, we find the critical points.

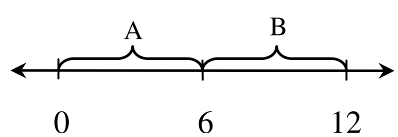
$T'(t) = -0.2t + 1.2$

$T'(t)$ exists for all real numbers. Solve

$$\begin{aligned} T'(t) &= 0 \\ -0.2t + 1.2 &= 0 \\ -0.2t &= -1.2 \\ t &= 6 \end{aligned}$$

The only critical value is 6. We use it to divide the interval $[0,12]$ into two intervals:

A: $[0,6)$ and B: $(6,12]$



Next, we test a point in each interval to determine the sign of the derivative.

A: Test $0, T'(0) = -0.2(0) + 1.2 = 1.2 > 0$

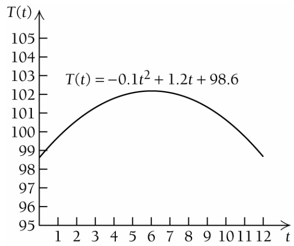
B: Test $7, T'(7) = -0.2(7) + 1.2 = -0.2 < 0$

Since, $T(t)$ is increasing on $[0,6)$ and decreasing on $(6,12]$, there is a relative maximum at $t = 6$.

$T(6) = -0.1(6)^2 + 1.2(6) + 98.6 = 102.2$

There is a relative maximum at $(6,102.2)$. We sketch the graph.

t	$T(t)$
0	98.6
3	101.3
5	102.1
7	102.1
8	101.8
12	98.6



80. $N(a) = -a^2 + 300a + 6, \qquad 0 \le a \le 300$

$N'(a) = -2a + 300$

$N'(a)$ exists for all real numbers. Solve,

$$N'(a) = 0$$
$$-2a + 300 = 0$$
$$-2a = -300$$
$$a = 150$$

The only critical value is 150. We divide the interval $[0,300]$ into two intervals,

A: $[0,150)$ and B:$(150,300]$.

A: Test 100,

$$N'(100) = -2(100) + 300 = 100 > 0$$

B: Test 200,

$$N'(200) = -2(200) + 300 = -100 < 0$$

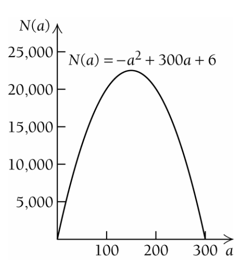
Since, $N(a)$ is increasing on $[0,150)$ and decreasing on $(150,300]$, there is a relative maximum at $x = 150$.

$N(150) = -(150)^2 + 300(150) + 6 = 22,506$

There is a relative maximum at $(150, 22, 506)$.

We sketch the graph.

a	$N(a)$
0	6
100	20,006
200	20,006
300	6



81. $A(t) = 0.0265t^3 - 0.453t^2 + 1.796t + 7.47,$

$0 \le t \le 10$

First, find the critical points.

$A'(t) = 0.0795t^2 - 0.906t + 1.796$

$A'(t)$ exists for all real numbers. We solve

$$A'(t) = 0$$

$0.0795t^2 - 0.906t + 1.796 = 0$

This is a quadratic equation with $a = 0.0795, b = -0.906,$ and $c = 1.796$

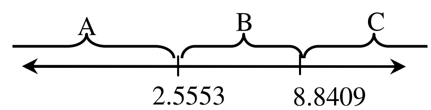
Applying the Quadratic formula we have:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-0.906) \pm \sqrt{(-0.906)^2 - 4(0.0795)(1.796)}}{2(0.0795)}$$
$$= \frac{0.906 \pm \sqrt{0.249708}}{0.159}$$

$t = 2.5553$ or $t = 8.8409$.

The critical values are 2.5553 and 8.8409. We use them to divide the real number line into three intervals, A: $(-\infty, 2.5553)$,

B: $(2.5553, 8.8409)$, and C:$(8.8409, \infty)$.



We use a test value in each interval to determine the sign of the derivative in each interval.

A: Test 0,

$$A'(0) = 0.0795(0)^2 - 0.906(0) + 1.796$$
$$= 1.796 > 0$$

B: Test 3,

$$A'(3) = 0.0795(3)^2 - 0.906(3) + 1.796$$
$$= -0.2065 < 0$$

C: Test 9,

$$A'(9) = 0.0795(9)^2 - 0.906(9) + 1.796$$
$$= 0.0815 > 0$$

We see that $A(t)$ is increasing on $(-\infty, 2.5553]$, decreasing on $[2.5553, 8.8409]$, and increasing again on $[8.8409, \infty)$. So there is a relative maximum at $t = 2.5553$ and a relative minimum at $t = 8.8409$.

We find $A(2.5553)$:

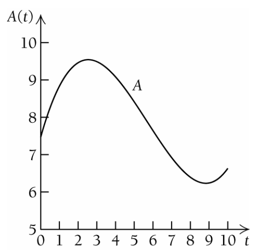
$$A(2.5553) = 0.0265(2.5553)^3 - 0.453(2.5553)^2 + 1.796(2.5553) + 7.47$$
$$\approx 9.5436$$

Then we find $A(8.8409)$:

$$A(8.8409) = 0.0265(8.8409)^3 - 0.453(8.8409)^2 + 1.796(8.8409) + 7.47 \approx 6.2531$$

There is a relative maximum at $(2.5553, 9.5436)$ and there is a relative minimum at $(8.8409, 6.2531)$. We use the information obtained to sketch the graph. Other function values are listed below.

x	$A(t)$
0	7.47
4	9.102
6	7.662
10	6.63



82. $f(x) = \sqrt{138.1 - 5.025x + 0.2902x^2}$

$$= \left(138.1 - 5.025x + 0.2902x^2\right)^{1/2}$$

$f'(x)$

$$= \frac{1}{2}\left(138.1 - 5.025x + 0.2902x^2\right)^{-1/2} \cdot (-5.025 + 0.5804x)$$

$$= \frac{-5.025 + 0.5804x}{2\sqrt{138.1 - 5.025x + 0.2902x^2}}$$

$f'(x)$ exists everywhere, so we solve

$$f'(x) = 0$$

$$\frac{-5.025 + 0.5804x}{2\sqrt{138.1 - 5.025x + 0.2902x^2}} = 0$$

$$-5.025 + 0.5804x = 0$$

$$0.5804x = 5.025$$

$$x = 8.6578$$

The only critical point is about 8.658 we use it to break up the interval $[0, 22]$ into two intervals A: $[0, 8.658)$ and B: $(8.658, 22]$.

A: Test 8,

$$f'(8) = \frac{-5.025 + 0.5804(8)}{2\sqrt{138.1 - 5.025(8) + 0.2902(8)^2}} = \frac{-0.3818}{2\sqrt{116.4728}} = -0.018 < 0$$

B: Test 9,

$$f'(9) = \frac{-5.025 + 0.5804(9)}{2\sqrt{138.1 - 5.025(9) + 0.2902(9)^2}} = \frac{0.1986}{2\sqrt{116.3812}} = 0.0092 > 0$$

We see that $f(x)$ is decreasing on $[0, 8.658)$ and increasing on $(8.658, 22]$, so there is a relative minimum at $x \approx 8.658$.

$f(8.658)$

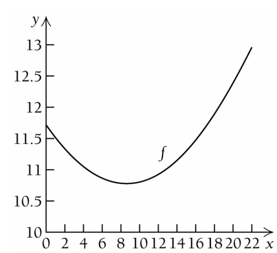
$$= \sqrt{138.1 - 5.025(8.658) + 0.2902(8.658)^2}$$

$$\approx 10.786$$

There is a relative minimum at about $(8.658, 10.786)$. Thus, the point farthest north on the path of the center of the eclipse is at a latitude of about 8.658 degrees East and a longitude of about 10.786 degrees South. The farthest point south on the path of the center of the eclipse is at a latitude of 22 degrees East and a longitude of about 12.962 degrees South

We use the information obtained above to sketch the graph. Other function values are listed below.

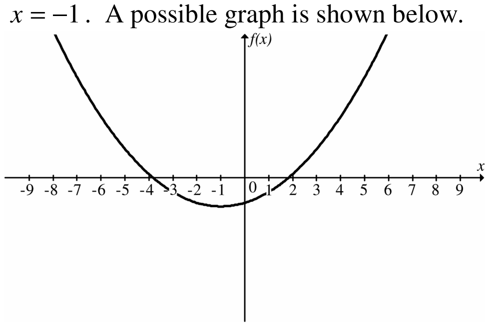
x	$f(x)$
0	11.752
5	10.965
10	10.811
15	11.315
20	12.397
22	12.962



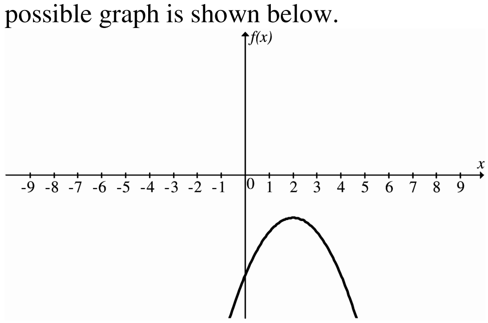
83. $\boxed{tw}$ The critical value of a function f is an interior value c of its domain at which the tangent to the graph is horizontal $\left(f'(c) = 0\right)$ or the tangent is vertical $\left(f'(c) \text{ does not exist}\right)$. The critical values for this graph are $x_1, x_3, x_4, x_5, x_6, x_7, x_8, x_{10}$.

84. $\boxed{tw}$ The function is increasing on intervals (a,b) and (c,d). A line tangent to the curve at any point on either of these intervals has a positive slope. Thus, the function is increasing on the intervals for which the first derivative is positive. Similarly, we see that on the intervals (b,c) and (d,e) the function is decreasing. A line tangent to the curve at any point on either of these intervals has a negative slope. Thus, the function is decreasing on the intervals for which first derivative is negative.

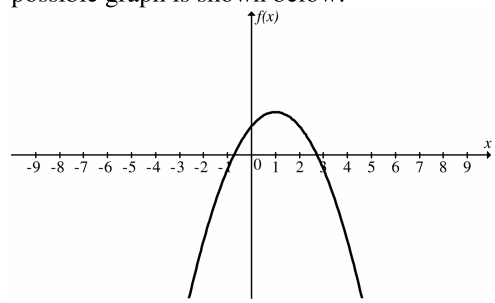
85. The derivative is negative over the interval $(-\infty,-1)$ and positive over the interval $(-1,\infty)$. Furthermore it is equal to zero when $x=-1$. This means that the function is decreasing over the interval $(-\infty,-1]$, increasing over the interval $[-1,\infty)$ and has a horizontal tangent at $x=-1$. A possible graph is shown below.



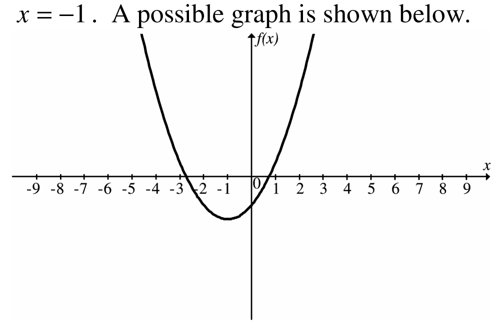
86. The derivative is positive over the interval $(-\infty,2)$ and negative over the interval $(2,\infty)$. Furthermore it is equal to zero when $x=2$. This means that the function is increasing over the interval $(-\infty,2)$, decreasing over the interval $(2,\infty)$ and has a horizontal tangent at $x=2$. A possible graph is shown below.



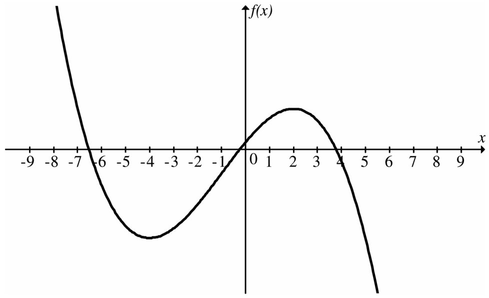
87. The derivative is positive over the interval $(-\infty,1)$ and negative over the interval $(1,\infty)$. Furthermore it is equal to zero when $x=1$. This means that the function is increasing over the interval $(-\infty,1]$, decreasing over the interval $[1,\infty)$ and has a horizontal tangent at $x=1$. A possible graph is shown below.



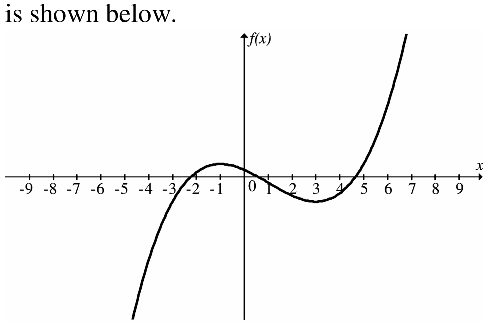
88. The derivative is negative over the interval $(-\infty,-1)$ and positive over the interval $(-1,\infty)$. Furthermore it is equal to zero when $x=-1$. This means that the function is decreasing over the interval $(-\infty,-1)$, increasing over the interval $(-1,\infty)$ and has a horizontal tangent at $x=-1$. A possible graph is shown below.



89. The derivative is positive over the interval $(-4,2)$ and negative over the intervals $(-\infty,-4)$ and $(2,\infty)$. Furthermore it is equal to zero when $x=-4$ and $x=2$. This means that the function is decreasing over the interval $(-\infty,-4]$, then increasing over the interval $[-4,2]$, and then decreasing again over the interval $[2,\infty)$. The function has horizontal tangents at $x=-4$ and $x=2$. A possible graph is shown on the next page.

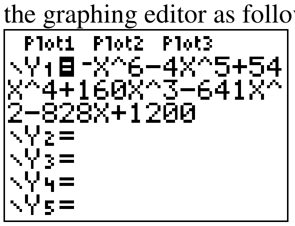


90. The derivative is negative over the interval $(-1,3)$ and intervals and positive over the intervals $(-\infty,-1)$ and $(3,\infty)$. Furthermore it is equal to zero when $x=-1$ and $x=3$. This means that the function is increasing over the interval $(-\infty,-1)$, then decreasing over the interval $(-1,3)$, and then increasing again over the interval $(3,\infty)$. The function has horizontal tangents at $x=-1$ and $x=3$. A possible graph is shown below.

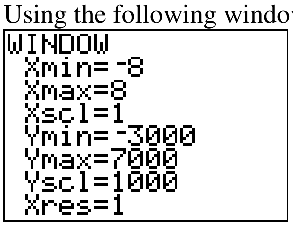


91. $f(x)=-x^6-4x^5+54x^4+160x^3-641x^2$
$-828x+1200$

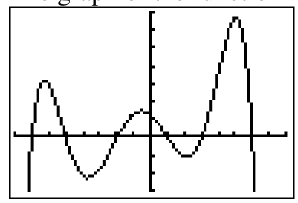
Using the calculator we enter the function into the graphing editor as follows:



Using the following window:



The graph of the function is:

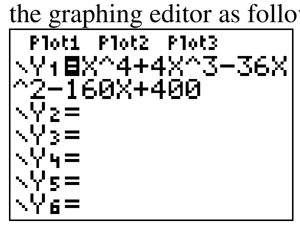


We find the relative extrema using the minimum/maximum feature on the calculator. There are relative minima at $(-3.683,-2288.03)$ and $(2.116,-1083.08)$.

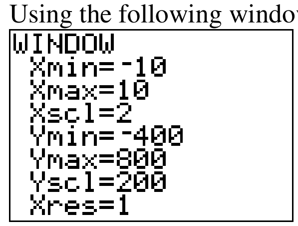
There are relative maxima at $(-6.262,3213.8)$, $(-0.559,1440.06)$, and $(5.054,6674.12)$.

92. $f(x)=x^4+4x^3-36x^2-160x+400$

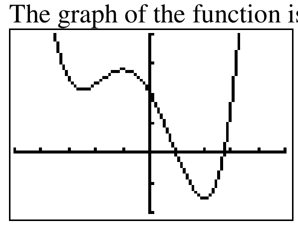
Using the calculator we enter the function into the graphing editor as follows:



Using the following window:



The graph of the function is:



We find the relative extrema using the minimum/maximum feature on the calculator. There are relative minima at $(-5,425)$ and $(4,-304)$.

There is a relative maximum at $(-2,560)$.

93. $f\left(x\right)=\sqrt[3]{\left|4-x^2\right|}+1$

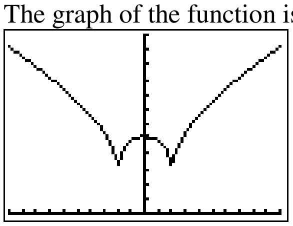
Using the calculator we enter the function into the graphing editor as follows:

```
Plot1 Plot2 Plot3
\Y1=³√(abs(4-X^2
))+1
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

Using the following window:

```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=6
 Yscl=.5
 Xres=1
```

The graph of the function is:



We find the relative extrema using the minimum/maximum feature on the calculator. There are relative minima at $\left(-2,1\right)$ and $\left(2,1\right)$.

There is a relative maximum at $\left(0,2.587\right)$.

94. $f\left(x\right)=x\sqrt{9-x^2}$

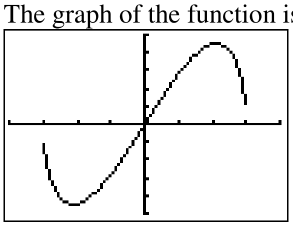
Using the calculator we enter the function into the graphing editor as follows:

```
Plot1 Plot2 Plot3
\Y1=X√(9-X^2)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Using the following window:

```
WINDOW
 Xmin=-4
 Xmax=4
 Xscl=1
 Ymin=-5
 Ymax=5
 Yscl=1
 Xres=1
```

The graph of the function is:



Notice, the calculator has trouble drawing the graph. The graph should continue to the x-intercepts at $\left(-3,0\right)$ and $\left(3,0\right)$. Fortunately, this does not hinder our efforts to find the extrema. We find the relative extrema using the minimum/maximum feature on the calculator. There is a relative minimum at $\left(-2.12,-4.5\right)$.

There is a relative maximum at $\left(2.12,4.5\right)$.

95. $\boxed{tw}$

a) We enter the data into the calculator and run a cubic regression. The calculator returns

```
CubicReg
 y=ax³+bx²+cx+d
 a=3.5708082E-8
 b=-3.319682E-4
 c=1.016994624
 d=-948.1034996
```

When we try to run a quartic regression, the calculator returns a domain error. Therefore, the cubic regression fits best.

b) The domain of the function is the set of nonnegative real numbers. Realistically, there would be some upper limit upon daily caloric intake.

c) The cubic regression model appears to have a relative minimum at $\left(3430,75.57\right)$ and it appears to have a relative maximum at $\left(2768,80.75\right)$.

96. $\boxed{tw}$

a) The cubic function fits best. In fact some calculators will return an error message when an attempt is made to fit a quartic function to the data.

```
CubicReg
 y=ax³+bx²+cx+d
 a=-1.047703E-7
 b=9.6298796E-4
 c=-2.91598216
 d=2916.68317
```

b) The domain of the function is the set of nonnegative real numbers. Realistically, there would be some upper limit upon daily caloric intake.

c) The cubic regression model appears to have a relative minimum at $\left(2732.84,1.404\right)$ and it appears to have a relative maximum at $\left(3394.80,16.597\right)$.

97. $\boxed{tw}$

a) Answers will vary. In Exercises 1-16 the function is given in equation form. The most accurate way to select an appropriate viewing window, one should first determine the domain, because that will help determine the x-range. For polynomials the domain is all real numbers, so we will typically select a x-range that is symmetric about 0. Next, you should find the critical values and make sure that your x-range contains them. Finally, you should determine the x-intercepts and make sure the x-range includes them. To find the y-range, you should find the y-values of the critical points and make sure the y-range includes those values. You should also make sure that the y-range includes the y-intercept.
To avoid the calculations required to find the relative extrema and the zeros as described above, we can determine a good window by using the table screen on the calculator and observing the appropriate y-values for selected x-values.

b) Answers will vary. When the equations are somewhat complex, the best way to determine a viewing window is to use the table screen on the calculator and observing appropriate y-values for selected x-values. You will need to set your table to accept selected x-values. Enter the table set up feature on your calculator and turn on the ask feature for your independent variable. This will allow you to enter an x-value and the calculator will return the y-value. You should make your ranges large enough so that all the data points will be easily viewed in the window.