57. Let T_4 be the time it takes for a population to quadruple, then according to the exponential growth function:

$4 \cdot P_0 = P_0 e^{kT_4}$

Solve this equation for T_4.

$$\frac{4P_0}{P_0} = e^{kT_4}$$
$$\ln 4 = \ln e^{kT_4}$$
$$\ln 4 = kT_4$$
$$\frac{\ln 4}{k} = T_4$$
$$T_4 = \frac{\ln 4}{k}$$

58. $3 \cdot P_0 = P_0 e^{kT_3}$

$$3 = e^{kT_3}$$
$$\ln 3 = kT_3$$
$$\frac{\ln 3}{k} = T_3$$

59. Let k_1 and k_2 represent the growth rates of Q_1 and Q_2. Using the Theorem 9, we know that:

$$k_1 = \frac{\ln 2}{1} \quad \text{and } k_2 = \frac{\ln 2}{2}$$

Since the initial amounts of both quantities are the same we have:

$$Q_1 = Q_0 e^{\ln 2 \cdot t} \text{ and } Q_2 = Q_0 e^{\left((\ln 2)/2\right)t}$$

Q_1 is twice the size of Q_2 when

$$Q_1 = 2Q_2$$
$$Q_0 e^{t \cdot \ln 2} = 2Q_0 e^{t\left((\ln 2)/2\right)}$$
$$\frac{Q_0 e^{t \cdot \ln 2}}{Q_0 e^{t\left((\ln 2)/2\right)}} = 2$$
$$e^{t\left(\ln 2 - \left((\ln 2)/2\right)\right)} = 2$$
$$\ln\left[e^{t\left((\ln 2)/2\right)}\right] = \ln 2$$
$$t\left((\ln 2)/2\right) = \ln 2$$
$$t = \frac{\ln 2}{(\ln 2)/2}$$
$$t = \frac{1}{\frac{1}{2}} = 2$$

It will take 2 years for Q_1 to be twice the size of Q_2.

60. If the amount grows at a rate of 100% per day, then it doubles each day. There are 24 hours in a day, so doubling time is 24 hours.

$$k = \frac{\ln 2}{24} \approx 0.028881$$

A growth rate of 100% per day, corresponds to an hourly growth rate of 0.028881 or approximately 2.888% per hour.

61. $i = e^k - 1$

Substitute 0.073 into the equation for *k.*

$$i = e^{0.073} - 1$$
$$\approx 1.075731 - 1$$
$$\approx 0.07573$$

The effective annual yield is 7.57%.

62. $i = e^{0.08} - 1 \approx 0.083287$

The effective annual yield is 8.33%.

63. $i = e^k - 1$

Substitute 0.0942 into the equation for *i.*

$$0.0942 = e^k - 1$$
$$1.0942 = e^k$$
$$\ln 1.0942 = \ln e^k$$
$$0.09002 \approx k$$

The interest rate was 9.0%.

64. $0.0661 = e^k - 1$

$$1.0661 = e^k$$
$$\ln 1.0661 = k$$
$$0.064 \approx k$$

The interest rate was 6.4%.

65. Consider the system of equations.

$$y_1 = Ce^{kt_1}$$
$$y_2 = Ce^{kt_2}$$

Divide the first equation by the second equation.

$$\frac{y_1}{y_2} = \frac{Ce^{kt_1}}{Ce^{kt_2}}$$
$$\frac{y_1}{y_2} = e^{k(t_1 - t_2)}$$
$$\ln\left(\frac{y_1}{y_2}\right) = \ln e^{k(t_1 - t_2)}$$
$$\ln\left(\frac{y_1}{y_2}\right) = k(t_1 - t_2) \qquad (\text{P5})$$

Solving the equation for k we have:

$$\frac{\ln\left(\frac{y_1}{y_2}\right)}{t_1 - t_2} = k$$

If we initially divide y_2 by y_1, we find,

equivalently, that $k = \dfrac{\ln\left(\frac{y_2}{y_1}\right)}{t_2 - t_1}$.

66. $\boxed{tw}$

Growth Rate k (% per year)	Doubling Time T (in years)
1%	69.3
2%	34.7
4.6%	15
6.9%	10
14%	5

67. $\boxed{tw}$ The graph of an exponential function increases or decreases without bound, while the graph of a logistic function approaches a limiting value.

68. $\boxed{tw}$ The Rule of 70 is used to estimate how long it will take an investment to double. In order to apply the Rule of 70, divide 70 by the interest rate. This will give you the time in years it takes for the investment to double. Since inflation takes away purchasing power, the rule of 70 could help someone determine how much money they will need to purchase a good or service in the future. Suppose the inflation rate is 4%. The Rule of 70 tells us that it takes $\frac{70}{4} = 17.5$ years for the overall price level to double. That means that in 17.5 years, you will need \$200 to purchase what cost \$100 today.

Exercise Set 3.4

1. a) $N(t) = N_0 e^{-kt}$

We substitute 0.096 in for k.

$N(t) = N_0 e^{-0.096t}$

b) $N_0 = 500$

$$N(t) = 500e^{-0.096t}$$
$$N(4) = 500e^{-0.096(4)}$$
$$= 500e^{-0.384}$$
$$\approx 340.565713$$
$$\approx 341$$

There will be approximately 341g of Iodine -131 present after 4 days.

c) From Theorem 10, we know that the half-life T and the decay rate k are related by:

$T = \dfrac{\ln 2}{k}$.

Substitute $k = .096$

$T = \dfrac{\ln 2}{0.096} \approx 7.22028$

It will take about 7.2 days for half of the 500g of Iodine - 131 to remain.

2. a) $N(t) = N_0 e^{-kt}$

$N(t) = N_0 e^{-0.0001205t}$

b) $N_0 = 200$

$$N(t) = 200e^{-0.0001205t}$$
$$N(800) = 200e^{-0.0001205(800)}$$
$$= 200e^{-0.0964}$$
$$\approx 181.62$$
$$\approx 182$$

There will be approximately 182g of Carbon-14 present after 800 years.

c) $T = \dfrac{\ln 2}{k}$.

Substitute $k = .0001205$

$T = \dfrac{\ln 2}{0.0001205} \approx 5752.26$

It will take about 5752 years for half of the Carbon-14 to remain.

3. a) When A decomposes at a rate proportional to the amount of A present, we know that

$\dfrac{dA}{dt} = -kA$.

The solution to this equation is

$A(t) = A_0 e^{-kt}$.

b) First, we find k. The half-life of A is 3.3 hrs. From theorem 10 we know:

$k = \dfrac{\ln 2}{T}$

$k = \dfrac{\ln 2}{3.3} \approx 0.21$

The initial amount $A_0 = 10$, so the exponential decay function is:
$A(t) = 10e^{-0.21t}$.
To determine how long it will take to reduce A to 1 lb we solve $A(t) = 1$ for t.

$$10e^{-0.21t} = 1$$
$$e^{-0.21t} = 0.1$$
$$\ln\left(e^{-0.21t}\right) = \ln(0.1)$$
$$-0.21t = \ln(0.1)$$
$$t = \frac{\ln 0.1}{-0.21}$$
$$t \approx 10.96469$$
$$t \approx 11$$

It will take approximately 11 hours for 10 lbs of substance A to reduce to 1 lb.

4. a) $A(t) = A_0 e^{-kt}$.

b) First, we find k. The half-life of A is 3 hrs.

$$k = \frac{\ln 2}{3} \approx 0.23$$

Then,
$A(t) = 8e^{-0.23t}$.

$$8e^{-0.23t} = 1$$
$$e^{-0.23t} = 0.125$$
$$\ln\left(e^{-0.23t}\right) = \ln(0.125)$$
$$-0.23t = \ln(0.125)$$
$$t = \frac{\ln 0.125}{-0.23}$$
$$t \approx 9$$

It will take approximately 9 hours for 8g of substance A to reduce to 1g.

5. From theorem 10 we know:

$$k = \frac{\ln 2}{T}$$
$$k = \frac{\ln 2}{3} \approx 0.231$$

The decay rate is 0.231 or 23.1% per minute.

6. From theorem 10 we know:

$$k = \frac{\ln 2}{T}$$
$$k = \frac{\ln 2}{4560} \approx 0.000152$$

The decay rate is 0.000152 or 0.0152% per yr.

7. From theorem 10 we know:

$$T = \frac{\ln 2}{k}$$
$$T = \frac{\ln 2}{.0315} \approx 22.0$$

The half-life is 22.0 years.

8. From theorem 10 we know:

$$T = \frac{\ln 2}{k}$$
$$T = \frac{\ln 2}{0.0277} \approx 25.0$$

The half-life is 25 years.

9. $P(t) = P_0 e^{-kt}$

We substitute 1000 for P_0, 0.0315 for k, and 100 for t.

$$P(100) = 1000e^{-0.0315(100)}$$
$$= 1000e^{-3.15}$$
$$\approx 42.9$$

Approximately 42.9 grams of lead-210 will remain after 100 years.

10. $P(20) = 1000e^{-0.231(20)} \approx 9.9$

Approximately 9.9 grams of polonium-218 will remain after 20 minutes.

11. If an ivory tusk has lost 40% of its carbon-14 from its initial amount P_0, then 60% of the initial amount remains. To find the age of the tusk, we solve the following equation for t:

$$0.60P_0 = P_0 e^{-0.0001205t}$$
$$0.60 = e^{-0.0001205t}$$
$$\ln 0.60 = \ln e^{-0.0001205t}$$
$$\ln 0.60 = -0.0001205t$$
$$\frac{\ln 0.60}{-0.0001205} = t$$
$$4239.216 \approx t$$
$$4239 \approx t$$

The ivory tusk is approximately 4239 years old.

12. The amount of carbon-14 present is 10% of P_0

$$0.10P_0 = P_0e^{-0.0001205t}$$
$$0.10 = e^{-0.0001205t}$$
$$\ln 0.10 = -0.0001205t$$
$$\frac{\ln 0.10}{-0.0001205} = t$$
$$19{,}108.589 \approx t$$

The piece of wood is approximately 19,109 years old.

13. First, we find k. The half-life is 60.1 days. From Theorem 10 we know.

$$k = \frac{\ln 2}{T}$$
$$k = \frac{\ln 2}{60.1}$$
$$k \approx 0.0115$$

The decay rate is approximately 0.0115 or 1.15% per day.
If the initial amount A_0 decreased by 25%, then 75% of A_0 remains. We solve the following equation for t.

$$0.75A_0 = A_0e^{-0.0115t}$$
$$0.75 = e^{-0.0115t}$$
$$\ln 0.75 = \ln e^{-0.0115t}$$
$$\ln 0.75 = -0.0115t$$
$$\frac{\ln 0.75}{-0.0115} = t$$
$$25.015 \approx t$$
$$25 \approx t$$

The sample was sitting on the shelf for approximately 25 days.

14. The amount of carbon-14 present is 40% of P_0

$$0.40P_0 = P_0e^{-0.0001205t}$$
$$0.40 = e^{-0.0001205t}$$
$$\ln 0.40 = -0.0001205t$$
$$\frac{\ln 0.40}{-0.0001205} = t$$
$$7604 \approx t$$

The artifact is approximately 7604 years old.

15. Since the corn pollen had lost 38.1% of its carbon-14, then 61.9% of the initial carbon-14 remains. The decay rate of carbon-14 is $k = 0.0001205$. In order to find out the age of the corn pollen, we must solve the following equation for t.

$$0.619P_0 = P_0e^{-0.0001205t}$$
$$0.619 = e^{-0.0001205t}$$
$$\ln 0.619 = -0.0001205t$$
$$\frac{\ln 0.619}{-0.0001205} = t$$
$$3980 \approx t$$

The corn pollen was approximately 3980 years old.

16. Using the exponential growth function $P(t) = P_0e^{kt}$ and the future value $P(20) = 30{,}000$. We solve for the initial investment P_0

$$30{,}000 = P_0e^{0.06(20)}$$
$$\frac{30{,}000}{e^{1.2}} = P_0$$
$$9035.83 \approx P_0$$

The parents should invest \$9035.83 in order to have \$30,000 on their child's 20th birthday.

17. Use the exponential growth function $P(t) = P_0e^{kt}$. The interest rate is 5.3% so $k = 0.053$. When $t = 20$ the parents wish the future value to be \$40,000, substituting this information into the function gives us the equation:

$$40{,}000 = P_0e^{0.053(20)}$$
$$\frac{40{,}000}{e^{1.06}} = P_0$$
$$13{,}858.23 \approx P_0$$

The parents should invest \$13,858.23 in order to have \$40,000 on their child's 20th birthday.

18.

$$15{,}000 = P_0e^{0.061(5)}$$
$$\frac{15{,}000}{e^{0.305}} = P_0$$
$$11{,}056.85 \approx P_0$$

The home owner should invest \$11,056.85 in order to have the required amount in 5 years.

19. The interest rate is 5.7%, so $k = 0.057$. In 6 years, the athletes salary will be \$9 million so $P(6) = 9$ million dollars. Substituting this information into the exponential growth function we get:

$$9 = P_0 e^{0.057(6)}$$
$$9 = P_0 e^{0.342}$$
$$\frac{9}{e^{0.342}} = P_0$$
$$6.393134 \approx P_0$$

The present value is \$6.393134 million or \$6,393,134.

20.
$$P(t) = P_0 e^{kt}$$
$$12 = P_0 e^{0.062(3)}$$
$$12 = P_0 e^{0.186}$$
$$\frac{12}{e^{0.186}} = P_0$$
$$9.963283 \approx P_0$$

The present value of the payment is \$9.963283 million or \$9,963,283.

21. Using the exponential growth function $P(t) = P_0 e^{kt}$. It is known that the interest rate is 8.3%, therefore $k = 0.083$. In 13 years the value of the trust fund will be \$80,000, so $P(13) = 80,000$. Substitute this information into the exponential growth function and solve for the present value P_0.

$$80,000 = P_0 e^{0.083(13)}$$
$$80,000 = P_0 e^{1.079}$$
$$\frac{80,000}{e^{1.079}} = P_0$$
$$27,194.82 \approx P_0$$

The present value of the trust fund is \$27,194.82.

22. Equilibrium occurs when supply equals demand

$$S(x) = D(x)$$
$$\ln x = \ln \frac{163,000}{x}$$
$$\ln x = \ln 163,000 - \ln x$$
$$2\ln x = \ln 163,000$$
$$\ln x^2 = \ln 163,000$$
$$x^2 = 163,000$$
$$x \approx 403.73$$

The equilibrium price of stereos is \$403.73. Substitute this price into the supply equation to find the equilibrium quantity.

$S(403.73) = \ln 403.73 \approx 6$.

Therefore, the equilibrium point is $(\$403.73, 6 \text{ stereos})$.

23. a) $V(0) = 40,000e^{-0} = 40,000$

The initial cost of the machinery was \$40,000.

b) $V(2) = 40,000e^{-2} \approx 5413.41$

The salvage value after 2 years is approximately \$5413.41.

c) $\boxed{tw}$ $V(t) = 40,000e^{-t}$

$V'(t) = -40,000e^{-t}$

The value of the machinery is decreasing at a rate of $40,000e^{-t}$ dollars per year t years after it was purchased.

24. a) $P(t) = P_0 e^{-kt}$

$$42 = 100e^{-k(27)} \qquad (t = 1967 - 1940 = 27)$$
$$\frac{42}{100} = e^{-27k}$$
$$\ln 0.42 = \ln e^{-27k}$$
$$\ln 0.42 = -27k$$
$$\frac{\ln 0.42}{-27} = k$$
$$0.032 \approx k$$

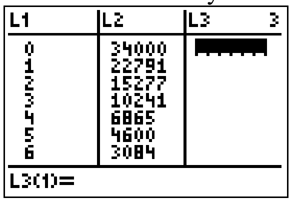
Thus, $P(t) = 100e^{-0.032t}$, where t is the number years that precede 1967.

b) In 1900, $t = 1967 - 1900 = 67$.

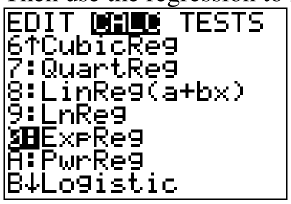
$P(67) = 100e^{-0.032(67)} \approx 11.72$

The same goods and services that cost \$100 in 1967 would have cost \$11.72 in 1900.

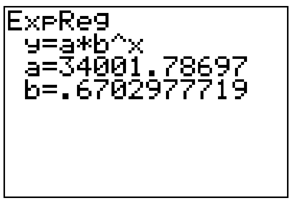
25. a) Enter the data into your calculator.



Then use the regression to fit the data.



We have:



The calculator determined the exponential function to be

$y = 34{,}001.78697\left(0.6702977719\right)^x$.

Using $b^x = e^{x(\ln b)}$, we have

$$0.6702977719^x = e^{x\left(\ln(0.6702977719)\right)}$$
$$= e^{-0.4000332297\cdot x}$$

Converting the calculator equation to $V(t) = V_0 e^{-kt}$ we have:

$$V(t) = 34{,}001.78697e^{-0.4000332297t}$$

b) $V(7) = 34{,}001.78697e^{-0.4000332297(7)}$

≈ 2067.17

The salvage value of the copier is approximately \$2067.17 after 7 years.

$V(10) = 34{,}001.78697e^{-0.4000332297(10)}$

≈ 622.56

The salvage value of the copier is approximately \$622.56 after 10 years.

c) Solve $V(t) = 1000$ for t.

$$34{,}001.78697e^{-0.4000332297t} = 1000$$
$$e^{-0.4000332297t} = \frac{1000}{34{,}001.78697}$$
$$\ln e^{-0.4000332297t} = \ln\left(\frac{1000}{34{,}001.78697}\right)$$
$$-0.4000332297t = \ln\left(\frac{1000}{34{,}001.78697}\right)$$
$$t = \frac{\ln\left(\frac{1000}{34{,}001.78967}\right)}{-0.4000332297}$$
$$t \approx 8.82$$

The salvage value of the copier will be \$1000 approximately 8.8 years after the purchase.

d) From Theorem 10 we know:

$$T = \frac{\ln 2}{k}$$
$$T = \frac{\ln 2}{0.4} \approx 1.73$$

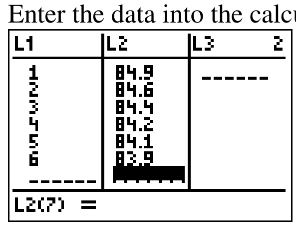
The copier will be worth half of its original value approximately 1.7 years after it was purchased.

e) $\boxed{tw}$ $V'(t) = 34{,}001.79(-0.4)e^{-0.4t}$

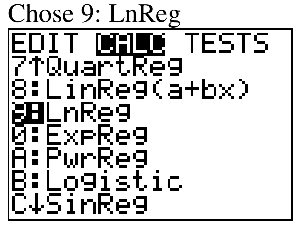
$= -13{,}600.716e^{-0.4t}$

The value of the copier is decreasing at a rate of $13{,}600.716e^{-0.4t}$ dollars per year t years after is was purchased.

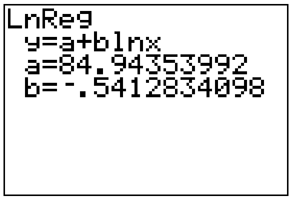
26. a) Enter the data into the calculator.



Chose 9: LnReg



We have:



The calculator determined the logarithmic function to be:

$y = 84.94353992 - 0.5412834098 \ln x$

b) After 8 months,

$$\begin{aligned} y &= 84.94353992 - 0.5412834098 \ln 8 \\ &\approx 83.81 \end{aligned}$$

The average test score will be 83.8%

After 10 months,

$$\begin{aligned} y &= 84.94353992 - 0.5412834098 \ln 10 \\ &\approx 83.69 \end{aligned}$$

The average test score will be 83.7%.

After 24 months,

$$\begin{aligned} y &= 84.94353992 - 0.5412834098 \ln 24 \\ &\approx 83.22 \end{aligned}$$

The average test score will be 83.2%

After 30 months,

$$\begin{aligned} y &= 84.94353992 - 0.5412834098 \ln 36 \\ &\approx 83.00 \end{aligned}$$

The average test score will be 83.0%

c) Set $y = 82$ and solve for x.

$$\begin{aligned} 84.94353992 - 0.5412834098 \ln x &= 82 \\ -0.5412834098 \ln x &= -2.94353992 \\ \ln x &= 5.438075261 \\ x &= e^{5.438075261} \\ x &\approx 229.999 \end{aligned}$$

After approximately 230 months the average test score will fall below 82%.

d)
$$\begin{aligned} \frac{dy}{dx} &= -0.5412834098\left(\frac{1}{x}\right) \\ &= -\frac{0.5412834098}{x} \end{aligned}$$

The average test scores are changing at a rate of $-\dfrac{0.5412834098}{x}$ percent per month, x months after the students took the exam.

27. a) Use the exponential-decay model $B(t) = B_0 e^{-kt}$ Let t be years since 1985, this gives us the initial consumption of beef $B_0 = 80$. Using the fact that in 1996, $t = 1996 - 1985 = 11$, and the beef consumption was 67 lbs, we can substitute into the exponential-decay function to find the decay rate k.

$$\begin{aligned} 67 &= 80e^{-k(11)} \\ \frac{67}{80} &= e^{-11k} \\ \ln\left(\frac{67}{80}\right) &= \ln e^{-11k} \\ \ln\left(\frac{67}{80}\right) &= -11k \\ \frac{\ln\left(\frac{67}{80}\right)}{-11} &= k \\ 0.016 &\approx k \end{aligned}$$

The decay rate is 0.016, or 1.6% per year. Substituting this into the exponential decay function we get:

$B(t) = 80e^{-0.016t}$.

b) In 2000, $t = 2000 - 1985 = 15$.

$B(15) = 80e^{-0.016(15)} \approx 62.9$

In the year 2000, annual consumption of beef will be approximately 62.9 pounds per person.

c) Set $B(t) = 20$ and solve for t.

$$\begin{aligned} 80e^{-0.016t} &= 20 \\ e^{-0.016t} &= \frac{20}{80} \\ \ln e^{-0.016t} &= \ln\left(\tfrac{1}{4}\right) \\ -0.016t &= \ln\left(\tfrac{1}{4}\right) \\ t &= \frac{\ln\left(\frac{1}{4}\right)}{-0.016} \\ t &\approx 86.6 \end{aligned}$$

Theoretically, consumption of beef will be 20lbs per person about 86.6 years after 1985, or in the year 2072.

28. a) $P(t) = P_0 e^{-kt}$

$$\begin{aligned} 142.9 &= 150e^{-k(11)} \quad (t = 2006 - 1995 = 11) \\ 0.953 &= e^{-11k} \\ \ln(0.953) &= -11k \\ 0.0044 &\approx k \end{aligned}$$

We have $P(t) = 150e^{-0.0044t}$, where $P(t)$ is in millions of people and t is the number of years since 1995.

b) In 2012, $t = 2012 - 1995 = 17$.

$P(17) = 150e^{-0.0044(17)} \approx 139.2$

Russia's population will be approximately 139.2 million people in 2012.

c) $150e^{-0.0044t} = 100$

$$e^{-0.0044t} = \frac{100}{150}$$

$$\ln e^{-0.0044t} = \ln\left(\tfrac{2}{3}\right)$$

$$-0.0044t = \ln\left(\tfrac{2}{3}\right)$$

$$t = \frac{\ln\left(\tfrac{2}{3}\right)}{-0.0044}$$

$$t \approx 92.15$$

Russia's population will reach 100 million people approximately 92 years after 1995, or in the year 2087.

29. a) Use the exponential decay model $P(t) = P_0 e^{-kt}$. If we let $t = 0$ correspond to 1995, then $P_0 = 51.9$ million people. In 2006, the population of Ukraine was 46.7 million, so $P(11) = 46.7$. Substituting this information into the model we get:

$$46.7 = 51.9e^{-k(11)}$$

$$\frac{46.7}{51.9} = e^{-11k}$$

$$\ln\left(\frac{46.7}{51.9}\right) = \ln e^{-11k}$$

$$\ln\left(\frac{46.7}{51.9}\right) = -11k$$

$$\frac{\ln\left(\frac{46.7}{51.9}\right)}{-11} = k$$

$$0.0096 \approx k$$

Thus, $P(t) = 51.9e^{-0.0096t}$, where $P(t)$ is in millions of people and t is the number of years since 1995.

b) In 2015, $t = 2015 - 1995 = 20$.

$P(20) = 51.9e^{-0.0096(20)} \approx 42.8$

The population of the Ukraine will be approximately 42.8 million in 2015.

c) Note that 1 = 0.000001 million.
Set $P(t) = 0.000001$ and solve for t.

$$51.9e^{-0.0096t} = 0.000001$$

$$e^{-0.0096t} = \frac{0.000001}{51.9}$$

$$\ln e^{-0.0096t} = \ln\left(\frac{0.000001}{51.9}\right)$$

$$-0.0096t = \ln\left(\frac{0.000001}{51.9}\right)$$

$$t = \frac{\ln\left(\frac{0.000001}{51.9}\right)}{-0.0096}$$

$$t \approx 1851$$

According to the model, the population of the Ukraine will be 1 person 1851 years after 1995, or in the year 3846.

30. a) $T(t) = ae^{-kt} + C$

$$100 = ae^{-k(0)} + 40$$

$$100 = a + 40$$

$$60 = a$$

b) $T(t) = 60e^{-kt} + 40$

$$95 = 60e^{-k(5)} + 40$$

$$55 = 60e^{-5k}$$

$$\frac{55}{60} = e^{-5k}$$

$$\ln\left(\frac{55}{60}\right) = -5k$$

$$\frac{\ln\left(\frac{55}{60}\right)}{-5} = k$$

$$0.01740 \approx k$$

c) $T(t) = 60e^{-0.01740t} + 40$

$T(10) = 60e^{-0.01740(10)} + 40 \approx 90.4$

The water temperature will be 90 degrees in 10 minutes.

d) Solve $T(t)=41$ for t.

$$60e^{-0.01740t}+40=41$$
$$60e^{-0.01740t}=1$$
$$e^{-0.01740t}=\frac{1}{60}$$
$$-0.01740t=\ln\left(\frac{1}{60}\right)$$
$$t=\frac{\ln\left(\frac{1}{60}\right)}{-0.01740}$$
$$t\approx 235.3$$

It will take approximately 235 minutes for the water temperature to reach 41 degrees.

e) $\boxed{tw}$ $T'(t)=60(-0.01740)e^{-0.01740t}$

$$=-1.044e^{-0.01740t}$$

The temperature of the water is decreasing is $1.044e^{-0.01740t}$ degrees per minute, t minutes after the heating element fails.

31. a) According to Newton's Law of Cooling $T(t)=ae^{-kt}+C$. The room temperature is 75 degrees so $C=75$. At $t=0$, $T=102$ degrees. We substitute these values into Newton's law of Cooling.

$$102=ae^{-k(0)}+75$$
$$102=ae^{0}+75$$
$$102=a+75$$
$$27=a$$

b) From part 'a' we have $T(t)=27e^{-kt}+75$.

Using the fact that when $t=10$, $T=90$ we have:

$$90=27e^{-k(10)}+75$$
$$15=27e^{-10k}$$
$$\frac{15}{27}=e^{-10k}$$
$$\ln\left(\frac{15}{27}\right)=\ln\left(e^{-10k}\right)$$
$$\ln\left(\frac{15}{27}\right)=-10k$$
$$\frac{\ln\left(\frac{15}{27}\right)}{-10}=k$$
$$0.05878\approx k$$

c) From 'a' and 'b' parts we have

$$T(t)=27e^{-0.05878t}+75$$
$$T(20)=27e^{-0.05878(20)}+75$$
$$=27e^{-1.1756}+75$$
$$\approx 83.3$$

After 20 minutes the water temperature is approximately 83 degrees.

d) Solve $T(t)=80$ for t.

$$27e^{-0.05878t}+75=80$$
$$27e^{-0.05878t}=5$$
$$e^{-0.05878t}=\frac{5}{27}$$
$$\ln e^{-0.05878t}=\ln\left(\frac{5}{27}\right)$$
$$-0.05878t=\ln\left(\frac{5}{27}\right)$$
$$t=\frac{\ln\left(\frac{5}{27}\right)}{-0.05878}$$
$$t\approx 28.7$$

It will take approximately 29 minutes for the water temperature to cool to 80 degrees.

e) $\boxed{tw}$ $T'(t)=27(-0.05878)e^{-0.05878t}$

$$=-1.58706e^{-0.05878t}$$

At time t in minutes after the water has started to cool, the rate of change of the water temperature is $-1.58706e^{-0.05878t}$ degrees per minute.

32. Newton's Law of Cooling states $T(t)=ae^{-kt}+C$. Find the constant a first.

Assume 98.6 degrees at the time of death and $C=10$ the temperature of the meat freezer.

$$98.6=ae^{-k(0)}+10$$
$$98.6=a+10$$
$$88.6=a$$

Next, we use the two temperature readings to find k.

$$61.6=88.6e^{-kt}+10$$
$$57.2=88.6e^{-k(t+1)}+10$$

Subtracting 10 from each equation we get:

$$51.6=88.6e^{-kt}$$
$$47.2=88.6e^{-k(t+1)}$$

Therefore,

$$\frac{51.6}{47.2} = \frac{88.6e^{-kt}}{88.6e^{-k(t+1)}}$$

$$\frac{51.6}{47.2} = e^{k}$$

$$\ln\left(\frac{51.6}{47.2}\right) = k$$

$$0.09 \approx k$$

Now we can solve for *t*.

$$51.6 = 88.6e^{-0.09t}$$

$$\frac{51.6}{88.6} = e^{-0.09t}$$

$$\ln\left(\frac{51.6}{88.6}\right) = -0.09t$$

$$\frac{\ln\left(\frac{51.6}{88.6}\right)}{-0.09} = t$$

$$6 \approx t$$

Therefore, the body had been dead for 6 hours. Since the temperature was taken at 2 A.M, the time of death was 8 P.M.

33. Newton's law of Cooling states

$$T(t) = ae^{-kt} + C.$$

Assume the body had a normal temperature of 98.6 degrees at the time of death and the room remained a constant 60 degrees giving us the constant $C = 60$. We find the constant *a* first, using the fact that 98.6 degrees is normal body temperature.

$$98.6 = ae^{-k(0)} + 60$$

$$98.6 = a + 60$$

$$38.6 = a$$

So $T(t) = 38.6e^{-kt} + 60$

Next, we use the two temperature readings to find *k*. We want to find the number of hours since death, *t*. When the corner took the first temperature reading *t* hours since death, the body temperature was 85.9 degrees. One hour later at $t+1$ hours since death, the body temperature was 83.4 degrees. Using these two pieces of information we get two equations.

$$85.9 = 38.6e^{-kt} + 60$$

$$83.4 = 38.6e^{-k(t+1)} + 60$$

Subtracting 60 from each equation we get:

$$25.9 = 38.6e^{-kt}$$

$$23.4 = 38.6e^{-k(t+1)}$$

A quick way to solve this system of equations is to divide the first equation by the second equation. This gives us:

$$\frac{25.9}{23.4} = \frac{38.6e^{-kt}}{38.6e^{-k(t+1)}}$$

$$\frac{25.9}{23.4} = e^{-kt-(-kt-k)}$$

$$\frac{25.9}{23.4} = e^{k}$$

$$\ln\left(\frac{25.9}{23.4}\right) = \ln e^{k}$$

$$0.10 \approx k$$

Now we substitute 0.10 in for *k* into the equation $25.9 = 38.6e^{-kt}$ and solve for *t*.

$$25.9 = 38.6e^{-0.10t}$$

$$\frac{25.9}{38.6} = e^{-0.1t}$$

$$\ln\left(\frac{25.9}{38.6}\right) = \ln e^{-0.1t}$$

$$\ln\left(\frac{25.9}{38.6}\right) = -0.1t$$

$$\frac{\ln\left(\frac{25.9}{38.6}\right)}{-0.10} = t$$

$$4 \approx t$$

Therefore, the body had been dead for 4 hours. Since the temperature was taken at 11 P.M, the time of death was 7 P.M.

34. $W = 140e^{-0.009t}$

a) $W(25) = 140e^{-0.009(25)} \approx 111.79$

The prisoner weighs approximately 112 pounds after 25 days.

b) $W'(t) = 140(-0.009)e^{-0.009t}$

$= -1.26e^{-0.009t}$

$W'(25) = -1.26e^{-0.009(25)} \approx -1.006$

The prisoner is losing approximately 1 pound per day after 25 days.

35. $W = 170e^{-0.008t}$

a) We substitute 20 in for *t*.

$W(20) = 170e^{-0.008(20)}$

$= 170e^{-0.16}$

≈ 144.86

The monk weighs approximately 145 pounds after 20 days.

b) We take the derivative of the function to find the rate of change.
$$W'(t) = 170(-0.008)e^{-0.008t}$$
$$= -1.36e^{-0.008t}$$
Now, we substitute 20 in for *t*.
$$W'(20) = -1.36e^{-0.009(20)} \approx -1.16$$
The monk is losing approximately 1.2 pounds per day after 20 days.

36. $P(a) = 14.7e^{-0.00005a}$

a) $P(1000) = 14.7e^{-0.00005(1000)} \approx 14.0$

The pressure at an altitude of 1000 ft is approximately 14.0 pounds per square inch.

b) $P(20{,}000) = 14.7e^{-0.00005(20{,}000)} \approx 5.4$

The pressure at an altitude of 20,000 ft is approximately 5.4 pounds per square inch.

c) Set $P(a) = 14.7$
$$14.7 = 14.7e^{-0.00005a}$$
$$\frac{14.7}{14.7} = e^{-0.00005a}$$
$$1 = e^{-0.00005a}$$
$$\ln 1 = -0.00005a$$
$$0 = a$$
Note: It would have been easier to realize that the pressure at sea level is $P_0 = 14.7$, therefore the altitude that yields this pressure must be sea level or 0 ft.

d) $\boxed{tw}$ $P'(a) = -0.000735e^{-0.0005a}$. At an altitude of *a* feet, the pressure is changing at $-0.000735e^{-0.0005a}$ pounds per square inch per foot of altitude.

37. $P(t) = 50e^{-0.004t}$

a) We substitute 375 for *t*.
$$P(375) = 50e^{-0.004(375)}$$
$$= 50e^{-1.5}$$
$$\approx 11.2$$
After 375 days, approximately 11.2 watts of power will be available.

b) From theorem 10 we know:
$$T = \frac{\ln 2}{k}$$
$$T = \frac{\ln 2}{0.004} \approx 173$$
The half-life of the power supply is approximately 173 days.

c) Set $P(t) = 10$ and solve for *t*.
$$50e^{-0.004t} = 10$$
$$e^{-0.004t} = \frac{10}{50}$$
$$\ln e^{-0.004t} = \ln 0.2$$
$$-0.004t = \ln 0.2$$
$$t = \frac{\ln 0.2}{-0.004}$$
$$t \approx 402.36$$
The satellite can stay in operation for 402 days.

d) When $t = 0$
$$P(0) = 50e^{-0.004(0)}$$
$$= 50e^0$$
$$= 50$$
At the beginning, the satellite had 50 watts of power.

e) $\boxed{tw}$ $P'(t) = -0.2e^{-0.004t}$

On day *t*, the satellites power is decreasing at a rate of $0.2e^{-0.004t}$ watts per day.

38. (c)

39. (a)

40. (e)

41. (c)

42. (f)

43. (d)

44. (d)

45. (f)

46. (a)

47. (b)

48. Solve $D(x) = S(x)$
$$480e^{-0.003x} = 150e^{0.004x}$$
$$\frac{480}{150} = \frac{e^{0.004x}}{e^{-0.003x}}$$
$$3.2 = e^{0.007x}$$
$$\ln 3.2 = 0.007x$$
$$166.16 \approx x$$
The equilibrium price is \$166.16.

Therefore,
$D(166.16) = 480e^{-0.003(166.16)} \approx 291.57$
The equilibrium quantity is 292 units.
The equilibrium point is $(166.16, 292)$.

Note: Due to the context of the problem we can not use the true equilibrium point. We rounded the printers up to 292 units while keeping the equilibrium price the same, even though demand at this price is slightly less than 292 printers. It is not possible to make or sell a fraction of a printer, so the supplier would need to determine if 291 or 292 units would result in greater profits. In the interest of the student we decided to take the most logical course of action by rounding the equilibrium quantity to the proper integer quantity.

49. $\boxed{tw}$ $I = I_0 e^{-\mu x}$
Substituting $\mu = 0.01$ we get
$I = I_0 e^{-0.01x}$
Substituting $x = 100$ we get
$I = I_0 e^{-0.01(100)} = I_0(0.3679)$
At $x = 90$
$I = I_0 e^{-0.01(90)} = I_0(0.4066)$
Dropping the pollution from
$100 \,{}^{\text{mcg}}\!/\!_{\text{m}^3}$ to $90 \,{}^{\text{mcg}}\!/\!_{\text{m}^3}$
results in a change in intensity of
$(0.4066 - 0.3679)I_0 = 0.0387I_0$
3.8% of the original intensity.
At $x = 60$
$I = I_0 e^{-0.01(60)} = I_0(0.5488)$
At $x = 50$
$I = I_0 e^{-0.01(50)} = I_0(0.6065)$
Dropping the pollution from $60 \,{}^{\text{mcg}}\!/\!_{\text{m}^3}$ to $50 \,{}^{\text{mcg}}\!/\!_{\text{m}^3}$
results in a change in intensity of
$(0.6065 - 0.5488)I_0 = 0.0577I_0$
5.8% of the original intensity.
There will be a more significant change in intensity as from $60 \,{}^{\text{mcg}}\!/\!_{\text{m}^3}$ to $50 \,{}^{mcg}\!/\!_{m^3}$.
Alternatively, using the derivative,
$\frac{dI}{dx} = -0.01I_0 e^{-0.01x}$. The derivative negative for all values of $x > 0$, which means that the function is decreasing.
Moreover, the second derivative
$\frac{d^2I}{dx^2} = 0.0001I_0 e^{-0.01x}$ is positive for all values of $x > 0$, so the function is concave up (graph is getting flatter for the decreasing function) on its domain. Therefore, we will see a more significant change for lower values of *x*.

50. $\boxed{tw}$ $I = I_0 e^{-1.4x}$; Using the derivative,
$\frac{dI}{dx} = -1.4I_0 e^{-1.4x}$. The derivative is negative for all values of $x > 0$, which means that the function is decreasing. Moreover, the second derivative $\frac{d^2I}{dx^2} = 1.96I_0 e^{-1.4x}$ is positive for all values of $x > 0$, so the function is concave up (graph is getting flatter for the decreasing function) on its domain. Therefore, we will see a more significant change for lower values of *x*. There is a more significant change in intensity when the depth drops from 2 meters to 5 meters.

51. $\boxed{tw}$
a) – e) Answers will vary.
f) No; $\lim_{t\to\infty} T = \lim_{t\to\infty}\left(ae^{-kt} + C\right) = C$, for $k > 0$ implies that *T* will approach room temperature *C*, but never actually equal the temperature of the room.
g) $\frac{dT}{dt} = -ake^{-kt}$. This means that in minute *t* after the water starts cooling the water temperature is changing at $-ake^{-kt}$ degrees per minute.

52. $\boxed{tw}$ The interest rate decrease causes the interest earned on the investment to decrease. Thus, a larger deposit must be made in order to grow to the future value of the account. In other words the decrease in the interest rate increases in the present value of the amount.

53. $\frac{dP}{dt} = 0.009P$ implies that
$P(t) = P_0 e^{0.009t}$
The doubling time for the U.S. population is
$T = \frac{\ln 2}{0.009} \approx 77.02$
Therefore the U.S. population doubles approximately every 77 years. This means that the 2007 population will double in the year 2084.

Exercise Set 3.5

1. $y = 7^x$

$\frac{dy}{dx} = (\ln 7)7^x$ Theorem 12: $\frac{dy}{dx}a^x = (\ln a)a^x$

2. $y = 6^x$

$\frac{dy}{dx} = (\ln 6)6^x$

3. $f(x) = 8^x$

$f'(x) = (\ln 8)8^x$ Theorem 12

4. $f(x) = 15^x$

$f'(x) = (\ln 15)15^x$

5. $g(x) = x^3(5.4)^x$

Using the Product Rule, we get

$$g'(x) = x^3\left(\frac{dy}{dx}(5.4)^x\right) + \left(\frac{dy}{dx}x^3\right)(5.4)^x$$

$$= x^3\left(\underbrace{\ln(5.4)(5.4)^x}_{\text{Theorem 12}}\right) + 3x^2(5.4)^x$$

$$= x^2(5.4)^x(x\ln(5.4)+3) \quad \text{Factoring}$$

6. $g(x) = x^5(3.7)^x$

$$g'(x) = x^5\left(\ln(3.7)(3.7)^x\right) + 5x^4(3.7)^x$$

$$= x^4(3.7)^x(x\ln(3.7)+5)$$

7. $y = 7^{x^4+2}$

Using the Chain Rule and Theorem 12, we get

$$\frac{dy}{dx} = (\ln 7)7^{x^4+2}\left(\frac{d}{dx}(x^4+2)\right)$$

$$= (\ln 7)7^{x^4+2}(4x^3)$$

$$= 4x^3(\ln 7)7^{x^4+2}$$

8. $y = 4^{x^2+5}$

$$\frac{dy}{dx} = (\ln 4)4^{x^2+5}(2x)$$

$$= 2x(\ln 4)4^{x^2+5}$$

9. $y = e^{8x}$

$\frac{dy}{dx} = 8e^{8x}$ Theorem 2

10. $y = e^{x^2}$

$\frac{dy}{dx} = 2xe^{x^2}$

11. $f(x) = 3^{x^4+1}$

Using the Chain Rule and Theorem 12, we get

$$\frac{dy}{dx} = (\ln 3)3^{x^4+1}\left(\frac{d}{dx}(x^4+1)\right)$$

$$= (\ln 3)3^{x^4+1}(4x^3)$$

$$= 4x^3(\ln 3)3^{x^4+1}$$

12. $f(x) = 12^{7x-4}$

$$f'(x) = (\ln 12)12^{7x-4}(7)$$

$$= 7(\ln 12)12^{7x-4}$$

13. $y = \log_4 x$

$$\frac{dy}{dx} = \frac{1}{\ln 4}\cdot\frac{1}{x} \quad \text{Theorem 14}$$

$$= \frac{1}{x\ln 4}$$

14. $y = \log_8 x$

$$\frac{dy}{dx} = \frac{1}{\ln 8}\cdot\frac{1}{x}$$

$$= \frac{1}{x\ln 8}$$

15. $y = \log_{17} x$

$$\frac{dy}{dx} = \frac{1}{\ln 17}\cdot\frac{1}{x} \quad \text{Theorem 14.}$$

$$= \frac{1}{x\ln 17}$$

16. $y = \log_{23} x$

$$\frac{dy}{dx} = \frac{1}{\ln 23}\cdot\frac{1}{x}$$

$$= \frac{1}{x\ln 23}$$

17. $g(x)=\log_6(5x+1)$

Using the Chain Rule and Theorem 14, we get

$$g'(x)=\frac{1}{\ln 6}\cdot\frac{1}{5x+1}\cdot\frac{d}{dx}(5x+1)$$
$$=\frac{1}{\ln 6}\cdot\frac{1}{5x+1}\cdot 5$$
$$=\frac{5}{(5x+1)\ln 6}$$

18. $g(x)=\log_{32}(9x-2)$

$$g'(x)=\frac{1}{\ln 32}\cdot\frac{1}{9x-2}\cdot\frac{d}{dx}(9x-2)$$
$$=\frac{9}{(9x-2)\ln 32}$$

19. $F(x)=\log(6x-7)$ $\quad(\log x=\log_{10}x)$

Using the Chain Rule and Theorem 14 we get

$$F'(x)=\frac{1}{\ln 10}\cdot\frac{1}{6x-7}\cdot\frac{d}{dx}(6x-7)$$
$$=\frac{1}{\ln 10}\cdot\frac{1}{6x-7}\cdot 6$$
$$=\frac{6}{(6x-7)\ln 10}$$

20. $G(x)=\log(5x+4)$

$$G'(x)=\frac{1}{\ln 10}\cdot\frac{1}{5x+4}\cdot\frac{d}{dx}(5x+4)$$
$$=\frac{5}{(5x+4)\ln 10}$$

21. $y=\log_8(x^3+x)$

Using the Chain Rule and Theorem 14, we get

$$\frac{dy}{dx}=\frac{1}{\ln 8}\cdot\frac{1}{x^3+x}\cdot\frac{d}{dx}(x^3+x)$$
$$=\frac{1}{\ln 8}\cdot\frac{1}{x^3+x}\cdot(3x^2+1)$$
$$=\frac{3x^2+1}{(x^3+x)\ln 8}$$

22. $y=\log_9(x^4-x)$

$$\frac{dy}{dx}=\frac{1}{\ln 9}\cdot\frac{1}{x^4-x}\cdot\frac{d}{dx}(x^4-x)$$
$$=\frac{4x^3-1}{(x^4-x)\ln 9}$$

23. $f(x)=4\log_7(\sqrt{x}-2)$

$$f'(x)=4\frac{d}{dx}\log_7(\sqrt{x}-2)$$

Using the Chain Rule and Theorem 14, we get:

$$f'(x)=4\cdot\frac{1}{\ln 7}\cdot\frac{1}{\sqrt{x}-2}\cdot\frac{d}{dx}(\sqrt{x}-2)$$
$$=\frac{4}{(\sqrt{x}-2)\ln 7}\cdot\frac{d}{dx}\left(x^{1/2}-2\right)\quad\left[\sqrt[n]{x}=x^{1/n}\right]$$
$$=\frac{4}{(\sqrt{x}-2)\ln 7}\left(\frac{1}{2}x^{-1/2}\right)\quad\text{Power Rule}$$
$$=\frac{4}{(\sqrt{x}-2)\ln 7}\cdot\frac{1}{2x^{1/2}}\quad\text{Properties of Exponents}$$
$$=\frac{4}{2\sqrt{x}(\sqrt{x}-2)\ln 7}$$
$$=\frac{2}{(x-2\sqrt{x})\ln 7}$$

24. $g(x)=-\log_6(\sqrt[3]{x}+5)$

$$g'(x)=-\frac{1}{\ln 6}\cdot\frac{1}{\sqrt[3]{x}+5}\cdot\frac{d}{dx}\left(x^{1/3}+5\right)$$
$$=-\frac{1}{\ln 6}\cdot\frac{1}{\sqrt[3]{x}+5}\cdot\left(\frac{1}{3}x^{-2/3}\right)$$
$$=-\frac{1}{\ln 6}\cdot\frac{1}{\sqrt[3]{x}+5}\cdot\left(\frac{1}{3x^{2/3}}\right)$$
$$=-\frac{1}{3\sqrt[3]{x^2}(\sqrt[3]{x}+5)\ln 6}$$

25. $y=6^x\cdot\log_7 x$

Since y is of the form $f(x)\cdot g(x)$, we apply the Product Rule.

$$\frac{dy}{dx}=6^x\cdot\left(\frac{1}{\ln 7}\cdot\frac{1}{x}\right)+(\ln 6)6^x\cdot\log_7 x$$

Next, we use the commutative property of multiplication to rearrange the derivative:

$$\frac{dy}{dx}=\frac{6x}{x\ln 7}+6^x\ln 6\cdot\log_7 x$$

26. $y=5^x\cdot\log_2 x$

$$\frac{dy}{dx}=5^x\left(\frac{1}{\ln 2}\cdot\frac{1}{x}\right)+(\ln 5)5^x\log_2 x$$
$$=\frac{5^x}{x\ln 2}+5^x\ln 5\cdot\log_2 x$$

27. $G(x)=(\log_{12} x)^5$

Using the Extended Power Rule, we have:

$$G'(x)=5\cdot(\log_{12} x)^4\cdot\frac{d}{dx}(\log_{12} x)$$
$$=5\cdot(\log_{12} x)^4\cdot\left(\frac{1}{\ln 12}\cdot\frac{1}{x}\right) \quad \text{Theorem 14}$$
$$=5(\log_{12} x)^4\left(\frac{1}{x\ln 12}\right)$$

28. $F(x)=(\log_9 x)^7$

$$F'(x)=7\cdot(\log_9 x)^6\cdot\left(\frac{1}{\ln 9}\cdot\frac{1}{x}\right)$$
$$=7(\log_9 x)^6\left(\frac{1}{x\ln 9}\right)$$

29. $g(x)=\dfrac{7^x}{4x+1}$

Since $g(x)$ is in the form $g(x)=\dfrac{f(x)}{h(x)}$ we apply the Quotient Rule.

$$g'(x)=\frac{(4x+1)\left(\frac{d}{dx}7^x\right)-7^x\left(\frac{d}{dx}(4x+1)\right)}{(4x+1)^2}$$
$$=\frac{(4x+1)(\ln 7)7^x-7^x(4)}{(4x+1)^2}$$
$$=\frac{7^x\left((4x+1)\ln 7-4\right)}{(4x+1)^2} \quad \text{Factor out } 7^x$$
$$=\frac{7^x(4x\ln 7+\ln 7-4)}{(4x+1)^2} \quad \text{Distribute } \ln 7$$

30. $f(x)=\dfrac{6^x}{5x-1}$

$$f'(x)=\frac{(5x-1)(\ln 6)6^x-6^x(5)}{(5x-1)^2} \quad \text{Quotient Rule}$$
$$=\frac{6^x(5x\ln 6-\ln 6-5)}{(5x-1)^2}$$

31. $y=5^{2x^3-1}\cdot\log(6x+5)$

Using the Product Rule we have:

$$\frac{dy}{dx}=5^{2x^3-1}\left(\frac{d}{dx}\log(6x+5)\right)$$
$$+\left(\frac{d}{dx}5^{2x^3-1}\right)\log(6x+5)$$

Next, using the Chain Rule, we have:

$$\frac{dy}{dx}=5^{2x^3-1}\left(\frac{1}{\ln 10}\cdot\frac{1}{6x+5}\cdot 6\right)$$
$$+(\ln 5)5^{2x^3-1}(6x^2)\cdot\log(6x+5)$$

Next, using properties of multiplication, we have:

$$\frac{dy}{dx}$$
$$=\frac{6\cdot 5^{2x^3-1}}{(6x+5)\ln 10}+(\ln 5)5^{2x^3-1}\cdot 6x^2\log(6x+5)$$

32. $y=\log(7x+3)\cdot 4^{2x^4+8}$

$$\frac{dy}{dx}=\log(7x+3)\cdot(\ln 4)4^{2x^4+8}(8x^3)$$
$$+4^{2x^4+8}\cdot\left(\frac{1}{\ln 10}\cdot\frac{1}{7x+3}\cdot 7\right)$$
$$\frac{dy}{dx}=\frac{7\cdot 4^{2x^4+8}}{(7x+3)\ln 10}+$$
$$(\ln 4)4^{2x^4+8}\cdot 8x^3\log(7x+3)$$

33. $F(x)=7^x(\log_4 x)^9$

Using the Product Rule, the Extended Power Rule and Theorem 12, we have:

$$F'(x)$$
$$=7^x\cdot\frac{d}{dx}\left[(\log_4 x)^9\right]+(\log_4 x)^9\cdot\frac{d}{dx}\left[7^x\right]$$
$$=7^x\cdot\left[9(\log_4 x)^8\cdot\left(\frac{1}{x\cdot\ln 4}\right)\right]+$$
$$(\log_4 x)^9\cdot\left[(\ln 7)7^x\right]$$
$$=\frac{7^x\cdot 9\cdot(\log_4 x)^8}{x\cdot\ln 4}+(\ln 7)7^x\cdot(\log_4 x)^9$$

34. $G(x)=\log_9 x\cdot(4^x)^6=\log_9 x\cdot 4^{6x}$

$$G'(x)=\log_9 x(\ln 4)4^{6x}\cdot 6+\left(\frac{1}{\ln 9}\cdot\frac{1}{x}\right)4^{6x}$$
$$=6(\ln 4)4^{6x}\log_9 x+\frac{4^{6x}}{x\ln 9}$$

35. $f(x)=(3x^5+x)^5\log_3 x$

First, using the Product Rule, we have:

$$f'(x)=(3x^5+x)^5\frac{d}{dx}\log_3 x+\left(\frac{d}{dx}(3x^5+x)^5\right)\log_3 x$$

Next, we will apply the Chain Rule:

$$f'(x)=(3x^5+x)^5\left(\frac{1}{\ln 3}\cdot\frac{1}{x}\right)+5(3x^5+x)^4\frac{d}{dx}(3x^5+x)\cdot\log_3 x$$

Which gives us:

$$f'(x)=(3x^5+x)^5\left(\frac{1}{x\ln 3}\right)+5(3x^5+x)^4(15x^4+1)\log_3 x$$

36. $g(x)=\sqrt{x^3-x}\cdot(\log_5 x)$

$$g(x)=(x^3-x)^{1/2}(\log_5 x)$$

$$g'(x)=(x^3-x)^{1/2}\left(\frac{1}{\ln 5}\cdot\frac{1}{x}\right)+\frac{1}{2}(x^3-x)^{-1/2}(3x^2-1)\log_5 x$$

$$g'(x)=\frac{(x^3-x)^{1/2}}{x\ln 5}+\frac{(3x^2-1)\log_5 x}{2(x^3-x)^{1/2}}$$

$$=\frac{\sqrt{x^3-x}}{x\ln 5}+\frac{(3x^2-1)\log_5 x}{2\sqrt{x^3-x}}$$

37. a) $V(t)=5200(0.80)^t$

$$V'(t)=5200\frac{d}{dt}(0.80)^t$$

$$=5200(\ln 0.80)(0.80)^t \quad \text{Theorem 12}$$

b) $\boxed{tw}$ The value of the office machine t years after purchase is changing at a rate of $5200(\ln 0.80)(0.80)^t$ dollars per year.

38. a) $N(t)=250{,}000(0.45)^t$

$$N'(t)=250{,}000(\ln 0.45)(0.45)^t$$

b) $\boxed{tw}$ The number of cans still in use, t years after the initial distribution of 250,000 cans, is changing at a rate of $250{,}000(\ln 0.45)(0.45)^t$ cans per year.

39. a) $L(t)=1547(1.083)^t$

Since t is the number of years since 1980, the year 2012 corresponds to $t=2012-1980=32$ years.

$$L(32)=1547(1.083)^{32}$$
$$=1547(12.82657226)$$
$$=19{,}842.7072800$$
$$\approx 19{,}842.71$$

In the year 2012. total financial liability of U.S. households will be approximately \$19,842.71 billion.

b) First, we find the derivative using Theorem 12:

$$L'(t)=1547(\ln 1.083)(1.083)^t$$

Next, we evaluate the derivative at $t=25$

$$L'(25)=1547(\ln 1.083)(1.083)^{25}$$
$$=1547(\ln 1.083)(7.34025953)$$
$$=905.42098032$$
$$\approx 905.42$$

c) $\boxed{tw}$ 25 years after 1980, or in the year 2005, the total financial liability of U.S. Households was growing at a rate of 905.42 billion dollars per year.

40. a) $N(t)=8400\ln t-10{,}500$

In the year 2014 $t=2014-1970=44$.

$$N(44)=8400\ln(44)-10{,}500$$
$$\approx 21{,}287.19$$

In the year 2014, there will be 21,287.19 thousand, or 21.3 million, non-farm proprietorships in the United States.

b) $N'(t)=8400\cdot\frac{1}{t}$

$$N'(45)=8400\cdot\frac{1}{45}\approx 186.667$$

c) $\boxed{tw}$ $N'(45)$ represents the number of new non-farm proprietorships in thousands per year in the United States 45 years after 1970 or in 2015. Specifically in the model, approximately 187,000 new non-farm proprietorships will be formed in the year 2015.

41. a) $P(t)=(0.98)^t$

$P(10)=(0.98)^{10}=0.81707281$

Ten years after the neighboring farm begins to use the GMO seed, the GMO-free farm will be 81.7 percent GMO-free.

b) First, we find the derivative

$P'(t)=\ln(0.98)(0.98)^t$.

Next, we evaluate the derivative at $t=15$:

$$\begin{aligned}P'(15)&=(\ln 0.98)(0.98)^{15}\\&=(\ln 0.98)\cdot 0.73856910\\&=-0.01492110\\&\approx -0.0149\end{aligned}$$

c) $\boxed{tw}$ $P'(15)$ represents the percent decline per year of the portion of the crop that remains GMO free crops 15 years after a neighboring farm begins to use GMO seed. Specifically, in this problem 15 years after the neighboring farm begins to use the GMO seed, the GMO-free portion of the crop is being reduced by approximately 1.49 percent per year.

42. $R=\log\dfrac{I}{I_0}$

$R=\log\dfrac{10^5\cdot I_0}{I_0}=\log 10^5=5.0$

43. $R=\log\dfrac{I}{I_0}$

We substitute $10^{6.9}\cdot I_0$ for I

$$\begin{aligned}R&=\log\frac{10^{6.9}\cdot I_0}{I_0}\\&=\log 10^{6.9}\\&=6.9 \qquad (\text{P5})\end{aligned}$$

44. $I=I_0\cdot 10^R=10^R\cdot I_0$

a) $I=10^7\cdot I_0$ Substituting 7 for R.

b) $I=10^8\cdot I_0$ Substituting 8 for R.

c) Comparing parts (a) and (b) we have:

$10^8\cdot I_0=10\cdot 10^7\cdot I_0$

The intensity of (b) is 10 times that of (a).

d) $I=I_0 10^R$

$$\begin{aligned}\frac{dI}{dR}&=I_0(\ln 10)10^R \qquad \text{Theorem 12}\\&=(I_0\cdot\ln 10)10^R\end{aligned}$$

e) $\boxed{tw}$ For a given magnitude R on the Richter scale, the intensity is changing at a rate of $(I_0\cdot\ln 10)10^R$.

45. $I=I_0 10^{0.1L}$

a) Substituting 100 for L we have:

$$\begin{aligned}I&=I_0 10^{0.1(100)}\\&=I_0 10^{10}\\&=10^{10}\cdot I_0\end{aligned}$$

The intensity of a power mower is $10^{10}\cdot I_0$.

b) Substituting 10 for L we have:

$$\begin{aligned}I&=I_0 10^{0.1(10)}\\&=I_0 10^1\\&=10\cdot I_0\end{aligned}$$

The intensity of a just audible sound is $10\cdot I_0$

c) Comparing the intensity in parts (a) and (b) we have $10^{10}\cdot I_0=10^9(10\cdot I_0)$. The intensity of (a) is 10^9 times more than the intensity of (b).

d) $I=I_0 10^{0.1L}$

$\dfrac{dI}{dL}=I_0\dfrac{d}{dL}10^{0.1L}$ $\qquad I_0$ is a constant.

Next, using Theorem 12 and the Chain Rule we have:

$$\begin{aligned}\frac{dI}{dL}&=I_0(\ln 10)10^{0.1L}\cdot\left(\frac{d}{dL}0.1L\right)\\&=I_0(\ln 10)10^{0.1L}(0.1)\\&=0.1\cdot\ln 10\cdot I_0 10^{0.1L}\end{aligned}$$

e) $\boxed{tw}$ For a given loudness L. The intensity is changing at a rate of $0.1\cdot\ln 10\cdot I_0 10^{0.1L}$ per decibel.

46. a) $R=\log\dfrac{I}{I_0}$

$$\begin{aligned}\frac{dR}{dI}&=\frac{d}{dI}(\log I-\log I_0) \qquad (\text{P2})\\&=\frac{1}{\ln 10}\cdot\frac{1}{I}\\&=\frac{1}{(\ln 10)I}\end{aligned}$$

b) $\boxed{tw}$ The magnitude is changing at a rate of $\dfrac{1}{(\ln 10)I}$.

47. a) $L = 10\log\frac{I}{I_0}$

First, we rearrange the equation using Property 2.

$$L = 10\left(\log I - \log I_0\right)$$
$$= 10\log I - 10\log I_0$$

Next, we take the derivative using the Difference Rule:

$$\frac{dL}{dI} = 10\frac{d}{dI}\log I - \frac{d}{dI}10\log I_0$$

Using Theorem 14 we have:

$$\frac{dL}{dI} = 10\frac{1}{\ln 10}\cdot\frac{1}{I} - 0 \qquad I_0 \text{ is a constant}$$
$$= \frac{10}{\ln 10}\cdot\frac{1}{I}$$

b) $\boxed{tw}$ The loudness is changing at a rate of $\frac{10}{\ln 10}\cdot\frac{1}{I}$ decibels per intensity level.

48. a) $y = m\log x + b$

$$\frac{dy}{dx} = m\cdot\frac{1}{\ln 10}\cdot\frac{1}{x} + 0$$
$$= \frac{m}{x\ln 10}$$

b) The response of the patient to the drug is changing at a rate of $\frac{m}{x\ln 10}$ response per dosage.

49. By Theorem 13: $\lim_{h\to 0}\frac{3^h - 1}{h} = \ln 3$

50. $f(x) = 3^{\left(2^x\right)}$

$$f'(x) = (\ln 3)3^{2^x}\cdot\frac{d}{dx}2^x$$
$$= (\ln 3)3^{2^x}\cdot(\ln 2)2^x$$
$$= (\ln 3)(\ln 2)2^x 3^{2^x}$$

51. $y = 2^{x^4}$

$$\frac{dy}{dx} = (\ln 2)2^{x^4}\frac{d}{dx}x^4$$
$$= (\ln 2)2^{x^4}\cdot 4x^3$$

52. $y = x^x,\ x > 0$

$$y = e^{x\ln x} \qquad \left[a^x = e^{x\ln a}\right]$$
$$\frac{dy}{dx} = e^{x\ln x}\left(\frac{d}{dx}x\ln x\right) \qquad \text{Chain Rule}$$
$$= e^{x\ln x}\left(x\left(\frac{1}{x}\right) + (1)\ln x\right) \qquad \text{Product Rule}$$
$$= e^{x\ln x}(1 + \ln x)$$
$$= (\ln x + 1)e^{x\ln x}$$
$$= (\ln x + 1)x^x \qquad \left[a^x = e^{x\ln a}\right]$$

53. $y = \log_3(\log x)$

Using the Chain Rule and Theorem 14, we have:

$$\frac{dy}{dx} = \frac{1}{\ln 3}\cdot\frac{1}{\log x}\cdot\frac{d}{dx}\log x$$
$$= \frac{1}{\ln 3}\cdot\frac{1}{\log x}\cdot\frac{1}{\ln 10}\cdot\frac{1}{x}$$
$$= \frac{1}{\ln 3\cdot\log x\cdot\ln 10\cdot x}$$

54. $f(x) = x^{e^x}$

$$f(x) = e^{e^x\ln x} \qquad \left[a^x = e^{x\ln a}\right]$$
$$f'(x) = e^{e^x\ln x}\left(\frac{d}{dx}e^x\ln x\right) \qquad \text{Chain Rule}$$
$$= e^{e^x\ln x}\left(e^x\left(\frac{1}{x}\right) + e^x\ln x\right) \qquad \text{Product Rule}$$
$$= \left(\frac{e^x}{x} + e^x\ln x\right)e^{e^x\ln x}$$

55. $y = a^{f(x)}$

$$y = e^{f(x)\ln a} \qquad \left[a^x = e^{x\ln a}\right]$$
$$\frac{dy}{dx} = e^{f(x)\ln a}\frac{d}{dx}f(x)\ln a \qquad \text{Chain Rule}$$
$$\frac{dy}{dx} = e^{f(x)\ln a}\cdot f'(x)\ln a$$
$$\frac{dy}{dx} = \ln a\cdot a^{f(x)}\cdot f'(x) \qquad \left[a^x = e^{x\ln a}\right]$$

56. $y = \log_a f(x),\ f(x) > 0$

$$\frac{dy}{dx} = \frac{1}{\ln a}\cdot\frac{1}{f(x)}\cdot f'(x)$$
$$= \frac{f'(x)}{\ln a\cdot f(x)}$$

57. $y=\left[f(x)\right]^{g(x)},\ f(x)>0$

$y=e^{g(x)\cdot\ln f(x)}$ $\qquad \left[a^x=e^{x\ln a}\right]$

$\frac{dy}{dx}=e^{g(x)\cdot\ln f(x)}\frac{d}{dx}g(x)\ln f(x)$ Chain Rule

Next, using the Product Rule, we have:

$$\frac{dy}{dx}=e^{g(x)\cdot\ln f(x)}\left(g(x)\cdot\frac{1}{f(x)}\cdot f'(x)+g'(x)\ln f(x)\right)$$

Simplifying we get:

$$\frac{dy}{dx}=e^{g(x)\ln f(x)}\left(\frac{g(x)f'(x)}{f(x)}+g'(x)\ln f(x)\right)$$
$$=\left[f(x)\right]^{g(x)}\left(\frac{g(x)f'(x)}{f(x)}+g'(x)\ln f(x)\right)$$

58. $\boxed{tw}$ Since a^x can be written as $e^{x\ln a}$, we can find the derivative of $f(x)=a^x$ using the rule for differentiating an exponential function, base *e*.

59. $\boxed{tw}$ Using the Change of Base formula, we can write $f(x)=\log_a x$ as $f(x)=\frac{\ln x}{\ln a}=\frac{1}{\ln a}\cdot\ln x$. Now we can find the derivative of $f(x)=\log_a x$ by using the rules for differentiating $f(x)=c\cdot\ln x$

Exercise Set 3.6

1. a) The demand function is $q=D(x)=400-x$.

The definition of the elasticity of demand is given by: $E(x)=-\frac{x\cdot D'(x)}{D(x)}$. In order to find the elasticity of demand, we need to find the derivative of the demand function first.

$\frac{dq}{dx}=D'(x)=\frac{d}{dx}(400-x)=-1$.

Next, we substitute -1 for $D'(x)$, and $400-x$ for $D(x)$ into the expression for elasticity.

$$E(x)=-\frac{x\cdot(-1)}{400-x}=\frac{x}{400-x}$$

b) Substituting $x=125$ into the expression found in part (a) we have:

$$E(125)=\frac{(125)}{400-(125)}=\frac{125}{275}=\frac{5}{11}$$

Since $E(125)=\frac{5}{11}$ is less than one, the demand is inelastic.

c) The values of x for which $E(x)=1$ will maximize total revenue. We solve:

$$E(x)=1$$
$$\frac{x}{400-x}=1$$
$$x=400-x$$
$$2x=400$$
$$x=200$$

A price of \$200 will maximize total revenue.

2. $q=D(x)=500-x;\ x=38$

a) $D'(x)=-1$

$$E(x)=-\frac{x\cdot D'(x)}{D(x)}=-\frac{x\cdot(-1)}{500-x}=\frac{x}{500-x}$$

b) $E(38)=\frac{38}{500-38}=\frac{19}{231}$

$E(38)<1$, so demand is inelastic.

c) Solve $E(x)=1$

$$\frac{x}{500-x}=1$$
$$x=500-x$$
$$2x=500$$
$$x=250$$

A price of \$250 will maximize total revenue.

3. $q=D(x)=200-4x;\ x=46$

a) $D'(x)=-4$

$$E(x)=-\frac{x\cdot D'(x)}{D(x)}=-\frac{x\cdot(-4)}{200-4x}=\frac{4x}{200-4x}$$
$$=\frac{x}{50-x}$$

b) Substituting $x=46$ into the expression found in part (a) we have:

$$E(46)=\frac{46}{50-46}=\frac{46}{4}=\frac{23}{2}=11.5$$

Since $E(46)>1$, demand is elastic.

c) We solve $E(x)=1$

$$\frac{x}{50-x}=1$$
$$x=50-x$$
$$2x=50$$
$$x=25$$

A price of \$25 will maximize total revenue.

4. $q=D(x)=500-2x;\ x=57$

a) $D'(x)=-2$

$$E(x)=-\frac{x\cdot D'(x)}{D(x)}-\frac{x\cdot(-2)}{500-2x}=\frac{2x}{500-2x}$$
$$=\frac{x}{250-x}$$

b) $E(57)=\dfrac{57}{250-57}=\dfrac{57}{193}$

$E(57)<1$, so demand is inelastic.

c) Solve $E(x)=1$

$$\frac{x}{250-x}=1$$
$$x=250-x$$
$$2x=250$$
$$x=125$$

A price of \$125 will maximize total revenue.

5. $q=D(x)=\dfrac{400}{x};\ x=50$

a) First, we rewrite the demand function.

$D(x)=\dfrac{400}{x}=400x^{-1}$.

Next, we take the derivative of the demand function, using the Power Rule.

$D'(x)=400(-1)x^{-2}=-400x^{-2}$

Making the appropriate substitutions into the elasticity function, we have

$$E(x)=-\frac{x\cdot D'(x)}{D(x)}=-\frac{x\cdot\left(-400x^{-2}\right)}{400x^{-1}}=$$
$$=\frac{400x^{-1}}{400x^{-1}}=1$$

Therefore, $E(x)=1$ for all values of x.

b) $E(50)=1$, so demand is unit elastic.

c) $E(x)=1$ for all values of x. Therefore, total revenue is maximized for all values of x. In other words, total revenue is the same regardless of the price.

6. $q=D(x)=\dfrac{3000}{x};\ x=60$

a) $D(x)=\dfrac{3000}{x}=3000x^{-1}$

$D'(x)=-3000x^{-2}$

Making the appropriate substitutions into the elasticity function, we have

$$E(x)=-\frac{x\cdot D'(x)}{D(x)}=-\frac{x\cdot\left(-3000x^{-2}\right)}{3000x^{-1}}=$$
$$=\frac{3000x^{-1}}{3000x^{-1}}=1$$

Therefore, $E(x)=1$ for all values of x.

b) $E(60)=1$, so demand is unit elastic.

c) $E(x)=1$ for all values of x. Therefore, total revenue is maximized for all values of x. In other words, total revenue is the same regardless of the price.

7. $q=D(x)=\sqrt{600-x};\ x=100$

a) First rewrite the demand function:

$D(x)=(600-x)^{1/2}$.

Next, we take the derivative of the demand function, using the Chain Rule:

$$D'(x)=\frac{1}{2}(600-x)^{-1/2}\cdot\frac{d}{dx}(600-x)$$
$$=\frac{-1}{2\sqrt{600-x}}$$

Making the appropriate substitutions into the elasticity function, we have

$$E(x)=-\frac{x\cdot D'(x)}{D(x)}=-\frac{x\cdot\left(\frac{-1}{2\sqrt{600-x}}\right)}{\sqrt{600-x}}=$$
$$=\frac{\frac{x}{2\sqrt{600-x}}}{\sqrt{600-x}}=\frac{x}{2(600-x)}$$
$$=\frac{x}{1200-2x}$$

b) Substituting $x=100$ into the expression found in part (a) we have:

$$E(100)=\frac{100}{1200-2(100)}=\frac{100}{1000}=\frac{1}{10}$$

Since $E(100)<1$, demand is inelastic.

c) Solve $E(x)=1$

$$\frac{x}{1200-2x}=1$$
$$x=1200-2x$$
$$3x=1200$$
$$x=400$$

A price of \$400 will maximize total revenue.

8. $q=D(x)=\sqrt{300-x};\ x=250$

a) $D(x)=(300-x)^{1/2}$.

$$D'(x)=\frac{1}{2}(300-x)^{-1/2}\left(\frac{d}{dx}300-x\right)$$
$$=\frac{-1}{2\sqrt{300-x}}$$
$$E(x)=-\frac{x\cdot D'(x)}{D(x)}=-\frac{x\cdot\left(\frac{-1}{2\sqrt{300-x}}\right)}{\sqrt{300-x}}=$$
$$=\frac{\frac{x}{2\sqrt{300-x}}}{\sqrt{300-x}}=\frac{x}{2(300-x)}$$
$$=\frac{x}{600-2x}$$

b) $E(250)=\dfrac{250}{600-2(250)}=\dfrac{5}{2}$

Since $E(250)>1$, demand is elastic.

c) Solve $E(x)=1$

$$\frac{x}{600-2x}=1$$
$$x=600-2x$$
$$3x=600$$
$$x=200$$

A price of \$200 will maximize total revenue.

9. $q=D(x)=100e^{-0.25x};\ x=10$

a) Using the Chain Rule we have:

$$D'(x)=100e^{-0.25x}(-0.25)$$
$$=-25e^{-0.25x}$$

Making the appropriate substitutions into the elasticity function, we have

$$E(x)=-\frac{x\cdot D'(x)}{D(x)}=-\frac{x\cdot\left(-25e^{-0.25x}\right)}{100e^{-0.25x}}$$
$$=\frac{25xe^{-0.25x}}{100e^{-0.25x}}=\frac{x}{4}$$

b) Substituting $x=10$ into the expression found in part (a) we have:

$$E(10)=\frac{10}{4}=\frac{5}{2}=2.5$$

Since $E(10)>1$, demand is elastic.

c) Solve $E(x)=1$

$$\frac{x}{4}=1$$
$$x=4$$

A price of \$4 will maximize total revenue.

10. $q=D(x)=200e^{-0.05x};\ x=80$

a) $D'(x)=200e^{-0.05x}(-0.05)=-10e^{-0.05x}$

$$E(x)=-\frac{x\cdot D'(x)}{D(x)}=-\frac{x\cdot\left(-10e^{-0.05x}\right)}{200e^{-0.05x}}$$
$$=\frac{10xe^{-0.05x}}{200e^{-0.05x}}=\frac{x}{20}$$

b) $E(80)=\dfrac{80}{20}=4$

Since $E(80)>1$, demand is elastic.

c) Solve $E(x)=1$

$$\frac{x}{20}=1$$
$$x=20$$

A price of \$20 will maximize total revenue.

11. $q=D(x)=\dfrac{100}{(x+3)^2};\ x=1$

a) First, we rewrite the demand function:

$$D(x)=100(x+3)^{-2}.$$

Next, we take the derivative of the demand function, using the Chain Rule:

$$D'(x)=100(-2)(x+3)^{-3}\left(\frac{d}{dx}(x+3)\right)$$
$$=-200(x+3)^{-3}$$
$$=-\frac{200}{(x+3)^3}$$

Making the appropriate substitutions into the elasticity function, we have:

$$E(x) = -\frac{x \cdot D'(x)}{D(x)} = -\frac{x \cdot \left(-\frac{200}{(x+3)^3}\right)}{\frac{100}{(x+3)^2}}$$

$$= x \cdot \left(\frac{200}{(x+3)^3}\right)\frac{(x+3)^2}{100}$$

$$= \frac{2x}{x+3}$$

b) Substituting $x = 1$ into the expression found in part (a) we have:

$$E(1) = \frac{2 \cdot (1)}{1+3} = \frac{2}{4} = \frac{1}{2}$$

Since $E(1) < 1$, demand is inelastic.

c) Solve $E(x) = 1$

$$\frac{2x}{x+3} = 1$$
$$2x = x + 3$$
$$x = 3$$

A price of \$3 will maximize total revenue.

12. $q = D(x) = \frac{500}{(2x+12)^2}$; $x = 8$

a) $D(x) = 500(2x+12)^{-2}$.

$$D'(x) = 500(-2)(2x+12)^{-3}(2)$$
$$= -2000(2x+12)^{-3}$$
$$= -\frac{2000}{(2x+12)^3}$$

Therefore,

$$E(x) = -\frac{x \cdot D'(x)}{D(x)} = -\frac{x \cdot \left(-\frac{2000}{(2x+12)^3}\right)}{\frac{500}{(2x+12)^2}} =$$

$$= x \cdot \left(\frac{2000}{(2x+12)^3}\right)\frac{(2x+12)^2}{500} = \frac{2x}{x+6}$$

b) $E(8) = \frac{2 \cdot (8)}{(8)+6} = \frac{16}{14} = \frac{8}{7}$

Since $E(8) > 1$, demand is elastic.

c) Solve $E(x) = 1$

$$\frac{2x}{x+6} = 1$$
$$2x = x + 6$$
$$x = 6$$

A price of \$6 will maximize total revenue.

13. $q = D(x) = 967 - 25x$

a) $D'(x) = -25$

$$E(x) = -\frac{x \cdot D'(x)}{D(x)} = -\frac{x \cdot (-25)}{967 - 25x}$$
$$= \frac{25x}{967 - 25x}$$

b) We set $E(x) = 1$ and solve for *x*.

$$\frac{25x}{967 - 25x} = 1$$
$$25x = 967 - 25x$$
$$50x = 967$$
$$x = \frac{967}{50}$$
$$x = 19.34$$

Demand is unitary elastic when price is 19.34 cents.

c) Demand is elastic when $E(x) > 1$.

Testing a value on each side of 19.34 cents, we have:

$$E(19) = \frac{25 \cdot 19}{967 - 25 \cdot 19} \approx 0.97 < 1$$
$$E(20) = \frac{25 \cdot 20}{967 - 25 \cdot 20} \approx 1.07 > 1$$

Therefore, the demand for cookies is elastic for prices greater than 19.34 cents.

d) Demand is inelastic when $E(x) < 1$.

Using the calculations from part (c), we see that the demand for cookies is inelastic for prices less than 19.34 cents.

e) Total revenue is maximized when $E(x) = 1$.

In part (b) we showed that $E(x) = 1$ when price was 19.34 cents. Therefore, revenue will be maximized when price is 19.34 cents.

f) We have shown that the demand for cookies is elastic when the price of cookies is 20 cents. Therefore a small increase in price will cause total revenue to decrease.

14. $q = D(x) = 63{,}000 + 50x - 25x^2;\ 0 \le x \le 50$

a) $D'(x) = 50 - 50x$

$$E(x) = -\frac{x \cdot D'(x)}{D(x)} = -\frac{x \cdot (50 - 50x)}{63{,}000 + 50x - 25x^2}$$
$$= \frac{50x^2 - 50x}{63{,}000 + 50x - 25x^2}$$

b) $$E(10) = \frac{50(10)^2 - 50(10)}{63{,}000 + 50(10) - 25(10)^2}$$
$$= \frac{9}{122}$$
$$\approx 0.7377$$

Since $E(10) < 1$, the demand for oil is inelastic at \$10 a barrel.

c) $$E(20) = \frac{50(20)^2 - 50(20)}{63{,}000 + 50(20) - 25(20)^2}$$
$$= \frac{19}{54}$$
$$\approx 0.35185$$

Since $E(20) < 1$, the demand for oil is inelastic at \$20 a barrel.

d) $$E(30) = \frac{50(30)^2 - 50(30)}{63{,}000 + 50(30) - 25(30)^2}$$
$$= \frac{29}{28}$$
$$\approx 1.03571$$

Since $E(30) > 1$, the demand for oil is elastic at \$30 a barrel.

e) Revenue will be maximized when $E(x) = 1$. Therefore, we solve:

$$\frac{50x^2 - 50x}{63{,}000 + 50x - 25x^2} = 1$$
$$50x^2 - 50x = 63{,}000 + 50x - 25x^2$$
$$75x^2 - 100x - 63{,}000 = 0$$

Using the quadratic formula, we find that the only solution in the interval $0 \le x \le 50$ is $x \approx 29.66$. Thus, oil revenues will be maximized when price is \$29.66 a barrel.

f) $$D(29.66) = 63{,}000 + 50(29.66) - 25(29.66)^2$$
$$\approx 42{,}490.11$$

The demand for oil is 42,490 million barrels per day at a price of \$29.66 a barrel.

g) We determined in part (c) that the demand for oil is elastic at \$30 a barrel. Therefore an increase in price will result in a decrease in total revenue.

15. $q = D(x) = \sqrt{200 - x^3}$

a) First, we rewrite the demand function to make it easier to find the derivative.

$$D(x) = \left(200 - x^3\right)^{1/2}.$$

Next, using the Chain Rule, we have:

$$D'(x) = \frac{1}{2}\left(200 - x^3\right)^{-1/2}\left(-3x^2\right)$$
$$= \frac{-3x^2}{2\sqrt{200 - x^3}}$$

Now, substituting in the elasticity function we get:

$$E(x) = -\frac{x \cdot D'(x)}{D(x)} = -\frac{x \cdot \left(\frac{-3x^2}{2\sqrt{200 - x^3}}\right)}{\sqrt{200 - x^3}}$$
$$= \frac{3x^3}{2\left(\sqrt{200 - x^3}\right)^2}$$
$$= \frac{3x^3}{2\left(200 - x^3\right)}$$
$$= \frac{3x^3}{400 - 2x^3}$$

b) $$E(3) = \frac{3(3)^3}{400 - 2(3)^3}$$
$$= \frac{81}{346}$$
$$\approx 0.2341$$

Since $E(3) < 1$, the demand for computer games is inelastic when price is \$3.

c) From part (b) we know that the demand for computer games is inelastic at a price of \$3. Therefore an increase in the price of computer games will lead to an increase in the total revenue.

16. $q = D(x) = \dfrac{2x+300}{10x+11}$

a) Using the Quotient Rule, we have:

$$D'(x) = \frac{(10x+11)\cdot 2 - (2x+300)\cdot 10}{(10x+11)^2}$$
$$= \frac{20x+22-(20x+3000)}{(10x+11)^2}$$
$$= \frac{-2978}{(10x+11)^2}$$

Therefore,

$$E(x) = -\frac{x\cdot D'(x)}{D(x)} = -\frac{x\cdot\dfrac{-2978}{(10x+11)^2}}{\dfrac{2x+300}{10x+11}}$$
$$= \frac{2978x}{(10x+11)^2}\cdot\frac{10x+11}{2x+300}$$
$$= \frac{1489x}{(10x+11)(x+150)}$$

b) $E(3) = \dfrac{1489(3)}{(10(3)+11)((3)+150)}$

$$= \frac{1489}{2091}$$
$$\approx 0.7121$$

Since $E(3) < 1$, the demand for tomato plants is inelastic when the price is \$3 per plant.

c) We determined in part (b) that the demand for tomato plants was inelastic at \$3. Therefore, an increase in the price of tomato plants will lead to an increase in revenue.

17. $q = D(x) = \dfrac{k}{x^n}$

a) First, we rewrite the demand function to make it easier to find the derivative.

$D(x) = k\cdot x^{-n}$

Using the Power Rule, we have:

$D'(x) = k\cdot(-n)x^{-n-1} = -nkx^{-n-1}$

Substituting into the elasticity function we have:

$$E(x) = -\frac{x\cdot D'(x)}{D(x)} = -\frac{x\cdot(-nkx^{-n-1})}{kx^{-n}}$$
$$= \frac{nk(x\cdot x^{-n-1})}{kx^{-n}}$$
$$= \frac{nk(x^{-n})}{kx^{-n}}$$
$$= n$$

b) No, the elasticity of demand is constant for all prices. $E(x) = n$.

c) Total revenue is maximized when $E(x) = 1$. Since $E(x) = n$, total revenue will be maximized when $n = 1$.

18. a) $D(x) = Ae^{-kx}$

$D'(x) = Ae^{-kx}\cdot(-k) = -Ake^{-kx}$

Therefore,

$$E(x) = -\frac{x\cdot D'(x)}{D(x)} = -\frac{x\cdot(-Ake^{-kx})}{Ae^{-kx}}$$
$$= kx$$

b) Yes, $E(x) = kx$ is dependent upon the price x.

c) Solve $E(x) = 1$.

$$kx = 1$$
$$x = \frac{1}{k}$$

Therefore, revenue will be maximized when price is equal to $\dfrac{1}{k}$ dollars per unit.

19. $L(x) = \ln D(x)$

$$L'(x) = \frac{1}{D(x)}\cdot D'(x) = \frac{D'(x)}{D(x)}$$

The formula for elasticity of demand is:

$$E(x) = -\frac{x\cdot D'(x)}{D(x)} = -x\left(\frac{D'(x)}{D(x)}\right)$$

Substituting $L'(x)$ for $\dfrac{D'(x)}{D(x)}$ we have:

$$E(x) = -x\cdot L'(x)$$

20. $\boxed{tw}$ Answers will vary. The elasticity of demand is a measure of the responsiveness of quantity demanded to changes in price. This measure allows economists to determine how sensitive quantity demand is to price changes and help predict the effect of price changes on total revenue.

21. $\boxed{tw}$ Answers will vary. In general, the greater the availability of substitutes and the better the substitutes are will cause goods to have a higher elasticity of demand. For example, the demand for tea is relatively elastic because of the wide range of substitutes available. Substitutes like coffee, soda, or water. However, a diabetic's demand for insulin is very inelastic because there are no close substitutes for insulin.