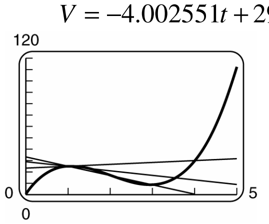
$$V - V_1 = m(t - t_1)$$
$$V - 25 = -8.169873(t - 1)$$
$$V - 25 = -8.169873t + 8.169873$$
$$V = -8.169873t + 33.169873$$

For $(1, V(1.5))$ and $(1.5, V(1.5))$:
From part (b) we know $V(1) = 25$.

$$V(1.5) = 5(1.5)^3 - 30(1.5)^2 + 45(1.5) + 5\sqrt{1.5}$$
$$\approx 22.998724$$
$$m = \frac{V(1.5) - V(1)}{1.5 - 1}$$
$$= \frac{22.998724 - 25}{1.5 - 1}$$
$$= \frac{-2.0012756}{0.5}$$
$$= -4.002551$$
$$V - V_1 = m(t - t_1)$$
$$V - 25 = -4.002551(t - 1)$$
$$V - 25 = -4.002551t + 4.002551$$
$$V = -4.002551t + 29.002551$$



e) Looking at the graph, the slope of the line tangent to the graph at the point $(1, V(1))$ appears to be 0.

f) The value of the investment is changing at approximately \$0 per year after 1 year.

49. There is a vertical tangent at $x = 5$, therefore, $f'(x)$ does not exist at $x = 5$.

Exercise Set 1.5

1. $y = x^7$
$$\frac{dy}{dx} = \frac{d}{dx}x^7$$
$$= 7x^{7-1} \qquad \text{Theorem 1}$$
$$= 7x^6$$

2. $y = x^8$
$$\frac{dy}{dx} = 8x^{8-1} = 8x^7$$

3. $y = -3x$
$$\frac{dy}{dx} = \frac{d}{dx}(-3x)$$
$$= -3\frac{d}{dx}x \qquad \text{Theorem 3}$$
$$= -3(1x^{1-1}) \qquad \text{Theorem 1}$$
$$= -3(x^0)$$
$$= -3 \qquad [a^0 = 1]$$

4. $y = -0.5x$
$$\frac{dy}{dx} = -0.5$$

5. $y = 12$ Constant function
$$\frac{dy}{dx} = \frac{d}{dx}12$$
$$= 0 \qquad \text{Theorem 2}$$

6. $y = 7$
$$\frac{dy}{dx} = 0$$

7. $y = 2x^{15}$
$$\frac{dy}{dx} = \frac{d}{dx}(2x^{15})$$
$$= 2\frac{d}{dx}(x^{15}) \qquad \text{Theorem 3}$$
$$= 2(15x^{15-1}) \qquad \text{Theorme 1}$$
$$= 30x^{14}$$

8. $y = 3x^{10}$

$\dfrac{dy}{dx} = 3\left(10x^{10-1}\right) = 30x^9$

9. $y = x^{-6}$

$\dfrac{dy}{dx} = \dfrac{d}{dx}x^{-6}$

$= -6x^{-6-1}$ Theorem 1

$= -6x^{-7}$

10. $y = x^{-8}$

$\dfrac{dy}{dx} = -8x^{-8-1} = -8x^{-9}$

11. $y = 4x^{-2}$

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(4x^{-2}\right)$

$= 4\dfrac{d}{dx}\left(x^{-2}\right)$ Theorem 3

$= 4\left(-2x^{-2-1}\right)$ Theorem 1

$= -8x^{-3}$

12. $y = 3x^{-5}$

$\dfrac{dy}{dx} = 3\left(-5x^{-5-1}\right) = -15x^{-6}$

13. $y = x^3 + 3x^2$

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(x^3 + 3x^2\right)$

$= \dfrac{d}{dx}x^3 + \dfrac{d}{dx}3x^2$ Theorem 4

$= \dfrac{d}{dx}x^3 + 3\dfrac{d}{dx}x^2$ Theorem 3

$= 3x^{3-1} + 3\left(2x^{2-1}\right)$ Theorem 1

$= 3x^2 + 6x$

14. $y = x^4 - 7x$

$\dfrac{dy}{dx} = 4x^{4-1} - 7 \cdot 1x^{1-1} = 4x^3 - 7$

15. $y = 8\sqrt{x} = 8x^{1/2}$

$\dfrac{dy}{dx} = \dfrac{d}{dx}8x^{1/2}$

$= 8\dfrac{d}{dx}x^{1/2}$ Theorem 3

$\dfrac{dy}{dx} = 8\left(\dfrac{1}{2}x^{1/2-1}\right)$ Theorem 1

$= 4x^{-1/2}$

$= \dfrac{4}{x^{1/2}} = \dfrac{4}{\sqrt{x}}$ Properties of exponents

16. $y = 4\sqrt{x} = 4x^{1/2}$

$\dfrac{dy}{dx} = 4\left(\dfrac{1}{2}x^{1/2-1}\right) = 2x^{-1/2} = \dfrac{2}{x^{1/2}} = \dfrac{2}{\sqrt{x}}$

17. $y = x^{0.9}$

$\dfrac{dy}{dx} = \dfrac{d}{dx}x^{0.9}$

$= 0.9x^{0.9-1}$ Theorem 1

$= 0.9x^{-0.1}$

18. $y = x^{0.7}$

$\dfrac{dy}{dx} = 0.7x^{0.7-1} = 0.7x^{-0.3}$

19. $y = \dfrac{1}{2}x^{4/5}$

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}x^{4/5}\right)$

$= \dfrac{1}{2} \cdot \dfrac{d}{dx}\left(x^{4/5}\right)$ Theorem 3

$= \dfrac{1}{2}\left(\dfrac{4}{5}x^{4/5-1}\right)$ Theorem 1

$= \dfrac{2}{5}x^{-1/5}$

20. $y = -4.8x^{1/3}$

$\dfrac{dy}{dx} = -4.8\left(\dfrac{1}{3}x^{1/3-1}\right) = -1.6x^{-2/3}$

21. $y=\frac{7}{x^3}=7x^{-3}$

$\frac{dy}{dx}=\frac{d}{dx}\left(7x^{-3}\right)$

$=7\frac{d}{dx}\left(x^{-3}\right)$ Theorem 3

$=7\left(-3x^{-3-1}\right)$ Theorem 1

$=-21x^{-4}$

$=-\frac{21}{x^4}$ Properties of exponents

22. $y=\frac{6}{x^4}=6x^{-4}$

$\frac{dy}{dx}=6\left(-4x^{-4-1}\right)=-24x^{-5}=-\frac{24}{x^5}$

23. $y=\frac{4x}{5}=\frac{4}{5}x$

$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{4}{5}x\right)$

$=\frac{4}{5}\cdot\frac{d}{dx}(x)$ Theorem 3

$=\frac{4}{5}\cdot\left(1x^{1-1}\right)$ Theorem 1

$=\frac{4}{5}$

24. $y=\frac{3x}{4}=\frac{3}{4}x$

$\frac{dy}{dx}=\frac{3}{4}\cdot 1x^{1-1}=\frac{3}{4}$

25. $\frac{d}{dx}\left(\sqrt[4]{x}-\frac{3}{x}\right)$

$=\frac{d}{dx}\sqrt[4]{x}-\frac{d}{dx}\frac{3}{x}$ Theorem 4

$=\frac{d}{dx}x^{1/4}-\frac{d}{dx}3x^{-1}$ Properties of exponents

$=\frac{d}{dx}x^{1/4}-3\frac{d}{dx}x^{-1}$ Theorem 3

$=\frac{1}{4}x^{1/4-1}-3\left(-1x^{-1-1}\right)$ Theorem 1

$=\frac{1}{4}x^{-3/4}+3x^{-2}$

$=\frac{1}{4x^{3/4}}+\frac{3}{x^2}$

$=\frac{1}{4\sqrt[4]{x^3}}+\frac{3}{x^2}$

26. $\frac{d}{dx}\left(\sqrt[5]{x}-\frac{2}{x}\right)$

$=\frac{d}{dx}\sqrt[5]{x}-\frac{d}{dx}\frac{2}{x}$

$=\frac{d}{dx}x^{1/5}-\frac{d}{dx}2x^{-1}$

$=\frac{1}{5}x^{1/5-1}-2\left(-1x^{-1-1}\right)$

$=\frac{1}{5}x^{-4/5}+2x^{-2}$

$=\frac{1}{5\sqrt[5]{x^4}}+\frac{2}{x^2}$

27. $\frac{d}{dx}\left(\sqrt{x}-\frac{2}{\sqrt{x}}\right)$

$=\frac{d}{dx}\sqrt{x}-\frac{d}{dx}\frac{2}{\sqrt{x}}$ Theorem 4

$=\frac{d}{dx}x^{1/2}-\frac{d}{dx}2x^{-1/2}$ Properties of exponents

$=\frac{d}{dx}x^{1/2}-2\frac{d}{dx}x^{-1/2}$ Theorem 3

$=\frac{1}{2}x^{1/2-1}-2\left(-\frac{1}{2}x^{-1/2-1}\right)$ Theorem 1

$=\frac{1}{2}x^{-1/2}+x^{-3/2}$

$=\frac{1}{2x^{1/2}}+\frac{1}{x^{3/2}}$

$=\frac{1}{2\sqrt{x}}+\frac{1}{\sqrt{x^3}}$

28. $\frac{d}{dx}\left(\sqrt[3]{x}+\frac{2\backslash 4}{\sqrt{x}}\right)$

$=\frac{d}{dx}\sqrt[3]{x}+\frac{d}{dx}\frac{4}{\sqrt{x}}$

$=\frac{d}{dx}x^{1/3}+\frac{d}{dx}4x^{-1/2}$

$=\frac{1}{3}x^{-2/3}-2x^{-3/2}$

$=\frac{1}{3x^{2/3}}-\frac{2}{x^{3/2}}$

$=\frac{1}{3\sqrt[3]{x^2}}-\frac{2}{\sqrt{x^3}}$

29. $\frac{d}{dx}\left(-2\sqrt[3]{x^5}\right)$

$= -2\frac{d}{dx}\left(\sqrt[3]{x^5}\right)$ Theorem 3

$= -2\frac{d}{dx}\left(x^{5/3}\right)$

$= -2\left(\frac{5}{3}x^{5/3-1}\right)$ Theorem 1

$= -\frac{10}{3}x^{2/3} = \frac{10\sqrt[3]{x^2}}{3}$

30. $\frac{d}{dx}\left(-\sqrt[4]{x^3}\right)$

$= -\frac{d}{dx}\left(x^{3/4}\right) = -\frac{3}{4}x^{-1/4} = -\frac{3}{4x^{1/4}} = -\frac{3}{4\sqrt[4]{x}}$

31. $\frac{d}{dx}\left(5x^2 - 7x + 3\right)$

$= \frac{d}{dx}5x^2 - \frac{d}{dx}7x + \frac{d}{dx}3$ Theorem 4

$= 5\frac{d}{dx}x^2 - 7\frac{d}{dx}x + \frac{d}{dx}3$ Theorem 3

$= 5\left(2x^{2-1}\right) - 7\left(1x^{1-1}\right) + 0$ Theorems 1 and 2

$= 10x - 7$

32. $\frac{d}{dx}\left(6x^2 - 5x + 9\right)$

$= \frac{d}{dx}6x^2 - \frac{d}{dx}5x + \frac{d}{dx}9 = 12x - 5$

33. $f(x) = 0.6x^{1.5}$

$f'(x) = \frac{d}{dx}0.6x^{1.5}$

$= 0.6\frac{d}{dx}x^{1.5}$ Theorem 3

$= 0.6\left(1.5x^{1.5-1}\right)$ Theorem 1

$= 0.9x^{0.5}$

34. $f(x) = 0.3x^{1.2}$

$f'(x) = 0.3\left(1.2x^{1.2-1}\right) = 0.36x^{0.2}$

35. $f(x) = \frac{2x}{3} = \frac{2}{3}x$

$f'(x) = \frac{d}{dx}\left(\frac{2}{3}x\right)$

$= \frac{2}{3}\frac{d}{dx}(x)$

$= \frac{2}{3}\left(1x^{1-1}\right)$

$= \frac{2}{3}$

36. $f(x) = \frac{3x}{4} = \frac{3}{4}x$

$f'(x) = \frac{3}{4}\left(1x^{1-1}\right) = \frac{3}{4}$

37. $f(x) = \frac{4}{7x^3} = \frac{4x^{-3}}{7} = \frac{4}{7}x^{-3}$

$f'(x) = \frac{d}{dx}\left(\frac{4}{7}x^{-3}\right)$

$= \frac{4}{7}\frac{d}{dx}\left(x^{-3}\right)$

$= \frac{4}{7}\left(-3x^{-3-1}\right)$

$= \frac{-12}{7}x^{-4}$

$= -\frac{12}{7x^4}$

38. $f(x) = \frac{2}{5x^6} = \frac{2}{5}x^{-6}$

$f'(x) = \frac{2}{5}\left(-6x^{-6-1}\right) = \frac{-12}{5}x^{-7} = -\frac{12}{5x^7}$

39. $f(x) = \frac{5}{x} - x^{2/3} = 5x^{-1} - x^{2/3}$

$f'(x) = \frac{d}{dx}\left(5x^{-1} - x^{2/3}\right)$

$= \frac{d}{dx}\left(5x^{-1}\right) - \frac{d}{dx}\left(x^{2/3}\right)$

$= 5\frac{d}{dx}\left(x^{-1}\right) - \frac{d}{dx}\left(x^{2/3}\right)$

$= 5\left(-1x^{-1-1}\right) - \frac{2}{3}x^{2/3-1}$

$= -5x^{-2} - \frac{2}{3}x^{-1/3}$

$= -\frac{5}{x^2} - \frac{2}{3}x^{-1/3}$

40. $f(x)=\frac{4}{x}-x^{3/5}=4x^{-1}-x^{3/5}$

$$f'(x)=4\left(-1x^{-1-1}\right)-\left(\frac{3}{5}x^{3/5-1}\right)$$
$$=-4x^{-2}-\frac{3}{5}x^{-2/5}$$
$$=-\frac{4}{x^2}-\frac{3}{5}x^{-2/5}$$

41. $f(x)=4x-7$

$$f'(x)=\frac{d}{dx}(4x-7)$$
$$=\frac{d}{dx}(4x)-\frac{d}{dx}(7)$$
$$f'(x)=4\frac{d}{dx}(x)-\frac{d}{dx}(7)$$
$$=4\left(1x^{1-1}\right)-0$$
$$=4$$

42. $f(x)=7x-14$

$$f'(x)=7\left(1x^{1-1}\right)-0=7$$

43. $f(x)=\frac{x^{4/3}}{4}=\frac{1}{4}x^{4/3}$

$$f'(x)=\frac{d}{dx}\left(\frac{1}{4}x^{4/3}\right)$$
$$=\frac{1}{4}\frac{d}{dx}\left(x^{4/3}\right)$$
$$=\frac{1}{4}\left(\frac{4}{3}x^{4/3-1}\right)$$
$$=\frac{1}{3}x^{1/3},\text{ or }\frac{\sqrt[3]{x}}{3}$$

44. $f(x)=\frac{x^{3/2}}{3}=\frac{1}{3}x^{3/2}$

$$f'(x)=\frac{1}{3}\left(\frac{3}{2}x^{3/2-1}\right)=\frac{1}{2}x^{1/2},\text{ or }\frac{\sqrt{x}}{2}$$

45. $f(x)=-0.01x^2-0.5x+70$

$$f'(x)=\frac{d}{dx}\left(-0.01x^2-0.5x+70\right)$$
$$=\frac{d}{dx}\left(-0.01x^2\right)-\frac{d}{dx}(0.5x)+\frac{d}{dx}(70)$$
$$=-0.01\frac{d}{dx}\left(x^2\right)-0.5\frac{d}{dx}(x)+\frac{d}{dx}(70)$$
$$=-0.01\left(2x^{2-1}\right)-0.5\left(1x^{1-1}\right)+0$$
$$=-0.02x-0.5$$

46. $f(x)=-0.01x^2+0.4x+50$

$$f'(x)=-0.01\left(2x^{2-1}\right)+0.4\left(1x^{1-1}\right)+0$$
$$=-0.02x+0.4$$

47. $y=3x^{-2/3}+x^{3/4}+x^{6/5}+\frac{8}{x^3}$

$$y=3x^{-2/3}+x^{3/4}+x^{6/5}+8x^{-3}$$
$$y'=\frac{d}{dx}\left(3x^{-2/3}+x^{3/4}+x^{6/5}+8x^{-3}\right)$$
$$=\frac{d}{dx}\left(3x^{-2/3}\right)+\frac{d}{dx}\left(x^{3/4}\right)+\frac{d}{dx}\left(x^{6/5}\right)+\frac{d}{dx}\left(8x^{-3}\right)$$
$$=3\left(\frac{-2}{3}x^{-2/3-1}\right)+\left(\frac{3}{4}x^{3/4-1}\right)+$$
$$\left(\frac{6}{5}x^{6/5-1}\right)+8\left(-3x^{-3-1}\right)$$
$$=-2x^{-5/3}+\frac{3}{4}x^{-1/4}+\frac{6}{5}x^{1/5}-24x^{-4}$$
$$=-2x^{-5/3}+\frac{3}{4}x^{-1/4}+\frac{6}{5}x^{1/5}-\frac{24}{x^4}$$

48. $y=x^{-3/4}-3x^{2/3}+x^{5/4}+\frac{2}{x^4}$

$$y=x^{-3/4}-3x^{2/3}+x^{5/4}+2x^{-4}$$
$$y'=\left(\frac{-3}{4}x^{-3/4-1}\right)-3\left(\frac{2}{3}x^{2/3-1}\right)+$$
$$\left(\frac{5}{4}x^{5/4-1}\right)+2\left(-4x^{-4-1}\right)$$
$$=\frac{-3}{4}x^{-7/4}-2x^{-1/3}+\frac{5}{4}x^{1/4}-8x^{-5}$$
$$=\frac{-3}{4}x^{-7/4}-2x^{-1/3}+\frac{5}{4}x^{1/4}-\frac{8}{x^5}$$

49. $y=\frac{2}{x}-\frac{x}{2}=2x^{-1}-\frac{1}{2}x$

$y'=\frac{d}{dx}\left(2x^{-1}-\frac{1}{2}x\right)$

$=\frac{d}{dx}\left(2x^{-1}\right)-\frac{d}{dx}\left(\frac{1}{2}x\right)$

$=2\frac{d}{dx}\left(x^{-1}\right)-\frac{1}{2}\frac{d}{dx}(x)$

$=2\left(-1x^{-1-1}\right)-\frac{1}{2}\left(1x^{1-1}\right)$

$=-2x^{-2}-\frac{1}{2}$

$=-\frac{2}{x^2}-\frac{1}{2}$

50. $y=\frac{x}{7}+\frac{7}{x}=\frac{1}{7}x+7x^{-1}$

$y'=\frac{1}{7}\left(1x^{1-1}\right)+7\left(-1x^{-1-1}\right)$

$=\frac{1}{7}-7x^{-2}=\frac{1}{7}-\frac{7}{x^2}$

51. $f(x)=x^2+4x-5$

First, we find $f'(x)$

$f'(x)=\frac{d}{dx}\left(x^2+4x-5\right)$

$=\frac{d}{dx}\left(x^2\right)+4\frac{d}{dx}(x)-\frac{d}{dx}5$

$=\left(2x^{2-1}\right)+4\left(1x^{1-1}\right)-0$

$=2x+4$

Therefore,

$f'(10)=2(10)+4$

$=24$

52. $f(x)=\sqrt{x}=x^{1/2}$

$f'(x)=\frac{1}{2}x^{1/2-1}=\frac{1}{2}x^{-1/2}=\frac{1}{2x^{1/2}}=\frac{1}{2\sqrt{x}}$

Therefore,

$f'(4)=\frac{1}{2\sqrt{4}}=\frac{1}{2\cdot 2}=\frac{1}{4}$

53. $y=\frac{4}{x^2}=4x^{-2}$

Find $\frac{dy}{dx}$ first.

$\frac{dy}{dx}=\frac{d}{dx}\left(4x^{-2}\right)$

$=4\frac{d}{dx}\left(x^{-2}\right)$

$=4\left(-2x^{-2-1}\right)$

$=-8x^{-3}$

$=-\frac{8}{x^3}$

Therefore,

$\left.\frac{dy}{dx}\right|_{x=-2}=-\frac{8}{(-2)^3}$

$=-\frac{8}{(-8)}$

$=1$

54. $y=x+\frac{2}{x^3}=x+2x^{-3}$

$\frac{dy}{dx}=1x^{1-1}+2\left(-3x^{-3-1}\right)=1-6x^{-4}=1-\frac{6}{x^4}$

$\left.\frac{dy}{dx}\right|_{x=1}=1-\frac{6}{(1)^4}$

$=1-6=-5$

55. We will need the derivative to find the slope of the tangent line at each of the indicated points. We find the derivative first.

$f(x)=x^3-2x+1$

$f'(x)=\frac{d}{dx}\left(x^3-2x+1\right)$

$=\frac{d}{dx}\left(x^3\right)-\frac{d}{dx}(2x)+\frac{d}{dx}(1)$

$=\left(3x^{3-1}\right)-2\left(1x^{1-1}\right)+0$

$=3x^2-2$

a) Using the derivative, we find the slope of the line tangent to the curve at point $(2,5)$ by evaluating the derivative at $x=2$. $f'(2)=3(2)^2-2=10$. Therefore the slope of the tangent line is 10. We use the point-slope equation to find the equation of the tangent line on the next page.

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 10(x - 2)$$
$$y - 5 = 10x - 20$$
$$y = 10x - 15$$

b) Using the derivative, we find the slope of the line tangent to the curve at point $(-1,2)$ by evaluating the derivative at $x = -1$.

$f'(-1) = 3(-1)^2 - 2 = 1$. Therefore the slope of the tangent line is 1. We use the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 1(x - (-1))$$
$$y - 2 = x + 1$$
$$y = x + 3$$

c) Using the derivative, we find the slope of the line tangent to the curve at point $(0,1)$ by evaluating the derivative at $x = 0$.

$f'(0) = 3(0)^2 - 2 = -2$. Therefore the slope of the tangent line is -2. We use the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -2(x - 0)$$
$$y - 1 = -2x$$
$$y = -2x + 1$$

56. $f(x) = x^2 - \sqrt{x} = x^2 - x^{1/2}$

$$f'(x) = 2x^{2-1} - \frac{1}{2}x^{1/2-1}$$
$$= 2x - \frac{1}{2x^{1/2}}$$
$$= 2x - \frac{1}{2\sqrt{x}}$$

a) At $(1,0)$: $f'(1) = 2(1) - \frac{1}{2\sqrt{1}} = 2 - \frac{1}{2} = \frac{3}{2}$

The equation of the tangent line is:

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{3}{2}(x - 1)$$
$$y = \frac{3}{2}x - \frac{3}{2}$$

b) At $(4,14)$:

$$f'(1) = 2(4) - \frac{1}{2\sqrt{4}} = 8 - \frac{1}{4} = \frac{31}{4}$$

The equation of the tangent line is:

$$y - y_1 = m(x - x_1)$$
$$y - 14 = \frac{31}{4}(x - 4)$$
$$y - 14 = \frac{31}{4}x - 31$$
$$y = \frac{31}{4}x - 17$$

c) At $(9,78)$:

$$f'(9) = 2(9) - \frac{1}{2\sqrt{9}} = 18 - \frac{1}{6} = \frac{107}{6}$$

The equation of the tangent line is:

$$y - y_1 = m(x - x_1)$$
$$y - 78 = \frac{107}{6}(x - 9)$$
$$y - 78 = \frac{107}{6}x - \frac{321}{2}$$
$$y = \frac{107}{6}x - \frac{165}{2}$$

57. $y = x^2 - 3$

A horizontal tangent line has slope equal to 0, so we first find the values of *x* that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 - 3)$$
$$\frac{dy}{dx} = \frac{d}{dx}x^2 - \frac{d}{dx}3$$
$$= 2x - 0$$
$$= 2x$$

Next, we set the derivative equal to zero and solve for *x*.

$$\frac{dy}{dx} = 0$$
$$2x = 0$$
$$x = \frac{0}{2} = 0$$

So the horizontal tangent will occur when $x = 0$. Next we find the point on the graph. For $x = 0$, $y = (0)^2 - 3 = -3$, so there is a horizontal tangent at the point $(0,-3)$.

58. $y = -x^2 + 4$

$\frac{dy}{dx} = -2x$

Solve:

$$\frac{dy}{dx} = 0$$
$$-2x = 0$$
$$x = 0$$

For $x = 0,\ y = -(0)^2 + 4 = 4$.

Therefore, there is a horizontal tangent at the point $(0,4)$.

59. $y = -x^3 + 1$

A horizontal tangent line has slope equal to 0, so we first find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(-x^3 + 1\right)\\ &= -\frac{d}{dx}x^3 + \frac{d}{dx}1\\ &= -3x^2 - 0\\ &= -3x^2\end{aligned}$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx} = 0$$
$$-3x^2 = 0$$
$$x^2 = 0$$
$$x = 0$$

So the horizontal tangent will occur when $x = 0$. Next we find the point on the graph. For $x = 0,\ y = -(0)^3 + 1 = 1$, so there is a horizontal tangent at the point $(0,1)$.

60. $y = x^3 - 2$

$\frac{dy}{dx} = 3x^2$

Solve:

$$\frac{dy}{dx} = 0$$
$$3x^2 = 0$$
$$x = 0$$

For $x = 0,\ y = (0)^3 - 2 = -2$.

Therefore, there is a horizontal tangent at the point $(0,-2)$.

61. $y = 3x^2 - 5x + 4$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(3x^2 - 5x + 4\right)\\ &= \frac{d}{dx}3x^2 - \frac{d}{dx}5x + \frac{d}{dx}4\\ &= 3\left(2x^{2-1}\right) - 5\left(1x^{1-1}\right) + 0\\ &= 6x - 5\end{aligned}$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx} = 0$$
$$6x - 5 = 0$$
$$6x = 5$$
$$x = \frac{5}{6}$$

So the horizontal tangent will occur when $x = \frac{5}{6}$. Next we find the point on the graph.

For $x = \frac{5}{6}$,

$$\begin{aligned}y &= 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) + 4\\ &= 3\left(\frac{25}{36}\right) - \frac{25}{6} + 4\\ &= \frac{25}{12} - \frac{25}{6} + 4\\ &= \frac{25}{12} - \frac{25}{6}\cdot\frac{2}{2} + \frac{4}{1}\cdot\frac{12}{12}\\ &= \frac{25}{12} - \frac{50}{12} + \frac{48}{12}\\ &= \frac{25 - 50 + 48}{12}\\ &= \frac{23}{12}\end{aligned}$$

Therefore, there is a horizontal tangent at the point $\left(\frac{5}{6}, \frac{23}{12}\right)$.

62. $y = 5x^2 - 3x + 8$

$\frac{dy}{dx} = 10x - 3$

Solve:

$$\frac{dy}{dx} = 0$$
$$10x - 3 = 0$$
$$10x = 3$$
$$x = \frac{3}{10}$$

For $x = \frac{3}{10}$,

$$y = 5\left(\frac{3}{10}\right)^2 - 3\left(\frac{3}{10}\right) + 8$$
$$= 5\left(\frac{9}{100}\right) - \frac{9}{10} + 8$$
$$= \frac{45}{100} - \frac{90}{100} + \frac{800}{100}$$
$$= \frac{755}{100}$$

Therefore, there is a horizontal tangent at the point $\left(\frac{3}{10}, \frac{755}{100}\right)$, or $(0.3, 7.55)$.

63. $y = -0.01x^2 - 0.5x + 70$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(-0.01x^2 - 0.5x + 70\right)$$
$$= -0.02x - 0.5 \qquad \text{See Exercise 45.}$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx} = 0$$
$$-0.02x - 0.5 = 0$$
$$-0.02x = 0.5$$
$$x = \frac{0.5}{-0.02}$$
$$x = -25$$

So the horizontal tangent will occur when $x = -25$. Next we find the point on the graph.

For $x = -25$,

$$y = -0.01(-25)^2 - 0.5(-25) + 70$$
$$= -0.01(625) + 12.5 + 70$$
$$= -6.25 + 12.5 + 70$$
$$= 76.25$$

Therefore, there is a horizontal tangent at the point $(-25, 76.25)$.

64. $y = -0.01x^2 + 0.4x + 50$

$\frac{dy}{dx} = -0.02x + 0.4$ See Exercise 46

$$\frac{dy}{dx} = 0$$
$$-0.02x + 0.4 = 0$$
$$-0.02x = -0.4$$
$$x = \frac{-0.4}{-0.02}$$
$$x = 20$$

For $x = 20$,

$$y = -0.01(20)^2 + 0.4(20) + 50$$
$$= -0.01(400) + 8 + 50$$
$$= -4 + 8 + 50$$
$$= 54$$

Therefore, there is a horizontal tangent at the point $(20, 54)$.

65. $y = 2x + 4$ Linear function

$\frac{dy}{dx} = 2$ Slope is 2

There are no values of x for which $\frac{dy}{dx} = 0$, so there are no points on the graph at which there is a horizontal tangent.

66. $y = -2x + 5$

$\frac{dy}{dx} = -2$

There are no values of x for which $\frac{dy}{dx} = 0$, so there are no points on the graph at which there is a horizontal tangent.

67. $y = 4$ Constant Function

$\frac{dy}{dx} = 0$ Theorem 2

$\frac{dy}{dx} = 0$ for all values of x, so the tangent line is horizontal for all points on the graph.

68. $y = -3$

$\frac{dy}{dx} = 0$ Theorem 2

$\frac{dy}{dx} = 0$ for all values of x, so the tangent line is horizontal for all points on the graph.

69. $y = -x^3 + x^2 + 5x - 1$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(-x^3 + x^2 + 5x - 1\right)$$
$$= -\frac{d}{dx}\left(x^3\right) + \frac{d}{dx}\left(x^2\right) + \frac{d}{dx}(5x) - \frac{d}{dx}(1)$$
$$= -3x^2 + 2x + 5$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx} = 0$$
$$-3x^2 + 2x + 5 = 0$$
$$3x^2 - 2x - 5 = 0 \quad \text{Multiply both sides by -1.}$$
$$(3x-5)(x+1) = 0 \quad \text{Factor the left hand side.}$$
$$3x - 5 = 0 \quad \text{or} \quad x + 1 = 0$$
$$3x = 5 \quad \text{or} \quad x = -1$$
$$x = \frac{5}{3} \quad \text{or} \quad x = -1$$

There are two horizontal tangents. One at $x = \frac{5}{3}$ and one at $x = -1$.

Next we find the points on the graph where the horizontal tangents occur.

For $x = -1$

$$y = -(-1)^3 + (-1)^2 + 5(-1) - 1$$
$$y = -(-1) + (1) - 5 - 1$$
$$y = -4$$

For $x = \frac{5}{3}$

$$y = -\left(\frac{5}{3}\right)^3 + \left(\frac{5}{3}\right)^2 + 5\left(\frac{5}{3}\right) - 1$$
$$y = -\left(\frac{125}{27}\right) + \left(\frac{25}{9}\right) + \frac{25}{3} - 1$$
$$y = -\frac{125}{27} + \frac{75}{27} + \frac{225}{27} - \frac{27}{27}$$
$$y = \frac{148}{27} = 5\tfrac{13}{27}$$

Therefore, there are horizontal tangents at the points $\left(\frac{5}{3}, 5\tfrac{13}{27}\right)$ and $(-1, -4)$.

70. $y = -\frac{1}{3}x^3 + 6x^2 - 11x - 50$

$\frac{dy}{dx} = -x^2 + 12x - 11$

Solve:

$$\frac{dy}{dx} = 0$$
$$-x^2 + 12x - 11 = 0$$
$$x^2 - 12x + 11 = 0$$
$$(x-1)(x-11) = 0$$
$$x - 1 = 0 \quad \text{or} \quad x - 11 = 0$$
$$x = 1 \quad \text{or} \quad x = 11$$

For $x = 1$,

$$y = -\frac{1}{3}(1)^3 + 6(1)^2 - 11(1) - 50$$
$$= -\frac{1}{3} + 6 - 11 - 50$$
$$= -\frac{166}{3} = -55\tfrac{1}{3}$$

For $x = 11$,

$$y = -\frac{1}{3}(11)^3 + 6(11)^2 - 11(11) - 50$$
$$= -\frac{1331}{3} + 726 - 121 - 50$$
$$= \frac{334}{3} = 111\tfrac{1}{3}$$

Therefore, there are horizontal tangents at the points $\left(1, -55\tfrac{1}{3}\right)$ and $\left(11, 111\tfrac{1}{3}\right)$.

71. $y = \frac{1}{3}x^3 - 3x + 2$

A horizontal tangent line has slope equal to 0, so we need to find the values of *x* that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^3 - 3x + 2\right)$$
$$= \frac{d}{dx}\left(\frac{1}{3}x^3\right) - \frac{d}{dx}(3x) + \frac{d}{dx}(2)$$
$$= x^2 - 3$$

Next, we set the derivative equal to zero and solve for *x*.

$$\frac{dy}{dx} = 0$$
$$x^2 - 3 = 0$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

There are two horizontal tangents. One at $x = -\sqrt{3}$ and one at $x = \sqrt{3}$. Next we find the points on the graph where the horizontal tangents occur.

For $x = -\sqrt{3}$

$$y = \frac{1}{3}\left(-\sqrt{3}\right)^3 - 3\left(-\sqrt{3}\right) + 2$$
$$= \frac{1}{3}\left(-3\sqrt{3}\right) - 3\left(-\sqrt{3}\right) + 2$$
$$= -\sqrt{3} + 3\sqrt{3} + 2$$
$$= 2 + 2\sqrt{3}$$

For $x = \sqrt{3}$

$$y = \frac{1}{3}\left(\sqrt{3}\right)^3 - 3\left(\sqrt{3}\right) + 2$$
$$= \frac{1}{3}\left(3\sqrt{3}\right) - 3\left(\sqrt{3}\right) + 2$$
$$= +\sqrt{3} - 3\sqrt{3} + 2$$
$$= 2 - 2\sqrt{3}$$

Therefore, there are horizontal tangents at the points $\left(-\sqrt{3}, 2 + 2\sqrt{3}\right)$ and $\left(\sqrt{3}, 2 - 2\sqrt{3}\right)$.

72. $y = x^3 - 6x + 1$

$$\frac{dy}{dx} = 3x^2 - 6$$

Solve:

$$\frac{dy}{dx} = 0$$
$$3x^2 - 6 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

For $x = -\sqrt{2}$,

$$y = \left(-\sqrt{2}\right)^3 - 6\left(-\sqrt{2}\right) + 1$$
$$= -2\sqrt{2} + 6\sqrt{2} + 1$$
$$= 1 + 4\sqrt{2}$$

For $x = \sqrt{2}$,

$$y = \left(\sqrt{2}\right)^3 - 6\left(\sqrt{2}\right) + 1$$
$$= 2\sqrt{2} - 6\sqrt{2} + 1$$
$$= 1 - 4\sqrt{2}$$

Therefore, there are horizontal tangents at the points $\left(-\sqrt{2}, 1 + 4\sqrt{2}\right)$ and $\left(\sqrt{2}, 1 - 4\sqrt{2}\right)$.

73. $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2$

A horizontal tangent line has slope equal to 0, so we need to find the values of *x* that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 - 2\right)$$
$$= \frac{d}{dx}\left(\frac{1}{3}x^3\right) + \frac{d}{dx}\left(\frac{1}{2}x^2\right) - \frac{d}{dx}(2)$$
$$= x^2 + x$$

Next, we set the derivative equal to zero and solve for *x*.

$$\frac{dy}{dx} = 0$$
$$x^2 + x = 0$$
$$x(x + 1) = 0$$
$$x = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 0 \quad \text{or} \quad x = -1$$

There are two horizontal tangents. One at $x = 0$ and one at $x = -1$.

Next we find the points on the graph where the horizontal tangents occur.

For $x = 0$

$$y = \frac{1}{3}(0)^3 + \frac{1}{2}(0)^2 - 2$$
$$y = -2$$

For $x = -1$

$$y = \frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2 - 2$$
$$= -\frac{1}{3} + \frac{1}{2} - 2$$
$$= -\frac{2}{6} + \frac{3}{6} - \frac{12}{6}$$
$$= -\frac{11}{6}$$

Therefore, there are horizontal tangents at the points $(0,-2)$ and $\left(-1,-\frac{11}{6}\right)$.

74. $y = \frac{1}{3}x^3 - 3x^2 + 9x - 9$

$\frac{dy}{dx} = x^2 - 6x + 9$

Solve:

$$\frac{dy}{dx} = 0$$
$$x^2 - 6x + 9 = 0$$
$$(x-3)^2 = 0$$
$$x - 3 = 0$$
$$x = 3$$

For $x = 3$,

$$y = \frac{1}{3}(3)^3 - 3(3)^2 + 9(3) - 9$$
$$= 9 - 27 + 27 - 9 = 0$$

Therefore, there is horizontal tangent at the point $(3,0)$.

75. $y = 20x - x^2$

To find the tangent line that has slope equal to 1, we need to find the values of x that make $\frac{dy}{dx} = 1$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(20x - x^2\right)$$
$$= \frac{d}{dx}20x - \frac{d}{dx}x^2$$
$$= 20 - 2x$$

Next, we set the derivative equal to 1 and solve for x.

$$\frac{dy}{dx} = 1$$
$$20 - 2x = 1$$
$$-2x = 1 - 20$$
$$-2x = -19$$
$$x = \frac{19}{2}$$

So the desired tangent line will occur when $x = \frac{19}{2}$. Next we find the point on the graph.

For $x = \frac{19}{2}$,

$$y = 20\left(\frac{19}{2}\right) - \left(\frac{19}{2}\right)^2$$
$$y = 190 - \left(\frac{361}{4}\right)$$
$$= \frac{760}{4} - \frac{361}{4}$$
$$= \frac{399}{4}$$

The tangent line has slope 1 at the point $\left(\frac{19}{2}, \frac{399}{4}\right)$.

76. $y = 6x - x^2$

$\frac{dy}{dx} = 6 - 2x$

Solve:

$$\frac{dy}{dx} = 1$$
$$6 - 2x = 1$$
$$-2x = -5$$
$$x = \frac{5}{2}$$

For $x = \frac{5}{2}$,

$$y = 6\left(\frac{5}{2}\right) - \left(\frac{5}{2}\right)^2$$
$$= 15 - \frac{25}{4} = \frac{35}{4}$$

The tangent line has slope 1 at $\left(\frac{5}{2}, \frac{35}{4}\right)$.

77. $y = -0.025x^2 + 4x$

To find the tangent line that has slope equal to 1, we need to find the values of *x* that make $\frac{dy}{dx} = 1$.

First, we find the derivative.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(-0.025x^2 + 4x\right)\\ &= \frac{d}{dx}(-0.025x^2) + \frac{d}{dx}(4x)\\ &= -0.025(2x) + 4\\ &= -0.05x + 4\end{aligned}$$

Next, we set the derivative equal to 1 and solve for *x*.

$$\begin{aligned}\frac{dy}{dx} &= 1\\ -0.05x + 4 &= 1\\ -0.05x &= -3\\ x &= \frac{-3}{-0.05}\\ x &= 60\end{aligned}$$

So the tangent line with slope equal to 1 will occur when $x = 60$. Next we find the point on the graph.

For $x = 60$,

$$\begin{aligned}y &= -0.025(60)^2 + 4(60)\\ &= -0.025(3600) + 240\\ &= -90 + 240\\ &= 150\end{aligned}$$

The tangent line has slope 1 at the point $(60, 150)$.

78. $y = -0.01x^2 + 2x$

$\frac{dy}{dx} = -0.02x + 2$

Solve:

$$\begin{aligned}\frac{dy}{dx} &= 1\\ -0.02x + 2 &= 1\\ -0.02x &= -1\\ x &= 50\end{aligned}$$

For $x = 50$,

$$\begin{aligned}y &= -0.01(50)^2 + 2(50)\\ &= -25 + 100 = 75\end{aligned}$$

The tangent line has slope 1 at $(50, 75)$.

79. $y = \frac{1}{3}x^3 + 2x^2 + 2x$

To find the tangent line that has slope equal to 1, we need to find the values of *x* that make $\frac{dy}{dx} = 1$.

First, we find the derivative.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1}{3}x^3 + 2x^2 + 2x\right)\\ &= \frac{d}{dx}\left(\frac{1}{3}x^3\right) + \frac{d}{dx}2x^2 + \frac{d}{dx}2x\\ &= x^2 + 4x + 2\end{aligned}$$

Next, we set the derivative equal to 1 and solve for *x*.

$$\begin{aligned}\frac{dy}{dx} &= 1\\ x^2 + 4x + 2 &= 1\\ x^2 + 4x + 1 &= 0\end{aligned}$$

This is a quadratic equation, not readily factorable, so we use the quadratic formula where $a = 1, b = 4,$ and, $c = 1$.

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ x &= \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(1)}}{2(1)} \quad \text{Substituting}\\ &= \frac{-4 \pm \sqrt{12}}{2}\\ &= \frac{-4 \pm 2\sqrt{3}}{2} \qquad \left[\sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}\right]\\ &= \frac{2\left(-2 \pm \sqrt{3}\right)}{2}\\ &= -2 \pm \sqrt{3}\end{aligned}$$

There are two tangent lines that have slope equal to 1. The first one occurs at $x = -2 + \sqrt{3}$ and the second one occurs at $x = -2 - \sqrt{3}$. We use the original equation to find the point on the graph.

For $x = -2 + \sqrt{3}$,

$$\begin{aligned}y &= \frac{1}{3}\left(-2 + \sqrt{3}\right)^3 + 2\left(-2 + \sqrt{3}\right)^2 + 2\left(-2 + \sqrt{3}\right)\\ &= \frac{1}{3}\left(-26 + 15\sqrt{3}\right) + 2\left(7 - 4\sqrt{3}\right) - 4 + 2\sqrt{3}\\ &= -\frac{26}{3} + 5\sqrt{3} + 14 - 8\sqrt{3} - 4 + 2\sqrt{3}\\ &= \frac{4}{3} - \sqrt{3}\end{aligned}$$

For $x = -2 - \sqrt{3}$,

$$y = \frac{1}{3}\left(-2-\sqrt{3}\right)^3 + 2\left(-2-\sqrt{3}\right)^2 + 2\left(-2-\sqrt{3}\right)$$
$$= \frac{1}{3}\left(-26-15\sqrt{3}\right) + 2\left(7+4\sqrt{3}\right) - 4 - 2\sqrt{3}$$
$$= -\frac{26}{3} - 5\sqrt{3} + 14 + 8\sqrt{3} - 4 - 2\sqrt{3}$$
$$= \frac{4}{3} + \sqrt{3}$$

The tangent lines have slope 1 at the points $\left(-2+\sqrt{3}, \frac{4}{3}-\sqrt{3}\right)$ and $\left(-2-\sqrt{3}, \frac{4}{3}+\sqrt{3}\right)$.

80. $y = \frac{1}{3}x^3 - x^2 - 4x + 1$

$$\frac{dy}{dx} = x^2 - 2x - 4$$

Solve:

$$\frac{dy}{dx} = 1$$
$$x^2 - 2x - 4 = 1$$
$$x^2 - 2x - 5 = 0$$

Using the quadratic formula,

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4+20}}{2}$$
$$= \frac{2 \pm \sqrt{24}}{2}$$
$$= \frac{2 \pm 2\sqrt{6}}{2}$$
$$= 1 \pm \sqrt{6}$$

For $x = 1 + \sqrt{6}$,

$$y = \frac{1}{3}\left(1+\sqrt{6}\right)^3 - \left(1+\sqrt{6}\right)^2 - 4\left(1+\sqrt{6}\right) + 1$$
$$= \frac{1}{3}\left(19+9\sqrt{6}\right) - \left(7+2\sqrt{6}\right) - 4 - 4\sqrt{6} + 1$$
$$= \frac{19}{3} + 3\sqrt{6} - 7 - 2\sqrt{6} - 4 - 4\sqrt{6} + 1$$
$$= -\frac{11}{3} - 3\sqrt{6}$$

For $x = 1 - \sqrt{6}$,

$$y = \frac{1}{3}\left(1-\sqrt{6}\right)^3 - \left(1-\sqrt{6}\right)^2 - 4\left(1-\sqrt{6}\right) + 1$$
$$= \frac{1}{3}\left(19-9\sqrt{6}\right) - \left(7-2\sqrt{6}\right) - 4 + 4\sqrt{6} + 1$$
$$= \frac{19}{3} - 3\sqrt{6} - 7 + 2\sqrt{6} - 4 + 4\sqrt{6} + 1$$
$$= -\frac{11}{3} + 3\sqrt{6}$$

Therefore, the tangent line has slope 1 at the points $\left(1+\sqrt{6}, -\frac{11}{3}-3\sqrt{6}\right)$ and $\left(1-\sqrt{6}, -\frac{11}{3}+3\sqrt{6}\right)$.

81. a) In order to find the rate of change of the area with respect to the radius, we must find the derivative of the function with respect to *r*.

$$A'(r) = \frac{d}{dr}\left(3.14r^2\right)$$
$$= 3.14\left(2r^{2-1}\right)$$
$$= 6.28r$$

b) $\boxed{tw}$ Answers will vary. $A'(r) = 6.28r$ means that the area of the wound with a radius *r* cm will increase at a rate of $6.28r$ cm^2 for each centimeter increase in the radius.

82. a) $C(r) = 6.28r$

$$C'(r) = 6.28$$

b) $\boxed{tw}$ Answers will vary. $C'(r) = 6.28$ means that the circumference of the wound with a radius *r* cm will increase at a rate of 6.28 cm for each centimeter increase in the radius.

83. $w(t) = 8.15 + 1.82t - 0.0596t^2 + 0.000758t^3$

a) In order to find the rate of change of weight with respect to time, we take the derivative of the function with respect to *t*.

$$w'(t)$$
$$= \frac{d}{dt}\left(8.15 + 1.82t - 0.0596t^2 + 0.000758t^3\right)$$
$$= 0 + 1.82 - 0.0596(2t) + 0.000758\left(3t^2\right)$$
$$= 1.82 - 0.1192t + 0.002274t^2$$

Therefore, the rate of change of weight with respect to time is given by:

$$w'(t) = 1.82 - 0.1192t + 0.002274t^2$$

b) The weight of the baby at age 10 months can be found by evaluating the function when $t = 10$.

$$w(10) = 8.15 + 1.82(10) - 0.0596(10)^2 + 0.000758(10)^3$$
$$\approx 21.148 \quad \text{Using a calculator}$$

Therefore, a 10 month old boy weighs approximately 21.148 pounds.

c) The rate of change of the baby's weight with respect to time at age of 10 months can be found by evaluating the derivative when $t = 10$.

$$w'(10)$$
$$= 1.82 - 0.1192(10) + 0.002274(10)^2$$
$$\approx 0.8554$$

A 10 month old boys weight will be increasing at a rate of 0.8554 pounds per month.

84. $T(t) = -0.1t^2 + 1.2t + 98.6$

a) $T'(t) = -0.2t + 1.2$

b) Evaluate T when $t = 1.5$

$$T(1.5) = -0.1(1.5)^2 + 1.2(1.5) + 98.6$$
$$= 100.175$$

The temperature of the ill person after 1.5 days is 100.175 degrees Fahrenheit.

c) Evaluate $T'(t)$ when $t = 1.5$.

$$T'(1.5) = -0.2(1.5) + 1.2$$
$$= 0.9$$

The ill person's temperature is increasing 0.9 degrees Fahrenheit per day after 1.5 days.

85. $R(v) = \dfrac{6000}{v} = 6000v^{-1}$

a) Using the power rule, we take the derivative of R with respect to v.

$$R'(v) = 6000\left(-1v^{-1-1}\right)$$
$$= -6000v^{-2}$$
$$= -\frac{6000}{v^2}$$

The rate of change of heart rate with respect to the output per beat is

$$R'(v) = -\frac{6000}{v^2}.$$

b) To find the heart rate at $v = 80$ ml per beat, we evaluate the function $R(v)$ when $v = 80$.

$$R(80) = \frac{6000}{80} = 75.$$

The heart rate is 75 beats per minute when the output per beat is 80 ml per beat.

c) To find the rate of change of the heart beat at $v = 80$ ml per beat, we evaluate the derivative $R'(v)$ at $v = 80$.

$$R'(80) = -\frac{6000}{80^2}$$
$$R'(80) = -\frac{15}{16}$$
$$\approx -0.9375$$

The heart rate is decreasing at a rate of 0.9375 beats per minute when the output per beat is 80 ml per beat.

86. $S(r) = \dfrac{1}{r^4} = r^{-4}$

a) $S'(r) = \dfrac{d}{dr}\left(r^{-4}\right) = -4r^{-5} = -\dfrac{4}{r^5}$

b) $S(1.2) = \dfrac{1}{(1.2)^4} \approx 0.48225309$

The resistance when $r = 1.2$ mm is 0.48225309.

c) $S'(r) = -\dfrac{4}{(0.8)^5} \approx -12.20703125$

The resistance, S, is changing with respect to r at an approximate rate of -12.21 per mm when $r = 0.8$ mm

87. a) Using the power rule, we find the growth rate $\dfrac{dP}{dt}$.

$$\frac{dP}{dt} = \frac{d}{dt}\left(100{,}000 + 2000t^2\right)$$
$$= 0 + 2000(2t)$$
$$= 4000t$$

b) Evaluate the function P when $t = 10$.

$$P(10) = 100{,}000 + 2000(10)^2$$
$$= 100{,}000 + 2000(100)$$
$$= 300{,}000$$

The population of the city will be 300,000 people after 10 years.

c) Evaluate the derivative $P'(t)$ when $t = 10$.

$$\left.\frac{dP}{dt}\right|_{t=10} = P'(10) = 4000(10) = 40,000$$

The population's growth rate after 10 years is 40,000 people per year.

d) $\boxed{tw}$ Answers will vary.

$P'(10) = 40,000$ means that after 10 years, the city's population is growing at a rate of 40,000 people per year.

88. $A(t) = 0.08t + 19.7$

a) $A'(t) = 0.08$

b) $\boxed{tw}$ Answers will vary. The median age, *A*, of women marrying for the first time has been increasing at a rate 0.08 year per year since 1950.

89. $V = 1.22\sqrt{h} = 1.22h^{1/2}$

a) Using the power rule,

$$\frac{dV}{dh} = \frac{d}{dh}\left(1.22h^{1/2}\right)$$
$$= 1.22\left(\frac{1}{2}h^{1/2-1}\right)$$
$$= 0.61h^{-1/2}$$
$$= \frac{0.61}{h^{1/2}} = \frac{0.61}{\sqrt{h}}$$

b) Evaluate the function *V* when $h = 40,000$.

$$V = 1.22\sqrt{40,000}$$
$$= 244$$

A person would be able to see 244 miles to the horizon from a height of 40,000 feet.

c) Evaluate the derivative $\frac{dV}{dh}$ when $h = 40,000$.

$$\left.\frac{dV}{dh}\right|_{h=40,000} = \frac{0.61}{\sqrt{40,000}}$$
$$= \frac{0.61}{200}$$
$$\approx 0.00305$$

The rate of change at $h = 40,000$ is 0.00305 miles per foot.

d) $\boxed{tw}$ Answers will vary. From part (a), we find that the distance that one can see to the horizon from height *h* increases $\frac{0.61}{\sqrt{h}}$ miles for every one foot increase in height. From part (c) we find that, at a height of 40,000 feet, the distance that a person can see to the horizon increases at a rate of 0.00305 miles per foot.

90. $p(x) = 9.41 - 0.19x + 0.09x^2$

a) $\frac{dp}{dx} = p'(x) = -0.19 + 0.18x$

b) In 2007, $t = 2007 - 1990 = 17$. Evaluating the function at $t = 17$, we have

$$p(17) = 9.41 - 0.19(17) + 0.09(17)^2 = 32.19$$

The average ticket price in 2007 is \$32.19.

c) Evaluate the derivative at $t = 17$.

$$p'(17) = -0.19 + 0.18(17) = 2.87$$

In 2007, the average ticket price is increasing at a rate of \$2.87 per year.

91. $f(x) = x^2 - 4x + 1$

The derivative is positive when $f'(x) > 0$.

Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left(x^2 - 4x + 1\right)$$
$$= 2x - 4$$

Next, we solve the inequality.

$$f'(x) > 0$$
$$2x - 4 > 0$$
$$2x > 4$$
$$x > 2$$

Therefore, the interval for which $f'(x)$ is positive is $(2, \infty)$.

92. $f(x) = x^2 + 7x + 2$

$f'(x) = 2x + 7$

Solve

$$f'(x) > 0$$
$$2x + 7 > 0$$
$$x > -\frac{7}{2}$$

Therefore, the interval for which $f'(x)$ is positive is $\left(-\frac{7}{2}, \infty\right)$.

93. $f(x)=\frac{1}{3}x^3-x^2-3x+5$

The derivative is positive when $f'(x)>0$.

Find $f'(x)$.

$$f'(x)=\frac{d}{dx}\left(\frac{1}{3}x^3-x^2-3x+5\right)$$
$$=x^2-2x-3$$

Next, we solve the inequality.

$$f'(x)>0$$
$$x^2-2x-3>0$$

First we find where the quadratic is equal to zero, in order to determine the intervals that we will need to test.

$$x^2-2x-3=0$$
$$(x-3)(x+1)=0$$
$$x+1=0 \quad \text{or} \quad x-3=0$$
$$x=-1 \quad \text{or} \quad x=3$$

Now we will test a value to the left of -1, between -1 and 3 and to the right of 3 to determine where the quadratic is positive or negative. We choose the values $x=-2,\ x=0,$ and $x=4$ to test.

When $x=-2$, the derivative $f'(x)$ is

$$f'(-2)=(-2)^2-2(-2)-3=5.$$

When $x=0$, the derivative $f'(x)$ is

$$f'(0)=(0)^2-2(0)-3=-3.$$

When $x=4$, the derivative $f'(x)$ is

$$f'(4)=(4)^2-2(4)-3=5.$$

We organize the results in the table below.

Test point x	Test $x=-2$	-1	Test $x=0$	3	Test $x=4$
$f'(x)$	$f'(-2)=5$	0	$f'(0)=-3$	0	$f'(4)=5$

From the table, we can see that $f'(x)$ is positive on the intervals. $(-\infty,-1)\cup(3,\infty)$.

94. $y=x^4-\frac{4}{3}x^2-4$

$$\frac{dy}{dx}=4x^3-\frac{8}{3}x$$

Solve $\frac{dy}{dx}=0$

$$4x^3-\frac{8}{3}x=0$$
$$4x\left(x^2-\frac{2}{3}\right)=0$$
$$4x=0 \quad \text{or} \quad x^2-\frac{2}{3}=0$$
$$x=0 \quad \text{or} \quad x^2=\frac{2}{3}$$
$$x=0 \quad \text{or} \quad x=\pm\sqrt{\frac{2}{3}}$$

For $x=0$,

$$y=(0)^4-\frac{4}{3}(0)^2-4=-4.$$

For $x=\sqrt{\frac{2}{3}}$

$$y=\left(\sqrt{\frac{2}{3}}\right)^4-\frac{4}{3}\left(\sqrt{\frac{2}{3}}\right)^2-4$$
$$=-\frac{40}{9}$$

For $x=-\sqrt{\frac{2}{3}}$

$$y=\left(-\sqrt{\frac{2}{3}}\right)^4-\frac{4}{3}\left(-\sqrt{\frac{2}{3}}\right)^2-4$$
$$=-\frac{40}{9}$$

There are three points on the graph for which the tangent line is horizontal.

$(0,-4)$, $\left(\sqrt{\frac{2}{3}},-\frac{40}{9}\right)$, and, $\left(-\sqrt{\frac{2}{3}},-\frac{40}{9}\right)$.

95. $y = 2x^6 - x^4 - 2$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(2x^6 - x^4 - 2\right)\\ &= \frac{d}{dx}2x^6 - \frac{d}{dx}x^4 - \frac{d}{dx}2\\ &= 2\left(6x^{6-1}\right) - \left(4x^{4-1}\right) + 0\\ &= 12x^5 - 4x^3\end{aligned}$$

Next, we set the derivative equal to zero and solve for x.

$$\begin{aligned}\frac{dy}{dx} &= 0\\ 12x^5 - 4x^3 &= 0\\ 4x^3\left(3x^2 - 1\right) &= 0\end{aligned}$$

$$\begin{array}{lcl} 4x^3 = 0 & \text{or} & 3x^2 - 1 = 0\\ x = 0 & \text{or} & 3x^2 = 1\\ x = 0 & \text{or} & x^2 = \frac{1}{3}\\ x = 0 & \text{or} & x = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}\end{array}$$

So the horizontal tangent will occur when $x = 0$, $x = \frac{1}{\sqrt{3}}$, and $x = -\frac{1}{\sqrt{3}}$. Next we find the points on the graph.

For $x = 0$,

$$\begin{aligned}y &= 2(0)^6 - (0) - 2\\ &= -2\end{aligned}$$

For $x = \frac{1}{\sqrt{3}}$,

$$\begin{aligned}y &= 2\left(\frac{1}{\sqrt{3}}\right)^6 - \left(\frac{1}{\sqrt{3}}\right)^4 - 2\\ &= 2\left(\frac{1}{27}\right) - \frac{1}{9} - 2\\ &= \frac{2}{27} - \frac{3}{27} - \frac{54}{27}\\ &= -\frac{55}{27}\end{aligned}$$

For $x = -\frac{1}{\sqrt{3}}$,

$$\begin{aligned}y &= 2\left(-\frac{1}{\sqrt{3}}\right)^6 - \left(-\frac{1}{\sqrt{3}}\right)^4 - 2\\ &= 2\left(\frac{1}{27}\right) - \frac{1}{9} - 2\\ &= \frac{2}{27} - \frac{3}{27} - \frac{54}{27}\\ &= -\frac{55}{27}\end{aligned}$$

Therefore, there are horizontal tangents at the points $(0, -2)$, $\left(\frac{1}{\sqrt{3}}, -\frac{55}{27}\right)$, and $\left(-\frac{1}{\sqrt{3}}, -\frac{55}{27}\right)$.

96. $y = (x+3)(x-2) = x^2 + x - 6$

$$\frac{dy}{dx} = 2x + 1$$

97. First, we multiply the two binomials.

$y = (x-1)(x+1) = x^2 - 1$

Therefore,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^2 - 1\right)\\ &= \frac{d}{dx}\left(x^2\right) - \frac{d}{dx}(1)\\ &= 2x\end{aligned}$$

98. $y = \frac{x^5 - x^3}{x^2}$

First, we separate the fraction.

$$\begin{aligned}y &= \frac{x^5}{x^2} - \frac{x^3}{x^2}\\ &= x^{5-2} - x^{3-2} \qquad \left[\frac{a^m}{a^n} = a^{m-n}\right]\\ &= x^3 - x^1\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^3 - x\right)\\ &= \frac{d}{dx}x^3 - \frac{d}{dx}x\\ &= 3x^2 - 1\end{aligned}$$

99. $y = \dfrac{5x^2 - 8x + 3}{8}$

First we separate the fraction.

$$y = \frac{5x^2}{8} - \frac{8x}{8} + \frac{3}{8}$$
$$= \frac{5}{8}x^2 - x + \frac{3}{8}$$

Therefore,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{5}{8}x^2 - x + \frac{3}{8}\right)$$
$$= \frac{d}{dx}\left(\frac{5}{8}x^2\right) - \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{3}{8}\right)$$
$$= \frac{5}{8}\left(2x^{2-1}\right) - 1 + 0$$
$$= \frac{5}{4}x - 1.$$

100. $y = \dfrac{x^5 + x}{x^2}$

First, we separate the fraction.

$$y = \frac{x^5}{x^2} + \frac{x}{x^2}$$
$$= x^{5-2} + x^{1-2}$$
$$= x^3 + x^{-1}$$

Therefore,

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 + x^{-1}\right)$$
$$= 3x^2 + \left(-x^{-1-1}\right)$$
$$= 3x^2 - x^{-2}$$
$$= 3x^2 - \frac{1}{x^2}$$

101. $y = \dfrac{x^5 - 3x^4 + 2x + 4}{x^2}$

First, we separate the fraction.

$$y = \frac{x^5}{x^2} - \frac{3x^4}{x^2} + \frac{2x}{x^2} + \frac{4}{x^2}$$
$$= x^{5-2} - 3x^{4-2} + 2x^{1-2} + 4x^{-2}$$
$$= x^3 - 3x^2 + 2x^{-1} + 4x^{-2}$$

Therefore,

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 - 3x^2 + 2x^{-1} + 4x^{-2}\right)$$
$$= \frac{d}{dx}\left(x^3\right) - \frac{d}{dx}\left(3x^2\right) + \frac{d}{dx}\left(2x^{-1}\right) + \frac{d}{dx}\left(4x^{-2}\right)$$
$$= 3x^2 - 3\left(2x^1\right) + 2\left(-1x^{-1-1}\right) + 4\left(-2x^{-2-1}\right)$$
$$= 3x^2 - 6x - 2x^{-2} - 8x^{-3}$$
$$= 3x^2 - 6x - \frac{2}{x^2} - \frac{8}{x^3}.$$

102. $y = (-4x)^3$

$$y = (-4)^3 \cdot x^3 = -64x^3$$

Therefore,

$$\frac{dy}{dx} = -64\left(3x^{3-1}\right) = -192x^2$$

103. $y = \sqrt{7x}$

First, we simplify the radical.

$$y = \sqrt{7 \cdot x}$$
$$= \sqrt{7}\sqrt{x} \qquad \left[\sqrt{m \cdot n} = \sqrt{m}\sqrt{n}\right]$$
$$= \sqrt{7}\left(x^{1/2}\right) \qquad \left[\sqrt[m]{a} = a^{1/m}; m = 2\right]$$

Therefore,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{7}\,(x)^{1/2}\right)$$
$$= \sqrt{7}\,\frac{d}{dx}\left(x^{1/2}\right)$$
$$= \sqrt{7}\left(\frac{1}{2}x^{1/2-1}\right)$$
$$= \frac{\sqrt{7}}{2}x^{-1/2}$$
$$= \frac{\sqrt{7}}{2x^{1/2}}$$
$$= \frac{\sqrt{7}}{2\sqrt{x}}.$$

104. $y = \sqrt[3]{8x} = (8x)^{1/3} = 8^{1/3} \cdot x^{1/3} = 2x^{1/3}$

$$\frac{dy}{dx} = \frac{2}{3}x^{-\frac{2}{3}} = \frac{2}{3x^{2/3}} = \frac{2}{3\sqrt[3]{x^2}}$$