Chapter 4

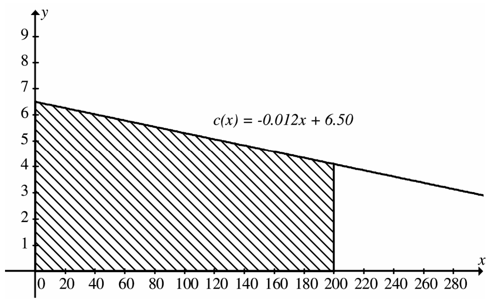
Integration

Exercise Set 4.1

1. $c(x) = -0.012x + 6.50, \quad \text{for } x \le 300$

Note that the cost per pound of roasting coffee decreases as the number of pounds of coffee increases. We use the area under the graph to find the total cost of roasting 200 lb of coffee.

Shading the area under $c(x)$ on the interval $0 \le x \le 200$ we see:



The area under the curve is a trapezoid; therefore, calculating the total cost of roasting 200 lb of coffee will require calculating the area of the trapezoid.

The formula for calculating the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is the height of the trapezoid and b_1 and b_2 are the lengths of the respective bases. If we view the trapezoid sideways, we see that

$h = 200$

$b_1 = c(0) = -0.012(0) + 6.50 = 6.50$

$b_2 = c(200) = -0.012(200) + 6.50 = 4.1$

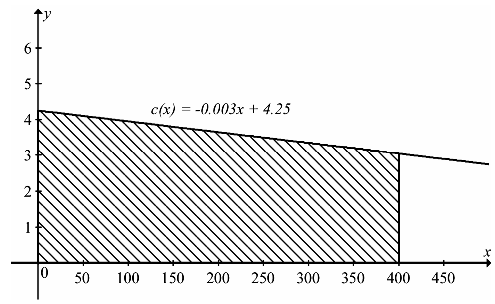
Substituting these values into the formula, we have:

$$\begin{aligned}\text{Total Cost} &= \text{Area of trapezoid}\\ &= \frac{1}{2}h(b_1 + b_2)\\ &= \frac{1}{2}(200)(6.5 + 4.1)\\ &= 100(10.6)\\ &= 1060\end{aligned}$$

The total cost of roasting 200 pounds of coffee is \$1060.

2. $c(x) = -0.003x + 4.25, \text{ for } x \le 500$

Shading the area under $c(x)$ on the interval $0 \le x \le 400$ we see:



We use the area under the graph to find the total cost of producing 400 kg. of cheese. Viewing the trapezoid sideways we have:

$h = 400$

$b_1 = c(0) = -0.003(0) + 4.25 = 4.25$

$b_2 = c(400) = -0.003(400) + 4.25 = 3.05$

Substituting these values into the formula, we have:

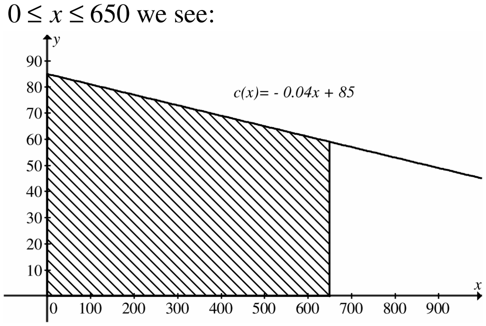
$$\begin{aligned}\text{Total Cost} &= \text{Area of trapezoid}\\ &= \frac{1}{2}h(b_1 + b_2)\\ &= \frac{1}{2}(400)(4.25 + 3.05)\\ &= 1460\end{aligned}$$

The total cost of producing 400 kg. of cheese is \$1460.

3. $c(x) = -0.04x + 85, \quad \text{for } x \le 1000$

Note that the cost per card decreases as the number of cards produced increases. We use the area under the graph to find the total cost of producing 650 cards.

Shading the area $c(x)$ on the interval $0 \le x \le 650$ we see:



The area under the curve is a trapezoid; therefore, calculating the total cost of producing 650 note cards will require calculating the area of the trapezoid.
The formula for calculating the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is the height of the trapezoid and b_1 and b_2 are the lengths of the respective bases. If we view the trapezoid sideways, we see that

$h = 650$

$b_1 = c(0) = -0.04(0) + 85 = 85$

$b_2 = c(650) = -0.04(650) + 85 = 59$

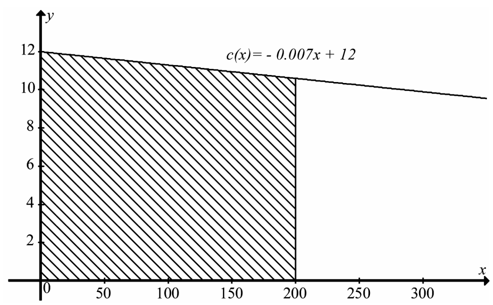
Substituting these values into the formula, we have:

$$\begin{aligned}\text{Total Cost} &= \text{Area of trapezoid} \\ &= \frac{1}{2}h(b_1 + b_2) \\ &= \frac{1}{2}(650)(85 + 59) \\ &= 325(144) \\ &= 46{,}800\end{aligned}$$

The total cost of producing 650 cards is 46,800 cents or \$468.

4. $c(x) = -0.007x + 12$

Shading the area under $c(x)$ on the interval $0 \le x \le 200$ we see:



We use the area under the graph to find the total cost of producing 200 yards of fabric. Viewing the trapezoid sideways we have:

$h = 200$

$b_1 = c(0) = -0.007(0) + 12 = 12$

$b_2 = c(200) = -0.007(200) + 12 = 10.6$

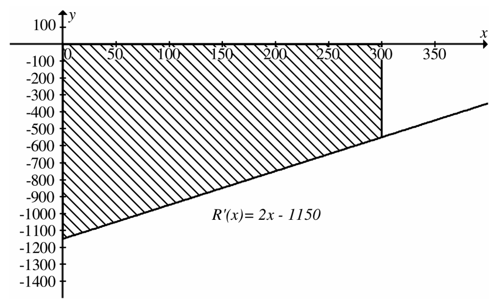
Substituting these values into the formula, we have:

$$\begin{aligned}\text{Total Cost} &= \text{Area of trapezoid} \\ &= \frac{1}{2}h(b_1 + b_2) \\ &= \frac{1}{2}(200)(12 + 10.6) \\ &= 2260\end{aligned}$$

The total cost of producing 200 yards of fabric is \$2260.

5. $R'(x) = 2x - 1150,\ x \ge 0$

Note that the marginal revenue is rate of change of total revenue with respect to the number of tickets. We use the area under the graph to find the total revenue of selling 300 tickets.
Shading the area under $R'(x)$ on the interval $0 \le x \le 300$ we see:



The area between the x-axis and the curve is a trapezoid; therefore, the total revenue will be equal to the area of the trapezoid.
The formula for calculating the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is the height of the trapezoid and b_1 and b_2 are the lengths of the respective bases. If we view the trapezoid sideways, we see that

$h = 200$

$b_1 = R'(0) = 2(0) - 1150 = -1150$

$b_2 = R'(300) = 2(300) - 1150 = -550$

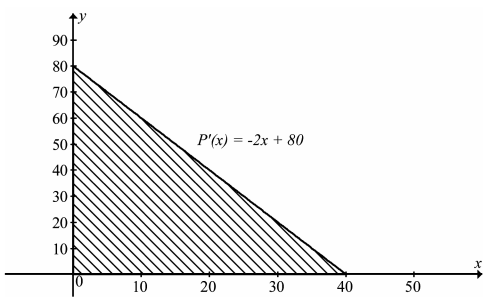
Because the marginal revenue function is below the x-axis, the values for b_1 and b_2 are negative. Due to the nature of the application, we will keep these negative values. The result will be a negative area of the trapezoid. This simply means that the total revenue will be negative, which is a counterintuitive result, but we can calculate it just the same.
Substituting these values into the formula, we have:

$$\begin{aligned}\text{Total Cost} &= \text{Area of trapezoid}\\ &= \frac{1}{2}h(b_1+b_2)\\ &= \frac{1}{2}(300)(-1150+(-550))\\ &= 150(-1700)\\ &= -255{,}000\end{aligned}$$

The total revenue from the sale of the first 300 tickets is $-\$255{,}000$.

6. $P'(x) = -2x+80$

Shading the area under $P'(x)$ on the interval $0 \le x \le 40$ we see:



We use the area under the graph to find the total profit from the sale of the first 40 units. The area under the graph is equal to the area of a triangle with

$h = P'(0) = 80$

$b = 40$

Substituting these values into the formula, we have:

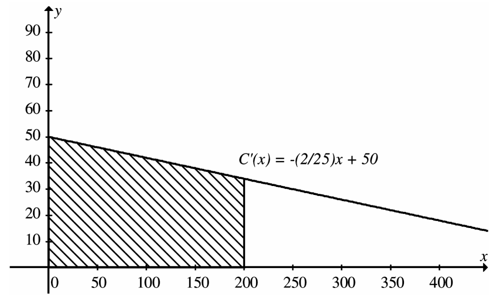
$$\begin{aligned}\text{Total Cost} &= \text{Area of triangle}\\ &= \frac{1}{2}bh\\ &= \frac{1}{2}(40)(80)\\ &= 1600\end{aligned}$$

The total profit from the sale of the first 40 units is \$1600.

7. $C'(x) = -\frac{2}{25}x+50, \quad \text{for } x \le 450$

We use the area under the graph to find the total cost of producing the first 200 dresses.

Shading the area under $C'(x)$ on the interval $0 \le x \le 200$ we see:



The area under the curve is a trapezoid; therefore, calculating the total cost of producing 200 dresses will require calculating the area of the trapezoid.

The formula for calculating the area of a trapezoid is $A = \frac{1}{2}h(b_1+b_2)$, where h is the height of the trapezoid and b_1 and b_2 are the lengths of the respective bases. If we view the trapezoid sideways, we see that

$h = 200$

$b_1 = C'(0) = -\frac{2}{25}(0)+50 = 50$

$b_2 = C'(200) = -\frac{2}{25}(200)+50 = 34$

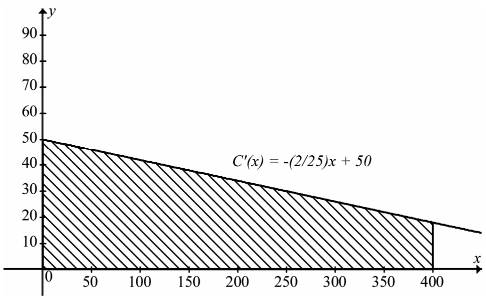
Substituting these values into the formula, we have:

$$\begin{aligned}\text{Total Cost} &= \text{Area of trapezoid}\\ &= \frac{1}{2}h(b_1+b_2)\\ &= \frac{1}{2}(200)(50+34)\\ &= 100(84)\\ &= 8400\end{aligned}$$

The total cost of producing the first 200 dresses is \$8400.

8. From Exercise 7, we know that the total cost of producing the first 200 dresses is \$8400. In order to find the total cost of producing the 201st dress to the 400th dress, we can calculate the total cost of producing the first 400 dresses and then subtract the total cost of producing the first 200 dresses.

Shading the area under $C'(x)$ on the interval $0 \le x \le 400$ we see:



If we view the trapezoid sideways, we see that $h = 400$

$$b_1 = C'(0) = -\frac{2}{25}(0) + 50 = 50$$

$$b_2 = C'(400) = -\frac{2}{25}(400) + 50 = 18$$

Substituting these values into the formula, we have:

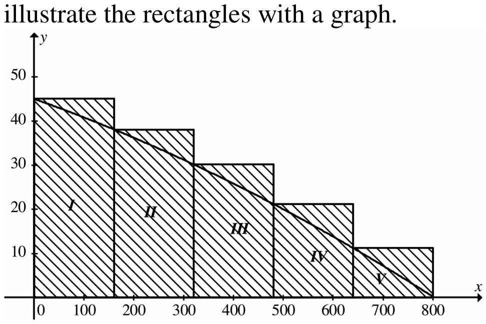
$$\begin{aligned}\text{Total Cost} &= \text{Area of trapezoid}\\ &= \frac{1}{2}h(b_1 + b_2)\\ &= \frac{1}{2}(400)(50+18)\\ &= 13{,}600\end{aligned}$$

The total cost of producing the first 400 dresses is \$13,600.
Therefore, the total cost of producing the 201^{st} dress to the 400^{th} dress is $\$13{,}600 - \$8400 = \$5200$.

9. $C'(x) = -0.00002x^2 - 0.04x + 45$

We divide the interval $[0, 800]$ into five subintervals, each of width $\Delta x = \dfrac{800}{5} = 160.$

To determine the height of each rectangle, we use the left endpoint of each subinterval. We illustrate the rectangles with a graph.



Then, we have

$$\text{Total Cost} \approx \text{Area I} + \text{Area II} + \text{Area III} + \text{Area IV} + \text{Area V}$$

The five left endpoints are 0, 160, 320, 480, and 640. Evaluating $C'(x)$ at each of these endpoints will determine the height of each rectangle and the width was determined to be 160 ft. Therefore, the area of each rectangle is:

$$\begin{aligned}\text{Area I } &= C'(0)\cdot 160 = 45\cdot 160 = 7200\\ \text{Area II } &= C'(160)\cdot 160 = 38.088\cdot 160 = 6094.08\\ \text{Area III } &= C'(320)\cdot 160 = 30.152\cdot 160 = 4824.32\\ \text{Area IV } &= C'(480)\cdot 160 = 21.192\cdot 160 = 3390.72\\ \text{Area V } &= C'(640)\cdot 160 = 11.208\cdot 160 = 1793.28\end{aligned}$$

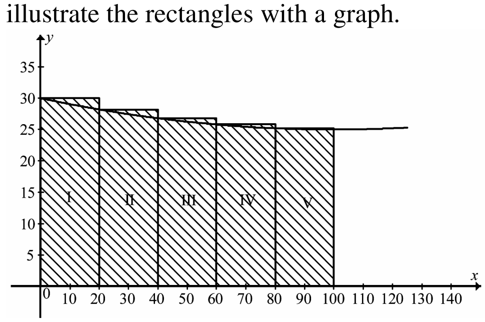
Summing the area of each of the five rectangles yields:

$$\begin{aligned}\text{Total Cost } &\approx 7200 + 6094.08 + 4824.32 + 3390.72 + 1793.28\\ &\approx 23{,}302.4\end{aligned}$$

Thus, the total cost of manufacturing 800 feet of molding is approximately 23,302.4 cents or about \$233.02.

10. $C'(x) = 0.0005x^2 - 0.1x + 30, \quad \text{for } x \le 125$

We divide the interval $[0, 100]$ into five subintervals, each of width $\Delta x = \dfrac{100}{5} = 20.$ To determine the height of each rectangle, we use the left endpoint of each subinterval. We illustrate the rectangles with a graph.



Then, we have

$$\text{Total Cost} \approx \text{Area I} + \text{Area II} + \text{Area III} + \text{Area IV} + \text{Area V}$$

The five left endpoints are 0, 20, 40, 60, and 80. Therefore, the area of each rectangle is

$$\begin{aligned}\text{Area I } &= C'(0)\cdot 20 = 600\\ \text{Area II } &= C'(20)\cdot 20 = 564\\ \text{Area III } &= C'(40)\cdot 20 = 536\\ \text{Area IV } &= C'(60)\cdot 20 = 516\\ \text{Area V } &= C'(80)\cdot 20 = 504\end{aligned}$$

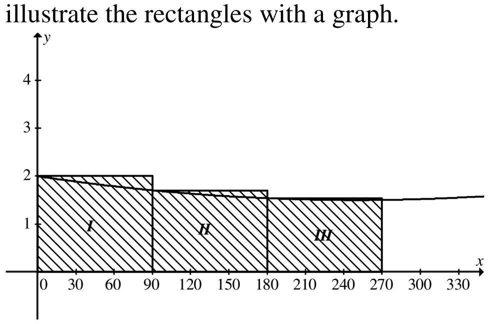
Summing the area of each of the five rectangles yields:

$$\text{Total Cost } \approx 600+564+536+516+504 \approx 2720$$

Thus, the total cost of producing 100 ounces of a new fragrance is approximately $2720.

11. $C'(x)=0.000008x^2-0.004x+2, \text{ for } x \le 350$

We divide the interval $[0,270]$ into three subintervals, each of width $\Delta x=\frac{270}{3}=90.$ To determine the height of each rectangle, we use the left endpoint of each subinterval. We illustrate the rectangles with a graph.



Then, we have

Total Cost ≈ Area I + Area II + Area III

The three left endpoints are 0, 90, and 180. Evaluating $C'(x)$ at each of these endpoints will determine the height of each rectangle and the width was determined to be 160 ft. Therefore, the area of each rectangle is:

$$\text{Area I } = C'(0)\cdot 90 = 2\cdot 90 = 180$$
$$\text{Area II } = C'(90)\cdot 90 = 1.7048\cdot 90 = 153.432$$
$$\text{Area III } = C'(180)\cdot 90 = 1.5392\cdot 90 = 138.528$$

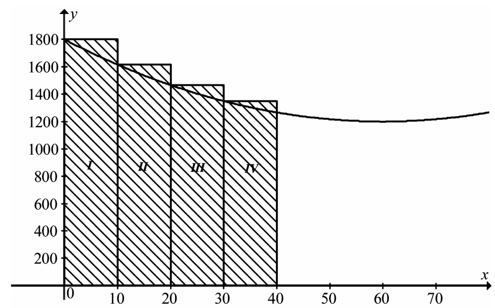
Summing the area of each of the three rectangles yields:

$$\text{Total Cost } \approx 180+153.432+138.528 \approx 471.96$$

Thus, the total cost of producing 270 pints of fresh-squeezed orange juice is approximately $471.96.

12. $C'(x)=\frac{1}{6}x^2-20x+1800, \quad \text{for } x \le 80$

We divide the interval $[0,40]$ into four intervals, each of width $\Delta x=\frac{40}{4}=10.$ To determine the height of each rectangle, we use the left endpoint of each subinterval. We illustrate the rectangles with a graph.



Then, we have

Total Cost ≈ Area I + Area II + Area III + Area IV

The four left endpoints are 0, 10, 20, and 30. Therefore, the area of each rectangle is

$$\text{Area I } = C'(0)\cdot 10 = 18{,}000$$
$$\text{Area II } = C'(10)\cdot 10 \approx 16{,}166.667$$
$$\text{Area III } = C'(20)\cdot 10 \approx 14{,}666.667$$
$$\text{Area IV } = C'(30)\cdot 10 = 13{,}500$$

Summing the area of each of the four rectangles yields:

$$\text{Total Cost } \approx 18{,}000+16{,}166.667+14{,}666.667+13{,}500 \approx 62{,}333.33$$

Thus, the total cost of paving 4000 feet of road surface is approximately $62,333.33.

13. Note that we are adding consecutive multiples of 3:

$$3+6+9+12+15+18=\sum_{i=1}^{6} 3i.$$

14. $5+10+15+20+25+30+35=\sum_{i=1}^{7} 5i.$

15. $f(x_1)+f(x_2)+f(x_3)+f(x_4)=\sum_{i=1}^{4} f(x_i).$

16. $g(x_1)+\cdots+g(x_5)=\sum_{i=1}^{5} g(x_i).$

17. $G(x_1)+G(x_2)+\cdots+G(x_{15})=\sum_{i=1}^{15} G(x_i).$

18. $F(x_1)+F(x_2)+\cdots+F(x_{17})=\sum_{i=1}^{17} F(x_i).$

19. $\sum_{i=1}^{4} 2^i = 2^1+2^2+2^3+2^4$

$= 2+4+8+16, \text{or } 30$

20. $\sum_{i=0}^{5}(-2)^i$
$= (-2)^0 + (-2)^1 + (-2)^2 + (-2)^3 + (-2)^4 + (-2)^5$
$= 1 - 2 + 4 - 8 + 16 - 32$, or -21.

21. $\sum_{i=1}^{5} f(x_i)$
$= f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)$.

22. $\sum_{i=1}^{4} g(x_i) = g(x_1) + g(x_2) + g(x_3) + g(x_4)$.

23. $f(x) = \frac{1}{x^2}$

a) In the drawing in the text the interval $[1,7]$ has been divided into 6 subintervals, each having width 1 $\left[\Delta x = \frac{7-1}{6} = 1\right]$.

The heights of the rectangles shown are

$f(1) = \frac{1}{1^2} = 1$

$f(2) = \frac{1}{2^2} = \frac{1}{4} = 0.2500$

$f(3) = \frac{1}{3^2} = \frac{1}{9} \approx 0.1111$

$f(4) = \frac{1}{4^2} = \frac{1}{16} = 0.0625$

$f(5) = \frac{1}{5^2} = \frac{1}{25} = 0.0400$

$f(6) = \frac{1}{6^2} = \frac{1}{36} \approx 0.0278$

Therefore, the area of each rectangle is:

Rectangle I $= f(1) \cdot \Delta x = 1 \cdot 1 = 1$

Rectangle II $= f(2) \cdot \Delta x = 0.2500 \cdot 1 = 0.2500$

Rectangle III $= f(3) \cdot \Delta x = 0.1111 \cdot 1 = 0.1111$

Rectangle IV $= f(4) \cdot \Delta x = 0.0625 \cdot 1 = 0.0625$

Rectangle V $= f(5) \cdot \Delta x = 1 \cdot 0.0400 = 0.0400$

Rectangle VI $= f(6) \cdot \Delta x = 1 \cdot 0.0278 = 0.0278$

The area of the region under the curve over $[1,7]$ is approximately the sum of the areas of the 6 rectangles. Therefore, the total area is approximately:

$1 + 0.2500 + 0.1111 + 0.0625 +$
$0.0400 + 0.0278 \approx 1.4914$.

b) In the drawing in the text the interval $[1,7]$ has been divided into 12 subintervals, each having width 0.5
$\left[\Delta x = \frac{7-1}{12} = \frac{6}{12} = 0.5\right]$.

The heights of 6 of the rectangles were computed in part (a). The heights of the other 6 rectangles are computed below:

$f(1.5) = \frac{1}{1.5^2} = \frac{1}{2.25} \approx 0.4444$

$f(2.5) = \frac{1}{2.5^2} = \frac{1}{6.25} = 0.1600$

$f(3.5) = \frac{1}{3.5^2} = \frac{1}{12.25} \approx 0.00816$

$f(4.5) = \frac{1}{4.5^2} = \frac{1}{20.25} \approx 0.0494$

$f(5.5) = \frac{1}{5.5^2} = \frac{1}{30.25} \approx 0.0331$

$f(6.5) = \frac{1}{6.5^2} = \frac{1}{42.25} \approx 0.0237$

Therefore, the area of each rectangle is:

Rectangle I: $f(1) \cdot \Delta x = 1(0.5) = 0.5000$

Rectangle II: $f(1.5) \cdot \Delta x = 0.4444(0.5) \approx 0.2222$

Rectangle III: $f(2) \cdot \Delta x = 0.2500(0.5) = 0.1250$

Rectangle IV: $f(2.5) \cdot \Delta x = 0.1600(0.5) = 0.0800$

Rectangle V: $f(3) \cdot \Delta x = 0.1111(0.5) \approx 0.0556$

Rectangle VI: $f(3.5) \cdot \Delta x = 0.0816(0.5) \approx 0.0408$

Rectangle VII: $f(4) \cdot \Delta x = 0.0625(0.5) \approx 0.0313$

Rectangle VIII: $f(4.5) \cdot \Delta x = 0.0494(0.5) \approx 0.0247$

Rectangle IX: $f(5) \cdot \Delta x = 0.0400(0.5) = 0.0200$

Rectangle X: $f(5.5) \cdot \Delta x = 0.0331 \approx 0.0165$

Rectangle XI: $f(6) \cdot \Delta x = 0.0278(0.5) \approx 0.0139$

Rectangle XII: $f(6.5) \cdot \Delta x = 0.0237(0.5) \approx 0.0118$

The area of the region under the curve over $[1,7]$ is approximately the sum of the areas of the 12 rectangles. Therefore, the total area is approximately:

$0.5 + 0.2222 + 0.1250 + 0.0800 +$
$0.0556 + 0.0408 + 0.0313 +$
$0.0247 + 0.0200 + 0.0165 +$
$0.0139 + 0.0118 \approx 1.1418$.

Answers may vary slightly depending on when rounding was done.

24. $f(x) = x^2 + 1$

a) $\Delta x = \dfrac{5-0}{5} = 1$

$$\begin{aligned} A &\approx \sum_{i=1}^{5} f(x_i)\Delta x \\ &= f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1 \\ &= (0^2+1)\cdot 1 + (1^2+1)\cdot 1 + (2^2+1)\cdot 1 + (3^2+1)\cdot 1 + (4^2+1)\cdot 1 \\ &= 1+2+5+10+17 \\ &= 35 \end{aligned}$$

b) $\Delta x = \dfrac{5-0}{10} = 0.5$

$$\begin{aligned} A &\approx \sum_{i=1}^{10} f(x_i)\Delta x \\ &= f(0)\cdot 0.5 + f(0.5)\cdot 0.5 + f(1)\cdot 0.5 + f(1.5)\cdot 0.5 + f(2)\cdot 0.5 + f(2.5)\cdot 0.5 + f(3)\cdot 0.5 + f(3.5)\cdot 0.5 + f(4)\cdot 0.5 + f(4.5)\cdot 0.5 \\ &= (0^2+1)\cdot 0.5 + ((0.5)^2+1)\cdot 0.5 + (1^2+1)\cdot 1 + ((1.5)^2+1)\cdot 0.5 + (2^2+1)\cdot 0.5 + ((2.5)^2+1)\cdot 0.5 + (3^2+1)\cdot 0.5 + ((3.5)^2+1)\cdot 0.5 + (4^2+1)\cdot 0.5 + ((4.5)^2+1)\cdot 0.5 \\ &= 40.625 \end{aligned}$$

25. $P'(x) = -0.0006x^3 + 0.28x^2 + 55.6x$

We divide $[0,300]$ into 6 subintervals of width $\Delta x = 50$. Therefore, the values of x_i are:
$x_1 = 0$; $x_2 = 50$; $x_3 = 100$;
$x_4 = 150$; $x_5 = 200$; $x_6 = 250$.
Then, the area under the curve is approximately:

$$\begin{aligned} \sum_{i=1}^{6} P'(x_i)\Delta x &= P'(x_1)\cdot 50 + P'(x_2)\cdot 50 + P'(x_3)\cdot 50 + P'(x_4)\cdot 50 + P'(x_5)\cdot 50 + P'(x_6)\cdot 50 \\ &= P'(0)\cdot 50 + P'(50)\cdot 50 + P'(100)\cdot 50 + P'(150)\cdot 50 + P'(200)\cdot 50 + P'(250)\cdot 50 \\ &= 0\cdot 50 + 3405\cdot 50 + 7760\cdot 50 + 12{,}615\cdot 50 + 17{,}520\cdot 50 + 22{,}025\cdot 50 \\ &= 0 + 170{,}250 + 388{,}000 + 630{,}750 + 876{,}000 + 1{,}101{,}250 \\ &= 3{,}166{,}250. \end{aligned}$$

The health club's total profit when 300 members are enrolled is approximately 3,166,250 cents or $31,662.50.

26. $C'(x) = 0.0003x^2 - 0.2x + 50$

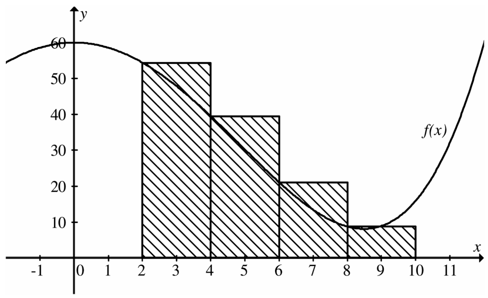
We divide $[0,400]$ into 4 subintervals of width $\Delta x = 100$. Therefore, the values of x_i range from $x_1 = 0$ to $x_4 = 300$.
The, the area under the curve is approximately

$$\begin{aligned} \sum_{i=1}^{4} C'(x_i)\Delta x &= C'(0)\cdot 100 + C'(100)\cdot 100 + C'(200)\cdot 100 + C'(300)\cdot 100 \\ &= 5000 + 3300 + 2200 + 1700 \\ &= 12{,}200 \end{aligned}$$

The total cost of producing 400 jackets is approximately $12,200.

27. $f(x) = 0.01x^4 - 1.44x^2 + 60$

Dividing the interval $[2,10]$ into four subintervals, we calculate the width of each subinterval to be $\Delta x = \dfrac{10-2}{4} = \dfrac{8}{4} = 2$, with x_i ranging from $x_1 = 2$ to $x_4 = 8$. Although a drawing is not required, we can make one to help visualize the area.

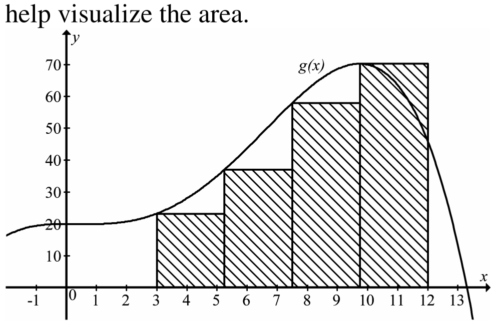


The area under the curve from 2 to 10 is approximately

$$\sum_{i=1}^{4} f(x_i)\Delta x = f(2)\cdot 2 + f(4)\cdot 2 + f(6)\cdot 2 + f(8)\cdot 2$$
$$= 54.4\cdot 2 + 39.52\cdot 2 + 21.12\cdot 2 + 8.8\cdot 2$$
$$= 247.68.$$

28. $g(x) = -0.02x^4 + 0.28x^3 - 0.3x^2 + 20$

Dividing the interval $[3,12]$ into four subintervals, we calculate the width of each subinterval to be $\Delta x = \frac{12-3}{4} = \frac{9}{4} = 2.25$, with x_i ranging from $x_1 = 3$ to $x_4 = 9.75$. Although a drawing is not required, we can make one to help visualize the area.



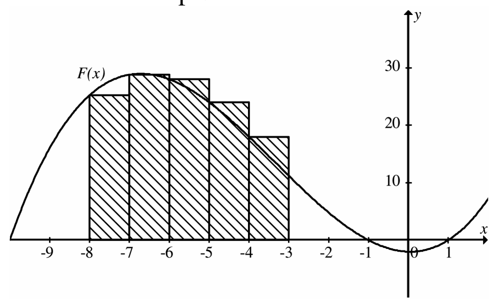
The area under the curve from 3 to 12 is approximately

$$\sum_{i=1}^{4} g(x_i)\Delta x = g(3)\cdot 2.25 + g(5.25)\cdot 2.25 + \text{g}(7.5)\cdot 2.25 + g(9.75)\cdot 2.25$$
$$= 424.19.$$

29. $F(x) = 0.2x^3 + 2x^2 - 0.2x - 2$

Dividing the interval $[-8,-3]$ into five subintervals, we calculate the width of each subinterval to be $\Delta x = \frac{-3-(-8)}{5} = \frac{5}{5} = 1$, with x_i ranging from $x_1 = -8$ to $x_5 = -4$.

Although a drawing is not required, we can make one to help visualize the area.

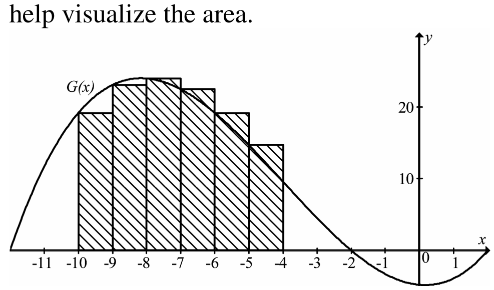


The area under the curve from -8 to -3 is approximately

$$\sum_{i=1}^{5} F(x_i)\Delta x = F(-8)\cdot 1 + F(-7)\cdot 1 + F(-6)\cdot 1 + F(-5)\cdot 1 + F(-4)\cdot 1$$
$$= 25.2\cdot 1 + 28.8\cdot 1 + 28\cdot 1 + 24\cdot 1 + 18\cdot 1$$
$$= 124.$$

30. $G(x) = 0.1x^3 + 1.2x^2 - 0.4x - 4.8$

Dividing the interval $[-10,-4]$ into six subintervals, we calculate the width of each subinterval to be $\Delta x = \frac{-4-(-10)}{6} = \frac{6}{6} = 1$, with x_i ranging from $x_1 = -10$ to $x_6 = -5$. Although a drawing is not required, we can make one to help visualize the area.



The area under the curve from -10 to -4 is approximately

$$\sum_{i=1}^{6} G(x_i)\Delta x = G(-10)\cdot 1 + G(-9)\cdot 1 + G(-8)\cdot 1$$
$$G(-7)\cdot 1 + G(-6)\cdot 1 + G(-5)\cdot 1$$
$$= 122.7.$$

31. For the specific case, begin by expanding the sum:

$$\sum_{i=1}^{4} k\cdot f(x_i) = k\cdot f(x_1) + k\cdot f(x_2) + k\cdot f(x_3) + k\cdot f(x_4)$$

Next, we factor out the common factor of k to get:

$$= k\left[f(x_1)+f(x_2)+f(x_3)+f(x_4)\right]$$
$$= k\cdot\left[\sum_{i=1}^{4} f(x_i)\right]$$

Thus:

$$\sum_{i=1}^{4} k\cdot f(x_i) = k\sum_{i=1}^{4} f(x_i).$$

Similarly for the general case, we have:

$$\sum_{i=1}^{n} k\cdot f(x_i)$$
$$= k\cdot f(x_1) + k\cdot f(x_2) + \cdots + k\cdot f(x_n)$$
$$= k\left[f(x_1)+f(x_2)+\cdots+f(x_n)\right]$$
$$= k\left[\sum_{i=1}^{n} f(x_i)\right]$$

Thus:

$$\sum_{i=1}^{n} k\cdot f(x_i) = k\sum_{i=1}^{n} f(x_i).$$

32. $f(x) = \dfrac{1}{x^2}$

$$\Delta x = \frac{7-1}{6} = 1$$

Using the Trapezoidal Rule, the area under the graph of $f(x)$ over the interval $[1,7]$ is approximately:

$$\text{Area} \approx \Delta x\left[\frac{f(1)}{2} + f(2) + \cdots + f(6) + \frac{f(7)}{2}\right]$$
$$\approx 1\cdot\left[\frac{\frac{1}{1^2}}{2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{\frac{1}{7^2}}{2}\right]$$
$$\approx \frac{1}{2} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \frac{1}{98}$$
$$\approx 1.0016$$

33. $f(x) = x^2 + 1$

$$\Delta x = \frac{5-0}{5} = 1$$

Using the Trapezoidal Rule, the area under the graph of $f(x)$ over the interval $[0,5]$ is approximately:

$$\text{Area} \approx \Delta x\left[\frac{f(0)}{2} + f(1) + \cdots + f(4) + \frac{f(5)}{2}\right]$$

The function values are:

$$f(0) = 0^2 + 1 = 1$$
$$f(1) = 1^2 + 1 = 2$$
$$f(2) = 2^2 + 1 = 5$$
$$f(3) = 3^2 + 1 = 10$$
$$f(4) = 4^2 + 1 = 17$$
$$f(5) = 5^2 + 1 = 26$$

Substituting these values into the Trapezoidal Rule, we have:

$$\text{Area} \approx 1\cdot\left[\frac{1}{2} + 2 + 5 + 10 + 17 + \frac{26}{2}\right]$$
$$\approx \frac{1}{2} + 2 + 5 + 10 + 17 + 13$$
$$\approx 47.5$$

34. $f(x) = \sqrt{25 - x^2}$

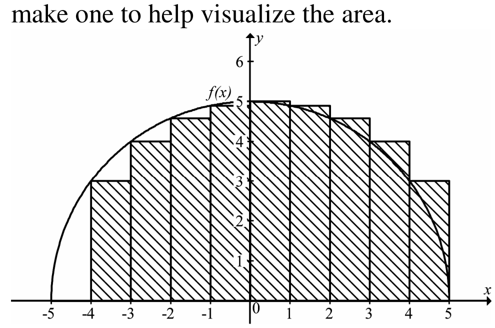
Notice that this semi-circle has a radius of five. Therefore the exact area is given by:

$$A = \frac{1}{2}\pi(5)^2 = \frac{25}{2}\pi = 12.5\pi$$

To approximate the area under the graph of $f(x) = \sqrt{25-x^2}$ using 10 rectangles, we first find the width of each rectangle.

$$\Delta x = \frac{5-(-5)}{10} = 1.$$

The x_i will range from $x_1 = -5$ to $x_{10} = 4$. Although a drawing is not required, we can make one to help visualize the area.



The area under the curve from -5 to 5 is approximately

$$\sum_{i=1}^{10} f(x_i)\Delta x = f(-5)\cdot 1 + f(-4)\cdot 1 + f(-3)\cdot 1$$
$$f(-2)\cdot 1 + f(-1)\cdot 1 + f(0)\cdot 1$$
$$f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1 +$$
$$f(4)\cdot 1$$
$$\approx 37.9631$$

In order to compare, if we round to 4 decimal places we have:
$A = 12.5\pi = 39.2699$.

35. $g(x) = \sqrt{49 - x^2}$

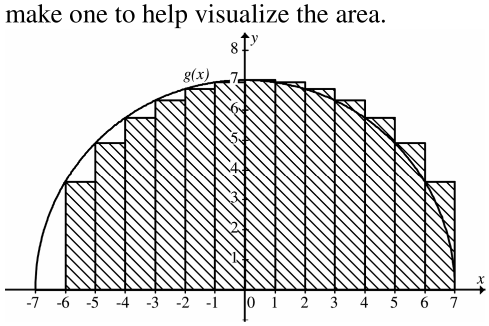
Notice that this semi-circle has a radius of seven. Therefore the exact area is given by:

$$A = \frac{1}{2}\pi(7)^2 = \frac{49}{2}\pi = 24.5\pi$$

To approximate the area under the graph of $g(x) = \sqrt{49 - x^2}$ using 14 rectangles, we first find the width of each rectangle.

$$\Delta x = \frac{7-(-7)}{14} = \frac{14}{14} = 1 .$$

The x_i will range from $x_1 = -7$ to $x_{10} = 6$.
Although a drawing is not required, we can make one to help visualize the area.



The area under the curve from -7 to 7 is approximately

$$\sum_{i=1}^{14} f(x_i)\Delta x = f(-7)\cdot 1 + f(-6)\cdot 1 +$$
$$f(-5)\cdot 1 + f(-4)\cdot 1 +$$
$$f(-3)\cdot 1 + f(-2)\cdot 1 +$$
$$f(-1)\cdot 1 + f(0)\cdot 1 + f(1)\cdot 1 +$$
$$f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1 +$$
$$f(5)\cdot 1 + f(6)\cdot 1$$
$$\approx 75.4201$$

In order to compare, if we round to 4 decimal places we have:
$A = 24.5\pi = 76.9690$.

Exercise Set 4.2

1. $\int x^6 dx$

$$= \frac{x^{6+1}}{6+1} + C \qquad \left[\int x^r dx = \frac{x^{r+1}}{r+1} + C\right]$$
$$= \frac{x^7}{7} + C$$ Don't forget the C.

2. $\int x^7 dx = \frac{x^8}{8} + C$

3. $\int 2dx$

$$= 2x + C \qquad \left[\int kdx = kx + C\right]$$

4. $\int 4dx = 4x + C$

5. $\int x^{1/4} dx$

$$= \frac{x^{1/4+1}}{\frac{1}{4}+1} + C \qquad \left[\int x^r dx = \frac{x^{r+1}}{r+1} + C\right]$$
$$= \frac{x^{5/4}}{\frac{5}{4}} + C$$
$$= \frac{4}{5}x^{5/4} + C$$

6. $\int x^{1/3} dx = \frac{3}{4}x^{4/3} + C$

7. $\int (x^2 + x - 1) dx$

$$= \int x^2 dx + \int x dx - \int 1 dx$$ The integral of a sum is the sum of the integrals.

$$= \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} - x + C \leftarrow$$ DON'T FORGET THE C!

$$\left[\int x^r dx = \frac{x^{r+1}}{r+1} + C\right]$$
$$\left[\int kdx = kx + C\right]$$

$$= \frac{x^3}{3} + \frac{x^2}{2} - x + C$$

8. $\int (x^2 - x + 2) dx = \frac{x^3}{3} - \frac{x^2}{2} + 2x + C$

9. $\int\left(2t^2+5t-3\right)dt$

$=\int 2t^2dt+\int 5tdt-\int 3dt$ The integral of a sum is the sum of the integrals.

$=2\cdot\frac{t^{2+1}}{2+1}+5\cdot\frac{t^{1+1}}{1+1}-3t+C$

$\left[\int x^r dx=\frac{x^{r+1}}{r+1}+C\right]$

$\left[\int kdx=kx+C\right]$

$=\frac{2}{3}t^3+\frac{5}{2}t^2-3t+C$

10. $\int\left(3t^2-4t+7\right)dt=t^3-2t^2+7t+C$

11. $\int\frac{1}{x^3}dx=\int x^{-3}dx$

$=\frac{x^{-3+1}}{-3+1}+C$ $\left[\int x^r dx=\frac{x^{r+1}}{r+1}+C\right]$

$=-\frac{x^{-2}}{2}+C$

12. $\int\frac{1}{x^5}dx=-\frac{x^{-4}}{4}+C$

13. $\int\sqrt[3]{x}dx=\int x^{1/3}dx$

$=\frac{x^{1/3+1}}{\frac{1}{3}+1}+C$ $\left[\int x^r dx=\frac{x^{r+1}}{r+1}+C\right]$

$=\frac{x^{4/3}}{\frac{4}{3}}+C$

$=\frac{3}{4}x^{4/3}+C$

14. $\int\sqrt{x}dx=\frac{2}{3}x^{3/2}+C$

15. $\int\sqrt{x^5}dx=\int x^{5/2}dx$

$=\frac{x^{5/2+1}}{\frac{5}{2}+1}+C$ $\left[\int x^r dx=\frac{x^{r+1}}{r+1}+C\right]$

$=\frac{x^{7/2}}{\frac{7}{2}}+C$

$=\frac{2}{7}x^{7/2}+C$

16. $\int\sqrt[3]{x^2}dx=\int x^{2/3}dx=\frac{3}{5}x^{5/3}+C$

17. $\int\frac{dx}{x^4}=\int\frac{1}{x^4}dx=\int x^{-4}dx$

$=\frac{x^{-4+1}}{-4+1}+C$ $\left[\int x^r dx=\frac{x^{r+1}}{r+1}+C\right]$

$=-\frac{x^{-3}}{3}+C$

18. $\int\frac{dx}{x^2}=\int x^{-2}dx=-x^{-1}+C$

19. $\int\frac{1}{x}dx$

$=\ln x+C,\ x>0$ $\left[\int\frac{1}{x}dx=\ln x+C, x>0\right]$

20. $\int\frac{2}{x}dx=2\ln x+C,\quad x>0$

21. $\int\left(\frac{3}{x}+\frac{5}{x^2}\right)dx$

$=\int\frac{3}{x}dx+\int\frac{5}{x^2}dx$ The integral of a sum is the sum of the integrals.

$=3\int x^{-1}dx+\int 5x^{-2}dx$

$=3\cdot\ln x+5\cdot\frac{x^{-2+1}}{-2+1}+C,\quad x>0$

$\left[\int x^{-1}dx=\ln x, x>0\right]$

$\left[\int x^r dx=\frac{x^{r+1}}{r+1}+C\right]$

$=3\ln x-5x^{-1}+C$

22. $\int\left(\frac{4}{x^3}+\frac{7}{x}\right)dx$

$=\int 4x^{-3}dx+\int 7x^{-1}dx$

$=-2x^{-2}+7\ln x+C,\quad x>0$

$=7\ln x-2x^{-2}+C$

23. $\int \frac{-7}{\sqrt[3]{x^2}}\,dx = \int \frac{-7}{x^{2/3}} = \int -7x^{-2/3}\,dx$

$-7\int x^{-2/3}\,dx$

$= -7\cdot\frac{x^{-2/3+1}}{-\frac{2}{3}+1} + C \qquad \left[\int x^r\,dx = \frac{x^{r+1}}{r+1} + C\right]$

$= -7\cdot\frac{x^{1/3}}{\frac{1}{3}} + C$

$= -21x^{1/3} + C$

24. $\int \frac{5}{\sqrt[4]{x^3}}\,dx = \int 5x^{-3/4}\,dx$

$= 5\cdot\frac{x^{1/4}}{\frac{1}{4}} + C = 20x^{1/4} + C$

25. $\int 2e^{2x}\,dx$

$= \frac{2}{2}e^{2x} + C \qquad \left[\int be^{ax}\,dx = \frac{b}{a}e^{ax} + C\right]$

$= e^{2x} + C$

26. $\int 4e^{4x}\,dx = e^{4x} + C$

27. $\int e^{3x}\,dx$

$= \frac{1}{3}e^{3x} + C \qquad \left[\int be^{ax}\,dx = \frac{b}{a}e^{ax} + C\right]$

28. $\int e^{5x}\,dx = \frac{1}{5}e^{5x} + C$

29. $\int e^{7x}\,dx$

$= \frac{1}{7}e^{7x} + C \qquad \left[\int be^{ax}\,dx = \frac{b}{a}e^{ax} + C\right]$

30. $\int e^{6x}\,dx = \frac{1}{6}e^{6x} + C$

31. $\int 5e^{3x}\,dx$

$= \frac{5}{3}e^{3x} + C \qquad \left[\int be^{ax}\,dx = \frac{b}{a}e^{ax} + C\right]$

32. $\int 2e^{5x}\,dx = \frac{2}{5}e^{5x} + C$

33. $\int 6e^{8x}\,dx$

$= \frac{6}{8}e^{8x} + C \qquad \left[\int be^{ax}\,dx = \frac{b}{a}e^{ax} + C\right]$

$= \frac{3}{4}e^{8x} + C$

34. $\int 12e^{3x}\,dx = \frac{12}{3}e^{3x} + C = 4e^{3x} + C$

35. $\int \frac{2}{3}e^{-9x}\,dx$

$= \frac{2}{3}\cdot\frac{1}{-9}e^{-9x} + C \qquad \left[\int be^{ax}\,dx = \frac{b}{a}e^{ax} + C\right]$

$= -\frac{2}{27}e^{-9x} + C$

36. $\int \frac{4}{5}e^{-10x}\,dx$

$= \frac{4}{5}\cdot\frac{1}{-10}e^{-10x} + C = -\frac{2}{25}e^{-10x} + C$

37. $\int \left(5x^2 - 2e^{7x}\right)dx$

$= \int 5x^2\,dx - \int 2e^{7x}\,dx$ The integral of a sum is the sum of the integrals.

$= 5\cdot\frac{x^{2+1}}{2+1} - \frac{2}{7}e^{7x} + C$

$\left[\int x^r\,dx = \frac{x^{r+1}}{r+1} + C\right]$

$\left[\int be^{ax}\,dx = \frac{b}{a}e^{ax} + C\right]$

$= \frac{5}{3}x^3 - \frac{2}{7}e^{7x} + C$

38. $\int \left(2x^5 - 4e^{3x}\right)dx = \frac{1}{3}x^6 - \frac{4}{3}e^{3x} + C$

39. $\int\left(x^2-\frac{3}{2}\sqrt{x}+x^{-4/3}\right)dx$

$=\int\left(x^2-\frac{3}{2}x^{1/2}+x^{-4/3}\right)dx$

$=\int x^2dx-\int\frac{3}{2}x^{1/2}dx+\int x^{-4/3}dx$

$=\frac{x^{2+1}}{2+1}-\frac{3}{2}\cdot\frac{x^{1/2+1}}{\frac{1}{2}+1}+\frac{x^{-4/3+1}}{-\frac{4}{3}+1}+C$

$\left[\int x^r\,dx=\frac{x^{r+1}}{r+1}+C\right]$

$=\frac{x^3}{3}-\frac{3}{2}\cdot\frac{x^{3/2}}{\frac{3}{2}}+\frac{x^{-1/3}}{-\frac{1}{3}}+C$

$=\frac{x^3}{3}-x^{3/2}-3x^{-1/3}+C$

40. $\int\left(x^4+\frac{1}{8\sqrt{x}}-\frac{4}{5}x^{-2/5}\right)dx$

$=\int\left(x^4+\frac{1}{8}x^{-1/2}-\frac{4}{5}x^{-2/5}\right)dx$

$=\frac{x^5}{5}+\frac{1}{8}\cdot\frac{x^{1/2}}{\frac{1}{2}}-\frac{4}{5}\frac{x^{3/5}}{\frac{3}{5}}+C$

$=\frac{x^5}{5}-\frac{4}{3}x^{3/5}+\frac{1}{4}x^{1/2}+C$

41. $\int(3x+2)^2\,dx=\int\left(9x^2+12x+4\right)dx$

$=\int 9x^2dx+\int 12x\,dx+\int 4dx$

$=9\cdot\frac{x^{2+1}}{2+1}+12\cdot\frac{x^{1+1}}{1+1}+4x+C$

$=\frac{9}{3}x^3+\frac{12}{2}x^2+4x+C$

$=3x^3+6x^2+4x+C$

42. $\int(x+4)^2\,dx=\int\left(x^2+8x+16\right)dx$

$=\frac{x^3}{3}+\frac{8}{2}x^2+16x+C$

$=\frac{x^3}{3}+4x^2+16x+C$

43. $\int\left(\frac{3}{x}-5e^{2x}+\sqrt{x^7}\right)dx$

$=\int\left(\frac{3}{x}-5e^{2x}+x^{7/2}\right)dx$

$=\int\frac{3}{x}dx-\int 5e^{2x}dx+\int x^{7/2}dx$

$=3\ln x-\frac{5}{2}\cdot e^{2x}+\frac{x^{7/2+1}}{\frac{7}{2}+1}+C$

$=3\ln x-\frac{5}{2}e^{2x}+\frac{2}{9}x^{9/2}+C$

44. $\int\left(2e^{6x}-\frac{3}{x}+\sqrt[3]{x^4}\right)dx$

$=\int\left(2e^{6x}-\frac{3}{x}+x^{4/3}\right)dx$

$=\frac{2}{6}\cdot e^{6x}-3\ln x+\frac{x^{7/3}}{\frac{7}{3}}+C$

$=\frac{1}{3}\cdot e^{6x}-3\ln x+\frac{3}{7}x^{7/3}+C$

45. $\int\left(\frac{7}{\sqrt{x}}-\frac{2}{3}e^{5x}-\frac{8}{x}\right)dx$

$=\int\left(7x^{-1/2}-\frac{2}{3}e^{5x}-\frac{8}{x}\right)dx$

$=\int 7x^{-1/2}dx-\int\frac{2}{3}e^{5x}dx-\int\frac{8}{x}dx$

$=7\cdot\frac{x^{-1/2+1}}{-\frac{1}{2}+1}-\frac{2}{3}\cdot\frac{1}{5}e^{5x}-8\ln x+C$

$=14x^{1/2}-\frac{2}{15}e^{5x}-8\ln x+C$

46. $\int\left(\frac{4}{\sqrt[5]{x}}+\frac{3}{4}e^{6x}-\frac{7}{x}\right)dx$

$=\int\left(4x^{-1/5}+\frac{3}{4}e^{6x}-\frac{7}{x}\right)dx$

$=4\cdot\frac{x^{4/5}}{\frac{4}{5}}+\frac{3}{4}\cdot\frac{1}{6}e^{6x}-7\ln x+C$

$=5x^{4/5}+\frac{1}{8}e^{6x}-7\ln x+C$

47. Find the function $f(x)$, such that

$f'(x) = x - 3,\ f(2) = 9$

We first find $f(x)$ by integrating:

$$\begin{aligned} f(x) &= \int (x-3)\,dx \\ &= \int x\,dx - \int 3\,dx \\ &= \frac{1}{2}x^2 - 3x + C. \end{aligned}$$

The condition $f(2) = 9$ allows us to find *C*:

$$\begin{aligned} f(2) &= 9 \\ \frac{1}{2}(2)^2 - 3(2) + C &= 9 \\ 2 - 6 + C &= 9 \\ -4 + C &= 9 \\ C &= 13. \end{aligned}$$

Thus, $f(x) = \frac{1}{2}x^2 - 3x + 13.$

48. $f'(x) = x - 5,\quad f(1) = 6$

$$\begin{aligned} f(x) &= \int (x-5)\,dx \\ &= \frac{1}{2}x^2 - 5x + C \\ f(1) &= 6 \\ \frac{1}{2}(1)^2 - 5(1) + C &= 6 \\ -\frac{9}{2} + C &= 6 \\ C &= \frac{21}{2} \end{aligned}$$

Thus, $f(x) = \frac{1}{2}x^2 - 5x + \frac{21}{2}.$

49. Find the function $f(x)$, such that

$f'(x) = x^2 - 4,\ f(0) = 7$.

We first find $f(x)$ by integrating:

$$\begin{aligned} f(x) &= \int (x^2-4)\,dx \\ &= \int x^2\,dx - \int 4\,dx \\ &= \frac{1}{3}x^3 - 4x + C. \end{aligned}$$

The condition $f(0) = 7$ allows us to find *C*:

$$\begin{aligned} f(0) &= 7 \\ \frac{1}{3}(0)^3 - 4(0) + C &= 7 \\ C &= 7. \end{aligned}$$

Thus, $f(x) = \frac{1}{3}x^3 - 4x + 7.$

50. $f'(x) = x^2 + 1,\quad f(0) = 8$

$$\begin{aligned} f(x) &= \int (x^2+1)\,dx \\ &= \frac{1}{3}x^3 + x + C \\ f(0) &= 8 \\ \frac{1}{3}(0)^3 + (0) + C &= 8 \\ C &= 8 \end{aligned}$$

Thus, $f(x) = \frac{1}{3}x^3 + x + 8.$

51. Find the function $f(x)$, such that

$f'(x) = 5x^2 + 3x - 7,\ f(0) = 9$.

We first find $f(x)$ by integrating:

$$\begin{aligned} f(x) &= \int (5x^2 + 3x - 7)\,dx \\ &= \int 5x^2\,dx + \int 3x\,dx - \int 7\,dx \\ &= \frac{5}{3}x^3 + \frac{3}{2}x^2 - 7x + C. \end{aligned}$$

The condition $f(0) = 9$ allows us to find *C*:

$$\begin{aligned} f(0) &= 9 \\ \frac{5}{3}(0)^3 + \frac{3}{2}(0)^2 - 7(0) + C &= 9 \\ C &= 9. \end{aligned}$$

Thus, $f(x) = \frac{5}{3}x^3 + \frac{3}{2}x^2 - 7x + 9.$

52. $f'(x) = 8x^2 + 4x - 2,\quad f(0) = 6$

$$\begin{aligned} f(x) &= \int (8x^2 + 4x - 2)\,dx \\ &= \frac{8}{3}x^3 + 2x^2 - 2x + C \\ f(0) &= 6 \\ \frac{8}{3}(0)^3 + 2(0)^2 - 2(0) + C &= 6 \\ C &= 6 \end{aligned}$$

Thus, $f(x) = \frac{8}{3}x^3 + 2x^2 - 2x + 6.$

53. Find the function $f(x)$, such that

$f'(x) = 3x^2 - 5x + 1,\ f(1) = \frac{7}{2}$.

We first find $f(x)$ by integrating:

$$f(x) = \int (3x^2 - 5x + 1)\,dx$$
$$= \int 3x^2 dx - \int 5x dx + \int dx$$
$$= x^3 - \frac{5}{2}x^2 + x + C.$$

The condition $f(1) = \frac{7}{2}$ allows us to find C:

$$f(1) = \frac{7}{2}$$
$$(1)^3 - \frac{5}{2}(1)^2 + (1) + C = \frac{7}{2}$$
$$-\frac{1}{2} + C = \frac{7}{2}$$
$$C = 4.$$

Thus, $f(x) = x^3 - \frac{5}{2}x^2 + x + 4$.

54. $f'(x) = 6x^2 - 4x + 2,\quad f(1) = 9$

$$f(x) = \int (6x^2 - 4x + 2)\,dx$$
$$= 2x^3 - 2x^2 + 2x + C$$
$$f(1) = 9$$
$$2(1)^3 - 2(1)^2 + 2(1) + C = 9$$
$$2 + C = 9$$
$$C = 7$$

Thus, $f(x) = 2x^3 - 2x^2 + 2x + 7$.

55. Find the function $f(x)$, such that

$f'(x) = 5e^{2x},\ f(0) = \frac{1}{2}$.

We first find $f(x)$ by integrating:

$$f(x) = \int 5e^{2x} dx$$
$$= \frac{5}{2}e^{2x} + C.$$

The condition $f(0) = \frac{1}{2}$ allows us to find C:

$$f(0) = \frac{1}{2}$$
$$\frac{5}{2}e^{2(0)} + C = \frac{1}{2}$$
$$\frac{5}{2}e^{0} + C = \frac{1}{2}$$
$$\frac{5}{2}\cdot 1 + C = \frac{1}{2}$$
$$C = -\frac{4}{2}$$
$$C = -2.$$

Thus, $f(x) = \frac{5}{2}e^{2x} - 2$.

56. $f'(x) = 3e^{4x},\quad f(0) = \frac{7}{4}$

$$f(x) = \int 3e^{4x} dx$$
$$= \frac{3}{4}e^{4x} + C$$
$$f(0) = \frac{7}{4}$$
$$\frac{3}{4}e^{4(0)} + C = \frac{7}{4}$$
$$\frac{3}{4} + C = \frac{7}{4}$$
$$C = 1$$

Thus, $f(x) = \frac{3}{4}e^{4x} + 1$.

57. Find the function $f(x)$, such that

$f'(x) = \frac{4}{\sqrt{x}},\ f(1) = -5$.

We first find $f(x)$ by integrating:

$$f(x) = \int \frac{4}{\sqrt{x}} dx$$
$$= \int 4x^{-1/2} dx$$
$$= 8x^{1/2} + C.$$

The condition $f(1) = -5$ allows us to find C:

$$f(1) = -5$$
$$8(1)^{1/2} + C = -5$$
$$8 + C = -5$$
$$C = -13.$$

Thus, $f(x) = 8x^{1/2} - 13$.

58. $f'(x)=\frac{2}{\sqrt[3]{x}},\quad f(1)=1$

$$f(x)=\int\frac{2}{\sqrt[3]{x}}\,dx$$
$$=\int 2x^{-1/3}dx$$
$$=3x^{2/3}+C$$
$$f(1)=1$$
$$3(1)^{2/3}+C=1$$
$$3+C=1$$
$$C=-2$$

Thus, $f(x)=3x^{2/3}-2.$

59. $D'(t)=1975-1190t+597t^2-71.3t^3$

We integrate to find $D(t)$.

$$D(t)=\int\left(1975-1190t+597t^2-71.3t^3\right)dt$$
$$=1975t-\frac{1190}{2}t^2+\frac{597}{3}t^3-\frac{71.3}{4}t^4+C$$
$$=1975t-595t^2+199t^3-17.825t^4+C$$

The condition $D(0)=17{,}198$ allows us to find C. Substituting, we have

$$D(0)=17{,}198$$
$$1975(0)-595(0)^2+199(0)^3-17.825(0)^4+C=17{,}198$$
$$C=17{,}198$$

Thus,

$$D(t)=17{,}198+1975t-595t^2+199t^3-17.825t^4$$

60. We use $D(t)$ found in Exercise 59.

In 2000, $t=2000-1994=6.$

$$D(6)=1975(6)-595(6)^2+199(6)^3-17.825(6)^4+17{,}198$$
$$=27{,}510.8.$$

Therefore, in 2000, the national credit market debt was about \$27,511 billion.

61. $C'(x)=x^3-2x$

We integrate to find $C(x)$, we use K for the constant of integration to avoid confusion with the cost function $C(x)$.

$$C(x)=\int C'(x)\,dx$$
$$=\int\left(x^3-2x\right)dx$$
$$=\frac{x^4}{4}-x^2+K$$

Fixed costs are \$7000. This means $C(0)=7000$. This allows us to determine K.

$$C(0)=7000$$
$$\frac{(0)^4}{4}-(0)^2+K=7000$$
$$K=7000$$

Thus, the total cost function is

$$C(x)=\frac{x^4}{4}-x^2+7000.$$

62. $C'(x)=x^3-x,\qquad \mathrm{C}(0)=6500$

$$C(x)=\int C'(x)\,dx$$
$$=\int\left(x^3-x\right)dx$$
$$=\frac{x^4}{4}-\frac{x^2}{2}+K$$
$$C(0)=6500$$
$$\frac{(0)^4}{4}-\frac{(0)^2}{2}+K=6500$$
$$K=6500$$

Thus, the total cost function is

$$C(x)=\frac{x^4}{4}-\frac{x^2}{2}+6500.$$

63. $R'(x)=x^2-3$

a) We integrate to find $R(x)$.

$$R(x)=\int R'(x)\,dx$$
$$=\int\left(x^2-3\right)dx$$
$$=\frac{x^3}{3}-3x+C$$

The condition $R(0)=0$ allows us to find C.

$$R(0)=0$$
$$\frac{(0)^3}{3}-3(0)+C=0$$
$$C=0$$

Thus, the total revenue function is

$$R(x)=\frac{x^3}{3}-3x.$$

b) $\boxed{tw}$ If the company does not sell any units, it will not generate any revenue.

64. $R'(x) = x^2 - 1$

a) We integrate to find $R(x)$.

$$R(x) = \int R'(x)\,dx = \int (x^2 - 1)\,dx = \frac{x^3}{3} - x + C$$

The condition $R(0) = 0$ allows us to find C.

$$R(0) = 0$$
$$\frac{(0)^3}{3} - (0) + C = 0$$
$$C = 0$$

Thus, the total revenue function is

$$R(x) = \frac{x^3}{3} - x.$$

b) $\boxed{tw}$ If the company does not sell any units, it will not generate any revenue.

65. $D'(x) = -\dfrac{4000}{x^2} = -4000x^{-2}$

We integrate to find $D(x)$.

$$D(x) = \int D'(x)\,dx = \int -4000x^{-2}\,dx = -4000 \cdot \frac{x^{-1}}{-1} + C = 4000x^{-1} + C = \frac{4000}{x} + C$$

When the price is \$4 per unit, the demand is 1003 units. This means $D(4) = 1003$.

Substituting 4 for x and 1003 for $D(x)$ we can determine C as follows:

$$D(4) = 1003$$
$$\frac{4000}{4} + C = 1003$$
$$1000 + C = 1003$$
$$C = 3.$$

Thus, the demand function is $D(x) = \dfrac{4000}{x} + 3$.

66. $S'(x) = 0.24x^2 + 4x + 10$

$$S(x) = \int S'(x)\,dx = \int (0.24x^2 + 4x + 10)\,dx = 0.08x^3 + 2x^2 + 10x + C$$

When the price is \$5 per unit, the supply is 121 units. This means $S(5) = 121$. We can determine C as follows:

$$S(5) = 121$$
$$0.08(5)^3 + 2(5)^2 + 10(5) + C = 121$$
$$10 + 50 + 50 + C = 121$$
$$110 + C = 121$$
$$C = 11.$$

Thus, the supply function is
$S(x) = 0.08x^3 + 2x^2 + 10x + 11$.

67. $\dfrac{dE}{dt} = 30 - 10t$

a) We find $E(t)$ by integrating

$$E(t) = \int E'(t)\,dt = \int (30 - 10t)\,dt = 30t - 10 \cdot \frac{t^2}{2} + C = 30t - 5t^2 + C$$

The condition $E(2) = 72$ allows us to find C as follows:

$$E(2) = 72$$
$$30(2) - 5(2)^2 + C = 72 \qquad \text{Substituting}$$
$$60 - 20 + C = 72$$
$$40 + C = 72$$
$$C = 32.$$

Thus, $E(t) = 30t - 5t^2 + 32$.

b) $E(t) = 32 + 30t - 5t^2$

Substituting 3 for t, we have:

$$E(3) = 32 + 30(3) - 5(3)^2 = 32 + 90 - 45 = 77.$$

After 3 hours, the operator's efficiency is 77%.

Substituting 5 for t, we have:

$$E(5) = 32 + 30(5) - 5(5)^2$$
$$= 32 + 150 - 125$$
$$= 57.$$

After 5 hours, the operator's efficiency is 57%.

68. $\dfrac{dE}{dt} = 40 - 10t$

a) We find $E(t)$ by integrating

$$E(t) = \int E'(t)\,dt$$
$$= \int (40 - 10t)\,dt$$
$$= 40t - 5t^2 + C$$

The condition $E(2) = 72$ allows us to find C as follows:

$$E(2) = 72$$
$$40(2) - 5(2)^2 + C = 72$$
$$60 + C = 72$$
$$C = 12.$$

Thus, $E(t) = 12 + 40t - 5t^2$.

b) Substituting 4 for t, we have:

$E(4) = 12 + 40(4) - 5(4)^2 = 92.$

After 4 hours, the operator's efficiency is 92%.

Substituting 8 for t, we have:

$E(8) = 12 + 40(8) - 5(8)^2 = 12.$

After 8 hours, the operator's efficiency is 12%.

69. $I'(t) = 3.389e^{0.1049t}$

a) We integrate to find $I(t)$.

$$I(t) = \int I'(t)\,dt$$
$$= \int 3.389e^{0.1049t}\,dt$$
$$= \frac{3.389}{0.1049}e^{0.1049t} + C$$
$$\approx 32.31e^{0.1049t} + C$$

The condition $I(0) = 0$ allows us to find C.

$$I(0) = 0$$
$$32.31e^{0.1049(0)} + C = 0 \qquad \text{Substituting}$$
$$32.31e^{0} + C = 0$$
$$32.31 + C = 0$$
$$C = -32.31$$

The total number per 100,000 who have contracted influenza by time t is given by $I(t) = 32.31e^{0.1049t} - 32.31.$

b) Using the function found in part (a), we substitute 27 for t.

$$I(27) = 32.31e^{0.1049(27)} - 32.31$$
$$= 32.31e^{2.8323} - 32.31$$
$$\approx 516.459$$

After 27 weeks, approximately 516 per 100,000 people have contracted influenza.

c) Using the function found in part (a), we substitute 34 for t.

$$I(34) = 32.31e^{0.1049(34)} - 32.31$$
$$= 32.31e^{3.5666} - 32.31$$
$$\approx 1111.34$$

After 34 weeks, approximately 1111 per 100,000 people have contracted influenza.

d) The number of people that contracted influenza during the last 7 weeks of the first 34 weeks is given by

$I(34) - I(27) \approx 1111 - 516 \approx 595$.

Approximately 595 per 100,000 people contracted influenza during the last 7 of the 34 weeks.

70. $M'(t) = 0.2t - 0.003t^2$

a) $M(t) = \int M'(t)\,dt$

$$M(t) = \int \left(0.2t - 0.003t^2\right)dt$$
$$= 0.1t^2 - 0.001t^3 + C$$

We use $M(0) = 0$ to find C.

$$M(0) = 0$$
$$0.1(0)^2 - 0.001(0)^3 + C = 0$$
$$C = 0$$

Thus, $M(t) = 0.1t - 0.001t^3$.

b) $M(8) = 0.1(8)^2 - 0.001(8)^3 = 5.888$

About 6 words are memorized in 8 minutes.

71. Find the function $f(t)$, such that

$$f'(t)=\sqrt{t}+\frac{1}{\sqrt{t}},\ f(4)=0.$$

We first find $f(t)$ by integrating:

$$f(t)=\int\left(\sqrt{t}+\frac{1}{\sqrt{t}}\right)dt$$

$$=\int\left(t^{1/2}+t^{-1/2}\right)dt$$

$$=\frac{2}{3}t^{3/2}+2t^{1/2}+C.$$

The condition $f(4)=0$ allows us to find C:

$$f(4)=0$$

$$\frac{2}{3}(4)^{3/2}+2(4)^{1/2}+C=0$$

$$\frac{2}{3}(8)+4+C=0$$

$$\frac{16}{3}+4+C=0$$

$$C=-\frac{28}{3}.$$

Thus, $f(t)=\frac{2}{3}t^{3/2}+2t^{1/2}-\frac{28}{3}$.

72. Find the function $f(t)$, such that

$$f'(t)=t^{\sqrt{3}},\ f(0)=8.$$

We first find $f(t)$ by integrating:

$$f(t)=\int t^{\sqrt{3}}dt$$

$$=\frac{t^{\sqrt{3}+1}}{\sqrt{3}+1}+C.$$

The condition $f(0)=8$ allows us to find C:

$$f(0)=8$$

$$\frac{(0)^{\sqrt{3}+1}}{\sqrt{3}+1}+C=8$$

$$0+C=8$$

$$C=8.$$

Thus, $f(t)=\frac{t^{\sqrt{3}+1}}{\sqrt{3}+1}+8$.

73. $\int(5t+4)^2\,t^4dx$

First, we will expand the binomial.

$$=\int(5t+4)(5t+4)t^4dt$$

$$=\int\left(25t^2+40t+16\right)t^4dt$$

Next, we distribute t^4.

$$\int\left(25t^6+40t^5+16t^4\right)dt$$

Now we can integrate as follows:

$$=\int 25t^6dt+\int 40t^5dt+\int 16t^4dt$$ The integral of a sum is the sum of the integrals.

$$=25\cdot\frac{t^{6+1}}{6+1}+40\cdot\frac{t^{5+1}}{5+1}+16\cdot\frac{t^{4+1}}{4+1}+C$$

$$\left[\int x^r dx=\frac{x^{r+1}}{r+1}+C\right]$$

$$=\frac{25}{7}t^7+\frac{40}{6}t^6+\frac{16}{5}t^5+C$$

$$=\frac{25}{7}t^7+\frac{20}{3}t^6+\frac{16}{5}t^5+C$$

74. $\int(x-1)^2\,x^3dx$

$$=\int\left(x^2-2x+1\right)x^3dx$$

$$=\int\left(x^5-2x^4+x^3\right)dx$$

$$=\frac{x^6}{6}-\frac{2}{5}x^5+\frac{x^4}{4}+C$$

75. $\int(1-t)\sqrt{t}\,dt$

$$=\int\left(\sqrt{t}-t\sqrt{t}\right)dt$$ Distributing $\sqrt{t}$

$$=\int\left(t^{1/2}-t^{3/2}\right)dt$$

$$=\frac{t^{3/2}}{3/2}-\frac{t^{5/2}}{5/2}+C$$

$$=\frac{2}{3}t^{3/2}-\frac{2}{5}t^{5/2}+C$$

76. $\int\frac{(t+3)^2}{\sqrt{t}}dt$

$$=\int\left(t^2+6t+9\right)t^{-1/2}dt$$

$$=\int\left(t^{3/2}+6t^{1/2}+9t^{-1/2}\right)dt$$

$$=\frac{t^{5/2}}{5/2}+6\cdot\frac{t^{3/2}}{3/2}+9\cdot\frac{t^{1/2}}{1/2}+C$$

$$=\frac{2}{5}t^{5/2}+4t^{3/2}+18t^{1/2}+C$$

77. $\int \frac{x^4-6x^2-7}{x^3}dx$

$= \int \left(x^4-6x^2-7\right)x^{-3}dx$

$= \int \left(x-6x^{-1}-7x^{-3}\right)dx$

$= \frac{x^2}{2}-6\ln x-7\cdot\frac{x^{-2}}{-2}+C$

$= \frac{x^2}{2}-6\ln x+\frac{7}{2}x^{-2}+C$

78. $\int (t+1)^3\,dt$

$= \int \left(t^3+3t^2+3t+1\right)dt$

$= \frac{t^4}{4}+3\cdot\frac{t^3}{3}+3\cdot\frac{t^2}{2}+t+C$

$= \frac{t^4}{4}+t^3+\frac{3}{2}t^2+t+C$

79. $\int \frac{1}{\ln 10}\cdot\frac{dx}{x}$

$= \frac{1}{\ln 10}\int \frac{1}{x}dx$

$= \frac{1}{\ln 10}\cdot \ln x + C$

$= \log x + C$ Properties of logarithms.

80. $\int be^{ax}dx = \frac{b}{a}e^{ax}+C$

81. $\int (3x-5)(2x+1)^2\,dx$

$= \int (3x-5)\left(4x^2+4x+1\right)dx$

$= \int \left(12x^3-8x^2-17x-5\right)dx$

$= 12\cdot\frac{x^4}{4}-8\cdot\frac{x^3}{3}-17\cdot\frac{x^2}{2}-5x+C$

$= 3x^4-\frac{8}{3}x^3-\frac{17}{2}x^2-5x+C$

82. $\int \sqrt[3]{64x^4}\,dx$

$= \int 4\sqrt[3]{x^4}\,dx$

$= 4\int x^{4/3}dx$

$= 4\cdot\frac{x^{7/3}}{7/3}+C$

$= \frac{12}{7}x^{7/3}+C$

83. $\int \frac{x^2-1}{x+1}dx$

$= \int \frac{(x-1)(x+1)}{x+1}dx$

$= \int (x-1)\,dx$

$= \frac{x^2}{2}-x+C$

84. $\int \frac{t^3+8}{t+2}dt$

$= \int \frac{(t+2)\left(t^2-2t+4\right)}{t+2}dt$

$= \int \left(t^2-2t+4\right)dt$

$= \frac{t^3}{3}-t^2+4t+C$

85. $\boxed{tw}$ The statement is not true. The indefinite integral of x^2, $\int x^2dx = \frac{x^3}{3}+C$, is a family of functions whose derivative is equal to x^2. These functions differ by only a constant; however, none of the functions is a unique integral of x^2.

86. $\boxed{tw}$ The graphs of the antiderivatives of a function $f(x)$ fill up the plane, with exactly one curve going through any given point. The slope of a line tangent to any of the curves at $x = a$ is $f(a)$.

Exercise Set 4.3

1. Find any antiderivative $F(x)$ of $y = 4$. We choose the simplest one for which the constant of integration is 0:

$$F(x) = \int 4dx$$
$$= 4x + C$$
$$= 4x. \qquad [C = 0]$$

The area under the curve over the interval $[1,3]$ is given by $F(3) - F(1)$. Substitute 3 and 1, and find the difference:

$$F(3) - F(1) = 4(3) - 4(1)$$
$$= 12 - 4$$
$$= 8.$$

2. Find any antiderivative $F(x)$ of $y = 5$. We choose the simplest one:

$$F(x) = \int 5dx = 5x. \qquad [C = 0]$$

Substitute 3 and 1, and find the difference $F(3) - F(1)$:

$$F(3) - F(1) = 5(3) - 5(1) = 10.$$

3. Find any antiderivative $F(x)$ of $y = 2x$. We choose the simplest one for which the constant of integration is 0:

$$F(x) = \int 2xdx$$
$$= x^2 + C$$
$$= x^2. \qquad [C = 0]$$

The area under the curve over the interval $[1,3]$ is given by $F(3) - F(1)$. Substitute 3 and 1, and find the difference:

$$F(3) - F(1) = (3)^2 - (1)^2$$
$$= 9 - 1$$
$$= 8.$$

4. Find any antiderivative $F(x)$ of $y = x^2$. We choose the simplest one:

$$F(x) = \int x^2 dx = \frac{x^3}{3}. \qquad [C = 0]$$

Substitute 3 and 0, and find the difference $F(3) - F(0)$:

$$F(3) - F(0) = \frac{(3)^3}{3} - \frac{(0)^3}{3} = 9.$$

5. Find any antiderivative $F(x)$ of $y = x^2$. We choose the simplest one for which the constant of integration is 0:

$$F(x) = \int x^2 dx$$
$$= \frac{x^3}{3} + C$$
$$= \frac{x^3}{3}. \qquad [C = 0]$$

The area under the curve over the interval $[0,5]$ is given by $F(5) - F(0)$. Substitute 5 and 0, and find the difference:

$$F(5) - F(0) = \frac{(5)^3}{3} - \frac{(0)^3}{3}$$
$$= \frac{125}{3}$$
$$41\tfrac{2}{3}.$$

6. Find any antiderivative $F(x)$ of $y = x^3$. We choose the simplest one:

$$F(x) = \int x^3 dx = \frac{x^4}{4}. \qquad [C = 0]$$

Substitute 2 and 0, and find the difference $F(2) - F(0)$:

$$F(2) - F(0) = \frac{(2)^4}{4} - \frac{(0)^4}{4} = 4.$$

7. Find any antiderivative $F(x)$ of $y = x^3$. We choose the simplest one for which the constant of integration is 0:

$$F(x) = \int x^3 dx$$
$$= \frac{x^4}{4} + C$$
$$= \frac{x^4}{4}. \qquad [C = 0]$$

The area under the curve over the interval $[0,1]$ is given by $F(1) - F(0)$. Substitute 1 and 0, and find the difference:

$$F(1) - F(0) = \frac{(1)^4}{4} - \frac{(0)^4}{4}$$
$$= \frac{1}{4}.$$

8. Find any antiderivative $F(x)$ of $y = 1 - x^2$. We choose the simplest one:

$$F(x) = \int(1-x^2)\,dx = x - \frac{x^3}{3}. \qquad [C=0]$$

Substitute 1 and -1, and find the difference $F(1) - F(-1)$:

$$\begin{aligned} F(1)-F(-1) &= \left((1) - \frac{(1)^3}{3}\right) - \left((-1) - \frac{(-1)^3}{3}\right) \\ &= \left(\frac{2}{3}\right) - \left(-\frac{2}{3}\right) \\ &= \frac{4}{3} \\ &= 1\tfrac{1}{3}. \end{aligned}$$

9. Find any antiderivative $F(x)$ of $y = 4 - x^2$. We choose the simplest one for which the constant of integration is 0:

$$\begin{aligned} F(x) &= \int(4-x^2)\,dx \\ &= 4x - \frac{x^3}{3} + C \\ &= 4x - \frac{x^3}{3}. \qquad [C=0] \end{aligned}$$

The area under the curve over the interval $[-2,2]$ is given by $F(2)-F(-2)$. Substitute 2 and -2, and find the difference:

$$\begin{aligned} &F(2)-F(-2) \\ &= \left(4(2) - \frac{(2)^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right) \\ &= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) \\ &= \frac{16}{3} - \left(-\frac{16}{3}\right) \\ &= \frac{32}{3} \\ &= 10\tfrac{2}{3}. \end{aligned}$$

10. Find any antiderivative $F(x)$ of $y = e^x$. We choose the simplest one:

$$F(x) = \int e^x\,dx = e^x. \qquad [C=0]$$

Substitute 2 and 0, and find the difference $F(2) - F(0)$:

$$\begin{aligned} F(2)-F(0) &= (e^2) - (e^0) \\ &= e^2 - 1 \\ &\approx 6.389. \end{aligned}$$

11. Find any antiderivative $F(x)$ of $y = e^x$. We choose the simplest one for which the constant of integration is 0:

$$\begin{aligned} F(x) &= \int e^x\,dx \\ &= e^x + C \\ &= e^x. \qquad [C=0] \end{aligned}$$

The area under the curve over the interval $[0,3]$ is given by $F(3)-F(0)$. Substitute 3 and 0, and find the difference:

$$\begin{aligned} F(3)-F(0) &= (e^3) - (e^0) \\ &= e^3 - 1 \\ &\approx 19.086. \end{aligned}$$

12. Find any antiderivative $F(x)$ of $y = \frac{2}{x}$. We choose the simplest one:

$$F(x) = \int \frac{2}{x}\,dx = 2\ln x. \qquad [C=0]$$

Substitute 4 and 1, and find the difference $F(4) - F(1)$:

$$\begin{aligned} F(4)-F(1) &= (2\ln 4) - (2\ln 1) \\ &= 2\ln 4 - 0 \\ &\approx 2.773. \end{aligned}$$

13. Find any antiderivative $F(x)$ of $y = \frac{3}{x}$. We choose the simplest one for which the constant of integration is 0:

$$\begin{aligned} F(x) &= \int \frac{3}{x}\,dx \\ &= 3\ln x + C \\ &= 3\ln x. \qquad [C=0] \end{aligned}$$

The area under the curve over the interval $[1,6]$ is given by $F(6)-F(1)$. Substitute 6 and 1, and find the difference:

$$\begin{aligned} F(6)-F(1) &= (3\ln 6) - (3\ln 1) \\ &= 3\ln 6 - 0 \\ &\approx 5.375. \end{aligned}$$

14. Find any antiderivative $F(x)$ of $y = x^2 - 4x$. We choose the simplest one:

$$F(x) = \int (x^2 - 4x)\,dx = \frac{x^3}{3} - 2x^2. \quad [C = 0]$$

Substitute -2 and -4, and find the difference $F(-2) - F(-4)$:

$$\begin{aligned} &F(-2) - F(-4) \\ &= \left(\frac{(-2)^3}{3} - 2(-2)^2\right) - \left(\frac{(-4)^3}{3} - 2(-4)^2\right) \\ &= \left(\frac{-8}{3} - 8\right) - \left(\frac{-64}{3} - 32\right) \\ &= \frac{128}{3} \\ &= 42\tfrac{2}{3}. \end{aligned}$$

15. The height of the area under the curve is total cost per day and the width of the area under the curve is time in days. Thus, area under the curve represents total cost in dollars, for t days.

$$\frac{\text{Total Cost}}{\text{days}} \cdot \text{days} = \text{Total cost.}$$

16. The area under the curve represents total number of miles traveled in t hours.

$$\frac{\text{Miles}}{\text{Hour}} \cdot \text{Hours} = \text{Miles.}$$

17. The height of the area under the curve is total number of kilowatts used per hour and the width of the area under the curve is time in hours. Thus, area under the curve represents Total number of kilowatts used in t hours.

$$\frac{\#\text{KW}}{\text{Hour}} \cdot \text{Hours} = \#\,\text{KW.}$$

18. The area under the curve represents total number of marriages in t years.

$$\frac{\text{Marriages}}{\text{Year}} \cdot \text{Years} = \text{Marriages.}$$

19. The height of the area under the curve is revenue in dollars per unit and the width of the area under the curve is number of units. Thus, area under the curve represents total revenue, in dollars, for x units produced. $\frac{\$}{\text{Unit}} \cdot \text{Units} = \$.$

20. The area under the curve represents total cost for x units produced. $\frac{\text{Cost}}{\text{unit}} \cdot \text{units} = \text{Cost.}$

21. The height of the area under the curve is milligrams per cubic centimeter and the width of the area under the curve is cubic centimeters. Thus, area under the curve represents total concentration of a drug, in milligrams, in v cubic centimeters of blood. $\frac{\text{mg}}{cm^3} \cdot cm^3 = \text{mg.}$

22. The area under the curve represents total sales for t days. $\frac{\text{Sales}}{\text{day}} \cdot \text{days} = \text{Sales.}$

23. The height of the area under the curve is number of memorized words per minute and the width of the area under the curve is time in minutes. Thus, area under the curve represents the total number of words memorized in t minutes.

$$\frac{\text{Words memorized}}{\text{Minute}} \cdot \text{Minutes} = \text{Words memorized.}$$

24. The area under the curve represents total number of orders in t hours.

$$\frac{\text{Orders}}{\text{Hour}} \cdot \text{Hours} = \text{Orders.}$$

25. Find any antiderivative $F(x)$ of $y = x^3$. We choose the simplest one for which the constant of integration is 0:

$$\begin{aligned} F(x) &= \int x^3\,dx \\ &= \frac{x^4}{4} + C \\ &= \frac{x^4}{4}. \qquad [C = 0] \end{aligned}$$

The area under the curve over the interval $[0, 2]$ is given by $F(2) - F(0)$. Substitute 2 and 0, and find the difference:

$$\begin{aligned} F(2) - F(0) &= \frac{(2)^4}{4} - \frac{(0)^4}{4} \\ &= \frac{16}{4} \\ &= 4. \end{aligned}$$

26. Find any antiderivative $F(x)$ of $y = x^4$. We choose the simplest one:

$F(x) = \int x^5 dx = \frac{x^5}{5}. \qquad [C = 0]$

Substitute 1 and 0, and find the difference $F(1) - F(0)$:

$F(1) - F(0) = \frac{(1)^5}{5} - \frac{(0)^5}{5} = \frac{1}{5}.$

27. Find any antiderivative $F(x)$ of $y = x^2 + x + 1$. We choose the simplest one for which the constant of integration is 0:

$F(x) = \int x^2 + x + 1 dx$

$= \frac{x^3}{3} + \frac{x^2}{2} + x + C$

$= \frac{x^3}{3} + \frac{x^2}{2} + x. \qquad [C = 0]$

The area under the curve over the interval $[2,3]$ is given by $F(3) - F(2)$. Substitute 3 and 2, and find the difference:

$F(3) - F(2)$

$= \frac{(3)^3}{3} + \frac{(3)^2}{2} + (3) - \left(\frac{(2)^3}{3} + \frac{(2)^2}{2} + (2)\right)$

$= \frac{27}{3} + \frac{9}{2} + 3 - \left(\frac{8}{3} + \frac{4}{2} + 2\right)$

$= \frac{54}{6} + \frac{27}{6} + \frac{18}{6} - \left(\frac{16}{6} + \frac{12}{6} + \frac{12}{6}\right)$

$= \frac{99}{6} - \left(\frac{40}{6}\right)$

$= \frac{59}{6}$

$= 9\tfrac{5}{6}.$

28. Find any antiderivative $F(x)$ of $y = 2 - x - x^2$. We choose the simplest one:

$F(x) = \int 2 - x - x^2 dx$

$= 2x - \frac{x^2}{2} - \frac{x^3}{3}. \qquad [C = 0]$

Substitute 1 and -2, and find the difference $F(1) - F(-2)$:

$F(1) - F(-2)$

$= \left(2(1) - \frac{(1)^2}{2} - \frac{(1)^3}{3}\right) - \left(2(-2) - \frac{(-2)^2}{2} - \frac{(-2)^3}{3}\right)$

$= \frac{9}{2} = 4\tfrac{1}{2}.$

29. Find any antiderivative $F(x)$ of $y = 5 - x^2$. We choose the simplest one for which the constant of integration is 0:

$F(x) = \int (5 - x^2) dx$

$= 5x - \frac{x^3}{3} + C$

$= 5x - \frac{x^3}{3}. \qquad [C = 0]$

The area under the curve over the interval $[-1,2]$ is given by $F(2) - F(-1)$. Substitute 2 and -1, and find the difference:

$F(2) - F(-1)$

$= \left(5(2) - \frac{(2)^3}{3}\right) - \left(5(-1) - \frac{(-1)^3}{3}\right)$

$= \left(10 - \frac{8}{3}\right) - \left(-5 + \frac{1}{3}\right)$

$= \frac{22}{3} - \left(-\frac{14}{3}\right)$

$= \frac{36}{3}$

$= 12.$

30. Find any antiderivative $F(x)$ of $y = e^x$. We choose the simplest one:

$F(x) = \int e^x dx = e^x. \qquad [C = 0]$

Substitute 3 and -2, and find the difference $F(3) - F(-2)$:

$F(3) - F(-2) = (e^3) - (e^{-2})$

$= e^3 - e^{-2}$

$\approx 19.950.$

31. Find any antiderivative $F(x)$ of $y = e^x$. We choose the simplest one for which the constant of integration is 0:

$F(x) = \int e^x dx$

$= e^x + C$

$= e^x. \qquad [C = 0]$

The area under the curve over the interval $[-1,5]$ is given by $F(5)-F(-1)$. Substitute 5 and -1, and find the difference:

$$\begin{aligned} F(5)-F(-1) &= (e^5)-(e^{-1}) \\ &= e^5-e^{-1} \\ &\approx 148.045. \end{aligned}$$

32. Find any antiderivative $F(x)$ of $y = 2x+\frac{1}{x^2}$.

We choose the simplest one:

$$\begin{aligned} F(x) &= \int 2x+\frac{1}{x^2}\,dx \\ &= \int 2x+x^{-2}dx \\ &= x^2-x^{-1}. \qquad [C=0] \end{aligned}$$

Substitute 4 and 1, and find the difference $F(4)-F(1)$:

$$\begin{aligned} F(4)-F(1) &= \left((4)^2-\frac{1}{(4)}\right)-\left((1)^2-\frac{1}{(1)}\right) \\ &= 16-\frac{1}{4}-0 \\ &= \frac{63}{4} = 15\tfrac{3}{4}. \end{aligned}$$

33. $\boxed{tw}$

a) $\int_a^b f(x)\,dx = 0$, because there is the same area above the x-axis as below. The area is $A-A=0$.

b) $\int_a^b f(x)\,dx < 0$, because there is more area below the x-axis than above. The area is $A-2A=-A<0$.

34. $\boxed{tw}$

a) $\int_a^b f(x)\,dx > 0$, because there is more area above the x-axis than below. The area is $3A-A=2A>0$.

b) $\int_a^b f(x)\,dx < 0$, because there is more area below the x-axis than above. The area is $-3A<0$.

35. $\int_0^{1.5}\left(x-x^2\right)dx$

$$\begin{aligned} &= \left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^{1.5} \\ &= \left(\frac{(1.5)^2}{2}-\frac{(1.5)^3}{3}\right)-\left(\frac{(0)^2}{2}-\frac{(0)^3}{3}\right) \\ &= \left(\frac{2.25}{2}-\frac{3.375}{3}\right)-0 \\ &= 1.125-1.125 \\ &= 0 \end{aligned}$$

The area above the x-axis is equal to the area below the x-axis.

36. $\int_0^{2}\left(x^2-x\right)dx$

$$\begin{aligned} &= \left[\frac{x^3}{3}-\frac{x^2}{2}\right]_0^{2} \\ &= \left(\frac{(2)^3}{3}-\frac{(2)^2}{2}\right)-\left(\frac{(0)^3}{3}-\frac{(0)^2}{2}\right) \\ &= \left(\frac{8}{3}-\frac{4}{2}\right)-0 \\ &= \frac{2}{3} \end{aligned}$$

The area above the x-axis is greater than the area below the x-axis.

37. $\int_{-1}^{1}\left(x^4-x^2\right)dx$

$$\begin{aligned} &= \left[\frac{x^5}{5}-\frac{x^3}{3}\right]_{-1}^{1} \\ &= \left(\frac{(1)^5}{5}-\frac{(1)^3}{3}\right)-\left(\frac{(-1)^5}{5}-\frac{(-1)^3}{3}\right) \\ &= \left(\frac{1}{5}-\frac{1}{3}\right)-\left(\frac{-1}{5}-\frac{-1}{3}\right) \\ &= \frac{-2}{15}-\left(\frac{2}{15}\right) \\ &= -\frac{4}{15} \end{aligned}$$

The area below the x-axis is greater than the area above the x-axis.

38. $\int_0^b -2e^{3x}\,dx$

$= \left[-\frac{2}{3}e^{3x}\right]_0^b$

$= -\frac{2}{3}\left(e^{3b} - e^0\right)$

$= -\frac{2}{3}\left(e^{3b} - 1\right) < 0,$ for all $b > 0.$

The area below the x-axis is greater than the area above the x-axis.

39 – 42. Left to the student.

43. $\int_1^3 \left(3t^2 + 7\right) dt$

$= \left[t^3 + 7t\right]_1^3$

$= \left(3^3 + 7(3)\right) - \left(1^3 + 7(1)\right)$

$= 48 - (8)$

$= 40$

44. $\int_1^2 \left(4t^3 - 1\right) dt$

$= \left[t^4 - t\right]_1^2$

$= \left(2^4 - (2)\right) - \left(1^4 - (1)\right)$

$= 14 - (0)$

$= 14$

45. $\int_1^4 \left(\sqrt{x} - 1\right) dx = \int_1^4 \left(x^{1/2} - 1\right) dx$

$= \left[\frac{2}{3}x^{3/2} - x\right]_1^4$

$= \left(\frac{2}{3}(4)^{3/2} - 4\right) - \left(\frac{2}{3}(1)^{3/2} - 1\right)$

$= \left(\frac{16}{3} - 4\right) - \left(\frac{2}{3} - 1\right)$

$= \left(\frac{4}{3}\right) - \left(-\frac{1}{3}\right)$

$= \frac{5}{3}$

46. $\int_1^8 \left(\sqrt[3]{x} - 2\right) dx = \int_1^8 \left(x^{1/3} - 2\right) dx$

$= \left[\frac{3}{4}x^{4/3} - 2x\right]_1^8$

$= \left(\frac{3}{4}(8)^{4/3} - 2(8)\right) - \left(\frac{3}{4}(1)^{4/3} - 2(1)\right)$

$= (12 - 16) - \left(\frac{3}{4} - 2\right)$

$= (-4) - \left(-\frac{5}{4}\right)$

$= -\frac{11}{4}$

47. $\int_{-2}^5 \left(2x^2 - 3x + 7\right) dx$

$= \left[\frac{2}{3}x^3 - \frac{3}{2}x^2 + 7x\right]_{-2}^5$

$= \left(\frac{2}{3}(5)^3 - \frac{3}{2}(5)^2 + 7(5)\right) -$

$\left(\frac{2}{3}(-2)^3 - \frac{3}{2}(-2)^2 + 7(-2)\right)$

$= \frac{485}{6} - \left(-\frac{152}{6}\right)$

$= \frac{637}{6}$

48. $\int_{-2}^3 \left(-x^2 + 4x - 5\right) dx$

$= \left[-\frac{x^3}{3} + 2x^2 - 5x\right]_{-2}^3$

$= \left(-\frac{(3)^3}{3} + 2(3)^2 - 5(3)\right) -$

$\left(-\frac{(-2)^3}{3} + 2(-2)^2 - 5(-2)\right)$

$= -6 - \left(\frac{62}{3}\right)$

$= -\frac{80}{3}$

49. $\int_{-5}^2 e^t\,dt$

$= \left[e^t\right]_{-5}^2$

$= e^2 - e^{-5}$

≈ 7.382

50. $\int_{-2}^{3} e^{-t}dt$
$=\left[-e^{-t}\right]_{-2}^{3}$
$=-e^{-3}-\left(-e^{-(-2)}\right)$
$=-e^{-3}+e^{2}$
≈ 7.339

51. $\int_a^b \frac{1}{2}x^2dx$
$=\left[\frac{1}{6}x^3\right]_a^b$
$=\frac{1}{6}(b)^3-\frac{1}{6}(a)^3$
$=\frac{b^3-a^3}{6}$

52. $\int_a^b \frac{1}{5}x^3dx$
$=\left[\frac{1}{20}x^4\right]_a^b$
$=\frac{1}{20}(b)^4-\frac{1}{20}(a)^4$
$=\frac{b^4-a^4}{20}$

53. $\int_a^b e^{2t}dt$
$=\left[\frac{1}{2}e^{2t}\right]_a^b$
$==\frac{1}{2}e^{2b}-\frac{1}{2}e^{2a}$
$=\frac{e^{2b}-e^{2a}}{2}$

54. $\int_a^b -e^{t}dt$
$=\left[-e^{t}\right]_a^b$
$=-e^{b}-\left(-e^{a}\right)$
$=e^{a}-e^{b}$

55. $\int_1^e \left(x+\frac{1}{x}\right)dx$
$=\left[\frac{x^2}{2}+\ln x\right]_1^e$
$=\left(\frac{e^2}{2}+\ln e\right)-\left(\frac{1^2}{2}+\ln 1\right)$
$=\frac{e^2}{2}+1-\left(\frac{1}{2}+0\right)$
$=\frac{e^2}{2}+\frac{1}{2}$
$=\frac{e^2+1}{2}$
≈ 4.195

56. $\int_1^e \left(x-\frac{1}{x}\right)dx$
$=\left[\frac{x^2}{2}-\ln x\right]_1^e$
$=\left(\frac{e^2}{2}-\ln e\right)-\left(\frac{1^2}{2}-\ln 1\right)$
$=\frac{e^2}{2}-1-\left(\frac{1}{2}-0\right)$
$=\frac{e^2}{2}-\frac{3}{2}$
$=\frac{e^2-3}{2}$
≈ 2.195

57. $\int_0^2 \sqrt{2x}\,dx=\int_0^2 \sqrt{2}\cdot x^{1/2}dx=\sqrt{2}\int_0^2 x^{1/2}dx$
$=\sqrt{2}\left[\frac{2}{3}x^{3/2}\right]_0^2$
$=\sqrt{2}\left[\frac{2}{3}(2)^{3/2}-\frac{2}{3}(0)^{3/2}\right]$
$=\sqrt{2}\left[\frac{2}{3}\sqrt{2^3}\right]$
$=\sqrt{2}\left[\frac{2}{3}\sqrt{8}\right]$
$=\frac{2}{3}\sqrt{16}$
$=\frac{8}{3}$

58. $\int_0^{27} \sqrt{3x}\,dx = \sqrt{3}\int_0^{27} x^{1/2}\,dx$

$= \sqrt{3}\left[\frac{2}{3}x^{3/2}\right]_0^{27}$

$= \sqrt{3}\left[\frac{2}{3}(27)^{3/2} - \frac{2}{3}(0)^{3/2}\right]$

$= \sqrt{3}\left[\frac{2}{3}\left(81\sqrt{3}\right)\right]$

$= 54\sqrt{9}$

$= 162$

59. We integrate to find:

$P(250) = \int_0^{250} P'(x)\,dx$

$= \int_0^{250} \sqrt[5]{x}\,dx$

$= \int_0^{250} x^{1/5}\,dx$

$= \left[\frac{5}{6}x^{6/5}\right]_0^{250}$

$= \frac{5}{6}(250)^{6/5} - \frac{5}{6}(0)^{6/5}$

$\approx 628.56.$ Using a calculator.

When a 250 foot well is drilled, Pure Water Enterprises profit is \$628.56.

60. We integrate to find $R(300)$:

$R(300) = \int_0^{300} R'(x)\,dx$

$= \int_0^{300} 6x^{-1/6}\,dx$

$= \left[\frac{36}{5}x^{5/6}\right]_0^{300}$

≈ 834.83

When 300 pounds of maple coated pecans are produced, Sally's revenue is \$834.83.

61. In order find the cost of producing an additional 14 feet of counter top after 50 feet have already been produced, we integrate $C'(x)$ over the interval $[50, 64]$.

$C(64) - C(50) = \int_{50}^{64} C'(x)\,dx$

$= \int_{50}^{64} 8x^{-1/3}\,dx$

$= \left[12x^{2/3}\right]_{50}^{64}$

$= 12(64)^{2/3} - 12(50)^{2/3}$

$\approx 29.13.$ Using a calculator.

The cost of installing an extra 14 feet of counter top after 50 feet has already been ordered is \$29.13.

62. We integrate $P'(x)$ over the interval $[1200, 1500]$ to find the additional profit:

$P(1500) - P(1200) = \int_{1200}^{1500} P'(x)\,dx$

$= \int_{1200}^{1500} 2.6x^{0.1}\,dx$

$= \left[\frac{2.6}{1.1}x^{1.1}\right]_{1200}^{1500}$

≈ 1603.42 Using a calculator

63. $S'(t) = 20e^t$

a) We integrate $S'(t)$ over the interval $[0, 5]$ to find the accumulated sales.

$S(5) = \int_0^5 S'(t)dt$

$= \int_0^5 20e^t\,dt$

$= \left[20e^t\right]_0^5$

$= 20e^5 - 20e^0$

$= 20e^5 - 20\cdot 1$

≈ 2948.26

The accumulated sales for the first 5 days are approximately \$2948.26.

b) We integrate $S'(t)$ over the interval $[1, 5]$ to find the accumulated sales for the 2nd day through the 5th day.

$S(5) = \int_1^5 S'(t)dt$

$= \int_1^5 20e^t\,dt$

$= \left[20e^t\right]_1^5$

$= 20e^5 - 20e^1$

≈ 2913.90

The sales from the 2nd day through the 5th day are approximately \$2913.90.

64. $S'(t) = 10e^t$

a) We integrate $S'(t)$ over the interval $[0,5]$ to find the accumulated sales.

$$S(5) = \int_0^5 S'(t)dt$$
$$= \int_0^5 10e^t\,dt$$
$$= \left[10e^t\right]_0^5$$
$$\approx 1474.13$$

The accumulated sales for the first 5 days are approximately \$1474.13.

b) We integrate $S'(t)$ over the interval $[1,5]$ to find the accumulated sales for the 2nd day through the 5th day.

$$S(5) = \int_1^5 S'(t)dt$$
$$= \int_1^5 10e^t\,dt$$
$$= \left[10e^t\right]_1^5$$
$$\approx 1456.95$$

The sales from the 2nd day through the 5th day are approximately \$1456.95.

65. In 1996, $t = 1$ and in 2000, $t = 5$. Therefore, we integrate $D'(t)$ over the interval $[1,5]$.

$$\int_1^5 D'(t)\,dt$$
$$= \int_1^5 \left(857.98 + 829.66t - 197.34t^2 + 15.36t^3\right)dt$$
$$= \left[857.98t + 414.83t^2 - 65.78t^3 + 3.84t^4\right]_1^5$$
$$\approx 7627.28$$

The credit market debt increased \$7627.28 billion from 1996 to 2000.

66. In 1999, $t = 4$ and in 2005, $t = 10$. Therefore, we integrate $D'(t)$ over the interval $[4,10]$.

$$\int_4^{10} D'(t)\,dt$$
$$= \int_4^{10} \left(857.98 + 829.66t - 197.34t^2 + 15.36t^3\right)dt$$
$$= \left[857.98t + 414.83t^2 - 65.78t^3 + 3.84t^4\right]_4^{10}$$
$$\approx 15{,}840.48$$

The credit market debt increase \$15,840.48 billion from 1999 to 2005.

67. We integrate $T(x)$ over the interval $[1,10]$.

$$\int_1^{10} T(x)\,dx$$
$$= \int_1^{10} \left(2 + 0.3x^{-1}\right)dx$$
$$= \left[2x + 0.3\ln x\right]_1^{10}$$
$$= 2(10) + 0.3\ln(10) - \left(2(1) + 0.3\ln(1)\right)$$
$$= 18 + 0.3\ln 10$$
$$\approx 18.69$$

It takes 18.69 hours for a new worker to produce units 1 through 10.

To find the time it takes a new worker to produce units 20 through 30, we integrate $T(x)$ over the interval $[20,30]$.

$$\int_{20}^{30} T(x)\,dx$$
$$= \int_{20}^{30} \left(2 + 0.3x^{-1}\right)dx$$
$$= \left[2x + 0.3\ln x\right]_{20}^{30}$$
$$= 2(30) + 0.3\ln(30) - \left(2(20) + 0.3\ln(20)\right)$$
$$\approx 20.12$$

It takes 20.12 hours for a new worker to produce units 20 through 30.

68. We integrate $T(x)$ over the interval $[1,20]$.

$$\int_1^{20} T(x)\,dx$$
$$= \int_1^{20} \left(2 + 0.3x^{-1}\right)dx$$
$$= \left[2x + 0.3\ln x\right]_1^{20}$$
$$\approx 38.9$$

It takes 38.9 hours for a new worker to produce units 1 through 20.

We integrate $T(x)$ over the interval $[20,40]$.

$$\int_{20}^{40} T(x)\,dx$$
$$= \int_{20}^{40} \left(2 + 0.3x^{-1}\right)dx$$
$$= \left[2x + 0.3\ln x\right]_{20}^{40}$$
$$\approx 40.21$$

It takes 40.21 hours for a new worker to produce units 20 through 40.

69. We integrate $M'(t)$ over the interval $[0,10]$.

$$\begin{aligned}M(10) &= \int_0^{10} M'(t)\\ &= \int_0^{10}\left(-0.009t^2+0.2t\right)dt\\ &= \left[-0.003t^3+0.1t^2\right]_0^{10}\\ &= \left(-0.003(10)^3+0.1(10)^2\right)\\ &\quad -\left(-0.003(0)^3+0.1(0)^2\right)\\ &= 7-0\\ &= 7\end{aligned}$$

In the first 10 minutes, 7 words are memorized.

70. We integrate $M'(t)$ over the interval $[0,10]$.

$$\begin{aligned}M(10) &= \int_0^{10} M'(t)\\ &= \int_0^{10}\left(-0.003t^2+0.2t\right)dt\\ &= \left[-0.001t^3+0.1t^2\right]_0^{10}\\ &= 9\end{aligned}$$

In the first 10 minutes, 9 words are memorized.

71. We integrate $M'(t)$ over the interval $[10,15]$.

$$\begin{aligned}M(15)-M(10) &= \int_{10}^{15} M'(t)\\ &= \int_{10}^{15}\left(-0.009t^2+0.2t\right)dt\\ &= \left[-0.003t^3+0.1t^2\right]_{10}^{15}\\ &= \left(-0.003(15)^3+0.1(15)^2\right)\\ &\quad -\left(-0.003(10)^3+0.1(10)^2\right)\\ &= 12.375-7\\ &= 5.375\end{aligned}$$

About 5 words are memorized during minutes 10 – 15.

72. We integrate $M'(t)$ over the interval $[10,17]$.

$$\begin{aligned}M(17)-M(10) &= \int_{10}^{17} M'(t)\\ &= \int_{10}^{17}\left(-0.003t^2+0.2t\right)dt\\ &= \left[-0.001t^3+0.1t^2\right]_{10}^{17}\\ &= 14.99\end{aligned}$$

About 15 words are memorized during minutes 10 – 17.

73. We first find $s(t)$ by integrating:

$$s(t)=\int v(t)\,dt=\int 3t^2dt=t^3+C.$$

Next we determine C by using the initial condition $s(0)=4$, which is the starting position for s at time $t=0$:

$$\begin{aligned}s(0)&=4\\ 0^3+C&=4\\ C&=4.\end{aligned}$$

Thus, $s(t)=t^3+4.$

74. We first find $s(t)$ by integrating:

$$s(t)=\int v(t)\,dt=\int 2tdt=t^2+C.$$

Next we determine C by using the initial condition $s(0)=10$:

$$\begin{aligned}s(0)&=10\\ 0^2+C&=10\\ C&=10.\end{aligned}$$

Thus, $s(t)=t^2+10.$

75. We first find $v(t)$ by integrating:

$$v(t)=\int a(t)\,dt=\int 4tdt=2t^2+C.$$

Next we determine C by using the initial condition $v(0)=20$:

$$\begin{aligned}v(0)&=20\\ 2\cdot 0^2+C&=20\\ C&=20.\end{aligned}$$

Thus, $v(t)=2t^2+20.$

76. We first find $v(t)$ by integrating:

$$v(t)=\int a(t)\,dt=\int 6tdt=3t^2+C.$$

Next we determine C by using the initial condition $v(0)=30$:

$$\begin{aligned}v(0)&=30\\ 3\cdot 0^2+C&=30\\ C&=30.\end{aligned}$$

Thus, $v(t)=3t^2+30.$

77. We first find $v(t)$ by integrating:

$$v(t) = \int a(t)\,dt = \int(-2t+6)\,dt = -t^2+6t+C_1.$$

Next we determine C_1 by using the initial condition $v(0)=6$:

$$v(0)=6$$
$$-(0)^2+6(0)+C_1=6$$
$$C_1=6.$$

Thus, $v(t)=-t^2+6t+6.$

Next, we find $s(t)$ by integrating:

$$s(t)=\int v(t)\,dt = \int\left(-t^2+6t+6\right)dt = -\frac{1}{3}t^3+3t^2+6t+C_2$$

Next we determine C_2 by using the initial condition $s(0)=10$:

$$s(0)=10$$
$$-\frac{1}{3}(0)^3+3(0)^2+6(0)+C_2=10$$
$$C_2=10.$$

Thus, $s(t)=-\frac{1}{3}t^3+3t^2+6t+10.$

78. We first find $v(t)$ by integrating:

$$v(t)=\int a(t)\,dt = \int(-6t+7)\,dt = -3t^2+7t+C_1.$$

Next we determine C_1 by using the initial condition $v(0)=10$:

$$v(0)=10$$
$$-3(0)^2+7(0)+C_1=10$$
$$C_1=10.$$

Thus, $v(t)=-3t^2+7t+10.$

Next, we find $s(t)$ by integrating $v(t)$ with respect to *t*:

$$s(t)=\int v(t)\,dt = \int\left(-3t^2+7t+10\right)dt = -t^3+\frac{7}{2}t^2+10t+C$$

Next we determine C_2 by using the initial condition $s(0)=20$:

$$s(0)=20$$
$$-(0)^3+\frac{7}{2}(0)^2+10(0)+C_2=20$$
$$C_2=20.$$

Thus, $s(t)=-t^3+\frac{7}{2}t^2+10t+20.$

79. a) We integrate $v(t)$ over the interval $[0,5]$:

$$s(5)=\int_0^5 v(t)\,dt = \int_0^5\left(-0.5t^2+10t\right)dt = \left[-\frac{1}{6}t^3+5t^2\right]_0^5 = \left(-\frac{1}{6}(5)^3+5(5)^2\right)-\left(-\frac{1}{6}(0)^3+5(0)^2\right) = \frac{625}{6}-0 \approx 104.17$$

The particle travels approximately 104.17 meters during the first 5 seconds.

b) We integrate $v(t)$ over the interval $[5,10]$:

$$s(10)-s(5)=\int_5^{10} v(t)\,dt = \left[-\frac{1}{6}t^3+5t^2\right]_5^{10} \quad \text{From part (a).}$$
$$\approx 229.17$$

The particle travels approximately 229.17 meters during the second 5 seconds.

80. a) We integrate $v(t)$ over the interval $[0,10]$:

$$s(10)=\int_0^{10} v(t)\,dt = \int_0^{10}\left(-0.3t^2+9t\right)dt = \left[-0.1t^3+\frac{9}{2}t^2\right]_0^{10} = 350$$

The particle travels 350 kilometers during the first 10 minutes.

b) We integrate $v(t)$ over the interval $[10, 20]$:

$$s(20) - s(10) = \int_{10}^{20} v(t)\,dt$$
$$= \left[-0.1t^3 + \frac{9}{2}t^2\right]_{10}^{20}$$
$$= 650$$

The particle travels approximately 650 kilometers during the second 10 minutes.

81. a) Converting 15 seconds into hours, we have $\frac{15}{3600} = \frac{1}{240}$. Thus, the motorcycle's acceleration function is

$a(t) = \dfrac{60 - 0}{\frac{1}{240} - 0} = 14,400$, where $a(t)$ is in miles per hour squared, and t is in hours. Thus, we can find the velocity function by integrating $a(t)$.

$$v(t) = \int a(t)\,dt = \int 14,400dt = 14,400t + C$$

We use the initial condition $v(0) = 0$ to find C.

$$v(0) = 0$$
$$14,400(0) + C = 0$$
$$C = 0$$

Thus, $v(t) = 14,400t$, where $v(t)$ is in miles per hour and t is in hours.

Now, substituting $\frac{1}{240}$ hour (15 seconds) for t, we have $v\left(\frac{1}{240}\right) = 14,400\left(\frac{1}{240}\right) = 60.$

The motorcycle is traveling at a speed of 60 miles per hour after 15 seconds.
Note: the intuitive solution to this problem is if the motorcycle accelerates at a constant rate from 0 mph to 60 mph in 15 seconds, then the motorcycle is obviously traveling at 60 mph after 15 seconds. We derive the velocity function to find the distance in part (b)

b) Using the information in Part (a), we integrate $v(t)$ over the interval $\left[0, \frac{1}{240}\right]$.

$$s\left(\frac{1}{240}\right) = \int_0^{1/240} 14,400t\,dt$$
$$= \left[7200t^2\right]_0^{1/240}$$
$$= 7200\left(\frac{1}{240}\right)^2 - 7200(0)$$
$$= \frac{1}{8}.$$

The motorcycle has traveled $\frac{1}{8}$ mi of a mile after 15 seconds.

82. a) The car is traveling at 60 mph after 30 seconds.

b) Converting seconds to hours, we have 30 seconds is $\frac{1}{120}$ hours. Therefore, the velocity function is $v(t) = 7200t$ (see Exercise 81.)

we integrate $v(t)$ over the interval $\left[0, \frac{1}{120}\right]$.

$$s\left(\frac{1}{120}\right) = \int_0^{1/120} 7200t\,dt$$
$$= \left[3600t^2\right]_0^{1/120}$$
$$= \frac{1}{4}.$$

The motorcycle has traveled 1/4 mile after 30 seconds.

83. a) Converting 45 seconds into hours, we have $\frac{45}{3600} = \frac{1}{80}$. Thus, the cyclist's acceleration function is $a(t) = \dfrac{30 - 0}{\frac{1}{80} - 0} = 2400$, where $a(t)$ is in kilometers per hour squared, and t is in hours. Thus, we can find the velocity function by integrating $a(t)$.

$$v(t) = \int a(t)\,dt = \int 2400dt = 2400t + C$$

We use the initial condition $v(0) = 0$ to find C.

$$v(0) = 0$$
$$2400(0) + C = 0$$
$$C = 0$$

Thus, $v(t) = 2400t$, where $v(t)$ is in kilometers per hour and t is in hours.

Now, substituting $\frac{1}{180}$ hour (20 seconds) for t, we have $v\left(\frac{1}{180}\right)=2400\left(\frac{1}{180}\right)=13.33$

The cyclist is traveling at a speed of 13.33 kilometers per hour after 20 seconds.

b) Using the information in Part (a), we integrate $v(t)$ over the interval $\left[0,\frac{1}{80}\right]$.

$$s\left(\frac{1}{80}\right)=\int_0^{1/80}2400t\,dt$$
$$=\left[1200t^2\right]_0^{1/80}$$
$$=1200\left(\frac{1}{80}\right)^2-1200(0)^2$$
$$=\frac{3}{16}\approx 0.1875.$$

The motorcycle has traveled 0.1875 kilometers after 45 seconds.

84. a) The cheetah is moving at a rate of 25 kilometers per hour after 10 seconds.

b) Converting seconds to hours, we have 20 seconds is $\frac{1}{180}$ hours. Therefore, the velocity function is $v(t)=9000t$.(See Exercise 81)

We integrate $v(t)$ over the interval $\left[0,\frac{1}{180}\right]$.

$$s\left(\frac{1}{180}\right)=\int_0^{1/180}9000t\,dt$$
$$=\left[4500t^2\right]_0^{1/180}$$
$$=\frac{5}{36}\approx 0.1389.$$

The cheetah has traveled 0.1389 kilometers after 20 seconds.

85. We first find $v(t)$ by integrating:

$$v(t)=\int a(t)\,dt$$
$$=\int(-32)\,dt$$
$$=-32t+C_1.$$

Next we determine C_1 by using the initial condition $v(0)=v_0$:

$$v(0)=v_0$$
$$-32(0)+C_1=v_0$$
$$C_1=v_0.$$

Thus, $v(t)=-32t+v_0$.

Next, we find $s(t)$ by integrating:

$$s(t)=\int v(t)\,dt$$
$$=\int(-32t+v_0)\,dt$$
$$=-16t^2+v_0\cdot t+C_2$$

Next we determine C_2 by using the initial condition $s(0)=s_0$:

$$s(0)=s_0$$
$$-16(0)^2+v_0(0)+C_2=s_0$$
$$C_2=s_0.$$

Thus, $s(t)=-16t^2+v_0t+s_0$.

86. From Exercise 85, we have $s(t)=-16t^2+v_0t+s_0$. Substituting 80 for v_0 and 10 for s_0, we have:

$$s(t)=-16t^2+80t+10$$

To determine when the ball hits the ground, we solve the equation:

$$s(t)=0$$
$$-16t^2+80t+10=0$$

Using the quadratic formula, we have:

$$t=\frac{-80\pm\sqrt{(80)^2-4(-16)(10)}}{2(-16)}$$
$$=\frac{10\pm\sqrt{110}}{4}$$
$$\approx 5.12 \qquad (t>0)$$

It will take approximately 5.12 seconds for the ball to hit the ground.

87. $a(t)=7200$

$$v(t)=\int a(t)\,dt$$
$$=\int 7200\,dt$$
$$=7200t+C$$

Since $v(0)=0$, we have

$$v(0)=0$$
$$7200(0)+C=0$$
$$C=0$$

Thus, $v(t)=7200t$

Integrating $v(t)$ from $\left[0, \frac{1}{120}\right]$ we have

$$s\left(\frac{1}{120}\right) = \int_0^{1/120} 7200t\,dt$$
$$= \left[3600t^2\right]_0^{1/120}$$
$$= \frac{1}{4}.$$

The car travels $\frac{1}{4}$ mile in $\frac{1}{2}$ minute.

88. $a(t) = 12{,}000$

$v(t) = 12{,}000t$ See Exercise 81.

We integrate $v(t)$ over the interval $\left[0, \frac{1}{240}\right]$.

$$s\left(\frac{1}{240}\right) = \int_0^{1/240} 12{,}000t\,dt$$
$$= \left[6000t^2\right]_0^{1/240}$$
$$= \frac{5}{48} \approx 0.104.$$

The motorcycle travels approximately 0.104 miles in 15 seconds.

89. We integrate $v(t)$ over the interval $[1,5]$:

$$s(5) - s(1) = \int_1^5 v(t)\,dt$$
$$= \int_1^5 \left(3t^2 + 2t\right)dt$$
$$= \left[t^3 + t^2\right]_1^5$$
$$= \left(5^3 + 5^2\right) - \left(1^3 + 1^2\right)$$
$$= 150 - 2$$
$$= 148$$

The particle travels approximately 148 miles from the 2nd hour to through the 5th hour.

90. We integrate $v(t)$ over the interval $[0,3]$:

$$s(3) = \int_0^3 v(t)\,dt$$
$$= \int_0^3 \left(4t^3 + 2t\right)dt$$
$$= \left[t^4 + t^2\right]_0^3$$
$$= \left(3^4 + 3^2\right) - \left(0^4 + 0^2\right)$$
$$= 90$$

The particle travels approximately 90 miles during the first 3 hours.

91. $S(t) = \int S'(t)\,dt$

$S(t) = \int 0.5e^t\,dt = 0.5e^t + C$

Assuming $S(0) = 0$, we have:

$$S(0) = 0$$
$$0.5e^0 + C = 0$$
$$0.5 + C = 0$$
$$C = -0.5$$

Thus, $S(t) = 0.5e^t - 0.5$

When Bluetape reaches $10,000 in sales, $S(t) = 10{,}000$. We solve the equation for *t*.

$$0.5e^t - 0.5 = 10{,}000$$
$$0.5e^t = 10{,}000.5$$
$$e^t = 20{,}001$$
$$\ln e^t = \ln 20{,}001$$
$$t = 9.9035$$

Therefore, they will reach $10,000 in sales on the 10th day.

92. $N'(t) = 280t^{3/2}$

a) Integrating over the interval $[0,16]$:

$$N(16) = \int_0^{16} N'(t)\,dt$$
$$= \int_0^{16} 280t^{3/2}\,dt$$
$$= \left[112t^{5/2}\right]_0^{16}$$
$$= 114{,}688$$

114,688 pounds of pollutants enter the lake during the first 16 months.

b) $N(T) = \int_0^T N'(t)\,dt = 50{,}000$

Therefore,

$$112(T)^{5/2} = 50{,}000$$
$$(T)^{5/2} = \frac{50{,}000}{112}$$
$$T = \left(\frac{50{,}000}{112}\right)^{2/5}$$
$$T \approx 11.48$$

The factory must begin cleanup procedures during the 11th month.

93. $\int_2^3 \frac{x^2-1}{x-1}dx = \int_2^3 \frac{(x-1)(x+1)}{(x-1)}dx$

$= \int_2^3 (x+1)\,dx$

$= \left[\frac{x^2}{2}+x\right]_2^3$

$= \left(\frac{3^2}{2}+3\right)-\left(\frac{2^2}{2}+2\right)$

$= \frac{15}{2}-4$

$= \frac{7}{2} = 3.5$

94. $\int_1^5 \frac{x^5-x^{-1}}{x^2}dx = \int_1^5 \left(x^3-x^{-3}\right)dx$

$= \left[\frac{x^4}{4}+\frac{x^{-2}}{2}\right]_1^5$

$= \left(\frac{(5)^4}{4}+\frac{(5)^{-2}}{2}\right)-\left(\frac{(1)^4}{4}+\frac{(1)^{-2}}{2}\right)$

$= \frac{3888}{25}$

$= 155.52$

95. $\int_4^{16} (x-1)\sqrt{x}dx = \int_4^{16}\left(x^{3/2}-x^{1/2}\right)dx$

$= \left[\frac{2}{5}x^{5/2}-\frac{2}{3}x^{3/2}\right]_4^{16}$

$= \left(\frac{2}{5}(16)^{5/2}-\frac{2}{3}(16)^{3/2}\right)-\left(\frac{2}{5}(4)^{5/2}-\frac{2}{3}(4)^{3/2}\right)$

$= \left(\frac{2048}{5}-\frac{128}{3}\right)-\left(\frac{64}{5}-\frac{16}{3}\right)$

$= \frac{5392}{15}$

$= 359\frac{7}{15}$

96. $\int_0^1 (x+2)^3\,dx$

$= \left[\frac{1}{4}(x+2)^4\right]_0^1$

$= \frac{1}{4}(1+2)^4-\frac{1}{4}(0+2)^4$

$= \frac{81}{4}-\frac{16}{4}$

$= \frac{65}{4} = 16.25$

97. $\int_1^8 \frac{\sqrt[3]{x^2}-1}{\sqrt[3]{x}}dx = \int_1^8\left(x^{2/3}-1\right)x^{-1/3}dx$

$= \int_1^8\left(x^{1/3}-x^{-1/3}\right)dx$

$= \left[\frac{3}{4}x^{4/3}-\frac{3}{2}x^{2/3}\right]_1^8$

$= \left(\frac{3}{4}(8)^{4/3}-\frac{3}{2}(8)^{2/3}\right)-\left(\frac{3}{4}(1)^{4/3}-\frac{3}{2}(1)^{2/3}\right)$

$= (12-6)-\left(\frac{3}{4}-\frac{3}{2}\right)$

$= 6-\left(-\frac{3}{4}\right)$

$= \frac{27}{4}$

$= 6.75$

98. $\int_0^1 \frac{x^3+8}{x+2}dx = \int_0^1\frac{(x+2)\left(x^2-2x+4\right)}{(x+2)}dx$

$= \int_0^1\left(x^2-2x+4\right)dx$

$= \left[\frac{x^3}{3}-x^2+4x\right]_0^1$

$= \left(\frac{(1)^3}{3}-(1)^2+4(1)\right)-\left(\frac{(0)^3}{2}-(0)^2+4(0)\right)$

$= \frac{10}{3} = 3\frac{1}{3}$

99. $\int_2^5 \left(t+\sqrt{3}\right)\left(t-\sqrt{3}\right)dt = \int_2^5 \left(t^2-3\right)dt$

$= \left[\frac{t^3}{3} - 3t\right]_2^5$

$= \left(\frac{(5)^3}{3} - 3(5)\right) - \left(\frac{(2)^3}{3} - 3(2)\right)$

$= \frac{80}{3} - \left(-\frac{10}{3}\right)$

$= \frac{90}{3}$

$= 30$

100. $\int_0^1 (t+1)^3 dt$

$= \left[\frac{1}{4}(t+1)^4\right]_0^1$

$= \frac{1}{4}(1+1)^4 - \frac{1}{4}(0+1)^4$

$= \frac{16}{4} - \frac{1}{4}$

$= \frac{15}{4} = 3.75$

101. $\int_1^3 \left(x - \frac{1}{x}\right)^2 dx = \int_1^3 \left(x - x^{-1}\right)^2 dx$

$= \int_1^3 \left(x^2 - 2 + x^{-2}\right)dx$ Expanding $\left(x - x^{-1}\right)^2$

$= \left[\frac{x^3}{3} - 2x - x^{-1}\right]_1^3$

$= \left(\frac{(3)^3}{3} - 2(3) - (3)^{-1}\right) - \left(\frac{(1)^3}{3} - 2(1) - (1)^{-1}\right)$

$= \frac{8}{3} - \left(-\frac{8}{3}\right)$

$= \frac{16}{3}$

$= 5\tfrac{1}{3}$

102. $\int_1^3 \frac{t^5 - t}{t^3} dt = \int_1^3 \left(t^2 - t^{-2}\right)dt$

$= \left[\frac{t^3}{3} + t^{-1}\right]_1^3$

$= \left(\frac{(3)^3}{3} + (3)^{-1}\right) - \left(\frac{(1)^3}{3} + (1)^{-1}\right)$

$= \frac{28}{3} - \frac{4}{3}$

$= \frac{24}{3}$

$= 8$

103. $\int_4^9 \frac{t+1}{\sqrt{t}} dt = \int_4^9 (t+1)t^{-1/2} dt$

$= \int_4^9 \left(t^{1/2} + t^{-1/2}\right)dt$

$= \left[\frac{2}{3}t^{3/2} + 2t^{1/2}\right]_4^9$

$= \left(\frac{2}{3}(9)^{3/2} + 2(9)^{1/2}\right) - \left(\frac{2}{3}(4)^{3/2} + 2(4)^{1/2}\right)$

$= 24 - \frac{28}{3}$

$= \frac{44}{3}$

$= 14\tfrac{2}{3}$

104. $\boxed{tw}$ The value of the antiderivative at $x = 1$ was not subtracted from the value of the antiderivative at $x = 2$.

105. $\boxed{tw}$ The antiderivative of $\ln x$ is not $\frac{1}{x}$. We are not able to find the antiderivative of $\ln x$ at this point in our studies.

106. Using the fnInt feature on a calculator, we get:
$\int_{-1.2}^{6.3} \left(x^3 - 9x^2 + 27x + 50\right)dx \approx 529.356.$

107. Using the fnInt feature on a calculator, we get:
$\int_{-8}^{1.4} \left(x^4 + 4x^3 - 36x^2 - 160x + 300\right)dx \approx 4068.789.$

108. Using the fnInt feature on a calculator, we get:
$\int_{-2}^{2} \sqrt{4 - x^2}\, dx \approx 6.283.$