Chapter 3

Exponential and Logarithmic Functions

Exercise Set 3.1

1. Graph: $y = 4^x$

First, we find some function values.

$x = -2, y = 4^{-2} = \frac{1}{4^2} = \frac{1}{16} = 0.0625$

$x = -1, y = 4^{-1} = \frac{1}{4} = 0.25$

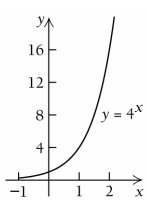
$x = 0, \quad y = 4^0 = 1$

$x = 1, \quad y = 4^1 = 4$

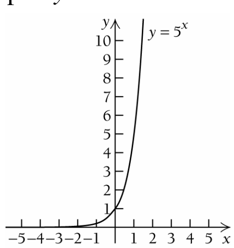
$x = 2, \quad y = 4^2 = 16$

x	y
-2	0.0625
-1	0.25
0	1
1	4
2	16

Next, we plot the points and connect them with a smooth curve.



2. Graph: $y = 5^x$



3. Graph: $y = (0.25)^x$

First note that:

$$\begin{aligned} y &= (0.25)^x \\ &= \left(\frac{1}{4}\right)^x \\ &= \left(4^{-1}\right)^x \\ &= 4^{-x} \end{aligned}$$

This will ease our work in calculating function values.

x	y
-2	16
-1	4
0	1
1	0.25
2	0.0625

$x = -2, y = 4^{-(-2)} = 4^2 = 16$

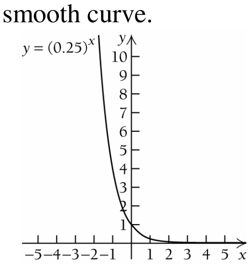
$x = -1, y = 4^{-(-1)} = 4^1 = 4$

$x = 0, \quad y = 4^{-(0)} = 1$

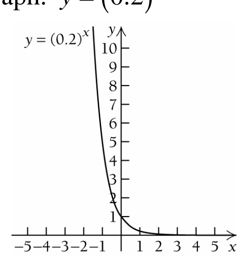
$x = 1, \quad y = 4^{-(1)} = \frac{1}{4} = 0.25$

$x = 2, \quad y = 4^{-(2)} = \frac{1}{4^2} = \frac{1}{16} = 0.0625$

Next, we plot the points and connect them with a smooth curve.



4. Graph: $y = (0.2)^x$



5. Graph: $f(x)=\left(\dfrac{3}{2}\right)^x$

First, we find some function values.

$$x=-2, f(-2)=\left(\frac{3}{2}\right)^{-2}=\frac{1}{\left(\frac{3}{2}\right)^2}=\frac{1}{\left(\frac{9}{4}\right)}=\frac{4}{9}=.44\overline{4}$$

$$x=-1, f(-1)=\left(\frac{3}{2}\right)^{-1}=\frac{1}{\left(\frac{3}{2}\right)}=\frac{2}{3}=.66\overline{6}$$

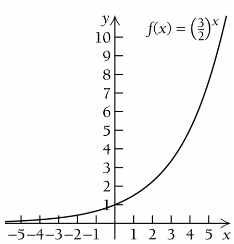
$$x=0,\quad f(0)=\left(\frac{3}{2}\right)^{0}=1$$

$$x=1,\quad f(1)=\left(\frac{3}{2}\right)^{1}=\frac{3}{2}=1.5$$

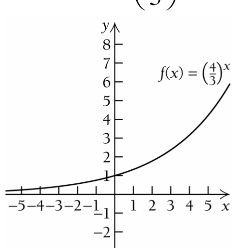
$$x=2,\quad f(2)=\left(\frac{3}{2}\right)^{2}=\frac{9}{4}=2.25$$

x	$f(x)$
-2	0.444
-1	0.666
0	1
1	1.5
2	2.25

Next, we plot the points and connect them with a smooth curve.



6. Graph: $f(x)=\left(\dfrac{4}{3}\right)^x$



7. Graph: $g(x)=\left(\dfrac{2}{3}\right)^x$

First, we find some function values.

$$x=-2, g(-2)=\left(\frac{2}{3}\right)^{-2}=\frac{1}{\left(\frac{2}{3}\right)^2}=\frac{1}{\frac{4}{9}}=\frac{9}{4}=2.25$$

$$x=-1, g(-1)=\left(\frac{2}{3}\right)^{-1}=\frac{1}{\left(\frac{2}{3}\right)}=\frac{3}{2}=1.5$$

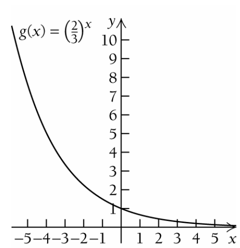
$$x=0,\quad g(0)=\left(\frac{2}{3}\right)^{0}=1$$

$$x=1,\quad g(1)=\left(\frac{2}{3}\right)^{1}=0.66\overline{6}$$

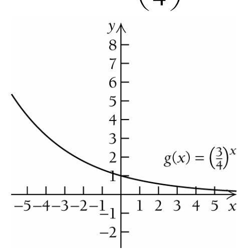
$$x=2,\quad g(2)=\left(\frac{2}{3}\right)^{2}=\frac{4}{9}=0.44\overline{4}$$

x	$g(x)$
-2	2.25
-1	1.5
0	1
1	0.666
2	0.444

Next, we plot the points and connect them with a smooth curve.



8. Graph: $g(x)=\left(\dfrac{3}{4}\right)^x$



9. Graph: $f(x)=(2.5)^x$

First, we find some function values.

$x=-2, f(-2)=(2.5)^{-2}=\dfrac{1}{(2.5)^2}=\dfrac{1}{6.25}=0.16$

$x=-1, f(-1)=(2.5)^{-1}=\dfrac{1}{2.5}=0.4$

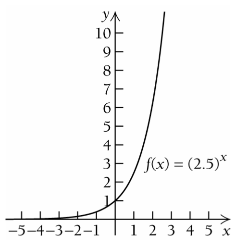
$x=0,\ \ f(0)=(2.5)^0=1$

$x=1,\ \ f(1)=(2.5)^1=2.5$

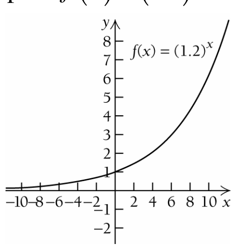
$x=2,\ \ f(2)=(2.5)^2=6.25$

x	$f(x)$
-2	0.16
-1	0.4
0	1
1	2.5
2	6.25

Next, we plot the points and connect them with a smooth curve.



10. Graph: $f(x)=(1.2)^x$



11. $f(x)=e^x$

By Theorem 1, $\dfrac{d}{dx}e^x=e^x$. Therefore,

$f'(x)=e^x$

12. $f(x)=e^{-x}$

$f'(x)=-e^{-x}$

13. $g(x)=e^{2x}$

Using the chain rule $\dfrac{d}{dx}e^{f(x)}=f'(x)e^{f(x)}$, we have $g'(x)=e^{2x}\cdot 2=2e^{2x}$.

14. $g(x)=e^{3x}$

$g'(x)=3e^{3x}$

15. $f(x)=6e^x$

Using $\dfrac{d}{dx}\left[c\cdot f(x)\right]=c\cdot f'(x)$, we have:

$f'(x)=6e^x$.

16. $f(x)=4e^x$

$f'(x)=4e^x$

17. $F(x)=e^{-7x}$

$F'(x)=-7e^{-7x}$

18. $F(x)=e^{-4x}$

$F'(x)=-4e^{-4x}$

19. $G(x)=2e^{4x}$

We use $\dfrac{d}{dx}\left[c\cdot f(x)\right]=c\cdot f'(x)$:

$$G'(x)=\frac{d}{dx}\left[2\cdot e^{4x}\right]=2\cdot\frac{d}{dx}e^{4x}$$

Next, we use the Chain Rule:

$\dfrac{d}{dx}e^{f(x)}=f'(x)e^{f(x)}$; therefore,

$G'(x)=2\cdot 4e^{4x}$

$G'(x)=8e^{4x}$.

20. $g(x)=3e^{5x}$

$$\begin{aligned}g'(x)&=3\cdot 5e^{5x}\\&=15e^{5x}\end{aligned}$$

21. $f(x)=-3e^{-x}$

$$\begin{aligned}f'(x)&=\frac{d}{dx}\left[-3\cdot e^{-x}\right]=-3\cdot\frac{d}{dx}e^{-x}\\&=-3\cdot e^{-x}\cdot(-1)\\&=3e^{-x}\end{aligned}$$

22. $G(x) = -7e^{-x}$

$$G'(x) = -7 \cdot e^{-x} \cdot (-1) = 7e^{-x}$$

23. $g(x) = \frac{1}{2}e^{-5x}$

$$g'(x) = \frac{d}{dx}\left[\frac{1}{2} \cdot e^{-5x}\right] = \frac{1}{2} \cdot \frac{d}{dx}e^{-5x} = \frac{1}{2}e^{-5x} \cdot (-5) = -\frac{5}{2}e^{-5x}$$

24. $f(x) = \frac{1}{3}e^{-4x}$

$$f'(x) = \frac{1}{3} \cdot e^{-4x} \cdot (-4) = -\frac{4}{3}e^{-4x}$$

25. $F(x) = -\frac{2}{3}e^{x^2}$

$$F'(x) = \frac{d}{dx}\left[-\frac{2}{3} \cdot e^{x^2}\right] = -\frac{2}{3} \cdot \frac{d}{dx}e^{x^2} = -\frac{2}{3} \cdot e^{x^2} \cdot (2x) = -\frac{4x}{3}e^{x^2}$$

26. $g(x) = -\frac{4}{5}e^{x^3}$

$$g'(x) = -\frac{4}{5} \cdot e^{x^3} \cdot (3x^2) = -\frac{12x^2}{5}e^{x^3}$$

27. $G(x) = 7 + 3e^{5x}$

Using the Sum Rule, we have

$$G'(x) = \frac{d}{dx}\left[7 + 3e^{5x}\right] = \frac{d}{dx}7 + 3\frac{d}{dx}e^{5x}.$$

To differentiate the two terms remember that the derivative of a constant is zero. All that is left is to apply the Chain Rule.

$$G'(x) = 0 + 3 \cdot e^{5x} \cdot (5) = 15e^{5x}$$

28. $F(x) = 4 - e^{2x}$

$$F'(x) = 0 - e^{2x} \cdot (2) = -2e^{2x}$$

29. $f(x) = x^5 - 2e^{6x}$

First, we use the Difference Rule

$$f'(x) = \frac{d}{dx}x^5 - 2\frac{d}{dx}e^{6x}.$$

Next, we use the Power Rule on the first term and the Chain Rule on the second term

$$f'(x) = 5x^4 - 2 \cdot e^{6x} \cdot (6) = 5x^4 - 12e^{6x}$$

30. $G(x) = x^3 - 5e^{2x}$

$$G'(x) = 3x^2 - 5(2)e^{2x} = 3x^2 - 10e^{2x}$$

31. $g(x) = x^5e^{2x}$

First, we use the Product Rule.

$$g'(x) = x^5 \cdot \left[\frac{d}{dx}e^{2x}\right] + \left[\frac{d}{dx}x^5\right] \cdot e^{2x} = x^5 \cdot e^{2x} \cdot (2) + 5x^4 \cdot e^{2x} = 2x^5e^{2x} + 5x^4e^{2x} = (2x+5)x^4e^{2x} \quad \text{Factoring}$$

32. $f(x) = x^7e^{4x}$

$$f'(x) = x^7 \cdot e^{4x} \cdot (4) + 7x^6 \cdot e^{4x} = 4x^7e^{4x} + 7x^6 \cdot e^{4x} = (4x+7)x^6e^{4x}$$

33. $F(x) = \frac{e^{2x}}{x^4}$

First, we use the Quotient Rule.

$$F'(x) = \frac{x^4 \cdot (2)e^{2x} - e^{2x} \cdot 4x^3}{x^8}$$

$$= \frac{2x^3e^{2x}(x-2)}{x^3 \cdot x^5} \quad \text{Factoring}$$

$$= \frac{x^3}{x^3} \cdot \frac{2e^{2x}(x-2)}{x^5} \quad \text{Remove the factor equal to 1.}$$

$$= \frac{2e^{2x}(x-2)}{x^5}$$

34. $g(x)=\dfrac{e^{3x}}{x^6}$

$$g'(x)=\frac{x^6\cdot(3)e^{3x}-6x^5\cdot e^{3x}}{x^{12}}$$
$$=\frac{3x^5\cdot e^{3x}(x-2)}{x^{12}}$$
$$=\frac{3e^{3x}(x-2)}{x^7}$$

35. $f(x)=\left(x^2+3x-9\right)e^x$

$$f'(x)=(2x+3)e^x+\left(x^2+3x-9\right)e^x \quad \text{Product Rule}$$
$$=\left(2x+3+x^2+3x-9\right)e^x \quad \text{Factoring}$$
$$=\left(x^2+5x-6\right)e^x$$

36. $f(x)=\left(x^2-2x+2\right)e^x$

$$f'(x)=(2x-2)e^x+\left(x^2-2x+2\right)e^x$$
$$=\left(2x-2+x^2-2x+2\right)e^x$$
$$=x^2e^x$$

37. $f(x)=\dfrac{e^x}{x^4}$

$$f'(x)=\frac{x^4\cdot e^x-4x^3\cdot e^x}{x^8} \quad \text{By the Quotient Rule}$$
$$=\frac{x^3e^x(x-4)}{x^3\cdot x^5} \quad \text{Factoring}$$
$$=\frac{x^3}{x^3}\cdot\frac{e^x(x-4)}{x^5} \quad \text{Remove the factor equal to 1.}$$
$$=\frac{e^x(x-4)}{x^5}$$

38. $f(x)=\dfrac{e^x}{x^5}$

$$f'(x)=\frac{x^5\cdot e^x-5x^4\cdot e^x}{x^{10}}$$
$$=\frac{x^4\cdot e^x(x-5)}{x^{10}}$$
$$=\frac{e^x(x-5)}{x^6}$$

39. $f(x)=e^{-x^2+7x}$

$$f'(x)=e^{-x^2+7x}\cdot\left[\frac{d}{dx}\left(-x^2+7x\right)\right] \quad \text{By the Chain Rule}$$
$$=^{-x^2+7x}\cdot[-2x+7]$$
$$=(-2x+7)\cdot e^{-x^2+7x}$$

40. $f(x)=e^{-x^2+8x}$

$$f'(x)=(-2x+8)e^{-x^2+8x}$$

41. $f(x)=e^{-x^2/2}$

$$f'(x)=e^{-x^2/2}\cdot\left[\frac{d}{dx}\left(-\frac{x^2}{2}\right)\right]$$
$$=e^{-x^2/2}\left[\frac{-1}{2}(2x)\right]$$
$$=-x\cdot e^{-x^2/2}$$

42. $f(x)=e^{x^2/2}$

$$f'(x)=\tfrac{1}{2}(2x)e^{x^2/2}$$
$$=xe^{x^2/2}$$

43. $y=e^{\sqrt{x-7}}$

First note that

$y=e^{\sqrt{x-7}}=e^{(x-7)^{1/2}}$.

Using the Chain Rule, we have

$$\frac{dy}{dx}=e^{\sqrt{x-7}}\cdot\left[\tfrac{d}{dx}(x-7)^{1/2}\right].$$

Using the Chain Rule again, we have

$$\frac{dy}{dx}=e^{\sqrt{x-7}}\cdot\left[\frac{1}{2}(x-7)^{-1/2}\right]$$
$$=e^{\sqrt{x-7}}\cdot\frac{1}{2(x-7)^{1/2}} \quad \text{Properties of exponents}$$
$$=\frac{e^{\sqrt{x-7}}}{2\sqrt{x-7}}$$

44. $y = e^{\sqrt{x-4}}$

$$\frac{dy}{dx} = e^{\sqrt{x-4}} \cdot \frac{1}{2}(x-4)^{-1/2}$$
$$= \frac{e^{\sqrt{x-4}}}{2\sqrt{x-4}}$$

45. $y = \sqrt{e^x - 1}$

$$y = \sqrt{e^x - 1} = (e^x - 1)^{1/2}$$
$$\frac{dy}{dx} = \frac{1}{2}(e^x - 1)^{-1/2} \cdot \frac{d}{dx}(e^x - 1) \qquad \text{By the Chain Rule}$$
$$= \frac{1}{2}(e^x - 1)^{-1/2} \cdot (e^x - 0)$$
$$= \frac{e^x}{2(e^x - 1)^{1/2}}$$
$$= \frac{e^x}{2\sqrt{e^x - 1}}$$

46. $y = \sqrt{e^x + 1}$

$$\frac{dy}{dx} = \tfrac{1}{2}(e^x + 1)^{-1/2} \cdot (e^x)$$
$$= \frac{e^x}{2\sqrt{e^x + 1}}$$

47. $y = xe^{-2x} + e^{-x} + x^3$

Differentiate the function term by term. We apply the Product Rule to the first term, the Chain Rule to the second term, and the Power Rule to the third term.

$$\frac{dy}{dx} = \underbrace{x(-2)e^{-2x} + 1 \cdot e^{-2x}}_{\text{Product Rule}} + \underbrace{(-1)e^{-x}}_{\text{Chain Rule}} + \underbrace{3x^2}_{\text{Power Rule}}$$

Now we simplify:

$$\frac{dy}{dx} = -2xe^{-2x} + e^{-2x} - e^{-x} + 3x^2$$
$$= (1-2x)e^{-2x} - e^{-x} + 3x^2 \quad .$$

48. $y = e^x + x^3 - xe^x$

$$\frac{dy}{dx} = e^x + 3x^2 - \left((x \cdot e^x) + 1 \cdot e^x\right)$$
$$= e^x + 3x^2 - xe^x - e^x$$
$$= 3x^2 - xe^x$$

49. $y = 1 - e^{-x}$

$$\frac{dy}{dx} = 0 - e^{-x} \cdot (-1)$$
$$= e^{-x}$$

50. $y = 1 - e^{-3x}$

$$\frac{dy}{dx} = -e^{-3x} \cdot (-3) = 3e^{-3x}$$

51. $y = 1 - e^{-kx}$

We use the Chain Rule, remembering that k is a constant:

$$\frac{dy}{dx} = 0 - e^{-kx} \cdot (-k)$$
$$= ke^{-kx}$$

52. $y = 1 - e^{-mx}$

$$\frac{dy}{dx} = -e^{-mx} \cdot (-m) = me^{-mx}$$

53. $g(x) = (4x^2 + 3x)e^{x^2 - 7x}$

We use the Product Rule first, and apply the Chain Rule when taking the derivative of the exponential term.

$$g'(x) = (4x^2 + 3x)\underbrace{(2x-7)e^{x^2-7x}}_{\text{Chain Rule}} +$$
$$(8x+3)e^{x^2-7x}$$

Factoring out the common term, we have

$$g'(x) = \left[(4x^2 + 3x)(2x-7) + (8x+3)\right]e^{x^2-7x} \quad .$$

Simplifying inside the bracket, yields

$$g'(x) = \left[(8x^3 - 22x^2 - 21x) + (8x+3)\right]e^{x^2-7x}$$
$$= \left[8x^3 - 22x^2 - 13x + 3\right]e^{x^2-7x}.$$

54. $g(x) = (5x^2 - 8x)e^{x^2-4x}$

$$g'(x) = (5x^2 - 8x)(2x-4)e^{x^2-4x} +$$
$$(10x-8)e^{x^2-4x}$$
$$= \left[(10x^3 - 36x^2 + 32x) + (10x-8)\right]e^{x^2-4x}$$
$$= \left[10x^3 - 36x^2 + 42x - 8\right]e^{x^2-4x}$$

55. Graph: $f(x)=e^{2x}$

Using a calculator, we first find some function values.

$f(-2)=e^{2(-2)}=e^{-4}\approx 0.0183$

$f(-1)=e^{2(-1)}=e^{-2}\approx 0.1353$

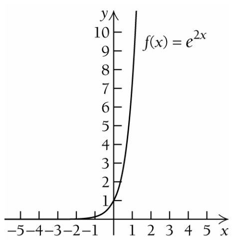
$f(0)=e^{2(0)}=e^{0}=1$

$f(1)=e^{2(1)}=e^{2}\approx 7.3891$

$f(2)=e^{2(2)}=e^{4}\approx 54.598$

x	$f(x)$
-2	0.0183
-1	0.1353
0	1
1	7.3891
2	54.598

Next, we plot the points and connect them with a smooth curve.



Derivatives. $f'(x)=2e^{2x}$ and $f''(x)=4e^{2x}$.

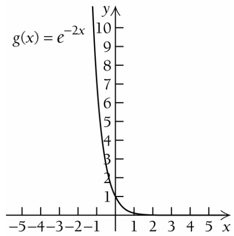
Critical values of f. Since $f'(x)>0$ for all real numbers x, we know that the derivative exists for all real numbers and there is no solution to the equation $f'(x)=0$. There are no critical values and therefore no maximum or minimum values.

Increasing. Since $f'(x)>0$ for all real numbers x, the function f is increasing over the entire real line.

Inflection points. Since $f''(x)>0$ for all real numbers x, the equation $f''(x)=0$ has no solution and there are no points of inflection.

Concavity. Since $f''(x)>0$ for all real numbers x, the function f' is increasing over the entire real line and the graph is concave up over the entire real line.

56. Graph: $g(x)=e^{-2x}$



Derivatives. $g'(x)=-2e^{-2x}$ and

$g''(x)=4e^{-2x}$.

Critical values of g. Since $g'(x)<0$ for all real numbers x, we know that the derivative exists for all real numbers and there is no solution to the equation $g'(x)=0$. There are no critical values and therefore no maximum or minimum values.

Decreasing. Since $g'(x)<0$ for all real numbers x, the function g is decreasing over the entire real line.

Inflection points. Since $g''(x)>0$ for all real numbers x, the equation $g''(x)=0$ has no solution and there are no points of inflection.

Concavity. Since $g''(x)>0$ for all real numbers x, the function g' is increasing over the entire real line and the graph is concave up over the entire real line.

57. Graph: $g(x)=e^{\left(\frac{1}{2}\right)x}$

First, we find some function values.

$g(-2)=e^{\left(\frac{1}{2}\right)(-2)}=e^{-1}\approx 0.3679$

$g(-1)=e^{\left(\frac{1}{2}\right)(-1)}=e^{-\frac{1}{2}}\approx 0.6065$

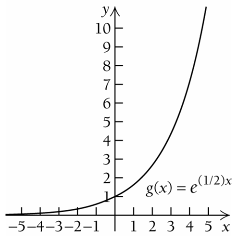
$g(0)=e^{\left(\frac{1}{2}\right)(0)}=e^{0}=1$

$g(1)=e^{\left(\frac{1}{2}\right)(1)}=e^{\frac{1}{2}}\approx 1.6487$

$g(2)=e^{\left(\frac{1}{2}\right)(2)}=e^{1}\approx 2.7183$

x	$g(x)$
-2	0.3679
-1	0.6065
0	1
1	1.6487
2	2.7183

Next, we plot the points and connect them with a smooth curve.



Derivatives. $g'(x) = \frac{1}{2}e^{\left(\frac{1}{2}\right)x}$ and

$$g''(x) = \tfrac{1}{4}e^{\left(\frac{1}{2}\right)x}.$$

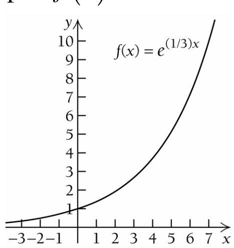
Critical values of g. Since $g'(x) > 0$ for all real numbers x, we know that the derivative exists for all real numbers and there is no solution to the equation $g'(x) = 0$. There are no critical values and therefore no maximum or minimum values.

Increasing. Since $g'(x) > 0$ for all real numbers x, the function g is increasing over the entire real line.

Inflection points. Since $g''(x) > 0$ for all real numbers x, the equation $g''(x) = 0$ has no solution and there are no points of inflection.

Concavity. Since $g''(x) > 0$ for all real numbers x, the function g' is increasing over the entire real line and the graph is concave up over the entire real line.

58. Graph: $f(x) = e^{\left(\frac{1}{3}\right)x}$



Derivatives. $f'(x) = \frac{1}{3}e^{\left(\frac{1}{3}\right)x}$ and

$$f''(x) = \tfrac{1}{9}e^{\left(\frac{1}{3}\right)x}.$$

Critical values of f. Since $f'(x) > 0$ for all real numbers x, we know that the derivative exists for all real numbers and there is no solution to the equation $f'(x) = 0$. There are no critical values and therefore no maximum or minimum values.

Increasing. Since $f'(x) > 0$ for all real numbers x, the function f is increasing over the entire real line.

Inflection points. Since $f''(x) > 0$ for all real numbers x, the equation $f''(x) = 0$ has no solution and there are no points of inflection.

Concavity. Since $f''(x) > 0$ for all real numbers x, the function f' is increasing over the entire real line and the graph is concave up over the entire real line.

59. Graph: $f(x) = \dfrac{1}{2}e^{-x}$

First, we find some function values.

$$f(-2) = \frac{1}{2}e^{-(-2)} = \frac{1}{2}e^{2} \approx 3.6945$$

$$f(-1) = \frac{1}{2}e^{-(-1)} = \frac{1}{2}e^{1} \approx 1.3591$$

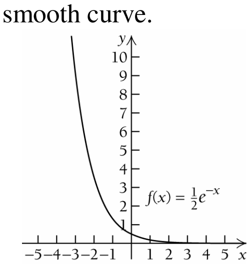
$$f(0) = \frac{1}{2}e^{-(0)} = \frac{1}{2}e^{0} = 0.5$$

$$f(1) = \frac{1}{2}e^{-(1)} = \frac{1}{2}e^{-1} \approx 0.1839$$

$$f(2) = \frac{1}{2}e^{-(2)} = \frac{1}{2}e^{-2} \approx 0.06767$$

x	$f(x)$
-2	3.6945
-1	1.3591
0	0.5
1	0.1839
2	0.06767

Next, we plot the points and connect them with a smooth curve.



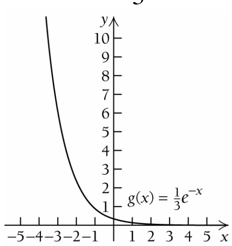
Derivatives. $f'(x) = -\dfrac{1}{2}e^{-x}$ and

$$f''(x) = \frac{1}{2}e^{-x}.$$

Critical values of f. Since $f'(x) < 0$ for all real numbers x, we know that the derivative exists for all real numbers and there is no solution to the equation $f'(x) = 0$. There are no critical values and therefore no maximum or minimum values.

Decreasing. Since $f'(x) < 0$ for all real numbers x, the function f is decreasing over the entire real line.
Inflection points. Since $f''(x) > 0$ for all real numbers x, the equation $f''(x) = 0$ has no solution and there are no points of inflection.
Concavity. Since $f''(x) > 0$ for all real numbers x, the function f' is increasing over the entire real line and the graph is concave up over the entire real line.

60. Graph: $g(x) = \frac{1}{3}e^{-x}$



Derivatives. $g'(x) = -\frac{1}{3}e^{-x}$ and

$$g''(x) = \frac{1}{3}e^{-x}.$$

Critical values of g. Since $g'(x) < 0$ for all real numbers x, we know that the derivative exists for all real numbers and there is no solution to the equation $g'(x) = 0$. There are no critical values and therefore no maximum or minimum values.
Decreasing. Since $g'(x) < 0$ for all real numbers x, the function g is decreasing over the entire real line.
Inflection points. Since $g''(x) > 0$ for all real numbers x, the equation $g''(x) = 0$ has no solution and there are no points of inflection.
Concavity. Since $g''(x) > 0$ for all real numbers x, the function g' is increasing over the entire real line and the graph is concave up over the entire real line.

61. Graph: $F(x) = -e^{\left(\frac{1}{3}\right)x}$

First, we find some function values.

$F(-2) = -e^{\left(\frac{1}{3}\right)(-2)} = -e^{-\frac{2}{3}} \approx -0.5134$

$F(-1) = -e^{\left(\frac{1}{3}\right)(-1)} = -e^{-\frac{1}{3}} \approx -0.7165$

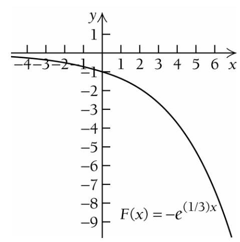
$F(0) = -e^{\left(\frac{1}{3}\right)(0)} = -e^{0} = -1$

$F(1) = -e^{\left(\frac{1}{3}\right)(1)} = -e^{\frac{1}{3}} \approx -1.3956$

$F(2) = -e^{\left(\frac{1}{3}\right)(2)} = -e^{\frac{2}{3}} \approx -1.9477$

x	$F(x)$
-2	- 0.5134
-1	- 0.7165
0	- 1
1	- 1.3956
2	- 1.9477

Next, we plot the points and connect them with a smooth curve.

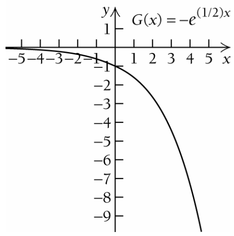


Derivatives $F'(x) = -\frac{1}{3}e^{\left(\frac{1}{3}\right)x}$ and

$$F''(x) = -\frac{1}{9}e^{\left(\frac{1}{3}\right)x}.$$

Critical values of F. Since $F'(x) < 0$ for all real numbers x, we know that the derivative exists for all real numbers and there is no solution to the equation $F'(x) = 0$. There are no critical values and therefore no maximum or minimum values.
Decreasing. Since $F'(x) < 0$ for all real numbers x, the function F is decreasing over the entire real line.
Inflection points. Since $F''(x) < 0$ for all real numbers x, the equation $F''(x) = 0$ has no solution and there are no points of inflection.
Concavity. Since $F''(x) < 0$ for all real numbers x, the function F' is decreasing over the entire real line and the graph is concave down over the entire real line.

62. Graph: $G(x) = -e^{(1/2)x}$



Derivative. $G'(x) = -\frac{1}{2}e^{(1/2)x}$ and

$$G''(x) = -\frac{1}{4}e^{(1/2)x}.$$

Critical values of G. Since $G'(x) < 0$ for all real numbers x, we know that the derivative exists for all real numbers and there is no solution to the equation $G'(x) = 0$. There are no critical values and therefore no maximum or minimum values.

Decreasing. Since $G'(x) < 0$ for all real numbers x, the function G is decreasing over the entire real line.

Inflection points. Since $G''(x) < 0$ for all real numbers x, the equation $G''(x) = 0$ has no solution and there are no points of inflection.

Concavity. Since $G''(x) < 0$ for all real numbers x, the function G' is decreasing over the entire real line and the graph is concave down over the entire real line.

63. Graph: $g(x) = 2(1 - e^{-x})$, for $x \geq 0$

First, we find some function values:

$g(0) = 2(1 - e^{-0}) = 0$

$g(1) = 2(1 - e^{-1}) \approx 1.2642$

$g(2) = 2(1 - e^{-2}) \approx 1.7293$

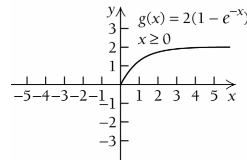
$g(3) = 2(1 - e^{-3}) \approx 1.9004$

$g(4) = 2(1 - e^{-4}) \approx 1.9634$

$g(5) = 2(1 - e^{-5}) \approx 1.9865$

x	$g(x)$
0	0
1	1.2642
2	1.7293
3	1.9004
4	1.9634
5	1.9865

Next, we plot the points and connect them with a smooth curve.



Derivatives. $g'(x) = 2e^{-x}$ and

$$g''(x) = -2e^{-x}.$$

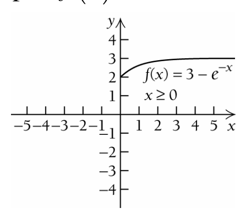
Critical values of g. Since $2e^{-x} > 0$ for all real numbers x, we know that there is no solution to the equation $g'(x) = 0$. There are no critical values on $(0, \infty)$.

Increasing. Since $2e^{-x} > 0$ for all real numbers x, we know that g is increasing over its entire domain $[0, \infty)$.

Inflection points. Since $-2e^{-x} < 0$ for all real numbers x, the equation $g''(x) = 0$ has no solution and there are no points of inflection.

Concavity. Since $-2e^{-x} < 0$ for all real numbers, the function g' is decreasing and the graph is concave down over the interval $(0, \infty)$.

64. Graph: $f(x) = 3 - e^{-x}$, for $x \geq 0$



Derivatives. $f'(x) = e^{-x}$ and $f''(x) = -e^{-x}$.

Critical values of f. Since $e^{-x} > 0$ for all real numbers x, we know that there is no solution to the equation $f'(x) = 0$. There are no critical values on $(0, \infty)$.

Increasing. Since $e^{-x} > 0$ for all real numbers x, we know that f is increasing over its entire domain, $[0, \infty)$.

Inflection points. Since $-e^{-x} < 0$ for all real numbers x, the equation $f''(x) = 0$ has no solution and there are no points of inflection.

Concavity. Since $-e^{-x} < 0$ for all real numbers, the function f' is decreasing and the graph is concave down over the interval $(0, \infty)$.

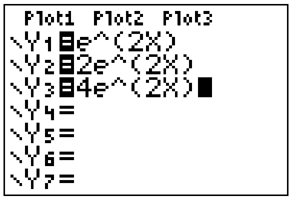
65. From exercise 55 we know that:

$f(x) = e^{2x}$

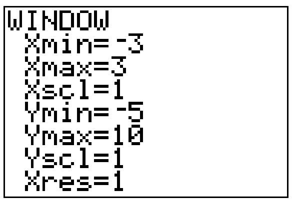
$f'(x) = 2e^{2x}$

$f''(x) = 4e^{2x}$

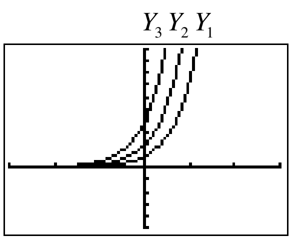
Enter each function into your graphing calculator.



Set the window on the calculator by pressing the Window button and changing the dimensions of the window.



Now press the graph key. The graphs are shown in the screen shot.

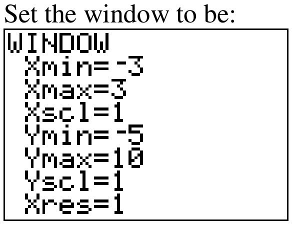


66. $Y_1 = g(x) = e^{-2x}$

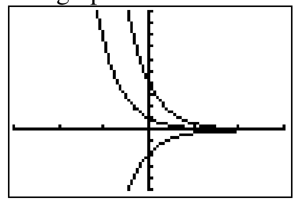
$Y_2 = g'(x) = -2e^{-2x}$

$Y_3 = g''(x) = 4e^{-2x}$

Set the window to be:



The graphs are shown in the screen shot.



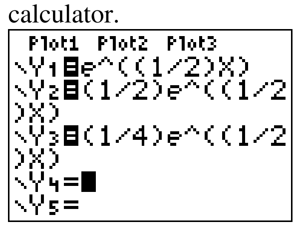
67. From Exercise 57 we know that:

$g(x) = e^{(1/2)x}$

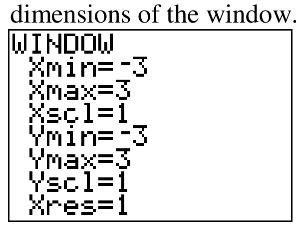
$g'(x) = \frac{1}{2}e^{(1/2)x}$

$g''(x) = \frac{1}{4}e^{(1/2)x}$

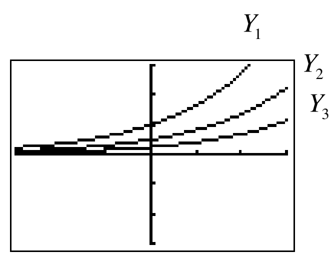
Enter each function into your graphing calculator.



Set the window on the calculator by pressing the Window button and changing the dimensions of the window.



Now press the graph key. The graphs are shown in the screen shot.



68. $Y_1 = f(x) = e^{(1/3)x}$

$Y_2 = f'(x) = \frac{1}{3}e^{(1/3)x}$

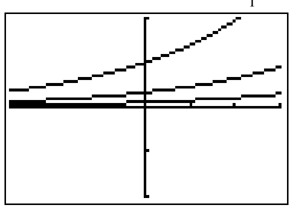
$Y_3 = f''(x) = \frac{1}{9}e^{(1/3)x}$

Set the window to be:

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-2
 Ymax=2
 Yscl=1
 Xres=1
```

The graphs are shown in the screen shot.

Y_1



Y_2

Y_3

69. From Exercise 59 we know that:

$f(x) = \frac{1}{2}e^{-x}$

$f'(x) = -\frac{1}{2}e^{-x}$

$f''(x) = \frac{1}{2}e^{-x}$

Enter each function into your graphing calculator.

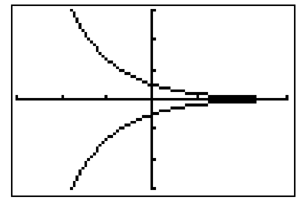
```
 Plot1 Plot2 Plot3
\Y1=(1/2)e^(-X)
\Y2=(-1/2)e^(-X)

\Y3=(1/2)e^(-X)
\Y4=
\Y5=
\Y6=
```

Set the window on the calculator by pressing the Window button and changing the dimensions of the window.

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-3
 Ymax=3
 Yscl=1
 Xres=1
```

Now press the graph key. The graphs are shown in the screen shot.

$Y_1 = Y_3$



Y_2

70. $Y_1 = g(x) = \frac{1}{3}e^{-x}$

$Y_2 = g'(x) = -\frac{1}{3}e^{-x}$

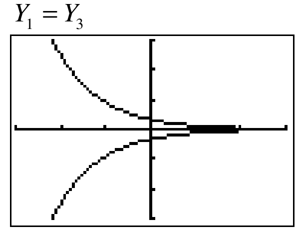
$Y_3 = g''(x) = \frac{1}{3}e^{-x}$

Set the window to be:

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-3
 Ymax=3
 Yscl=1
 Xres=1
```

The graphs are shown in the screen shot.

$Y_1 = Y_3$



Y_2

71. From Exercise 61 we know that:

$F(x) = -e^{(1/3)x}$

$F'(x) = -\frac{1}{3}e^{(1/3)x}$

$F''(x) = -\frac{1}{9}e^{(1/3)x}$

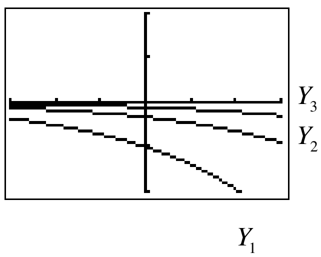
Enter each function into your graphing calculator.

```
 Plot1 Plot2 Plot3
\Y1=-e^((1/3)X)
\Y2=(-1/3)e^((1/
3)X)
\Y3=(-1/9)e^((1/
3)X)
\Y4=
\Y5=
```

Set the window on the calculator by pressing the Window button and changing the dimensions of the window.

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-2
 Ymax=2
 Yscl=1
 Xres=1
```

Now press the graph key. The graphs are shown in the screen shot.



72. $Y_1 = G(x) = -e^{(1/2)x}$

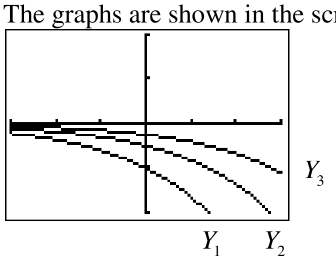
$Y_2 = G'(x) = -\frac{1}{2}e^{(1/2)x}$

$Y_3 = G''(x) = -\frac{1}{4}e^{(1/2)x}$

Set the window to be:

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-2
 Ymax=2
 Yscl=1
 Xres=1
```

The graphs are shown in the screen shot.



73. From Exercise 63 we know that:

$g(x) = 2(1-e^{-x}); \quad x \ge 0$

$g'(x) = 2e^{-x}; \quad x \ge 0$

$g''(x) = -2e^{-x}; \quad x \ge 0$

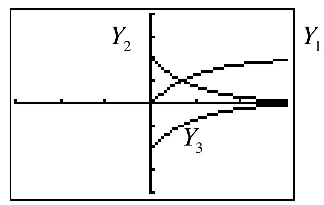
Enter each function into your graphing calculator. Notice how you can specify the domain in your calculator. Consult your owner's manual for a detailed explanation on how to do this.

```
 Plot1 Plot2 Plot3
\Y1=2(1-e^(-X))/
(X≥0)
\Y2=2e^(-X)/(X≥0
)
\Y3=-2e^(-X)/(X≥
0)
\Y4=
```

Set the window on the calculator by pressing the Window button and changing the dimensions of the window.

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-4
 Ymax=4
 Yscl=1
 Xres=1
```

Now press the graph key. The graphs are shown in the screen shot.



74. $Y_1 = g(x) = 3 - e^{-x}; \quad x \ge 0$

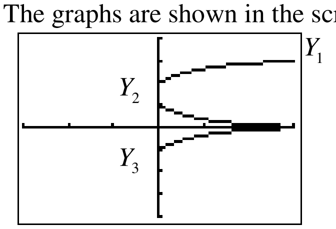
$Y_2 = f'(x) = e^{-x}; \quad x \ge 0$

$Y_3 = f''(x) = -e^{-x}; \quad x \ge 0$

Set the window to be:

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-4
 Ymax=4
 Yscl=1
 Xres=1
```

The graphs are shown in the screen shot.



75. First find the derivative of the function.

$f(x) = e^x$

$f'(x) = e^x$

Now, evaluate the derivative at $x = 0$.

$f'(0) = e^0 = 1$

The slope of the tangent line at the point $(0,1)$ is 1.

76. $f(x) = 2e^{-3x}$

$f'(x) = -6e^{-3x}$

$f'(0) = -6e^{0} = -6$

77. First find the slope of the tangent line at $(0,1)$, by evaluating the derivative at $x = 0$.

$g(x) = e^{-x}$

$g'(x) = -e^{-x}$

$g'(0) = -e^{-0} = -1$

Then we find the equation of the line with slope -1 and containing the point $(0,1)$.

$y - y_1 = m(x - x_1)$ Point-slope equation

$y - 1 = -1(x - 0)$

$y - 1 = -x$

$y = -x + 1$

78. $f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

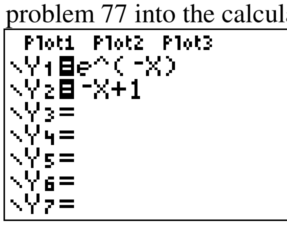
$f'(0) = 2e^{2\cdot 0} = 2$

$y - 1 = 2(x - 0)$

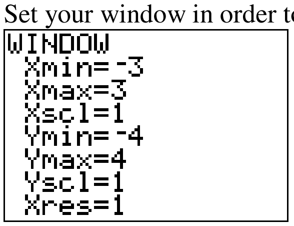
$y - 1 = 2x$

$y = 2x + 1$

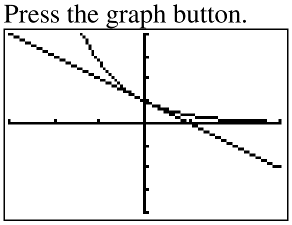
79. Enter the function and the tangent line found in problem 77 into the calculator.



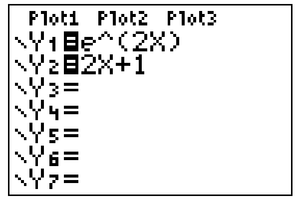
Set your window in order to see the functions.



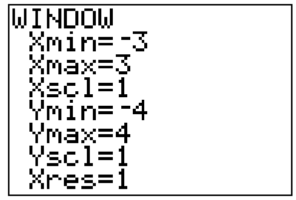
Press the graph button.



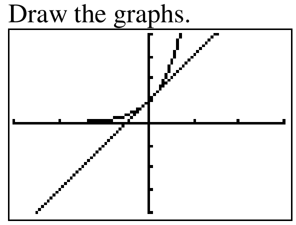
80. Enter the functions into the calculator.



Set the window.



Draw the graphs.



81. a) Evaluate $A(t)$ at $t = 14$. Note: 2009 is 14 years after 1995.

$A(14) = 2.43e^{0.18(14)}$

$= 2.43e^{2.52}$

$= 30.201490$

$A(14) \approx 30.2$

Americans will spend approximately \$30.2 billion on organic food and beverages in 2009.

b) Find the derivative of the function.

$A(t) = 2.43e^{0.18t}$

$A'(t) = 2.43(0.18)e^{0.18t}$

$A'(t) = 0.4374e^{0.18t}$

Find the rate of change in 2006 by evaluating the derivative at $t = 11$.

$A'(11) = 0.4374e^{0.18(11)}$

$= 0.4374e^{1.98}$

$= 3.167976$

$A'(t) \approx 3.17$

American expenditure on organic food and beverages is growing at a rate of \$3.17 billion per year in 2006.

82. a) First notice that

$$A(11) = 2.43e^{0.18(11)} \approx 17.6$$

$$A(12) = 2.43e^{0.18(12)} \approx 21.07$$

$$\Delta A = A(12) - A(11)$$
$$= 21.07 - 17.6$$
$$= 3.47$$

Organic food and beverages expenditures increased \$3.47 billion between 2006 and 2007.

b) $\boxed{tw}$ The answer in exercise 82a is the average increase in organic food expenditures between the year 2006 and 2007. In Exercise 81b, the instantaneous rate of growth was determined. This is the slope of the tangent line of the function at $t = 11$. Exercise 82a found the slope of the secant line intersecting the function at $t = 11$ and $t = 12$.

83. $T(x) = \dfrac{5200}{1 + 16.3e^{-0.7x}}$

a) In 2005, $x = 2005 - 1994 = 11$.

$$T(11) = \frac{5200}{1 + 16.3e^{-0.7(11)}}$$
$$= \frac{5200}{1 + 16.3e^{-7.7}}$$
$$\approx 5162$$

In 2005 there will be approximately 5162 thousand, or 5.162 million, cellular phones in Israel.

In 2010, $x = 2010 - 1994 = 16$.

$$T(16) = \frac{5200}{1 + 16.3e^{-0.7(16)}}$$
$$= \frac{5200}{1 + 16.3e^{-11.2}}$$
$$\approx 5199$$

In 2010 there will be approximately 5199 thousand, or 5.199 million, cellular phones in Israel.

b) Evaluate the limit of $T(x)$ as $x \to \infty$

$$\lim_{x\to\infty} T(x) = \lim_{x\to\infty} \frac{5200}{1 + 16.3e^{-0.7x}}$$
$$= \frac{\lim_{x\to\infty} 5200}{\lim_{x\to\infty} 1 + \lim_{x\to\infty} 16.3e^{-0.7x}}$$
$$= \frac{5200}{1 + 0}$$
$$= 5200$$

The limit is 5200 thousand cell phones or 5.2 million cell phones. This represents the saturation point for Israel's cellular phone market.

84. a) $T(x) = \dfrac{5200}{1 + 16.3e^{-0.7x}}$

$$T'(x) = \frac{\left(1 + 16.3e^{-0.7x}\right)(0) - \left[\left(16.3e^{-0.7x}\right)(-0.7)\right] \cdot 5200}{\left(1 + 16.3e^{-0.7x}\right)^2}$$

$$T'(x) = \frac{59{,}332e^{-0.7x}}{\left(1 + 16.3e^{-0.7x}\right)^2}$$

b) In 2009, $x = 2009 - 1994 = 15$.

$$T'(15) = \frac{59{,}332e^{-0.7(15)}}{\left(1 + 16.3e^{-0.7(15)}\right)^2} \approx 1.630$$

Approximately 1.630 thousand cell phones will be put into use in Israel between 2009 and 2010.

85. a) $C'(t) = 0 - 50(-1)e^{-t}$

$C'(t) = 50e^{-t}$

b) $C'(0) = 50e^{-0} = 50 \cdot 1 = 50$

Marginal cost when $t = 0$ is 50 million dollars per year.

c) $C'(4) = 50e^{-4} \approx 0.916$

Marginal cost when $t = 4$ is 0.916 million dollars per year or \$916,000 per year.

d) $\boxed{tw}$ $\lim_{t\to\infty} C(t) = 100$; $\lim_{t\to\infty} C'(t) = 0$

Early in the production process equipment might need to be purchased and or employees might need to be trained. Once this has taken place costs will tend to level off.

86. a) $C'(t) = 0 - 40(-1)e^{-t}$

$C'(t) = 40e^{-t}$

b) $C'(0) = 40e^{-0} = 40$

Marginal cost when $t = 0$ is 40 million dollars per year.

c) $C'(5) = 40e^{-5} \approx 0.270$

Marginal cost when $t = 5$ is 0.270 million dollars per year or \$270,000 per year.

d) $\boxed{tw}$ $\lim_{t\to\infty} C(t) = 200$; $\lim_{t\to\infty} C'(t) = 0$

Early in the production process equipment might need to be purchased and or employees might need to be trained. Once this has taken place costs will tend to level off.

87. $q = 240e^{-0.003x}$

a) When $x = 250$

$q = 240e^{-0.003(250)}$

$= 240e^{-0.75}$

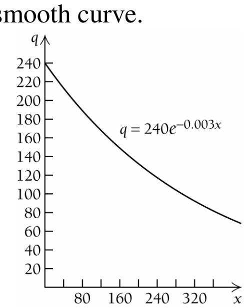
$q \approx 113$

At a price of \$250 the demand for this MP3 player is 113 thousand, or 113,000 units.

b) Create a table of values as needed. Find the function values for q as shown in part 'a'.

x	q
0	240
100	177.796
200	131.715
250	113.368
300	97.577
400	72.287

Next plot the points and connect the lines with a smooth curve.



c) Take the derivative of the demand function with respect to x.

$q'(x) = 240(-0.003)e^{-0.003x}$

$= -0.72e^{-0.003x}$

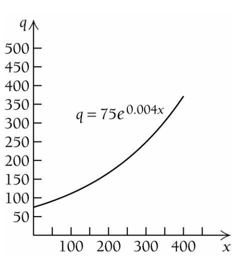
d) $\boxed{tw}$ The derivative represents the rate of change in quantity demanded of MP3 players with respect to the price of MP3 players.

88. a) $q = 75e^{0.004(250)} = 75e^{1} \approx 203.871 \approx 204$

Approximately 204 thousand, or 204,000 MP3 players will be supplied at a price of \$250.

b)

x	q
0	75
100	111.887
200	166.916
250	203.871
300	249.009
400	371.477



c) $q'(x) = 0.3e^{0.004x}$

d) $\boxed{tw}$ The derivative represents the rate of change in quantity supplied of MP3 players with respect to the price of MP3 players.

89. a) $C(0) = 10 \cdot 0^2 e^{-0} = 0$

The initial concentration is 0 parts per million (ppm).

$C(1) = 10 \cdot 1^2 e^{-1}$

$\approx 10(0.367879)$

≈ 3.68

After 1 hour, the concentration is approximately 3.7 ppm.

$C(2) = 10 \cdot 2^2 e^{-2}$

$\approx 40(0.135335)$

≈ 5.41

After 2 hours, the concentration is approximately 5.4 ppm.

$C(3) = 10 \cdot 3^2 e^{-3}$

$\approx 90(0.049787)$

≈ 4.48

After 3 hours, the concentration is approximately 4.5 ppm.

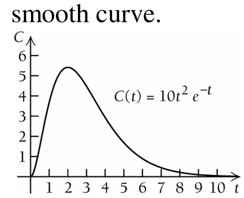
$C(10) = 10 \cdot 10^2 e^{-10}$

$\approx 1000(0.000045)$

≈ 0.05

After 10 hours, the concentration is approximately 0.05 ppm.

b) Plot the points $(0,0);(1,3.7);(2,5.4);$ $(3,4.5)$ and $(10,0.05)$ and other points as needed. Then we connect the points with a smooth curve.



c) $C'(t) = 10t^2(-1)e^{-t} + 20te^{-t}$

$= (20t - 10t^2)e^{-t}$

d) Since $C'(x)$ exists for all values of $x \geq 0$, the only critical values are where $C'(x)=0$. Solve:

$$C'(x)=0$$
$$\left(20t-10t^2\right)e^{-t}=0$$
$$20t-10t^2=0 \qquad \left(e^{-t}\neq 0\right)$$
$$10t(2-t)=0$$
$$t=0 \text{ or } t=2$$

The are two critical values. The critical value $t=2$ is the obvious maximum. Find $C(2)$.

$$C(2)=10\cdot 2^2 e^{-2}$$
$$\approx 40(0.135335)$$
$$\approx 5.41$$

The maximum value of the concentration will be 5.41 parts per million and will occur 2 hours after the drug has been administered.

e) $\boxed{tw}$ The derivative represents the rate of change of the concentration of the medication with respect to the time t.

90. a) In 2010, $t=2010-1900=110$

$$D(110)=34.4-\frac{30.48}{1+29.44e^{-0.072(110)}}$$
$$\approx 4.24$$

The death rate was 4.24 deaths per thousand people per year in 2010.

In 2050, $t = 2050 - 1900 = 150$

$$D(150)=34.4-\frac{30.48}{1+29.44e^{-0.072(150)}}$$
$$\approx 3.94$$

The death rate was 3.94 deaths per thousand people per year in 2050.

b) $$\lim_{t\to\infty} D(t)=\lim_{t\to\infty}\left[34.4-\frac{30.48}{1+29.44e^{-0.072t}}\right]$$
$$=34.4-\frac{30.48}{1+0}$$
$$=3.92$$

The limiting value of Mexico's death rate over time is 3.92 deaths per thousand people. The death rate stabilizes at this limit.

91. $$D(t)=34.4-\frac{30.48}{1+29.44e^{-0.072t}}$$

a) Take the derivative using the quotient rule.

$$D'(t)=0-\frac{d}{dt}\left[\frac{30.48}{1+29.44e^{-0.072t}}\right]$$
$$=-\frac{\left(1+29.44e^{-0.072t}\right)(0)-29.44e^{-0.072t}(-0.072)\cdot 30.48}{\left(1+29.44e^{-0.072t}\right)^2}$$

Simplifying the numerator the derivative is

$$D'(t)\approx -\frac{64.6e^{-0.072t}}{\left(1+29.44e^{-0.072t}\right)^2}$$

The derivative represents the rate of change of the death rate per thousand people per year in Mexico t years after 1990.

b) In 2009, $t=2009-1900=109$.

$$D'(109)\approx -\frac{64.6e^{-0.072(109)}}{\left(1+29.44e^{-0.072(109)}\right)^2}$$
$$\approx -\frac{64.6e^{-7.848}}{\left(1+29.44e^{-7.848}\right)^2}$$
$$\approx -0.025$$

Between 2009 and 2010, Mexico's death rate will decline approximately 0.025 deaths per thousand people.

92. Insert the given constants into the equation.

$$P(t)=40+(100-40)e^{-0.7t}$$
$$=40+(60)e^{-0.7t}$$

a) $$P(0)=40+60e^{-0.7(0)}$$
$$=100$$

The initial percentage retained is 100%.

$$P(1)=40+60e^{-0.7(1)}$$
$$\approx 40+29.795$$
$$\approx 69.8$$

The percentage retained after 1 week is 69.8%.

$$P(2)=40+60e^{-0.7(2)}$$
$$\approx 40+14.796$$
$$\approx 54.8$$

The percentage retained after 2 weeks is 54.8%.

$$P(6)=40+60e^{-0.7(6)}$$
$$\approx 40+0.900$$
$$\approx 40.9$$

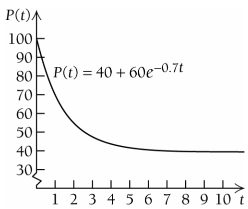
The percentage retained after 6 weeks is 40.9%.

$$P(10) = 40 + 60e^{-0.7(10)}$$
$$\approx 40 + 0.0547$$
$$\approx 40.1$$

The percentage retained after 10 weeks is 40.1%.

b) $\lim_{t\to\infty} P(t) = 40 + 0 = 40$

c)



d) $P'(t) = 60(-0.7)e^{-0.7t}$
$$= -42e^{-0.7t}$$

e) $\boxed{tw}$ The derivative represents the rate of change of the percentage of knowledge that is retained with respect to the time t weeks after the knowledge was learned.

93. $y = \left(e^{3x}+1\right)^5$

Using the Extended Power Rule.

$$\frac{dy}{dx} = 5\left(e^{3x}+1\right)^4 \cdot 3e^{3x}$$
$$= 15e^{3x}\left(e^{3x}+1\right)^4$$

94. $y = \left(e^{x^2}-2\right)^4$

Using the Extended Power Rule.

$$\frac{dy}{dx} = 4\left(e^{x^2}-2\right)^3 \cdot 2xe^{x^2}$$
$$= 8xe^{x^2}\left(e^{x^2}-2\right)^3$$

95. $y = \dfrac{e^{3t}-e^{7t}}{e^{4t}}$

Simplify the expression.

$$y = \frac{e^{3t}\left(1-e^{4t}\right)}{e^{3t}\cdot e^{t}}$$
$$= \frac{1-e^{4t}}{e^{t}}$$

Using the Quotient Rule.

$$\frac{dy}{dt} = \frac{e^t\left(-4e^{4t}\right)-e^t\left(1-e^{4t}\right)}{\left(e^t\right)^2}$$
$$= \frac{e^t\left(-4e^{4t}-1+e^{4t}\right)}{e^t\cdot e^t}$$
$$= \frac{-1-3e^{4t}}{e^t}$$
$$\frac{dy}{dt} = -e^{-t}-3e^{3t}$$

96. $y = \sqrt[3]{e^{3t}+t} = \left(e^{3t}+t\right)^{1/3}$

$$\frac{dy}{dx} = \frac{1}{3}\left(e^{3t}+t\right)^{-2/3}\cdot\left(3e^{3t}+1\right)$$
$$= \frac{3e^{3t}+1}{3\cdot\left(e^{3t}+t\right)^{2/3}}$$

97. $y = \dfrac{e^x}{x^2+1}$

$$\frac{dy}{dx} = \frac{\left(x^2+1\right)e^x - e^x(2x)}{\left(x^2+1\right)^2}$$ Quotient Rule

$$= \frac{e^x\left(x^2+1-2x\right)}{\left(x^2+1\right)^2}$$ Factoring the Numerator

$$= \frac{e^x\left(x^2-2x+1\right)}{\left(x^2+1\right)^2}$$
$$= \frac{e^x(x-1)^2}{\left(x^2+1\right)^2}$$

98. $y = \dfrac{e^x}{1-e^x}$

$$\frac{dy}{dx} = \frac{\left(1-e^x\right)e^x - e^x\left(-e^x\right)}{\left(1-e^x\right)^2}$$
$$= \frac{e^x-e^{2x}+e^{2x}}{\left(1-e^x\right)^2}$$
$$\frac{dy}{dx} = \frac{e^x}{\left(1-e^x\right)^2}$$

99. $f(x) = e^{\sqrt{x}} + \sqrt{e^x}$

$= e^{x^{1/2}} + e^{x/2}$

$f'(x) = e^{x^{1/2}} \cdot \frac{1}{2} x^{-1/2} + \frac{1}{2} e^{x/2}$ Chain Rule

$= \frac{e^{\sqrt{x}}}{2\sqrt{x}} + \frac{\sqrt{e^x}}{2}$

100. $f(x) = \frac{1}{e^x} + e^{1/x} = e^{-x} + e^{1/x}$

$f'(x) = -e^{-x} + \left(-x^{-2}\right) e^{1/x}$

$= -e^{-x} - x^{-2} e^{1/x}$

$= \frac{-1}{e^x} - \frac{e^{1/z}}{x^2}$

101. $f(x) = e^{x/2} \cdot \sqrt{x-1}$

First, we note that

$f(x) = e^{x/2} \cdot (x-1)^{1/2}$.

Next, we use the Product Rule.

$f'(x) = e^{x/2} \cdot \frac{1}{2}(x-1)^{-1/2} + \frac{1}{2} e^{x/2} (x-1)^{1/2}$

$= \frac{1}{2} e^{x/2} \left[(x-1)^{-1/2} + (x-1)^{1/2} \right]$ Factoring

$= \frac{1}{2} e^{x/2} \left[\frac{1}{\sqrt{x-1}} + \sqrt{x-1} \right]$

$= \frac{1}{2} e^{x/2} \left[\frac{1}{\sqrt{x-1}} + \sqrt{x-1} \cdot \frac{\sqrt{x-1}}{\sqrt{x-1}} \right]$ Multiplying by 1

$= \frac{1}{2} e^{x/2} \left[\frac{1}{\sqrt{x-1}} + \frac{x-1}{\sqrt{x-1}} \right]$

$= \frac{1}{2} e^{x/2} \left[\frac{x}{\sqrt{x-1}} \right]$ Adding Fractions

$= e^{x/2} \left[\frac{x}{2\sqrt{x-1}} \right]$

102. $f(x) = \frac{xe^{-x}}{1+x^2}$

$f'(x) = \frac{\left(1+x^2\right)\left[x\left(-e^{-x}\right)+e^{-x}\right] - 2x\left(xe^{-x}\right)}{\left(1+x^2\right)^2}$

$= \frac{-xe^{-x} + e^{-x} - x^3e^{-x} + x^2e^{-x} - 2x^2e^{-x}}{\left(1+x^2\right)^2}$

$= \frac{-x^3e^{-x} - x^2e^{-x} - xe^{-x} + e^{-x}}{\left(1+x^2\right)^2}$

$= \frac{-e^{-x}\left(x^3 + x^2 + x - 1\right)}{\left(1+x^2\right)^2}$

103. $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Using the Quotient Rule

$f'(x)$

$= \frac{\left(e^x + e^{-x}\right)\left(e^x + e^{-x}\right) - \left(e^x - e^{-x}\right)\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}$

$= \frac{\left(e^{2x} + e^0 + e^0 + e^{-2x}\right) - \left(e^{2x} - e^0 - e^0 + e^{-2x}\right)}{\left(e^x + e^{-x}\right)^2}$

$= \frac{e^{2x} + e^0 + e^0 + e^{-2x} - e^{2x} + e^0 + e^0 - e^{-2x}}{\left(e^x + e^{-x}\right)^2}$

$= \frac{4}{\left(e^x + e^{-x}\right)^2}$

104. $f(x) = e^{e^x}$

$f'(x) = e^x \cdot e^{e^x} = e^{x+e^x}$

105. $e = \lim_{t \to 0} f(t);\ f(t) = (1+t)^{1/t}$

$$f(1) = (1+1)^{1/1} = 2^1 = 2$$

$$f(0.5) = (1+0.5)^{1/0.5} = 1.5^2 = 2.25$$

$$f(0.2) = (1+0.2)^{1/0.2} = 1.2^5 = 2.48832$$

$$f(0.1) = (1+0.1)^{1/0.1} = 1.1^{10} \approx 2.59374$$

$$f(0.001) = (1+0.001)^{1/0.001} = 1.001^{1000} \approx 2.71692$$

106. $e = \lim_{t \to 1} g(t); g(t) = t^{1/(t-1)}$

$g(0.5) = (0.5)^{1/(0.5-1)} = 4$

$g(0.9) = (0.9)^{1/(0.9-1)} \approx 2.86797$

$g(0.99) = (0.99)^{1/(0.99-1)} \approx 2.73200$

$g(0.999) = (0.999)^{1/(0.999-1)} \approx 2.71964$

$g(0.9998) = (0.9998)^{1/(0.9998-1)} \approx 2.71855$

107. $f(x) = x^2 e^{-x}; [0,4]$

$f'(x) = x^2 \cdot (-1)e^{-x} + 2xe^{-x}$

$= -x^2 e^{-x} + 2xe^{-x}$

Since $f'(x)$ exists for all values of x in $[0,4]$, the only critical values are where $f'(x) = 0$.

$f'(x) = 0$

$-x^2 e^{-x} + 2xe^{-x} = 0$

$-e^{-x}(x^2 - 2x) = 0$

$x^2 - 2x = 0 \qquad (-e^{-x} \neq 0)$

$x(x-2) = 0$

$x = 0 \;\; or \;\; x = 2$

Use the Max-Min Principle 1. Find the function values at $x = 0, x = 2,$ and $x = 4$.

$f(x) = x^2 e^{-x}$

$f(0) = 0^2 \cdot e^{-0} = 0 \cdot 1 = 0$

$f(2) = 2^2 \cdot e^{-2} = 4 \cdot (0.135335) \approx 0.5413$

$f(4) = 4^2 \cdot e^{-4} = 16(0.018315) \approx 0.2930$

The maximum value of $f(x)$ is $4e^{-2}$ or approximately 0.5413 when $x = 2$.

108. $f(x) = xe^{-x}; [-2,0]$

$f'(x) = xe^x + e^x$

$f'(x)$ exists for all x in $[-2,0]$. Solve:

$f'(x) = 0$

$xe^x + e^x = 0$

$e^x(x+1) = 0$

$x + 1 = 0 \qquad (e^x \neq 0)$

$x = -1$

The critical value and the endpoints are $-2, -1,$ and 0.

Finding the function values of each critical value we have.

$f(-2) = -2e^{-2} \approx -0.2707$

$f(-1) = -e^{-1} \approx -0.3679$

$f(0) = 0 \cdot e^{-0} = 0$

On $[-2,0]$, the minimum value is $-e^{-1} \approx -0.3679$ when $x = -1$.

109. $\boxed{tw}$ The Power Rule cannot be used to find the derivative of an exponential function. The correct statement is $\frac{d}{dx} e^x = e^x$.

110 $\boxed{tw}$ Although the domain of both functions is $(-\infty, \infty)$, the range of $f(x)$ is $(0, \infty)$ while the range of $g(x)$ is $(-\infty, \infty)$. $f(x)$ is concave up for all values of x, while $g(x)$ is concave down over the interval $(-\infty, 0)$ and concave up over the interval $(0, \infty)$. $f(x)$ has no *x*-intercept; its *y*-intercept is $(0,1)$. The *x*-intercept and *y*-intercept of $g(x)$ is $(0,0)$. The *x*-axis is the horizontal asymptote for $f(x)$. $g(x)$ has no horizontal asymptote.

111. Graph: $y = x^2 e^{-x}$

From Exercise 107 we see that here is a relative maximum at $(2, 4e^{-2})$, or approximately $(2, 0.5413)$.

Using our work from Exercise 107 and the First-Derivative Test we find that there is a relative minimum at $(0,0)$. Also observe that $y \geq 0$ for all *x*.

For:

$x = -3, \;\; y = (-3)^2 e^{-(-3)} = 9e^3 \approx 180.77$

$x = -2, \;\; y = (-2)^2 e^{-(-2)} = 4e^2 \approx 29.56$

$x = -1, \;\; y = (-1)^2 e^{-(-1)} = 1e^1 \approx 2.72$

$x = 0, \;\; y = (0)^2 e^{-(0)} = 0 \cdot 1 = 0$

$x = 1, \;\; y = (1)^2 e^{-(1)} = 1e^{-1} \approx 0.37$

$x = 2, \;\; y = (2)^2 e^{-(2)} = 4e^{-2} \approx 0.54$

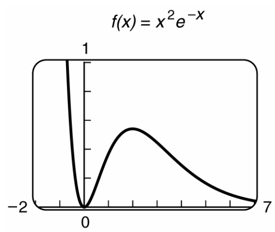
$x = 3, \;\; y = (3)^2 e^{-(3)} = 9e^{-3} \approx 0.45$

$x = 4,\ y = (4)^2 e^{-(4)} = 16e^{-4} \approx 0.29$

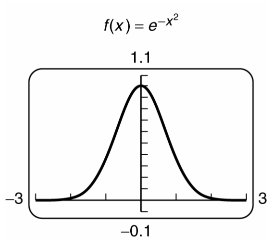
$x = 5,\ y = (5)^2 e^{-(5)} = 25e^{-5} \approx 0.17$

$x = 10,\ y = (10)^2 e^{-(10)} = 100e^{-10} \approx 0.005$

Next, we plot these points and connect them with a smooth curve.



112.



$$\frac{dy}{dx} = -2xe^{-x^2}$$

$\frac{dy}{dx}$ exists for all real numbers. Solve:

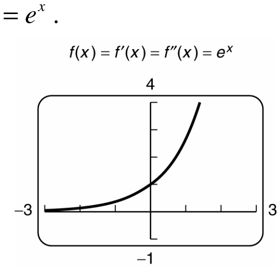
$$\frac{dy}{dx} = 0$$

$$-2xe^{-x^2} = 0$$

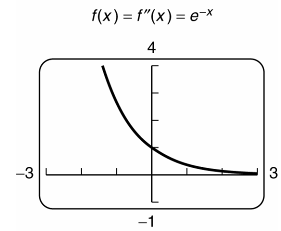
$$x = 0 \quad \left(-2e^{-x^2} \neq 0\right)$$

For $x < 0$, $\frac{dy}{dx} > 0$ and for $x > 0$, $\frac{dy}{dx} < 0$. Thus, y is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$, so there is a relative maximum at $(0,1)$. The function has no minimum.

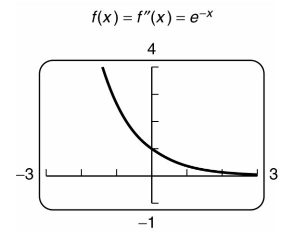
113. See page 310 in the text. The graphs of $f(x), f'(x),$ and $f''(x)$ are all the graph of $y = e^x$.



114. $f(x) = f''(x) = e^{-x}$



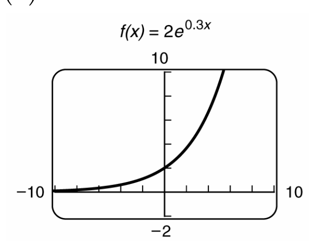
$f'(x) = -e^{-x}$

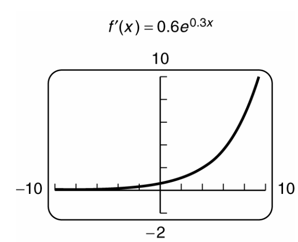


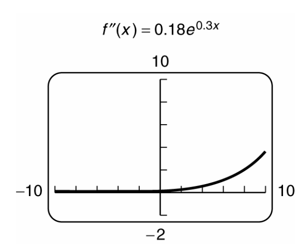
115. $f(x) = 2e^{0.3x}$

$f'(x) = 0.6e^{0.3x}$

$f''(x) = 0.18e^{0.3x}$



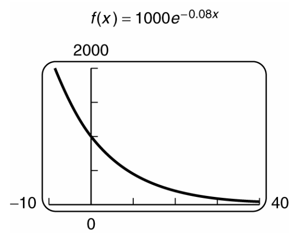


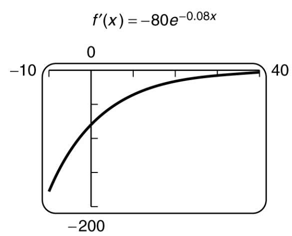


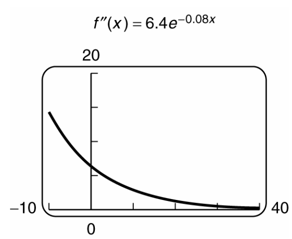
116. $f(x) = 1000e^{-0.08x}$

$f'(x) = -80e^{-0.08x}$

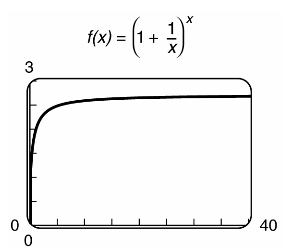
$f''(x) = 6.4e^{-0.08x}$







117.



Exercise Set 3.2

1. $\log_2 8 = 3$ Logarithmic equation.

$2^3 = 8$ Exponential equation;
2 is the base, 3 is the exponent.

2. $3^4 = 81$

3. $\log_8 2 = \dfrac{1}{3}$ Logarithmic equation.

$8^{1/3} = 2$ Exponential equation;
8 is the base, 3 is the exponent.

4. $27^{1/3} = 3$

5. $\log_a K = J$ Logarithmic equation.

$a^J = K$ Exponential equation;
a is the base, J is the exponent.

6. $a^K = J$

7. $-\log_{10} h = p$ Logarithmic equation.

$\log_{10} h = -p$ Multiply by -1.

$10^{-p} = h$ Exponential equation;
10 is the base,
$-p$ is the exponent.

8. $-\log_b V = w$

$\log_b V = -w$

$b^{-w} = V$

9. $e^M = b$ Exponential equation;
e is the base, M is the exponent.

$\log_e b = M$ Logarithmic equation.

or $\ln b = M$ $\ln b$ is the abbreviation for $\log_e b$.

10. $\log_e p = t$, or $\ln p = t$

11. $10^2 = 100$ Exponential equation;
10 is the base,
2is the exponent.

$\log_{10} 100 = 2$ Logarithmic equation.

12. $\log_{10} 1000 = 3$

13. $10^{-1} = 0.1$ Exponential equation;
10 is the base,
-1is the exponent.

$\log_{10} 0.1 = -1$ Logarithmic equation.

14. $\log_{10} 0.01 = -2$

15. $M^p = V$ Exponential equation;
M is the base, p is the exponent.

$\log_M V = p$ Logarithmic equation.

16. $\log_Q T = n$

17. $\log_b\left(\frac{5}{3}\right) = \log_b 5 - \log_b 3$ (P2)
$= 1.609 - 1.099$
$= 0.51$

18. $\log_b\left(\frac{1}{5}\right) = \log_b 1 - \log_b 5$
$= 0 - 1.609 = -1.609$

19. $\log_b 15 = \log_b(3\cdot 5)$
$= \log_b 3 + \log_b 5$ (P1)
$= 1.099 + 1.609$
$= 2.708$

20. $\log_b\sqrt{b^3} = \log_b(b)^{3/2} = \frac{3}{2}$

21. $\log_b 5b = \log_b 5 + \log_b b$ (P1)
$= 1.609 + 1$ (P5)
$= 2.609$

22. $\log_b 75 = \log_b(5^2\cdot 3)$
$= \log_b 5^2 + \log_b 3$
$= 2\log_b 5 + \log_b 3$
$= 2(1.609) + 1.099$
$= 4.317$

23. $\ln 20 = \ln(4\cdot 5)$
$= \ln 4 + \ln 5$ (P1)
$= 1.3863 + 1.6094$
$= 2.9957$

24. $\ln\left(\frac{5}{4}\right) = \ln 5 - \ln 4$
$= 1.6094 - 1.3863$
$= 0.2231$

25. $\ln\left(\frac{1}{5}\right) = \ln 1 - \ln 5$ (P2)
$= 0 - 1.6094$ (P6)
$= -1.6094$

26. $\ln(4e) = \ln 4 + \ln e$
$= 1.3863 + 1$
$= 2.3863$

27. $\ln\sqrt{e^8} = \ln e^{8/2}$
$= \ln e^4$
$= 4$ (P5)

28. $\ln 80 = \ln(4^2\cdot 5)$
$= \ln 4^2 + \ln 5$
$= 2\ln 4 + \ln 5$
$= 2(1.3863) + 1.6094$
$= 4.382$

29. Using a calculator and rounding to six decimal places, we have

$\ln 5894 \approx 8.681690$.

30. $\ln 99{,}999 \approx 11.512915$

31. $\ln 0.0182 \approx -4.006334$

32. $\ln 0.00087 \approx -7.047017$

33. $\ln 8100 \approx 8.999619$

34. $\ln 0.011 \approx -4.509860$

35. $e^t = 80$
$\ln e^t = \ln 80$ Taking the natural log on both sides
$t = \ln 80$ (P5)
$t \approx 4.382027$ Using a calculator
$t \approx 4.382$

36. $e^t = 10$
$\ln e^t = \ln 10$
$t = \ln 10$
$t \approx 2.303$

37. $e^{2t} = 1000$
$\ln e^{2t} = \ln 1000$ Taking the natural log on both sides
$2t = \ln 1000$ (P5)
$t = \frac{\ln 1000}{2}$
$t \approx \frac{6.907755}{2}$ Using a calculator
$t \approx 3.453878$
$t \approx 3.454$

38.
$$\begin{aligned} e^{3t} &= 900 \\ \ln e^{3t} &= \ln 900 \\ 3t &= \ln 900 \\ t &= \frac{\ln 900}{3} \\ t &\approx 2.267 \end{aligned}$$

39.
$$\begin{aligned} e^{-t} &= 0.1 \\ \ln e^{-t} &= \ln 0.1 && \text{Taking the natural log on both sides} \\ -t &= \ln 0.1 && \text{(P5)} \\ t &= -\ln 0.1 \\ t &\approx 2.302585 && \text{Using a calculator} \\ t &\approx 2.303 \end{aligned}$$

40.
$$\begin{aligned} e^{-t} &= 0.01 \\ \ln e^{-t} &= \ln 0.01 \\ -t &= \ln 0.01 \\ t &= -\ln 0.01 \\ t &\approx 4.605 \end{aligned}$$

41.
$$\begin{aligned} e^{-0.02t} &= 0.06 \\ \ln e^{-0.02t} &= \ln 0.06 && \text{Taking the natural log on both sides} \\ -0.02t &= \ln 0.06 && \text{(P5)} \\ t &= \frac{\ln 0.06}{-0.02} \\ t &\approx \frac{-2.813411}{-0.02} && \text{Using a calculator} \\ t &\approx 140.671 \end{aligned}$$

42.
$$\begin{aligned} e^{0.07t} &= 2 \\ \ln e^{0.07t} &= \ln 2 \\ 0.07t &= \ln 2 \\ t &= \frac{\ln 2}{0.07} \\ t &\approx 9.902 \end{aligned}$$

43. $y = -8\ln x$

$$\frac{dy}{dx} = -8 \cdot \frac{1}{x} \qquad \left[\tfrac{d}{dx}\left[c \cdot f(x)\right] = c \cdot f'(x)\right]$$

$$\frac{dy}{dx} = \frac{-8}{x}$$

44. $y = -9\ln x$

$$\frac{dy}{dx} = -9 \cdot \frac{1}{x} = \frac{-9}{x}$$

45. $y = x^4 \ln x - \frac{1}{2}x^2$

Differentiate this function term by term. We apply the Product Rule to the first term, and the Power Rule to the second term.

$$\frac{dy}{dx} = \underbrace{x^4 \cdot \frac{1}{x} + 4x^3 \cdot \ln x}_{\text{Product Rule}} - \underbrace{\frac{1}{2} \cdot 2x}_{\text{Power Rule}}$$

$$\frac{dy}{dx} = x^3 + 4x^3 \ln x - x$$

46. $y = x^6 \ln x - \frac{1}{4}x^4$

$$\begin{aligned} \frac{dy}{dx} &= x^6 \cdot \frac{1}{x} + 6x^5 \cdot \ln x - \frac{1}{4} \cdot 4x^3 \\ &= x^5 + 6x^5 \ln x - x^3 \end{aligned}$$

47. $f(x) = \ln(6x)$

Using Theorem 7, we have

$$\frac{d}{dx} \ln f(x) = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}$$

$$\begin{aligned} f'(x) &= \frac{1}{6x} \cdot 6 \\ &= \frac{1}{x} \end{aligned}$$

48. $f(x) = \ln(9x)$

$$f'(x) = \frac{1}{9x} \cdot 9 = \frac{1}{x}$$

49. $g(x) = x^2 \ln(7x)$

We apply the Product Rule. Remember to use Theorem 7 when taking the derivative of the logarithm.

$$g'(x) = \underbrace{x^2 \cdot \underbrace{\frac{1}{7x}}_{\text{Theorem 7}} \cdot 7 + 2x \cdot \ln(7x)}_{\text{Product Rule}}$$

$$\begin{aligned} g'(x) &= x^2 \cdot \frac{1}{x} + 2x \ln(7x) \\ &= x + 2x \ln(7x) \end{aligned}$$

50. $g(x) = x^5 \ln(3x)$

$$\begin{aligned} g'(x) &= x^5 \cdot \frac{1}{3x} \cdot 3 + 5x^4 \ln(3x) \\ &= x^4 + 5x^4 \ln(3x) \end{aligned}$$

51. $y = \dfrac{\ln x}{x^4}$

Using the Quotient Rule, we have

$$\frac{dy}{dx} = \frac{x^4 \cdot \frac{1}{x} - 4x^3 \cdot \ln x}{x^8}$$
$$= \frac{x^3 - 4x^3 \ln x}{x^8}$$
$$= \frac{x^3(1 - 4\ln x)}{x^3 \cdot x^5} \qquad \text{Factoring}$$
$$= \frac{x^3}{x^3} \cdot \frac{1 - 4\ln x}{x^5} \qquad \text{Removing a factor equal to 1}$$
$$= \frac{1 - 4\ln x}{x^5}$$

52. $y = \dfrac{\ln x}{x^5}$

$$\frac{dy}{dx} = \frac{x^5 \cdot \dfrac{1}{x} - 5x^4 \cdot \ln x}{x^{10}}$$
$$= \frac{x^4 - 5x^4 \ln x}{x^{10}}$$
$$= \frac{x^4(1 - 5\ln x)}{x^4 \cdot x^6}$$
$$= \frac{1 - 5\ln x}{x^6}$$

53. $y = \ln\left(\dfrac{x^2}{4}\right)$

$$y = \ln x^2 - \ln 4 \qquad \text{(P2)}$$
$$\frac{dy}{dx} = \frac{1}{x^2} \cdot 2x - 0 \qquad \text{Theorem 7}$$
$$= \frac{2}{x}$$

54. $y = \ln\left(\dfrac{x^4}{2}\right)$

$$y = \ln x^4 - \ln 2$$
$$\frac{dy}{dx} = \frac{1}{x^4} .4x^3 - 0$$
$$= \frac{4}{x}$$

55. $y = \ln\left(3x^2 + 2x - 1\right)$

Using Theorem 7, we note:

$$\frac{d}{dx} \ln g(x) = \frac{g'(x)}{g(x)}$$
$$g(x) = 3x^2 + 2x - 1$$
$$g'(x) = 6x + 2$$

Therefore,

$$\frac{dy}{dx} = \frac{6x + 2}{3x^2 + 2x - 1}$$
$$\frac{dy}{dx} = \frac{2(3x + 1)}{3x^2 + 2x - 1}$$

56. $y = \ln\left(7x^2 + 5x + 2\right)$

$$\frac{dy}{dx} = \frac{14x + 5}{7x^2 + 5x + 2}$$

57. $f(x) = \ln\left(\dfrac{x^2 - 7}{x}\right)$

Using the Theorem 7.

$$f'(x) = \frac{\left(\dfrac{d}{dx}\left[\dfrac{x^2 - 7}{x}\right]\right)}{\dfrac{x^2 - 7}{x}}$$
$$= \left(\frac{d}{dx}\left[\frac{x^2 - 7}{x}\right]\right) \cdot \frac{x}{x^2 - 7} \qquad \text{Dividing fractions}$$

We apply the Quotient Rule to take the derivative of the inside function to get

$$\frac{d}{dx}\left[\frac{x^2 - 7}{x}\right] = \frac{x \cdot 2x - \left(x^2 - 7\right) \cdot 1}{x^2}$$
$$= \frac{2x^2 - x^2 + 7}{x^2}$$
$$= \frac{x^2 + 7}{x^2}$$

Now we substitute back into the original derivative.

$$f'(x) = \frac{x^2 + 7}{x^2} \cdot \frac{x}{x^2 - 7}$$
$$= \frac{x^2 + 7}{x\left(x^2 - 7\right)}$$

An alternate solution to this problem involves using property P2.

$$\begin{aligned} f(x) &= \ln\left(\frac{x^2-7}{x}\right) \\ &= \ln(x^2-7)-\ln x && \text{(P2)} \\ &= \frac{2x}{(x^2-7)}-\frac{1}{x} && \text{Theorem 7} \\ &= \frac{2x}{x^2-7}-\frac{1}{x} \end{aligned}$$

Combine the fractions by finding a common denominator.

$$\begin{aligned} f'(x) &= \frac{2x}{x^2-7}\cdot\frac{x}{x}-\frac{1}{x}\cdot\frac{x^2-7}{x^2-7} && \text{Multiply by 1} \\ &= \frac{2x^2}{x(x^2-7)}-\frac{x^2-7}{x(x^2-7)} \\ &= \frac{x^2+7}{x(x^2-7)} \end{aligned}$$

58. $f(x)=\ln\left(\frac{x^2+5}{x}\right)=\ln(x^2+5)-\ln x$

$$\begin{aligned} f'(x) &= \frac{2x}{x^2+5}-\frac{1}{x} \\ &= \frac{2x}{x^2+5}\cdot\frac{x}{x}-\frac{1}{x}\cdot\frac{x^2+5}{x^2+5} \\ &= \frac{2x^2-(x^2+5)}{x(x^2+5)} \\ &= \frac{x^2-5}{x(x^2+5)} \end{aligned}$$

59. $g(x)=e^x\ln x^2$

$$\begin{aligned} g'(x) &= e^x\cdot\underbrace{\frac{2x}{x^2}}_{\text{Theorem 7}}+e^x\cdot\ln x^2 && \text{By the Product Rule} \\ &= e^x\cdot\frac{2}{x}+e^x\ln x^2 \\ &= \frac{2e^x}{x}+2e^x\ln x && \text{(P3)} \end{aligned}$$

60. $g(x)=e^{2x}\ln x$

$$\begin{aligned} g'(x) &= e^{2x}\cdot\frac{1}{x}+2e^{2x}\cdot\ln x \\ &= \frac{e^{2x}}{x}+2e^{2x}\ln x \end{aligned}$$

61. $f(x)=\ln(e^x+1)$

$$\begin{aligned} f'(x) &= \frac{(e^x+0)}{e^x+1} && \text{Theorem 7} \\ &= \frac{e^x}{e^x+1} \end{aligned}$$

62. $f(x)=\ln(e^x-2)$

$$f'(x)=\frac{e^x}{e^x-2}$$

63. $g(x)=(\ln x)^4$

Using the Extended Power Rule, we have

$$\begin{aligned} g'(x) &= 4(\ln x)^3\cdot\frac{1}{x} \\ &= \frac{4(\ln x)^3}{x} \end{aligned}$$

64. $g(x)=(\ln x)^3$

$$g'(x)=3(\ln x)^2\cdot\frac{1}{x}=\frac{3(\ln x)^2}{x}$$

65. $f(x)=\ln(\ln(8x))$

First we apply Theorem 7.

$$f'(x)=\frac{1}{\ln(8x)}\cdot\left(\frac{d}{dx}\ln(8x)\right)$$

Using Theorem 7 again, we have

$$\begin{aligned} f'(x) &= \frac{1}{\ln(8x)}\cdot\frac{1}{8x}\cdot 8 \\ &= \frac{1}{x\ln(8x)} \end{aligned}$$

66. $f(x)=\ln(\ln(3x))$

$$\begin{aligned} f'(x) &= \frac{1}{\ln(3x)}\cdot\frac{1}{3x}\cdot 3 \\ &= \frac{1}{x\ln(3x)} \end{aligned}$$

67. $g(x)=\ln(5x)\cdot\ln(3x)$

Using the Product Rule along with Theorem 7, we have

$$g'(x)=\ln(5x)\cdot\underbrace{\left(\frac{3}{3x}\right)}_{\text{Theorem 7}}+\underbrace{\left(\frac{5}{5x}\right)}_{\text{Theorem 7}}\cdot\ln(3x)$$

$$=\ln(5x)\frac{1}{x}+\frac{1}{x}\ln(3x)$$

$$=\frac{\ln(5x)+\ln(3x)}{x}$$

If we wanted to simplify the expression, we could use Property 1 to combine the logarithms.

$$g'(x)=\frac{\ln(5x\cdot 3x)}{x}=\frac{\ln(15x^2)}{x}$$

68. $g(x)=\ln(2x)\cdot\ln(7x)$

$$g'(x)=\ln(2x)\cdot\left(\frac{7}{7x}\right)+\left(\frac{2}{2x}\right)\ln(7x)$$

$$=\ln(2x)\cdot\frac{1}{x}+\frac{1}{x}\cdot\ln(7x)$$

$$=\frac{\ln(2x)+\ln(7x)}{x}$$

If we wanted to simplify the expression, we could use Property 1 to combine the logarithms.

$$g'(x)=\frac{\ln(2x\cdot 7x)}{x}$$

$$=\frac{\ln(14x^2)}{x}$$

69. First, we find the point of tangency by evaluating the function at $x=2$.

$x=2;$

$y=(2^2-2)\ln(6\cdot 2)=2\ln(12)\approx 4.9698$

Point of tangency: $(2,4.9698)$

Now, we find the slope of the function at $x=2$ by taking the derivative.

$$\frac{dy}{dx}=(x^2-x)\left(6\cdot\frac{1}{6x}\right)+(2x-1)\ln(6x)$$

$$=(x^2-x)\left(\frac{1}{x}\right)+(2x-1)\ln(6x)$$

$$=x-1+(2x-1)\ln(6x)$$

Next, we evaluate the derivative at $x=2$

$$\left.\frac{dy}{dx}\right|_{x=2}=2-1+(2\cdot 2-1)\ln(6\cdot 2)$$

$$=1+3\ln(12)\approx 8.4547$$

Now we have the slope and a point on the tangent line. We use the Point-Slope formula to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$

$$y-4.970=8.455(x-2)$$

$$y-4.970=8.455x-16.91$$

$$y=8.455x-11.94$$

70. First, we find the point of tangency.

$x=1;$

$y=e^{3(1)}\cdot\ln(4\cdot 1)\approx 27.84446$

$(1,27.844)$

Then, we find the slope of the tangent line at $x=1$.

$$\frac{dy}{dx}=e^{3x}\cdot 4\frac{1}{4x}+3e^{3x}\ln(4x)$$

$$=\frac{e^{3x}}{x}+3e^{3x}\ln(4x)$$

Next, evaluate the derivative at $x=1$.

$$\left.\frac{dy}{dx}\right|_{x=1}=\frac{e^{3\cdot 1}}{1}+3e^{3\cdot 1}\ln(4\cdot 1)$$

$$=e^3+3e^3\ln 4\approx 103.619$$

Now, we use the Point-Slope Formula to find the equation of the tangent line.

$$y-27.844=103.619(x-1)$$

$$y-27.844=103.619x-103.619$$

$$y=103.619x-75.775$$

71. First, we find the point of tangency by evaluating the function at $x=3$.

$x=3;$

$y=(\ln 3)^2\approx 1.207$

Point of tangency: $(3,1.207)$

Now, we find the slope of the function at $x=3$ by taking the derivative of the function using the Extended Power Rule.

$$\frac{dy}{dx}=2(\ln(x))\cdot\frac{1}{x}$$

$$=\frac{2\ln x}{x}$$

Next, we evaluate the derivative at $x=3$

$$\left.\frac{dy}{dx}\right|_{x=3}=\frac{2\ln(3)}{3}\approx 0.732$$

Now we have the slope and a point on the tangent line. We use the Point-Slope formula to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 1.207 = 0.732(x - 3)$$
$$y - 1.207 = 0.732x - 2.196$$
$$y = 0.732x - 0.989$$

72. First, we find the point of tangency.
$x = 2$;
$$y = \ln\left(4(2)^2 - 7\right) \approx 2.197$$
$$(2, 2.197)$$
Next, we find the slope of the tangent line at $x = 2$.
$$\frac{dy}{dx} = 8x \cdot \frac{1}{4x^2 - 7} = \frac{8x}{4x^2 - 7}$$
We evaluate the derivative at $x = 2$.
$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{8(2)}{4(2)^2 - 7} = \frac{16}{9} \approx 1.778$$
We use the Point-Slope Formula to find the equation of the tangent line.
$$y - 2.197 = 1.778(x - 2)$$
$$y - 2.197 = 1.778x - 3.556$$
$$y = 1.778x - 1.359$$

73. $N(a) = 2000 + 500\ln a,\ a \ge 1$

a) We substitute 1 in for a.
$$N(1) = 2000 + 500 \cdot \ln 1 = 2000 + 500 \cdot 0 = 2000$$
Thus, 2000 units were sold after spending \$1000 on advertising.

b) Taking the derivative with respect to a, we have
$$N'(a) = 0 + 500 \cdot \frac{1}{a} = \frac{500}{a}.$$
Therefore,
$$N'(10) = \frac{500}{10} = 50.$$

c) $N'(a) > 0$ for all $a \ge 1$. Thus $N(a)$ is an increasing function and has a minimum value of 2000 when $a = 1$ thousand dollars. There is no maximum, because there is not an upper limit on the advertising budget.

d) $\boxed{tw}$ $\lim_{a\to\infty} N'(a) = \lim_{a\to\infty} \frac{500}{a} = 0$
It does not make sense to spend more and more dollars on advertising since the rate of change of the number of units sold approaches 0 for large expenditures on advertising.

74. $N(a) = 1000 + 200\ln a,\ a \ge 1$

a) Substitute 1 in for a.
$$N(1) = 1000 + 200 \cdot \ln 1 = 1000 + 200 \cdot 0 = 1000$$
Thus 1000 units were sold after spending \$1000 on advertising.

b) Taking the derivative with respect to a.
$$N'(a) = 0 + 200 \cdot \frac{1}{a} = \frac{200}{a}$$
Therefore,
$$N'(10) = \frac{200}{10} = 20.$$

c) $N'(a) > 0$ for all $a \ge 1$. Thus $N(a)$ is an increasing function and has a minimum value of 1000 when $a = 1$ thousand dollars. There is no maximum, because there is not an upper limit on the advertising budget.

d) $\boxed{tw}$ $\lim_{a\to\infty} N'(a) = \lim_{a\to\infty} \frac{500}{a} = 0$
It does not make sense to spend more and more dollars on advertising since the rate of change of the number of units sold approaches 0 for large expenditures on advertising.

75. How long should the campaign last in order to maximize profit? We need to find the revenue and cost functions in order to find the profit. We find the revenue function first.

$$R(t)=\begin{pmatrix}\text{Price}\\ \text{Per}\\ \text{Unit}\end{pmatrix}\cdot\begin{pmatrix}\text{Target}\\ \text{Market}\end{pmatrix}\cdot\begin{pmatrix}\text{Percentage}\\ \text{Buying}\end{pmatrix}$$

$$R(t)=0.50(1{,}000{,}000)\left(1-e^{-0.04t}\right)$$
$$=500{,}000-500{,}000e^{-0.04t}$$

Next, we find the cost function.

$$C(t)=\begin{pmatrix}\text{Advertising cost}\\ \text{per day}\end{pmatrix}\cdot\begin{pmatrix}\text{Number of}\\ \text{days}\end{pmatrix}$$
$$C(t)=2000\cdot t$$

Now, we find the profit function.

$$P(t)=R(t)-C(t)$$
$$=500{,}000-500{,}000e^{-0.04t}-2000t$$

We take the derivative of the profit function.

$$P'(t)=0-500{,}000(-0.04)e^{-0.04t}-2000$$
$$=20{,}000e^{-0.04t}-2000$$

Next, we set the derivative of the profit function equal to zero and solve for t to find the critical values.

$$\begin{aligned}P'(t)&=0\\ 20{,}000e^{-0.04t}-2000&=0\\ 20{,}000e^{-0.04t}&=2000\\ e^{-0.04t}&=\frac{2000}{20{,}000}\\ e^{-0.04t}&=0.1\\ \ln\left(e^{-0.04t}\right)&=\ln(0.1)\\ -0.04t&=-2.30258\\ t&\approx\frac{-2.30258}{-0.04}\approx 57.565\end{aligned}$$

Rounding up, we see that the only critical value is 58 days. We will apply the second derivative test to see if we have a maximum.

The second derivative of the profit function is $P''(t)=-800e^{-0.04t}$.

Evaluating this at the critical value we get $P''(58)=-800e^{-0.04\cdot 58}<0$ so we have a maximum.

The length of the advertising campaign must be 58 days to result in maximum profit.

76. $R(t)=500{,}000-500{,}000e^{-0.04t}$

$C(t)=4000t$

$P(t)=500{,}000-500{,}000e^{-0.04t}-4000t$

$P'(t)=20{,}000e^{-0.04t}-4000$

Find the critical values. $P'(t)$ exist for all $t\ge 0$.

Solve:

$$\begin{aligned}P'(t)&=0\\ 20{,}000e^{-0.04t}-4000&=0\\ e^{-0.04t}&=0.2\\ \ln e^{-0.04t}&=\ln 0.2\\ t&=\frac{\ln 0.2}{-0.04}\\ t&\approx 40\end{aligned}$$

Using the Second Derivative Test.

$P''(t)=-800e^{-0.04t}$

$P''(40)=-800e^{-0.04\cdot 40}<0$, so we have a maximum.

The length of the advertising campaign must be 40 days to maximize profit.

77. $V(t)=58\left(1-e^{-1.1t}\right)+20$

Where $V(t)$ is the value of the stock after time t, in months.

a) $$\begin{aligned}V(1)&=58\left(1-e^{-1.1(1)}\right)+20\\ &=58(1-0.332871)+20\\ &=58(0.667129)+20\\ &=38.693482+20\\ &=58.693482\\ &\approx 58.69\end{aligned}$$

The value of the stock one month after purchase is \$58.69

$$\begin{aligned}V(12)&=58\left(1-e^{-1.1(12)}\right)+20\\ &=58(1-0.000002)+20\\ &=58(0.999998)+20\\ &=57.999884+20\\ &=77.999884\\ &\approx 78.00\end{aligned}$$

The value of the stock 12 months after purchase is \$78.00

b) $$\begin{aligned}V'(t)&=58\left(0-(-1.1)e^{-1.1t}\right)+0\\ &=58\left(1.1e^{-1.1t}\right)\\ V'(t)&=63.8e^{-1.1t}\end{aligned}$$

c) Solve: $V(t) = 75$

$$58\left(1-e^{-1.1t}\right)+20 = 75$$
$$58\left(1-e^{-1.1t}\right) = 55$$
$$\left(1-e^{-1.1t}\right) = \frac{55}{58}$$
$$1-e^{-1.1t} = 0.948276$$
$$-e^{-1.1t} = -0.051724$$
$$e^{-1.1t} = 0.051724$$
$$\ln e^{-1.1t} = \ln(0.051724)$$
$$-1.1t = -2.96183$$
$$t = \frac{-2.96183}{-1.1}$$
$$t \approx 2.6926$$

The stock will first reach $75 approximately 2.7 months after the purchase.

d) $\boxed{tw}$ $\lim_{t\to\infty} V(t) = \lim_{t\to\infty} 58\left(1-e^{-1.1t}\right)+20 = 78$

Over a long period of time, the value of the stock will level off at about $78 per share.

78. a) $R(x) = x\cdot p(x)$

$$R(x) = x\cdot(53.5 - 8\ln x)$$
$$R(x) = 53.5x - 8x\ln x$$

b) $R'(x) = 53.5 - 8x\left(\frac{1}{x}\right) - 8(1)\ln x$

$$= 45.5 - 8\ln x$$

c) $\boxed{tw}$ $R'(x) = 0$

$$45.5 - 8\ln x = 0$$
$$\ln x = 5.6875$$
$$x = e^{5.6875} \approx 295.15$$

Notice that $R''(x) = \frac{-8}{x} < 0$, for all values of $x > 0$. Therefore any positive critical value will result in a maximum revenue.
Substitute the revenue maximizing quantity into the demand function in order to find the price that will yield a maximum revenue.

$p(295) = 53.5 - 8\ln 295 = 8.00$

A price of $8 will yield a maximum profit.

79. a) Taking the derivative of the profit function $P(x) = 2x - 0.3x\ln x$ will give us marginal profit.

$$P'(x) = 2 - \left[0.3x\left(\frac{1}{x}\right) + 0.3\ln x\right]$$
$$= 2 - 0.3 - 0.3\ln x$$
$$= 1.7 - 0.3\ln x$$

b) $\boxed{tw}$ $P'(150) = 1.7 - 0.3\ln 150 \approx 0.19681$

Since quantity is in 1000's of mechanical pencils and profit is in 1000's of dollars. That a production level of 150,000 pencils, the additional profit earned by producing another 1000 pencils will be 196.81 dollars. That is roughly 19 cents per pencil.

c) Since $P'(x)$ is defined for all values of $x > 0$. We find the critical values by setting $P'(x) = 0$.

$$1.7 - 0.3\ln x = 0$$
$$-0.3\ln x = -1.7$$
$$\ln x = \frac{-1.7}{-0.3}$$
$$\ln x = 5.666667$$
$$x = e^{5.66667}$$
$$x \approx 289.069$$

Notice that:

$P''(x) = \frac{-0.3}{x} < 0$ for all values of $x > 0$

The critical value results in a maximum.

Therefore, 289,069 mechanical pencils should be sold to maximize profit.

80. a) $P(t) = 100\left(1-e^{-0.2t}\right)$

$$P(1) = 100\left(1-e^{-0.2(1)}\right)$$
$$= 100\left(1-e^{-0.2}\right)$$
$$= 100(0.181269)$$
$$\approx 18.1$$

18% of doctors prescribe the new medicine one month after approval.

$$P(6) = 100\left(1-e^{-0.2(6)}\right)$$
$$= 100\left(1-e^{-1.2}\right)$$
$$= 100(0.698806)$$
$$\approx 69.9$$

70% of doctors prescribe the new medicine 6 months after approval.

b) $P(t) = 100\left(1 - e^{-0.2t}\right)$

$P'(t) = 100\left[-(-0.2)e^{-0.2t}\right]$

$= 20e^{-0.2t}$

c) Solve:

$$P(t) = 90$$

$$100\left(1 - e^{-0.2t}\right) = 90$$

$1 - e^{-0.2t} = 0.9$ Divide by 100

$-e^{-0.2t} = -0.1$ Subtract 1

$e^{-0.2t} = 0.1$ Multiply by -1

$\ln e^{-0.2t} = \ln 0.1$ Take the Natural log on both sides

$-0.2t = \ln 0.1$

$t = \dfrac{\ln 0.1}{-0.2}$ Divide by -0.2

$t \approx 11.5$

It will take approximately 11.5 months for 90% of doctors to prescribe the new medicine.

d) $\boxed{tw}$ $\lim_{t\to\infty} P(t) = \lim_{t\to\infty} 100\left(1 - e^{-0.2t}\right) = 100$

This indicates that 100% of doctors will eventually prescribe this medication.

81. $R = A\ln r - Br$

$$\frac{dR}{dr} = \frac{A}{r} - B$$

Since $\dfrac{dR}{dr}$ exists for all $r > 0$, we find the critical values by solving $\dfrac{dR}{dr} = 0$.

$$\frac{dR}{dr} = 0$$

$$\frac{A}{r} - B = 0$$

$$\frac{A}{r} = B$$

$$\frac{A}{B} = r$$

The critical value occurs when $r = \dfrac{A}{B}$.

Use the Second-Derivative Test to see if it is a maximum.

$\dfrac{d^2R}{dr^2} = -Ar^{-2} = \dfrac{-A}{r^2} < 0$ for all $r > 0$.

Thus, we have maximum at $r = \dfrac{A}{B}$. The maximum function value will be:

$$R = A\ln\left(\frac{A}{B}\right) - B\left(\frac{A}{B}\right)$$

$$= A(\ln A - \ln B) - A$$

$$= A\ln\left(\frac{A}{B}\right) - A$$

or

$$= A\ln A - A\ln B - A$$

82. $S(t) = 68 - 20\ln(t+1), \quad t \ge 0$

a) $S(0) = 68 - 20\ln(0+1)$

$= 68 - 20\ln(1)$

$= 68 - 20 \cdot 0$

$= 68$

The average score when they initially took the exam was 68%.

b) $S(4) = 68 - 20\ln(4+1)$

$= 68 - 20\ln(5)$

$= 68 - 20(1.609438)$

$= 68 - 32.18876$

≈ 35.8

The average score 4 months after they took the exam was 35.8%.

c) $S(24) = 68 - 20\ln(24+1)$

$= 68 - 20\ln(25)$

$= 68 - 20(3.218876)$

$= 68 - 64.37752$

≈ 3.6

The average score 24 months after they took the exam was 3.6%.

d) First, we reword the question:

"3.6 (the average score after 24 months) is what percent of 68 (the average score on the initial exam)?

Next, we translate the sentence into an equation and solve:

$$3.6 = x \cdot 68$$

$$\frac{3.6}{68} = x$$

$$0.052941 = x$$

$$5.3\% \approx x$$

The students retained approximately 5.3% of their original answers after 2 years.

e) $S(t) = 68 - 20\ln(t+1)$

$$S'(t) = 0 - 20(1)\left(\frac{1}{t+1}\right)$$
$$= -\frac{20}{t+1}$$

f) $S'(t) < 0$ for all values of $t \geq 0$. Therefore, $S(t)$ is a decreasing function and has a maximum value of 68% when $t = 0$. The function does not have a minimum value.

g) $\boxed{tw}$ $\lim_{t\to\infty} S(t) = \lim_{t\to\infty}\left[68 - 20\ln(t+1)\right] = -\infty$
Clearly, the score can not be less than 0, but the limit suggests that everything will be forgotten eventually.

83. $S(t) = 78 - 15\ln(t+1), \quad t \geq 0$

a) $S(0) = 78 - 15\ln(0+1)$
$$= 78 - 15\ln(1)$$
$$= 78 - 15\cdot 0$$
$$= 78$$
The average score when they initially took the exam was 78%.

b) $S(4) = 78 - 15\ln(4+1)$
$$= 78 - 15\ln(5)$$
$$= 78 - 15(1.609438)$$
$$= 78 - 24.141569$$
$$\approx 53.9$$
The average score 4 months after they took the exam was 53.9%.

c) $S(24) = 78 - 15\ln(24+1)$
$$= 78 - 15\ln(25)$$
$$= 78 - 15(3.218876)$$
$$= 78 - 48.283137$$
$$\approx 29.7$$
The average score 24 months after they took the exam was 29.7%.

d) First, we reword the question:
"29.7 (the average score after 24 months) is what percent of 78 (the average score on the initial exam)?
Next, we translate the sentence into an equation and solve:
$$29.7 = x\cdot 78$$
$$\frac{29.7}{78} = x$$
$$0.381 = x$$
$$38.1\% \approx x$$
The students retained approximately 38.1% of their original answers after 2 years.

e) $S(t) = 78 - 15\ln(t+1)$
$$S'(t) = 0 - 15(1)\left(\frac{1}{t+1}\right)$$
$$= -\frac{15}{t+1}$$

f) $S'(t) < 0$ for all values of $t \geq 0$. Therefore, $S(t)$ is a decreasing function and has a maximum value of 78% when $t = 0$. The function does not have a minimum value.

g) $\boxed{tw}$ $\lim_{t\to\infty} S(t) = \lim_{t\to\infty}\left[78 - 15\ln(t+1)\right] = -\infty$
Clearly, the score can not be less than 0, but the limit suggests that everything will be forgotten eventually.

84. $v(p) = 0.37\ln p + 0.05$

a) $v(571) = 0.37\ln 571 + 0.05$
$$= 0.37(6.347389) + 0.05$$
$$= 2.398534$$
$$\approx 2.4$$
The average walking speed of a person living in Seattle is 2.40 feet per second.

b) $v(8100) = 0.37\ln 8100 + 0.05$
$$= 0.37(8.999619) + 0.05$$
$$= 3.379859$$
$$\approx 3.4$$
The average walking speed of a person living in New York is 3.4 feet per second.

c) $v'(p) = 0.37\frac{1}{p} + 0 = \frac{0.37}{p}$

d) $\boxed{tw}$ $v'(p)$ is the rate of change of the average walking speed with respect to the city's population. This study concludes that as the city's population grows, the average walking speed of the city will increase.

85. $W(t)=100\left(1-e^{-0.3t}\right)$

a) $W(1)=100\left(1-e^{-0.3(1)}\right)$

$$=100\left(1-e^{-0.3}\right)$$
$$=100(1-0.740818)$$
$$=100(0.259182)$$
$$=25.91812$$
$$\approx 26$$

After 1 week of practice, a keyboarder will type approximately 26 words per minute.

$W(8)=100\left(1-e^{-0.3(8)}\right)$

$$=100\left(1-e^{-2.4}\right)$$
$$=100(1-0.090718)$$
$$=100(0.909282)$$
$$=90.9282$$
$$\approx 91$$

After 8 weeks of practice, a keyboarder will type approximately 91 words per minute.

b) $W(t)=100\left(1-e^{-0.3t}\right)$

$W'(t)=100\left[-(-0.3)e^{-0.3t}\right]$

$=30e^{-0.3t}$

c) Solve:

$W(t)=95$

$100\left(1-e^{-0.3t}\right)=95$

$\left(1-e^{-0.3t}\right)=0.95$ Divide by 100

$-e^{-0.3t}=-0.05$ Subtract 1

$e^{-0.3t}=0.05$ Multiply by -1

$\ln\left(e^{-0.3t}\right)=\ln(0.05)$ Taking the Natural log on both sides

$-0.3t=\ln(0.05)$

$t=\dfrac{\ln(0.05)}{-0.3}$ Simplifying

$t\approx 9.985774$

$t\approx 10.0$

It will take a keyboarder approximately 10 weeks of practice in order to type 95 words per minute.

d) $\boxed{tw}$ $\lim_{t\to\infty}W(t)=\lim_{t\to\infty}\left[100\left(1-e^{-0.3t}\right)\right]=100$

This indicates that a keyboarder's speed will eventually level off at 100 words per minute.

86. $P=P_o e^{-kt}$

$$\frac{P}{P_0}=e^{-kt}$$
$$\ln\left(\tfrac{P}{P_0}\right)=\ln\left(e^{-kt}\right)$$
$$\ln\left(\tfrac{P}{P_0}\right)=-kt$$
$$\frac{\ln\left(\frac{P}{P_0}\right)}{-k}=t$$
$$-\frac{\ln\left(\frac{P}{P_0}\right)}{k}=t$$

Note: The previous expression can be rearranged using property of logarithms.

$$-\frac{\ln\left(\frac{P}{P_0}\right)}{k}=t$$
$$\frac{-\ln\left(\frac{P}{P_0}\right)}{k}=t$$
$$\frac{\ln\left(\frac{P}{P_0}\right)^{-1}}{k}=t \qquad (\text{P3})$$
$$\frac{\ln\left(\frac{P_0}{P}\right)}{k}=t$$

87. $P=P_0e^{kt}$

$\dfrac{P}{P_0}=e^{kt}$ Divide by P_0

$\ln\left(\tfrac{P}{P_0}\right)=\ln\left(e^{kt}\right)$ Take the Natural Log on both sides

$\ln\left(\tfrac{P}{P_0}\right)=kt$ (P5)

$\dfrac{\ln\left(\frac{P}{P_0}\right)}{k}=t$ Divide by *k*.

88. $f(x)=\ln\left(x^3+1\right)^5$

$f(x)=5\ln\left(x^3+1\right)$ (P3)

$f'(x)=5\left(\dfrac{3x^2}{x^3+1}\right)$ By Theorem 7

$f'(x)=\dfrac{15x^2}{x^3+1}$ Simplifying