Chapter 9

More Tests

9.1 Chi-Square Goodness-of-Fit Tests

9.1–2
$$\begin{aligned} q_4 &= \frac{(224-232)^2}{232}+\frac{(119-116)^2}{116}+\frac{(130-116)^2}{116}+\frac{(48-58)^2}{58}+\frac{(59-58)^2}{58} \\ &= 3.784. \end{aligned}$$

The null hypothesis will not be rejected at any reasonable significance level. Note that $E(Q_4) = 4$ when H_0 is true.

9.1–4
$$\begin{aligned} q_3 &= \frac{(124-117)^2}{117}+\frac{(30-39)^2}{39}+\frac{(43-39)^2}{39}+\frac{(11-13)^2}{13} \\ &= 0.419+2.077+0.410+0.308 = 3.214 < 7.815 = \chi^2_{0.05}(3). \end{aligned}$$

Thus we do not reject the Mendelian theory with these data.

9.1–6 Using Table II in Appendix B with $p = 0.30$, the hypothesized probabilities are $p_0 = P(X=0) = 0.343,\ p_1 = P(X=1) = 0.441,\ p_2 = P(X=2) = 0.189,\ p_3 = P(X=3) = 0.027$. Thus the respective expected values are 68.6, 88.2, 37.8, and 5.4. The value of the chi-square goodness of fit statistic is:

$$\begin{aligned} q &= \frac{(57-68.6)^2}{68.6}+\frac{(95-88.2)^2}{88.2}+\frac{(38-37.8)^2}{37.8}+\frac{(10-5.4)^2}{5.4} \\ &= 6.405 < 7.815 = \chi^2_{0.05}(3). \end{aligned}$$

Do not reject the hypothesis that X is $b(3,\ 0.30)$ at a 5% significance level. Limits for the p-value are $0.05 < p\text{-value} < 0.10$ because $\chi^2_{0.10}(3) = 6.251 < 6.405 < \chi^2_{0.05}(3) = 7.815$.

We find that $\widehat{p} = 201/600 = 0.335$. Thus a 95% confidence interval for p is

$$0.335 \pm 1.96\sqrt{(0.335)(0.665)/600} \quad \text{or} \quad [0.297, 0.373].$$

The pennies that were used were minted 1998 or earlier. See Figure 9.1-6. Repeat this experiment with similar pennies or with newer pennies and compare your results with those obtained by these students.

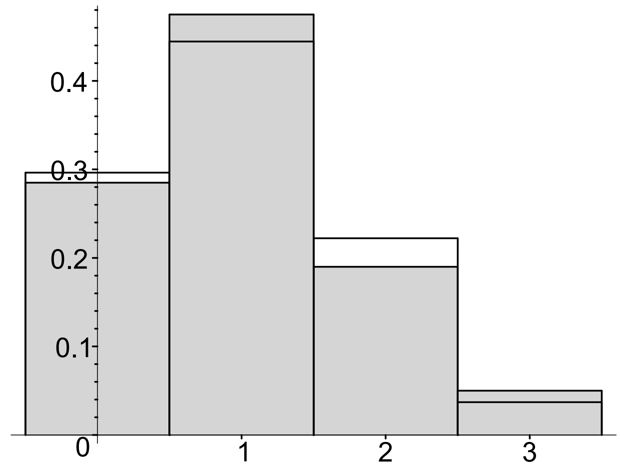

Figure 9.1–6: The $b(3,\ 3/10)$ probability histogram and the relative frequency histogram (shaded)

9.1–8 **(a)** Estimate λ by $\overline{x} = 50/40 = 1.25$. Then

$$\begin{aligned} q_3 &= \frac{[15-(0.2865)(40)]^2}{(0.2865)(40)} + \frac{[12-(0.3581)(40)]^2}{(0.3581)(40)} + \\ &\quad \frac{[7-(0.2238)(40)]^2}{(0.2238)(40)} + \frac{[6-(0.1316)(40)]^2}{(0.1316)(40)} \\ &= 1.999 < \chi^2_{0.05}(2) = 5.991; \end{aligned}$$

do not reject H_0;

(b) Estimate p by $1/\overline{y} = 1/2.25 = 4/9$. Then

$$\begin{aligned} q_3 &= \frac{[15-(4/9)(40)]^2}{(4/9)(40)} + \frac{[12-(20/81)(40)]^2}{(20/81)(40)} + \\ &\quad \frac{[7-(100/729)(40)]^2}{(100/729)(40)} + \frac{[6-(125/729)(40)]^2}{(125/729)(40)} \\ &= 1.415 < \chi^2_{0.05}(2) = 5.991; \end{aligned}$$

do not reject H_0.

9.1–10 The respective probabilities and expected frequencies are 0.050, 0.149, 0.224, 0.224, 0.168, 0.101, 0.050, 0.022, 0.012 and 15.0, 44.7, 67.2, 67.2, 50.4, 30.3, 15.0, 6.6, 3.6. The last two cells could be combined to give an expected frequency of 10.2. From Exercise 3.5–12, the respective frequencies are 17, 47, 63, 63, 49, 28, 21, and 12 giving

$$q_7 = \frac{(17-15.0)^2}{15.0} + \frac{(47-44.7)^2}{44.7} + \cdots + \frac{(12-10.2)^2}{10.2} = 3.841.$$

Since $3.841 < 14.07 = \chi^2_{0.05}(7)$, do not reject. The sample mean is $\overline{x} = 3.03$ and the sample variance is $s^2 = 3.19$ which also supports the hypothesis. The following figure compares the probability histogram with the relative frequency histogram of the data.

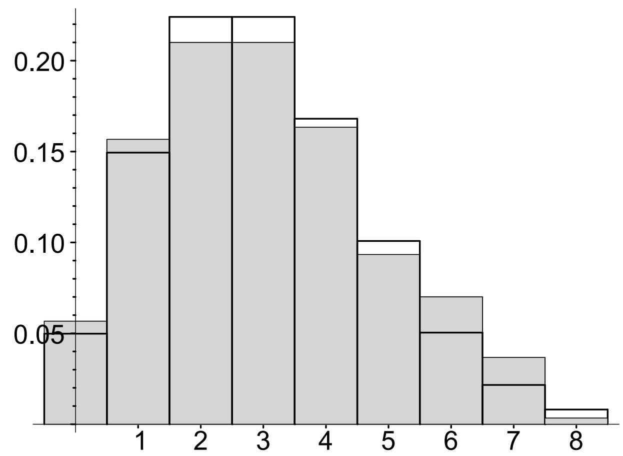

Figure 9.1–10: The Poisson probability histogram, $\lambda = 3$, and relative frequency histogram (shaded)

9.1–12 **(a)** For the infected snails, $\overline{x} = 84.74$, $s_x = 64.79$;
For the control snails, $\overline{y} = 113.16$, $s_y = 87.02$;

(b)

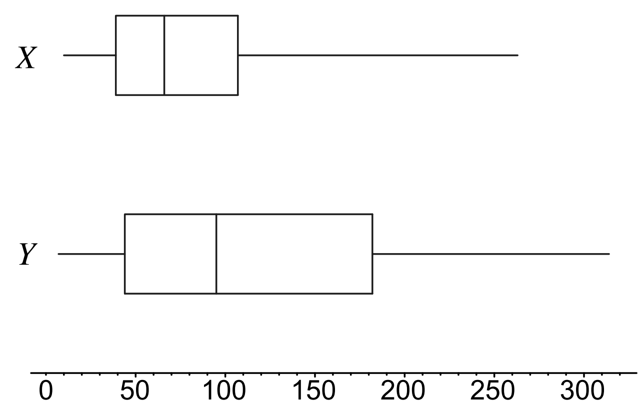

Figure 9.1–12: Box-and-whisker diagrams for infected (X) and control (Y)

(c) We shall use 5 classes with equal probability for the control snails.

A_i	Observed	Expected	q
[0, 25.25)	4	6.2	0.781
[25.25, 57.81)	9	6.2	1.264
[57.81, 103.69)	5	6.2	0.232
[103.69, 182.13)	6	6.2	0.006
[182.13, ∞)	7	6.2	0.103
	31	31.0	2.386

The p-value for $5 - 1 - 1 = 3$ degrees of freedom is 0.496 so we fail to reject the null hypothesis;

(d) We shall use 10 classes with equal probability for the infected snalis.

A_i	Observed	Expected	q
[0, 22.34)	3	3.9	0.208
[22.34, 34.62)	3	3.9	0.208
[34.62, 46.09)	8	3.9	4.310
[46.09, 57.81)	4	3.9	0.003
[57.81, 70.49)	2	3.9	0.926
[70.49, 84.94)	4	3.9	0.003
[84.94, 102.45)	4	3.9	0.003
[102.45, 125.76)	4	3.9	0.003
[125.76, 163.37)	2	3.9	0.926
[163.37, ∞)	5	3.9	0.310
	39	39.0	6.900

The p-value for $10 - 1 = 9$ degrees of freedom is 0.648 so we fail to reject the null hypothesis.

9.2 Contingency Tables

9.2–2 $10.18 < 20.48 = \chi^2_{0.025}(10)$, accept H_0.

9.2–4 In the combined sample of 45 observations, the lower third includes those with scores of 61 or lower, the middle third have scores from 62 through 78, and the higher third are those with scores of 79 and above.

	low	middle	high	Totals
Class U	9 (5)	4 (5)	2 (5)	15
Class V	5 (5)	5 (5)	5 (5)	15
Class W	1 (5)	6 (5)	8 (5)	15
Totals	15	15	15	45

Thus

$$q = 3.2 + 0.2 + 1.8 + 0 + 0 + 0 + 3.2 + 0.2 + 1.8 = 10.4.$$

Since

$$q = 10.4 > 9.488 = \chi^2_{0.05}(4),$$

we reject the equality of these three distributions. (p-value = 0.034.)

9.2–6 $q = 8.410 < 9.488 = \chi^2_{0.05}(4)$, fail to reject H_0. (p-value = 0.078.)

9.2–8 $q = 4.268 > 3.841 = \chi^2_{0.05}(1)$, reject H_0. (p-value = 0.039.)

9.2–10 $q = 7.683 < 9.210 = \chi^2_{0.01}(2)$, fail to reject H_0. (p-value = 0.021.)

9.2–12 $q = 8.792 > 7.378 = \chi^2_{0.025}(2)$, reject H_0. (p-value = 0.012.)

9.2–14 The contingency table is:

Eyes	A_1	A_2	A_3	A_4	Totals
Blue	22 (20)	17 (20)	21 (20)	20 (20)	80
Brown	14 (17.50)	20 (17.50)	20 (17.50)	16 (17.0)	70
Other	14 (12.50)	13 (12.50)	9 (12.50)	14 (12.50)	50
Totals	50	50	50	50	200

Because $q = 3.603 < 12.59 = \chi^2_{0.05}(6)$, do not reject the hypothesis of independent attributes. (p-value $= 0.730$.)

9.3 One-Factor Analysis of Variance

9.3–2

Source	SS	DF	MS	F	p-value
Treatment	388.2805	3	129.4268	4.9078	0.0188
Error	316.4597	12	26.3716		
Total	704.7402	15			

$F = 4.9078 > 3.49 = F_{0.05}(3, 12)$, reject H_0.

9.3–4 **(a)**

Source	SS	DF	MS	F	p-value
Treatment	184.8	2	92.4	15.4	0.00015
Error	102.0	17	6.0		
Total	286.8	19			

$F = 15.4 > 3.59 = F_{0.05}(2, 17)$, reject H_0;

(b)

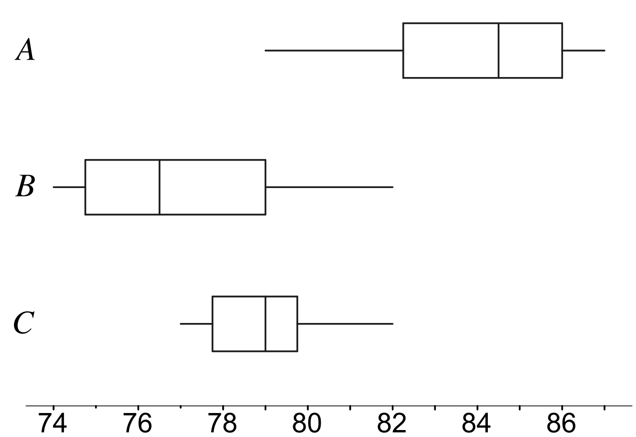

Figure 9.3–4: Box-and-whisker diagrams of strengths of different beams

9.3–6 **(a)** $F \geq F_{0.05}(3, 24) = 3.01$;

(b)

Source	SS	DF	MS	F	p-value
Treatment	12,280.86	3	4,093.62	3.455	0.0323
Error	28,434.57	24	1,184.77		
Total	40,715.43	27			

$F = 3.455 > 3.01$, reject H_0;

(c) $0.025 < p\text{-value} < 0.05$.

(d)

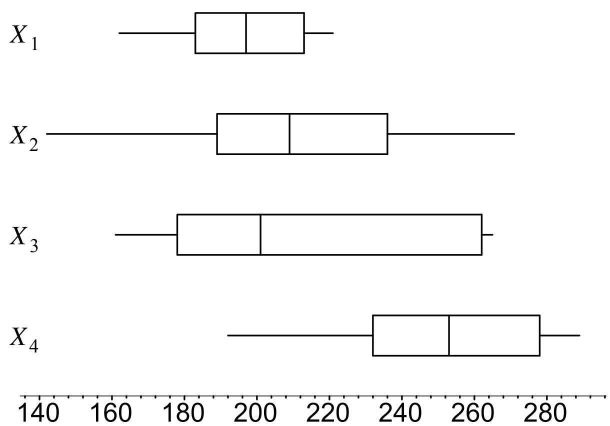

Figure 9.3–6: Box-and-whisker diagrams for cholesterol levels

9.3–8 **(a)** $F \geq F_{0.05}(4, 30) = 2.69$;

(b)

Source	SS	DF	MS	F	p-value
Treatment	0.00442	4	0.00111	2.85	0.0403
Error	0.01157	30	0.00039		
Total	0.01599	34			

$F = 2.85 > 2.69$, reject H_0;

(c)

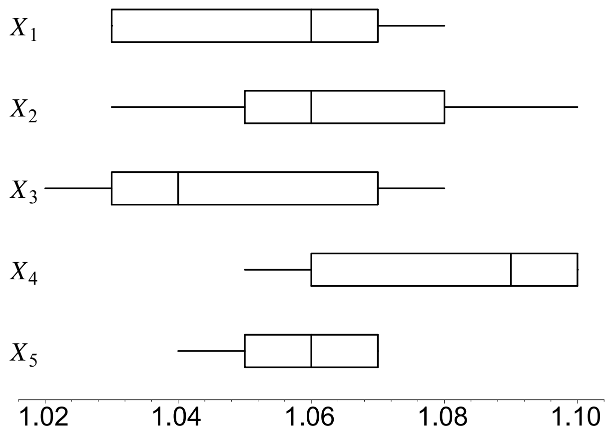

Figure 9.3–8: Box-and-whisker diagrams for nail weights

9.3–10 **(a)** $t = \dfrac{92.143 - 103.000}{\sqrt{\dfrac{6(69.139) + 6(57.669)}{12}\left(\dfrac{1}{7} + \dfrac{1}{7}\right)}} = -2.55 < -2.179$, reject H_0.

$F = \dfrac{412.517}{63.4048} = 6.507 > 4.75$, reject H_0.

The F and the t tests give the same results since $t^2 = F$;

(b) $F = \dfrac{86.3336}{114.8889} = 0.7515 < 3.55$, do not reject H_0.

9.3–12 **(a)**

Source	SS	DF	MS	F	p-value
Treatment	122.1956	2	61.0978	2.130	0.136
Error	860.4799	30	28.6827		
Total	982.6755	32			

$F = 2.130 < 3.32 = F_{0.05}(2, 30)$, fail to reject H_0;

(b)

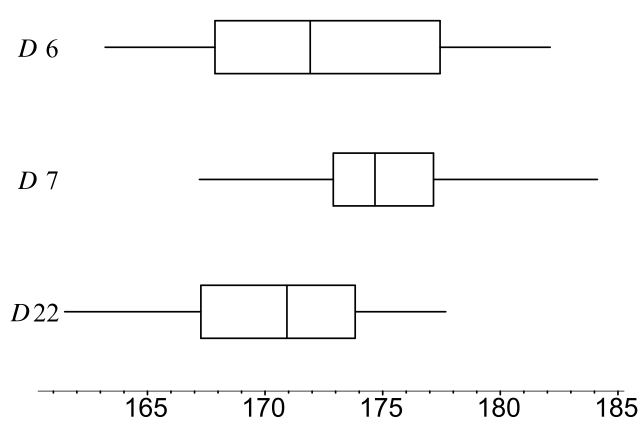

Figure 9.3–12: Box-and-whisker diagrams for resistances on three days

9.3–14 **(a)**

Source	SS	DF	MS	F	p-value
Worker	1.5474	2	0.7737	1.0794	0.3557
Error	17.2022	24	0.7168		
Total	18.7496	26			

$F = 1.0794 < 3.40 = F_{0.05}(2, 24)$, fail to reject H_0;

(b)

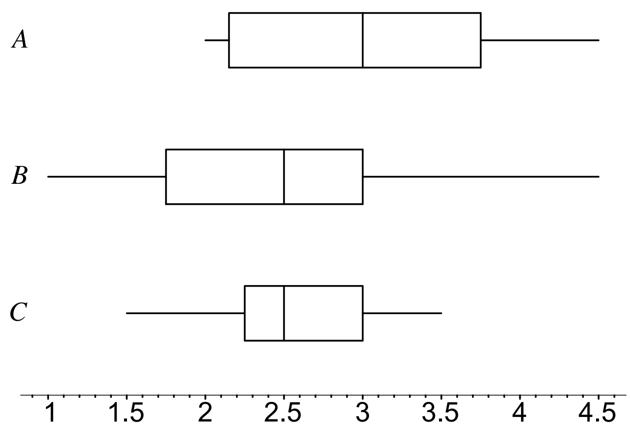

Figure 9.3–14: Box-and-whisker diagrams for workers A, B, and C

The box plot confirms the answer from part (a).

9.4 Two-Way Analysis of Variance

9.4–2

					$\mu + \alpha_i$
	6	3	7	8	6
	10	7	11	12	10
	8	5	9	10	8
$\mu + \beta_j$	8	5	9	10	$\mu = 8$

So $\alpha_1 = -2,\ \alpha_2 = 2,\ \alpha_3 = 0$ and $\beta_1 = 0,\ \beta_2 = -3,\ \beta_3 = 1,\ \beta_4 = 2.$

9.4–4 $\displaystyle\sum_{i=1}^{a}\sum_{j=1}^{b}(\overline{X}_{i\cdot} - \overline{X}_{\cdot\cdot})(X_{ij} - \overline{X}_{i\cdot} - \overline{X}_{\cdot j} + \overline{X}_{\cdot\cdot})$

$$= \sum_{i=1}^{a}(\overline{X}_{i\cdot} - \overline{X}_{\cdot\cdot})\sum_{j=1}^{b}[(X_{ij} - \overline{X}_{i\cdot}) - (\overline{X}_{\cdot j} - \overline{X}_{\cdot\cdot})]$$

$$= \sum_{i=1}^{a}(\overline{X}_{i\cdot} - \overline{X}_{\cdot\cdot})\left\{\sum_{j=1}^{b}(X_{ij} - \overline{X}_{i\cdot}) - \sum_{j=1}^{b}(\overline{X}_{\cdot j} - \overline{X}_{\cdot\cdot})\right\}$$

$$= \sum_{i=1}^{a}(\overline{X}_{i\cdot} - \overline{X}_{\cdot\cdot})(0 - 0) = 0;$$

$\displaystyle\sum_{i=1}^{a}\sum_{j=1}^{b}(\overline{X}_{\cdot j} - \overline{X}_{\cdot\cdot})(X_{ij} - \overline{X}_{i\cdot} - \overline{X}_{\cdot j} + \overline{X}_{\cdot\cdot}) = 0$, similarly;

$$\sum_{i=1}^{a}\sum_{j=1}^{b}(\overline{X}_{i\cdot} - \overline{X}_{\cdot\cdot})(\overline{X}_{\cdot j} - \overline{X}_{\cdot\cdot}) = \left\{\sum_{i=1}^{a}(\overline{X}_{i\cdot} - \overline{X}_{\cdot\cdot})\right\}\left\{\sum_{j=1}^{b}(\overline{X}_{\cdot j} - \overline{X}_{\cdot\cdot})\right\} = (0)(0) = 0\ .$$

9.4–6

					$\mu + \alpha_i$
	6	7	7	12	8
	10	3	11	8	8
	8	5	9	10	8
$\mu + \beta_j$	8	5	9	10	$\mu = 8$

So $\alpha_1 = \alpha_2 = \alpha_3 = 0$ and $\beta_1 = 0,\ \beta_2 = -3,\ \beta_3 = 1,\ \beta_4 = 2$ as in Exercise 9.4–2. However, $\gamma_{11} = -2$ because $8 + 0 + 0 + (-2) = 6$. Similarly we obtain the other γ_{ij}'s :

$$\begin{array}{rrrr} -2 & 2 & -2 & 2 \\ 2 & -2 & 2 & -2 \\ 0 & 0 & 0 & 0 \end{array}$$

9.4–8

Source	SS	DF	MS	F	p-value
Row (A)	5,103.0000	1	5,103.0000	4.307	0.049
Col (B)	6,121.2857	1	6,121.2857	5.167	0.032
Int(AB)	1,056.5714	1	1,056.5714	0.892	0.354
Error	28,434.5714	24	1,184.7738		
Total	40,715.4286	27			

(a) Since $F = 0.892 < F_{0.05}(1, 24) = 4.26$, do not reject H_{AB};

(b) Since $F = 4.307 > F_{0.05}(1, 24) = 4.26$, reject H_A;

(c) Since $F = 5.167 > F_{0.05}(1, 24) = 4.26$, reject H_B.

9.5 General Factorial and 2^k Factorial Designs

9.5–4 **(a)** [A] $= -28.4/8 = -3.55$, [B] $= -1.45$, [C] $= 3.2$, [AB] $= -1.525$, [AC] $= -0.525$, [BC] $= 0.375$, [ABC] $= -1.2$;

(b)

Identity of Effect	Ordered Effect	Percentile	Percentile from N(0,1)
[A]	−3.550	12.5	−1.15
[AB]	−1.525	25.0	−0.67
[B]	−1.450	37.5	−0.32
[ABC]	−1.200	50.0	0.00
[AC]	−0.525	62.5	0.32
[BC]	0.375	75.0	0.67
[C]	3.20	87.5	1.15

The main effects of temperature (A) and concentration (C) are significant.

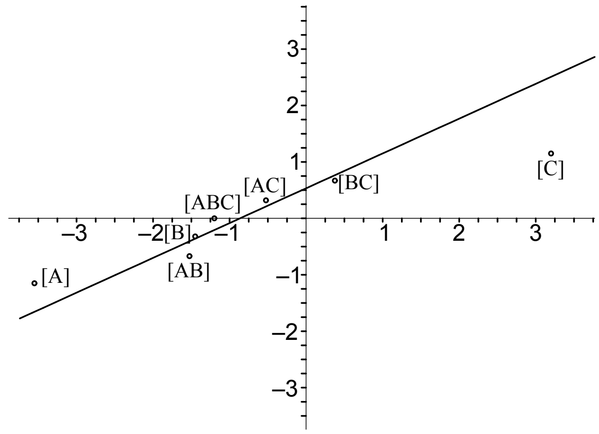

Figure 9.5–4: q-q plot

9.6 Tests Concerning Regression and Correlation

9.6–2 The critical region is $t_1 \ge t_{0.025}(8) = 2.306$. From Exercise 6.5–4,

$$\widehat{\beta} = 4.64/5.04 \text{ and } n\widehat{\sigma^2} = 1.84924; \quad \text{also } \sum_{i=1}^{10}(x_i - \overline{x})^2 = 5.04, \text{ so}$$

$$t_1 = \frac{4.64/5.04}{\sqrt{\dfrac{1.84924}{8(5.04)}}} = \frac{0.9206}{0.2142} = 4.299.$$

Since $t_1 = 4.299 > 2.306$, we reject H_0.

9.6–4 For these data, $r = -0.413$. Since $|r| = 0.413 < 0.7292$, do not reject H_0.

9.6–6 Following the suggestion given in the hint, the expression equals

$$\begin{aligned}(n-1)S_Y^2 - \frac{2Rs_XS_Y}{s_X^2}(n-1)Rs_XS_Y + \frac{R^2s_X^2S_Y^2(n-1)s_X^2}{s_X^4} &= (n-1)S_Y^2(1 - 2R^2 + R^2) \\ &= (n-1)S_Y^2(1 - R^2).\end{aligned}$$

9.6–8

$$\begin{aligned}u(R) &\approx u(\rho) + (R-\rho)u'(\rho), \\ \mathrm{Var}[u(\rho) + (R-\rho)u'(\rho)] &= [u'(\rho)]^2\mathrm{Var}(R) \\ &= [u'(\rho)]^2\,\frac{(1-\rho^2)^2}{n} = c, \quad \text{which is free of } \rho, \\ u'(\rho) &= \frac{k/2}{1-\rho} + \frac{k/2}{1+\rho}, \\ u(\rho) &= -\frac{k}{2}\ln(1-\rho) + \frac{k}{2}\ln(1+\rho) = \frac{k}{2}\ln\left(\frac{1+\rho}{1-\rho}\right).\end{aligned}$$

Thus, taking $k = 1$,

$$u(R) = \left(\frac{1}{2}\right)\ln\left[\frac{1+R}{1-R}\right]$$

has a variance almost free of ρ.

9.6–10 **(a)** $r = -0.4906, |\,r\,| = 0.4906 > 0.4258$, reject H_0 at $\alpha = 0.10$;

(b) $|\,r\,| = 0.4906 < 0.4973$, fail to reject H_0 at $\alpha = 0.05$.

9.6–12 **(a)** $r = 0.339, |\,r\,| = 0.339 < 0.5325 = r_{0.025}(12)$, fail to reject H_0 at $\alpha = 0.05$;

(b) $r = -0.821 < -0.6613 = r_{0.005}(12)$, reject H_0 at $\alpha = 0.005$;

(c) $r = 0.149, |\,r\,| = 0.149 < 0.5325 = r_{0.025}(12)$, fail to reject H_0 at $\alpha = 0.05$.

9.7 Statistical Quality Control

9.7–2 **(a)** $\overline{\overline{x}} = 67.44$, $\overline{s} = 2.392$, $\overline{R} = 5.88$;

(b) $\text{UCL} = \overline{\overline{x}} + 1.43(\overline{s}) = 67.44 + 1.43(2.392) = 70.86$;

$\text{LCL} = \overline{\overline{x}} - 1.43(\overline{s}) = 67.44 - 1.43(2.392) = 64.02$;

(c) $\text{UCL} = 2.09(\overline{s}) = 2.09(2.392) = 5.00$; $\text{LCL} = 0$;

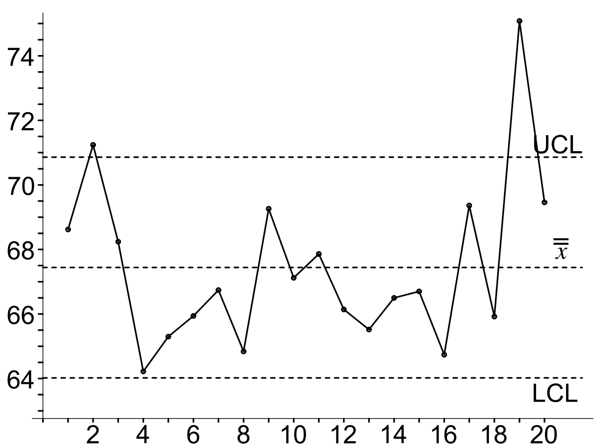

Figure 9.7–2: **(b)** $\overline{x}$-chart using $\overline{s}$ and **(c)** s-chart

(d) UCL $= \overline{\overline{x}} + 0.58(\overline{R}) = 67.44 + 0.58(5.88) = 70.85$;

LCL $= \overline{\overline{x}} - 0.58(\overline{R}) = 67.44 - 0.58(5.88) = 64.03$;

(e) UCL $= 2.11(\overline{R}) = 2.11(5.88) = 12.41$; LCL $= 0$;

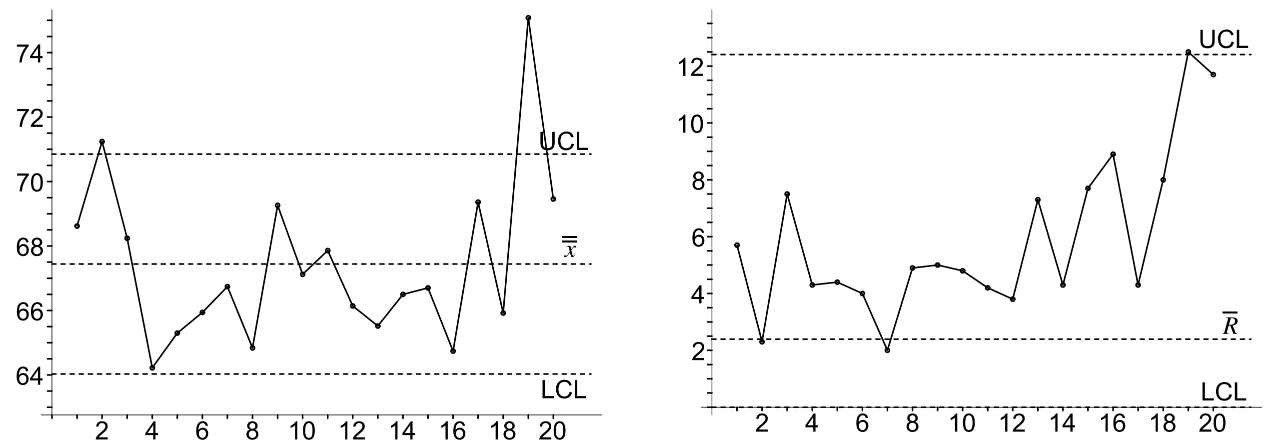

Figure 9.7–2: **(d)** $\overline{x}$-chart using $\overline{R}$ and **(e)** R-chart

(f) Quite well until near the end.

9.7–4 With $\overline{p} = 0.0254$, UCL $= \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/100} = 0.073$;

with $\overline{p} = 0.02$, UCL $= \overline{p} + 3\sqrt{\overline{p}(1-\overline{p})/100} = 0.062$;

In both cases we see that problems are arising near the end.

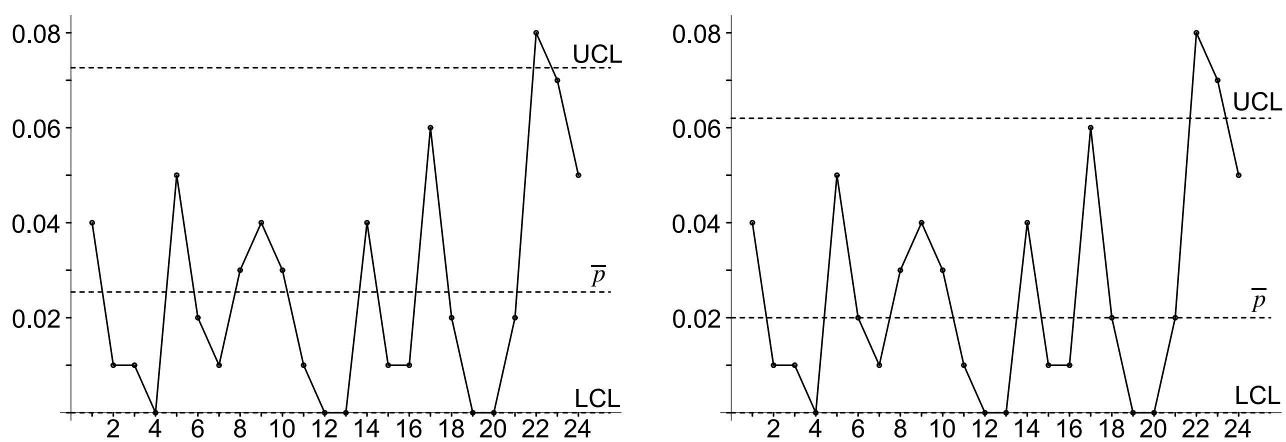

Figure 9.7–4: p-charts using $\overline{p} = 0.0254$ and $\overline{p} = 0.02$

9.7–6 **(a)** $\text{UCL} = \overline{c} + 3\sqrt{\overline{c}} = 1.80 + 3\sqrt{1.80} = 5.825; \quad \text{LCL} = 0;$

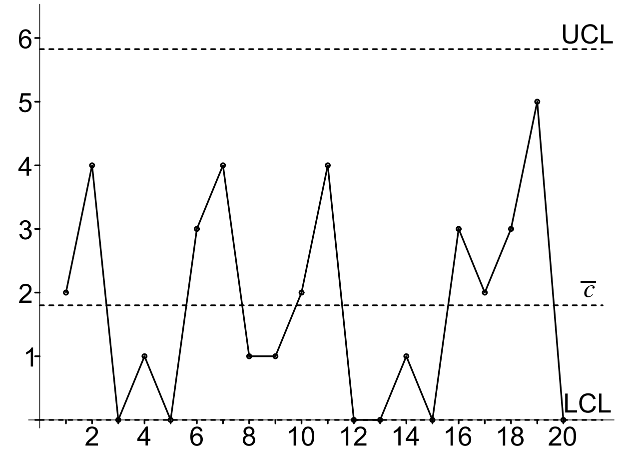

Figure 9.7–6: c-chart

(b) The process is in statistical control.

9.7–8 **(a)** $\overline{\overline{x}} = \dfrac{5101}{150} = 34.0067$, $\overline{s} = 2.8244$, $\overline{R} = 7$;

(b) $\text{UCL} = \overline{\overline{x}} + 1.43(\overline{s}) = 38.046$ for the $\overline{x}$ chart;
$\text{LCL} = \overline{\overline{x}} - 1.43(\overline{s}) = 29.968$ for the $\overline{x}$ chart;

(c) $\text{UCL} = 2.09(\overline{s}) = 5.903$ for the s chart;
$\text{LCL} = 0(\overline{s})$ for the s chart;

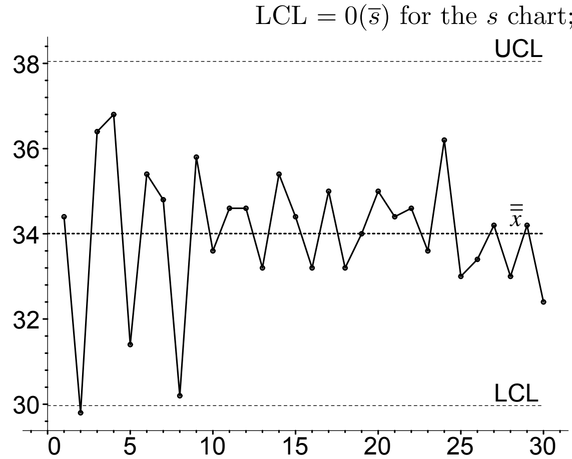

Figure 9.7–8: **(b)** $\overline{x}$-chart using $\overline{s}$ and **(c)** s-chart

(d) UCL $= \overline{\overline{x}} + 0.58(\overline{R}) = 38.0667$;

LCL $= \overline{\overline{x}} - 0.58(\overline{R}) = 29.9467$;

(e) UCL $= 2.11(\overline{R}) = 2.11(7) = 14.77$; LCL $= 0(\overline{R}) = 0$;

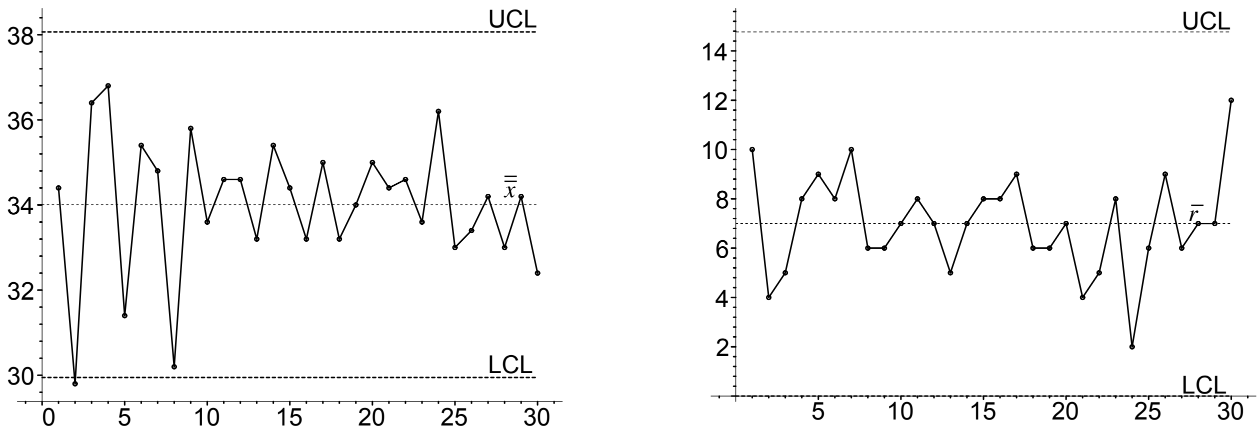

Figure 9.7–8: **(d)** $\overline{x}$-chart using $\overline{R}$ and **(e)** R-chart

(f) Yes.

9.7–10 $LCL = 1.4 - 3\sqrt{1.4} < 0$ so $LCL = 0$;
$UCL = 1.4 + 3\sqrt{1.4} = 4.9496$;

(a) If $\lambda = 3, P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.815 = 0.185$;

(b) Y is $b(10, 0.185)$. Thus

$$\begin{aligned} P(\text{at least } 1) &= 1 - P(Y = 0) \\ &= 1 - p^0 q^{10} \\ &= 1 - 0.815^{10} \\ &= 0.871. \end{aligned}$$