Instructor's Solutions Manual

John E. Freund's

MATHEMATICAL STATISTICS

Sixth Edition

Seventh

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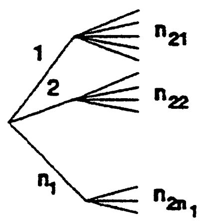
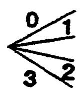
CHAPTER 1

1.1



(a) $\sum_{i=1}^{n_1} n_{2i}$

(b)

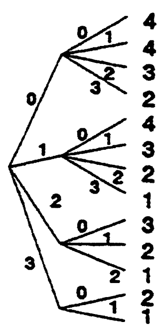


0 1 2 3
0 1 2 3
0 1 2
0 1

$\sum = 13$

1.2 $\sum_{i=1}^{n_1} n_{2i} = \sum_{i=1}^{n_1} n_2 = n_1 n_2$

1.3



$n_{300} = 4$
$n_{301} = 4$
$n_{302} = 3$
$n_{303} = 2$
$n_{310} = 4$
$n_{311} = 3$
$n_{312} = 2$
$n_{313} = 1$

$n_{320} = 3$
$n_{321} = 2$
$n_{322} = 1$
$n_{330} = 2$
$n_{331} = 1$

$\sum = 32$

1.4 $\sum_{i=1}^{n_1} \sum_{j=1}^{n_2} n_3 = n_1 n_2 n_3$

1.5 (b) 6, 20, and 70

"2 out of 3" m = 2 $\quad 2[\binom{1}{1} + \binom{2}{1}] = 2(1 + 2) = 6$

"3 out of 5" m = 3 $\quad 2[\binom{2}{2} + \binom{3}{2} + \binom{4}{2}] = 2(1 + 3 + 6) = 20$

"4 out of 7" m = 4 $\quad 2[\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \binom{6}{3}] = 2(1 + 4 + 10 + 20)$
$= 70$

1.6 (a) $\sqrt{20\pi}\left(\frac{10}{e}\right)^{10} = (7.92665)(3.678797)^{10} = (7.92665)(454{,}002.09)$

$= 3{,}598{,}716$

$n! = 3.6288 \cdot 10^{6} \approx \frac{3.6288 - 3.5987}{3.6288} \cdot 100 = 0.83\%$

$\sqrt{24\pi}\left(\frac{12}{e}\right)^{12} = (8.683215)(4.41455)^{12}$

$12! = \frac{47568718}{479} \approx \frac{4.7900 - 4.7568}{4.7900} \cdot 100 = 0.69\%$

(b) $$\binom{52}{13} = \frac{52!}{13!\ 39!} = \frac{\sqrt{104\pi}\left(\frac{52}{e}\right)^{52}}{\sqrt{26\pi}\ \sqrt{78\pi}\ \left(\frac{13}{e}\right)^{13}\left(\frac{39}{e}\right)^{39}}$$

$$= \frac{13^{52} \cdot 4^{52}}{\sqrt{19.5\pi}\ \ 13^{13} \cdot 13^{39} \cdot 3^{39}} = \frac{4^{52}}{\sqrt{19.5\pi}\ \ 3^{39}} \approx 639 \text{ billion}$$

1.7 Using Stirling's formula in $\binom{2n}{n} = \frac{2n!}{n!\ n!}$ yields

$$\frac{\binom{2n}{n}\sqrt{\pi n}}{2^{2n}} = \frac{\sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n}}{\left[\sqrt{2\pi n}\left(\frac{n}{e}\right)^{n}\right]^{2}} \cdot \frac{\sqrt{\pi n}}{2^{2n}} = 1$$

1.8 n^r and $12^3 = 1{,}728$

1.9 $\binom{r+n-1}{r}$ and $\binom{5+3-1}{5} = \binom{7}{5} = 21$

1.10 Substitute r-n for r into result of 1.9

$\binom{r-n+n-1}{r-n} = \binom{r-1}{r-n}$ and $\binom{5-1}{5-3} = \binom{4}{2} = 6$

1.11 (b) Seventh row is 1, 6, 15, 20, 15, 6, 1

Eighth row is 1, 7, 21, 35, 35, 21, 7, 1

$(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

$(x+y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

1.14 (a) Set $x = 1$ and $y = 1$

(b) Set $x = 1$ and $y = -1$

(c) Set $x = 1$ and $y = a - 1$

1.19 (a) $\dfrac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{24} = -\dfrac{15}{384}$ and $\dfrac{(-3)(-4)(-5)}{6} = -10$

(b) $\sqrt{5} = 2(1+\frac{1}{4})^{1/2} = 2\left[1 + \frac{1}{2}(\frac{1}{4}) + \frac{1}{2}(-\frac{1}{2})(\frac{1}{4})^2 + \frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})(\frac{1}{4})^3\right]$

$$= 2\left[1 + \frac{1}{8} - \frac{1}{64} + \frac{3}{512} \cdots\right] \approx 2\cdot\frac{512 + 64 - 8 + 3}{512}$$

$$= 2\cdot\frac{571}{512} = 2.23$$

$$\frac{1142}{512} = 2.230$$

1.20 (a) $\dfrac{(-1)(-2)\ldots(-r)}{r!} = (-1)^r$

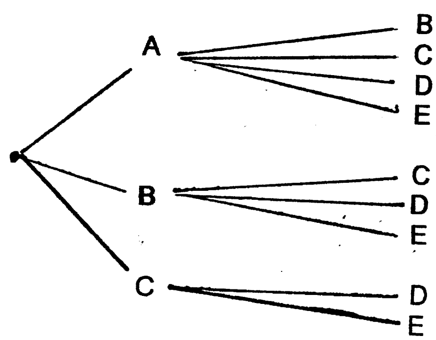
(b) $\binom{-n}{r} = \dfrac{(-n)(-n-1)\ldots(-n-r+1)}{r!} = (-1)^r\,\dfrac{n(n+1)\ldots(n+r-1)}{r!}$

$$= (-1)^r\,\frac{(n+r-1)\ldots(n+1)n}{r!} = (-1)^r\binom{n+r-1}{r}$$

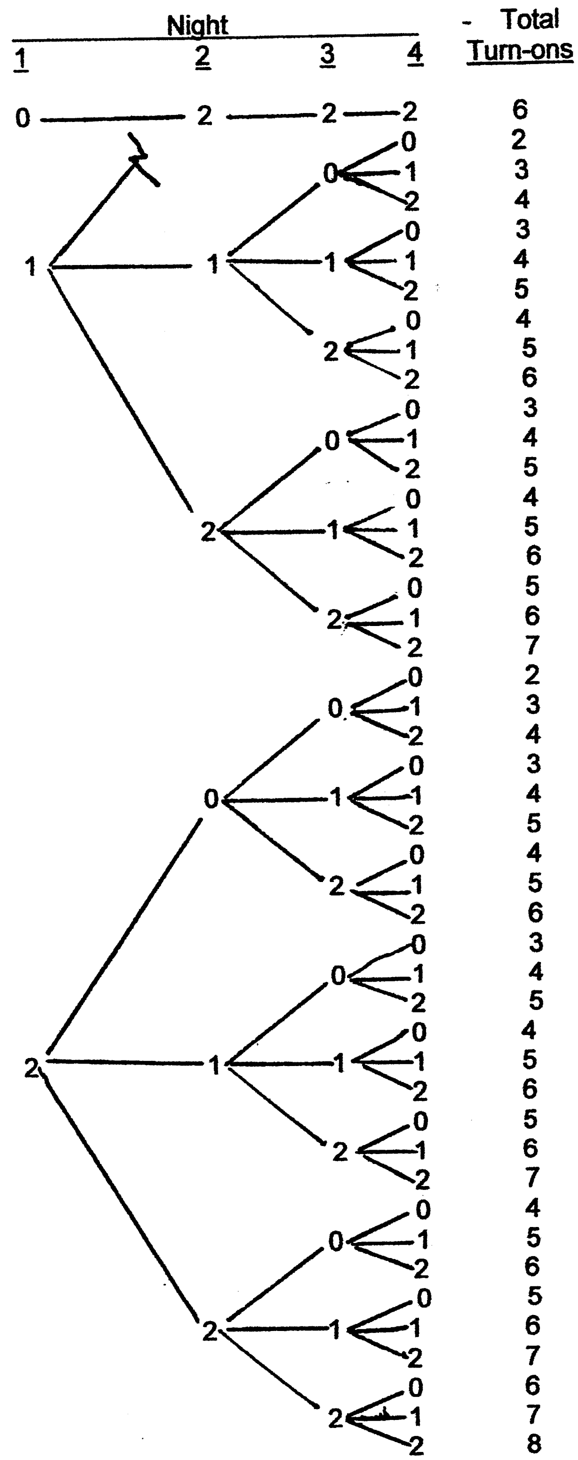
1.21 $\dfrac{8!}{2!\;3!\;3!} = \dfrac{8\cdot7\cdot6\cdot5\cdot4}{2\cdot6} = 560$

1.22 $\dfrac{9!}{3!\;2!\;3!}\cdot2^3\cdot3^2\cdot(-4)^3 = -\dfrac{9\cdot8\cdot7\cdot6\cdot5\cdot4}{12}\cdot8\cdot9\cdot64 = -23{,}224{,}320$

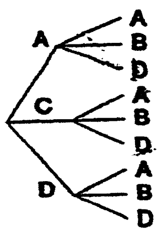
1.24



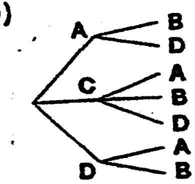
1.25 Note: If there are 0 turn-ons the first night, 6 turn-ons in four nights can only occur if there are 2 turn-ons on each of the subsequent three nights. Thus, we need to show only that part of the tree following this event.



1.26 (a)



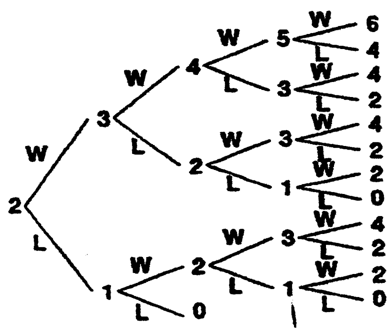
(b)



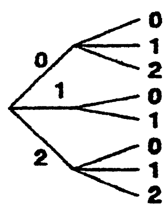
1.27 (a) 5

(b) 4

1.28



1.29



1.30 (a) 6; (b) 6•5 = 30; (c) 5•4 = 20 first one fixed; (d) 6 + 30 + 20 = 56

1.31 (a) 6•5 = 30; (b) 6•6 = 36

1.32 (a) 5•4 = 20; (b) 5•4•3 = 60

1.33 (a) 4•5•2 = 40; (b) 5•6•3 = 90

1.34 $3^{15} = 14{,}348{,}907$

1.35 $\dfrac{15\cdot 14}{2\cdot 1} = 105$

1.36 (a) $\dfrac{14\cdot 13}{2\cdot 1} = 91$; (b) $\dfrac{14\cdot 13\cdot 12}{3\cdot 2\cdot 1} = 364$

1.37 (a) 10•9•8•7 = 5040; (b) $\dfrac{5040}{24} = 210$

1.38 6! = 720

1.39 $\dfrac{6!}{2!\ 2!\ 2!} = \dfrac{720}{8} = 90$

1.40 $\frac{6!}{3!\ 3!} = \frac{720}{36} = 20$

1.41 $5! = 120$ and $120 - 2 \cdot 4! = 72$

1.42 $7! = 5040$

1.43 (a) $5! = 120$; (b) $\frac{5!}{2!} = 60$

1.44 $\frac{10!}{3!\ 3!\ 2!} = \frac{3628800}{72} = 50{,}400$ and $\frac{8!}{3!\ 2!} = \frac{40320}{12} = 3360$

1.45 $\frac{10!}{5!\ 4!} = \frac{3628800}{120 \cdot 24} = 1{,}260$

1.46 $\frac{8!}{3!\ 4!} = \frac{40320}{6 \cdot 24} = 280$

1.47 (a) $\binom{20}{7} = 77{,}520$; (b) $\binom{20}{10} = 184{,}756$

(c) $\binom{20}{17} + \binom{20}{18} + \binom{20}{19} + \binom{20}{20} = 1140 + 190 + 20 + 1 = 1351$

1.48 (a) $\binom{7}{2} = 21$; (b) $\binom{4}{2} = 6$; (c) $3 \cdot 4 = 12$

1.49 $\binom{3}{2}\binom{7}{2} + \binom{3}{3}\binom{7}{1} = 3 \cdot 21 + 1 \cdot 7 = 63 + 7 = 70$

1.50 $\binom{4}{2}\binom{7}{3}\binom{3}{1} = 6 \cdot 35 \cdot 3 = 630$

1.51 $\binom{13}{5}\binom{13}{3}\binom{13}{3}\binom{13}{2} = 1287 \cdot 286 \cdot 286 \cdot 78 = 8{,}211{,}173{,}256$

1.52 $\frac{7!}{3!\ 2!} = \frac{5040}{12} = 420$

1.53 $3^{10} = 59{,}049$

1.54 $5^6 = 15{,}625$

1.55 $\binom{12+6-1}{12} = \binom{17}{12} = \binom{17}{5} = 6{,}188$

1.56 $\binom{12-1}{6} = \binom{11}{6} = 462$

1.57 $\binom{14+3-1}{14} = \binom{16}{14} = 120$

1.58 $\binom{r-2n+n-1}{n-1} = \binom{r-n-1}{n-1}$

$\binom{r-n-1}{n-1} = \binom{10}{2} = 45$