CHAPTER 1
FUNDAMENTAL CONCEPTS

1.1 We use the first three steps of Eq. 1.11

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

Adding the above, we get

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

Adding and subtracting $\nu\frac{\sigma_x}{E}$ from the first equation,

$$\varepsilon_x = \frac{1+\nu}{E}\sigma_x - \frac{\nu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

Similar expressions can be obtained for ε_y, and ε_z.

From the relationship for γ_{yz} and Eq. 1.12,

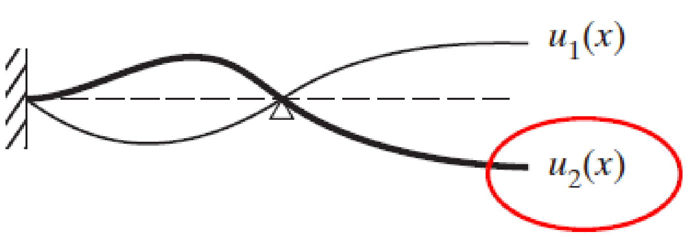
$$\tau_{yz} = \frac{E}{2(1+\nu)}\gamma_{yz} \qquad \text{etc.}$$

Above relations can be written in the form

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}$$

where **D** is the material property matrix defined in Eq. 1.15. ■

1.2 Note that $u_2(x)$ satisfies the zero slope boundary condition at the support.



■

1.3 Plane strain condition implies that

$$\varepsilon_z = 0 = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

which gives

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right)$$

We have, $\sigma_x = 20000\,\text{psi} \quad \sigma_y = -10000\,\text{psi} \quad E = 30\times 10^6\ \text{psi} \quad \nu = 0.3$.

On substituting the values,

$$\sigma_z = 3000\,\text{psi} \qquad \blacksquare$$

1.4 Displacement field

$$u = 10^{-4}\left(-x^2 + 2y^2 + 6xy\right)$$

$$v = 10^{-4}\left(3x + 6y - y^2\right)$$

$$\frac{\partial u}{\partial x} = 10^{-4}\left(-2x + 6y\right) \quad \frac{\partial u}{\partial y} = 10^{-4}\left(4y + 6x\right)$$

$$\frac{\partial v}{\partial x} = 3\times 10^{-4} \qquad \frac{\partial v}{\partial y} = 10^{-4}\left(6 + 2y\right)$$

$$\varepsilon = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix}$$

at $x = 1, y = 0$

$$\varepsilon = 10^{-4}\begin{Bmatrix} -2 \\ 6 \\ 9 \end{Bmatrix}$$

$\blacksquare$

1.5 On inspection, we note that the displacements *u* and *v* are given by

$$u = 0.1\,y + 4$$

$$v = 0$$

It is then easy to see that

$$\varepsilon_x = \frac{\partial u}{\partial x} = 0$$
$$\varepsilon_y = \frac{\partial v}{\partial y} = 0$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0.1$$

■

1.6 The displacement field is given as

$$u = 1 + 3x + 4x^3 + 6xy^2$$
$$v = xy - 7x^2$$

(a) The strains are then given by

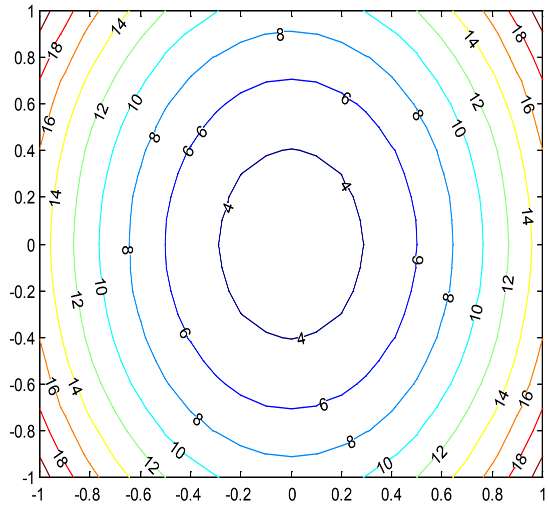
$$\varepsilon_x = \frac{\partial u}{\partial x} = 3 + 12x^2 + 6y^2$$
$$\varepsilon_y = \frac{\partial v}{\partial y} = x$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 12xy + y - 14x$$

(b) In order to draw the contours of the strain field using MATLAB, we need to create a script file, which may be edited as a text file and save with ".m" extension. The file for plotting ε_x is given below

file "prob1p5b.m"

```
[X,Y] = meshgrid(-1:.1:1,-1:.1:1);
Z = 3.+12.*X.^2+6.*Y.^2;
[C,h] = contour(X,Y,Z);
clabel(C,h);
```

On running the program, the contour map is shown as follows:



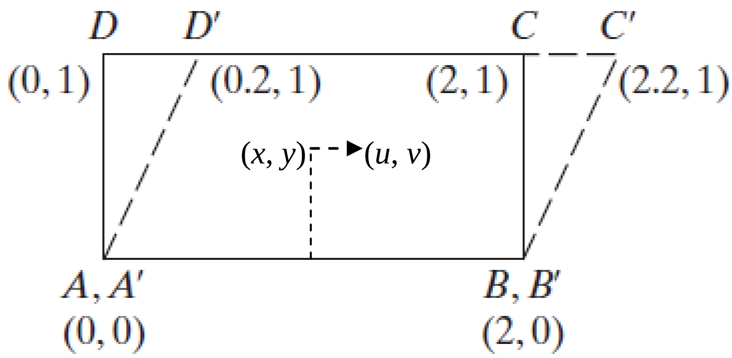
Contours of ε_x

Contours of ε_y and γ_{xy} are obtained by changing Z in the script file. The numbers on the contours show the function values.

(c) The maximum value of ε_x is at any of the corners of the square region. The maximum value is 21.

■

1.7



a) $u = \dfrac{0.2}{1} y \;\Rightarrow\; u = 0.2y \qquad v = 0$

b) $\varepsilon_x = \dfrac{\partial u}{\partial x} = 0 \quad \varepsilon_y = \dfrac{\partial v}{\partial y} = 0 \quad \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} = 0.2$

1.8

$$\sigma_x = 40\text{ MPa} \quad \sigma_y = 20\text{ MPa} \quad \sigma_z = 30\text{ MPa}$$
$$\tau_{yz} = -30\text{ MPa} \quad \tau_{xz} = 15\text{ MPa} \quad \tau_{xy} = 10\text{ MPa}$$
$$\mathbf{n} = \left[\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{\sqrt{2}}\right]^{\mathrm{T}}$$

From Eq. 1.8 we get

$$T_x = \sigma_x n_x + \tau_{xy} n_y + \tau_{xz} n_z$$
$$= 35.607\text{ MPa}$$
$$T_y = \tau_{xy} n_x + \sigma_y n_y + \tau_{yz} n_z$$
$$= -6.213\text{ MPa}$$
$$T_z = \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z$$
$$= 13.713\text{ MPa}$$
$$\sigma_n = T_x n_x + T_y n_y + T_z n_z$$
$$= 24.393\text{ MPa}$$

■

1.9 From the derivation made in P1.1, we have

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu\varepsilon_y + \nu\varepsilon_z\right]$$

which can be written in the form

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-2\nu)\varepsilon_x + \nu\varepsilon_v\right]$$

and

$$\tau_{yz} = \frac{E}{2(1+\nu)}\gamma_{yz}$$

Lame's constants λ and μ are defined in the expressions

$$\sigma_x = \lambda\varepsilon_v + 2\mu\varepsilon_x$$
$$\tau_{yz} = \mu\gamma_{yz}$$

On inspection,

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$
$$\mu = \frac{E}{2(1+\nu)}$$

μ is same as the shear modulus G. ■

1.10

$$\varepsilon = 1.2\times10^{-5}$$
$$\Delta T = 30^0 C$$
$$E = 200\,\text{GPa}$$
$$\alpha = 12\times10^{-6} /^0 C$$
$$\varepsilon_0 = \alpha\Delta T = 3.6\times10^{-4}$$
$$\sigma = E(\varepsilon-\varepsilon_0) = -69.6\,\text{MPa}$$

■

1.11

$$\varepsilon_x = \frac{du}{dx} = 1 + 2x^2$$
$$\delta = \int_0^L \frac{du}{dx}dx = \left(x + \frac{2}{3}x^3\right)\Bigg|_0^L$$
$$= L\left(1 + \frac{2}{3}L^2\right)$$

■

1.12 Following the steps of Example 1.1, we have

$$\begin{bmatrix} (80+40+50) & -80 \\ -80 & 80 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} 60 \\ 50 \end{Bmatrix}$$

Above matrix form is same as the set of equations:

$$170\, q_1 - 80\, q_2 = 60$$

$$-80\, q_1 + 80\, q_2 = 50$$

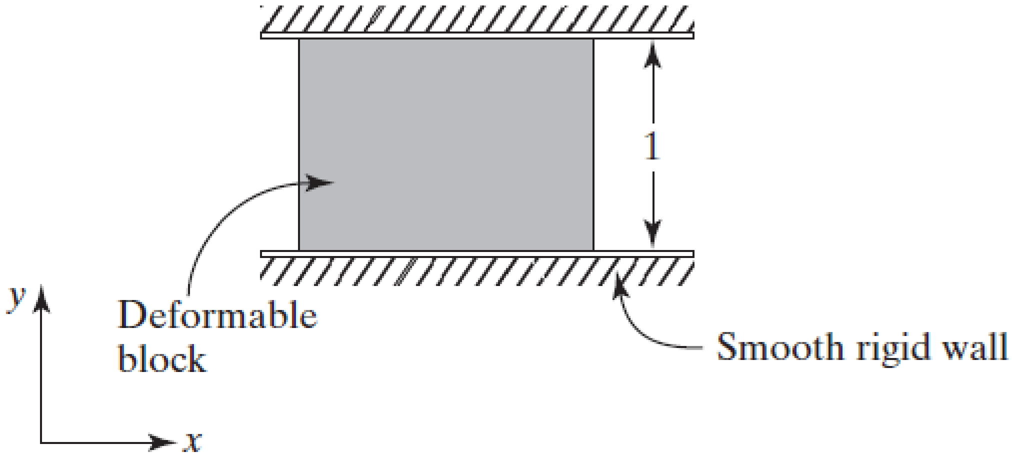
Solving for q_1 and q_2, we get

$$q_1 = 1.222 \text{ mm}$$

$$q_2 = 1.847 \text{ mm}$$

■

1.13



When the wall is smooth, $\sigma_x = 0$. ΔT is the temperature rise.

a) When the block is thin in the *z* direction, it corresponds to plane stress condition. The rigid walls in the *y* direction require $\varepsilon_y = 0$. The generalized Hooke's law yields the equations

$$\varepsilon_x = -\nu \frac{\sigma_y}{E} + \alpha \Delta T$$

$$\varepsilon_y = \frac{\sigma_y}{E} + \alpha \Delta T$$

From the second equation, setting $\varepsilon_y = 0$, we get $\sigma_y = -E\alpha\Delta T$. ε_x is then calculated using the first equation as $(1-\nu)\alpha\Delta T$.

b) When the block is very thick in the *z* direction, plain strain condition prevails. Now we have $\varepsilon_z = 0$, in addition to $\varepsilon_y = 0$. σ_z is not zero.

$$\varepsilon_x = -\nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} + \alpha \Delta T$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} + \alpha \Delta T = 0$$

$$\varepsilon_z = -\nu \frac{\sigma_y}{E} + \frac{\sigma_z}{E} + \alpha \Delta T = 0$$

From the last two equations, we get

$$\sigma_y = \frac{-E\alpha\Delta T}{1+\nu} \quad \sigma_z = -\frac{1+2\nu}{1+\nu} E\alpha\Delta T$$

ε_x is now obtained from the first equation. ■