1

Vector Spaces

1.1 INTRODUCTION

2. **(b)** $x = (2, 4, 0) + t(-5, -10, 0)$ **(d)** $x = (-2, -1, 5) + t(5, 10, 2)$

3. **(b)** $x = (3, -6, 7) + s(-5, 6, -11) + t(2, -3, -9)$
(d) $x = (1, 1, 1) + s(4, 4, 4) + t(-7, 3, 1)$

4. $(0, 0)$

1.2 VECTOR SPACES

2. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

4. **(b)** $\begin{pmatrix} 1 & -1 \\ 3 & -5 \\ 3 & 8 \end{pmatrix}$ **(d)** $\begin{pmatrix} 30 & -20 \\ -15 & 10 \\ -5 & -40 \end{pmatrix}$

(f) $-x^3 + 7x^2 + 4$ **(h)** $3x^5 - 6x^3 + 12x + 6$

5. $\begin{pmatrix} 8 & 3 & 1 \\ 3 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 9 & 1 & 4 \\ 3 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 17 & 4 & 5 \\ 6 & 0 & 0 \\ 4 & 1 & 0 \end{pmatrix}$

16. Yes **18.** No, (VS 1) fails. **19.** No, (VS 8) fails.

1.3 SUBSPACES

2. **(b)** $\begin{pmatrix} 0 & 3 \\ 8 & 4 \\ -6 & 7 \end{pmatrix}$ **(d)** $\begin{pmatrix} 10 & 2 & -5 \\ 0 & -4 & 7 \\ -8 & 3 & 6 \end{pmatrix}$ **(f)** $\begin{pmatrix} -2 & 7 \\ 5 & 0 \\ 1 & 1 \\ 4 & -6 \end{pmatrix}$

The trace is 12.

(h) $\begin{pmatrix} -4 & 0 & 6 \\ 0 & 1 & -3 \\ 6 & -3 & 5 \end{pmatrix}$

The trace is 2.

8. **(b)** No **(d)** Yes **(f)** No

9. $\mathsf{W}_1 \cap \mathsf{W}_3 = \{(0, 0, 0)\}$, $\mathsf{W}_1 \cap \mathsf{W}_4 = \mathsf{W}_1$,
$\mathsf{W}_3 \cap \mathsf{W}_4 = \{(a_1, a_2, a_3) \in \mathsf{R}^3 \colon a_1 = -11a_3 \text{ and } a_2 = -3a_3\}$

1.4 LINEAR COMBINATIONS AND SYSTEMS OF LINEAR EQUATIONS

2. **(b)** $(-2, -4, -3)$
(d) $\{x_3(-8, 3, 1, 0) + (-16, 9, 0, 2)\colon\ x_3 \in R\}$
(f) $(3, 4, -2)$

3. **(a)** $(-2, 0, 3) = 4(1, 3, 0) - 3(2, 4, -1)$
(b) $(1, 2, -3) = 5(-3, 2, 1) + 8(2, -1, -1)$
(d) No
(f) $(-2, 2, 2) = 4(1, 2, -1) + 2(-3, -3, 3)$

4. **(a)** $x^3 - 3x + 5 = 3(x^3 + 2x^2 - x + 1) - 2(x^3 + 3x^2 - 1)$
(b) No
(c) $-2x^3 - 11x^2 + 3x + 2 = 4(x^3 - 2x^2 + 3x - 1) - 3(2x^3 + x^2 + 3x - 2)$
(d) $x^3 + x^2 + 2x + 13 = -2(2x^3 - 3x^2 + 4x + 1) + 5(x^3 - x^2 + 2x + 3)$
(f) No

5. **(b)** No **(d)** Yes **(f)** No **(h)** No

11. The span of $\{x\}$ is $\{\mathit{0}\}$ if $x = \mathit{0}$ and is the line through the origin of R^3 in the direction of x if $x \neq \mathit{0}$.

17. if W is finite

1.5 LINEAR DEPENDENCE AND LINEAR INDEPENDENCE

2. **(b)** Linearly independent **(d)** Linearly dependent
(f) Linearly independent **(h)** Linearly independent
(j) Linearly dependent

10. $(1, 0, 0)$, $(0, 1, 0)$, $(1, 1, 0)$

1.6 BASES AND DIMENSION

2. **(b)** Not a basis **(d)** Basis

3. **(b)** Basis **(d)** Basis

4. No, $\dim(\mathsf{P}_3(R)) = 4$. **5.** No, $\dim(\mathsf{R}^3) = 3$.

8. $\{u_1, u_3, u_5, u_7\}$

10. **(b)** $12 - 3x$ **(d)** $2x^3 - x^2 - 6x + 15$

14. $\{(0, 1, 0, 0, 0), (0, 0, 0, 0, 1), (1, 0, 1, 0, 0), (1, 0, 0, 1, 0)\}$ and $\{(-1, 0, 0, 0, 1), (0, 1, 1, 1, 0)\}$; $\dim(\mathsf{W}_1) = 4$ and $\dim(\mathsf{W}_2) = 2$.

16. $\dim(\mathsf{W}) = \frac{1}{2}n(n+1)$

18. Let σ_j be the sequence such that

$$\sigma_j(i) = \begin{cases} 1 & i = j \\ 0 & i \neq j. \end{cases}$$

Then $\{\sigma_j\colon\ j = 1, 2, \ldots\}$ is a basis for the vector space in Example 5 of Section 1.2.

22. $\mathsf{W}_1 \subseteq \mathsf{W}_2$

23. **(a)** $v \in \mathsf{W}_1$ **(b)** $\dim(\mathsf{W}_2) = \dim(\mathsf{W}_1) + 1$

25. mn

27. If n is even, then $\dim(\mathsf{W}_1) = \dim(\mathsf{W}_2) = \dfrac{n}{2}$; and if n is odd, then $\dim(\mathsf{W}_1) = \dfrac{n+1}{2}$ and $\dim(\mathsf{W}_2) = \dfrac{n-1}{2}$.

32. **(a)** Take $\mathsf{W}_1 = \mathsf{R}^3$ and $\mathsf{W}_2 = \operatorname{span}(\{e_1\})$.
(b) Take $\mathsf{W}_1 = \operatorname{span}(\{e_1, e_2\})$ and $\mathsf{W}_2 = \operatorname{span}(\{e_3\})$.
(c) Take $\mathsf{W}_1 = \operatorname{span}(\{e_1, e_2\})$ and $\mathsf{W}_2 = \operatorname{span}(\{e_2, e_3\})$.

35. **(b)** $\dim(\mathsf{V}) = \dim(\mathsf{W}) + \dim(\mathsf{V}/\mathsf{W})$