5.1. For the beam cross-section shown in Fig. 5.33 determine whether the failure of the beam will be initiated by crushing of concrete or yielding of steel. Given:

$$f'_c = 4000 \text{ psi } (27.6 \text{ MPa}) \text{ for case (a)}, A_s = 10 \text{ in.}^2$$
$$f'_c = 7000 \text{ psi } (48.3 \text{ MPa}) \text{ for case (b)}, A_s = 5 \text{ in.}^2$$
$$f_y = 60{,}000 \text{ psi } (414 \text{ MPa})$$

Also determine whether the section satisfies ACI Code requirements.

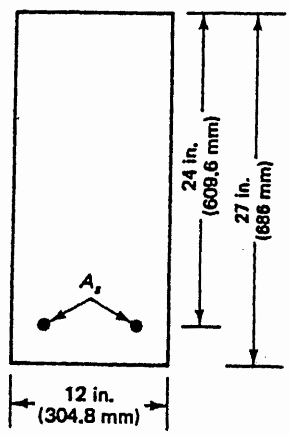

Figure 5.33

Solution:

a) $\beta_1 = 0.85$

$A_s = 10 \text{ in}^2$

$f_y = 60{,}000 \text{ psi}$

$f'_c = 4{,}000 \text{ psi}$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(10)(60{,}000)}{0.85(4{,}000)(12)} = 14.71 \text{ inches}$$

$$c = \frac{a}{\beta_1} = \frac{14.71}{0.85} = 17.31 \text{ inches}$$

$$\frac{c}{d_t} = \frac{17.31}{24} = 0.72 > 0.60$$

$\therefore$ Compression-controlled and concrete crushes before tension steel yields

b) $\beta_1 = 0.85 - 0.05\left(\frac{7{,}000-4{,}000}{1{,}000}\right) = 0.70$

$f'_c = 7{,}000$ psi

$A_s = 5\ in^2$

$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(5)(60{,}000)}{0.85(7{,}000)(12)} = 4.20$ inches

$c = \frac{a}{\beta_1} = \frac{4.2}{0.70} = 6.0$ inches

$\frac{c}{d_t} = \frac{6}{24} = 0.25 < 0.375$ $\therefore$ Tension - controlled

and steel yields before concrete crushes

$\rho = \frac{A_s}{bd} = \frac{5}{(12)(24)} = 0.017$ in/in.

$\rho_{min} = \max\left\{\frac{3\sqrt{7{,}000}}{60{,}000} = 0.0042,\ \frac{200}{60{,}000} = 0.0033\right\} = 0.0042$ in/in.

$0.017 > 0.0042$ $\therefore$ O.K. Satisfies ACI CODE

5.2. Calculate the nominal moment strength of the beam sections shown in Fig. 5.34. Given:

f'_c = 5000 psi (20.7 MPa) for case (a)

f'_c = 6000 psi (41.4 MPa) for case (b)

f_y = 60,000 psi (414 MPa)

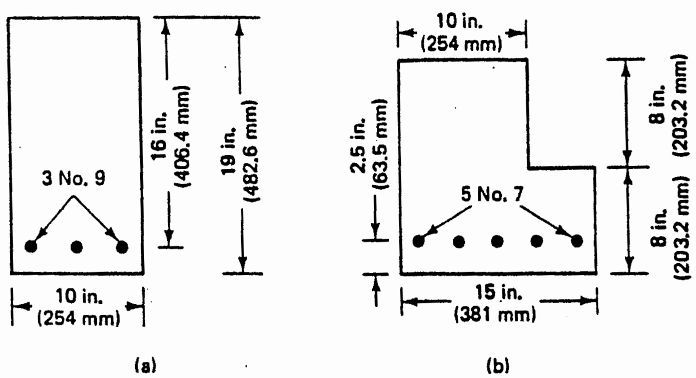

Figure 5.34

Solution:

a) $f'_c = 5{,}000$ psi

$\beta_1 = 0.80$

$A_s = 3 \times 0.9 = 3 \text{ in}^2$

$b = 10$ in.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3)(60{,}000)}{0.85(5{,}000)(10)} = 4.24 \text{ inches}$$

$$c = \frac{a}{\beta_1} = \frac{4.24}{0.80} = 5.29 \text{ in.}$$

$$\frac{c}{d_t} = \frac{5.29}{16} = 0.33 < 0.375$$

$\therefore$ Tension-Controlled

$\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{3}{(10)(16)} = 0.019$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} = 0.0035 < 0.019 \qquad \underline{\underline{\text{O.K.}}}$$

$$M_n = A_s f_y (d - a/2) = (3)(60{,}000)\left(16 - \frac{4.24}{2}\right) = 2{,}498{,}824 \text{ in-lb}$$

$$M_u = \phi M_n = 0.90\,(2{,}498{,}824) = 2{,}248{,}941 \text{ in-lb.}$$

b) $f'_c = 6{,}000$ psi
$\beta_1 = 0.75$
$A_s = 5 \times 0.7 = 3 \text{ in}^2$
$b = 10$ in (assuming neutral axis is w/in top 8 inches)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3)(60{,}000)}{0.85(6{,}000)(10)} = 3.53 \text{ in.}$$

$$c = a/\beta_1 = 4.71 \text{ in} < 8 \text{ in.} \quad \therefore\ b = 10 \text{ in.} \qquad \underline{\underline{\text{O.K.}}}$$

$$\frac{c}{d_t} = \frac{4.71}{13.5} = 0.35 < 0.375 \quad \therefore \text{ Tension-Controlled}$$

$$\phi = 0.90$$

$$\rho = \frac{A_s}{bd} = 0.022 \qquad \rho_{min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} = 0.0039 < 0.022 \quad \underline{\underline{\text{O.K.}}}$$

$$M_n = A_s f_y (d - a/2) = (3)(60{,}000)\left(13.5 - \frac{3.53}{2}\right) = 2{,}112{,}353 \text{ in-lb}$$

$$M_u = \phi M_n = 0.9\,M_n = 1{,}901{,}118 \text{ in-lb.}$$