CALCULUS AND ITS APPLICATIONS

Name:

Final Exam, Form A

1. Write an equation of the line with slope -5 and containing the point (2, 3).
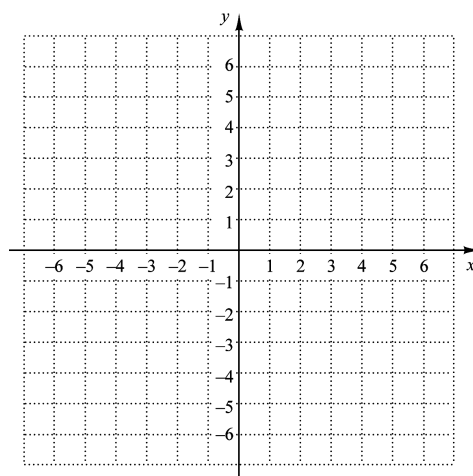
1. _____

2. For $f(x) = 7 - 3x^2$, find $f(x+h)$.

2. _____

3. **(a)** Graph:

$$f(x) = \begin{cases} -x^2 + 2, & \text{for } x \neq 1, \\ 0, & x = 1. \end{cases}$$

3. **(a)** _____

(b) Find $\lim\limits_{x \to 1} f(x)$.

(c) Find $f(1)$.

(d) Is f continuous at 1?

(b) _____

(c) _____

(d) _____

Find the limit, if it exists.

4. $\lim\limits_{x \to -2} (x^5 - 7x^2 + 3)$

4. _____

5. $\lim\limits_{x \to 0} \dfrac{9}{x}$

5. _____

6. $\lim\limits_{x \to 3} \dfrac{x-3}{x^2 + x - 12}$

6. _____

7. $\lim\limits_{x\to\infty} \dfrac{25x-6}{5x+4}$

7. _____

8. $\lim\limits_{x\to\infty} \dfrac{2x^2-5x}{7x^4+2x+1}$

8. _____

9. If $f(x)=7-3x^2$, find $f'(x)$ by determining $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

9. _____

Differentiate.

10. $y=-5x+8$

10. _____

11. $y=3x^4+5x^2+4$

11. _____

12. $y=x^{7/5}$

12. _____

13. $f(x)=x^{-11}$

13. _____

14. $f(x)=\sqrt[3]{3x^3+2}$

14. _____

15. $f(x)=\dfrac{3x+2}{x^2-8}$

15. _____

16. $y=\ln(x^2-6x+1)$

16. _____

17. $y=e^{5\ln x}$

17. _____

18. $y=11e^x$

18. _____

19. $y = e^{3x^2 + 2x}$

19. _____

20. $f(x) = \ln(e^x - 8)$

20. _____

21. For $y = 4x^4 + 3x^2$, find $\dfrac{d^2y}{dx^2}$.

21. _____

22. *Business: average cost.* Daniel's Digital Documents finds that the cost, in dollars, of producing x boxes of trifold brochures is given by

$$C(x) = 400 + 6\sqrt{x}.$$

Find the rate at which the average cost is changing when 25 boxes of trifold brochures have been produced.

22. _____

23. Differentiate implicitly to find $\dfrac{dy}{dx}$ if $\sqrt{x} + 8 = 2xy^2$.
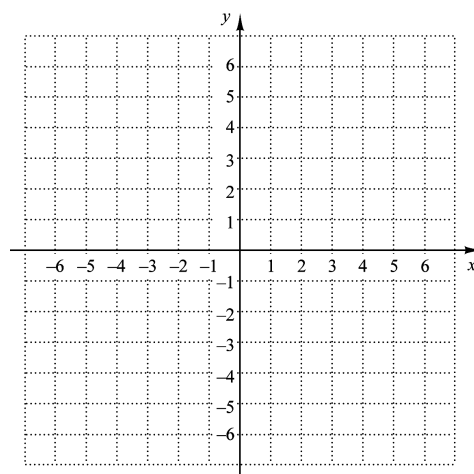
23. _____

24. Find an equation of the tangent line to the graph of $y = x^4 - \ln x + 1$ the point $(1, 2)$.

24. _____

Sketch the graph of each function. Identify the coordinates of any extrema and points of inflection. State where the function is increasing or decreasing, where it is concave up or down, and where any asymptotes occur.
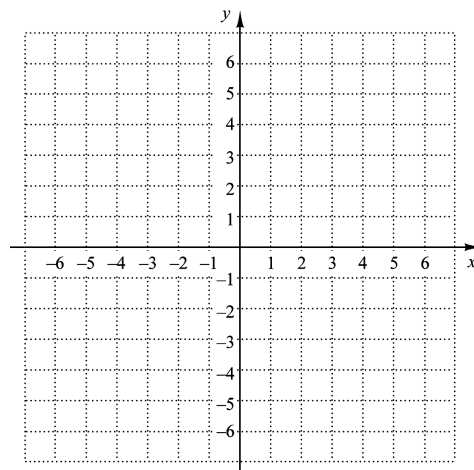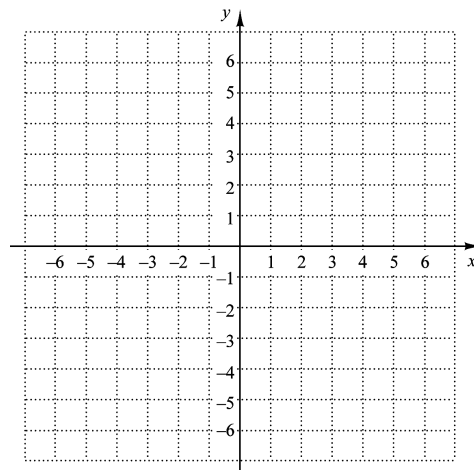
25. $f(x) = x^3 + 6x^2 + 9x + 1$

25. _____
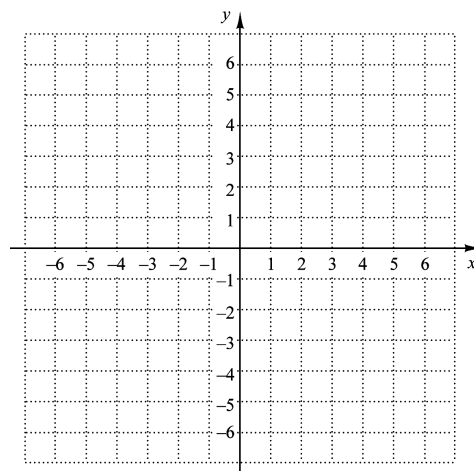
26. $f(x) = 8x^2 - x^4 - 8$

26. _____

27. $f(x) = \dfrac{20x}{5 + x^2}$

27. _____

28. $f(x) = \dfrac{4}{x^2 - 4}$

28. _____

**Find the absolute maximum and minimum values, if
they exist, over the indicated interval. If no interval is
indicated, use the real line.**

29. $f(x) = 3x^2 - 6x + 5$ 29. _____

30. $f(x) = 8x + 7$ 30. _____

                                                                                    31. _____
31. $f(x) = \dfrac{1}{3}x^3 + x^2 - 8x + 2; [-1, 3]$

32. *Business: maximizing profit.* Find the maximum profit   32. _____
       and the number of units x that must be produced and
       sold in order to yield maximum profit. Assume that
       $R(x)$ and $C(x)$ are the revenue and cost, in dollars,
       where x units are produced:

$$R(x) = 5x^2 + 12x + 150$$
$$C(x) = 5.4x^2 + 4x + 20.$$

33. *Business: minimizing inventory costs.* A department   33. _____
       store sells 600 Blu-ray players per year. It costs $3 to
       store one Blu-ray player for one year. To order Blu-ray
       players there is a fixed cost of $9 plus $2.50 for each
       Blu-ray player. How many times per year should the
       store reorder Blu-ray players, and in what lot size, in
       order to minimize inventory costs?

34. For $f(x) = 6x^2 + 2,$ $x = 2,$ and $\Delta x = 0.01,$ find Δy and   34. _____
       $f'(x)\Delta x.$

35. *Life Science: bacterial population.* The number of bacteria in a population increases at a rate of 12% per hour; that is,

$$\frac{dP}{dt} = 0.12P,$$

where P is the number of bacteria and t is the time, in hours.

(a) When $t = 0,$ there are 40,000 bacteria present. Find a function that satisfies the equation.

(b) What will the bacteria population be after 6 hr?

(c) What is the doubling time of the bacteria population?

35. (a) _____

(b) _____

(c) _____

36. *Economics: elasticity of demand.* Consider the demand function

$$q = D(x) = 100 - 8x$$

(a) Find the elasticity.

(b) Find the elasticity at $x = 5$ and state whether the demand is elastic or inelastic.

(c) Find the elasticity at $x = 8$ and state whether the demand is elastic or inelastic.

(d) At a price of \$5, will a small increase in price cause the total revenue to increase or decrease?

(e) Find the price for which the total revenue is a maximum.

36. (a) _____

(b) _____

(c) _____

(d) _____

(e) _____

Evaluate.

37. $\int 3x^8 dx$

37. _____

38. $\int_0^2 (4e^x + 3x)\,dx$

38. _____

39. $\int \dfrac{x}{(2x+3)^2}\,dx$ (Use Table 1.)

39. _____

40. $\int 12xe^{x^2+4}\,dx$ (Do not use Table 1.)

40. _____

41. $\int (x-6)\ln x\,dx$

41. _____

42. $\int \dfrac{23}{x}\,dx$ (Assume $x>0$.)

42. _____

43. $\int_0^9 \sqrt[2]{x}\,dx$

43. _____

44. Find the area under the graph of $y=-x^4+8x$ over the interval [0, 2].

44. _____

45. *Business: present value*. Find the present value of $140,000 due in 25 yr at 3.8% compounded continuously.

45. _____

46. *Accumulated present value of a continuous income stream.* A couple wants to have $50,000 in 10 yr for a down payment on a house. Find the amount they need to save, at $R(t)$ dollars per year, at 6.21% compounded continuously, to achieve the desired future value.

46. _____

47. Determine whether the improper integral is convergent or divergent, and calculate its value if it is convergent:

$$\int_4^\infty \frac{1}{x^6}\,dx.$$

47. _____

48. Given the probability density function   **48.** _____

$$f(x) = \frac{x^3}{2} \text{ over } [0, 2]$$

find $E(x)$.

49. Let x be a continuous random variable that is normally   **49.** _____
distributed with mean $\mu = 5$ and standard deviation
$\sigma = 1.6$. Using Table 2, find $P(4.6 \leq x \leq 6.2)$.

50. *Economics: supply and demand.* Given the demand   **50.** _____
and supply functions,

$$P = D(x) = (x - 25)^2 \text{ and } p = S(x) = x^2 + 20x + 65,$$

where p is the price per unit, in dollars, when x units
are sold, find the equilibrium point and the consumer's
surplus at the equilibrium point.

51. Find the volume of the solid of revolution generated by   **51.** _____
rotating the region under the graph of

$$e^{x/4} \text{ from } x = 0 \text{ to } x = 8.$$

about the x-axis.

52. Solve the differential equation $\dfrac{dy}{dx} = 2x^3 y$.   **52.** _____

53. Consider the data in the table.

Age of Business (in years)	3	5	6
Profit (in thousands of dollars)	4	8	11

(a) Find the regression line, $y = mx + b$.   **53. (a)** _____

(b) Use the regression line to predict the profit when   **(b)** _____
the business is 12 years old.

Given $f(x, y) = \dfrac{3x}{y} + e^x + y^2$, **find each of the following.**

54. f_y 54. _____

55. f_{xx} 55. _____

56. Find the relative maximum and minimum values of   56. _____
$f(x, y) - 2x^2 - 5y^2 + 6.$

57. Maximize $f(x, y) = 4x + 2y - x^2 + y^2 + 3$ subject to the   57. _____
constraint $x + 3y = 5.$

58. Evaluate: 58. _____

$$\int_{-1}^{1} \int_{0}^{x} (2y^2 + e^y)dy\, dx$$

59. Integrate: $\displaystyle\int \frac{2e^x}{1 + e^x}dx.$ 59. _____

60. Find $\displaystyle\lim_{x \to -3} \frac{x^4 - 5x^2 - 36}{x + 3}.$ 60. _____

61. Use your calculator to approximate the integral   61. _____

$$\int_{-\infty}^{\infty} \frac{4}{2 + x^2}dx.$$

Final Exam, Form B

1. Write an equation of the line with slope $-\frac{2}{3}$ and containing the point (6, 1).
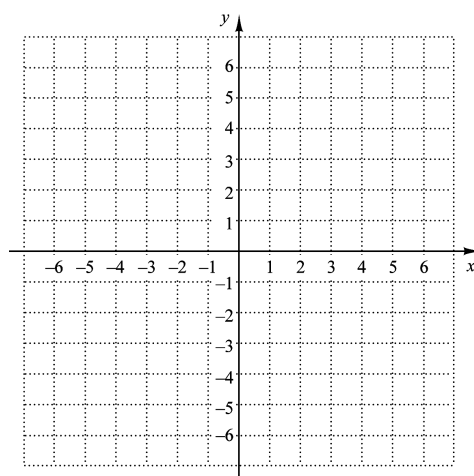
1. _____

2. For $f(x) = 2x^2 + 4$, find $f(x+h)$.

2. _____

3. (a) Graph:

$$f(x) = \begin{cases} x^2 - 5, & \text{for } x \neq 2, \\ 0, & \text{for } x = 2. \end{cases}$$

3. (a) _____

(b) Find $\lim_{x \to 2} f(x)$.

(b) _____

(c) Find $f(2)$.

(c) _____

(d) Is f continuous at 2?

(d) _____

Find the limit, if it exists.

4. $\lim_{x \to -3} (2x^4 + 5x - 6)$

4. _____

5. $\lim_{x \to 0} \dfrac{3}{x}$

5. _____

6. $\lim_{x \to -1} \dfrac{x+1}{x^2 - 4x - 5}$

6. _____

7. $\lim\limits_{x\to\infty} \dfrac{48x^3 - 11}{6x^3 - 7}$

7. _____

8. $\lim\limits_{x\to\infty} \dfrac{2x + 7}{x^3 - 6x^2}$

8. _____

9. If $f(x) = 2x^2 + 4$, find $f'(x)$ by determining

$\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$.

9. _____

Differentiate.

10. $y = 7x + 3$

10. _____

11. $y = x^5 + 3x - 12$

11. _____

12. $y = x^{5/6}$

12. _____

13. $f(x) = x^{-13}$

13. _____

14. $f(x) = \sqrt[4]{3x^2 + 8}$

14. _____

15. $f(x) = \dfrac{5x - 2}{3 - x^2}$

15. _____

16. $y = \ln(x^3 + 7x + 3)$

16. _____

17. $y = e^{3\ln x}$

17. _____

18. $y = 4e^x$

18. _____

19. $y = e^{x^2 + 8x}$

19. _____

20. $f(x) = \ln(-3 + e^x)$

20. _____

21. For $y = 13x^3 + 8x,$ find $\dfrac{d^2y}{dx^2}.$

21. _____

22. *Business: average cost.* Daniel's Digital Documents finds that the cost, in dollars, of producing x boxes of colored brochures is given by

$$C(x) = 200 + 5\sqrt{x}.$$

Find the rate at which the average cost is changing when 64 boxes of colored brochures have been produced.

22. _____

23. Differentiate implicitly to find $\dfrac{dy}{dx}$ if $x^2 + 4x = xy.$
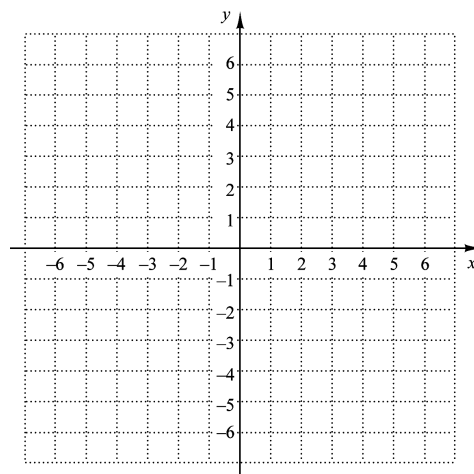
23. _____

24. Find an equation of the tangent line to the graph of $y = \ln x + x^2$ at the point $(1, 5).$

24. _____

Sketch the graph of each function. Identify the coordinates of any extrema and points of inflection. State where the function is increasing or decreasing, where it is concave up or down, and where any asymptotes occur.
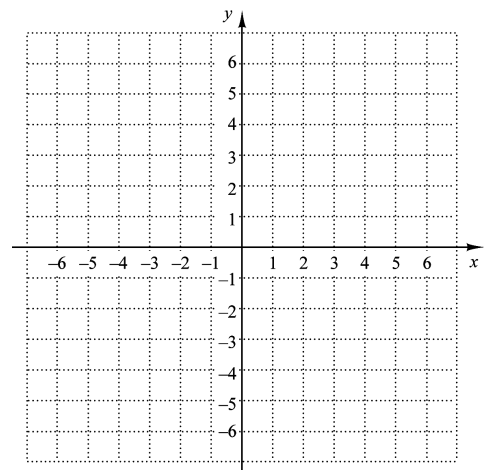
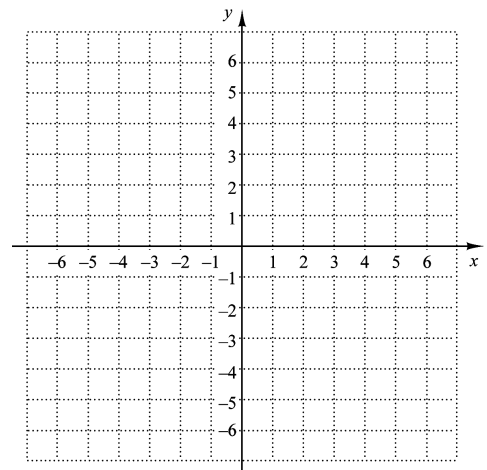**25.** $f(x) = 2x^3 + 12x^2 + 18x + 5$

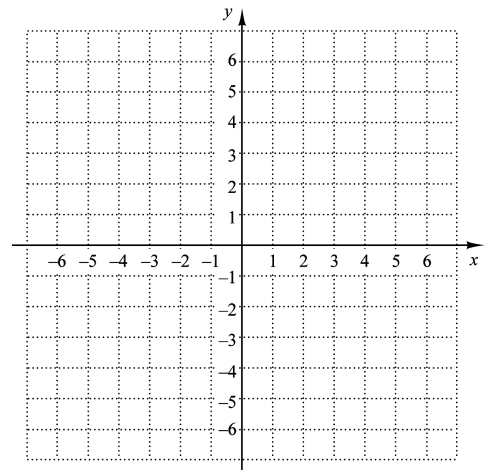25. _____

26. $f(x) = 8 - 8x^2 + x^4$

26. _____

27. $f(x) = \dfrac{12x}{x^2 + 4}$

27. _____

28. $f(x) = \dfrac{-10}{x^2 - 4}$

28. _____

Find the absolute maximum and minimum values, if they exist, over the indicated interval. If no interval is indicated, use the real line.

29. $f(x) = 6x^2 - 5x + 2$ **29.** _____

30. $f(x) = 7 - 2x$ **30.** _____

31. $f(x) = \dfrac{1}{3}x^3 + x^2 - 8x + 4; [0, 4]$ **31.** _____

32. *Business: maximizing profit.* Find the maximum profit   **32.** _____
and the number of units x that must be produced and
sold in order to yield the maximum profit. Assume that
$R(x)$ and $C(x)$ are the revenue and cost, in dollars,
where x units are produced:

$$R(x) = 3x^2 + 10x + 120$$
$$C(x) = 3.5x^2 + 6x + 20.$$

33. *Business: minimizing inventory costs.* A shoe store   **33.** _____
sells 800 pairs of boots per year. It costs $2 to store
one pair for one year. To order boots there is a fixed
cost of $8 plus $2 for each pair. How many time per
year should the store reorder sboots, and in what lot
size, in order to minimize inventory costs?

34. For $f(x) = 3x^2 - 2$, $x = 5$, and $\Delta x = 0.01$; find Δy and   **34.** _____
$f'(x)\Delta x$.

35. *Life Science: bacterial population.* The number of bacteria in a population increases at a rate of 10% per hour; that is,

$$\frac{dP}{dt} = 0.1P,$$

where P is the number of bacteria and t is the time, in hours.

(a) When $t = 0$, there are 10,000 bacteria present. Find a function that satisfies the equation.

(b) What will the bacteria population be after 3 hr?

(c) What is the doubling time of the bacteria population?

35. (a) _____

(b) _____

(c) _____

36. *Economics: elasticity of demand.* Consider the demand function

$$q = D(x) = 200 - 12x$$

(a) Find the elasticity.

(b) Find the elasticity at $x = 10$ and state whether the demand is elastic or inelastic.

(c) Find the elasticity at $x = 5$ and state whether the demand is elastic or inelastic.

(d) At a price of $10, will a small increase in price cause the total revenue to increase or decrease?

(e) Find the price for which the total revenue is a maximum.

36. (a) _____

(b) _____

(c) _____

(d) _____

(e) _____

Evaluate.

37. $\int 5x^7 \, dx$

37. _____

38. $\int_0^4 (3e^x + x) \, dx$

38. _____

39. $\int 12^x\,dx$ (Use Table 1.)

39. _____

40. $\int 2x^3 e^{x^4+2}\,dx$ (Do not use Table 1.)

40. _____

41. $\int (x+2)\ln x\,dx$

41. _____

42. $\int \dfrac{7}{x}\,dx$ (Assume $x>0$.)

42. _____

43. $\int_9^{16} \dfrac{1}{4}\sqrt{x}\,dx$

43. _____

44. Find the area under the graph of $y = x^3 + 2x$ over the interval [0, 2].

44. _____

45. *Business: present value.* Find the present value of $100,000 due in 20 yr at 4.1% compounded continuously.

45. _____

46. *Accumulated present value of a continuous income stream.* A small business owner wants to have $30,000 in 8 yr for equipment upgrades. Find the amount he needs to save, at $R(t)$ dollars per year, at 5.72% compounded continuously, to achieve the desired future value.

46. _____

47. Determine whether the improper integral is convergent or divergent, and calculate its value if it is convergent:

$$\int_2^\infty \frac{1}{x^3}\,dx.$$

47. _____

48. Given the probability density function

$$f(x) = \frac{x^3}{4} \text{ over } [0, 2]$$

find $E(x)$.

48. _____

49. Let x be a continuous random variable that is normally distributed with mean $\mu = 3$ and standard deviation $\sigma = 0.25$. Using Table 2, find $P(3.1 \le x \le 3.5)$.

49. _____

50. *Economics: supply and demand.* Given the demand and supply functions,

$$P = D(x) = (x-8)^2 \text{ and } p = S(x) = x^2 + 9x + 14,$$

where p is the price per unit, in dollars, when x units are sold, find the equilibrium point and the consumer's surplus at the equilibrium point.

50. _____

51. Find the volume of the solid of revolution generated by rotating the region under the graph of

$$y = e^{x/10} \text{ from } x = 2 \text{ to } x = 5$$

about the x-axis.

51. _____

52. Solve the differential equation $\dfrac{dy}{dx} = 6x^3 y.$

52. _____

53. Consider the data in the table.

Age of Business (in years)	2	3	7
Profit (in thousands of dollars)	5	8	12

(a) Find the regression line, $y = mx + b$.

(b) Use the regression line to predict the profit when the business is 10 years old.

53. (a) _____

(b) _____

Given $f(x, y) = 6y - 2x^3y + 4e^x$, **find each of the following.**

54. f_y

54. _____

55. f_{xx}

55. _____

56. Find the relative maximum and minimum values of $f(x, y) = -x^2 - 2y^2 + 8$.

56. _____

57. Maximize $f(x, y) = 40x + 10y - x^2 + 6$ subject to the constraint $x + 3y = 45$.

57. _____

58. Evaluate:

$$\int_{-1}^{0}\int_{1}^{2} \frac{1}{x} + y^2 dx\, dy.$$

58. _____

59. Find f_t: $f(x, t) = \dfrac{2t - 3x^2}{x + 2t}$.

59. _____

60. Integrate: $\displaystyle\int \frac{e^x}{e^x + 1} dx$.

60. _____

61. Use a calculator to approximate the area between

$$y = x^3 - 3x^2 - 5x \quad \text{and}$$
$$y = x - 2x^2,$$

on the interval [0, 3].

61. _____

Final Exam, Form C

1. Write an equation of the line with slope $\frac{3}{4}$ and containing the point $(2, -5)$.
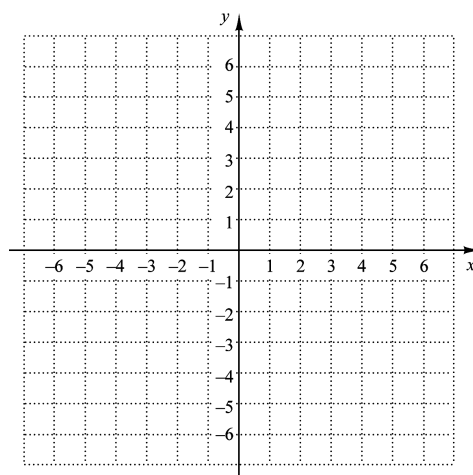
1. _____

2. For $f(x) = 7 - x^2$. find $f(x + h)$.

2. _____

3. **(a)** Graph:

$$f(x) = \begin{cases} x^2 + 1, & \text{for } x \neq 0, \\ 0, & \text{for } x = 0. \end{cases}$$

3. **(a)** _____

(b) Find $\lim\limits_{x \to 0} f(x)$.

(b) _____

(c) Find $f(0)$.

(c) _____

(d) Is f continuous at 0?

(d) _____

Find the limit, if it exists.

4. $\lim\limits_{x \to 4} (-x^4 + 7x - 1)$

4. _____

5. $\lim\limits_{x \to -5} \dfrac{7}{x + 5}$

5. _____

6. $\lim\limits_{x \to -2} \dfrac{x + 2}{x^2 - 4}$

6. _____

7. $\lim\limits_{x\to\infty}\dfrac{36x^2-13}{4x^2-7}$

7. _____

8. $\lim\limits_{x\to\infty}\dfrac{8x^2+13}{2x^4+7x}$

8. _____

9. If $f(x)=7-x^2$, find $f'(x)$ by determining $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

9. _____

Differentiate.

10. $y=-7x+6$

10. _____

11. $y=x^5-4x^3+1$

11. _____

12. $y=x^{2/5}$

12. _____

13. $f(x)=x^{-3}$

13. _____

14. $f(x)=\sqrt[5]{3x^5-1}$

14. _____

15. $f(x)=\dfrac{3x-1}{x^2+6}$

15. _____

16. $y=\ln(x^3-2x+7)$

16. _____

17. $y=e^{7\ln x}$

17. _____

18. $y=21e^x$

18. _____

19. $y = e^{5x^2 + x}$

19. _____

20. $f(x) = \ln(5 + e^x)$

20. _____

21. For $y = 5x^4 + 16x$, find $\dfrac{d^2 y}{dx^2}$.

21. _____

22. *Business: average cost.* Daniel's Digital Documents finds that the cost, in dollars, of producing x boxes of fourfold brochures is given by

$$C(x) = 500 + 8\sqrt{x}.$$

Find the rate at which the average cost is changing when 25 boxes of fourfold brochures have been produced.

22. _____

23. Differentiate implicitly to find $\dfrac{dy}{dx}$ if $4 - \sqrt{x} = 2xy$.
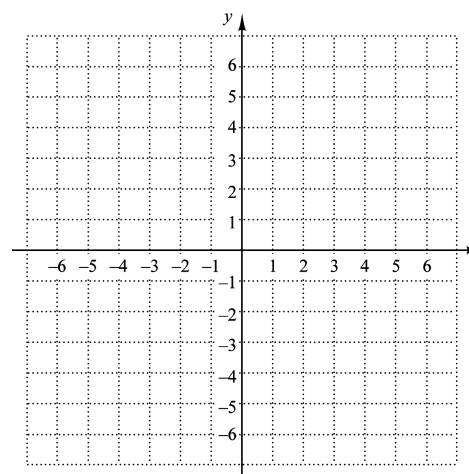
23. _____

24. Find an equation of the tangent line to the graph of $y = 4\ln x + x^2 - 2$ at the point $(1, -1)$.

24. _____

Sketch the graph of each function. Identify the coordinates of any extrema and points of inflection. State where the function is increasing or decreasing, where it is concave up or down, and where any asymptotes occur.
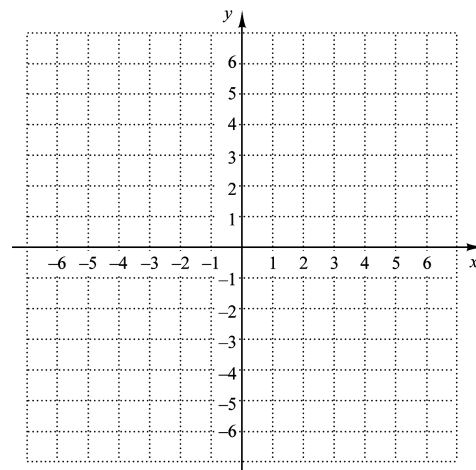
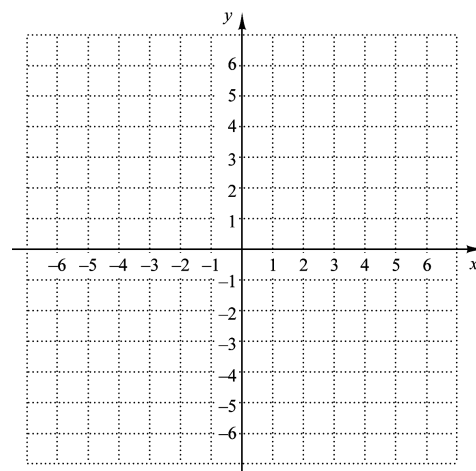**25.** $f(x) = x^3 + 6x^2 + 9x + 2$

25. _____
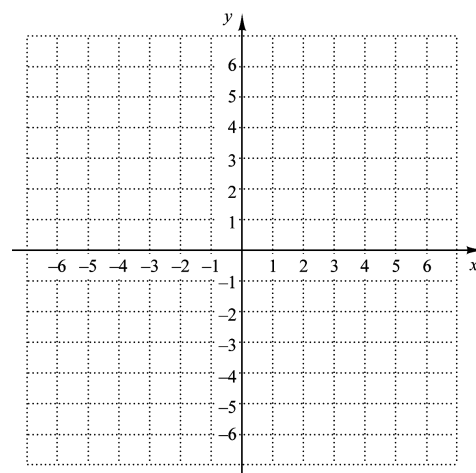
26. $f(x) = 4 - 5x^2 + x^4$

26. _____

27. $f(x) = \dfrac{6x}{3 + x^2}$

27. _____

28. $f(x) = \dfrac{12}{x^2 - 4}$

28. _____

Find the absolute maximum and minimum values, if they exist, over the indicated interval. If no interval is indicated, use the real line.

29. $f(x) = -4x^2 + 6x - 5$ **29.** _____

30. $f(x) = \dfrac{1}{x} - 1$ **30.** _____

31. $f(x) = \dfrac{2}{3}x^3 - x^2 - 4x + 6; [-3, 3]$ **31.** _____

32. *Business: maximizing profit.* Find the maximum profit and the number of units x that must be produced and sold in order to yield maximum profit. Assume that $R(x)$ and $C(x)$ are the revenue and cost, in dollars, where x units are produced:

$$R(x) = 6x^2 + 8x + 120$$
$$C(x) = 6.4x^2 + 4x + 20.$$

     32. _____

33. *Business: minimizing inventory costs.* A department store sells 1600 packages of ball-point pens per year. It costs $0.10 to store one package for one year. To order the pens there is a fixed cost of $5 plus $0.05 for each package. How many times per year should the store reorder the ball-point pens, and in what lot size, in order to minimize inventory costs?

     33. _____

34. For $f(x) = 3x^2 + 4$, $x = 5$, and $\Delta x = 0.01$; find Δy and $f'(x)\Delta x$.

     34. _____

35. *Social science: population growth.* The population of a
town is increasing at a rate of 3% per year; that is

$$\frac{dP}{dt} = 0.03P,$$

where P is the population of the town and t is the time,
in years, and $t = 0$ corresponds to 2008.

(a) When $t = 0$, the town's population was 14,000.
Find a function that satisfies the equation.

(b) What will the population be after 10 yr?

(c) What is the doubling time of the population?

35. (a) _____

(b) _____

(c) _____

36. *Economics: elasticity of demand.* Consider the demand
function

$$q = D(x) = 500 - 40x$$

(a) Find the elasticity.

(b) Find the elasticity at $x = 4$ and state whether the
demand is elastic or inelastic.

(c) Find the elasticity at $x = 12$ and state whether the
demand is elastic or inelastic.

(d) At a price of $12, will a small increase in price
cause the total revenue to increase or decrease?

(e) Find the price for which the total revenue is a
maximum.

36. (a) _____

(b) _____

(c) _____

(d) _____

(e) _____

Evaluate.

37. $\int 8x^3 dx$.

37. _____

38. $\int_{-1}^{0} (2 + 5e^x) dx$

38. _____

39. $\int \dfrac{1}{x(3x+4)} dx$ (Use Table 1.)

39. _____

40. $\int x^3 e^{x^4+3} dx$ (Do not use Table 1.)

40. _____

41. $\int (x-2) \ln x \, dx$

41. _____

42. $\int \dfrac{3}{x} dx$ (Assume $x > 0$.)

42. _____

43. $\int_1^4 \dfrac{1}{\sqrt{x}} dx$

43. _____

44. Find the area under the graph of $y = 5x - x^2$ over the interval [2, 3].

44. _____

45. *Business: present value.* Find the present value of $50,000 due in 10 yr at 5.2% compounded continuously.

45. _____

46. *Accumulated present value of a continuous income stream.* A young man wants to have $40,000 in 10 yr for a down payment on a house. Find the amount he needs to save, at $R(t)$ dollars per year, at 6.03% compounded continuously, to achieve the desired future value.

46. _____

47. Determine whether the improper integral is convergent or divergent, and calculate its value if it is convergent:

$$\int_2^\infty \dfrac{1}{x^4} dx.$$

47. _____

48. Given the probability density function   48. _____

$$f(x) = \frac{4x^3}{125} \text{ over } [0, 5]$$

find $E(x)$.

49. Let x be a continuous random variable that is normally   49. _____
distributed with mean $\mu = 10$ and standard deviation
$\sigma = 0.5$. Using Table 2, find $P(9.15 \le x \le 10)$.

50. *Economics: supply and demand.* Given the demand   50. _____
and supply functions,

$$P = D(x) = (x - 15)^2 \text{ and } p = S(x) = x^2 + 8x + 35,$$

where p is the price per unit, in dollars, when x units
are sold, find the equilibrium point and the consumer's
surplus at the equilibrium point.

51. Find the volume of the solid of revolution generated by   51. _____
rotating the region under the graph of

$$y = e^{-x} \text{ from } x = 1 \text{ to } x = 3$$

about the x-axis.

                                                                52. _____
52. Solve the differential equation $\dfrac{dy}{dx} = 4x^5 y.$

53. Consider the data in the table.

Age of Business (in years)	3	5	7
Profit (in thousands of dollars)	5	10	12

(a) Find the regression line, $y = mx + b$.   53. **(a)** _____

(b) Use the regression line to predict the profit when   **(b)** _____
the business is 15 years old.

Given $f(x, y) = 6x^2 + xy^5 - 7e^y$, **find each of the following.**

54. f_y 54. _____

55. f_{xx} 55. _____

56. Find the relative maximum and minimum values of   56. _____
      $f(x, y) = 2x^2 + 5y^2 + 6$.

57. Maximize $f(x, y) = 12x + 6y - x^2 + y^2 + 2$ subject to   57. _____
      the constraint $x - 4y = -12$.

58. Evaluate: 58. _____

$$\int_{-1}^{1}\int_{0}^{1}(e^y + 2)\,dx\,dy.$$

59. Differentiate: $f(x) = e^x \ln(xe^x)$.   59. _____

60. Integrate: $\int \dfrac{e^x}{5 + e^x}\,dx$.   60. _____

61. Use your calculator to graph $f(x) = 3x^3 - 2x^4$. Sketch   61. _____
      the graph.

Name:

Final Exam, Form D

1. Write an equation of the line with slope -3 and containing the point $\left(\frac{1}{2}, -4\right)$.
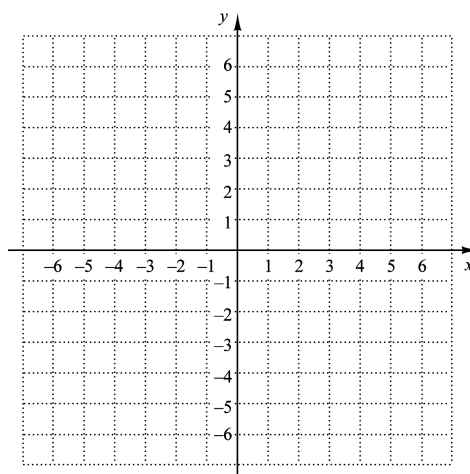
1. _____

2. For $f(x) = 8 + 5x^2$, find $f(x+h)$.

2. _____

3. (a) Graph:

$$f(x) = \begin{cases} x^2 - 4, & \text{for } x \neq 2, \\ 4, & \text{for } x = 2. \end{cases}$$

3. (a) _____

(b) Find $\lim\limits_{x \to 2} f(x)$.

(c) Find $f(2)$.

(d) Is f continuous at 2?

(b) _____

(c) _____

(d) _____

Find the limit, if it exists.

4. $\lim\limits_{x \to -2} 3x^4 - x + 10$

4. _____

5. $\lim\limits_{x \to -4} \dfrac{5}{x+4}$

5. _____

6. _____

6. $\displaystyle\lim_{x\to-3}\frac{x+3}{x^2-x-12}$

7. $\displaystyle\lim_{x\to\infty}\frac{15x^3+4}{5x^3-6}$

7. _____

8. _____

8. $\displaystyle\lim_{x\to\infty}\frac{2x+7}{9x^3-12x}$

9. If $f(x)=8+5x^2$, find $f'(x)$ by

9. _____

determining $\displaystyle\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$.

Differentiate.

10. $y=-11x-4$

10. _____

11. $y=x^7+5x^3-3$

11. _____

12. $y=x^{3/7}$

12. _____

13. $f(x)=x^{-2}$

13. _____

14. $f(x)=\sqrt[3]{2x^8+5}$

14. _____

15. $f(x)=\dfrac{3x+4}{x^2-2}$

15. _____

16. $y=\ln(x^2+8x+5)$

16. _____

17. $y=8e^{\ln x}$

17. _____

18. $y=-2e^x$

18. _____

19. $y = e^{3x^2 - 2x}$

19. _____

20. $f(x) = \ln(4 + e^x)$

20. _____

21. For $y = 10x^3 + 4x$, find $\dfrac{d^2y}{dx^2}$.

21. _____

22. *Business: average cost*. Daniel's Digital
Documents finds that the cost, in dollars, of
producing x boxes of accordion brochures
is given by

$$C(x) = 600 + 7\sqrt{x}.$$

Find the rate at which the average cost is
changing when 4 boxes of accordion
brochures have been produced.

22. _____

23. Differentiate implicitly to find $\dfrac{dy}{dx}$ if

$x^4 - 2xy^3 = 3.$

23. _____

24. Find an equation of the tangent line to the
graph of $y = x^2 - 5 + 3e^x$ at the point
$(0, -2)$.

24. _____

**Sketch the graph of each function. Identify
the coordinates of any extrema and points of
inflection. State where the function is
increasing or decreasing, where it is concave
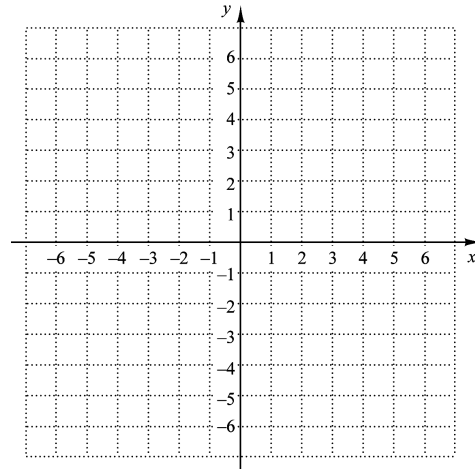up or down, and where any asymptotes
occur.**

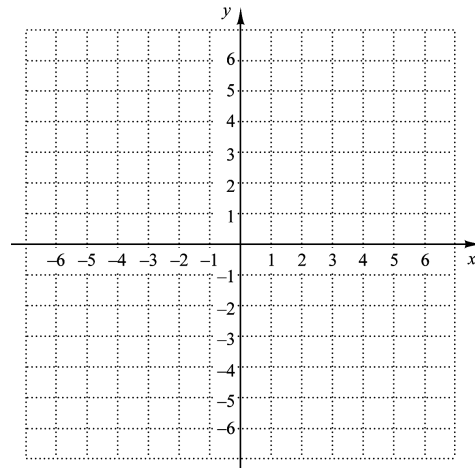**25.** $f(x) = x^3 - 6x^2 + 9x - 8$

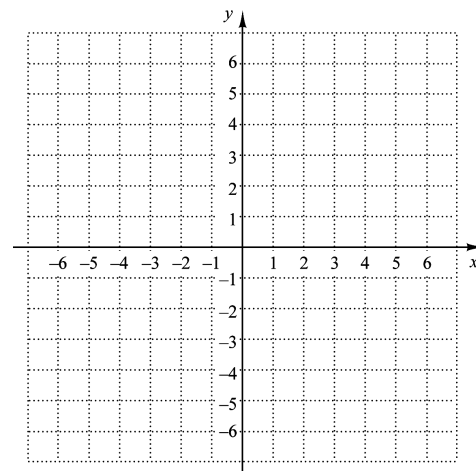**25.** _____

26. $f(x) = -2x^2 - 8 + x^4$

26. _____

27. $f(x) = \dfrac{-6x}{x^2 + 4}$

27. _____

28. $f(x) = \dfrac{-8}{x^2 - 4}$

28.
_____

Find the absolute maximum and minimum values, if they exist, over the indicated interval. If no interval is indicated, use the real line.

29. $f(x) = -9x^2 + 6x - 1$

29. _____

30. $f(x) = 3x + 1$

30. _____

31. $f(x) = \dfrac{2}{3}x^3 - 4x^2 - 10x + 7; [1, 10]$

31. _____

32. *Business: maximizing profit.* Find the maximum profit and the number of units x that must be produced and sold in order to yield the maximum profit. Assume that $R(x)$ and $C(x)$ are the revenue and cost, in dollars, where x units are produced:

$$R(x) = 5x^2 + 12x + 130$$
$$C(x) = 5.2x^2 + 4x + 30.$$

32. _____

33. *Business: minimizing inventory costs.* An outdoor sports store sells 300 lanterns per year. It costs $1 to store one lantern for one year. To order lanterns there is a fixed cost of $6 plus $0.50 for each lantern. How many times per year should the store reorder lanterns, and in what lot size, in order to minimize inventory costs?

33. _____

34. For $f(x) = 4x^2 + 3$, $x = 3$, and $\Delta x = 0.01$, find Δy and $f'(x)\Delta x$.

34. _____

35. *Social Science: population growth.* The
population of a town is increasing at a rate
of 8% per year; that is

$$\frac{dP}{dt} = 0.08P,$$

where P is the population of the town and t
is the time, in years, and $t = 0$ corresponds
to 2008.

(a) When $t = 0,$ the town's population was
400. Find a function that satisfies the
equation.

(b) What will the population be after 10 yr?

(c) What is the doubling time of the
population?

35. (a) _____

(b) _____

(c) _____

36. *Economics: elasticity of demand.* Consider
the demand function

$$q = D(x) = 300 - 60x,$$

(a) Find the elasticity.

(b) Find the elasticity at $x = 1.75$ and state
whether the demand is elastic or
inelastic.

(c) Find the elasticity at $x = 3$ and state
whether the demand is elastic or
inelastic.

(d) At a price of $3, will a small increase in
price cause the total revenue to increase
or decrease?

(e) Find the price for which the total
revenue is a maximum.

36. (a) _____

(b) _____

(c) _____

(d) _____

(e) _____

Evaluate.

37. $\int 4x^5 dx$

37. _____

38. $\int_0^2 (2e^x + 2)\, dx$

38. _____

39. $\int \dfrac{1}{x(3x+1)}dx$ (Use Table 1.)

39. _____

40. $\int 15x^4 e^{x^5+2}dx$ (Do not use Table 1.)

40. _____

41. $\int (x+5)\ln x \, dx$

41. _____

42. $\int \dfrac{-11dx}{x}$ (Assume $x>0$.)

42. _____

43. $\int_1^9 \dfrac{2}{\sqrt{x}} dx$

43. _____

44. Find the area under the graph of $y = 5x - x^3$ over the interval $[0, 2]$.

44. _____

45. *Business: present value.* Find the present value of \$100,000 due in 20 yr at 3.7% compounded continuously.

45. _____

46. *Accumulated present value of a continuous income stream.* A small business owner wants to have \$25,000 in 5 yr for equipment upgrades. Find the amount she needs to save, at $R(t)$ dollars per year, at 4.76% compounded continuously, to achieve the desired future value.

46. _____

47. Determine whether the improper integral is convergent or divergent, and calculate its value if it is convergent:

$$\int_4^\infty \frac{1}{x^2}dx.$$

47. _____

48. Given the probability density function   48. _____

$$f(x) = \frac{x^2}{8} \text{ over } [0, 3]$$

find $E(x)$.

49. Let x be a continuous random variable that   49. _____
is normally distributed with mean $\mu = 20$
and standard deviation $\sigma = 2.5$. Using
Table 2, find $P(18 \le x \le 21)$.

50. *Economics: supply and demand.* Given the   50. _____
demand and supply functions,

$$P = D(x) = (x - 11)^2 \text{ and } p = S(x) = x^2 + 6x + 9,$$

where p is the price per unit, in dollars,
when x units are sold, find the equilibrium
point and the consumer's surplus at the
equilibrium point.

51. Find the volume of the solid of revolution   51. _____
generated by rotating the region under the
graph of

$$y = e^{2x} \text{ from } x = 0 \text{ to } x = 5$$

about the x-axis.

52. Solve the differential equation $\dfrac{dy}{dx} = -x^3 y.$

                                                       52. _____

53. Consider the data in the table.

Age of Business (in years)	2	4	8
Profit (in thousands of dollars)	6	9	15

(a) Find the regression line, $y = mx + b$.   53. (a) _____

(b) Use the regression line to predict the   (b) _____
profit when the business is 15 years old.

Given $f(x, y) = 2xy + y^2 + 3e^x$, find each of the following.

54. f_y

54. _____

55. f_{xx}

55. _____

56. Find the relative maximum and minimum values of $f(x, y) = 2x^2 + y^2 + 6$.

56. _____

57. Maximize $f(x, y) = 2x - 3y + x^2 - y^2 + 6$ subject to the constraint $x + 2y = 11$.

57. _____

58. Evaluate:

$$\int_{-1}^{1}\int_{0}^{1}(3e^y + x)\,dx\,dy.$$

58. _____

59. Differentiate: $f(x) = \dfrac{\ln(e^x x)}{e^x}$.

59. _____

60. Find $\lim\limits_{x \to 3} \dfrac{x^3 - 27}{x - 3}$.

60. _____

61. Use a calculator to approximate the area between the curves.

$$y = x^3 - 3x^2 - 4x \text{ and}$$
$$y = x^2 - 4x$$

on the interval $[0, 4]$.

61. _____

Final Exam, Form E

1. Write an equation of the line with slope $\frac{1}{3}$ and containing the point $(-2, 1)$.
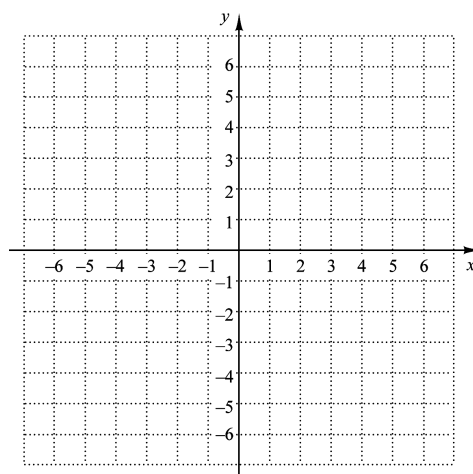
1. _____

2. For $f(x) = 9x^2 - 1$, find $f(x + h)$.

2. _____

3. (a) Graph:

$$f(x) = \begin{cases} -x^2 + 1, & \text{for } x \neq -1, \\ -1, & \text{for } x = -1. \end{cases}$$

3. (a) _____

(b) _____

(b) Find $\lim\limits_{x \to -1} f(x)$.

(c) _____

(c) Find $f(-1)$.

(d) _____

(d) Is f continuous at -1?

Find the limit, if it exists.

4. $\lim\limits_{x \to -1} (5x^4 + 7x^2 - 9)$

4. _____

5. $\lim\limits_{x \to 7} \dfrac{7}{x - 7}$

5. _____

6. $\lim\limits_{x \to -3} \dfrac{x - 3}{x^2 + x - 12}$

6. _____

7. $\lim\limits_{x\to\infty}\dfrac{8x^2+5}{2x^2-3}$

7. _____

8. $\lim\limits_{x\to\infty}\dfrac{5x^2-11}{x^4-3x^3+1}$

8. _____

9. If $f(x)=9x^2-1$, find $f'(x)$ by determining

   $\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}$.

9. _____

Differentiate.

10. $y=13x-7$

10. _____

11. $y=5x^4+2x-6$

11. _____

12. $y=x^{5/8}$

12. _____

13. $f(x)=x^{-12}$

13. _____

14. $f(x)=\sqrt[4]{x^7+4}$

14. _____

15. $f(x)=\dfrac{3x-4}{x^2+5}$

15. _____

16. $y=\ln(x^5-3x^2)$

16. _____

17. $y=10e^{\ln x}$

17. _____

18. $y=-8e^x$

18. _____

19. $y = e^{x^3 + 3x}$ 19. _____

20. $f(x) = \ln(e^x - 7)$ 20. _____

21. For $y = 2x^5 - 3x^2$, find $\dfrac{d^2 y}{dx^2}$.   21. _____

22. *Business: average cost*. Daniel's Digital Documents   22. _____
finds that the cost, in dollars, of producing x boxes of
bifold brochures is given by

$$C(x) = 200 + 11\sqrt{x}.$$

Find the rate at which the average cost is changing
when 25 boxes of bifold brochures have been
produced.

23. Differentiate implicitly to find $\dfrac{dy}{dx}$ if $x^{10} - 5x^2 y^2 = 4$.   23. _____
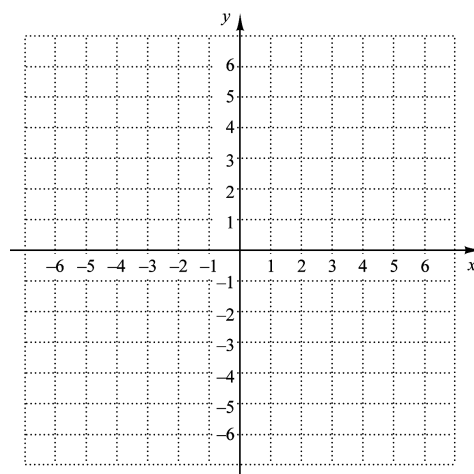
24. Find an equation of the tangent line to the graph of   24. _____
$y = 2e^x + x^3 - 4$ the point $(0, -2)$.

**Sketch the graph of each function. Identify the
coordinates of any extrema and points of inflection.
State where the function is increasing or decreasing,
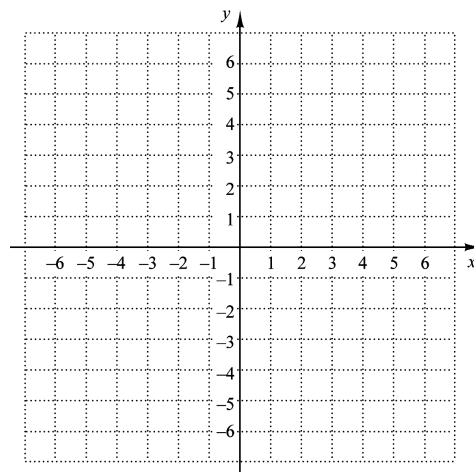where it is concave up or down, and where any
asymptotes occur.**

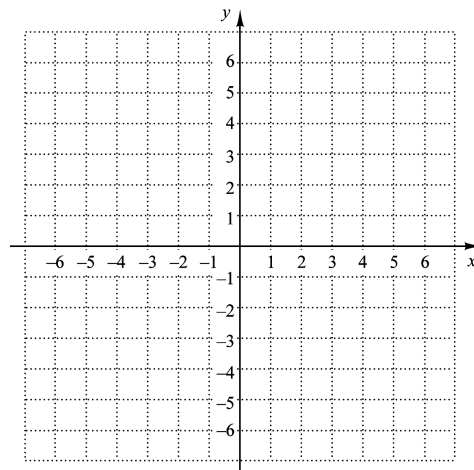**25.** $f(x) = -x^3 - 6x^2 - 9x + 5$ 25. _____
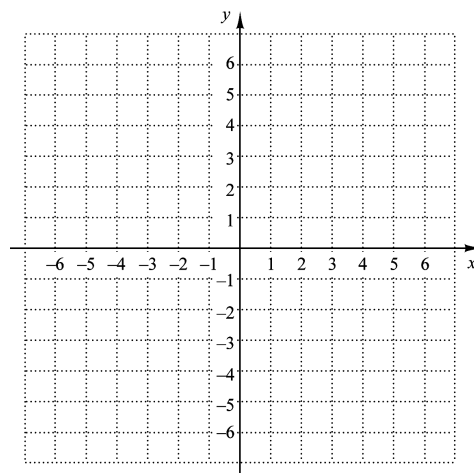
26. $f(x) = x^4 - 6$

26. _____

27. $f(x) = \dfrac{-3x}{x^2 + 2}$

27. _____

28. $f(x) = \dfrac{12}{x^2 - 9}$

28. _____

Find the absolute maximum and minimum values, if they exist, over the indicated interval. If no interval is indicated, use the real line.

29. $f(x) = 5x^2 + 8x - 6$

29. _____

30. $f(x) = 13 - 2x$

30. _____

31. $f(x) = \dfrac{1}{3}x^3 - 2x^2 + 3x + 2; [-2, 2]$

31. _____

32. *Business: maximizing profit.* Find the maximum profit and the number of units x that must be produced and sold in order to yield maximum profit. Assume that $R(x)$ and $C(x)$ are the revenue and cost, in dollars, where x units are produced:

$$R(x) = 3x^2 + 12x + 100$$
$$C(x) = 3.1x^2 + 6x + 20.$$

32. _____

33. *Business: minimizing inventory costs.* An office supply store sells 400 boxes of printer cartridges per year. It costs $2 to store one box for one year. To order printer cartridges there is a fixed cost of $4 plus $1.25 for each box. How many times per year should the store reorder printer cartridges, and in what lot size, in order to minimize inventory costs?

33. _____

34. For $f(x) = 3x^2 - 2$, $x = 5$, and $\Delta x = 0.01$; find Δy and $f'(x)\Delta x$.

34. _____

35. *Social Science: population growth.* The population of a town is increasing at a rate of 6% per year; that is

$$\frac{dP}{dt} = 0.06P,$$

where P is the population of the town and t is the time, in years, and $t = 0$ corresponds to 2008.

(a) When $t = 0$, the town's population was 8500. Find a function that satisfies the equation.

(b) What will the population be after 23 yr?

(c) What is the doubling time of the population?

35. (a) _____

(b) _____

(c) _____

36. *Economics: elasticity of demand.* Consider the demand function

$$q = D(x) = 50 - 2x$$

(a) Find the elasticity.

(b) Find the elasticity at $x = 15$ and state whether the demand is elastic or inelastic.

(c) Find the elasticity at $x = 12$ and state whether the demand is elastic or inelastic.

(d) At a price of $12, will a small increase in price cause the total revenue to increase or decrease?

(e) Find the price for which the total revenue is a maximum.

36. (a) _____

(b) _____

(c) _____

(d) _____

(e) _____

Evaluate.

37. $\int 2x^6 dx$.

37. _____

38. $\int_1^2 (4e^x + 6x)\, dx$

38. _____

39. $\int \dfrac{1}{\sqrt{x^2+9}}\,dx$ (Use Table 1.)

39. _____

40. $\int x^2 e^{x^3+1}\,dx$ (Do not use Table 1.)

40. _____

41. $\int (x-8)\ln x\,dx$

41. _____

42. $\int \dfrac{5}{x}\,dx$ (Assume $x>0$.)

42. _____

43. $\int_1^8 2\sqrt[3]{x}\,dx$

43. _____

44. Find the area under the graph of $y=x^3-4x$ over the interval $[-2,0]$.

44. _____

45. *Business: present value.* Find the present value of $200,000 due in 30 yr at 5.4% compounded continuously.

45. _____

46. *Accumulated present value of a continuous income stream.* A young woman wants to have $42,000 in 7 yr for a down payment on a house. Find the amount she needs to save, at $R(t)$ dollars per year, at 5.25% compounded continuously, to achieve the desired future value.

46. _____

47. _____

47. Determine whether the improper integral is convergent or divergent, and calculate its value if it is convergent.

$$\int_3^\infty \frac{1}{x^5}\,dx.$$

48. Given the probability density function   **48.** _____

$$f(x) = \frac{2x^2}{27} \text{ over } [0, 3]$$

find $E(x)$.

49. Let x be a continuous random variable that is normally   **49.** _____
distributed with mean $\mu = 12$ and standard deviation
$\sigma = 1.5$. Using Table 2, find $P(9 \le x \le 12.6)$.

50. *Economics: supply and demand.* Given the demand   **50.** _____
and supply functions,

$$P = D(x) = (x - 18)^2 \text{ and } p = S(x) = x^2 + 3x + 51,$$

where p is the price per unit, in dollars, when x units
are sold, find the equilibrium point and the consumer's
surplus at the equilibrium point.

51. Find the volume of the solid of revolution generated by   **51.** _____
rotating the region under the graph of

$$y = e^{x/2} \text{ from } x = 0 \text{ to } x = 10$$

about the x-axis.

52. _____

52. Solve the differential equation $\dfrac{dy}{dx} = -8x^7 y$.

53. Consider the data in the table.

Age of Business (in years)	12	15	18
Profit (in thousands of dollars)	25	30	36

53. (a) _____

(a) Find the regression line, $y = mx + b$.

(b) _____

(b) Use the regression line to predict the profit when
the business is 25 years old.

Given $f(x, y) = 2y^2 + xy^3 + 2e^x$, **find each of the following.**

54. f_y

54. _____

55. f_{xx}

55. _____

56. Find the relative maximum and minimum values of $f(x, y) = 3x^2 - 5y^2 + 4$.

56. _____

57. Minimize $f(x, y) = 4x + 2y - x^2 - 3y^2 + 2$ subject to the constraint $x + 2y = 5$.

57. _____

58. Evaluate:

$$\int_{-1}^{0} \int_{0}^{2} (e^x - 3)\, dy\, dx.$$

58. _____

59. Find f_x: $f(x, t) = \dfrac{4x^3 - t}{xt + 2}$.

59. _____

60. Integrate: $\displaystyle\int \frac{7e^x}{7 + e^x}\, dx$.

60. _____

61. Use a calculator to graph $f(x) = 8x^3 - 3x^5$. Then sketch the graph.

61. _____

Final Exam, Form F

1. Write an equation of the line with slope 7 and containing the point $(-3, 4)$.
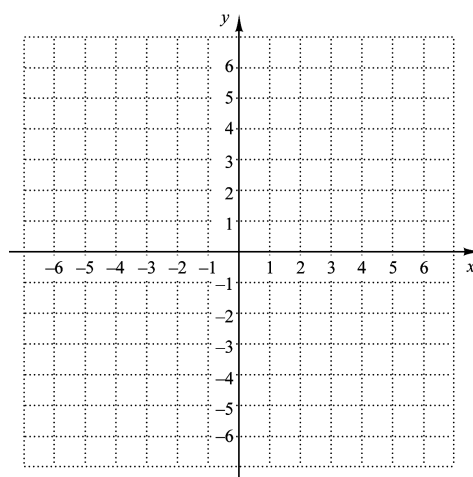
1. _____

2. For $f(x) = 3 - 6x^2$, find $f(x+h)$.

2. _____

3. **(a)** Graph:

$$f(x) = \begin{cases} 4 - x^2, & \text{for } x \neq 1, \\ 1, & \text{for } x = 1. \end{cases}$$

3. **(a)** _____

(b) Find $\lim_{x \to 1} f(x)$.

(c) Find $f(1)$.

(d) Is f continuous at 1?

(b) _____

(c) _____

(d) _____

Find the limit, if it exists.

4. $\lim_{x \to -3} (x^5 - 4x + 9)$

4. _____

5. $\lim_{x \to 3} \dfrac{10}{x - 3}$

5. _____

6. $\lim_{x \to -5} \dfrac{x + 5}{x^2 + 3x - 10}$

6. _____

7. $\lim\limits_{x\to\infty}\dfrac{26x+3}{13x-1}$

7. _____

8. $\lim\limits_{x\to\infty}\dfrac{12x^2+1}{5x^6-x+3}$

8. _____

9. If $f(x)=7-x^2$, find $f'(x)$ by determining $\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}$.

9. _____

Differentiate.

10. $y=6x+6$

10. _____

11. $y=3x^7-x^2+5$

11. _____

12. $y=x^{2/9}$

12. _____

13. $f(x)=x^{-7}$

13. _____

14. $f(x)=\sqrt[6]{x^5-2}$

14. _____

15. $f(x)=\dfrac{8-5x}{x^3+3}$

15. _____

16. $y=\ln(x^2+3x+9)$

16. _____

17. $y=e^{15\ln x}$

17. _____

18. $y=-6e^x$

18. _____

19. $y = e^{x^4 - 2x}$

19. _____

20. $f(x) = \ln(e^x + 8)$

20. _____

21. For $y = 6x^3 + 9x$, find $\dfrac{d^2y}{dx^2}$.

21. _____

22. *Business: average cost.* Daniel's Digital Documents finds that the cost, in dollars, of producing x boxes of glossy brochures is given by

$$C(x) = 700 + 7\sqrt{x}.$$

Find the rate at which the average cost is changing when 100 boxes of glossy brochures have been produced.

22. _____

23. Differentiate implicitly to find $\dfrac{dy}{dx}$ if $2xy^4 - 3 = 4x^2$.
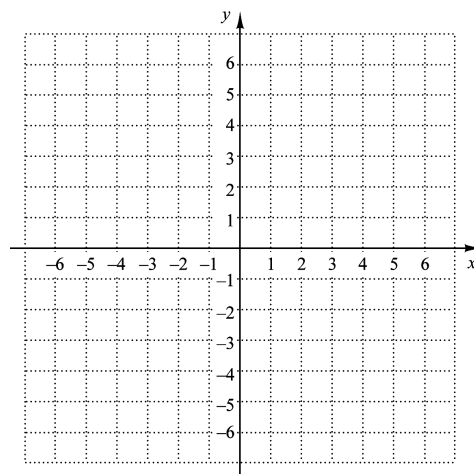
23. _____

24. Find an equation of the tangent line to the graph of $y = 3x^2 - 2x + 4e^{-x}$ at the point $(0, 4)$.

24. _____

Sketch the graph of each function. Identify the coordinates of any extrema and points of inflection. State where the function is increasing or decreasing, where it is concave up or down, and where any asymptotes occur.
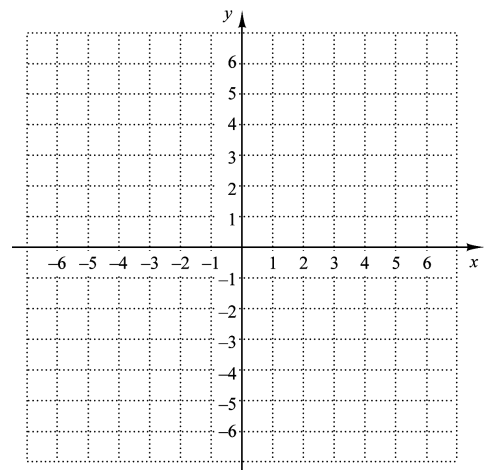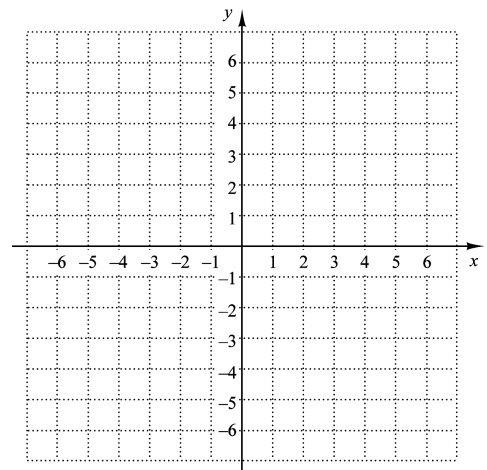
25. $f(x) = -2x^3 + 12x^2 - 18x + 6$
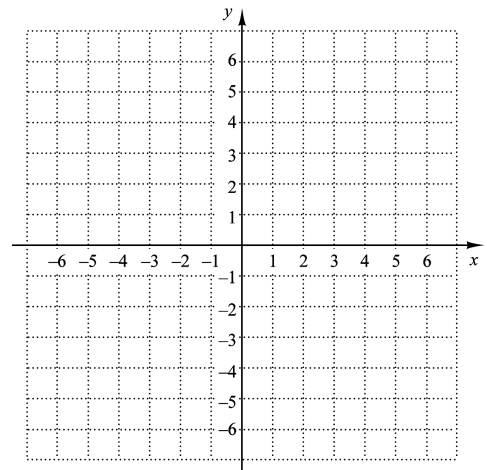
25. _____

26. $f(x) = 2x^2 + 5 - x^4$

26. _____

27. $f(x) = \dfrac{-12x}{x^2 + 9}$

27. _____

28. $f(x) = \dfrac{-18}{x^2 - 9}$

28. _____

Find the absolute maximum and minimum values, if they exist, over the indicated interval. If no interval is indicated, use the real line.

29. $f(x) = 3x^2 + 6x - 5$

29. _____

30. $f(x) = -5x + 7$

30. _____

31. _____

31. $f(x) = \dfrac{2}{3}x^3 - x^2 - 4x + 5; [-3, 3]$

32. *Business: maximizing profit.* Find the maximum profit and the number of units x that must be produced and sold in order to yield maximum profit. Assume that $R(x)$ and $C(x)$ are the revenue and cost, in dollars, where x units are produced:

$$R(x) = 2x^2 + 4x + 80$$
$$C(x) = 2.2x^2 + 2x + 70.$$

32. _____

33. _____

33. *Business: minimizing inventory costs.* A department store sells 240 coffee makers per year. It costs $4.80 to store one coffee maker for one year. To order coffee makers there is a fixed cost of $9 plus $2 for each coffee makers. How many times per year should she store reorder coffee maker and in what lot size, in order to minimize inventory costs?

34. _____

34. For $f(x) = 8x^2 - 5$, $x = 3$, and $\Delta x = 0.01$; find Δy and $f'(x)\Delta x$.

35. *Life Science: bacterial population.* The number of bacteria in population increases at a rate of 15% per hour, that is,

$$\frac{dP}{dt} = 0.15P,$$

where P is the number of bacteria and t is the time, in hours.

(a) When $t = 0$, there are 8000 bacteria present. Find a function that satisfies the equation.

(b) What will the bacteria population be after 6 hr?

(c) What is the doubling time of the bacteria population?

35. (a) _____

(b) _____

(c) _____

36. *Economics: elasticity of demand.* Consider the demand function

$$q = D(x) = 60 - 5x.$$

(a) Find the elasticity.

(b) Find the elasticity at $x = 7.50$ and state whether the demand is elastic or inelastic.

(c) Find the elasticity at $x = 5$ and state whether the demand is elastic or inelastic.

(d) At a price of $5, will a small increase in price cause the total revenue to increase or decrease?

(e) Find the price for which the total revenue is a maximum.

36. (a) _____

(b) _____

(c) _____

(d) _____

(e) _____

Evaluate.

37. $\int 6x^4 dx$.

37. _____

38. $\int_1^2 (8e^x + 3x^2)\, dx$

38. _____

39. $\int \dfrac{1}{\sqrt{x^2 - 25}}\, dx$ (Use Table 1.)

39. _____

40. $\int x^3 e^{x^4+6}\, dx$ (Do not use Table 1.)

40. _____

41. $\int (x+6)\ln x\, dx$

41. _____

42. $\int \dfrac{-9}{x}\, dx$ (Assume $x > 0$.)

42. _____

43. $\int_0^1 \dfrac{1}{2}\sqrt[3]{x}\, dx$

43. _____

44. Find the area under the graph of $y = x^3 + 2x$ over the interval $[1, 3]$.

44. _____

45. *Business: present value.* Find the present value of $200,000 due in 20 yr at 4.7% compounded continuously.

45. _____

46. *Accumulated present value of a continuous income stream.* A small business owner wants to have $36,000 in 7 yr for equipment upgrades. Find the amount she needs to save, at $R(t)$ dollars per year, at 3.8% compounded continuously, to achieve the desired future value.

46. _____

47. Determine whether the improper integral is convergent or divergent, and calculate its value if it is convergent:

$$\int_1^\infty \frac{1}{x^5}\, dx.$$

47. _____

48. Given the probability density function

$$f(x) = \frac{5x^3}{16} \text{ over } [0, 2]$$

find $E(x)$.

48. _____

49. Let x be a continuous random variable that is normally distributed with mean $\mu = 5$ and standard deviation $\sigma = 0.8$. Using Table 2, find $P(3 \leq x \leq 5.2)$.

49. _____

50. *Economics: supply and demand.* Given the demand and supply functions,

$$P = D(x) = (x - 12)^2 \text{ and } p = S(x) = x^2 + 12x + 36,$$

where p is the price per unit, in dollars, when x units are sold, find the equilibrium point and the consumer's surplus at the equilibrium point.

50. _____

51. Find the volume of the solid of revolution generated by rotating the region under the graph of

$$e^{3x} \text{ from } x = 0 \text{ to } x = 6$$

about the x-axis.

51. _____

52. Solve the differential equation $\dfrac{dy}{dx} = 3x^4 y$.

52. _____

53. Consider the data in the table.

Age of Business (in years)	10	15	20
Profit (in thousands of dollars)	40	50	65

(a) Find the regression line, $y = mx + b$.

(b) Use the regression line to predict the profit when the business is 30 years old.

53. (a) _____

(b) _____

Given $f(x, y) = \frac{y}{x} + 2x - 6y^2$, **find each of the following.**

54. _____

54. f_y

55. _____

55. f_{xx}

56. Find the relative maximum and minimum values of   56. _____
   $f(x, y) = 5x^2 + 6y^2 + 2$.

57. Maximize $f(x, y) = x + 2y - 2x^2 + 5y^2 + 1$ subject to   57. _____
   the constraint $x + 2y = 3$.

58. Evaluate: 58. _____

$$\int_2^4 \int_0^1 (2e^x + 1)\, dy\, dx.$$

59. Find $\lim\limits_{x \to 3} \dfrac{x^4 - 10x^2 + 9}{x - 3}$.   59. _____

60. Differentiate: $f(x) = x^2 \ln(xe^x)$ 60. _____

61. Use a calculator to approximate the integral: 61. _____

$$\int_{-\infty}^{\infty} \frac{3}{x^2 + 2}\, dx$$