Name:

Chapter 5, Form A

Let $D(x) = (x-5)^2$ be the price, in dollars per unit, that consumers are willing to pay for x units of an item, and let $S(x) = x^2 + 2x + 1$ be the price, in dollars per unit, that producers are willing to accept for x units. Find:

1. The equilibrium point — **1.** ____________

2. The consumer surplus at the equilibrium point — **2.** ____________

3. The producer surplus at the equilibrium point — **3.** ____________

4. *Business: future value*. Find the future value of $9,000 invested for 8 yr at an annual percentage rate of 3.9%, compounded continuously. — **4.** ____________

5. *Business: future value of a continuous income stream.* Find the accumulated future value of $5,000 per year, at an interest rate of 4.25%, compounded continuously, for 8 yr. — **5.** ____________

6. *Physical science: demand for a gem*. In 2004 $(t = 0)$, the world use of a certain gemstone was 15.9 million metric tons and the demand was increasing at a rate of 3.8% per year. If the demand continues to grow at this rate, how much of this gemstone will the world use from 2004 to 2020? — **6.** ____________

7. *Physical science: depletion of a gem*. See Question 6. The world reserves of the gemstone in 2004 were approximately 3200 million metric tons. Assuming the demand for the gemstone continues to grow at the rate of 3.8% per year and no new reserves are discovered, when will the reserves be depleted? — **7.** ____________

8. *Business: accumulated present value of a continuous income stream*. New parents want to have $80,000 in 12 yr to help with college costs. Find the amount they need to save, at $R(t)$ dollars per year, at 5.215%, compounded continuously, to achieve the desired future value. — **8.** ____________

9. *Business: contract buyout*. An executive is working under a contract that pays him \$250,000 each year for 5 yr. After 3 yr, the company offers to buy out the remainder of his contract. What is the least amount the executive should accept, if the going interest rate is 6.8% compounded continuously?

9. ______________________________

10. *Business: future value of a noncontinuous income stream*. Nick signs a contract that will pay him an income given by $R(t) = 80,000 + 7500t$, where t is in years and $0 \le t \le 10$. If he invests this money at 7%, compounded continuously. What is the future value of the income stream?

10. ______________________________

Determine whether the improper integral is convergent or divergent, and calculate its value if it is convergent.

11. $\int_2^\infty \frac{dx}{x^3}$

11. ______________________________

12. $\int_0^\infty \frac{5}{3+x}\,dx$

12. ______________________________

13. Find k such that $f(x) = kx^3$ is a probability density function over the interval [0, 3]. Then find the probability density function.

13. ______________________________

14. *Business*: *length of wait*. A car dealership determines that the length of time t, in minutes, that a customer must wait to meet a sales representative is an exponentially distributed random variable with probability density function

$$f(t) = \frac{1}{8}e^{-0.125t}, 0 \le t < \infty.$$

Find the probability that a customer will wait in line no more than 3 min.

14. ______________________________

Given the probability density function $f(x) = \frac{1}{10}x$ over [4, 6], find each of the following.

15. $E(x)$

15. ______________________________

16. $E(x^2)$

16. ______________________________

17. The mean

17. ______________________________

Given the probability density function $f(x) = \frac{1}{10}x$ over [4, 6], find each of the following.

18. The variance — **18.** ________

19. The standard deviation — **19.** ________

20. The percentile corresponding to $x = 5$ — **20.** ________

Let x be a continuous random variable with a standard normal distribution. Using Table 2, find each of the following.

21. $P(0 \le x \le 2.3)$ — **21.** ________

22. $P(-0.56 \le x \le -0.2)$ — **22.** ________

23. $P(-1.4 \le x \le 2.02)$ — **23.** ________

24. The price per pound p of coffee at various stores in a certain city is normally distributed with mean $\mu = \$6.25$ and standard deviation $\sigma = \$0.80$. What is the probability that the price per pound is \$7.50 or more? — **24.** ________

25. *Test score distribution.* In a large class, students' test scores had a mean of $\mu = 73$ and a standard deviation of $\sigma = 11$. If the top 10% get an *A*, what is the minimum score needed to get an *A*? Round to the appropriate integer. — **25.** ________

Find the volume generated by revolving about the x-axis the regions bounded by the following graphs.

26. $y = \dfrac{1}{x},\ x = 1,\ x = 6$ — **26.** ________

27. $y = \sqrt{x-3},\ x = 3,\ x = 8$ — **27.** ________

Solve each differential equation.

28. $\dfrac{dy}{dx} = 4x^7 y$ — **28.** ________

29. $\dfrac{dy}{dx} = \dfrac{8}{y}$ — **29.** ________

30. $\dfrac{dy}{dt} = 3y;\ y = 7$ when $t = 0$ — **30.** ________

31. $y' = 5x^{10} - x^{10}y$ — **31.** ________

Solve each differential equation.

32. $\frac{dr}{dt} = -4r^{-5}$ **32.** ____________

33. $y' = 7y + xy$ **33.** ____________

34. *Economics: elasticity.* Find the demand function $q = D(x)$, given the elasticity function **34.** ____________

$$E(x) = 10, \text{ for all } x > 0.$$

35. *Business: stock growth.* The growth rate of a stock, in dollars per month, can be modeled by

$$\frac{dV}{dt} = k(L - V),$$

where V is the value of the stock per share, in dollars, after t months; $L = \$21$, the limiting value of the stock; k is a constant, and $V(0) = 0$.

(a) Write the solution $V(t)$ in terms of L and k. **35.(a)** ____________

(b) If $V(4) = 10$, determine k to the nearest hundredth. **(b)** ____________

(c) Rewrite $V(t)$ in terms of t and k using the value of k found in part (b). **(c)** ____________

(d) Use the equation in part (c) to find $V(15)$, the value of the stock after 15 months. **(d)** ____________

(e) In how many months will the value be \$15? **(e)** ____________

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36. The function $f(x) = x^{11}$ is a probability density function over the interval [0, b]. What is b? **36.** ____________

37. Determine whether the following improper integral is convergent or divergent, and calculate its value if it is convergent: **37.** ____________

$$\int_1^{\infty} 8x^3 e^{-x^4}\, dx.$$

---

38. Approximate the integral: **38.** ____________

$$\int_{-\infty}^{\infty} \frac{5}{1 + 3x^2}\, dx\ .$$

 Name:

Chapter 5, Form B

Let $D(x) = (x-6)^2$ be the price, in dollars per unit, that consumers are willing to pay for x units of an item, and let $S(x) = x^2 + 6x$ be the price, in dollars per unit, that producers are willing to accept for x units. Find:

1. The equilibrium point — **1.** ______________

2. The consumer surplus at the equilibrium point — **2.** ______________

3. The producer surplus at the equilibrium point — **3.** ______________

4. *Business: future value*. Find the future value of \$15,000 invested for 5 yr at an annual percentage rate of 4.51%, compounded continuously. — **4.** ______________

5. *Business: future value of a continuous income stream.* Find the accumulated future value of \$7,500 per year, at an interest rate of 3.92%, compounded continuously, for 10 yr. — **5.** ______________

6. *Physical: science: demand for a gem.* In 2004 $(t = 0)$, the world use of a certain gemstone was 25 million metric tons and the demand was increasing at a rate of 5.2% per year. If the demand continues to grow at this rate, how much of this gemstone will the world use from 2004 to 2018? — **6.** ______________

7. *Physical science: depletion of a gem*. See Question 6. The world reserves of the gemstone in 2004 were approximately 3800 million metric tons. Assuming the demand for the gemstone continues to grow at a rate of 5.2% per year and no new reserves are discovered, when will the reserves be depleted? — **7.** ______________

8. *Business: accumulated present value of a continuous income stream*. Ethan Clark wants to have \$30,000 in 5 yr for a down payment on a house. Find the amount he needs to save, at $R(t)$ dollars per year, at 7.19%, compounded continuously, to achieve the desired future value. — **8.** ______________

9. *Business: contract buyout*. A professional athlete signs a 5-yr contract to play baseball at a salary of $525,000 per year. After 3yr, his team offers to buy out the remainder of his contract. What is the least amount this athlete should accept, if the going interest rate is 5.9%, compounded continuously?

9. ______________________

10. *Business: future value of a noncontinuous income stream*. Stephen signs a contract that will pay him an income given by $R(t) = 50,000 + 3000t$, where t is in years and $0 \leq t \leq 7$. If he invests this money at 6%, compounded continuously, what is the future value of the income stream?

10. ______________________

Determine whether each improper integral is convergent or divergent, and calculate its value if it is convergent.

11. $\int_1^\infty \frac{dx}{x^6}$

11. ______________________

12. $\int_0^\infty \frac{4}{3+x}\,dx$

12. ______________________

13. Find k such that $f(x) = kx^2$ is a probability density function over the interval [0, 4]. Then find the probability density function.

13. ______________________

14. *Business: length of wait*. A bookstore determines that the length of time t, in weeks, that a customer must wait for a special order is an exponentially distributed random variable with probability density function

$$f(t) = 0.85e^{-0.85t}, 0 \leq t < \infty.$$

Find the probability that a customer will wait for an order no more than 3 weeks.

14. ______________________

Given the probability density function $f(x) = \frac{1}{6}x$ over [2, 4], find each of the following.

15. $E(x)$

15. ______________________

16. $E(x^2)$

16. ______________________

17. The mean

17. ______________________

Given the probability density function $f(x) = \frac{1}{6}x$ over [2, 4], find each of the following.

18. The variance **18.** ______________

19. The standard deviation **19.** ______________

20. The percentile corresponding to $x = 3$ **20.** ______________

Let x be a continuous random variable with a standard normal distribution. Using Table 2, find each of the following.

21. $P(0 \le x \le 1.3)$ **21.** ______________

22. $P(-0.9 \le x \le -0.15)$ **22.** ______________

23. $P(-1.1 \le x \le 0.78)$ **23.** ______________

24. The price per pound p of cheddar cheese at various stores in a certain city is normally distributed with mean $\mu = \$5.25$ and standard deviation $\sigma = \$0.45$. What is the probability that the price per pound is \$4.50 or less? **24.** ______________

25. *Business: price distribution.* If the price per pound of Alaskan salmon is normally distributed, with mean $\mu = \$13.50$ and standard deviation $\sigma = \$1.75$, what is the lowest price in the top 20% of salmon prices? **25.** ______________

Find the volume generated by revolving about the x-axis the regions bounded by the following graphs.

26. $y = 5x,\ x = 1,\ x = 2$ **26.** ______________

27. $y = \sqrt{3x+1},\ x = 0,\ x = 4$ **27.** ______________

Solve each differential equation.

28. $\dfrac{dy}{dx} = 5x^3y$ **28.** ______________

29. $\dfrac{dy}{dx} = \dfrac{4}{y}$ **29.** ______________

30. $\dfrac{dy}{dt} = 7y;\ y = 3$ when $t = 0$ **30.** ______________

31. $y' = 5x^5 - x^5y$ **31.** ______________

Solve each differential equation.

32. $\frac{dr}{dt} = -2r^{-8}$ **32.** ____________

33. $y' = 11y + xy$ **33.** ____________

34. *Economics: elasticity.* Find the demand function $q = D(x)$, given the elasticity condition **34.** ____________

$$E(x) = 3, \text{ for all } x > 0.$$

35. *Business: stock growth.* The growth rate of a stock, in dollars per month, can be modeled by

$$\frac{dV}{dt} = k(L - V),$$

where V is the value of the stock per share, in dollars, after t months; $L = \$50$, the limiting value of the stock; k is a constant, and $V(0) = 0$.

(a) Write the solution $V(t)$ in terms of L and k. **35.(a)** ____________

(b) If $V(9) = 17.5$, determine k to the nearest hundredth. **(b)** ____________

(c) Rewrite $V(t)$ in terms of t and k using the value of k found in part (b). **(c)** ____________

(d) Use the equation in part (c) to find $V(15)$, the value of the stock after 15 months. **(d)** ____________

(e) In how many months will the value be \$35? **(e)** ____________

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36. The function $f(x) = x^7$ is a probability density function over the interval [0, b]. What is b? **36.** ____________

37. Determine whether the following improper integral is convergent or divergent, and calculate its value if it is convergent: **37.** ____________

$$\int_1^{\infty} 2x^4 e^{x^5}\,dx.$$

---

38. Approximate the integral: **38.** ____________

$$\int_{-\infty}^{\infty} \frac{4}{x^2 + 2}\,dx\,.$$

Name:

Chapter 5, Form C

Let $D(x) = (x-8)^2$ be the price, in dollars per unit, that consumers are willing to pay for x units of an item, and let $S(x) = x^2 + 5x + 1$ be the price, in dollars per unit, that producers are willing to accept for x units. Find:

1. The equilibrium point
2. The consumer surplus at the equilibrium point
3. The producer surplus at the equilibrium point
4. *Business: future value*. Find the future value of \$4000 invested for 6 yr at an annual percentage rate of 4.32%, compounded continuously.
5. *Business: future value of a continuous income stream*. Find the accumulated future value of \$11,000 per year, at an interest rate of 4.71%, compounded continuously, for 8 yr.
6. *Physical science: demand for a mineral*. In 2005 $(t = 0)$, the world use of a certain mineral was 51.9 million metric tons and the demand was increasing at a rate of 8.3% per year. If the demand continues to grow at this rate, how much of this mineral will the world use from 2005 to 2016?
7. *Physical science: depletion of a mineral*. See Question 6. The world reserves of the mineral in 2005 were approximately 9200 million metric tons. Assuming the demand for the mineral continues to grow at a rate of 8.3% per year and no new reserves are discovered, when will the reserves be depleted?
8. *Business: accumulated present value of a continuous income stream*. Grandparents want to have \$100,000 in 15 yr to help their grandchild with college costs. Find the amount they need to save, at $R(t)$ dollars per year, at 6.75%, compounded continuously, to achieve the desired future value.

1. ______________________________
2. ______________________________
3. ______________________________
4. ______________________________
5. ______________________________
6. ______________________________
7. ______________________________
8. ______________________________

9. *Business: contract buyout*. A talk-show host signs a 7-yr contract to host a radio talk-show at a salary of \$375,000 per year. After 4 yr, the company offers to buy out the remainder of his contract. What is the least amount the host should accept, if the going interest rate is 4.3%, compounded continuously?

9. ______________________________

10. *Business: future value of noncontinuous income stream*. Paul signs a contract that will pay him an income given by $R(t) = 90{,}000 + 6000t$, where t is in years and $0 \le t \le 8$. If the invests this money at 4%, compounded continuously, what is the future value of the income stream?

10. ______________________________

Determine whether each improper integral is convergent or divergent, and calculate its value if it is convergent.

11. $\int_2^\infty \frac{dx}{x^4}$

11. ______________________________

12. $\int_0^\infty \frac{3}{x+6}\,dx$

12. ______________________________

13. Find k such that $f(x) = kx^5$ is a probability density function over the interval [0, 2]. Then find the probability density function.

13. ______________________________

14. *Business: length of wait*. A restaurant determines that the length of time t, in minutes, that a customer must wait for an order is an exponentially distributed random variable with probability density function

$$f(t) = 0.02e^{-0.02t}, 0 \le t < \infty.$$

Find the probability that a customer will wait for an order no more than 20 min.

14. ______________________________

Given the probability density function $f(x) = \frac{1}{12}x$ over [1, 5], find each of the following.

15. $E(x)$

15. ______________________________

16. $E(x^2)$

16. ______________________________

17. The mean

17. ______________________________

Given the probability density function $f(x) = \frac{1}{12}x$ over [1, 5], find each of the following.

18. The variance **18.** ______________

19. The standard deviation **19.** ______________

20. The percentile corresponding to $x = 4$ **20.** ______________

Let x be a continuous random variable with standard normal distribution. Using Table 2, find each of the following.

21. $P(0 \le x \le 1.5)$ **21.** ______________

22. $P(-1.2 \le x \le -0.35)$ **22.** ______________

23. $P(-1.4 \le x \le 2.15)$ **23.** ______________

24. The price per dozen eggs at various stores in a certain city is normally distributed with mean $\mu = \$2.30$ and standard deviation $\sigma = \$0.20$. What is the probability that the price per dozen is \$1.95 or more? **24.** ______________

25. *Test score distribution*. In a large class, students' test scores had a mean of $\mu = 76$ and a standard deviation of $\sigma = 9$. If the top 15% get an A, what is the minimum score needed to get an A? Round to the appropriate integer. **25.** ______________

Find the volume generated by revolving about the x-axis the regions bounded by the following graphs.

26. $y = \dfrac{2}{\sqrt[4]{x}}, \ x = 4, \ x = 9$ **26.** ______________

27. $y = \sqrt{x+5}, \ x = 1, \ x = 8$ **27.** ______________

Solve each differential equation.

28. $\dfrac{dy}{dx} = 4x^3 y$ **28.** ______________

29. $\dfrac{dy}{dx} = \dfrac{3}{y}$ **29.** ______________

30. $\dfrac{dy}{dt} = 5y; \ y = 4$ when $t = 0$ **30.** ______________

31. $y' = 6x^3 - x^3 y$ **31.** ______________

Solve each differential equation.

32. $\frac{dr}{dt} = 6r^{-9}$ **32.** ____________

33. $y' = 5y - xy$ **33.** ____________

34. *Economics: elasticity*. Find the demand function $q = D(x)$, given the elasticity condition **34.** ____________

$$E(x) = 8, \text{ for all } x > 0.$$

35. *Business: stock growth*. The growth rate of a stock, in dollars per month, can be modeled by

$$\frac{dV}{dt} = k(L - V),$$

where V is the value of the stock per share, in dollars, after t months; $L = \$25$, the limiting value of the stock; k is a constant, and $V(0) = 0$.

(a) Write the solution $V(t)$ in terms of L and k. **35.(a)** ____________

(b) If $V(8) = 12$, determine k to the nearest hundredth. **(b)** ____________

(c) Rewrite $V(t)$ in terms of t and k using the value of k found in part (b). **(c)** ____________

(d) Use the equation in part (c) to find $V(15)$, the value of the stock after 15 months. **(d)** ____________

(e) In how many months will the value be \$18? **(e)** ____________

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36. The function $f(x) = x^5$ is a probability density function over the interval [0, b]. What is b? **36.** ____________

37. Determine whether the following improper integral is convergent or divergent, and calculate its value if it is convergent: **37.** ____________

$$\int_1^\infty \frac{1}{2}x^4 e^{-x^5}\,dx.$$

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38. Approximate the integral: **38.** ____________

$$\int_{-\infty}^{\infty} \frac{2}{1 + x^2}\,dx\,.$$

 Name:

Chapter 5, Form D

Let $D(x) = (x - 9)^2$ be the price, in dollars per unit, that consumers are willing to pay for x units of an item, and let $S(x) = x^2 + 2x + 1$ be the price, in dollars per unit, that producers are willing to accept for x units. Find:

1. The equilibrium point — **1.** ______

2. The consumer surplus at the equilibrium point — **2.** ______

3. The producer surplus at the equilibrium point — **3.** ______

4. *Business: future value*. Find the future value of \$2500 invested for 4 yr at an annual percentage rate of 4.78%, compounded continuously. — **4.** ______

5. *Business: future value of a continuous income stream*. Find the accumulated future value of \$20,000 per year, at an interest rate of 4.38%, compounded continuously, for 7 yr. — **5.** ______

6. *Physical science: demand for a mineral*. In 2005 $(t = 0)$, the world use of a certain mineral was 38.5 million metric tons and the demand was increasing at a rate of 6.2% per year. If the demand continues to grow at this rate, how much of this mineral will the world use from 2005 to 2015? — **6.** ______

7. *Physical science: depletion of a mineral*. See Question 6. The world reserves of the mineral in 2005 were approximately 6800 million metric tons. Assuming the demand for the mineral continues to grow at a rate of 6.2% per year and no new reserves are discovered, when will the reserves be depleted? — **7.** ______

8. *Business: accumulated present value of a continuous income stream*. Mark Aaron wants to have \$28,000 in 4 yr for a down payment on a house. Find the amount he needs to save, at $R(t)$ dollars per year, at 5.62%, compounded continuously, to achieve the desired future value. — **8.** ______

9. *Business: contact buyout*. A business executive is working under a contract that pays her \$450,000 each year for 6 yr. After 2 yr, the company offers to buy out the remainder of her contract. What is the least amount the executive should accept, if the going interest rate is 5.2% compounded continuously?

9. ______________________

10. *Business: future value of a noncontinuous income stream*. Lisa signs a contract that will pay her an income given by $R(t) = 125{,}000 + 8000t$, where t is in years and $0 \le t \le 5$. If he invests this money at 5%, compounded continuously, what is the future value of the income stream?

10. ______________________

Determine whether each improper integral is convergent or divergent, and calculate its value if it is convergent.

11. $\int_2^\infty \frac{dx}{x^2}$

11. ______________________

12. $\int_0^\infty \frac{1}{2x+3}\,dx$

12. ______________________

13. Find k such that $f(x) = kx^4$ is a probability density function over the interval [0, 3]. Then find the probability density function.

13. ______________________

14. *Business*: *length of wait*. A grocery store determines that the length of time t, in minutes, that a customer must wait in line is an exponentially distributed random variable with probability density function

$$f(t) = 0.3e^{-0.3t},\ 0 \le t < \infty.$$

Find the probability that a customer will wait in line no more than 5 min.

14. ______________________

Given the probability density function $f(x) = \frac{1}{8}x$ over [0, 4], find each of the following.

15. $E(x)$

15. ______________________

16. $E(x^2)$

16. ______________________

17. The mean

16. ______________________

Given the probability density function $f(x) = \frac{1}{8}x$ over [0, 4], find each of the following.

18. The variance **18.** ________________

19. The standard deviation **19.** ________________

20. The percentile corresponding to $x = 2$ **20.** ________________

Let x be a continuous random variable with standard normal distribution. Using Table 2, find each of the following.

21. $P(0 \le x \le 0.8)$ **21.** ________________

22. $P(0.42 \le x \le 2.86)$ **22.** ________________

23. $P(-2.2 \le x \le 1.8)$ **23.** ________________

24. The price per gallon of milk at various stores in a certain city is normally distributed with mean $\mu = \$2.80$ and standard deviation $\sigma = \$0.24$. What is the probability that the price per gallon is \$3.00 or less? **24.** ________________

25. *Business: price distribution*. If the price per pound of King Crab legs is normally distributed, with $\mu = \$24.95$ and standard deviation, $\sigma = \$3.50$, what is the lowest price in the top 15% of King Crab leg prices? **25.** ________________

Find the volume generated by revolving about the x-axis the regions bounded by the following graphs.

26. $y = 3x,\ x = 1,\ x = 6$ **26.** ________________

27. $y = \sqrt{3+x},\ x = 2,\ x = 4$ **27.** ________________

Solve each differential equation.

28. $\frac{dy}{dx} = 10x^4 y$ **28.** ________________

29. $\frac{dy}{dx} = \frac{5}{y}$ **29.** ________________

30. $\frac{dy}{dt} = 6y;\ y = 5$ when $t = 0$ **30.** ________________

31. $y' = 2x^3 - x^3 y$ **31.** ________________

Solve each differential equation.

32. $\dfrac{dr}{dt} = -5r^{-2}$ **32.** ______________

33. $y' = 9y - xy$ **33.** ______________

34. *Economics: elasticity*. Find the demand function $q = D(x)$, given the elasticity condition **34.** ______________

$$E(x) = 3, \text{ for all } x > 0.$$

35. *Business: stock growth*. The growth rate of a stock, in dollars per month, can be modeled by

$$\frac{dV}{dt} = k(L - V),$$

where V is the value of the stock per share, in dollars, after t months; $L = \$15$, the limiting value of the stock; k is a constant, and $V(0) = 0$.

(a) Write the solution $V(t)$ in terms of L and k. **35.(a)** ______________

(b) If $V(4) = 6$, determine k to the nearest hundredth. **(b)** ______________

(c) Rewrite $V(t)$ in terms of t and k using the value of k found in part (b). **(c)** ______________

(d) Use the equation in part (c) to find $V(15)$, the value of the stock after 15 months. **(d)** ______________

(e) In how many months will the value be \$12? **(e)** ______________

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36. The function $f(x) = x^6$ is a probability density function over the interval [0, *b*]. What is *b*? **36.** ______________

37. Determine whether the following improper integral is convergent or divergent, and calculate its value if it is convergent: **37.** ______________

$$\int_1^\infty 2x^2 e^{-x^3}\,dx.$$

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38. Approximate the integral **38.** ______________

$$\int_{-\infty}^{\infty} \frac{7}{x^2 + 3}\,dx.$$

Chapter 5, Form E

Let $D(x) = (x - 4)^2$ be the price, in dollars per unit, that consumers are willing to pay for x units of an item, and let $S(x) = x^2$ be the price, in dollars per unit, that producers are willing to accept for x units. Find:

1. The equilibrium point **1.** ______________________________

2. The consumer surplus at the equilibrium point **2.** ______________________________

3. The producer surplus at the equilibrium point **3.** ______________________________

4. *Business: future value.* Find the future value of \$6500 invested for 12 yr at an annual percentage rate of 3.85%, compounded continuously. **4.** ______________________________

5. *Business: future value of a continuous income stream.* Find the accumulated future value of \$5,200 per year, at an interest rate of 4.46%, compounded continuously, for 5 yr. **5.** ______________________________

6. *Physical science: demand for a gem.* In 2006 $(t = 0)$, the world use of a certain gemstone was 29.9 million metric tons and the demand was increasing at a rate of 8.7% per year. If the demand continues to grow at this rate, how much of this mineral will the world use from 2006 to 2020? **6.** ______________________________

7. *Physical science: depletion of a gem.* See Question 6. The world reserves of a certain gemstone in 2006 were 3100 million metric tons. Assuming the demand for the gemstone continues to grow at a rate of 8.7% per year and no new reserves are discovered, when will the reserves be depleted? **7.** ______________________________

8. *Business: accumulated present value of a continuous income stream.* Parents want to have \$75,000 in 10 yr to help their child with college costs. Find the amount they need to save, at $R(t)$ dollars per year, at 4.31%, compounded continuously, to achieve the desired future value. **8.** ______________________________

9. *Business: contract buyout*. A professional athlete signs a 3-yr contract to play football at a salary of $400,000 per year. After 1 yr, his team offers to buy out the remainder of his contract. What is the least amount this athlete should accept, if the going interest rate is 6.7%, compounded continuously?

9. ______________________________

10. *Business: future value of a noncontinuous income stream*. Dan signs a contract that will pay him an income given by $R(t) = 70{,}000 + 5000t,$ where t is in years and $0 \le t \le 8.$ If he invests this money at 5%, compounded continuously, what is the future value of the income stream?

10. ______________________________

Determine whether each improper integral is convergent or divergent, and calculate its value if it is convergent.

11. $\int_3^\infty \frac{dx}{x^3}$

11. ______________________________

12. $\int_0^\infty \frac{5}{5+x}dx$

12. ______________________________

13. Find k such that $f(x) = kx^6$ is a probability density function over the interval [0, 2]. Then find the probability density function.

13. ______________________________

14. *Business*: *length of wait*. An online retail store determines that the length of time t, in days, that a customer must wait for an order is an exponentially distributed random variable with probability density function

$$f(t) = 0.11e^{-0.11t}, 0 \le t < \infty.$$

Find the probability that a customer will wait no more than 8 days for an order.

14. ______________________________

Given the probability density function $f(x) = \frac{1}{14}x$ over [6, 8], find each of the following.

15. $E(x)$

15. ______________________________

16. $E(x^2)$

16. ______________________________

17. The mean

17. ______________________________

Given the probability density function $f(x)=\frac{1}{14}x$ over [6, 8], find each of the following.

18. The variance **18.** ____________

19. The standard deviation **19.** ____________

20. The percentile corresponding to $x=7$ **20.** ____________

Let *x* be a continuous random variable with a standard normal distribution. Using Table 2, find each of the following.

21. $P(0\le x\le 2.1)$ **21.** ____________

22. $P(0.46\le x\le 1.43)$ **22.** ____________

23. $P(-2.2\le x\le 1.83)$ **23.** ____________

24. The price per dozen roses at various stores in a certain city is normally distributed with mean $\mu=\$18.00$ and standard deviation $\sigma=\$3.00$. What is the probability that the price per dozen is \$25.00 or more? **24.** ____________

25. *Test score distribution*. In a large class, students' test scores had a mean of $\mu=74$ and a standard deviation of $\sigma=8$. If the top 12% get an *A*, what is the minimum score needed to get an *A*? Round to the appropriate integer. **25.** ____________

Find the volume generated by revolving about the *x*-axis the regions bounded by the following graphs.

26. $y=\dfrac{5}{x\sqrt{x}},\ x=1,\ x=9$ **26.** ____________

27. $y=e^x,\ \ x=-3,\ \ x=5$ **27.** ____________

Solve each differential equation.

28. $\dfrac{dy}{dx}=3x^5y$ **28.** ____________

29. $\dfrac{dy}{dx}=\dfrac{6}{y}$ **29.** ____________

30. $\dfrac{dy}{dt}=8y;\ \ y=3$ when $t\ =\ 0$ **30.** ____________

31. $y'=10x^4-x^4y$ **31.** ____________

Solve each differential equation.

32. $\dfrac{dr}{dt} = 6r^{-7}$ **32.** ______________

33. $y' = 6y + xy$ **33.** ______________

34. *Economics: elasticity*. Find the demand function $q = D(x)$, given the elasticity condition **34.** ______________

$$E(x) = 7, \text{ for all } x > 0.$$

35. *Business: stock growth*. The growth rate of a stock, in dollars per month, can be is modeled by

$$\frac{dV}{dt} = k(L - V),$$

where V is the value of the stock per share, in dollars, after t months; $L = \$30$, the limiting value of the stock; k is a constant, and $V(0) = 0$.

(a) Write the solution $V(t)$ in terms of L and k. **35.(a)** ______________

(b) If $V(4) = 18$, determine k to the nearest hundredth. **(b)** ______________

(c) Rewrite $V(t)$ in terms of t and k using the value of k found in part (b). **(c)** ______________

(d) Use the equation in part (c) to find $V(15)$, the value of the stock after 15 months. **(d)** ______________

(e) In how many months will the value be $20? **(e)** ______________

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36. The function $f(x) = x^4$ is a probability density function over the interval [0, b]. What is b? **36.** ______________

37. Determine whether the following improper integral is convergent or divergent, and calculate its value if it is convergent: **37.** ______________

$$\int_{-\infty}^{0} 3x^4 e^{x^5}\, dx.$$

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38. Approximate the integral: **38.** ______________

$$\int_{-\infty}^{\infty} \frac{5}{2x^2 + 1}\, dx\,.$$

CALCULUS AND ITS APPLICATIONS

Name:

Chapter 5, Form F

Let $D(x) = (x - 6)^2$ be the price, in dollars per unit, that consumers are willing to pay for x units of an item, and let $S(x) = x^2 + 12$ be the price, in dollars per unit, that producers are willing to accept for x units. Find:

1. The equilibrium — **1.** ____________________

2. The consumer surplus at the equilibrium point — **2.** ____________________

3. The producer surplus at the equilibrium point — **3.** ____________________

4. *Business: future value*. Find the future value of \$10,000 invested for 9 yr at an annual percentage rate of 4.61% compounded continuously. — **4.** ____________________

5. *Business: future value of a continuous income stream*. Find the accumulated future value of \$7,000 per year, at an interest rate of 3.91%, compounded continuously for 10 yr. — **5.** ____________________

6. *Physical science: demand for a mineral*. In 2006 $(t = 0)$, the world use of a certain mineral was 42.8 million metric tons and the demand was increasing at a rate of 7.1% per year. If the demand continues to grow at this rate, how much of this mineral will the world use from 2006 to 2015? — **6.** ____________________

7. *Physical science: depletion of a mineral*. See Question 6. The world reserves of the mineral in 2006 were approximately 5400 million metric tons. Assuming the demand for the mineral continues to grow at a rate of 7.1% per year and no new reserves are discovered, when will the reserves be depleted? — **7.** ____________________

8. *Business: accumulated present value of a continuous income stream*. Nicole Kent wants to have \$20,000 in 5 yr for a down payment on a condominium. Find the amount she needs to save, at $R(t)$ dollars per year, at 4.78%, compounded continuously, to achieve the desired future value. — **8.** ____________________

9. *Business: contract buyout*. A marketing executive is working under a contract that pays his \$275,000 each year for 6 yr. After 3 yr, the company offers to buy out the remainder of his contract. What is the least amount the executive should accept, if the going interest rate is 4.8% compounded continuously?

9. ______________________

10. *Business: future value of a noncontinuous income stream*. Mia signs a contract that will pay her an income given by $R(t) = 110,000 + 10,000,$ where t is in years and $0 \le t \le 15.$ If she invests this money at 6%, compounded continuously, what is the future value of the income stream?

10. ______________________

Determine whether each improper integral is convergent or divergent, and calculate its value if it is convergent.

11. $\int_3^{\infty} \frac{dx}{x^4}$

11. ______________________

12. $\int_0^{\infty} \frac{6}{2x+3}\,dx$

12. ______________________

13. Find k such that $f(x) = kx^5$ is a probability density function over the interval [0, 3]. Then find the probability density function.

13. ______________________

14. *Business*: *length of wait*. A bagel shop determines that the length of time t, in minutes, that a customer must wait in line is an exponentially distributed random variable with probability density function

$$f(t) = 0.55e^{-0.55t}, 0 \le t < \infty.$$

Find the probability that a customer will wait in line no more than 2 min.

14. ______________________

Given the probability density function $f(x) = \frac{2}{3}x$ over [1, 2], find each of the following.

15. $E(x)$

15. ______________________

16. $E(x^2)$

16. ______________________

17. The mean

17. ______________________

Given the probability density function $f(x) = \frac{2}{3}x$ over [1, 2], find each of the following.

18. The variance — **18.** ______

19. The standard deviation — **19.** ______

20. The percentile corresponding to $x = \frac{3}{2}$ — **20.** ______

Let x be a continuous random variable with standard normal distribution. Using Table 2, find each of the following.

21. $P(0 \le x \le 0.84)$ — **21.** ______

22. $P(0.75 \le x \le 2.15)$ — **22.** ______

23. $P(-1.2 \le x \le 2.05)$ — **23.** ______

24. The price per dozen doughnuts at various stores in a certain city is normally distributed with mean $\mu = \$6.00$ and standard deviation $\sigma = \$1.30$. What is the probability that the price per dozen is \$4.50 or more? — **24.** ______

25. *Business: price distribution*. If the price per pound of domestically farmed tilapia is normally distributed, with $\mu = \$9.95$ and standard deviation, $\sigma = \$1.50$, what is the lowest price in the top 10% of tilapia prices? — **25.** ______

Find the volume generated by revolving about the x-axis the regions bounded by the following graphs.

26. $y = \dfrac{1}{2}x^4,\ x = 0,\ x = 2$ — **26.** ______

27. $y = \sqrt{3 + x},\ x = 4,\ x = 5$ — **27.** ______

Solve each differential equation.

28. $\dfrac{dy}{dx} = 6x^3 y$ — **28.** ______

29. $\dfrac{dy}{dx} = \dfrac{3}{y}$ — **29.** ______

30. $\dfrac{dy}{dt} = 9y;\ y = 2$ when $t = 0$ — **30.** ______

31. $y' = 4x^4 - x^4 y$ — **31.** ______

Solve each differential equation.

32. $\frac{dr}{dt} = 3r^{-6}$ **32.** ______________

33. $y' = 8y - xy$ **33.** ______________

34. *Economics: elasticity*. Find the demand function $q = D(x)$, given the elasticity condition **34.** ______________

$$E(x) = 9 \text{ for all } x > 0.$$

35. *Business: stock growth*. The growth rate of a stock, in dollars per month, can be modeled by

$$\frac{dV}{dt} = k(L - V),$$

where V is the value of the stock per share, in dollars, after t months; $L = \$35,$ the limiting value of the stock; k is a constant, and $V(0) = 0.$

(a) Write the solution $V(t)$ in terms of L and k. **35.(a)** ______________

(b) If $V(6) = 20,$ determine k to the nearest hundredth. **(b)** ______________

(c) Rewrite $V(t)$ in terms of t and k using the value of k found in part (b). **(c)** ______________

(d) Use the equation in part (c) to find $V(15),$ the value of the stock after 15 months. **(d)** ______________

(e) In how many months will the value be \$15? **(e)** ______________

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36. The function $f(x) = x^8$ is a probability density function over the interval [0, b]. What is b? **36.** ______________

37. Determine whether the following improper integral is convergent or divergent, and calculate its value if it is convergent: **37.** ______________

$$\int_1^\infty x^4 e^{-x^5} dx.$$

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38. Approximate the integral: **38.** ______________

$$\int_{-\infty}^{\infty} \frac{12}{1 + x^2} dx .$$