Chapter 6, Form A

Given $f(x, y) = 2x^2y^3 + 5e^x - 4y$, find each of the following.

1. $f(-1, 3)$ **1.** _____

2. $\dfrac{\partial f}{\partial x}$ **2.** _____

3. $\dfrac{\partial f}{\partial y}$ **3.** _____

4. $\dfrac{\partial^2 f}{\partial x^2}$ **4.** _____

5. $\dfrac{\partial^2 f}{\partial x\,\partial y}$ **5.** _____

6. $\dfrac{\partial^2 f}{\partial y\,\partial x}$ **6.** _____

7. $\dfrac{\partial^2 f}{\partial y^2}$ **7.** _____

Find the relative maximum and minimum values.

8. $f(x, y) = x^2 + xy + y^3 + 2x$ **8.** _____

9. $f(x, y) = -4x^2 - 3y^2$ **9.** _____

10. *Business*: *predicting total sales*. Consider the data in the following table regarding the total sales of a company during the first three years of operation.

Year, x	Sales, y (in millions)
1	6
2	11
3	14

(a) Find the regression line $y = mx + b$.

(b) Use the regression line to predict sales in the fourth year.

10. (a) _____

(b) _____

11. Find the maximum value of

$$f(x, y) = 2xy - x^2 - 2y^2$$

subject to the constraint $2x + y = 13$.

11. _____

12. Evaluate

$$\int_0^3 \int_1^2 4y^3x^2 dy\, dx.$$

12. _____

---

13. *Business*: *maximizing production*. An automobile company has the following Cobb-Douglas production function for a certain product:

$$p(x, y) = 50x^{1/3}y^{2/3},$$

where x is labor, measured in dollars, and y is capital, measured in dollars. Suppose that the company can make a total investment in labor and capital of \$4,200,000. How should it allocate the investment between labor and capital in order to maximize production?

13. _____

14. Find f_x and f_t:

$$f(x, t) = \frac{t - 3x^3}{2t + 3x^3}.$$

14. _____

---

15. Use a 3D graphics program to graph

$$f(x, y) = 5x - \frac{1}{5}y^2 - \frac{1}{2}x^2.$$

Sketch the graph.

15. _____

Chapter 6, Form B

Given $f(x, y) = \frac{1}{x} + 3xy^2 - y$, find each of the following.

1. $f(-1, 6)$ **1.** _____

2. $\frac{\partial f}{\partial x}$ **2.** _____

3. $\frac{\partial f}{\partial y}$ **3.** _____

4. $\frac{\partial^2 f}{\partial x^2}$ **4.** _____

5. $\frac{\partial^2 f}{\partial x \partial y}$ **5.** _____

6. $\frac{\partial^2 f}{\partial y \partial x}$ **6.** _____

7. $\frac{\partial^2 f}{\partial y^2}$ **7.** _____

Find the relative maximum and minimum values.

8. $f(x, y) = x^2 + 2y^2 - 4xy + y$ **8.** _____

9. $f(x, y) = 2x^2 + 5y^4 + y^2 + 6$ **9.** _____

10. *Business*: *predicting total sales*. Consider the data in the following table regarding the total sales of a company during the first three years of operation.

Year, x	Sales, y (in millions)
1	7
2	12
3	16

(a) Find the regression line $y = mx + b$.

(b) Use the regression line to predict sales in the fourth year.

10. (a) _____

(b) _____

11. Find the maximum value of

$$f(x, y) = 4xy - 2x^2 - 5y^2$$

subject to the constraint $x + 2y = 4$.

11. _____

12. Evaluate

$$\int_0^5 \int_1^3 6x^2y^4 dx\, dy.$$

12. _____

---

13. *Business*: *maximizing production*. An entertainment company has the following Cobb-Douglas production function for a certain product:

$$p(x, y) = 40x^{7/10}y^{3/10},$$

where x is labor, measured in dollars, and y is capital, measured in dollars. Suppose that the company can make a total investment in labor and capital of $300,000. How should it allocate the investment between labor and capital in order to maximize production?

13. _____

14. Find f_x and f_t:

$$f(x, t) = \frac{4x^2 - t}{4x^3 + t}.$$

14. _____

---

15. Use a 3D graphics program to graph

$$f(x, y) = 2x^2 - 3y^2 + x.$$

Sketch the graph.

15. _____

Given $f(x, y) = 4y + 3x^2y - \ln x$, find each of the following.

1. $f(1, -1)$

2. $\dfrac{\partial f}{\partial x}$

3. $\dfrac{\partial f}{\partial y}$

4. $\dfrac{\partial^2 f}{\partial x^2}$

5. $\dfrac{\partial^2 f}{\partial x \partial y}$

6. $\dfrac{\partial^2 f}{\partial y \partial x}$

7. $\dfrac{\partial^2 f}{\partial y^2}$

Find the relative maximum and minimum values.

8. $f(x, y) = y^3 - 6xy + x^2 - 48y$

9. $f(x, y) = 5x^2 - 3y^2$

10. *Business*: *predicting total sales*. Consider the data in the following table regarding the total sales of a company during the first three years of operation.

Year, x	Sales, y (in millions)
1	4
2	8
3	11

(a) Find the regression line $y = mx + b$.

(b) Use the regression line to predict sales in the fourth year.

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. (a) _____

(b) _____

11. Find the minimum value of

$$f(x, y) = 2xy - 4x^2 + 6y^2$$

subject to the constraint $2x - y = 24$.

11. _____

12. Evaluate

$$\int_0^2 \int_1^3 8y^3x^3 dy\, dx.$$

12. _____

---

13. *Business: maximizing production*. An internet provider company has the following Cobb-Douglas production function for a certain product:

$$p(x, y) = 40x^{3/5}y^{2/5},$$

where x is labor, measured in dollars, and y is capital, measured in dollars. Suppose that the company can make a total investment in labor and capital of $500,000. How should it allocate the investment between labor and capital in order to maximize production?

13. _____

14. Find f_x and f_t:

$$f(x, t) = \frac{x^4 - 5t}{x^3 + 5t}.$$

14. _____

---

15. Use a 3D graphics program to graph

$$f(x, y) = x + y^2 - \frac{1}{5}x^3.$$

Sketch the graph.

15. _____

Chapter 6, Form D

Given $f(x, y) = e^x + 3x^2y^2 + 5y$, find each of the following.

1. $f(2, -1)$ **1.** _____

2. $\dfrac{\partial f}{\partial x}$ **2.** _____

3. $\dfrac{\partial f}{\partial y}$ **3.** _____

4. $\dfrac{\partial^2 f}{\partial x^2}$ **4.** _____

5. $\dfrac{\partial^2 f}{\partial x\,\partial y}$ **5.** _____

6. $\dfrac{\partial^2 f}{\partial y\,\partial x}$ **6.** _____

7. $\dfrac{\partial^2 f}{\partial y^2}$ **7.** _____

Find the relative maximum and minimum values.

8. $f(x, y) = 3x^2 + y^2 + x - xy$ **8.** _____

9. $f(x, y) = 5x^2 - 6y^2$ **9.** _____

10. *Business*: *predicting total sales*. Consider the data in the following table regarding the total sales of a company during the first three years of operation.

Year, x	Sales, y (in millions)
1	11
2	12
3	16

(a) Find the regression line $y = mx + b$.

(b) Use the regression line to predict sales in the fourth year.

10. (a) _____

(b) _____

11. Find the maximum value of

$$f(x, y) = 2xy - x^2 - 2y^2$$

subject to the constraint $x - 4y = 11$.

11. _____

12. Evaluate

$$\int_0^3 \int_1^2 5x^4 y^2 \, dx \, dy.$$

12. _____

---

13. *Business*: *maximizing production*. A sales company has the following Cobb-Douglas production function for a certain product:

$$p(x, y) = 50x^{3/4} y^{1/4},$$

where x is labor, measured in dollars, and y is capital, measured in dollars. Suppose that the company can make a total investment in labor and capital of $200,000. How should it allocate the investment between labor and capital in order to maximize production?

13. _____

14. Find f_x and f_t:

$$f(x, t) = \frac{2t + 3x^2}{2t - 3x^3}.$$

14. _____

---

15. Use a 3D graphics program to graph

$$f(x, y) = 2x + \frac{1}{4}y^2 - \frac{1}{5}x^2.$$

Sketch the graph.

15. _____

Given $f(x, y) = 5y + 3x^2y + e^x$, find each of the following.

1. $f(3, -1)$ **2.** $\dfrac{\partial f}{\partial x}$

1. _____

2. _____

3. $\dfrac{\partial f}{\partial y}$ **4.** $\dfrac{\partial^2 f}{\partial x^2}$

3. _____

4. _____

5. $\dfrac{\partial^2 f}{\partial x \partial y}$ **6.** $\dfrac{\partial^2 f}{\partial y \partial x}$

5. _____

6. _____

7. $\dfrac{\partial^2 f}{\partial y^2}$

7. _____

Find the relative maximum and minimum values.

8. $f(x, y) = 2x^2 + 4xy - y^3 + x$

8. _____

9. $f(x, y) = -2xy$

9. _____

10. *Business*: *predicting total sales*. Consider the data in the following table regarding the total sales of a company during the first three years of operation.

Year, x	Sales, y (in millions)
1	6
2	9
3	11

(a) Find the regression line $y = mx + b$.

(b) Use the regression line to predict sales in the fourth year.

10. (a) _____

(b) _____

11. Find the maximum value of

$$f(x, y) = 2xy - 2x^2 + y^2$$

subject to the constraint $x + 2y = 11$.

11. _____

12. Evaluate

$$\int_0^2 \int_1^3 6x^5 y^3 \, dx \, dy.$$

12. _____

---

13. *Business: maximizing production*. A data collections company has the following Cobb-Douglas production function for a certain product:

$$p(x, y) = 80x^{5/8}y^{3/8},$$

where x is labor, measured in dollars, and y is capital, measured in dollars. Suppose that the company can make a total investment in labor and capital of $250,000. How should it allocate the investment between labor and capital in order to maximize production?

13. _____

14. Find f_x and f_t:

$$f(x, t) = \frac{x^3 - t}{x^4 + t}.$$

14. _____

---

15. Use a 3D graphics program to graph

$$f(x, y) = \frac{1}{2}x^3 - 8x^2 + y.$$

Sketch the graph.

15. _____

Chapter 6, Form F

Given $f(x, y) = 3e^x + 2x^2y + 5y$, find each of the following.

1. $f(-1, 2)$

2. $\dfrac{\partial f}{\partial x}$

3. $\dfrac{\partial f}{\partial y}$

4. $\dfrac{\partial^2 f}{\partial x^2}$

5. $\dfrac{\partial^2 f}{\partial x\,\partial y}$

6. $\dfrac{\partial^2 f}{\partial y\,\partial x}$

7. $\dfrac{\partial^2 f}{\partial y^2}$

Find the relative maximum and minimum values.

8. $f(x, y) = 2y^2 - 3x^2$

9. $f(x, y) = x^2 + 4y^2 - 6x$

10. *Business*: *predicting total sales*. Consider the data in the following table regarding the total sales of a company during the first three years of operation.

Year, x	Sales, y (in millions)
1	3
2	4
3	7

(a) Find the regression line $y = mx + b$.

(b) Use the regression line to predict sales in the fourth year.

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. (a) _____

(b) _____

11. Find the maximum value of

$$f(x, y) = 4xy - 3x^2 + 5y^2$$

subject to the constraint $x + 2y = 60$.

11. _____

12. Evaluate

$$\int_0^3 \int_1^3 8y^3x^4 dy\, dx.$$

12. _____

---

13. *Business*: *maximizing production*. A clothing company has the following Cobb-Douglas production function for a certain product:

$$p(x, y) = 30x^{4/5}y^{1/5},$$

where x is labor, measured in dollars, and y is capital, measured in dollars. Suppose that the company can make a total investment in labor and capital of \$150,000. How should it allocate the investment between labor and capital in order to maximize production?

13. _____

14. Find f_x and f_t:

$$f(x, t) = \frac{x^2 + 5t}{x^3 - 5t}.$$

14. _____

---

15. Use a 3D graphics program to graph

$$f(x, y) = y - \frac{2}{3}x^2 - \frac{3}{4}y^2.$$

15. _____