Chapter 2
Simple Comparative Experiments
Solutions

2.1. Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

Variable	N	Mean	SE Mean	Std. Dev.	Variance	Minimum	Maximum
Y	9	19.96	?	3.12	?	15.94	27.16

SE Mean = 1.04 Variance = 9.73

2.2. Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

Variable	N	Mean	SE Mean	Std. Dev.	Sum
Y	16	?	0.159	?	399.851

Mean = 24.991 Std. Dev. = 0.636

2.3. Suppose that we are testing H_0: $\mu = \mu_0$ versus H_1: $\mu \neq \mu_0$. Calculate the *P*-value for the following observed values of the test statistic:

(a) $Z_0 = 2.25$ *P*-value = 0.02445

(b) $Z_0 = 1.55$ *P*-value = 0.12114

(c) $Z_0 = 2.10$ *P*-value = 0.03573

(d) $Z_0 = 1.95$ *P*-value = 0.05118

(e) $Z_0 = -0.10$ *P*-value = 0.92034

2.4. Suppose that we are testing H_0: $\mu = \mu_0$ versus H_1: $\mu > \mu_0$. Calculate the *P*-value for the following observed values of the test statistic:

(a) $Z_0 = 2.45$ *P*-value = 0.00714

(b) $Z_0 = -1.53$ *P*-value = 0.93699

(c) $Z_0 = 2.15$ *P*-value = 0.01578

(d) $Z_0 = 1.95$ *P*-value = 0.02559

(e) $Z_0 = -0.25$ *P*-value = 0.59871

2.5. Consider the computer output shown below.

One-Sample Z

Test of mu = 30 vs not = 30

The assumed standard deviation = 1.2

N	Mean	SE Mean	95% CI	Z	P
16	31.2000	0.3000	(30.6120, 31.7880)	?	?

(a) Fill in the missing values in the output. What conclusion would you draw?

Z = 4 P = 0.00006; therefore, the mean is not equal to 30.

(b) Is this a one-sided or two-sided test?

Two-sided.

(c) Use the output and the normal table to find a 99 percent CI on the mean.

CI = 30.42725, 31.97275

(d) What is the *P*-value if the alternative hypothesis is H_1: $\mu > 30$

P-value = 0.00003

2.6. Suppose that we are testing H_0: $\mu_1 = \mu_2$ versus H_1: $\mu_1 = \mu_2$ with a sample size of $n_1 = n_2 = 12$. Both sample variances are unknown but assumed equal. Find bounds on the *P*-value for the following observed values of the test statistic:

(a) $t_0 = 2.30$ Table *P*-value = 0.02, 0.05 Computer *P*-value = 0.0313

(b) $t_0 = 3.41$ Table *P*-value = 0.002, 0.005 Computer *P*-value = 0.0025

(c) $t_0 = 1.95$ Table *P*-value = 0.1, 0.05 Computer *P*-value = 0.0640

(d) $t_0 = -2.45$ Table *P*-value = 0.05, 0.02 Computer *P*-value = 0.0227

Note that the degrees of freedom is (12 +12) – 2 = 22. This is a two-sided test

2.7. Suppose that we are testing H_0: $\mu_1 = \mu_2$ versus H_1: $\mu_1 > \mu_2$ with a sample size of $n_1 = n_2 = 10$. Both sample variances are unknown but assumed equal. Find bounds on the *P*-value for the following observed values of the test statistic:

(a) $t_0 = 2.31$ Table *P*-value = 0.01, 0.025 Computer *P*-value = 0.01648

(b) $t_0 = 3.60$ Table *P*-value = 0.001, 0.0005 Computer *P*-value = 0.00102

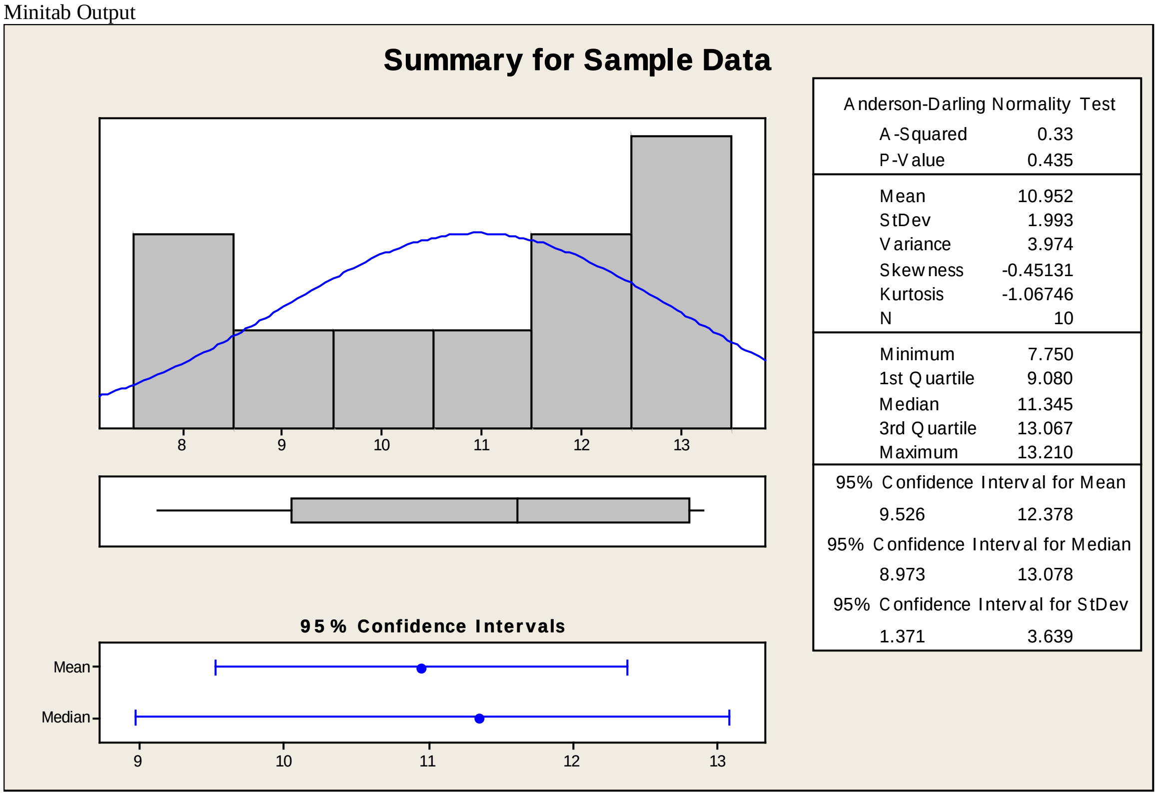
(c) $t_0 = 1.95$ Table *P*-value = 0.05, 0.025 Computer *P*-value = 0.03346

(d) $t_0 = 2.19$ Table P-value = 0.01, 0.025 Computer P-value = 0.02097

Note that the degrees of freedom is (10 +10) – 2 = 18. This is a one-sided test.

2.8. Consider the following sample data: 9.37, 13.04, 11.69, 8.21, 11.18, 10.41, 13.15, 11.51, 13.21, and 7.75. Is it reasonable to assume that this data is from a normal distribution? Is there evidence to support a claim that the mean of the population is 10?

Minitab Output



According to the output, the Anderson-Darling Normality Test has a P-Value of 0.435. The data can be considered normal. The 95% confidence interval on the mean is (9.526,12.378). This confidence interval contains 10, therefore there is evidence that the population mean is 10.

2.9. A computer program has produced the following output for the hypothesis testing problem:

```
Difference in sample means: 2.35
Degrees of freedom: 18
Standard error of the difference in the sample means: ?
Test statistic: t_o = 2.01
P-Value = 0.0298
```

(a) What is the missing value for the standard error?

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{2.35}{StdError} = 2.01$$
$$StdError = 2.35/2.01 = 1.169$$

(b) Is this a two-sided or one-sided test? One-sided test for a t_0 = 2.01 is a *P*-value of 0.0298.
(c) If α=0.05, what are your conclusions? Reject the null hypothesis and conclude that there is a difference in the two samples.
(d) Find a 90% two-sided CI on the difference in the means.

$$\bar{y}_1 - \bar{y}_2 - t_{\alpha/2,n_1+n_2-2}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_1 \le \bar{y}_1 - \bar{y}_2 + t_{\alpha/2,n_1+n_2-2}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$
$$\bar{y}_1 - \bar{y}_2 - t_{0.05,18}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_1 \le \bar{y}_1 - \bar{y}_2 + t_{0.05,18}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$
$$2.35 - 1.734(1.169) \le \mu_1 - \mu_1 \le 2.35 + 1.734(1.169)$$
$$0.323 \le \mu_1 - \mu_1 \le 4.377$$

2.10. A computer program has produced the following output for the hypothesis testing problem:

```
Difference in sample means: 11.5
Degrees of freedom: 24
Standard error of the difference in the sample means: ?
Test statistic: to = -1.88
P-Value = 0.0723
```

(a) What is the missing value for the standard error?

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{-11.5}{StdError} = -1.88$$
$$StdError = -11.5/-1.88 = 6.12$$

(b) Is this a two-sided or one-sided test? Two-sided test for a t_0 = -1.88 is a *P*-value of 0.0723.
(c) If α=0.05, what are your conclusions? Accept the null hypothesis, there is no difference in the means.
(d) Find a 90% two-sided CI on the difference in the means.

$$\bar{y}_1-\bar{y}_2-t_{\alpha/2,n_1+n_2-2}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\le\mu_1-\mu_1\le\bar{y}_1-\bar{y}_2+t_{\alpha/2,n_1+n_2-2}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$\bar{y}_1-\bar{y}_2-t_{0.05,24}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\le\mu_1-\mu_1\le\bar{y}_1-\bar{y}_2+t_{0.05,24}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$-11.5-1.711(6.12)\le\mu_1-\mu_1\le-11.5+1.711(6.12)$$

$$-21.97\le\mu_1-\mu_1\le-1.03$$

2.11. Suppose that we are testing H_0: $\mu = \mu_0$ versus H_1: $\mu > \mu_0$ with a sample size of n = 15. Calculate bounds on the *P*-value for the following observed values of the test statistic:

(a) t_0 = 2.35 Table *P*-value = 0.01, 0.025 Computer *P*-value = 0.01698

(b) t_0 = 3.55 Table *P*-value = 0.001, 0.0025 Computer *P*-value = 0.00160

(c) t_0 = 2.00 Table *P*-value = 0.025, 0.005 Computer *P*-value = 0.03264

(d) t_0 = 1.55 Table *P*-value = 0.05, 0.10 Computer *P*-value = 0.07172

The degrees of freedom are 15 – 1 = 14. This is a one-sided test.

2.12. Suppose that we are testing H_0: $\mu = \mu_0$ versus H_1: $\mu \ne \mu_0$ with a sample size of n = 10. Calculate bounds on the *P*-value for the following observed values of the test statistic:

(a) t_0 = 2.48 Table *P*-value = 0.02, 0.05 Computer *P*-value = 0.03499

(b) t_0 = -3.95 Table *P*-value = 0.002, 0.005 Computer *P*-value = 0.00335

(c) t_0 = 2.69 Table *P*-value = 0.02, 0.05 Computer *P*-value = 0.02480

(d) t_0 = 1.88 Table *P*-value = 0.05, 0.10 Computer *P*-value = 0.09281

(e) t_0 = -1.25 Table *P*-value = 0.20, 0.50 Computer *P*-value = 0.24282

2.13. Consider the computer output shown below.

One-Sample T: Y

Test of mu = 91 vs. not = 91

Variable	N	Mean	Std. Dev.	SE Mean	95% CI	T	P
Y	25	92.5805	?	0.4675	(91.6160, ?)	3.38	0.002

(a) Fill in the missing values in the output. Can the null hypothesis be rejected at the 0.05 level? Why?

Std. Dev. = 2.3365 UCI = 93.5450
Yes, the null hypothesis can be rejected at the 0.05 level because the *P*-value is much lower at 0.002.

(b) Is this a one-sided or two-sided test?

Two-sided.

(c) If the hypothesis had been H_0: $\mu = 90$ versus H_1: $\mu \neq 90$ would you reject the null hypothesis at the 0.05 level?

Yes.

(d) Use the output and the *t* table to find a 99 percent two-sided CI on the mean.

CI = 91.2735, 93.8875

(e) What is the *P*-value if the alternative hypothesis is H_1: $\mu > 91$?

P-value = 0.001.

2.14. Consider the computer output shown below.

One-Sample T: Y

Test of mu = 25 vs > 25

Variable	N	Mean	Std. Dev.	SE Mean	95% Lower Bound	T	P
Y	12	25.6818	?	0.3360	?	?	0.034

(a) How many degrees of freedom are there on the *t*-test statistic?

(N-1) = (12 – 1) = 11

(b) Fill in the missing information.

Std. Dev. = 1.1639 95% Lower Bound = 2.0292

2.15. Consider the computer output shown below.

Two-Sample T-Test and CI: Y1, Y2

Two-sample T for Y1 vs Y2

	N	Mean	Std. Dev.	SE Mean
Y1	20	50.19	1.71	0.38
Y2	20	52.52	2.48	0.55

Difference = mu (X1) – mu (X2)

Estimate for difference: -2.33341

95% CI for difference: (-3.69547, -0.97135)

T-Test of difference = 0 (vs not =) : T-Value = -3.47

P-Value = 0.01 DF = 38

Both use Pooled Std. Dev. = 2.1277

(a) Can the null hypothesis be rejected at the 0.05 level? Why?

Yes, the *P*-Value of 0.001 is much less than 0.05.

(b) Is this a one-sided or two-sided test?

Two-sided.

(c) If the hypothesis had been H_0: $\mu_1 - \mu_2 = 2$ versus H_1: $\mu_1 - \mu_2 \neq 2$ would you reject the null hypothesis at the 0.05 level?

Yes.

(d) If the hypothesis had been H_0: $\mu_1 - \mu_2 = 2$ versus H_1: $\mu_1 - \mu_2 < 2$ would you reject the null hypothesis at the 0.05 level? Can you answer this question without doing any additional calculations? Why?

Yes, no additional calculations are required because the test is naturally becoming more significant with the change from -2.33341 to -4.33341.

(e) Use the output and the *t* table to find a 95 percent upper confidence bound on the difference in means?

95% upper confidence bound = -1.21.

(f) What is the *P*-value if the alternative hypotheses are H_0: $\mu_1 - \mu_2 = 2$ versus H_1: $\mu_1 - \mu_2 \neq 2$?

P-value = 1.4E-07.

2.16. The breaking strength of a fiber is required to be at least 150 psi. Past experience has indicated that the standard deviation of breaking strength is $\sigma = 3$ psi. A random sample of four specimens is tested. The results are y_1=145, y_2=153, y_3=150 and y_4=147.

(a) State the hypotheses that you think should be tested in this experiment.

$$H_0\text{: } \mu = 150 \qquad H_1\text{: } \mu > 150$$

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

$n = 4$, $\sigma = 3$, $\bar{y} = 1/4\ (145 + 153 + 150 + 147) = 148.75$

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{148.75 - 150}{\frac{3}{\sqrt{4}}} = \frac{-1.25}{\frac{3}{2}} = -0.8333$$

Since $z_{0.05} = 1.645$, do not reject.

(c) Find the *P*-value for the test in part (b).

From the *z*-table: $P \cong 1 - \left[0.7967 + (2/3)(0.7995 - 0.7967)\right] = 0.2014$

(d) Construct a 95 percent confidence interval on the mean breaking strength.

The 95% confidence interval is

$$\bar{y} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{y} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$
$$148.75 - (1.96)(3/2) \le \mu \le 148.75 + (1.96)(3/2)$$

$$145.81 \le \mu \le 151.69$$

2.17. The viscosity of a liquid detergent is supposed to average 800 centistokes at 25°C. A random sample of 16 batches of detergent is collected, and the average viscosity is 812. Suppose we know that the standard deviation of viscosity is σ = 25 centistokes.

(a) State the hypotheses that should be tested.

H_0: $\mu = 800$ $\qquad$ H_1: $\mu \neq 800$

(b) Test these hypotheses using α = 0.05. What are your conclusions?

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{812 - 800}{\frac{25}{\sqrt{16}}} = \frac{12}{\frac{25}{4}} = 1.92$$

Since $z_{\alpha/2} = z_{0.025}$ = 1.96, do not reject.

(c) What is the P-value for the test?

(d) Find a 95 percent confidence interval on the mean.

The 95% confidence interval is

$$\bar{y} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{y} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$
$$812 - (1.96)(25/4) \le \mu \le 812 + (1.96)(25/4)$$
$$812 - 12.25 \le \mu \le 812 + 12.25$$
$$799.75 \le \mu \le 824.25$$

2.18. The diameters of steel shafts produced by a certain manufacturing process should have a mean diameter of 0.255 inches. The diameter is known to have a standard deviation of σ = 0.0001 inch. A random sample of 10 shafts has an average diameter of 0.2545 inches.

(a) Set up the appropriate hypotheses on the mean μ.

H_0: $\mu = 0.255$ $\qquad$ H_1: $\mu \neq 0.255$

(b) Test these hypotheses using α = 0.05. What are your conclusions?

$$n = 10, \quad \sigma = 0.0001, \quad \bar{y} = 0.2545$$

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{0.2545 - 0.255}{\frac{0.0001}{\sqrt{10}}} = -15.81$$

Since $z_{0.025} = 1.96$, reject H_0.

(c) Find the *P*-value for this test. $P = 2.6547\text{x}10^{-56}$

(d) Construct a 95 percent confidence interval on the mean shaft diameter.

The 95% confidence interval is

$$\bar{y} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{y} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$0.2545 - (1.96)\left(\frac{0.0001}{\sqrt{10}}\right) \le \mu \le 0.2545 + (1.96)\left(\frac{0.0001}{\sqrt{10}}\right)$$

$$0.254438 \le \mu \le 0.254562$$

2.19. A normally distributed random variable has an unknown mean μ and a known variance $\sigma^2 = 9$. Find the sample size required to construct a 95 percent confidence interval on the mean that has total length of 1.0.

Since $y \sim N(\mu, 9)$, a 95% two-sided confidence interval on μ is

If the total interval is to have width 1.0, then the half-interval is 0.5. Since $z_{\alpha/2} = z_{0.025} = 1.96$,

$$(1.96)\left(3/\sqrt{n}\right) = 0.5$$
$$\sqrt{n} = (1.96)(3/0.5) = 11.76$$
$$n = (11.76)^2 = 138.30 \cong 139$$

2.20. The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

Days	
108	138
124	163
124	159
106	134
115	139

(a) We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.

H_0: $\mu = 120$ H_1: $\mu > 120$

(b) Test these hypotheses using $\alpha = 0.01$. What are your conclusions?

$\bar{y} = 131$
$S^2 = 3438 / 9 = 382$
$S = \sqrt{382} = 19.54$

$$t_0 = \frac{\bar{y} - \mu_0}{S/\sqrt{n}} = \frac{131 - 120}{19.54/\sqrt{10}} = 1.78$$

since $t_{0.01,9} = 2.821$; do not reject H_0

Minitab Output

```
T-Test of the Mean

Test of mu = 120.00 vs mu > 120.00

Variable     N      Mean    StDev   SE Mean        T          P
Shelf Life  10    131.00    19.54      6.18     1.78      0.054

T Confidence Intervals

Variable     N      Mean    StDev  SE Mean       99.0 % CI
Shelf Life  10    131.00    19.54     6.18  (  110.91,  151.09)
```

(c) Find the *P*-value for the test in part (b). P=0.054

(d) Construct a 99 percent confidence interval on the mean shelf life.

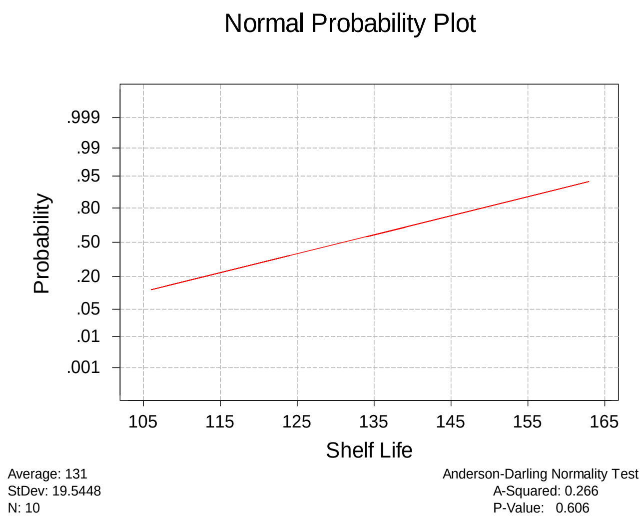
The 99% confidence interval is $\bar{y} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \le \mu \le \bar{y} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$ with $\alpha = 0.01$.

$$131 - (3.250)\left(\frac{19.54}{\sqrt{10}}\right) \le \mu \le 131 + (3.250)\left(\frac{19.54}{\sqrt{10}}\right)$$

$$110.91 \le \mu \le 151.08$$

2.21. Consider the shelf life data in Problem 2.20. Can shelf life be described or modeled adequately by a normal distribution? What effect would violation of this assumption have on the test procedure you used in solving Problem 2.20?

A normal probability plot, obtained from Minitab, is shown. There is no reason to doubt the adequacy of the normality assumption. If shelf life is not normally distributed, then the impact of this on the *t*-test in problem 2.20 is not too serious unless the departure from normality is severe.



2.22. The time to repair an electronic instrument is a normally distributed random variable measured in hours. The repair time for 16 such instruments chosen at random are as follows:

Hours			
159	280	101	212
224	379	179	264
222	362	168	250
149	260	485	170

(a) You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.

$$H_0\text{: } \mu = 225 \qquad H_1\text{: } \mu > 225$$

(b) Test the hypotheses you formulated in part (a). What are your conclusions? Use $\alpha = 0.05$.

$$\bar{y} = 241.50$$
$$S^2 = 146202 / (16 - 1) = 9746.80$$

$$S = \sqrt{9746.8} = 98.73$$

$$t_o = \frac{\bar{y} - \mu_o}{\frac{S}{\sqrt{n}}} = \frac{241.50 - 225}{\frac{98.73}{\sqrt{16}}} = 0.67$$

since $t_{0.05,15} = 1.753$; do not reject H_0

Minitab Output

T-Test of the Mean

```
Test of mu = 225.0 vs mu > 225.0

Variable      N      Mean     StDev   SE Mean        T         P
Hours        16     241.5      98.7      24.7     0.67      0.26
```

T Confidence Intervals

```
Variable      N      Mean     StDev  SE Mean         95.0 % CI
Hours        16     241.5      98.7     24.7   (    188.9,    294.1)
```

(c) Find the *P*-value for this test. *P*=0.26

(d) Construct a 95 percent confidence interval on mean repair time.

The 95% confidence interval is $\bar{y} - t_{\alpha/2,n-1}\dfrac{S}{\sqrt{n}} \le \mu \le \bar{y} + t_{\alpha/2,n-1}\dfrac{S}{\sqrt{n}}$

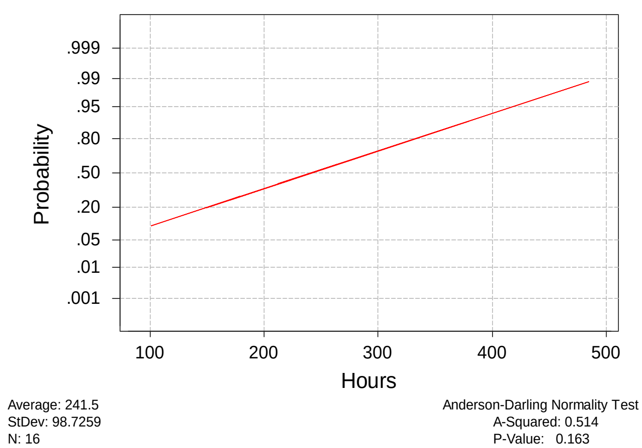
$$241.50-(2.131)\left(\frac{98.73}{\sqrt{16}}\right) \le \mu \le 241.50+(2.131)\left(\frac{98.73}{\sqrt{16}}\right)$$

$$188.9 \le \mu \le 294.1$$

2.23. Reconsider the repair time data in Problem 2.22. Can repair time, in your opinion, be adequately modeled by a normal distribution?

The normal probability plot below does not reveal any serious problem with the normality assumption.

Normal Probability Plot



2.24. Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The filling processes can be assumed to be normal, with standard deviation of σ_1 = 0.015 and σ_2 = 0.018. The quality engineering department suspects that both machines fill to the same net volume, whether or not this volume is 16.0 ounces. An experiment is performed by taking a random sample from the output of each machine.

Machine 1		Machine 2	
16.03	16.01	16.02	16.03
16.04	15.96	15.97	16.04
16.05	15.98	15.96	16.02
16.05	16.02	16.01	16.01
16.02	15.99	15.99	16.00

(a) State the hypotheses that should be tested in this experiment.

H_0: $\mu_1 = \mu_2$ $\quad\quad$ H_1: $\mu_1 \neq \mu_2$

(b) Test these hypotheses using α=0.05. What are your conclusions?

$$\begin{array}{ll} \bar{y}_1 = 16.015 & \bar{y}_2 = 16.005 \\ \sigma_1 = 0.015 & \sigma_2 = 0.018 \\ n_1 = 10 & n_2 = 10 \end{array}$$

$$z_o = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{16.015 - 16.018}{\sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}}} = 1.35$$

$z_{0.025}$ = 1.96; do not reject

(c) What is the *P*-value for the test? P = 0.1770

(d) Find a 95 percent confidence interval on the difference in the mean fill volume for the two machines.

The 95% confidence interval is

$$\bar{y}_1 - \bar{y}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{y}_1 - \bar{y}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(16.015 - 16.005) - (1.96)\sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}} \le \mu_1 - \mu_2 \le (16.015 - 16.005) + (1.96)\sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}}$$

$$-0.0045 \le \mu_1 - \mu_2 \le 0.0245$$

2.25. Two types of plastic are suitable for use by an electronic calculator manufacturer. The breaking strength of this plastic is important. It is known that $\sigma_1 = \sigma_2$ = 1.0 psi. From random samples of n_1 = 10 and n_2 = 12 we obtain $\bar{y}_1$ = 162.5 and $\bar{y}_2$ = 155.0. The company will not adopt plastic 1 unless its breaking strength exceeds that of plastic 2 by at least 10 psi. Based on the sample information, should they use plastic 1? In answering this questions, set up and test appropriate hypotheses using α = 0.01. Construct a 99 percent confidence interval on the true mean difference in breaking strength.

H_0: $\mu_1 - \mu_2 = 10$ $\quad\quad$ H_1: $\mu_1 - \mu_2 > 10$

$$\begin{array}{ll} \bar{y}_1 = 162.5 & \bar{y}_2 = 155.0 \\ \sigma_1 = 1 & \sigma_2 = 1 \\ n_1 = 10 & n_2 = 10 \end{array}$$

$$z_o = \frac{\bar{y}_1 - \bar{y}_2 - 10}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{162.5 - 155.0 - 10}{\sqrt{\frac{1^2}{10} + \frac{1^2}{12}}} = -5.84$$

$z_{0.01}$ = 2.325; do not reject

The 99 percent confidence interval is

$$\bar{y}_1 - \bar{y}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{y}_1 - \bar{y}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$(162.5-155.0)-(2.575)\sqrt{\frac{1^2}{10}+\frac{1^2}{12}} \le \mu_1 - \mu_2 \le (162.5-155.0)+(2.575)\sqrt{\frac{1^2}{10}+\frac{1^2}{12}}$$

$$6.40 \le \mu_1 - \mu_2 \le 8.60$$

2.26. The following are the burning times (in minutes) of chemical flares of two different formulations. The design engineers are interested in both the means and variance of the burning times.

Type 1		Type 2	
65	82	64	56
81	67	71	69
57	59	83	74
66	75	59	82
82	70	65	79

(a) Test the hypotheses that the two variances are equal. Use $\alpha = 0.05$.

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 \ne \sigma_2^2$$

Do not reject.

(b) Using the results of (a), test the hypotheses that the mean burning times are equal. Use $\alpha = 0.05$. What is the *P*-value for this test?

Do not reject.

From the computer output, t=0.05; do not reject. Also from the computer output P=0.96

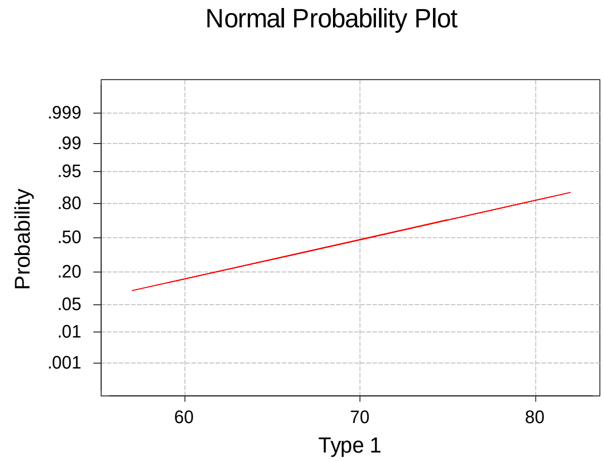
Minitab Output

Two Sample T-Test and Confidence Interval

```
Two sample T for Type 1 vs Type 2

         N      Mean     StDev   SE Mean
Type 1  10     70.40      9.26       2.9
Type 2  10     70.20      9.37       3.0

95% CI for mu Type 1 - mu Type 2: ( -8.6,  9.0)
T-Test mu Type 1 = mu Type 2 (vs not =): T = 0.05  P = 0.96  DF = 18
Both use Pooled StDev = 9.32
```

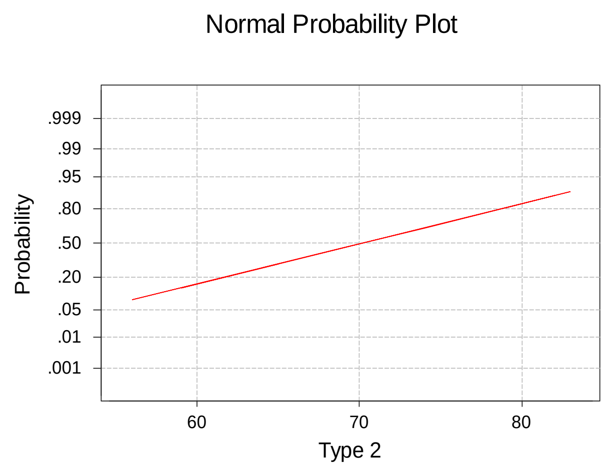
(c) Discuss the role of the normality assumption in this problem. Check the assumption of normality for both types of flares.

The assumption of normality is required in the theoretical development of the *t*-test. However, moderate departure from normality has little impact on the performance of the *t*-test. The normality assumption is more important for the test on the equality of the two variances. An indication of nonnormality would be of concern here. The normal probability plots shown below indicate that burning time for both formulations follow the normal distribution.



Average: 70.4
StDev: 9.26403
N: 10

Anderson-Darling Normality Test
A-Squared: 0.344
P-Value: 0.409



Average: 70.2
StDev: 9.36661
N: 10

Anderson-Darling Normality Test
A-Squared: 0.186
P-Value: 0.876

2.27. An article in *Solid State Technology*, "Orthogonal Design of Process Optimization and Its Application to Plasma Etching" by G.Z. Yin and D.W. Jillie (May, 1987) describes an experiment to determine the effect of C_2F_6 flow rate on the uniformity of the etch on a silicon wafer used in integrated circuit manufacturing. Data for two flow rates are as follows:

C_2F_6	Uniformity Observation					
(SCCM)	1	2	3	4	5	6
125	2.7	4.6	2.6	3.0	3.2	3.8
200	4.6	3.4	2.9	3.5	4.1	5.1

(a) Does the C_2F_6 flow rate affect average etch uniformity? Use $\alpha = 0.05$.

No, C_2F_6 flow rate does not affect average etch uniformity.

Minitab Output

```
Two Sample T-Test and Confidence Interval

Two sample T for Uniformity

Flow Rat    N      Mean     StDev   SE Mean
125         6     3.317     0.760      0.31
200         6     3.933     0.821      0.34

95% CI for mu (125) - mu (200): ( -1.63,  0.40)
T-Test mu (125) = mu (200) (vs not =): T = -1.35  P = 0.21  DF = 10
Both use Pooled StDev = 0.791
```

(b) What is the *P*-value for the test in part (a)? From the *Minitab* output, *P*=0.21

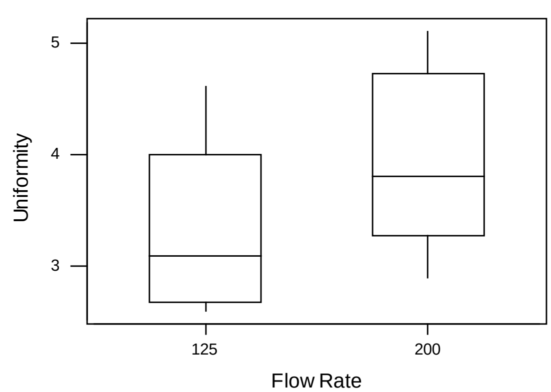
(c) Does the C_2F_6 flow rate affect the wafer-to-wafer variability in etch uniformity? Use $\alpha = 0.05$.

$$H_0: \sigma_1^2 = \sigma_2^2$$
$$H_1: \sigma_1^2 \neq \sigma_2^2$$
$$F_{0.025,5,5} = 7.15$$
$$F_{0.975,5,5} = 0.14$$
$$F_0 = \frac{0.5776}{0.6724} = 0.86$$

Do not reject; C_2F_6 flow rate does not affect wafer-to-wafer variability.

(d) Draw box plots to assist in the interpretation of the data from this experiment.

The box plots shown below indicate that there is little difference in uniformity at the two gas flow rates. Any observed difference is not statistically significant. See the *t*-test in part (a).



2.28. A new filtering device is installed in a chemical unit. Before its installation, a random sample yielded the following information about the percentage of impurity: $\bar{y}_1 = 12.5$, $S_1^2 = 101.17$, and $n_1 = 8$. After installation, a random sample yielded $\bar{y}_2 = 10.2$, $S_2^2 = 94.73$, $n_2 = 9$.

(a) Can you conclude that the two variances are equal? Use $\alpha = 0.05$.

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 \neq \sigma_2^2$$
$$F_{0.025,7,8} = 4.53$$
$$F_0 = \frac{S_1^2}{S_2^2} = \frac{101.17}{94.73} = 1.07$$

Do not reject. Assume that the variances are equal.

(b) Has the filtering device reduced the percentage of impurity significantly? Use $\alpha = 0.05$.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 > \mu_2$$
$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{(8-1)(101.17) + (9-1)(94.73)}{8+9-2} = 97.74$$
$$S_p = 9.89$$
$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{12.5 - 10.2}{9.89\sqrt{\frac{1}{8} + \frac{1}{9}}} = 0.479$$
$$t_{0.05,15} = 1.753$$

Do not reject. There is no evidence to indicate that the new filtering device has affected the mean.

2.29. Photoresist is a light-sensitive material applied to semiconductor wafers so that the circuit pattern can be imaged on to the wafer. After application, the coated wafers are baked to remove the solvent in the photoresist mixture and to harden the resist. Here are measurements of photoresist thickness (in kÅ) for eight wafers baked at two different temperatures. Assume that all of the runs were made in random order.

95 ºC	100 ºC
11.176	5.623
7.089	6.748
8.097	7.461
11.739	7.015
11.291	8.133
10.759	7.418
6.467	3.772
8.315	8.963

(a) Is there evidence to support the claim that the higher baking temperature results in wafers with a lower mean photoresist thickness? Use $\alpha = 0.05$.

$$
\begin{aligned}
&H_0 : \mu_1 = \mu_2 \\
&H_1 : \mu_1 > \mu_2 \\
&S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{(8-1)(4.41) + (8-1)(2.54)}{8+8-2} = 3.48 \\
&S_p = 1.86 \\
&t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{9.37 - 6.89}{1.86\sqrt{\frac{1}{8} + \frac{1}{8}}} = 2.65 \\
&t_{0.05,14} = 1.761
\end{aligned}
$$

Since $t_{0.05,14} = 1.761$, reject H_0. There appears to be a lower mean thickness at the higher temperature. This is also seen in the computer output.

Minitab Output

Two-Sample T-Test and CI: Thickness, Temp

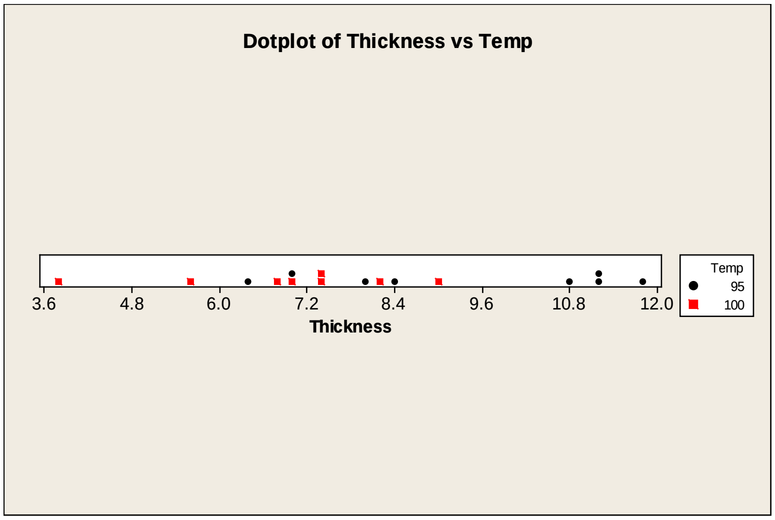
```
Two-sample T for Thick@95 vs Thick@100

          N      Mean     StDev   SE Mean
Thick@95  8      9.37      2.10      0.74
Thick@10  8      6.89      1.60      0.56

Difference = mu Thick@95 - mu Thick@100
Estimate for difference:  2.475
95% lower bound for difference: 0.833
T-Test of difference = 0 (vs >): T-Value = 2.65  P-Value = 0.009  DF = 14
Both use Pooled StDev = 1.86
```

(b) What is the P-value for the test conducted in part (a)? $P = 0.009$

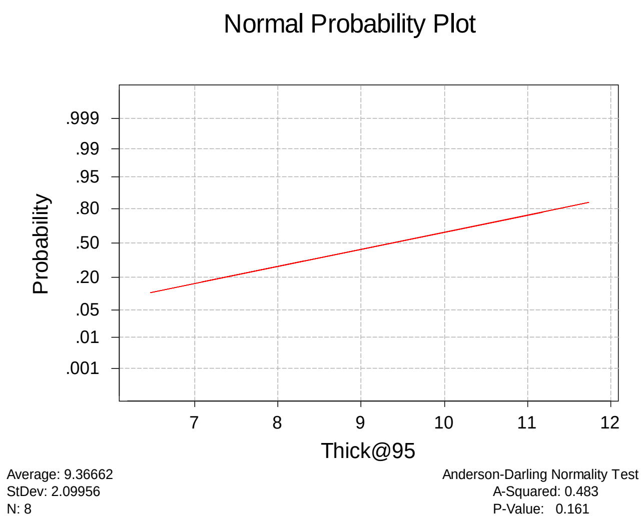
(c) Find a 95% confidence interval on the difference in means. Provide a practical interpretation of this interval.

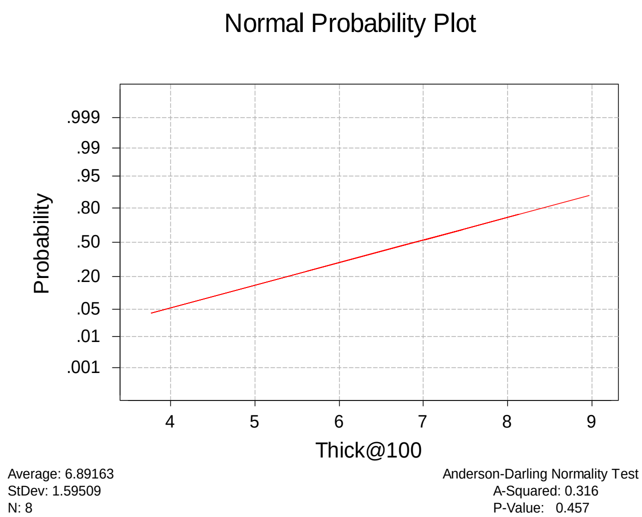
From the computer output the 95% lower confidence bound is $0.833 \le \mu_1 - \mu_2$. This lower confidence bound is greater than 0; therefore, there is a difference in the two temperatures on the thickness of the photoresist.

(d) Draw dot diagrams to assist in interpreting the results from this experiment.



(e) Check the assumption of normality of the photoresist thickness.





There are no significant deviations from the normality assumptions.

(f) Find the power of this test for detecting an actual difference in means of 2.5 kÅ.

Minitab Output

```
Power and Sample Size

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)
Calculating power for mean 1 = mean 2 + difference
Alpha = 0.05  Sigma = 1.86

            Sample
Difference    Size   Power
       2.5       8  0.7056
```

(g) What sample size would be necessary to detect an actual difference in means of 1.5 kÅ with a power of at least 0.9?.

Minitab Output

```
Power and Sample Size

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)
Calculating power for mean 1 = mean 2 + difference
Alpha = 0.05  Sigma = 1.86

            Sample  Target  Actual
Difference    Size   Power   Power
       1.5      34  0.9000  0.9060
```

This result makes intuitive sense. More samples are needed to detect a smaller difference.

2.30. Front housings for cell phones are manufactured in an injection molding process. The time the part is allowed to cool in the mold before removal is thought to influence the occurrence of a particularly troublesome cosmetic defect, flow lines, in the finished housing. After manufacturing, the housings are inspected visually and assigned a score between 1 and 10 based on their appearance, with 10 corresponding to a perfect part and 1 corresponding to a completely defective part. An experiment was conducted using two cool-down times, 10 seconds and 20 seconds, and 20 housings were evaluated at each level of cool-down time. All 40 observations in this experiment were run in random order. The data are shown below.

10 Seconds		20 Seconds	
1	3	7	6
2	6	8	9
1	5	5	5
3	3	9	7
5	2	5	4
1	1	8	6
5	6	6	8
2	8	4	5
3	2	6	8
5	3	7	7

(a) Is there evidence to support the claim that the longer cool-down time results in fewer appearance defects? Use $\alpha = 0.05$.

From the analysis shown below, there is evidence that the longer cool-down time results in fewer appearance defects.

Minitab Output

Two-Sample T-Test and CI: 10 seconds, 20 seconds

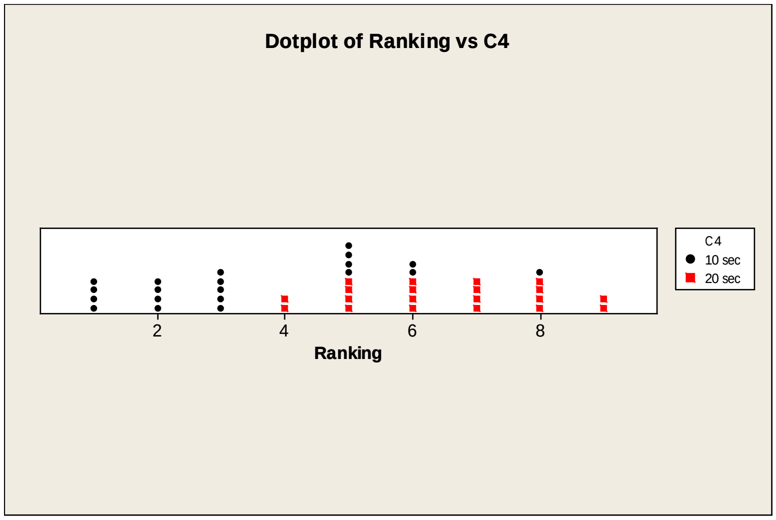
```
Two-sample T for 10 seconds vs 20 seconds

           N      Mean     StDev   SE Mean
10 secon  20      3.35      2.01      0.45
20 secon  20      6.50      1.54      0.34

Difference = mu 10 seconds - mu 20 seconds
Estimate for difference:  -3.150
95% upper bound for difference: -2.196
T-Test of difference = 0 (vs <): T-Value = -5.57  P-Value = 0.000  DF = 38
Both use Pooled StDev = 1.79
```

(b) What is the *P*-value for the test conducted in part (a)? From the *Minitab* output, $P = 0.000$

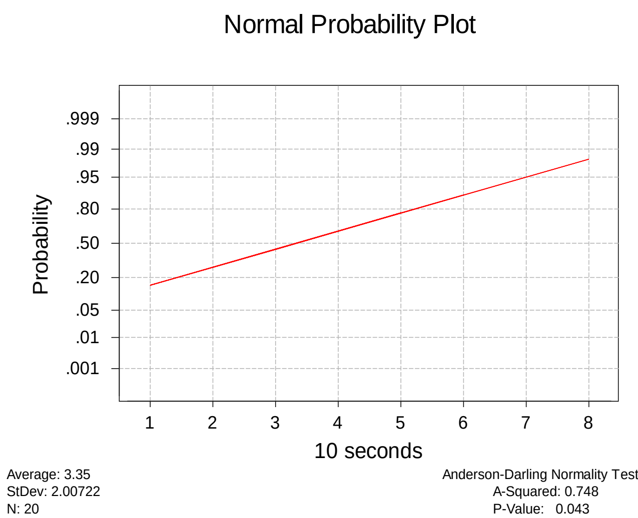
(c) Find a 95% confidence interval on the difference in means. Provide a practical interpretation of this interval.

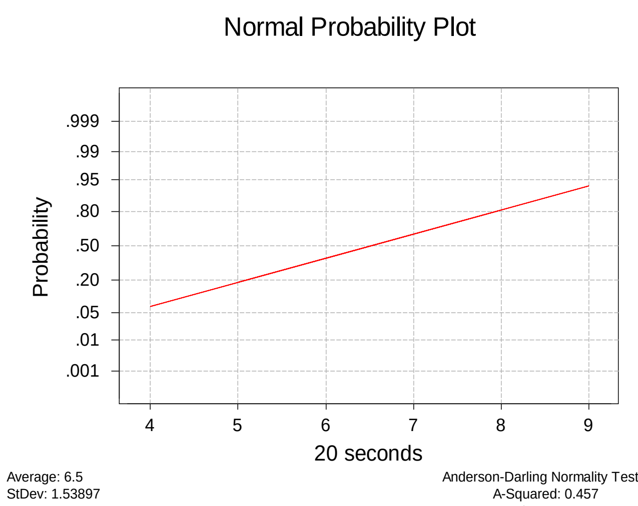
From the *Minitab* output, $\mu_1 - \mu_2 \leq -2.196$. This lower confidence bound is less than 0. The two samples are different. The 20 second cooling time gives a cosmetically better housing.

(d) Draw dot diagrams to assist in interpreting the results from this experiment.



(e) Check the assumption of normality for the data from this experiment.





There are no significant departures from normality.

2.31. Twenty observations on etch uniformity on silicon wafers are taken during a qualification experiment for a plasma etcher. The data are as follows:

Etch Uniformity				
5.34	6.65	4.76	5.98	7.25
6.00	7.55	5.54	5.62	6.21
5.97	7.35	5.44	4.39	4.98
5.25	6.35	4.61	6.00	5.32

(a) Construct a 95 percent confidence interval estimate of σ^2.

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{(1-\alpha/2),n-1}}$$

$$\frac{(20-1)(0.88907)^2}{32.852} \le \sigma^2 \le \frac{(20-1)(0.88907)^2}{8.907}$$

$$0.457 \le \sigma^2 \le 1.686$$

(b) Test the hypothesis that $\sigma^2 = 1.0$. Use $\alpha = 0.05$. What are your conclusions?

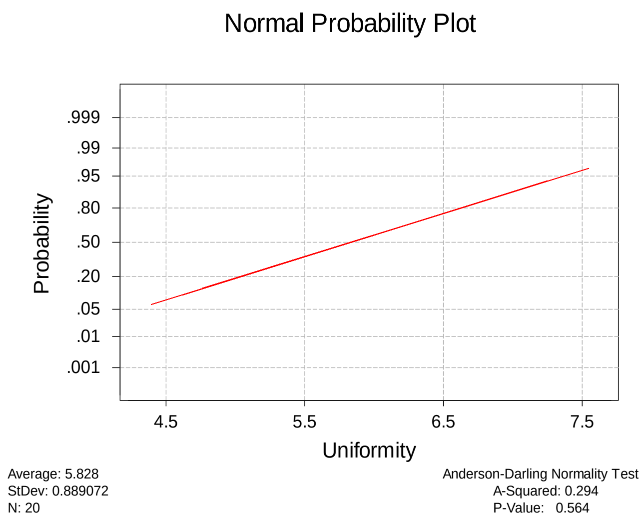
Do not reject. There is no evidence to indicate that $\sigma^2 \neq 1$

(c) Discuss the normality assumption and its role in this problem.

The normality assumption is much more important when analyzing variances then when analyzing means. A moderate departure from normality could cause problems with both statistical tests and confidence intervals. Specifically, it will cause the reported significance levels to be incorrect.

(d) Check normality by constructing a normal probability plot. What are your conclusions?

The normal probability plot indicates that there is not a serious problem with the normality assumption.



2.32. The diameter of a ball bearing was measured by 12 inspectors, each using two different kinds of calipers. The results were:

Inspector	Caliper 1	Caliper 2	Difference	Difference^2
1	0.265	0.264	0.001	0.000001
2	0.265	0.265	0.000	0
3	0.266	0.264	0.002	0.000004
4	0.267	0.266	0.001	0.000001
5	0.267	0.267	0.000	0
6	0.265	0.268	-0.003	0.000009
7	0.267	0.264	0.003	0.000009
8	0.267	0.265	0.002	0.000004
9	0.265	0.265	0.000	0
10	0.268	0.267	0.001	0.000001
11	0.268	0.268	0.000	0
12	0.265	0.269	-0.004	0.000016
			$\sum = 0.003$	$\sum = 0.000045$

(a) Is there a significant difference between the means of the population of measurements represented by the two samples? Use $\alpha = 0.05$.

$$\begin{aligned} H_0 &: \mu_1 = \mu_2 \\ H_1 &: \mu_1 \neq \mu_2 \end{aligned} \quad \text{or equivalently} \quad \begin{aligned} H_0 &: \mu_d = 0 \\ H_1 &: \mu_d \neq 0 \end{aligned}$$

Minitab Output

Paired T-Test and Confidence Interval

```
Paired T for Caliper 1 - Caliper 2

                 N      Mean     StDev   SE Mean
Caliper         12  0.266250  0.001215  0.000351
Caliper         12  0.266000  0.001758  0.000508
Difference      12  0.000250  0.002006  0.000579

95% CI for mean difference: (-0.001024, 0.001524)
T-Test of mean difference = 0 (vs not = 0): T-Value = 0.43  P-Value = 0.674
```

(b) Find the *P*-value for the test in part (a). *P*=0.674

(c) Construct a 95 percent confidence interval on the difference in the mean diameter measurements for the two types of calipers.

$$\bar{d} - t_{\alpha/2, n-1} \frac{S_d}{\sqrt{n}} \le \mu_D \left(= \mu_1 - \mu_2\right) \le \bar{d} + t_{\alpha/2, n-1} \frac{S_d}{\sqrt{n}}$$

$$0.00025 - 2.201 \frac{0.002}{\sqrt{12}} \le \mu_d \le 0.00025 + 2.201 \frac{0.002}{\sqrt{12}}$$

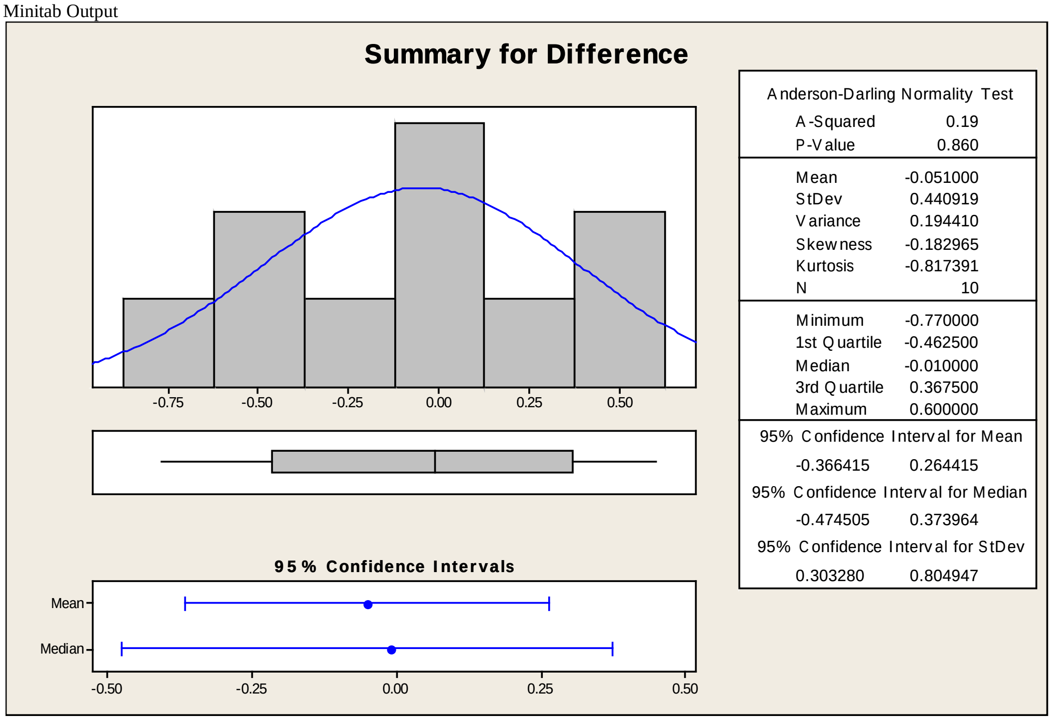
$$-0.00102 \le \mu_d \le 0.00152$$

2.33. An article in the journal of *Neurology* (1998, Vol. 50, pp.1246-1252) observed that the monozygotic twins share numerous physical, psychological and pathological traits. The investigators measured an intelligence score of 10 pairs of twins. The data are obtained as follows:

Pair	Birth Order: 1	Birth Order: 2
1	6.08	5.73
2	6.22	5.80
3	7.99	8.42
4	7.44	6.84
5	6.48	6.43
6	7.99	8.76
7	6.32	6.32
8	7.60	7.62
9	6.03	6.59
10	7.52	7.67

(a) Is the assumption that the difference in score is normally distributed reasonable?

Minitab Output



By plotting the differences, the output shows that the Anderson-Darling Normality Test shows a P-Value of 0.860. The data is assumed to be normal.

(b) Find a 95% confidence interval on the difference in the mean score. Is there any evidence that mean score depends on birth order?

The 95% confidence interval on the difference in mean score is (-0.366415, 0.264415) contains the value of zero. There is no difference in birth order.

(c) Test an appropriate set of hypothesis indicating that the mean score does not depend on birth order.

$$H_0: \mu_1 = \mu_2 \quad \text{or equivalently} \quad H_0: \mu_d = 0$$
$$H_1: \mu_1 \neq \mu_2 \quad \qquad \qquad \quad H_1: \mu_d \neq 0$$

Minitab Output

```
Paired T for Birth Order: 1 - Birth Order: 2

                 N    Mean  StDev  SE Mean
Birth Order: 1  10   6.967  0.810    0.256
Birth Order: 2  10   7.018  1.053    0.333
Difference      10  -0.051  0.441    0.139


95% CI for mean difference: (-0.366, 0.264)
T-Test of mean difference = 0 (vs not = 0): T-Value = -0.37  P-Value = 0.723
```

Do not reject. The *P*-value is 0.723.

2.34. An article in the *Journal of Strain Analysis* (vol.18, no. 2, 1983) compares several procedures for predicting the shear strength for steel plate girders. Data for nine girders in the form of the ratio of predicted to observed load for two of these procedures, the Karlsruhe and Lehigh methods, are as follows:

Girder	Karlsruhe Method	Lehigh Method	Difference	Difference^2
S1/1	1.186	1.061	0.125	0.015625
S2/1	1.151	0.992	0.159	0.025281
S3/1	1.322	1.063	0.259	0.067081
S4/1	1.339	1.062	0.277	0.076729
S5/1	1.200	1.065	0.135	0.018225
S2/1	1.402	1.178	0.224	0.050176
S2/2	1.365	1.037	0.328	0.107584
S2/3	1.537	1.086	0.451	0.203401
S2/4	1.559	1.052	0.507	0.257049
		Sum =	2.465	0.821151
		Average =	0.274	

(a) Is there any evidence to support a claim that there is a difference in mean performance between the two methods? Use $\alpha = 0.05$.

$$H_0: \mu_1 = \mu_2 \quad \text{or equivalently} \quad H_0: \mu_d = 0$$
$$H_1: \mu_1 \neq \mu_2 \quad \qquad \qquad \quad H_1: \mu_d \neq 0$$

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i = \frac{1}{9}(2.465) = 0.274$$

$$s_d = \left[\frac{\sum_{i=1}^{n} d_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} d_i\right)^2}{n-1}\right]^{1/2} = \left[\frac{0.821151 - \frac{1}{9}(2.465)^2}{9-1}\right]^{1/2} = 0.135$$

$$t_0 = \frac{\bar{d}}{\frac{S_d}{\sqrt{n}}} = \frac{0.274}{\frac{0.135}{\sqrt{9}}} = 6.08$$

$t_{\alpha/2,n-1} = t_{0.025,8} = 2.306$, reject the null hypothesis.

Minitab Output

Paired T-Test and Confidence Interval

Paired T for Karlsruhe - Lehigh

	N	Mean	StDev	SE Mean
Karlsruh	9	1.3401	0.1460	0.0487
Lehigh	9	1.0662	0.0494	0.0165
Difference	9	0.2739	0.1351	0.0450

95% CI for mean difference: (0.1700, 0.3777)
T-Test of mean difference = 0 (vs not = 0): T-Value = 6.08 P-Value = 0.000

(b) What is the *P*-value for the test in part (a)?

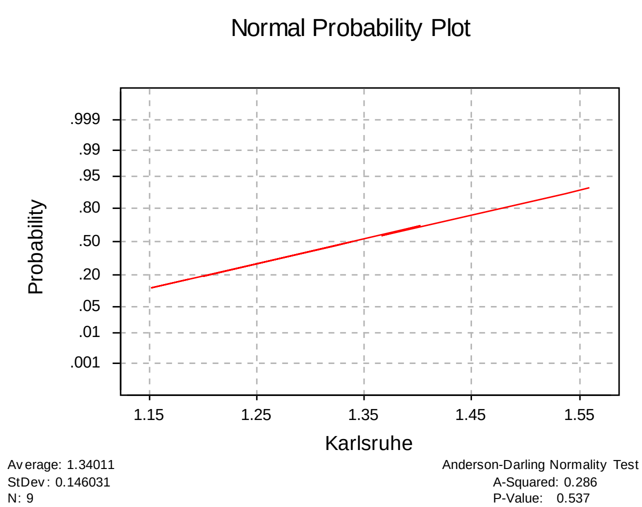
P=0.0002

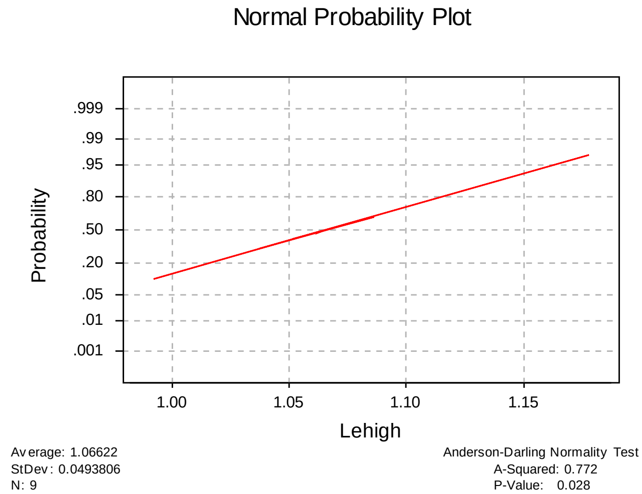
(c) Construct a 95 percent confidence interval for the difference in mean predicted to observed load.

$$\bar{d} - t_{\alpha/2,n-1}\frac{S_d}{\sqrt{n}} \le \mu_d \le \bar{d} + t_{\alpha/2,n-1}\frac{S_d}{\sqrt{n}}$$
$$0.274 - 2.306\frac{0.135}{\sqrt{9}} \le \mu_d \le 0.274 + 2.306\frac{0.135}{\sqrt{9}}$$
$$0.17023 \le \mu_d \le 0.37777$$

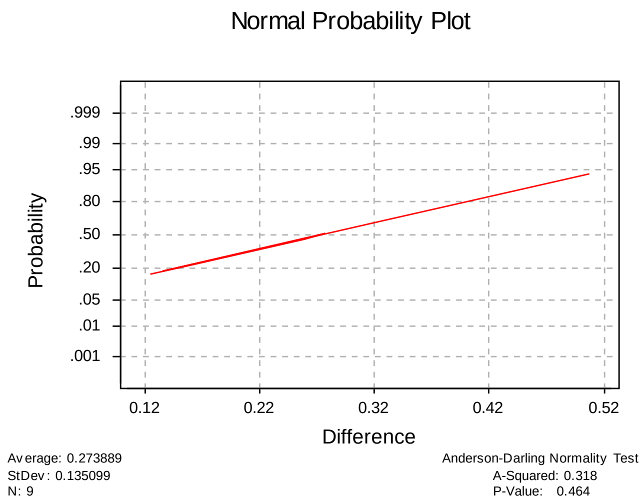
(d) Investigate the normality assumption for both samples.

The normal probability plots of the observations for each method follow. There are no serious concerns with the normality assumption, but there is an indication of a possible outlier (1.178) in the Lehigh method data.





(e) Investigate the normality assumption for the difference in ratios for the two methods.



There is no issue with normality in the difference of ratios of the two methods.

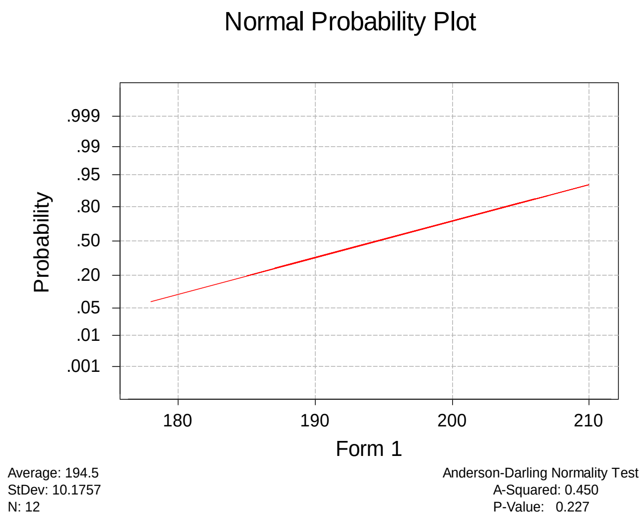
(f) Discuss the role of the normality assumption in the paired *t*-test.

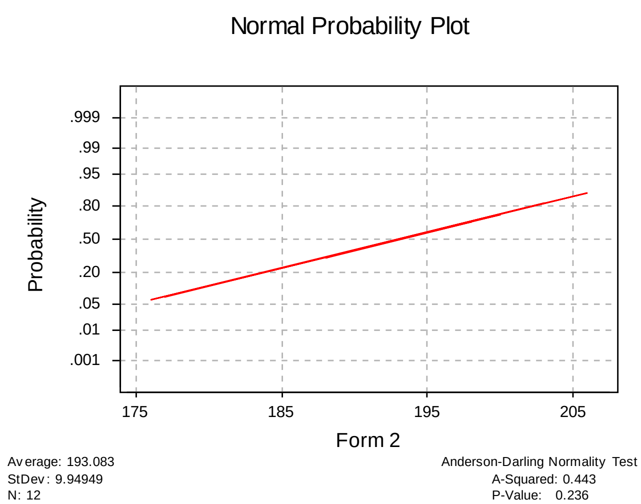
As in any *t*-test, the assumption of normality is of only moderate importance. In the paired *t*-test, the assumption of normality applies to the distribution of the differences. That is, the individual sample measurements do not have to be normally distributed, only their difference.

2.35. The deflection temperature under load for two different formulations of ABS plastic pipe is being studied. Two samples of 12 observations each are prepared using each formulation, and the deflection temperatures (in °F) are reported below:

Formulation 1			Formulation 2		
206	193	192	177	176	198
188	207	210	197	185	188
205	185	194	206	200	189
187	189	178	201	197	203

(a) Construct normal probability plots for both samples. Do these plots support assumptions of normality and equal variance for both samples?





(b) Do the data support the claim that the mean deflection temperature under load for formulation 1 exceeds that of formulation 2? Use $\alpha = 0.05$.

No, formulation 1 does not exceed formulation 2 per the *Minitab* output below.

Minitab Output

Two Sample T-Test and Confidence Interval

```
         N      Mean     StDev   SE Mean
Form 1  12     194.5      10.2       2.9
Form 2  12    193.08      9.95       2.9

Difference = mu Form 1 - mu Form 2
Estimate for difference:  1.42
95% lower bound for difference: -5.64
T-Test of difference = 0 (vs >): T-Value = 0.34  P-Value = 0.367  DF = 22
Both use Pooled StDev = 10.1
```

(c) What is the *P*-value for the test in part (a)?

$P = 0.367$

2.36. Refer to the data in problem 2.35. Do the data support a claim that the mean deflection temperature under load for formulation 1 exceeds that of formulation 2 by at least 3 °F?

No, formulation 1 does not exceed formulation 2 by at least 3 °F.

Minitab Output

Two-Sample T-Test and CI: Form1, Form2

```
Two-sample T for Form 1 vs Form 2

         N      Mean     StDev   SE Mean
Form 1  12     194.5      10.2       2.9
Form 2  12    193.08      9.95       2.9

Difference = mu Form 1 - mu Form 2
Estimate for difference:  1.42
95% lower bound for difference: -5.64
T-Test of difference = 3 (vs >): T-Value = -0.39  P-Value = 0.648  DF = 22
Both use Pooled StDev = 10.1
```

2.37. In semiconductor manufacturing, wet chemical etching is often used to remove silicon from the backs of wafers prior to metalization. The etch rate is an important characteristic of this process. Two different etching solutions are being evaluated. Eight randomly selected wafers have been etched in each solution and the observed etch rates (in mils/min) are shown below:

Solution 1		Solution 2	
9.9	10.6	10.2	10.6
9.4	10.3	10.0	10.2
10.0	9.3	10.7	10.4
10.3	9.8	10.5	10.3

(a) Do the data indicate that the claim that both solutions have the same mean etch rate is valid? Use α = 0.05 and assume equal variances.

No, the solutions do not have the same mean etch rate. See the Minitab output below.

Minitab Output

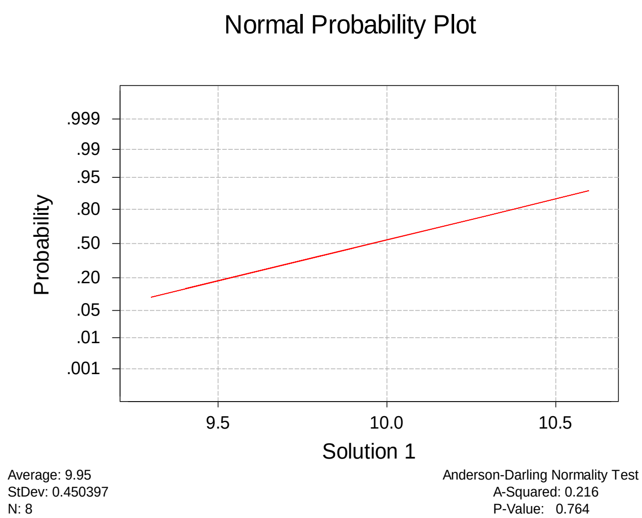
Two Sample T-Test and Confidence Interval

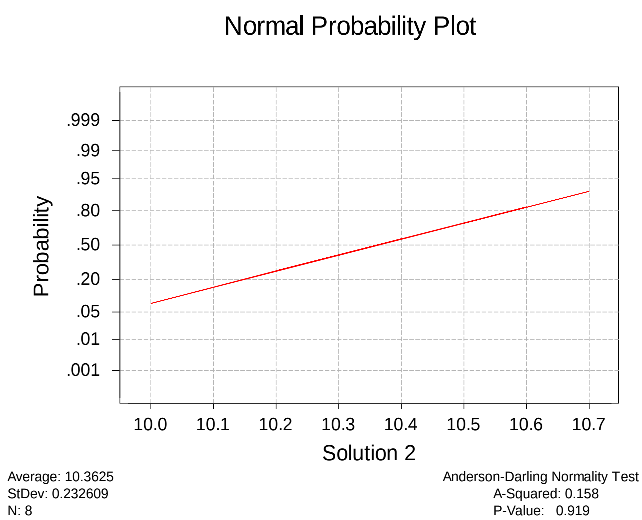
```
Two-sample T for Solution 1 vs Solution 2

          N      Mean     StDev   SE Mean
Solution  8     9.950     0.450      0.16
Solution  8    10.363     0.233     0.082

Difference = mu Solution 1 - mu Solution 2
Estimate for difference:  -0.413
95% CI for difference: (-0.797, -0.028)
T-Test of difference = 0 (vs not =): T-Value = -2.30  P-Value = 0.037  DF = 14
Both use Pooled StDev = 0.358
```

(b) Find a 95% confidence interval on the difference in mean etch rate.

From the Minitab output, -0.797 to –0.028.

(c) Use normal probability plots to investigate the adequacy of the assumptions of normality and equal variances.





Both the normality and equality of variance assumptions are valid.

2.38. Two popular pain medications are being compared on the basis of the speed of absorption by the body. Specifically, tablet 1 is claimed to be absorbed twice as fast as tablet 2. Assume that σ_1^2 and σ_2^2 are known. Develop a test statistic for

$$H_0\text{: } 2\mu_1 = \mu_2$$
$$H_1\text{: } 2\mu_1 \neq \mu_2$$

$2\bar{y}_1 - \bar{y}_2 \sim N\left(2\mu_1 - \mu_2, \dfrac{4\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}\right)$, assuming that the data is normally distributed.

The test statistic is: $z_o = \dfrac{2\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{4\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$, reject if $|z_o| > z_{\alpha/2}$

2.39. Continuation of Problem 2.38. An article in *Nature* (1972, pp.225-226) reported on the levels of monoamine oxidase in blood platelets for a sample of 43 schizophrenic patients resulting in $\bar{y}_1 = 2.69$ and $s_1 = 2.30$ while for a sample of 45 normal patients the results were $\bar{y}_2 = 6.35$ and $s_2 = 4.03$. The units are nm/mg protein/h. Use the results of the previous problem to test the claim that the mean monoamine oxidase level for normal patients is at least twice the mean level for schizophrenic patients. Assume that the sample sizes are large enough to use the sample standard deviations as the true parameter values.

$$z_o = \frac{2\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{4\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \frac{2(2.69) - 6.35}{\sqrt{\dfrac{4(2.30)^2}{43} + \dfrac{4.03^2}{45}}} = \frac{-0.97}{.92357} = -1.05$$

$z_0 = -1.05$; using $\alpha=0.05$, $z_{\alpha/2} = 1.96$, do not reject.

2.40. Suppose we are testing

H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$

where σ_1^2 and σ_2^2 are known. Our sampling resources are constrained such that $n_1 + n_2 = N$. How should we allocate the n_1, n_2 to the two samples that lead to the most powerful test?

The most powerful test is attained by the n_1 and n_2 that maximize z_o for given $\bar{y}_1 - \bar{y}_2$.
Thus, we chose n_1 and n_2 to $\max z_o = \dfrac{\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$, subject to $n_1 + n_2 = N$.

This is equivalent to min $L = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2} = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{N - n_1}$, subject to $n_1 + n_2 = N$.

Now $\dfrac{dL}{dn_1} = \dfrac{-\sigma_1^2}{n_1^2} + \dfrac{\sigma_2^2}{(N - n_1)^2} = 0$, implies that $n_1 / n_2 = \sigma_1 / \sigma_2$.

Thus n_1 and n_2 are assigned proportionally to the ratio of the standard deviations. This has intuitive appeal, as it allocates more observations to the population with the greatest variability.

2.41 Continuation of Problem 2.40. Suppose that we want to construct a 95% two-sided confidence interval on the difference in two means where the two sample standard deviations are known to be $\sigma_1 = 4$ and $\sigma_2 = 8$. The total sample size is restricted to $N = 30$. What is the length of the 95% CI if the sample sizes used by the experimenter are $n_1 = n_2 = 15$? How much shorter would the 95% CI have been if the experiment had used the optimal sample size calculation?

The 95% confidence interval for $n_1 = n_2 = 15$ is

$$(\bar{y}_1 - \bar{y}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$
$$(\bar{y}_1 - \bar{y}_2) - z_{\alpha/2}\sqrt{\frac{4^2}{15} + \frac{8^2}{15}} \le \mu_1 - \mu_2 \le (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2}\sqrt{\frac{4^2}{15} + \frac{8^2}{15}}$$
$$(\bar{y}_1 - \bar{y}_2) - z_{\alpha/2}(2.31) \le \mu_1 - \mu_2 \le (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2}(2.31)$$

The 95% confidence interval for the proportions is,

$$n_1 = 30 - n_2$$

$$\frac{n_1}{n_2} = \frac{\sigma_1}{\sigma_2} = \frac{30 - n_2}{n_2} = \frac{4}{8}$$

Therefore n_2 = 20 and n_1 = 10

$$(\bar{y}_1 - \bar{y}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$
$$(\bar{y}_1 - \bar{y}_2) - z_{\alpha/2}\sqrt{\frac{4^2}{10} + \frac{8^2}{20}} \le \mu_1 - \mu_2 \le (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2}\sqrt{\frac{4^2}{10} + \frac{8^2}{20}}$$
$$(\bar{y}_1 - \bar{y}_2) - z_{\alpha/2}(2.19) \le \mu_1 - \mu_2 \le (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2}(2.19)$$

The confidence interval decreases from a multiple of 2.31 to a multiple of 2.19.

2.42. Develop Equation 2.46 for a 100(1 - α) percent confidence interval for the variance of a normal distribution.

$\frac{SS}{\sigma^2} \sim \chi^2_{n-1}$. Thus, $P\left\{\chi^2_{1-\frac{\alpha}{2},n-1} \le \frac{SS}{\sigma^2} \le \chi^2_{\frac{\alpha}{2},n-1}\right\} = 1-\alpha$. Therefore,

$P\left\{\frac{SS}{\chi^2_{\frac{\alpha}{2},n-1}} \le \sigma^2 \le \frac{SS}{\chi^2_{1-\frac{\alpha}{2},n-1}}\right\} = 1-\alpha$,

so $\left[\frac{SS}{\chi^2_{\frac{\alpha}{2},n-1}}, \frac{SS}{\chi^2_{1-\frac{\alpha}{2},n-1}}\right]$ is the 100(1 - α)% confidence interval on σ^2.

2.43. Develop Equation 2.50 for a 100(1 - α) percent confidence interval for the ratio σ_1^2 / σ_2^2, where σ_1^2 and σ_2^2 are the variances of two normal distributions.

$$\frac{S_2^2/\sigma_2^2}{S_1^2/\sigma_1^2} \sim F_{n_2-1,n_1-1}$$
$$P\left\{F_{1-\alpha/2,n_2-1,n_1-1} \le \frac{S_2^2/\sigma_2^2}{S_1^2/\sigma_1^2} \le F_{\alpha/2,n_2-1,n_1-1}\right\} = 1-\alpha \quad \text{or}$$
$$P\left\{\frac{S_1^2}{S_2^2}F_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2}F_{\alpha/2,n_2-1,n_1-1}\right\} = 1-\alpha$$

2.44. Develop an equation for finding a 100(1 - α) percent confidence interval on the difference in the means of two normal distributions where $\sigma_1^2 \neq \sigma_2^2$. Apply your equation to the portland cement experiment data, and find a 95% confidence interval.

$$\frac{(\bar{y}_1-\bar{y}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}} \sim t_{\alpha/2,\upsilon}$$

$$t_{\alpha/2,\upsilon}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}} \le (\bar{y}_1-\bar{y}_2)-(\mu_1-\mu_2) \le t_{\alpha/2,\upsilon}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}$$

$$(\bar{y}_1-\bar{y}_2)-t_{\alpha/2,\upsilon}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}} \le (\mu_1-\mu_2) \le (\bar{y}_1-\bar{y}_2)+t_{\alpha/2,\upsilon}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}$$

$$\text{where} \quad \upsilon = \frac{\left(\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1-1}+\frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2-1}}$$

Using the data from Table 2.1

$$(16.764-17.343)-2.110\sqrt{\frac{0.100138}{10}+\frac{0.0614622}{10}} \le (\mu_1-\mu_2) \le$$
$$(16.764-17.343)+2.110\sqrt{\frac{0.100138}{10}+\frac{0.0614622}{10}}$$

$$\text{where} \quad \upsilon = \frac{\left(\frac{0.100138}{10}+\frac{0.0614622}{10}\right)^2}{\frac{\left(\frac{0.100138}{10}\right)^2}{10-1}+\frac{\left(\frac{0.0614622}{10}\right)^2}{10-1}} = 17.024 \cong 17$$

$$-1.426 \le (\mu_1-\mu_2) \le -0.889$$

2.45. Construct a data set for which the paired *t*-test statistic is very large, but for which the usual two-sample or pooled *t*-test statistic is small. In general, describe how you created the data. Does this give you any insight regarding how the paired *t*-test works?

A	B	delta
7.1662	8.2416	-1.0754
2.3590	2.4555	-0.0965
19.9977	21.1018	-1.1041
0.9077	2.3401	-1.4324
-15.9034	-15.0013	-0.9021
-6.0722	-5.5941	-0.4781
9.9501	10.6910	-0.7409
-1.0944	-0.1358	-0.9586
-4.6907	-3.3446	-1.3461
-6.6929	-5.9303	-0.7626

Minitab Output

Paired T-Test and Confidence Interval

```
Paired T for A - B
                  N      Mean     StDev   SE Mean
A                10      0.59     10.06      3.18
B                10      1.48     10.11      3.20
Difference       10    -0.890     0.398     0.126

95% CI for mean difference: (-1.174, -0.605)
T-Test of mean difference = 0 (vs not = 0): T-Value = -7.07  P-Value = 0.000
```

Two Sample T-Test and Confidence Interval

```
Two-sample T for A vs B

    N      Mean     StDev   SE Mean
A  10       0.6      10.1       3.2
B  10       1.5      10.1       3.2

Difference = mu A - mu B
Estimate for difference:  -0.89
95% CI for difference: (-10.37, 8.59)
T-Test of difference = 0 (vs not =): T-Value = -0.20  P-Value = 0.846  DF = 18
Both use Pooled StDev = 10.1
```

These two sets of data were created by making the observation for *A* and *B* moderately different within each pair (or block), but making the observations between pairs very different. The fact that the difference between pairs is large makes the pooled estimate of the standard deviation large and the two-sample *t*-test statistic small. Therefore the fairly small difference between the means of the two treatments that is present when they are applied to the same experimental unit cannot be detected. Generally, if the blocks are very different, then this will occur. Blocking eliminates the variability associated with the nuisance variable that they represent.

2.46. Consider the experiment described in problem 2.26. If the mean burning times of the two flames differ by as much as 2 minutes, find the power of the test. What sample size would be required to detect an actual difference in mean burning time of 1 minute with a power of at least 0.90?

From the *Minitab* output below, the power is 0.0740. This answer was obtained by using the pooled estimate of σ from Problem 2-11, $S_p = 9.32$. Because the difference in means is very small relative to the standard deviation, the power is very low.

Minitab Output

```
Power and Sample Size

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)
Calculating power for mean 1 = mean 2 + difference
Alpha = 0.05  Sigma = 9.32

            Sample
Difference    Size   Power
         2      10  0.0740
```

From the *Minitab* output below, the required sample size is 1827. The sample size is huge because the difference in means is very small relative to the standard deviation.

Minitab Output

```
Power and Sample Size

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)
Calculating power for mean 1 = mean 2 + difference
Alpha = 0.05  Sigma = 9.32

            Sample  Target  Actual
Difference    Size   Power   Power
         1    1827  0.9000  0.9001
```

2.47. Reconsider the bottle filling experiment described in Problem 2.24. Rework this problem assuming that the two population variances are unknown but equal.

Minitab Output

```
Two-Sample T-Test and CI: Machine 1, Machine 2


Two-sample T for Machine 1 vs Machine 2

          N      Mean     StDev   SE Mean
Machine  10   16.0150    0.0303    0.0096
Machine  10   16.0050    0.0255    0.0081

Difference = mu Machine 1 - mu Machine 2
Estimate for difference:  0.0100
95% CI for difference: (-0.0163, 0.0363)
T-Test of difference = 0 (vs not =): T-Value = 0.80  P-Value = 0.435  DF = 18
Both use Pooled StDev = 0.0280
```

The hypothesis test is the same: H_0: $\mu_1 = \mu_2$ H_1: $\mu_1 \neq \mu_2$

The conclusions are the same as Problem 2.19, do not reject H_0. There is no difference in the machines. The *P*-value for this analysis is 0.435.

The confidence interval is (-0.0163, 0.0363). This interval contains 0. There is no difference in machines.

2.48. Consider the data from problem 2.24. If the mean fill volume of the two machines differ by as much as 0.25 ounces, what is the power of the test used in problem 2.19? What sample size could result in a power of at least 0.9 if the actual difference in mean fill volume is 0.25 ounces?

The power is 1.0000 as shown in the analysis below.

Minitab Output

Power and Sample Size

```
2-Sample t Test

Testing mean 1 = mean 2 (versus not =)
Calculating power for mean 1 = mean 2 + difference
Alpha = 0.05  Sigma = 0.028

            Sample
Difference    Size   Power
      0.25      10  1.0000
```

The required sample size is 2 as shown below.

Minitab Output

Power and Sample Size

```
2-Sample t Test

Testing mean 1 = mean 2 (versus not =)
Calculating power for mean 1 = mean 2 + difference
Alpha = 0.05  Sigma = 0.028

            Sample  Target  Actual
Difference    Size   Power   Power
      0.25       2  0.9000  0.9805
```