Chapter 1
Functions Graphs, and Models; Linear
**Functions**

Toolbox

1. $\{1,2,3,4,5,6,7,8\}$ and
    $\{x \mid x < 9, x \in N\}$
    Remember that $x \in N$ means that x is a
    natural number.

2. Yes.

3. No. A is not a subset of B. A contains
    elements $2, 7$, and 10 which are not in B.

4. No. $N = \{1,2,3,4,...\}$. Therefore, $\frac{1}{2} \notin N$.

5. Yes. Every integer can be written as a
    fraction with the denominator equal to 1.

6. Yes. Irrational numbers are by definition
    numbers that are not rational.

7. Integers. However, note that this set of
    integers could also be considered as a set of
    rational numbers. See question 5.

8. Rational numbers

9. Irrational numbers
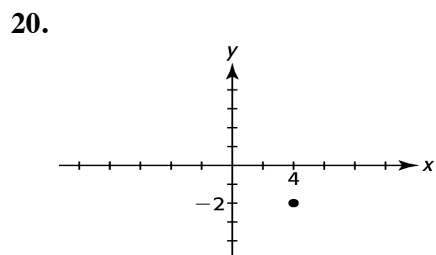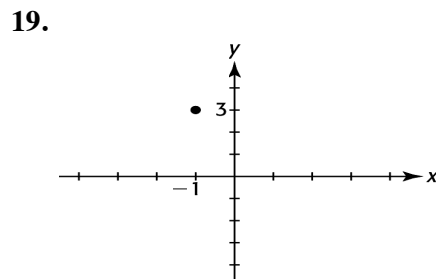
10. $x > -3$

11. $-3 \le x \le 3$

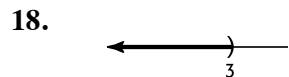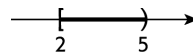12. $x \le 3$

13. $\left(-\infty, 7\right]$

14. $\left(3, 7\right]$

15. $\left(-\infty, 4\right)$

16.
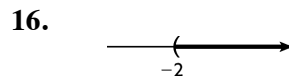
17. Note that $5 > x \ge 2$ implies $2 \le x < 5$,
    therefore:

18.

19.

20.

21.
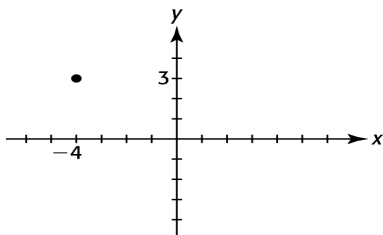
22.
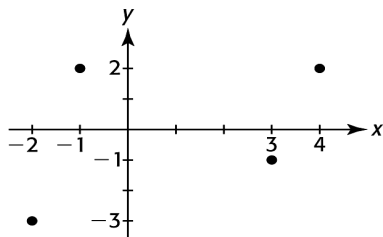
23.

24. The x^2 term has a coefficient of -3. The x term has a coefficient of -4. The constant term is 8.

25. The x^4 term has a coefficient of 5. The x^3 term has a coefficient of 7. The constant term is -3.

26. $\left(z^4 - 15z^2 + 20z - 6\right) + \left(2z^4 + 4z^3 - 12z^2 - 5\right)$
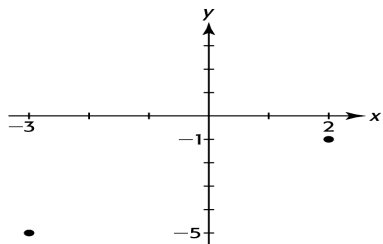$= \left(z^4 + 2z^4\right) + \left(4z^3\right) + \left(-15z^2 - 12z^2\right) +$
$\quad \left(20z\right) + \left(-6 - 5\right)$
$= 3z^4 + 4z^3 - 27z^2 + 20z - 11$

27. $-2x^3y^4 + \left(2y^4 + 5y^4\right) - 3y^2 +$
$\quad \left(3x - 5x\right) + \left(-119 + 110\right)$
$= -2x^3y^4 + 7y^4 - 3y^2 - 2x - 9$

28. $4\left(p + d\right)$
$= 4p + 4d$

29. $-2\left(3x - 7y\right)$
$= \left(-2 \cdot 3x\right) + \left(-2 \cdot -7y\right)$
$= -6x + 14y$

30. $-a\left(b + 8c\right)$
$= -ab - 8ac$

31. $4\left(x - y\right) - \left(3x + 2y\right)$
$= 4x - 4y - 3x - 2y$
$= 1x - 6y$
$= x - 6y$

32. $4\left(2x - y\right) + 4xy - 5\left(y - xy\right) - \left(2x - 4y\right)$
$= 8x - 4y + 4xy - 5y + 5xy - 2x + 4y$
$= \left(8x - 2x\right) + \left(-4y - 5y + 4y\right) + \left(4xy + 5xy\right)$
$= 6x - 5y + 9xy$

33. $2x\left(4yz - 4\right) - \left(5xyz - 3x\right)$
$= 8xyz - 8x - 5xyz + 3x$
$= \left(8xyz - 5xyz\right) + \left(-8x + 3x\right)$
$= 3xyz - 5x$

Section 1.1 Skills Check

1. Using Table A

    a. –5 is an x-value and therefore is an input
        into the function $f(x)$.

    b. $f(-5)$ represents an output from the
        function.

    c. The domain is the set of all inputs.
        D: $\{-9,-7,-5,6,12,17,20\}$. The range is
        the set of all outputs. R: $\{4,5,6,7,9,10\}$

    d. Each input x into the function f yields
        exactly one output $y = f(x)$.

2. Using Table B

    a. 0 is an x-value and therefore is an input
        into the function $g(x)$.

    b. $g(7)$ represents an output from the
        function.

    c. The domain is the set of all inputs.
        D: $\{-4,-1,0,1,3,7,12\}$. The range is the
        set of all outputs. R: $\{3,5,7,8,9,10,15\}$

    d. Each input x into the function g yields
        exactly one output $y = g(x)$.

3. $f(-9) = 5$
    $f(17) = 9$

4. $g(-4) = 5$
    $g(3) = 8$

5. No. In the given table, x is not a function of
    y. If y is considered the input variable, one
    input will correspond with more than one
    output. Specifically, if $y = 9$, then $x = 12$ or
    $x = 17$.

6. Yes. Each input y produces exactly one
    output x.

7. a. $f(2) = -1$, since $x = 2$ in the table
        corresponds with $f(x) = -1$.

    b. $f(2) = 10 - 3(2)^2$
        $= 10 - 3(4)$
        $= 10 - 12$
        $= -2$

    c. $f(2) = -3$, since $(2,-3)$ is a point on
        the graph.

8. a. $f(-1) = 5$, since $(-1,5)$ is a point on
        the graph.

    b. $f(-1) = -8$, since $x = -1$ in the table
        corresponds with $f(x) = -8$.

    c. $f(-1) = (-1)^2 + 3(-1) + 8$
        $= 1 - 3 + 8$
        $= 6$

9. | x | y |
    | --- | --- |
    | 0 | 2 |
    | –2 | –4 |

10. | x | y |
    | --- | --- |
    | 4 | 1 |
    | –4 | –3 |

11. Recall that $R(x) = 5x + 8$.

    a. $R(-3) = 5(-3) + 8 = -15 + 8 = -7$

    b. $R(-1) = 5(-1) + 8 = -5 + 8 = 3$

    c. $R(2) = 5(2) + 8 = 10 + 8 = 18$

12. Recall that $C(s) = 16 - 2s^2$.

    a. $\begin{aligned} C(3) &= 16 - 2(3)^2 \\ &= 16 - 2(9) \\ &= 16 - 18 \\ &= -2 \end{aligned}$

    b. $\begin{aligned} C(-2) &= 16 - 2(-2)^2 \\ &= 16 - 2(4) \\ &= 16 - 8 \\ &= 8 \end{aligned}$

    c. $\begin{aligned} C(-1) &= 16 - 2(-1)^2 \\ &= 16 - 2(1) \\ &= 16 - 2 \\ &= 14 \end{aligned}$

13. Yes. Each input corresponds with exactly one output. The domain is $\{-1,0,1,2,3\}$. The range is $\{-8,-1,2,5,7\}$.

14. No. Each input x does not match with exactly one output y. Specifically, if $x = 2$ then $y = -3$ or $y = 4$.

15. No. The graph fails the vertical line test. Each input does not match with exactly one output.

16. Yes. The graph passes the vertical line test. Each input matches with exactly one output.

17. Yes. The graph passes the vertical line test. Each input matches with exactly one output.

18. No. The graph fails the vertical line test. Each input does not match with exactly one output.

19. No. If $x = 3$, then $y = 5$ or $y = 7$. One input yields two outputs. The relation is not a function.

20. Yes. Each input x yields exactly one output y.

21. **a.** Not a function. If $x = 4$, then $y = 12$ or $y = 8$.

    b. Yes. Each input yields exactly one output.

22. **a.** Yes. Each input yields exactly one output.

    b. Not a function. If $x = 3$, then $y = 4$ or $y = 6$.

23. **a.** Not a function. If $x = 2$, then $y = 3$ or $y = 4$.

    b. Function. Each input yields exactly one output.

24. **a.** Function. Each input yields exactly one output.

    b. Not a function. If $x = -3$, then $y = 3$ or $y = -5$.

25. The domain is the set of all inputs.
D:$\{-3,-2,-1,1,3,4\}$. The range is the set
of all outputs. R:$\{-8,-4,2,4,6\}$

26. The domain is the set of all inputs.
D:$\{-6,-4,-2,0,2,4\}$. The range is the set
of all outputs. R:$\{-5,-2,0,1,4,6\}$

27. Considering y as a function of x, the domain
is the set of all inputs, x. Therefore the
domain is D:$[-10,8]$. The range is the set
of all outputs, y. Therefore, the range is
R:$[-12,2]$.

28. Considering y as a function of x, the domain
is the set of all inputs, x. Therefore the
domain is D:$[-4,3]$. The range is the set of
all outputs, y. Therefore, the range is
R:$[-1,4]$.

29. Since $y = \sqrt{3x-6}$ will not be a real number
if $3x-6 < 0$, the only values of x that yield
real outputs to the function are values that
satisfy $3x-6 \geq 0$. Isolating x yields:
$$3x-6+6 \geq 0+6$$
$$\frac{3x}{3} \geq \frac{6}{3}$$
$$x \geq 2$$
Therefore the domain is D: $[2,\infty)$.

30. Since $y = \sqrt{2x-8}$ will not be a real number
if $2x-8 < 0$, the only values of x that yield
real outputs to the function are values that
satisfy $2x-8 \geq 0$. Isolating x yields:
$$2x-8+8 \geq 0+8$$
$$\frac{2x}{2} \geq \frac{8}{2}$$
$$x \geq 4$$
Therefore the domain is D: $[4,\infty)$.

31. The denominator of the fractional part of the
function will be zero if $x+4 = 0$ or $x = -4$.
This implies $x \neq -4$. The domain is all real
numbers except -4 or in interval notation D:
$(-\infty,-4) \cup (-4,\infty)$.

32. The denominator of the fractional part of the
function will be zero if $2x-6 = 0$. Solving
for x yields:
$$2x-6+6 = 0+6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$
This implies $x \neq 3$. The domain is all real
numbers except 3 or in interval notation D:
$(-\infty,3) \cup (3,\infty)$.

33. No. If $x = 0$, then
$(0)^2 + y^2 = 4 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$. So,
one input of 0 corresponds with 2 outputs of
-2 and 2. Therefore the equation is not a
function.

34. Yes. Each input for x corresponds with
exactly one output for y.

35. $C = 2\pi r$, where C is the circumference and
r is the radius.

36. D is found by squaring E, multiplying the
result by 3, and subtracting 5.

Section 1.1 Exercises

37. a. No. Each input (x, given day) corresponds with multiple outputs (p, stock prices). Stock prices fluctuate throughout the trading day.

   b. Yes. Each input (x, given day) corresponds with exactly one output (p, the stock price at the end of the trading day). The independent variable is x, the given day, and the dependent variable is p, the stock price at the end of the trading day.

38. a. Yes. Each input (m, miles traveled) corresponds with exactly one output (s, odometer reading). The independent variable is m, miles traveled, and the dependent variable is s, the odometer reading.

   b. Yes. Each input (s, odometer reading) corresponds with exactly one output (m, miles traveled). The independent variable is s, odometer reading, and the dependent variable is m, miles traveled.

39. a. Yes. Each input (a, age in years) corresponds with exactly one output (p, life insurance premium). The independent variable is a, age in years, and the dependent variable is p, life insurance premium.

   b. No. One input of $11.81 corresponds with six outputs (a, age in years).

40. Yes. Each input (m, month) corresponds with exactly one output (p, average price). The independent variable is m, month, and the dependent variable is p, average price.

41. Yes. Each input (y, education level) corresponds with exactly one output (I, average income). The independent variable is y, education level, and the dependent variable is I, average income.

42. T, temperature, is a function of m, number of minutes after the power outage, since each value for m corresponds with exactly one value for T. The graph of the equation passes the vertical line test, which implies there is one temperature for each value of m, number of minutes after the power outage.

43. a. Yes. Each input (the barcode) corresponds with exactly one output (an item's price).

   b. No. Every input (an item's price) could correspond with more than one output (the barcode). Numerous items can have the same price but different barcodes.

44. a. Yes. Each input (a child's piano key) corresponds with exactly one output (a musical note). Since the domain is the set of all inputs into the function and there are 12 keys on the child's piano keyboard, there are 12 elements in the domain of the function.

   b. Yes. Each input (a note from the child's piano) corresponds with exactly one output (a piano key). Since the range is the set of all outputs from the function and there are 12 keys on the child's piano keyboard, there are 12 elements in the range of the function.

45. Yes. Each input (d, depth) corresponds with exactly one output (p, pressure). The graph of the equation passes the vertical line test.

46. Each input (x, years) corresponds with exactly one output (V, value of the property). The graph of the equation passes the vertical line test.

47. a. Yes. Each input (day of the month) corresponds with exactly one output (weight).

   b. The domain is the first 14 days of May or D: $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$.

   c. The range is $\{171,172,173,174,175,176,177,178\}$.

   d. The highest weights were on May 1 and May 3.

   e. The lowest weight was on May 14.

   f. Three days from May 8 until May 11.

48. a. No. One input of 75 matches with two outputs of 70 and 81.

   b. Yes. Each input (average score on the final exam) matches with exactly one output (average score on the math placement test).

49. a. $P(3) = \$1096.78$. If the car is financed over three years, the payment is $1096.78.

   b. $C(5) = \$42,580.80$.

   c. t = 4. If the total cost is $41,014.08, then the car has been financed over four years,

   d. Since C(5) = $42,580.80, and C(3) = $39,484.08, the savings would be C(5) – C(3) = $3096.72.

50. a. The couple must make payments for 20 years.
$$20 = f(103,000)$$

   b. $f(120,000) = 30$. It will take the couple 30 years to payoff a $120,000 mortgage at 7.5%.

   c. $f(3 \cdot 40,000) = f(120,000) = 30$

   d. If $A = 40,000$ then
$$f(A) = f(40,000) = 5 .$$

   e. $f(3 \cdot 40,000) = f(120,000) = 30$
$$3 \cdot f(40,000) = 3 \cdot 5 = 15$$
   The expressions are not equal.

51. a. When $t = 2005$, the ratio is approximately 4 to 1.

   b. $f(2005) = 4$. For year 2005 the projected ratio of the working-age population to the elderly is 4 to 1.

   c. The domain is the set of all possible inputs. In this example, the domain consists of all the years, t, represented in the figure. Specifically, the domain is $\{1995, 2000, 2005, 2010, 2015,$ $2020, 2025, 2030\}$.

   d. As the years, t, increase, the projected ratio of the working-age population to the elderly decreases. Notice that the bars in the figure grow smaller as the time increases.

52. a. Approximately 22 million

   b. $f(1930) = 11$. Approximately 11 million women were in the work force in 1930.

c. D: $\{1930,1940,1950,1960,1970,$
$1980,1990,2000,2005\}$

d. As the year increases, the number of women in the work force also increases.

53. a. $f(2005) = 419,640$

b. In year 2005 there were 419,640 nonfatal firearm crimes.

c. The maximum number of nonfatal firearm crimes is 1,060,800, occurring in year 1994. Note that $f(1994) = 1,060,800$.

54. a. $f(1890) = 26.1$
$f(2004) = 27.1$

b. $g(1940) = 21.5$
$g(2000) = 25.1$

c. If $x = 1980$, then $f(x) = 24.7$. The median age of first marriage for men in 1980 was 24.7 years.

d. $f(2004) = 27.1 > 22.8 = f(1960)$. Therefore, the median age of first marriage for men increased between 1960 and 2004.

55. a. In 2000, 41.5 million homes used the Internet.

b. $f(2003) = 52.4$. In 2003, 52.4 million U.S. homes used the Internet.

c. 1998. $f(1998) = 26.0$.

d. The function is increasing rapidly. Since 2003 home use of the Internet in the U.S. has continued to increase.

56. a. Yes. Each year, t, corresponds with exactly one percentage, p.

b. $f(1840) = 68.6$. $f(1840)$ represents the percentage of U.S. workers in a farm occupation in the year 1840.

c. If $f(t) = 27$, then $t = 1920$.

d. $f(1960) = 6.1$ implies that in 1960, 6.1% of U.S. workers were employed in a farm occupation.

e. As the time, t, increases, the percentage, p, of U.S. workers in farm occupations decreases. Note that the graph is trending downward if it is read from left to right.

57. a. $f(1990) = 3.4$. In 1990 there were 3.4 workers for each retiree.

b. 2030. $f(2030) = 2$.

c. As the years increase, the number of workers available to support retirees decreases. Therefore, funding for social security into the future is problematic. Workers will need to pay a larger portion of their salaries to fund payments to retirees.

58. a. When the input is 1995 the output is approximately 103. This implies that the pregnancy rate per 1000 girls in 1995 was approximately 103.

b. The rate was 113 in 1989 and 1992.

c. The rate increased from 1987-1991.

d. 1991. In 1991, the pregnancy rate per 1000 girls was approximately 117.

59. a. $R(200) = 32(200) = 6400$. The revenue generated from selling 200 golf hats is $6400.

b. $R(2500) = 32(2500) = \$80,000$

60. a. $C(200) = 4000 + 12(200) = 6400$. The production cost of manufacturing 200 golf hats is $6400.

b. $C(2500) = 4000 + 12(2500)$
$= \$34,000$

61. a. $f(1000) = 0.767(1000) + 16.37$
$= 767 + 16.37$
$= 783.37$
The monthly charge for using 1000 kilowatt hours is $783.37.

b. $f(1500) = 0.767(1500) + 16.37$
$= 1150.5 + 16.37$
$= 1166.87$
The monthly charge for using 1500 kilowatt hours is $1166.87.

62. a. $P(500) = 450(500) - 0.1(500)^2 - 2000$
$= 225,000 - 25,000 - 2000$
$= 198,000$
The profit generated from selling 500 iPod players is $198,000.

b. $P(4000)$
$= 450(4000) - 0.1(4000)^2 - 2000$
$= 1,800,000 - 1,600,000 - 2000$
$= 198,000$
$P(4000) = \$198,000$

63. a. $P(100) = 32(100) - 0.1(100)^2 - 1000$
$= 3200 - 1000 - 1000$
$= 1200$
The daily profit for producing 100 Blue Chief bicycles is $1200.

b. $P(160) = 32(160) - 0.1(160)^2 - 1000$
$= 5120 - 2560 - 1000$
$= 1560$
The daily profit for producing 160 Blue Chief bicycles is $1560.

64. a. $h(1) = 6 + 96(1) - 16(1)^2$
$= 6 + 96 - 16$
$= 86$
The height of the ball after one second is 86 feet.

b. $h(3) = 6 + 96(3) - 16(3)^2$
$= 6 + 288 - 144$
$= 150$
After three seconds the ball is 150 feet high.

c. Test $t = 2$.
$h(2) = 6 + 96(2) - 16(2)^2$
$= 6 + 192 - 64$
$= 134$

Test $t = 4$.
$h(4) = 6 + 96(4) - 16(4)^2$
$= 6 + 384 - 256$
$= 134$

Test $t = 5$.
$h(5) = 6 + 96(5) - 16(5)^2$
$= 6 + 480 - 400$
$= 86$

Since h(1) = 86, and h(2) = 134, and h(3) = 150, and h(4) = 134, and h(5) = 86, it appears that the ball stops climbing after 3 seconds and begins to fall. One might conclude that the ball reaches its maximum height at 3 seconds since at 1 and 5 seconds, and again at 2 and 4 seconds, the respective heights are the same.

65. a. Yes, since each value of s produces exactly one value of K_c.

b. Any input into the function must not create a negative number under the radical. Therefore, the radicand, $4s + 1$, must be greater than or equal to zero. Isolating s yields:
$$4s + 1 \geq 0$$
$$4s + 1 - 1 \geq 0 - 1$$
$$4s \geq -1$$
$$s \geq -\frac{1}{4}$$
Therefore, the domain defined by the equation is all real numbers greater than or equal to $-\frac{1}{4}$ or, in interval notation,
$$\left[-\frac{1}{4}, \infty \right).$$

c. Since s represents wind speed in the given function, and wind speed cannot be less than zero, the domain of the function is restricted based on the physical context of the problem. Even though the domain implied by the function is $\left[-\frac{1}{4}, \infty \right)$, the actual domain in the given physical context is $[0, \infty)$.

66. a.
$$0.3 + 0.7n = 0$$
$$0.7n = -0.3$$
$$\frac{0.7n}{0.7} = \frac{-0.3}{0.7}$$
$$n = -\frac{3}{7}$$
Therefore the domain of $R(n)$ is all real numbers except $-\frac{3}{7}$ or
$$\left(-\infty, -\frac{3}{7} \right) \cup \left(-\frac{3}{7}, \infty \right).$$

b. In the context of the problem, n represents the factor for increasing the number of questions on a test. Therefore it makes sense that n is positive ($n > 0$).

67. a. Since p is a percentage, $0 \leq p \leq 100$. However in the given function, the denominator, $100 - p$, cannot equal zero. Therefore, $p \neq 100$. The domain is $0 \leq p < 100$ or, in interval notation, $[0, 100)$.

b. $C(60) = \dfrac{237,000(60)}{100 - 60} = 355,500$

$C(90) = \dfrac{237,000(90)}{100 - 90} = 2,133,000$

68. a. Any input into the function must not create a negative number under the square root. Therefore, $2p + 1 \geq 0$. Isolating p yields
$$2p + 1 \geq 0$$
$$2p \geq -1$$
$$p \geq -\frac{1}{2}$$
Since the denominator cannot equal zero, $p \neq -\frac{1}{2}$.

Therefore the domain of q is $\left(-\dfrac{1}{2}, \infty\right)$.

b. In the context of the problem, p represents the price of a product. Since the price can not be negative, $p \geq 0$.

The domain is $[0, \infty)$. Also, since q represents the quantity of the product demanded by consumers, $q \geq 0$. The range is $(0, 100]$.

69. a. $V(12) = (12)^2 (108 - 4(12))$

$= 144(108 - 48)$

$= 144(60)$

$= 8640$

$V(18) = (18)^2 (108 - 4(18))$

$= 324(108 - 72)$

$= 324(36)$

$= 11,664$

b. First, since x represents a side length in the diagram, x must be greater than zero. Second, to satisfy postal restrictions, the length (longest side) plus the girth must be less than or equal to 108 inches. Therefore,

Length + Girth ≤ 108

Length $+ 4x \leq 108$

$4x \leq 108 - \text{Length}$

$x \leq \dfrac{108 - \text{Length}}{4}$

$x \leq 27 - \dfrac{\text{Length}}{4}$

Since x is greatest if the longest side is smallest, let the length equal zero to find the largest value for x.

$x \leq 27 - \dfrac{0}{4}$

$x \leq 27$

Therefore the conditions on x are $0 < x \leq 27$. If $x = 27$, the length would be zero and the package would not exist. Therefore, in the context of the question,

$0 < x < 27$ and the corresponding domain for the function $V(x)$ is $(0, 27)$.

c.

x	*Volume*
10	6800
12	8640
14	10192
16	11264
18	**11664**
20	11200
22	9680

The maximum volume occurs when $x = 18$. Therefore the dimensions that maximize the volume of the box are 18 inches by 18 inches by 36 inches.

70. a. $S(0) = -4.9(0)^2 + 98(0) + 2 = 2$

The initial height of the bullet is 2 meters.

b.

t	*Height*
9	487.1
9.5	490.78
10	492
10.5	490.78
11	487.1

$S(9) = 487.1$

$S(10) = 492$

$S(11) = 487.1$

c. The bullet seems to reach a maximum height at 10 seconds and then begins to fall. See the table in part b) for further verification, using 9.5 seconds and 10.5 seconds.

Section 1.2 Skills Check

1. a.

x	$y = x^3$	(x, y)
−3	−27	(−3, −27)
−2	−8	(−2, −8)
−1	−1	(−1, −1)
0	0	(0, 0)
1	1	(1, 1)
2	8	(2, 8)
3	27	(3, 27)
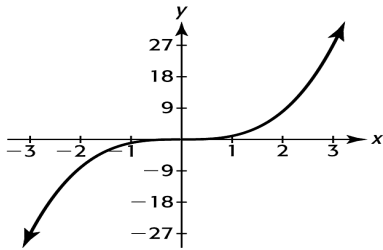
b.
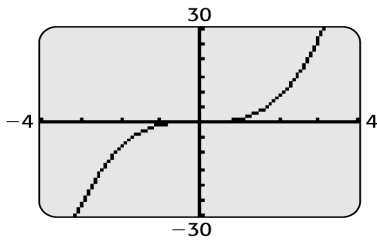
[−4, 4] by [−30, 30]

c. Your hand-drawn graph in part a) by plotting points from the table should match the calculator-drawn graph in part b).
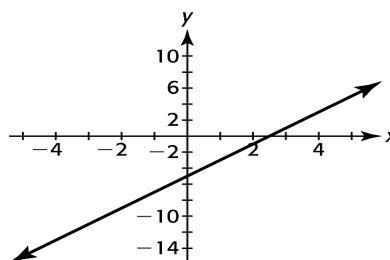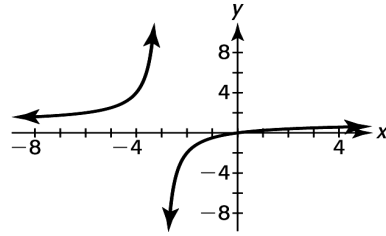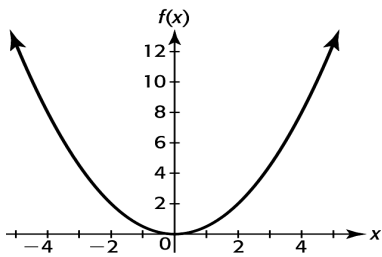
2. a.

x	$y = 2x^2 + 1$	(x, y)
−3	19	(−3, 19)
−2	9	(−2, 9)
−1	3	(−1, 3)
0	1	(0, 1)
1	3	(1, 3)
2	9	(2, 9)
3	19	(3, 19)
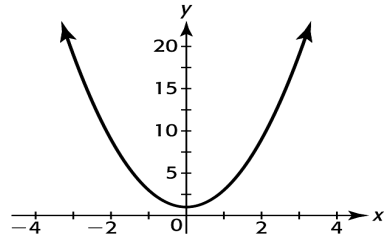
b.
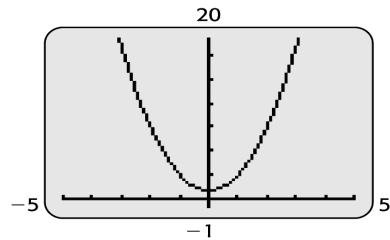
[−5, 5] by [−1, 20]

c. Your hand-drawn graph in part a) by plotting points from the table should match the calculator-drawn graph in part b).

3.

x	−2	−1	0	1	2
y	−7	−4	−1	2	5
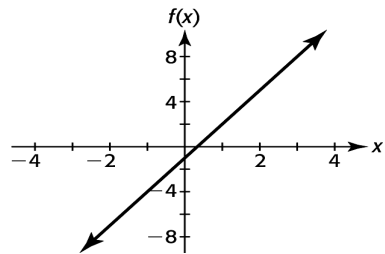
4.

x	−2	−1	0	1	2
y	−9	−7	−5	−3	−1

5.

x	-4	-2	0	2	4
y	8	2	0	2	8

6.
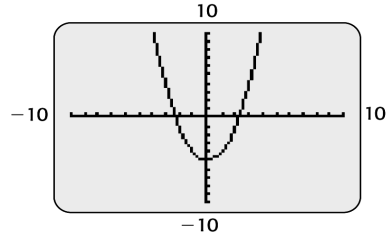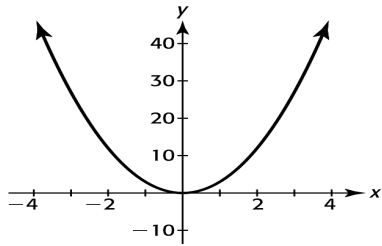
x	-2	-1	0	1	2
y	12	3	0	3	12
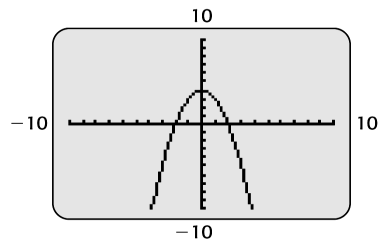
7.

x	0	1	2	3	4
y	$-\dfrac{1}{2}$	-1	undefined	1	$\dfrac{1}{2}$
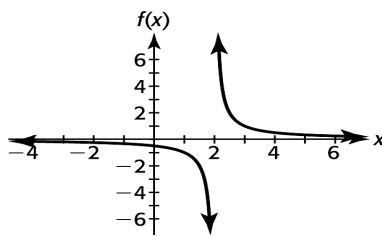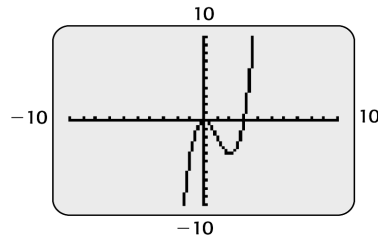
8.

x	-5	-4	-3	-2	-1
y	$\dfrac{5}{2}$	4	undefined	-2	$-\dfrac{1}{2}$

9. $y = x^2 - 5$

10. $y = 4 - x^2$

11. $y = x^3 - 3x^2$

12. $y = x^3 - 3x^2 + 4$
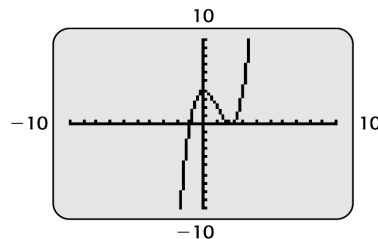
13. $y = 9/(x^2 + 1)$
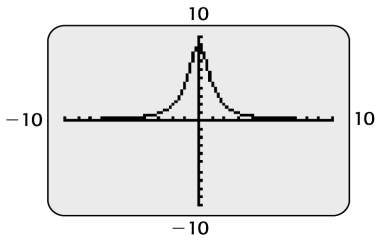
16. a. $y = x^3 - 3x + 13$

14. $y = 40/(x^2 + 4)$
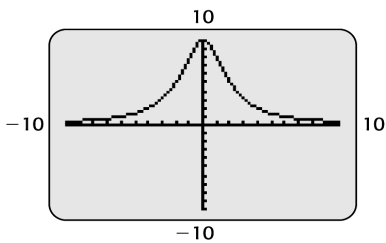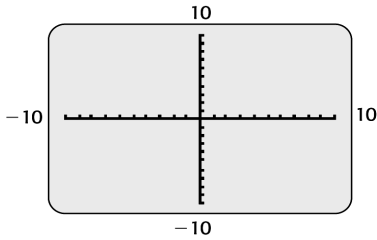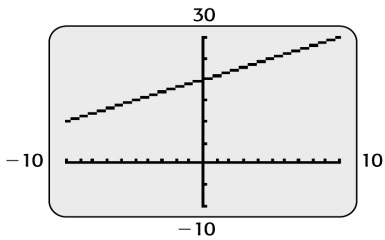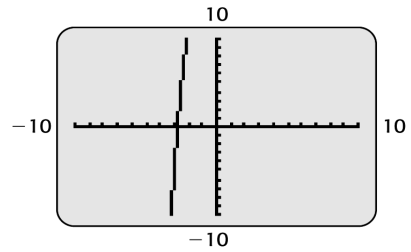
b. $y = x^3 - 3x + 13$

View b) is better.

15. a. $y = x + 20$
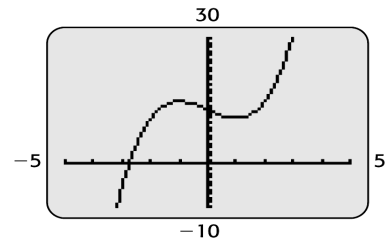
17. a. $y = \dfrac{0.04(x - 0.1)}{x^2 + 300}$

b. $y = x + 20$
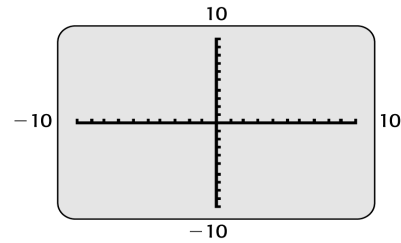
b. $y = \dfrac{0.04(x - 0.1)}{x^2 + 300}$
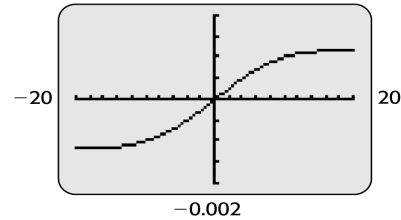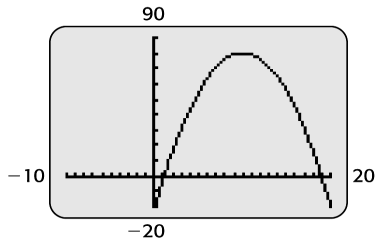
View b) is better.

View b) is better.

18. a. $y = -x^2 + 20x - 20$
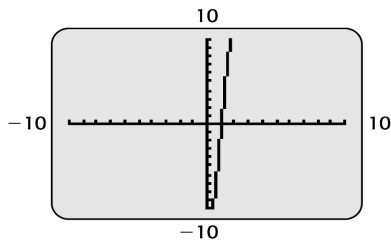
b. $y = -x^2 + 20x - 20$

View b) is better.

19. When $x = -3$ or $x = 3$, $y = 59$. When
$x = 0$, $y = 50$. Therefore, $[-3,3]$ by $[0,80]$
is an appropriate viewing window. (Note
that answers may vary.)
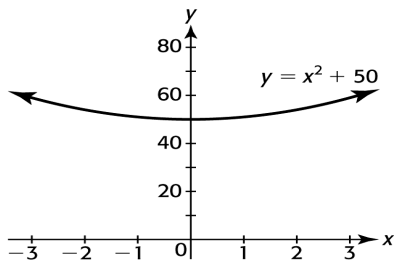
20. When $x = -60, y = 30$. When $x = 0, y = 30$.
When $x = -30, y = -870$. Therefore,
$[-60,0]$ by $[-1000,0]$ is an appropriate
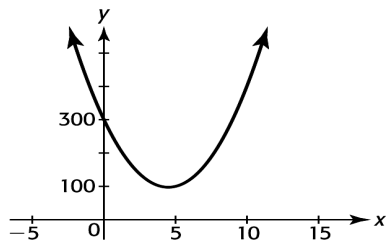viewing window. (Note that answers may
vary.)

$y = x^2 + 60x + 30$
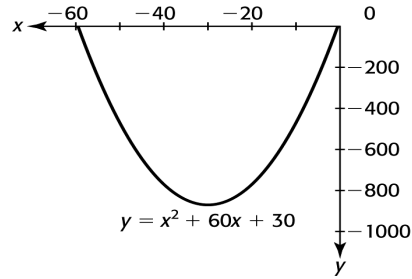
21. When $x = -10, y = -250$. When
$x = 10, y = 850$. When $x = 0, y = 0$.
Therefore, $[-10,10]$ by $[-200,200]$ is an
appropriate viewing window. (Note that
answers may vary.)

$y = x^3 + 3x^2 - 45x$
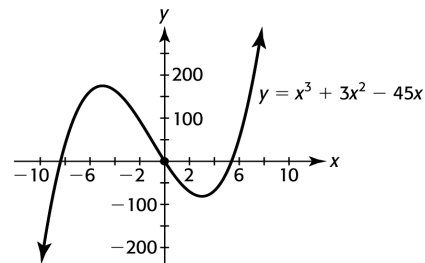
22. When $x = 28$, $y = 0$. When
$x = 25$, $y = -27$. When $x = 31$, $y = 27$.
Therefore, $[25,31]$ by $[-30,30]$ is an
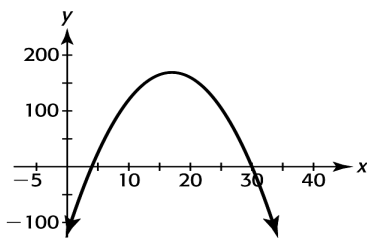appropriate viewing window. (Note that
answers may vary.)

$y = (x - 28)^3$
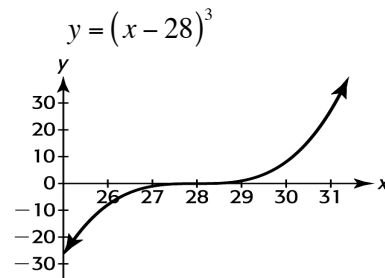
23. $y = 10x^2 - 90x + 300$

Note that answers may vary.

24. $y = -x^2 + 34x - 120$

Note that answers may vary.

25.

t	$S(t) = 5.2t - 10.5$
12	51.9
16	72.7
28	135.1
43	213.1

26.

q	$f(q) = 3q^2 - 5q + 8$
−8	240
−5	108
24	1616
43	5340

27.
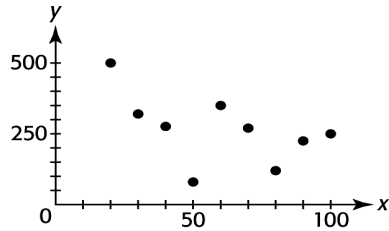
28.
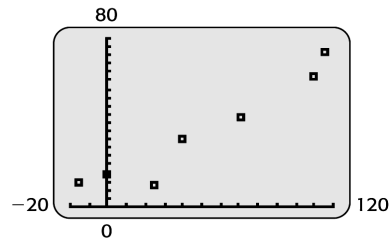
29. a.
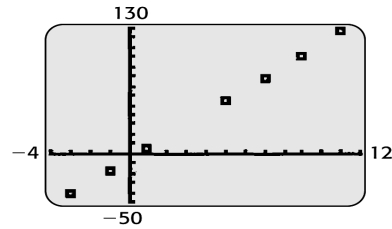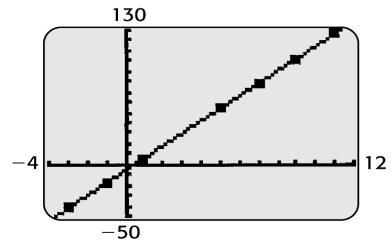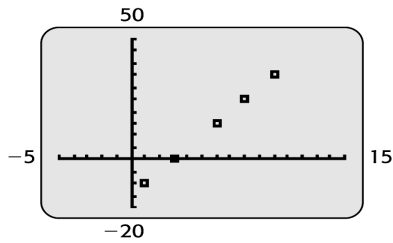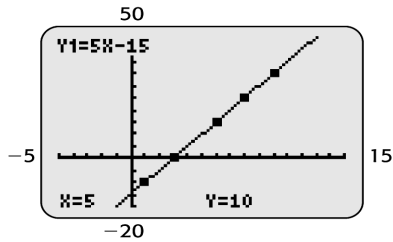

b.

c. Yes. Yes.

30. a.

b.

$[-5, 15]$ by $[-20, 50]$

c. Yes. Yes.

31. a. $f(20) = (20)^2 - 5(20)$
$= 400 - 100$
$= 300$

b. $x = 20$ implies 20 years after 2000. Therefore the answer to part a) yields the millions of dollars earned in 2020.

32. a. $f(10) = 100(10)^2 - 5(10)$
$= 10,000 - 50$
$= 9950$

b. In 2010, $x = 10$. Therefore, 9950 thousands of units or 9,950,000 units are produced in 2010.

Section 1.2 Exercises

33. a. $y = 5.74(44)^2 - 17.04(44) + 600.99$
$= 11,112.64 - 749.76 + 600.99$
$= 10,963.87$

In 1944 there were 10,963.87 thousand (10,963,870) women in the workforce.

b. In 2010, $x = 2010 - 1900 = 110$.
$y = 5.74(110)^2 - 17.04(110) + 600.99$
$= 69,454 - 1874.4 + 600.99$
$= 68,180.59$

Based on the model in 2010 there will be 68,180.59 thousand (68,180,590) women in the workforce.

34. a. $x =$ Year $- 1990$
For 1994, $x = 1994 - 1990 = 4$
For 1998, $x = 1998 - 1990 = 8$

b. $y = -112(8)^2 - 107(8) + 15,056 = 7032$

7032 represents the number of welfare cases in Niagara, Canada in 1998.

c. For 1995, $x = 5$. Therefore,
$y = -112(5)^2 - 107(5) + 15,056$
$= 11,721$

There were 11,721 welfare cases in Niagara, Canada in 1995.

35. a. $t =$ Year $- 1995$
For 1996, $t = 1996 - 1995 = 1$
For 2014, $t = 2014 - 1995 = 19$

b. $P = f(10)$ represents the value of P in 2005 $(1995 + 10 = 2005)$.
$f(10) = 6.9(10) - 3.18 = 65.82$

65.82 represents the percentage of households with Internet access in 2005.

c. $x_{min} = 1995 - 1995 = 0$

$x_{max} = 2015 - 1995 = 20$

36. a. $t = \text{Year} - 1980$

For 1982, $t = 1982 - 1980 = 2$

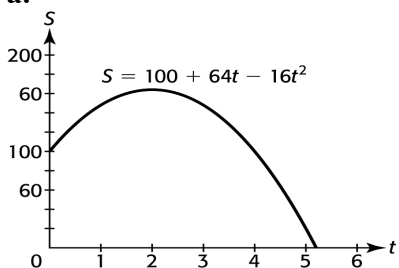For 1988, $t = 1988 - 1980 = 8$

For 2000, $t = 2000 - 1980 = 20$

b. $P = f(4)$ represents the value of P in 1984 $(1980 + 4 = 1984)$.

$f(4) = 35(4)^2 + 740(4) + 1207$

$= 4727$

4727 represents the cost of prizes and expenses in millions of dollars for state lotteries in 1984.

c. $x_{min} = 1980 - 1980 = 0$
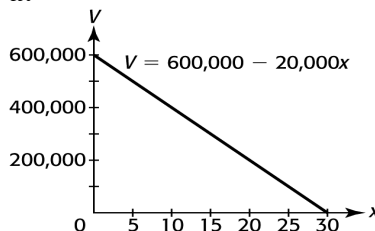
$x_{max} = 1997 - 1980 = 17$

37. a.

b.

Considering the table, $S = 148$ feet when x is 1 or when x is 3. The height is the

same for two different times because the height of the ball increases, reaches a maximum height, and then decreases.
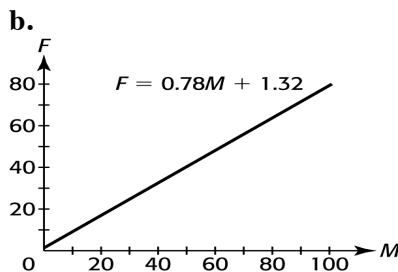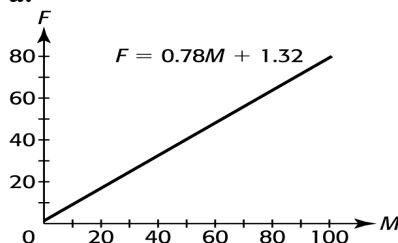
c.   From the table in part b), it appears the maximum height is 164 feet, occurring 2 seconds into the flight of the ball.

38. a.

b.   When $x = 10$,

$V = 600,000 - 20,000(10)$
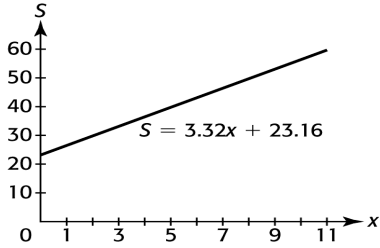
$= 600,000 - 200,000$

$= \$400,000$

39. a.

b.

$F = 0.78(63) + 1.32$

$= 49.14 + 1.32 = 50.46$

When $x = 63$, $y = 50.46$. Therefore, when the median male salary is $63,000, the median female salary is $50,460.
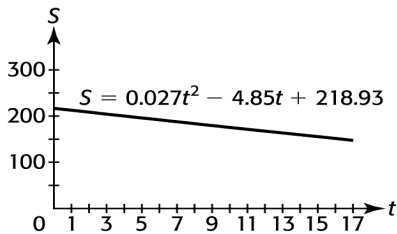
40. a.

b.

$$S = 3.32(11) + 23.16$$
$$= 36.52 + 23.16 = 59.68$$

In 2001, federal spending on education is approximately $59.68 billion.

41. a.

b. Use the Trace feature, and when $t = 15$, $S = 152.255$

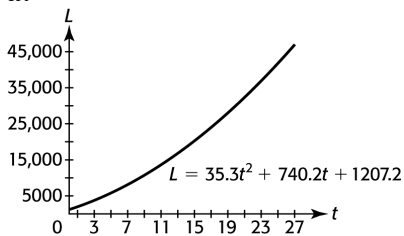**c.** If the year is 2005, then $t = 2005 - 1980 = 25$. When $t = 25$,
$$S = 0.027(25)^2 - 4.85(25) + 218.93$$
$$= 16.875 - 121.25 + 218.93$$
$$= 114.555$$
The estimated number of osteopathic students in 2005 is 114,555.

42. a.

b.

$$L(26) = 44,315$$

c. The cost in 2006 is approximately $44,315 million ($44,315,000,000).

43. a. $f(x) = -2.01x^2 + 54.84x + 87.31$
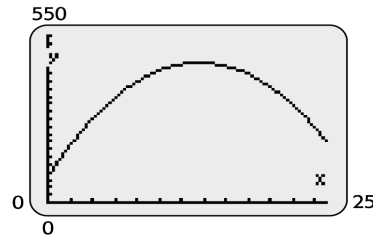
b. Since the base year is 1980, the graph in part a) shows years 1980 $(x = 0)$ through 2005 $(x = 25)$.

c. The graph in part a) decreases for $x \geq 14$. Therefore, the number of juvenile arrests per 100,000 people decreases after 1994.

d.

In 1994, the juvenile arrest rate is 461.11 per 100,000 people. In 2005, the juvenile arrest rate is 202.06 per 100,000 people.

e. Yes, the number of juvenile arrests decreases after 1994.

44. a. $B(t) = 17.69 + 2.25t$
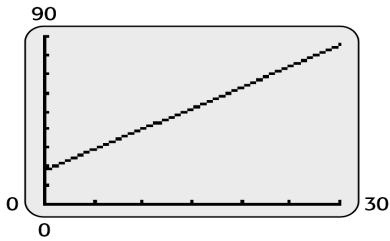
b. The tax burden increased. Reading the graph from left to right, as t increases $B(t)$ also increases.

45.
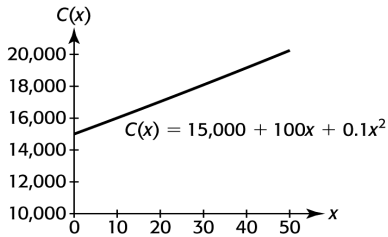
46.
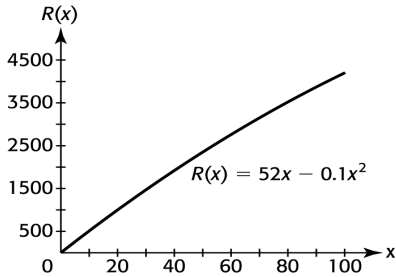
47.
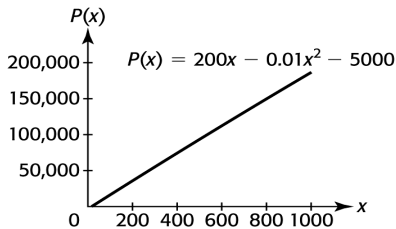
48.
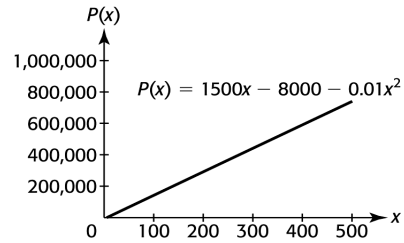
49. $f(t) = 982.06t + 32,903.77$

a. Since the base year is 1990, 1990-2005 correspond to values of t between 0 and 15 inclusive.

b. For 1990:

$$f(0) = 982.06(0) + 32,903.77$$
$$= 32,903.77$$

For 2005:

$$f(15) = 982.06(15) + 32,903.77$$
$$= 47,634.67$$

c.
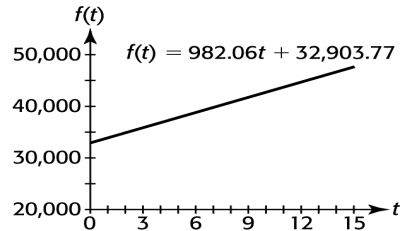
{Note that answers may vary.}

50. $y = 0.0035x^3 - 0.1666x^2 + 1.8689x + 9.5782$

a. Since the base year is 1975, 1975-2005 correspond to values of x between 0 and 30.

b. Since percentages are between 0 and 100, y must correspond to values between 0 and 100.

c.

c.

d.
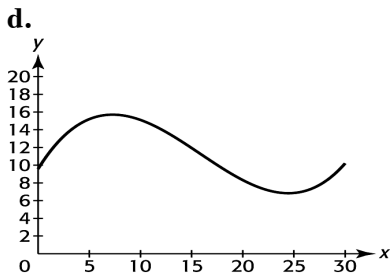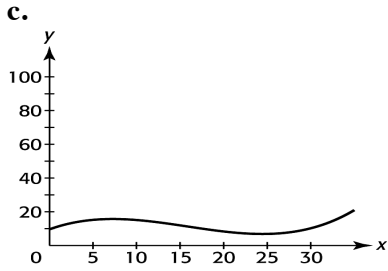
e. 2009 corresponds to $x = 2009 - 1975 = 34$. When $x = 34$, $y = .181$. Thus in 2009, approximately 18.1% of high school seniors had used cocaine.

51. a. 299.9 million or 299,900,000

b.

Years after 2000	Population (millions)
0	275.3
10	299.9
20	324.9
30	351.1
40	377.4
50	403.7
60	432.0

52.

53. a.

b.

54. a.

b.

$y = 5x^2 + 21.91x + 378.60$
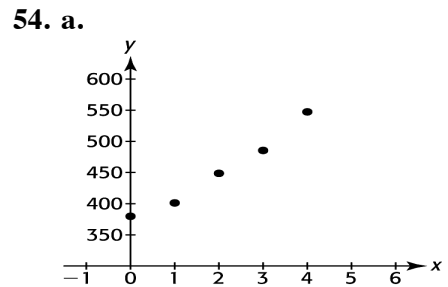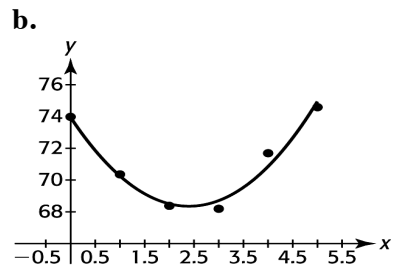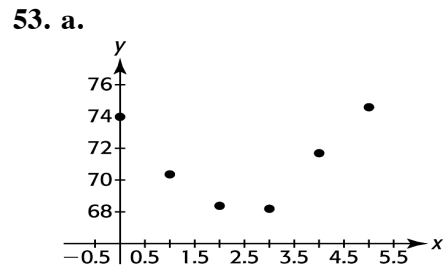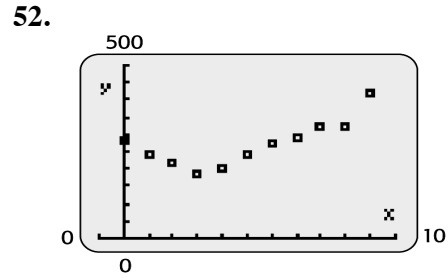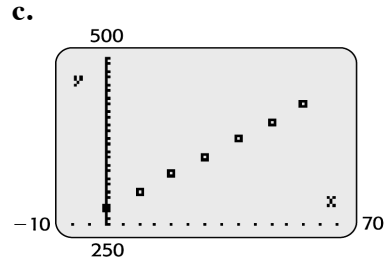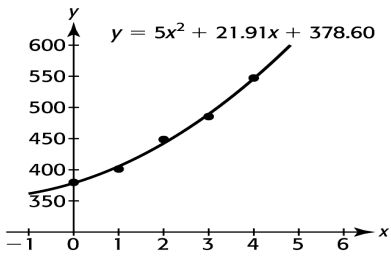
55. a. In 2003 the unemployment rate was 3.5%.

b.
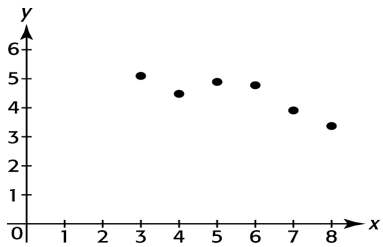
c.
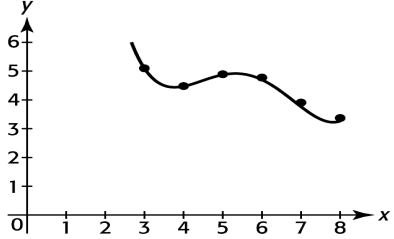
56. a. The dropout rate in 2004 is 5.6%.

b.
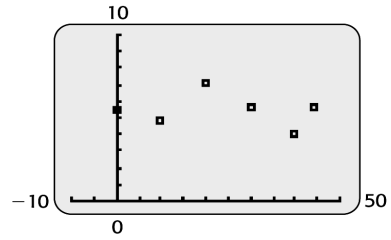
c.
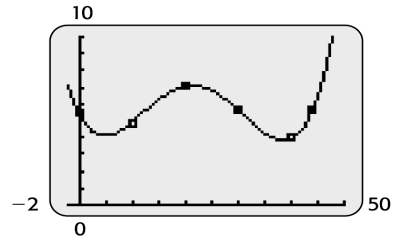
Section 1.3 Skills Check

1. Recall that linear functions must be in the
   form $f(x) = ax + b$.

   a. Not linear. The equation has a 2nd
      degree (squared) term.

   b. Linear.

   c. Not linear. The x-term is in the
      denominator of a fraction.

2. No. A vertical line is not a function.

3. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6 - 6}{28 - 4} = \dfrac{-12}{24} = -\dfrac{1}{2}$

4. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

   $= \dfrac{4 - (-10)}{8 - 8}$

   $= \dfrac{14}{0}$

   $=$ undefined

   Zero in the denominator creates an
   undefined expression.

5. The given line passes through $(-2, 0)$ and
   $(0, 4)$. Therefore the slope is

   $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 0}{0 - (-2)} = \dfrac{4}{2} = 2$

6. Since the line is horizontal, the slope of the
   line is zero. $m = 0$.

7. a. x-intercept: Let $y = 0$ and solve for x.

$5x - 3(0) = 15$

$5x = 15$

$x = 3$

y-intercept: Let $x = 0$ and solve for y.

$5(0) - 3y = 15$

$-3y = 15$

$y = -5$

x-intercept: $(3, 0)$, y-intercept: $(0, -5)$

   b.
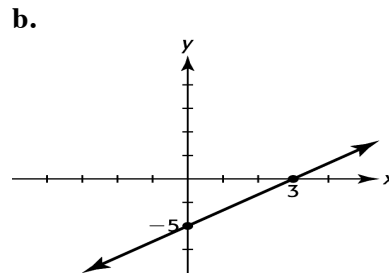
8. a. x-intercept: Let $y = 0$ and solve for x.

$x + 5(0) = 17$

$x = 17$

y-intercept: Let $x = 0$ and solve for y.

$0 + 5y = 17$

$5y = 17$

$y = \dfrac{17}{5}$

$y = 3.4$

x-intercept: $(17, 0)$, y-intercept: $(0, 3.4)$

   b.
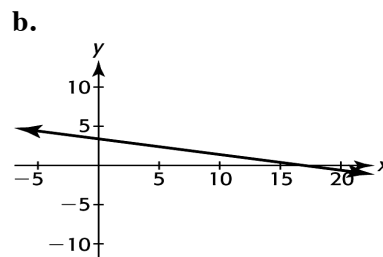
9. a. *x*-intercept: Let $y = 0$ and solve for *x*.

$$3(0) = 9 - 6x$$
$$0 = 9 - 6x$$
$$0 - 9 = 9 - 9 - 6x$$
$$-6x = -9$$
$$x = \frac{-9}{-6}$$
$$x = \frac{3}{2} = 1.5$$

y-intercept: Let $x = 0$ and solve for *y*.

$$3y = 9 - 6(0)$$
$$3y = 9$$
$$y = 3$$

x-intercept: $(1.5, 0)$, *y*-intercept: $(0, 3)$

b.
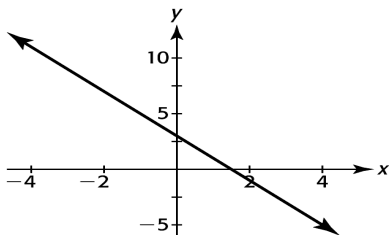
10. a. *x*-intercept: Let $y = 0$ and solve for *x*.

$$0 = 9x$$
$$x = 0$$

y-intercept: Let $x = 0$ and solve for *y*.

$$y = 9(0)$$
$$y = 0$$

x-intercept: $(0, 0)$, *y*-intercept: $(0, 0)$.
Note that the origin, $(0, 0)$, is both an *x*-
and *y*-intercept.

b.
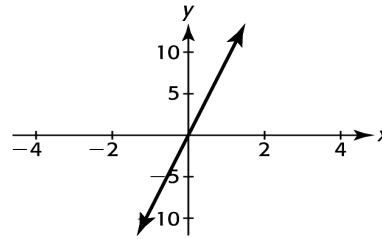
11. Horizontal lines have a slope of **zero**.
Vertical lines have an **undefined** slope.

12. Since the slope is undefined, the line is
vertical.

13. a. Positive. The graph is rising.

b. Undefined. The line is vertical.

14. a. Negative. The graph is falling.

b. Zero. The line is horizontal.

15. a. $m = 4, b = 8$

b.
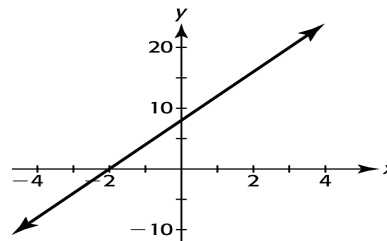
16. a. $3x + 2y = 7$

$$\frac{2y}{2} = \frac{-3x + 7}{2}$$
$$y = \frac{-3x + 7}{2}$$
$$y = -\frac{3}{2}x + \frac{7}{2}$$
$$m = -\frac{3}{2}, b = \frac{7}{2}$$

b.

c.
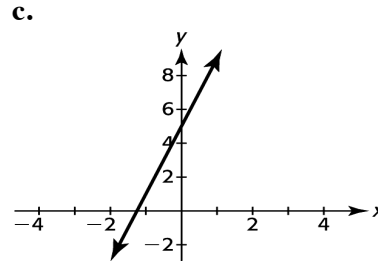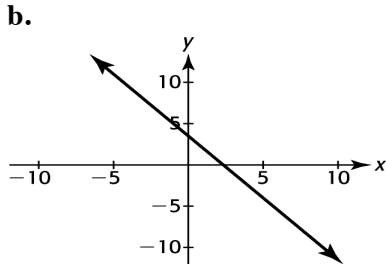
17. a. $5y = 2$

$y = \dfrac{2}{5}$, horizontal line

$m = 0, b = \dfrac{2}{5}$

b.

18. a. $x = 6$, vertical line

undefined slope, no y-intercept

b.

19. a. $m = 4, b = 5$

b.   Rising. The slope is positive

20. a. $m = 0.001, b = -0.03$

b.   Rising. The slope is positive.

c.

21. a. $m = -100, b = 50{,}000$

b.   Falling. The slope is negative.

c.

22. Steepness refers to the amount of vertical change compared to the amount of horizontal change between two points on a line, regardless of the direction of the line. The steeper line would have the greater absolute value of its slope. Since the slope in exercise 19 is 4, and in exercise 20 it is 0.001, and in exercise 21 it is -100, exercise 20 (m = .001) is the least steep, followed by exercise 19 (m = 4). Exercise

21 displays the greatest steepness since $m = -100$ which gives $|-100| = 100$.

23. For a linear function, the rate of change is equal to the slope. $m = 4$.

24. For a linear function, the rate of change is equal to the slope. $m = \dfrac{1}{3}$.

25. For a linear function, the rate of change is equal to the slope. $m = -15$.

26. For a linear function, the rate of change is equal to the slope. $m = 300$.

27. For a linear function, the rate of change is equal to the slope.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 3}{4 - (-1)} = \frac{-10}{5} = -2.$$

28. For a linear function, the rate of change is equal to the slope.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{6 - 2} = \frac{2}{4} = \frac{1}{2}.$$

29. a. The identity function is $y = x$. Graph *ii* represents the identity function.

  b. The constant function is $y = k$, where k is a real number. In this case, $k = 3$. Graph *i* represents a constant function.

30. The slope of the identity function is one ($m = 1$).

31. a. The slope of a constant function is zero ($m = 0$).

  b. The rate of change of a constant function equals the slope, which is zero.

32. The rate of change of the identity function equals the slope, which is one.

Section 1.3 Exercises

33. The function is linear since it is written in the form $f(x) = ax + b$.

34. Non-linear. The function does not fit the form $f(x) = ax + b$.

35. a. The function is linear since it is written in the form $f(x) = ax + b$.

  b. The slope is $m = -0.762$. The percent of unmarried women who get married has decreased by 0.762% each year since 1950.

36. a. The function is linear since it can be written in the form $f(x) = ax + b$.

  b. The slope is $m = -0.26$. The percent of the population voting in presidential elections since 1950 has decreased by 0.26% each year, or by 1.04% every 4 years.

37. a. *x*-intercept: Let $p = 0$ and solve for *x*.
$$30p - 19x = 30$$
$$30(0) - 19x = 30$$
$$-19x = 30$$
$$x = -\frac{30}{19}$$
The *x*-intercept is $\left(-\dfrac{30}{19}, 0\right)$.

b. *p*-intercept: Let $x = 0$ and solve for *p*.

$$30p - 19x = 30$$
$$30p - 19(0) = 30$$
$$30p = 30$$
$$p = 1$$

The *y*-intercept is $(0,1)$.
In 1990, the percentage of high school students using marijuana daily is 1%.

c. $x = 0$ corresponds to 1990, $x = 1$ corresponds 1991, etc. Integer values of $x \geq 0$ on the graph represent years 1990 and after.
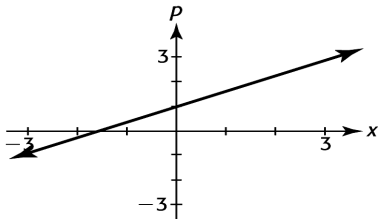
38. a. *y*-intercept: Let $x = 0$ and solve for *y*.

$$y = 828,000 - 2300(0) = 828,000$$

Initially the value of the building is $828,000.

b. *x*-intercept: Let $y = 0$ and solve for *x*.

$$0 = 828,000 - 2300x$$
$$-2300x = -828,000$$
$$x = \frac{-828,000}{-2300} = 360$$

The value of the building is zero (the building is completely depreciated) after 360 months or 30 years.

c.
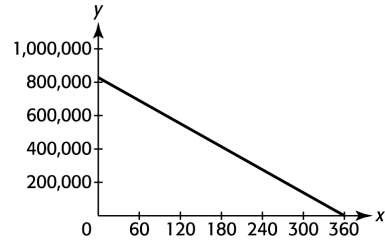
39. a. The data can be modeled by a constant function. Every input *x* yields the same output *y*.

b. $y = 11.81$

c. A constant function has a slope equal to zero.

d. For a linear function the rate of change is equal to the slope. $m = 0$.

40. a.
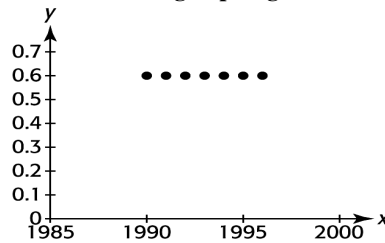
b. The data can be modeled by a constant function.

c. $y = 0.6$

d.
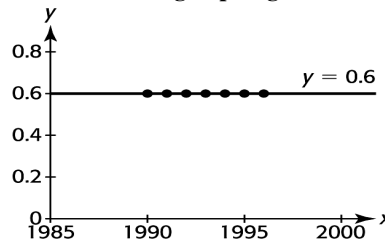
41. a. For a linear function, the rate of change is equal to the slope. $m = -0.743$. The slope is negative.

b. The percentage of high school seniors who have tried cigarettes decreased at a rate of 0.743 per year between 1991 and 2006.

42. a. $m = 26.5$

b. Each year, the percent of Fortune Global 500 firms recruiting via the Internet increased by 26.5%.

43. a. For a linear function, the rate of change is equal to the slope. $m = \dfrac{12}{7}$. The slope is positive.

b. For each one degree increase in temperature, there is a $\dfrac{12}{7}$ increase in the number of cricket chirps per minute. More generally, as the temperature increases, the number of chirps increases.

44. a. $m = 2.25$

b. The rate of growth is 2.25 hundred dollars, or $225, per year.

45. a. Yes, it is linear.

b. $m = 0.959$

c. For each one dollar increase in white median annual salaries, there is a 0.959 dollar increase in minority median annual salaries.

46. a. $m = -0.26$
For each one unit increase in x, the number of years since 1950, there is a 0.26 decrease in y, the percent of voters voting in presidential elections.

b. The rate of decrease is 0.26 percent per year.

47. a. To determine the slope, rewrite the equation in the form $f(x) = ax + b$ or $y = mx + b$.

$$30p - 19x = 30$$
$$30p = 19x + 30$$
$$\frac{30p}{30} = \frac{19x + 30}{30}$$
$$p = \frac{19}{30}x + 1$$
$$m = \frac{19}{30} \approx .633$$

b. Each year, the percentage of high school seniors using marijuana daily increases by approximately 0.63%.

48. a. $33p - 18d = 496$
Solving for p:
$$33p = 18d + 496$$
$$p = \frac{18d + 496}{33}$$
$$p = \frac{18}{33}d + \frac{496}{33}$$
$$p = \frac{6}{11}d + \frac{496}{33}$$
Therefore, $m = \dfrac{6}{11}$

b. For every one unit increase in depth, there is a corresponding $\dfrac{6}{11}$ pound per square inch increase in pressure.

49. x-intercept: Let $R = 0$ and solve for x.

$R = 3500 - 70x$

$0 = 3500 - 70x$

$70x = 3500$

$x = \dfrac{3500}{70} = 50$

The x-intercept is $(50,0)$.

R-intercept: Let $x = 0$ and solve for R.

$R = 3500 - 70x$

$R = 3500 - 70(0)$

$R = 3500$

The R-intercept is $(0,3500)$.
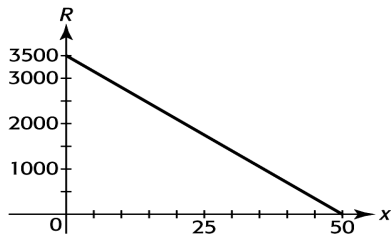
50. a. $D(50) = 0.137(50) - 5.09$

$= 1.76$

Based on the model, 50,000 ATM transactions correspond to a dollar volume of \$1.76 billion.

b. Fewer than approximately 37,154 ATMs. Test integers values of x in the following table to find which x's cause negative dollar values:

x = ATMs in 1000's	D(x) in \$Billions
35	-0.295
36	-0.158
37	-0.021
38	0.116
39	0.253
40	0.390

51. a. $m = 6.9$

y-intercept: $b = -3.18$

b. After 1995 the percentage of the U.S. population with Internet access increased by 6.9% each year.

52. a. $m = 11.23$

y-intercept: $b = 6.205$

b. The y-intercept represents the total amount spent for wireless communications in 1995. Therefore in 1995, the amount spent on wireless communication in the U.S. was 6.205 billion dollars.

c. The slope represents the annual change in the amount spent on wireless communications. Therefore, the amount spent on wireless communications in the U.S. increased by 11.23 billion each year.

53. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{700,000 - 1,310,000}{20 - 10}$

$= \dfrac{-610,000}{10}$

$= -61,000$

b. Based on the calculation in part a), the property value decreases by \$61,000 each year. The annual rate of change is $-61,000$.

54. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{76.0 - 18.1}{2005 - 1890}$

$= \dfrac{57.9}{115}$

$= 0.503$

b. Based on the calculation in part a), the number of men in the workforce increased by 0.503 million (or 503,000) each year.

55. Marginal profit is the rate of change of the profit function.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{9000 - 4650}{375 - 300}$$

$$= \frac{4350}{75}$$

$$= 58$$

The marginal profit is $58 per unit.

56. Marginal cost is the rate of change of the cost function.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3530 - 2690}{500 - 200}$$

$$= \frac{840}{300}$$

$$= 2.8$$

The marginal cost is $2.80 per unit.

57. a. $m = 0.56$

b. The marginal cost is $0.56 per unit.

c. Manufacturing one additional golf ball each month increases the cost by $0.56 or 56 cents.

58. a. $m = 98$

b. The marginal cost is $98 per unit.

c. Manufacturing one additional television each month increases the cost by $98.

59. a. $m = 1.60$

b. The marginal revenue is $1.60 per unit.

c. Selling one additional golf ball each month increases total revenue by $1.60.

60. a. $m = 198$

b. The marginal revenue is $198 per unit

c. Selling one additional television each month increases total revenue by $198.

61. The marginal profit is $19 per unit. Note that $m = 19$.

62. The marginal profit is $939 per unit. Note that $m = 939$.

Section 1.4 Skills Check

1. $m = 4, b = \dfrac{1}{2}$. The equation is $y = 4x + \dfrac{1}{2}$.

2. $m = 5, b = \dfrac{1}{3}$. The equation is $y = 5x + \dfrac{1}{3}$.

3. $m = \dfrac{1}{3}, b = 3$. The equation is $y = \dfrac{1}{3}x + 3$.

4. $m = -\dfrac{1}{2}, b = -8$. The equation is

$y = -\dfrac{1}{2}x - 8$.

5.
$$y - y_1 = m(x - x_1)$$
$$y - (-6) = -\dfrac{3}{4}(x - 4)$$
$$y + 6 = -\dfrac{3}{4}x + 3$$
$$y = -\dfrac{3}{4}x - 3$$

6.
$$y - y_1 = m(x - x_1)$$
$$y - 3 = -\dfrac{1}{2}(x - (-4))$$
$$y - 3 = -\dfrac{1}{2}(x + 4)$$
$$y - 3 = -\dfrac{1}{2}x - 2$$
$$y = -\dfrac{1}{2}x + 1$$

7. $x = 9$

8. $y = -10$

9. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 7}{-2 - (4)} = \dfrac{-6}{-6} = 1$

Equation:
$$y - y_1 = m(x - x_1)$$
$$y - 7 = 1(x - 4)$$
$$y - 7 = x - 4$$
$$y = x + 3$$

10. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6 - 3}{2 - (-1)} = \dfrac{3}{3} = 1$

Equation:
$$y - y_1 = m(x - x_1)$$
$$y - 6 = 1(x - 2)$$
$$y - 6 = x - 2$$
$$y = x + 4$$

11. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2 - 2}{5 - (-3)} = \dfrac{0}{8} = 0$

The line is horizontal. The equation of the line is $y = 2$.

12. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5 - 2}{9 - 9} = \dfrac{3}{0} =$ undefined

The line is vertical. The equation of the line is $x = 9$.

13. With the given intercepts, the line passes through the points $(-5, 0)$ and $(0, 4)$. The slope of the line is
$$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 0}{0 - (-5)} = \dfrac{4}{5}.$$

Equation:
$$y - y_1 = m(x - x_1)$$
$$y - 0 = \dfrac{4}{5}(x - (-5))$$
$$y = \dfrac{4}{5}(x + 5)$$
$$y = \dfrac{4}{5}x + 4$$

14. With the given intercepts, the line passes through the points $(4, 0)$ and $(0, -5)$. The slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 0}{0 - (4)} = \frac{-5}{-4} = \frac{5}{4}.$$

Equation: $y - y_1 = m(x - x_1)$

$$y - 0 = \frac{5}{4}(x - 4)$$

$$y = \frac{5}{4}(x - 4)$$

$$y = \frac{5}{4}x - 5$$

15. $3x + y = 4$

$$y = -3x + 4$$

$$m = -3$$

Since the new line is parallel with the given line, the slopes of both lines are the same.

Equation: $y - y_1 = m(x - x_1)$

$$y - (-6) = -3(x - 4)$$

$$y + 6 = -3x + 12$$

$$y = -3x + 6$$

16. $2x + y = -3$

$$y = -2x - 3$$

$$m = -2$$

Since the new line is parallel with the given line, the slopes of both lines are the same.

Equation: $y - y_1 = m(x - x_1)$

$$y - (-3) = -2(x - 5)$$

$$y + 3 = -2x + 10$$

$$y = -2x + 7$$

17. $2x + 3y = 7$

$$3y = -2x + 7$$

$$\frac{3y}{3} = \frac{-2x + 7}{3}$$

$$y = -\frac{2}{3}x + \frac{7}{3}$$

$$m = -\frac{2}{3}$$

Since the new line is perpendicular with the given line, the slope of the new line is

$m_\perp = -\dfrac{1}{m}$, where m is the slope of the

given line. $m_\perp = -\dfrac{1}{m} = -\dfrac{1}{\left(-\dfrac{2}{3}\right)} = \dfrac{3}{2}.$

Equation: $y - y_1 = m(x - x_1)$

$$y - 7 = \frac{3}{2}\left(x - (-3)\right)$$

$$y - 7 = \frac{3}{2}x + \frac{9}{2}$$

$$y - 7 + 7 = \frac{3}{2}x + \frac{9}{2} + 7$$

$$y = \frac{3}{2}x + \frac{23}{2}$$

18. $3x + 2y = -8$

$$2y = -3x - 8$$

$$\frac{2y}{2} = \frac{-3x - 8}{2}$$

$$y = -\frac{3}{2}x - 4$$

$$m = -\frac{3}{2}$$

Since the new line is perpendicular with the given line, the slope of the new line is

$m_\perp = -\dfrac{1}{m}$, where m is the slope of the

given line. $m_\perp = -\dfrac{1}{m} = -\dfrac{1}{\left(-\dfrac{3}{2}\right)} = \dfrac{2}{3}.$

Equation: $y - y_1 = m(x - x_1)$

$$y - 5 = \frac{2}{3}\left(x - (-4)\right)$$

$$y - 5 = \frac{2}{3}x + \frac{8}{3}$$

$$y - 5 + 5 = \frac{2}{3}x + \frac{8}{3} + 5$$

$$y = \frac{2}{3}x + \frac{23}{3}$$

19. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{13 - (-5)}{4 - (-2)} = \dfrac{18}{6} = 3$

Equation: $y - y_1 = m(x - x_1)$

$$y - 13 = 3(x - 4)$$

$$y - 13 = 3x - 12$$

$$y = 3x + 1$$

20. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-11 - 7}{2 - (-4)} = \dfrac{-18}{6} = -3$

Equation: $y - y_1 = m(x - x_1)$

$$y - 7 = -3\left(x - (-4)\right)$$

$$y - 7 = -3x - 12$$

$$y = -3x - 5$$

21. For a linear function, the rate of change is equal to the slope. Therefore, $m = -15$. The equation is

$$y - y_1 = m(x - x_1)$$

$$y - 12 = -15(x - 0)$$

$$y - 12 = -15x$$

$$y = -15x + 12.$$

22. For a linear function, the rate of change is equal to the slope. Therefore, $m = -8$. The equation is

23. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(2) - f(-1)}{2 - (-1)}$

$$= \frac{(2)^2 - (-1)^2}{3}$$

$$= \frac{4 - 1}{3}$$

$$= \frac{3}{3}$$

$$= 1$$

The average rate of change between the two points is 1.

24. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(2) - f(-1)}{2 - (-1)}$

$$= \frac{(2)^3 - (-1)^3}{3}$$

$$= \frac{8 + 1}{3}$$

$$= \frac{9}{3}$$

$$= 3$$

The average rate of change between the two points is 3.

25. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(1) - f(-2)}{1 - (-2)}$

$$= \frac{-2 - 7}{3}$$

$$= \frac{-9}{3}$$

$$= -3$$

26. $\dfrac{f(b)-f(a)}{b-a} = \dfrac{f(2)-f(-1)}{2-(-1)}$

$\quad\quad = \dfrac{-4-2}{3}$

$\quad\quad = \dfrac{-6}{3}$

$\quad\quad = -2$

27. $f(x+h) = 45 - 15(x+h)$

$\quad\quad = 45 - 15x - 15h$

$f(x+h) - f(x)$

$= 45 - 15x - 15h - \left[45 - 15x\right]$

$= 45 - 15x - 15h - 45 + 15x$

$= -15h$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{-15h}{h} = -15$

28. $f(x+h) = 32(x+h) + 12$

$\quad\quad = 32x + 32h + 12$

$f(x+h) - f(x)$

$= 32x + 32h + 12 - \left[32x + 12\right]$

$= 32x + 32h + 12 - 32x - 12$

$= 32h$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{32h}{h} = 32$

29. $f(x+h) = 2(x+h)^2 + 4$

$\quad\quad = 2\left(x^2 + 2xh + h^2\right) + 4$

$\quad\quad = 2x^2 + 4xh + 2h^2 + 4$

$f(x+h) - f(x)$

$= 2x^2 + 4xh + 2h^2 + 4 - \left[2x^2 + 4\right]$

$= 2x^2 + 4xh + 2h^2 + 4 - 2x^2 - 4$

$= 4xh + 2h^2$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{4xh+2h^2}{h}$

$\quad\quad = \dfrac{h(4x+2h)}{h}$

$\quad\quad = 4x + 2h$

30. $f(x+h) = 3(x+h)^2 + 1$

$\quad\quad = 3\left(x^2 + 2xh + h^2\right) + 1$

$\quad\quad = 3x^2 + 6xh + 3h^2 + 1$

$f(x+h) - f(x)$

$= 3x^2 + 6xh + 3h^2 + 1 - \left[3x^2 + 1\right]$

$= 3x^2 + 6xh + 3h^2 + 1 - 3x^2 - 1$

$= 6xh + 3h^2$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{6xh+3h^2}{h}$

$\quad\quad = \dfrac{h(6x+3h)}{h}$

$\quad\quad = 6x + 3h$

31. a. The difference in the y-coordinates is consistently 30, while the difference in the x-coordinates is consistently 10. Considering the scatter plot below, a line fits the data exactly.
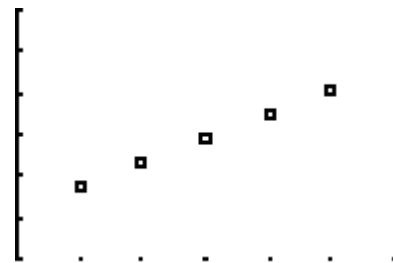
[0, 60] by [500, 800]

b. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{615 - 585}{20 - 10}$

$= \dfrac{30}{10}$

$= 3$

Equation: $y - y_1 = m(x - x_1)$

$y - 585 = 3(x - 10)$

$y - 585 = 3x - 30$

$y = 3x + 555$

32. a. The difference in the y-coordinates is consistently 9, while the difference in the x-coordinates is consistently 6. Considering the scatter plot below, a line fits the data exactly.
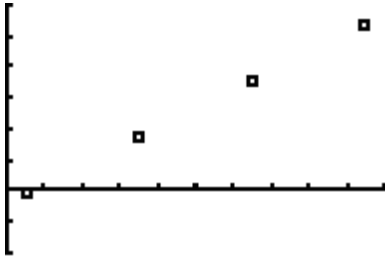
[0, 20] by [–10, 30]

b. Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{26.5 - 17.5}{19 - 13}$

$= \dfrac{9}{6}$

$= \dfrac{3}{2}$

Equation:

$y - y_1 = m(x - x_1)$

$y - 26.5 = \dfrac{3}{2}(x - 19)$

$y - 26.5 = \dfrac{3}{2}x - \dfrac{57}{2}$

$y = \dfrac{3}{2}x - 28.5 + 26.5$

$y = \dfrac{3}{2}x - 2$

Section 1.4 Exercises

33. Let x = KWh used and let y = monthly charge in dollars. Then the equation is $y = 0.0935x + 8.95$.

34. Let x = minutes used and let y = monthly charge in dollars. Then the equation is $y = 0.07x + 4.95$.

35. Let t = number of years, and let y = value of the machinery in dollars after t years. Then the equation is $y = 36{,}000 - 3{,}600t$.

36. a. Sleep needed $= 8 + \dfrac{1}{4}(18 - 10)$

$= 8 + \dfrac{1}{4}(8)$

$= 10$ hours

b. Sleep needed $= 8 + \dfrac{1}{4}(18 - 14)$

$= 8 + \dfrac{1}{4}(4)$

$= 9$ hours

c. Let x = age in years, and let y = hours of sleep. Using the results of parts a) and b), the two ordered pairs are (10, 10) and (14, 9).

The resulting slope is $\dfrac{9-10}{14-10} = -.25$

Then using the point-slope form of a linear equation and one of the ordered pairs, $(14, 9)$:

$$y - 9 = -0.25(x - 14)$$
$$y - 9 = -0.25x + 3.5$$
$$y = -0.25x + 12.5$$

d. Let $x = 18$.
$y = -0.25(18) + 12.5 = -4.5 + 12.5 = 8$

37. a. Let $x =$ the number of years since 1996, and let $P =$ the population of Del Webb's Sun City Hilton Head community. The linear equation modeling the population growth is $P = 705x + 198$.

b. To predict the population in 2002, let $x = 2002 - 1996 = 6$. The predicted population is $P = 705(6) + 198 = 4428$.

38. Let $x =$ the number of years past 1994, and let $y =$ the composite SAT score for the Beaufort County School District. The linear equation modeling the change in SAT score is $y = 952 + 0.51x$.

39. a. From year 0 to year 5, the automobile depreciates from a value of $26,000 to a value of $1,000. Therefore, the total depreciation is 26,000–1000 or $25,000.

b. Since the automobile depreciates for 5 years in a straight-line (linear) fashion, each year the value declines by $\dfrac{25,000}{5} = \$5,000$.

c. Let $t =$ the number of years, and let $s =$ the value of the automobile in dollars at the end of t years. Then, based on parts

a) and b) the linear equation modeling the value is $s = -5000t + 26,000$ dollars.

40. $P = 2.5\%(75,000)y$
$= 1875y$
where $y =$ number of years of service and $P =$ annual pension amount in dollars.

41. Notice that the x and y values are always match. The number of deputies always equals the number of patrol cars. Therefore the equation is $y = x$, where x represents the number of deputies, and y represents the number of patrol cars.

42. Notice that the y values are always the same, regardless of the x value. The premium is constant. Therefore the equation is $y = 11.81$, where x represents age, and y represents the premium in dollars.

43. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{9000 - 4650}{375 - 300}$

$= \dfrac{4350}{75} = 58$

Equation:
$$y - y_1 = m(x - x_1)$$
$$P - 4650 = 58(x - 300)$$
$$P - 4650 = 58x - 17,400$$
$$P = 58x - 12,750$$

44. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{3530 - 2680}{500 - 200}$

$= \dfrac{850}{300} = \dfrac{17}{6}$

Equation:

$y - y_1 = m(x - x_1)$

$y - 2680 = \dfrac{17}{6}(x - 200)$

$y - 2680 = \dfrac{17}{6}x - \dfrac{1700}{3}$

$y = \dfrac{17}{6}x - \dfrac{1700}{3} + \dfrac{8040}{3}$

$y = \dfrac{17}{6}x + \dfrac{6340}{3}$

$y \approx 2.83x + 2113.33$

45. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{700{,}000 - 1{,}310{,}000}{20 - 10}$

$= \dfrac{-610{,}000}{10}$

$= -61{,}000$

Equation:

$y - y_1 = m(x - x_1)$

$y - 1{,}920{,}000 = -61{,}000(x - 0)$

$y - 1{,}920{,}000 = -61{,}000x$

$y = -61{,}000x + 1{,}920{,}000$

$V = -61{,}000x + 1{,}920{,}000$

46. a. At $t = 0, y = 860{,}000.$

b. $(0, 860{,}000), (25, 0)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{0 - 860{,}000}{25 - 0}$

$= \dfrac{-860{,}000}{25}$

$= -34{,}400$

Equation:

$y - y_1 = m(x - x_1)$

$y - 0 = -34{,}400(x - 25)$

$y = -34{,}400x + 860{,}000$

$y = 860{,}000 - 34{,}400t$

where t = number of years and

y = property value.

47. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{47.1 - 63.5}{16 - 6}$

$= \dfrac{-16.4}{10}$

$= -1.64$

Equation:

$y - y_1 = m(x - x_1)$

$y - 63.5 = -1.64(x - 6)$

$y - 63.5 = -1.64x + 9.84$

$y - 63.5 + 63.5 = -1.64x + 9.84 + 63.5$

$y = -1.64x + 73.34$

$p = 73.34 - 1.64t$

where t = number of years beyond

1991, and p = percentage of cigarette use

48. Let x = median weekly income for whites, and y = median weekly income for blacks. The goal is to write $y = f(x)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{61.90}{100} = 0.619$$

Equation:
$$y - y_1 = m(x - x_1)$$
$$y - 527 = 0.619(x - 676)$$
$$y - 527 = 0.619x - 418.444$$
$$y = 0.619x + 108.556$$

49. a. Notice that the change in the x-values is consistently 1 while the change in the y-values is consistently 0.05. Therefore the table represents a linear function. The rate of change is the slope of the linear function.

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{0.05}{1} = 0.05$$

b. Let x = the number of drinks, and let y = the blood alcohol content. Using points $(0, 0)$ and $(1, 0.05)$, the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0.05 - 0}{1 - 0}$$
$$= \frac{0.05}{1} = 0.05.$$

Equation:
$$y - y_1 = m(x - x_1)$$
$$y - 0 = 0.05(x - 0)$$
$$y = 0.05x$$

50. a. Notice that the change in the x-values is consistently 1 while the change in the y-values is consistently 0.02. Therefore the table represents a linear function.

The rate of change is the slope of the linear function.

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{0.02}{1} = 0.02$$

b. Let x = the number of drinks, and let y = the blood alcohol content. Using points $(5, 0.11)$ and $(10, 0.21)$, the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0.21 - 0.11}{10 - 5}$$
$$= \frac{0.10}{5} = 0.02.$$

Equation:
$$y - y_1 = m(x - x_1)$$
$$y - 0.11 = 0.02(x - 5)$$
$$y - 0.11 = 0.02x - 0.1$$
$$y = 0.02x + 0.01$$

51. a. Let x = the year at the beginning of the decade, and let y = average number of men in the workforce during the decade. Using points $(1890, 18.1)$ and $(1990, 68.5)$ to calculate the slope yields:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{68.5 - 18.1}{1990 - 1890}$$
$$= \frac{50.4}{100} = 0.504$$

Equation:
$$y - y_1 = m(x - x_1)$$
$$y - 18.1 = 0.504(x - 1890)$$
$$y - 18.1 = 0.504x - 952.56$$
$$y - 18.1 + 18.1 = 0.504x - 952.56 + 18.1$$
$$y = 0.504x - 934.46$$

b. Yes. Consider the following table of values based on the equation in comparison to the actual data points.

x	y = Equation Values	Actual Values
1890	18.1	18.1
1900	23.14	22.6
1910	28.18	27
1920	33.22	32
1930	38.26	37
1940	43.3	40
1950	48.34	42.8
1960	53.38	47
1970	58.42	51.6
1980	63.46	61.4
1990	68.5	68.5
2000	73.54	72.9
2005	76.06	74.5
2010	78.58	76

c. They are the same since the points $(1890, 18.1)$ and $(1990, 68.5)$ were used to calculate the slope of the linear model.

52. a. Let t = the year, and let p = the percentage of workers in farm occupations. Using points (1820, 71.8) and (1994, 2.6) to calculate the slope yields:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2.6 - 71.8}{1994 - 1820}$$
$$= \frac{-69.2}{174}$$
$$= -0.3977011494 \approx -0.40$$

Equation:
$$y - y_1 = m(x - x_1)$$
$$y - 2.6 = -0.3977011494(x - 1994)$$
$$y - 2.6 = -0.3977011494x + 793.0160919$$
$$y = -0.3977011494x + 795.6160919$$
$$y \approx -0.40x + 795.62$$
$$p \approx -0.40t + 795.62$$

b. The line appears to be a reasonable fit to the data.

c. On average between 1820 and 1994, the percentage of workers in farm-related jobs decreased by 0.40% per year.

d. No. The percentage of farm workers would become negative.

53. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{40.1 - 23}{2001 - 1996}$$
$$= \frac{17.1}{5}$$
$$= 3.42$$

b. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(2001) - f(1996)}{2001 - 1996}$

$$= \frac{40.1 - 23}{5}$$
$$= \frac{17.1}{5}$$
$$= 3.42$$

The average rate of change is $3.42 billion dollars per year.

c. No. Note that change in education spending from one year to the next is not constant. It varies.

54. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{6704 - 5664}{2005 - 2000}$

$= \dfrac{1040}{5} = 208$

b.   The average rate of change is 208 students per year. Note that it is the same as the answer in part a).

c.   Since enrollment is projected to increase, additional buildings may be necessary.

$y - y_1 = m(x - x_1)$

$y - 76 = \dfrac{61}{36}(x - 46)$

$y - 76 = \dfrac{61}{36}x - \dfrac{2806}{36}$

$y = \dfrac{61}{36}x - \dfrac{2806}{36} + 76$

$y = \dfrac{61}{36}x - \dfrac{2806}{36} + \dfrac{2736}{36}$

$y = \dfrac{61}{36}x - \dfrac{70}{36}$

$y \approx 1.69x - 1.94$

$p \approx 1.69x - 1.94$

55. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{76 - 15}{46 - 10}$

$= \dfrac{61}{36} = 1.69\overline{4} \approx 1.69$

b. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{76 - 15}{46 - 10}$

$= \dfrac{61}{36} = 1.69\overline{4} \approx 1.69$

Note that it is the same as the answer in part a).

c.   On average between 1960 and 1996, the percentage of out-of-wedlock teenage births increased by approximately 1.69%. per year.

d.   Let $x =$ the number of years after 1950, and let $y =$ the percentage of out-of-wedlock births.

56. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{55.3 - 63.1}{2004 - 1960}$

$= \dfrac{-7.8}{44} = -0.177$

b.   It is the same as part a). On average the percentage of eligible people voting in presidential elections is decreasing at a rate of 0.177% per year.

c. $y - y_1 = m(x - x_1)$

$y - 63.1 = -0.177(x - 1960)$

$y - 63.1 = -0.177x + 346.92$

$y = -0.177x + 410.02$

$p = -0.177x + 410.02$

57. a. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(2005) - f(1960)}{2005 - 1960}$

$= \dfrac{1,512,823 - 212,953}{2005 - 1960}$

$= \dfrac{1,299,870}{45}$

$= 28,886$

b.   The slope of the line connecting the two points is the same as the average rate of

change between the two points. Based on part a), $m = 28,886$.

c. The equation of the secant line is given by:
$$y - y_1 = m(x - x_1)$$
$$y - 212,953 = 28,886(x - 1960)$$
$$y - 212,953 = 28,886x - 56,616,560$$
$$y - 212,953 = 28,886x - 56,616,560$$
$$+212,953 = \qquad + \quad 212,953$$
$$y = 28,886x - 56,403,607$$

d. No. The points on the scatter plot do not approximate a linear pattern.

e. Points corresponding to 1997 and 2005. The points on the scatter plot between those two years do approximate a linear pattern. Answers may vary.

58. a. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(5) - f(1)}{5 - 1}$
$$= \frac{1492 - 1083}{4}$$
$$= \frac{409}{4}$$
$$= 102.25$$

b. On average from year 1 to year 5, the worth of the investment increases by $102.25 per year.

c. The slope is the same as the average rate of change, 102.25.

d. $y - y_1 = m(x - x_1)$
$$y - 1083 = 102.25(x - 1)$$
$$y - 1083 = 102.25x - 102.25$$
$$y = 102.25x + 980.75$$

59. a. No.

b. Yes. The points seem to follow a straight line pattern for years between 2010 and 2030.

c. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(2030) - f(2010)}{2030 - 2010}$
$$= \frac{2.2 - 3.9}{2030 - 2010}$$
$$= \frac{-1.7}{20}$$
$$= -0.085$$

d. $y - y_1 = m(x - x_1)$
$$y - 3.9 = -0.085(x - 2010)$$
$$y - 3.9 = -0.085x + 170.85$$
$$y = -0.085x + 174.75$$

60. a. No. The points in the scatter plot do not lie approximately in a line.

b. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{f(1950) - f(1930)}{1950 - 1930}$
$$= \frac{16.443 - 10.519}{20}$$
$$= \frac{5.924}{20}$$
$$= 0.2962$$

The average rate of change is 0.2962 million (296,200) women per year.

c. $\dfrac{f(b) - f(a)}{b - a}$
$$= \frac{f(1990) - f(1950)}{1990 - 1950}$$
$$= \frac{59.531 - 16.443}{40}$$
$$= \frac{43.088}{40}$$
$$= 1.0772$$

The average rate of change is 1.0772 million (1,077,200) women per year.

d. Yes. Since the graph curves, the average rate of change is not constant. The points do not lie exactly along a line.

d. The data in the table can not be modeled exactly by a linear function.

61. a. Let x = the number of years since 1950, and let y = the U.S. population in thousands. Then, the average rate of change in U.S. population, in thousands, between 1950 and 2005 is given by:

$$\frac{f(b) - f(a)}{b - a} = \frac{f(55) - f(0)}{55 - 0}$$
$$= \frac{296,410 - 152,271}{55 - 0}$$
$$= \frac{144,139}{55}$$
$$\approx 2620.7 \approx 2621$$

The annual average increase in population is 2621 thousand people.

b.
$$y - y_1 = m(x - x_1)$$
$$y - 152,271 = 2621(x - 0)$$
$$y - 152,271 = 2621x$$
$$\underline{+152,271 = \qquad +152,271}$$
$$y = 2621x + 152,271$$

c. 1975 corresponds to x = 25.
$$y = 2621x + 152,271$$
$$y = 2621(25) + 152,271$$
$$y = 217,796 \text{ thousand}$$

The estimated U. S. population in 1975 is 217,796,000 people.

No. The values are different.

62. a. Let x represent the number of clients in Group 1, and y represent the number of clients in Group 2. Then,
Group 1 Expense +
  Group 2 Expense = Total Expense
$$300x + 200y = 100,000$$

b.
$$300x + 200y = 100,000$$
$$200y = -300x + 100,000$$
$$y = \frac{-300x + 100,000}{200}$$
$$y = \frac{-300}{200}x + \frac{100,000}{200}$$
$$y = -1.5x + 500$$

The y-intercept is 500. If no clients from the first group are served, then 500 clients from the second group can be served. The slope is –1.5. For each one person increase in the number of clients served from the first group there is a corresponding decrease of 1.5 clients served from the second group.

c. $10(-1.5) = -15$

Fifteen fewer clients can be served from the second group.

Chapter 1 Skills Check

b. $f(-3) = -10$

1. The table represents a function because each *x* matches with exactly one *y*.

7. $f(x) = -2x^3 + 5x$

2. Domain: $\{-3,-1,1,3,5,7,9,11,13\}$

Range: $\{9,6,3,0,-3,-6,-9,-12,-15\}$

3. $f(3) = 0$

4. Yes. The rate of change between any two pairs of values is constant.
The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-9}{-1-(-3)} = \frac{-3}{2} = -\frac{3}{2}.$$

Calculating the equation:

$$y - y_1 = m(x - x_1)$$

$$y - 9 = -\frac{3}{2}\left(x - (-3)\right)$$

$$y - 9 = -\frac{3}{2}x - \frac{9}{2}$$

$$y = -\frac{3}{2}x - \frac{9}{2} + 9$$

$$y = -\frac{3}{2}x - \frac{9}{2} + \frac{18}{2}$$

$$y = -\frac{3}{2}x + \frac{9}{2}$$

8. $y = 3x^2$

9. $y = -10x^2 + 400x + 10$

The second view is better.

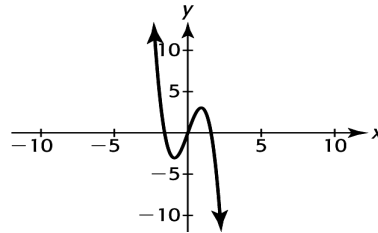**5. a.** $C(3) = 16 - 2(3)^2$
$= 16 - 2(9) = 16 - 18 = -2$

b. $C(-2) = 16 - 2(-2)^2$
$= 16 - 2(4) = 16 - 8 = 8$

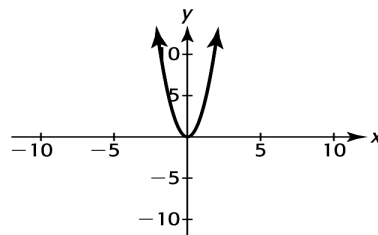**c.** $C(-1) = 16 - 2(-1)^2$
$= 16 - 2(1) = 16 - 2 = 14$
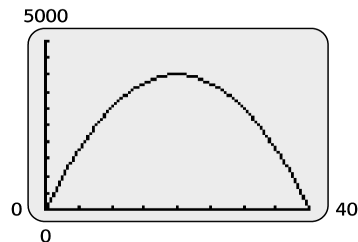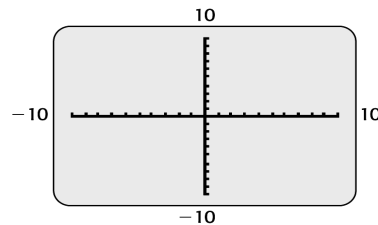
6. a. $f(-3) = 1$

10.
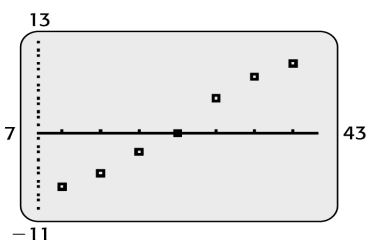
11. a. Since $y = \sqrt{2x - 8}$ will not be a real number if $2x - 8 < 0$, the only values of x that yield real outputs to the function are values that satisfy $2x - 8 \geq 0$. Isolating x yields:

$2x - 8 + 8 \geq 0 + 8$

$\dfrac{2x}{2} \geq \dfrac{8}{2}$

$x \geq 4$

Therefore the domain is D: $[4, \infty)$.

b. The denominator of the function will be zero if $x - 6 = 0$ or $x = 6$. This implies $x \neq 6$. The domain is all real numbers except 6 or in interval notation D: $(-\infty, 6) \cup (6, \infty)$.

12. The slope of the line through the two given points is: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{38 - 16}{-1 - (-12)} = \dfrac{22}{11} = 2$

The slope of the line from the given equation is: -1/2. Since the slopes are negative reciprocals of each other, the lines are perpendicular.

13. The slope of the given line is

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3 - 4}{5 - (-1)} = \dfrac{-7}{6} = -\dfrac{7}{6}$.

Since two parallel lines have the same slope, the slope of any parallel is also $m = -\dfrac{7}{6}$.

Since the slopes of perpendicular lines are negative reciprocals of one another,

$m_\perp = -\dfrac{1}{m} = -\dfrac{1}{\left(-\dfrac{7}{6}\right)} = -\left(-\dfrac{6}{7}\right) = \dfrac{6}{7}$

14. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-16 - 6}{8 - (-4)} = \dfrac{-22}{12} = -\dfrac{11}{6}$

15. a. x-intercept: Let $y = 0$ and solve for x.

$2x - 3(0) = 12$

$2x = 12$

$x = 6$

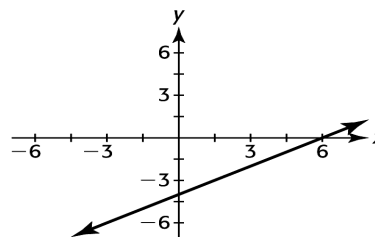y-intercept: Let $x = 0$ and solve for y.

$2(0) - 3y = 12$

$-3y = 12$

$y = -4$

x-intercept: $(6, 0)$, y-intercept: $(0, -4)$

b.

16. Using the intercepts found in 15 a),

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 - (-4)}{6 - 0} = \dfrac{4}{6} = \dfrac{2}{3}$

The slope, m, is $\dfrac{2}{3}$.

17. The slope is $m = -6$ The y-intercept is $(0, 3)$.

18. Since the function is linear, the rate of change is the slope. $m = -6$.

19. $y = mx + b$

$$y = \frac{1}{3}x + 3$$

20. $y - y_1 = m(x - x_1)$

$$y - (-6) = -\frac{3}{4}(x - 4)$$

$$y + 6 = -\frac{3}{4}x + 3$$

$$y = -\frac{3}{4}x - 3$$

21. The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 3}{2 - (-1)} = \frac{3}{3} = 1.$$

Solving for the equation:
$$y - y_1 = m(x - x_1)$$
$$y - 6 = 1(x - 2)$$
$$y - 6 = x - 2$$
$$y = x + 4$$

22. $y = x^2$

$x = 0:$ $y = (0)^2 = 0$ $(0,0)$
$x = 3:$ $y = (3)^2 = 9$ $(3,9)$

The average rate of change is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 0}{3 - 0} = \frac{9}{3} = 3.$$

23. a. $f(x + h) = 5 - 4(x + h)$
$$= 5 - 4x - 4h$$

b. $f(x + h) - f(x)$
$$= \left[5 - 4(x + h)\right] - \left[5 - 4x\right]$$
$$= 5 - 4x - 4h - 5 + 4x$$
$$= -4h$$

c. $\dfrac{f(x + h) - f(x)}{h}$
$$= \frac{-4h}{h}$$
$$= -4$$

24. a. $f(x + h)$
$$= 10(x + h) - 50$$
$$= 10x + 10h - 50$$

b. $f(x + h) - f(x)$
$$= \left[10x + 10h - 50\right] - \left[10x - 50\right]$$
$$= 10x + 10h - 50 - 10x + 50$$
$$= 10h$$

c. $\dfrac{f(x + h) - f(x)}{h}$
$$= \frac{10h}{h}$$
$$= 10$$

Chapter 1 Review Exercises

25. a. Yes. Each year matches with exactly one Democratic Party percentage.

b. $f(1992) = 82$. The table indicates that in 1992, 82% of African American voters supported a Democratic candidate for president.

c. When $f(y) = 94, y = 1964$. The table indicates that in 1964, 94% of African American voters supported a Democratic candidate for president.

26. a. The domain is $\{1960,1964,1968,1972,1976,1980,$
$1984,1992,1996\}$.

b. No. 1982 was not a presidential election year.

c. Discrete. The input values are the presidential election years. There are 4-year gaps between the inputs.

27.
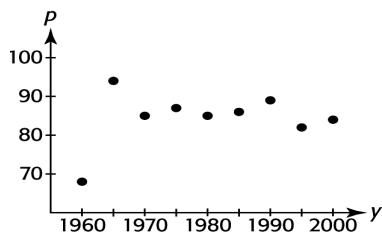
28. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{84 - 85}{1996 - 1968}$

$= \dfrac{-1}{28}$

≈ -0.0357

b. $\dfrac{f(b) - f(a)}{b - a} = \dfrac{84 - 85}{1996 - 1968}$

$= \dfrac{-1}{28}$

≈ -0.0357

c. No.

$\dfrac{f(b) - f(a)}{b - a} = \dfrac{86 - 85}{1980 - 1968}$

$= \dfrac{1}{12}$

≈ 0.0833

d. No. Consider the scatter plot created in problem 27. Since a line does not fit the data exactly, a linear function does not model the data exactly.

29. a. Each amount borrowed matches with exactly one monthly payment. The change in y is fixed at 89.62 for a fixed change in x of 5000.

b. $f(25,000) = 448.11$. Therefore, borrowing $25,000 to buy a car from the dealership results in a monthly payment of $448.11.

c. If $f(A) = 358.49$, then $A = 20,000$.

30. a. Domain: $\{10,000,15,000,20,000,$
$25,000,30,000\}$

Range: $\{179.25, 268.87, 358.49$
$448.11, 537.73\}$

b. No. $12,000 is not in the domain of the function.

c. Discrete. Gaps exist between the possible inputs.

31. a. $f(28,000) = 0.017924(28,000) + 0.01$

$\qquad = 501.872 + 0.01$

$\qquad = 501.882$

The predicted monthly payment on a car loan of $28,000 is $501.88.

b. Any positive input could be used for A. Borrowing a negative amount of money does not make sense in the context of the problem.

32. a. $f(1960) = 15.9$. A 65-year old woman in 1960 is expected to live 15.9 more years. Her overall life expectancy is 80.9 years.

b. $f(2010) = 19.4$. A 65-year old woman in 2010 has a life expectancy of 84.4 years.

c. Since $f(1990) = 19$, the average woman is expected to live 19 years past age 65 in 1990.

33. a. $g(2020) = 16.9$. A 65-year old man in 2020 is expected to live 16.9 more years. His overall life expectancy is 81.9 years.

b. Since $g(1950) = 12.8$, a 65-year old man in 1950 has a life expectancy of 77.8 years.

c. Since $g(1990) = 15$, the average man is expected to live 15 years past age 65 (or to age 80) in 1990. $g(1990) = 15$.

34. a. $t = 2000 - 1990 = 10$

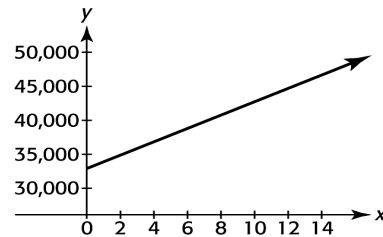$f(10) = 982.06(10) + 32,903.77$

$f(10) = 42,724.37$

b. $t = 15$

$f(15) = 982.06(15) + 32,903.77$

$f(15) = 47,634.67$

Based on the model in 2005 the average teacher salary is $47,634.67.

c. Increasing, since $m = 982.06$ is positive.

35. a.

b. Years from 1990 through 2005

36. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\qquad = \dfrac{14.5 - 12.0}{1999 - 1992}$

$\qquad = \dfrac{2.5}{7} \approx 0.357$

b. Assuming that drug use follows a linear model, the annual rate of change is equal to the slope calculated in part a). On average the number of people using illicit drugs increases by 0.357 million or 357,000 per year.

37. $f(x) = 4500$

38. a. Let x = the number of months past June 2006, and $f(x)$ = average weekly hours worked. The function is $f(x) = 33.8$.

b. Yes. The average rate of change is zero.

39. a. $R(120) = 564(120) = 67,680$. The revenue when 120 units are produced is $67,680.

b. $C(120) = 40,000 + 64(120) = 47,680$. The cost when 120 units are produced is $47,680.

c. Marginal Cost = $\overline{MC} = 64 = \$64$. Note that $\overline{MC}$ is the slope of the cost function.

Marginal Revenue = $\overline{MR} = 564 = \$564$. Note that $\overline{MR}$ is the slope of the revenue function.

d. $m = 64$

e.
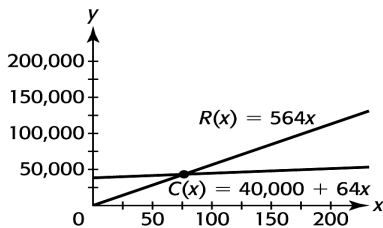
40. a. $P(x) = 564x - (40,000 + 64x)$
$= 564x - 40,000 - 64x$
$= 500x - 40,000$

b. $P(120) = 500(120) - 40,000$
$= 60,000 - 40,000$
$= 20,000$

c. Break-even occurs when $R(x) = C(x)$ or alternately $P(x) = R(x) - C(x) = 0$.

$500x - 40,000 = 0$
$500x = 40,000$
$x = \dfrac{40,000}{500}$
$x = 80$

Eighty units represent break-even for the company.

d. $\overline{MP}$ = the slope of $P(x) = 500$

e. $\overline{MP} = \overline{MR} - \overline{MC}$

41. a. Let $x = 0$, and solve for y.
$y + 3000(0) = 300,000$
$y = 300,000$
The initial value of the property is $300,000.

b. Let $y = 0$, and solve for x.
$0 + 3000x = 300,000$
$3000x = 300,000$
$x = 100$
The value of the property after 100 years is zero dollars.

42. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{895 - 455}{250 - 150} = \dfrac{440}{100} = 4.4$.
The average rate of change is $4.40 per unit.

b. For a linear function, the slope is the average rate of change. Referring to part a), the slope is 4.4.

c. $y - y_1 = m(x - x_1)$
$y - 455 = 4.4(x - 150)$
$y - 455 = 4.4x - 660$
$y = 4.4x - 205$
$P(x) = 4.4x - 205$

d. $\overline{MP}$ = the slope of $P(x) = 4.4$ or $4.40 per unit.

e. Break-even occurs when $R(x) = C(x)$ or alternately $P(x) = R(x) - C(x) = 0$.

$$4.4x - 205 = 0$$
$$4.4x = 205$$
$$x = \frac{205}{4.4}$$
$$x = 46.5\overline{09} \approx 47$$

The company will break even selling approximately 47 units.

Extended Applications

I. Body Mass Index

a. A person uses the table to determine his or her BMI by locating the entry in the table that corresponds to his or her height and weight. The entry in the table is the person's BMI.

b. If a person's BMI is 30 or higher, the person is considered obese and at risk for health problems.

c. **1.** Determine the heights and weights that produce a BMI of exactly 30 based on the table.

Height (inches)	Weight (pounds)
61	160
63	170
65	180
67	190
68	200
69	200
72	220
73	230

2.
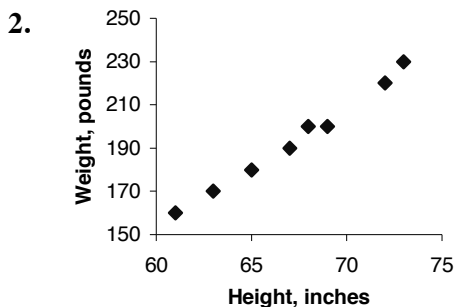
A linear model is reasonable, but not exact.

3.
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{230 - 160}{73 - 61}$$
$$= \frac{70}{12}$$
$$= 5.8\overline{3}$$
$$y - y_1 = m(x - x_1)$$
$$y - 160 = \frac{70}{12}(x - 61)$$
$$y - 160 = \frac{70}{12}x - \frac{4270}{12}$$
$$y = \frac{70}{12}x - \frac{4270}{12} + \frac{1920}{12}$$
$$y = \frac{70}{12}x - \frac{2350}{12}$$
$$y = 5.8\overline{3}x - 195.8\overline{3}$$

4.
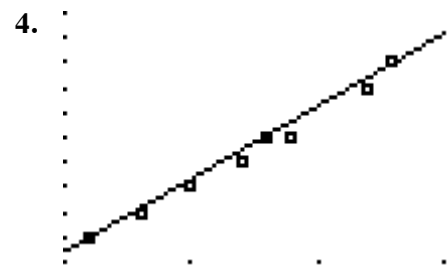
[60, 75] by [150, 250]

The line fits the data points well, but not perfectly.

5. Any data point that lies exactly along the line generated from the model will yield a BMI of 30. If a height is substituted into the model, the output weight would generate a BMI of 30. That weight or any higher weight for the given height would place a person at risk for health problems.

II. Total Revenue, Total Cost, and Profit

Let x represent the number of units produced and sold.

1. The revenue function is $R(x) = 98x$.

2. Marginal Revenue $= \overline{MR} = 98 = \$98$.

Note that $\overline{MR}$ is the slope of the revenue function.

3. $C(x) = 23x + 262{,}500$

4. Marginal Cost $= \overline{MC} = 23 = \$23$. Note that $\overline{MC}$ is the slope of the cost function.

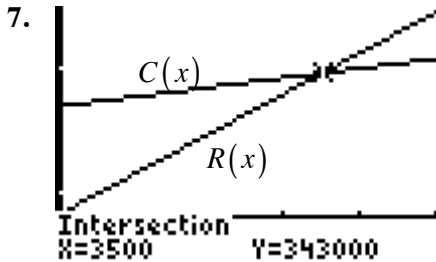Neither. $\overline{MC} \cdot x$ is the variable cost, VC.

5. $P(x) = R(x) - C(x)$

$\qquad = \big[98x\big] - \big[23x + 262{,}500\big]$
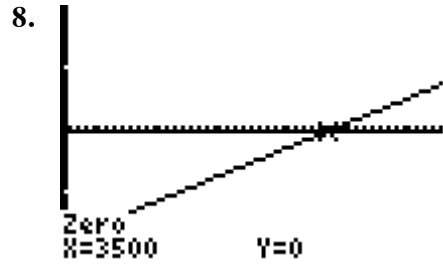
$\qquad = 75x - 262{,}500$

$\overline{MP} =$ the slope of $P(x) = 75$

6. If $x = 0$, then

$R(0) = 98(0) = 0$

$C(0) = 23(0) + 262{,}500 = 262{,}500$

$P(0) = 75(0) - 262{,}500 = -262{,}500$

7.

Intersection
X=3500   Y=343000

$[0, 5000]$ by $[0, 500{,}000]$

The intersection point is approximately $x = 3500$ units.

8.

Zero
X=3500   Y=0

$[0, 5000]$ by $[-300{,}000, 300{,}000]$

$x = 3500$ is the intersection point

9. The intersection points in questions 7 and 8 represent the value of x where $R(x) = C(x)$. Therefore, the points represent the break-even production level for the MP3 players. If the company produces and sells 3500 MP3 players, it will break even.